

Article

Reliability Analysis and Redundancy Optimization of a Command Post Phased-Mission System

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Abstract: This paper divides the execution process of the command post system into four stages: information acquisition, information processing, decision control and response execution. It combines multilayer complex networks with a phased-mission system. Most studies have only evaluated the reliability of phased-mission systems. This paper evaluates and optimizes the reliability of a phased-mission system. In order to improve the mission success rate and maximize the reliability of a command post system, this paper provides a multitasking node criticality index, and the index is used to identify the key nodes in the command post's four-stage network. Then, the hot backup system of the node is selected to determine the redundant structure of the key node. Under the constraints of the operation and maintenance costs of key nodes, with the goal of maximizing the reliability of the information processing network layer, the multitask redundancy optimization model of each stage is established. Finally, the reliability of the missions before and after redundancy optimization is compared, using the case analysis of the four-layer network to verify the effectiveness of the proposed model.



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1. Introduction

In modern warfare, the command post is the core facility of the combat force, and its survivability has a restrictive effect on the war process and the outcome of the war. In view of the threats faced by the command post system, it is of great significance to evaluate the importance of command post system nodes and improve the reliability of the system. The command post system includes four links: intelligence acquisition, intelligence processing, decision control and response execution, and each link exists independently. The mission reliability of the command post system is the probability that the mission information transmission between the nodes of the command post is completed within the specified time, under specified conditions.

1.1. Relevant Work

In terms of phased-mission system (PMS) reliability, the Markov chain model can describe the dynamic failure process of repairable and failure-related components in detail. Li et al. [1] explained the modeling process of probabilistic risk assessment (PRA) technology. PRA modeling is based on an event tree. According to the characteristics of intermediate events in the event tree model, fault tree, Bayesian network, Petri net and Markov chain methods are, respectively, selected for segment modeling. Li et al. [2] proposed a Markov regenerative process-based model for systems under mixed shocks and assessed the reliability of an Attitude and Orbit Control System in a spacecraft. Xing [3]

used the Markov process to assess the reliability of phased-mission systems, with imperfect fault coverage and common-cause failures. Wu and Yan [4] used the Forward Euler Method and Runge–Kutta Method to establish a sparse matrix of the transfer rate of the Markov model, which to some extent avoided the problem of model state space explosion. Using the Markov method, Xing et al. [5] proposed a recursive formula for calculating the reliability of a phased-mission k-out-of-n system with fault coverage, based on the conditional probability and recursive function method to calculate the reliability. A Bayesian network (BN) supports static PMS and dynamic PMS system analysis. Using the BN's unique reasoning mechanism, the PMS-BN model is also suitable for more complex applications, such as system fault diagnosis and importance analysis. Li et al. [6] combined BNs with event tree/fault tree analysis to analyze a PMS based on conditional probability, to give an expression of the phase-dependency, and further expanded it with dynamic Bayesian networks to cope with more complex time-dependency. Yu and Han [7] established the task reliability model of the system based on the task and used the Bayes method to evaluate the task reliability of each subnode in the model. The Petri net (PN) has flexible graphical representation and has been fully applied in dynamic system modeling and simulation. Chew et al. [8] realized the reliability building of a PMS based on a PN and completed the direct conversion from a phase fault tree to a PN. Yang et al. [9] combined the ideas of PN and Object Oriented technology and used the extended identification language to realize the automatic task reliability simulation generation algorithm for spacecraft. Li et al. [10] evaluated the mission reliability of a PMS considering both within-phase time redundancy and failed-task re-execution and proposed a PN simulation method. Yu et al. [11] proposed an extended object-oriented Petri net model to describe a PMS with time redundancy.

In terms of key node analysis, Dui et al. [12] proposed a joint comprehensive importance measure to guide preventive maintenance components, aiming to optimize system performance. Almoghathawi and Barker [13] proposed a component importance measure for analyzing changes in network recovery. Miziula and Navarro [14] extended the Birnbaum importance measure to the case of systems with dependent components to obtain relevant properties. Cai et al. [15] proposed a Birnbaum importance measure-based genetic algorithm to search for the near global optimal solution for linear consecutive k-out-of-n systems. Mao et al. [16] developed two metrics, including the cumulative performance loss and restoration rapidity, to optimize the resilience of supply chain networks. Tan et al. [17] used the gray correlation degree to measure the Euclidean distance and the coupling degree of the curve edge, which made the evaluation result of the importance degree more accurate. Zhang et al. [18] considered the influence of the Birnbaum importance measure and integrated the importance measure with the reliability changes of key components in a dissimilar redundancy actuation system. Guo et al. [19] used degree centrality, closeness centrality, betweenness centrality, toughness and cohesion as indicators to sort the importance of nodes. Wu et al. [20] used degree, betweenness, closeness and cohesion as indicators, and used a multiattribute decision-making method combined with a grey relational degree to evaluate node importance. Fan et al. [21] proposed a hybrid membership model and latent feature model to determine node importance. Fan et al. [22] used the Gini coefficient method to comprehensively sort nodes with multiple indicators. Kala [23] proposed a new importance measure in reliability-oriented global sensitivity analysis. Sun et al. [24] described the mathematical model of the parallel system and voting system and proposed that the parallel system can improve the reliability of the system more than the voting system. Ji et al. [25] analyzed the mechanism of fault cascading propagation in communication networks and proposed a fault prediction model based on the load capacity model. Zhang et al. [26] proposed a novel load capacity model with a tunable proportion. Guo et al. [27] proposed a development cascade fault model applied to complex network theory to simulate the disaster diffusion process in their project. Wang et al. [28] proposed a novel node importance evaluation method, considering the attack cost, the industry characteristics and the directionality of the network. Lin et al. [29] put forward a power network node importance model based on a critical method with degree

centrality, betweenness centrality, closeness centrality, eigenvector centrality and some power performance characteristics as indicators. This paper used the actual load of the node and the number of tasks completed by the node to measure the importance of the node.

In terms of redundancy design analysis, Dui et al. [30] proposed a resilience importance measure for performance loss to optimize the performance of the main coolant system. Xiao et al. [31] studied the reliability evaluation of missile test systems using different redundant configuration methods. Ma et al. [32] optimized the lower bound of the reconfigurable system reliability by reassigning the degrading components. Ni et al. [33] studied the reliability analysis and design optimization for nonlinear structures by using the Kriging-based method and First-order reliability method. Ling et al. [34] studied an optimal allocation strategy of components under the maximum allowable constraints in the system. Wu et al. [35] combined a heuristic algorithm with a famous genetic algorithm to solve the reliability–redundancy allocation problem of phased mission systems. Lima and Oliveira [36] conducted graph theory analysis and queuing theory analysis on foraging cooperation tasks in a multiagent problem. Wu et al. [37] proposed a variance-based global importance hybrid heuristic algorithm to evaluate and optimize the importance of a component to the mission reliability of a PMS when the reliabilities of all the components vary randomly. Feng et al. [38] proposed some importance measures to describe the effect of changes in the number of unmanned aerial vehicle swarms on phased-mission reliability. Wu et al. [39] investigated the reliability modelling and evaluation method of phased-mission systems with conflicting phase redundancy.

In Table 1, the work of related studies is summarized and compared.

Table 1. Comparison of the related literatures.

Attr-Ibute Studies	System			Reliability		Objective		Redundancy Configuration	
	Phased-Mission	Single Phase	Multi-task	Evaluation	Optimization	Single Objective	Multiobjective	Cold Standby	Hot Standby
					Redundancy Component				
Feng et al., 2022	✓			✓	✓	✓			
Li et al., 2021	✓			✓					
Xiao et al., 2020		✓		✓	✓	✓		✓	✓
Ni et al., 2020		✓		✓		✓			
Ma et al., 2021		✓		✓	✓		✓		
Fan et al., 2019		✓		✓	✓	✓		✓	
Cai et al., 2016		✓		✓	✓		✓		✓
Wu et al., 2017		✓	✓	✓					
This paper	✓		✓	✓	✓	✓			✓

In the reliability analysis of the phased-mission system, the above studies did not consider the reliability of the phased-mission system when each stage is a complex network. This paper used the characteristics of a complex network to obtain the system reliability of each stage and finally obtained the reliability of the whole multistage system. Most studies only evaluated the reliability of a phased-mission system and focus on the redundancy configuration optimization of single-stage systems. This paper evaluated and optimized the reliability of a phased-mission system through a redundancy strategy. A criticality index of nodes was used to identify the key nodes of the network at each stage. Then, the node's hot backup system was selected to determine the redundant key node structure. Under the constraints of the cost of the key nodes, with the goal of maximizing the reliability of the information processing network layer, the multitask redundancy optimization model of each stage was established.

1.2. Proposed Approach

We divided the command post system into four layers of different complex networks. Based on the load capacity model, we used the degree of the node to replace the betweenness to define the initial load of the node and to obtain the node capacity and actual load. Then, we defined the single-layer network reliability, the command post system reliability and the node criticality. The higher the node's criticality, the higher the cost. Finally, we proposed a redundancy optimization model for the single-layer network and command post system, which can obtain the optimal number of backups to maximize system reliability. The approach flowchart is shown in Figure 1.

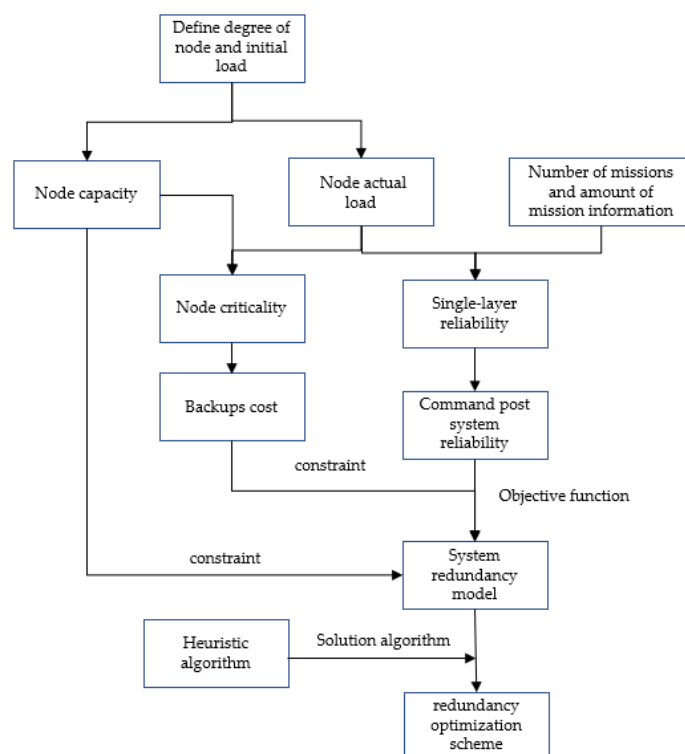


Figure 1. Content flowchart.

1.3. Overview

The rest of this paper is arranged as follows. Section 2 discusses the phased-mission reliability model of the command post system, based on the multilayer network. Section 3 analyzes key nodes and redundancy optimization. Section 4 uses the case of a four-layer network to verify the effectiveness of the proposed model. Section 5 closes the paper.

2. Reliability Analysis of Command Post System Based on Multilayer Network

The command post system includes four links: intelligence acquisition (IA), intelligence processing (IP), decision control (DC) and response execution (RE), and each link exists independently. The command post system can be regarded as a multilayer network system; each layer of the network represents a link in the system. The command system is goal-oriented to complete the command mission. Therefore, each layer of the network can be assumed to be a separate mission stage, and different missions need to be completed in different stages, as shown in Figure 2.

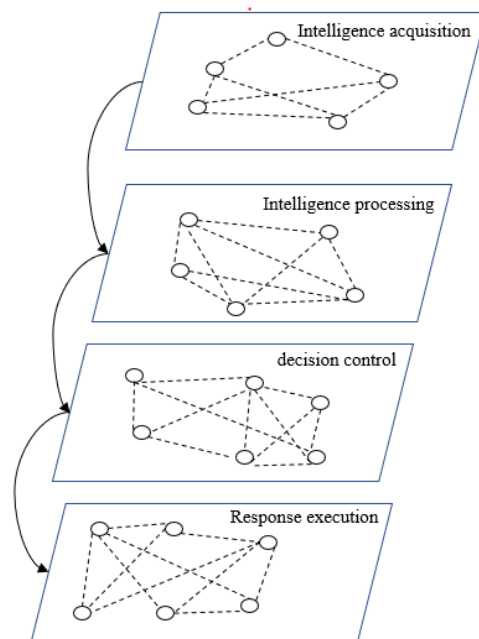


Figure 2. Missions at different layers.

In Figure 2, intelligence acquisition, intelligence processing, decision control and response execution, respectively, represent the four layers of the command post system. The intelligence acquisition layer is mainly used to complete information acquisition, the intelligence processing layer processes the information collected by the intelligence acquisition layer, the decision control layer analyzes the information processed by the intelligence processing layer, and the response execution layer is used to implement the decision information of the decision control layer.

Networks at different layers in the system are denoted by I , $I \in \{IA, IP, DC, RE\}$. Based on the load capacity model, this paper used the degree of the node to replace the betweenness to define the initial load of the node. The load capacity model of the node in the load network is introduced as follows: if the initial load of node i at the initial moment is only related to the degree of node i , the initial load of node i in a single-layer network is defined as

$$F_{I_i} = \rho k_{I_i}^\tau \quad (1)$$

F_{I_i} is the initial load value of node i in the network, G_I . k_{I_i} is the degrees of node i in the network, and G_I . ρ and τ are the parameters that control the initial load intensity of node i . When it is difficult to determine the actual physical load on the network, it is reasonable and effective to use this dimensionless “structural load” to study the reliability of the network.

Since the capacity is seriously limited by the cost, the load capacity model believes that the capacity of the node is proportional to its initial load, and the reliability of the network can be improved only by adding a certain percentage of redundant capacity, according to the load. Therefore, the information capacity of node i in the command post system network is

$$C_{I_i} = (1 + \alpha)F_{I_i}, \quad i = 1, 2, \dots, N_I \quad (2)$$

C_{I_i} is the information capacity of node i in the network and I . α is tolerance coefficient. As long as $\alpha \geq 0$, the network is in a free-flowing state. N_I is the number of nodes in the network I .

When the actual load F'_{I_i} of node i is equal to the information capacity C_{I_i} , node i completes one job. The actual load value of node i is the initial load value plus the increased amount of information in the network I during time (t_0, t) , which is expressed as

$$F'_{I_i}(t) = F_{I_i} + \frac{k_{I_i}}{\sum_{i=1}^{N_I} k_{I_i}} Q_I(t - t_0) \quad (3)$$

Q_I represents the amount of information that the network I can process per unit time. Because the ability of different layers of the network to process information is different, the Q_I of each network is different. The Q_I value is related to the sum of the total degrees of nodes in the network. The larger the $\sum_{i=1}^{N_I} k_{I_i}$ value is, the larger the Q_I value is, and the more information the network can process per unit time.

The multimission command system information processing is the work that the nodes in the single-layer network continuously perform. When the amount of information processed by the node reaches the capacity, the node completes one information processing job and continues to process the remaining information of this mission until the information processing of this mission is completed, and then the information processing of the next mission begins. The order of mission processing cannot be changed, and information must be processed according to the mission order. After the n -layer network processes the information of the f -th mission, the $n + 1$ -layer network can process the information of the f -th mission.

The reliability of the single mission network can be quantified as the ratio of completed information to total mission information, which is expressed as

$$R'_I = \frac{\sum_{i=1}^{N_I} F'_{I_i}(t)}{q} \quad (4)$$

R'_I is the mission reliability of the network I with a single mission. q is the amount of single mission information.

The reliability of a multimission network can be quantified as the ratio of the amount of information completed to the amount of information required to be completed by multiple missions, which is expressed as

$$R_I = \frac{\sum_{f=1}^{N_f} \sum_{i=1}^{N_I} F'_{I_i}(t)}{\sum_{f=1}^{N_f} Q_f} \quad (5)$$

R_I is the mission reliability of the network I with multiple missions. N_f is the number of missions. f is the number of missions. Q_f is the information quantity of the f -th mission.

The information acquisition, processing, response and execution process of multiple missions are as follows.

Step 1: All nodes in the intelligence acquisition network collect the first mission information, according to the node degree, and the first node that completes the first mission appears.

Step 2: If the running time reaches the specified time t , calculate the network reliability; otherwise, enter the next mission information.

Step 3: The first node that completes the first mission collects the second information, and the second node that completes the first mission appears.

Step 4: If the running time reaches the specified time t , calculate the network reliability; otherwise, the two nodes that complete the first mission collect the remaining information of the second mission.

Step 5: The third node that completes the first mission appears. Continue with the above steps.

Step 6: After the first mission information is collected by the first layer network, the information will be transmitted to other layer networks in order. Repeat step 1–5.

The above process is shown in Figure 3.

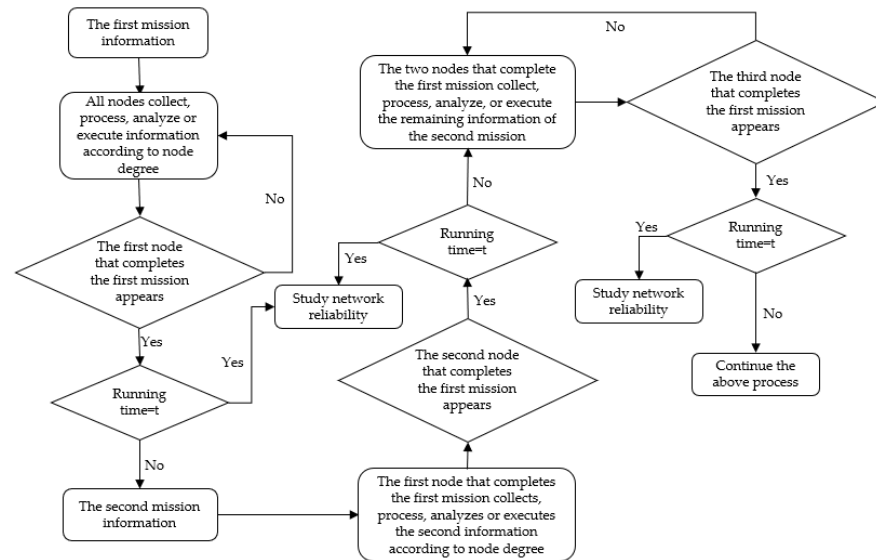


Figure 3. Information collection, processing, execution or response process of multiple missions.

According to the above characteristics, the command post system can be further analyzed according to the hierarchical network model, and each link can be regarded as each layer of the network, and the physical components required by each link to complete the mission can be regarded as the nodes on each layer of the network. The phased-mission system contains multiple phases, which do not overlap with each other and appear in order of time. In order to avoid the poor operation of the four links of intelligence acquisition, intelligence processing, decision control and response execution, the system support stage is added, and the final command post system reliability is

$$R = \prod_{I \in \{IA, IP, DC, RE\}} R_I \quad (6)$$

R is the phased-mission reliability of the command post system with multiple missions.

3. Redundancy Optimization

The criticality of node i is mainly related to the amount of information processed by the node and the number of times the capacity is reached in the multimission network I ; the criticality of node i in a single-layer network is

$$L_{I_i} = \delta \frac{F'_{I_i}}{\sum_{i=1}^{N_I} F'_{I_i}} + \omega \frac{p_{I_i}}{\sum_{i=1}^{N_I} p_{I_i}} \quad (7)$$

L_{I_i} represents the criticality of nodes in the single-layer network I under multiple missions. $\sum_{i=1}^{N_I} F'_{I_i}$ represents the sum of the information actually processed by all information nodes in the single-layer network I . p_{I_i} represents the times node i has completed the mission in the single-layer network I . δ, ω represents the weight coefficients.

After the node joins the hot backup, the information flow of the node changes and the actual load of different network nodes after the change is described as follows.

$$F'_{I_i} = F_{I_i} + (1 + j_{I_i}) \frac{k_{I_i}}{\sum_{i=1}^{N_I} k_{I_i}} Q_I \quad (8)$$

j_{I_i} is the number of backups of node i in network I . The addition of j_{I_i} backup nodes to the key node is equivalent to the parallel connection of j_{I_i} backup nodes to the master node,

at which time the amount of information through j_{I_i} backup nodes and the master node is equivalent to the amount of information through the backup nodes plus the amount of information through the master node.

The redundancy optimization model of network I is as follows.

$$\max R_I = \max \frac{\sum_{f=1}^{N_f} \sum_{i=1}^{N_I} F'_{I_i}(t)}{\sum_{f=1}^{N_f} Q_f} \quad (9)$$

$$\sum_{i=1}^{N_I} U_{I_i} j_{I_i} \leq U \quad (10)$$

$$U_{I_i} = B_{I_i} u_{I_i}, \quad B_{I_i} = 1, 2, 3, 4, 5 \quad (11)$$

$$u_{I_i} = P_I L_{I_i} \quad (12)$$

$$p_{I_i} = \sum_{i=1}^{N_I} \theta_{I_i} \quad (13)$$

$$F'_{I_i} = C_{I_i}, \quad \theta_{I_i} = 1 \quad (14)$$

$$F'_{I_i} < C_{I_i}, \quad \theta_{I_i} = 0 \quad (15)$$

The redundancy optimization model of the command post system is as follows.

$$\max R = \max \prod_{I \in \{IA, IP, DC, RE\}} R_I = \max \prod_{I \in \{IA, IP, DC, RE\}} \frac{\sum_{f=1}^{N_f} \sum_{i=1}^{N_I} F'_{I_i}(t)}{\sum_{f=1}^{N_f} Q_f} \quad (16)$$

$$\sum_{I \in \{IA, IP, DC, RE\}} \sum_{i=1}^{N_I} U_{I_i} j_{I_i} \leq U \quad (17)$$

$$U_{I_i} = B_{I_i} u_{I_i}, \quad B_{I_i} = 1, 2, 3, 4, 5 \quad (18)$$

$$u_{I_i} = P_I L_{I_i} \quad (19)$$

$$F'_{I_i} = C_{I_i}, \quad \theta_{I_i} = 1 \quad (20)$$

$$F'_{I_i} < C_{I_i}, \quad \theta_{I_i} = 0 \quad (21)$$

U_{I_i} is the cost of adding a spare part to node i in the network I . B_{I_i} is the geographical location of node i in the network I ; the more complex the geographical location, the larger the B_{I_i} and the higher the cost of adding a spare part. u_{I_i} is the cost of spare parts for node i in the network I . P_I is the unit cost required to operate and maintain each node in the network I . L_{I_i} is the criticality of node i in the network I . θ_{I_i} is 0, indicating that node i in the network I has not completed its mission. θ_{I_i} is 1, indicating that node i in the network has completed a mission. Equation (8) is the objective function to maximize the reliability of a single-layer network. Equation (9) means that the cost of backup nodes cannot exceed the total cost. Equation (10) indicates that the cost of adding a backup is related to its own cost and geographical location. Equation (11) indicates that the backup's cost is related to the node criticality. Equation (16) means that the cost of backup nodes in the four-layer network cannot exceed the total cost.

4. Case Study

According to a command post system, a four-layer network model diagram, as shown in Figure 4, was established. The first layer is five reconnaissance aircraft, the reconnaissance aircraft are responsible for the mission of information collection on the target. The second layer is the data center layer, responsible for processing and analyzing the information transmitted by the reconnaissance aircraft. The third layer is the control and decision layer,

responsible for making comprehensive decisions on the data processed and analyzed by the data center. The fourth layer is the fighter aircraft, responsible for responding to the commands issued by the decision-making layer. Each must be processed by the four-layer network. According to the model of the third part, the mission reliability of this command post system was analyzed.

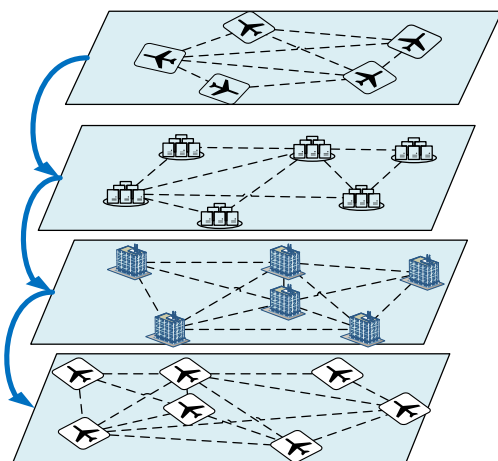


Figure 4. Command post system.

The assumptions for the command system are as follows:

Assumption 1: The missions of each phase in the command post system are continuous and nonoverlapping, and the order of each mission has been determined.

Assumption 2: Multiple missions are independent of each other, and nodes in the layer are independent of each other.

Assumption 3: The successful completion of missions in all phases of the command post system is considered as the success of the whole mission. That is, when a mission fails at a certain stage, the whole mission is considered as a failure.

The missions performed by the command post system are determined by the amount of information, and the amount of information of each mission was different. Based on the nature and type of missions, a set of mission information volume data was randomly generated, as shown in Table 2.

Table 2. Mission information data.

Number of missions	1	2	3	4	5	6	7	8
Amount of information	1605	959	1170	1034	1287	725	949	874

The degree of the network nodes is determined by the connections between the nodes in the layer. Based on the connections between the nodes in the intralayer network in the command post system shown in Figure 3, the degrees of the nodes in each layer of the network can be derived as shown in Table 3.

Table 3. Degree of nodes in each layer of the network.

	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7
Layer 1 network	3	3	4	4	2		
Layer 2 network	2	3	4	2	5	2	
Layer 3 network	3	5	4	3	5	4	
Layer 4 network	2	4	4	4	5	5	6

The initial load of the nodes in the intralayer network is related to the degree of the nodes and parameters ρ and τ . To simplify the calculation, $\rho = 3$ and $\tau = 2$. According to Equation (1), the initial load of the nodes in each layer of the network can be derived, as shown in Table 4.

Table 4. Initial load of the node.

	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7
Layer 1 network	27	27	48	48	12		
Layer 2 network	12	27	48	12	75	12	
Layer 3 network	27	75	48	27	75	48	
Layer 4 network	12	48	48	48	75	27	108

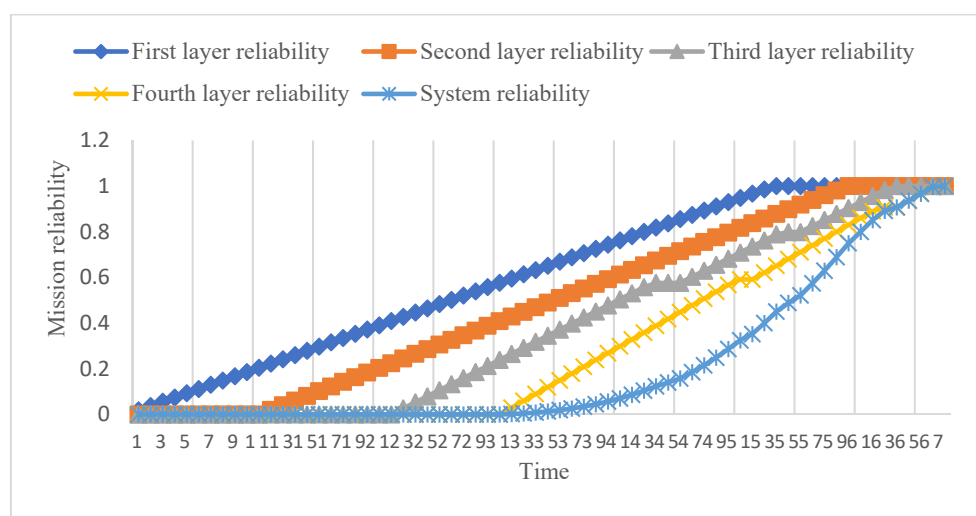
The capacity of a node is related to the initial load of the node and the parameter α . Therefore, the parameter $\alpha = 7$. According to Equation (2), the capacity of the nodes in each layer of the network within the system was derived, as shown in Table 5.

Table 5. Node capacity per layer.

	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7
Layer 1 network	216	216	384	384	96		
Layer 2 network	96	216	384	96	600	96	
Layer 3 network	216	600	384	216	600	384	
Layer 4 network	96	384	384	384	600	216	864

The amount of information processed per unit time in a single-layer network is fixed, which is based on the nature of the nodes. The amount of information processed by a node per unit time is determined by the degree of the node. Therefore, the sum of the degrees of the nodes within a layer determines the amount of information processed per unit time in a single-layer network. Based on the data in the above table, the mission processing process of the command post system was simulated and analyzed.

According to Equations (3)–(5), the process of changing mission reliability of the command post system was derived, as shown in Figure 5.



From Figure 5, it can be seen that the mission reliability at the intelligence acquisition layer increases with time and reaches the maximum value1 at time $t = 54$. The mission reliability at the intelligence processing layer begins to increase at time $t = 12$, indicating that at time $t = 11$, the intelligence acquisition layer finishes processing the information of the first mission and reaches the maximum value1 at time $t = 60$. The control decision layer begins to increase at time $t = 23$, indicating that the information processing layer finishes processing the information of the first mission and reaches the maximum value1 at $t = 64$. The response execution layer starts to increase at time $t = 32$ and reaches the maximum value at $t = 68$, when all missions are completed. The mission reliability of the command post system starts at $t = 32$ and gradually increases with the increase in time.

According to Equation (7), the node importance values of the nodes in the mission completion process were derived, as shown in Table 6.

Table 6. Node importance value.

	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7
Layer 1 network	0.1918	0.1918	0.2186	0.2156	0.1822		
Layer 2 network	0.1445	0.1968	0.2223	0.1445	0.2474	0.1445	
Layer 3 network	0.1557	0.1834	0.1701	0.1557	0.1823	0.1528	
Layer 4 network	0.1389	0.1390	0.1376	0.1360	0.1537	0.1261	0.1687

Without considering the value of the backup node itself, the more important the node is, the costlier it is to operate and maintain. The more complex the geographic location in which the node is located, the greater the cost of adding a redundancy to the node. Therefore, under the condition of randomly given geographic location complexity, when $P_I = 100$, the cost for each additional redundancy of nodes in each layer of the network is shown in Table 7.

Table 7. Cost of adding a redundancy to a node.

	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7
Layer 1 Network	38.03629	57.5443	43.722	43.1198	54.6486.		
Layer 2 Network	28.9012	39.3519	66.7037	57.8025	98.9505	57.8025	
Layer 3 Network	31.1489	18.3355	51.0274	62.2979	36.4544	61.1170	
Layer 4 Network	27.7836	41.6978	27.5149	13.6008	61.4925	63.0597	50.5970

In the redundancy optimization of the command post system, the second layer network is selected as the target for redundancy optimization, and the optimal redundancy strategy is selected to maximize the mission reliability of the information processing layer and the command post system. According to Equation (8), each eligible redundancy strategy is brought into the information processing of the command post system under the constraint of cost 225. When time $t = 50$, the optimal redundancy strategy is derived by comparing the mission reliability values of the command post system. The optimal redundancy strategies for the nodes in the second layer network are shown in Table 8.

Table 8. Optimal redundancy strategy for nodes in layer 2 networks.

	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6
Layer 2 network	2	4	0	0	0	0

The variation of mission reliability of the system under optimal redundancy backups is shown in Figure 6.

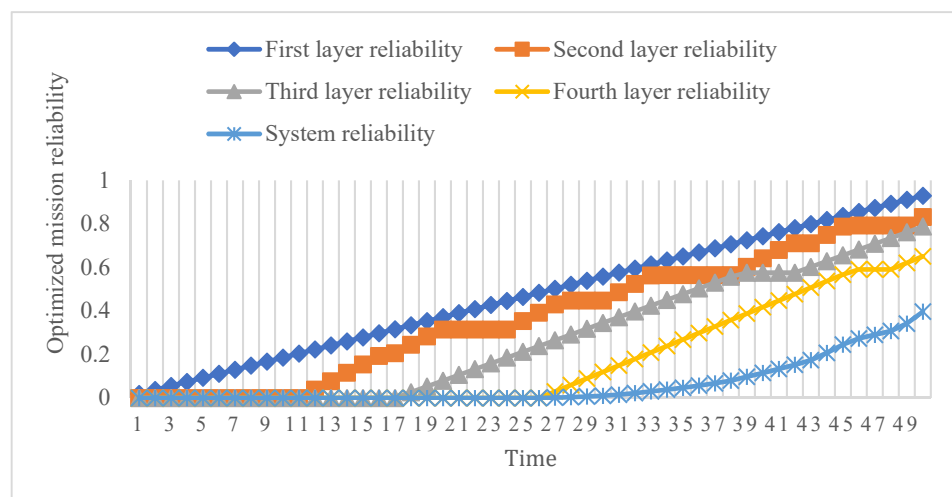


Figure 6. Optimized reliability after redundancy.

From Figure 6, the mission reliability of the information processing layer at $t = 50$ is 0.7966 and the mission reliability of the command post system is 0.2866. After optimization, the mission reliability of the information processing layer at $t = 50$ is 0.8317 and the mission reliability of the command post system is 0.3968. The mission reliability value of the information processing layer network is optimal. The mission reliability of optimized information processing layer and command post system is higher than that before optimization, so the solution proposed in this paper is correct.

5. Conclusions and Future Work

The reliability optimization analysis of the command post network system is the basis for guaranteeing the normal operation of the system for business processing, information transmission and other functions under general conditions and wartime tension. The reliability of the operation link of the command post seriously affects its accuracy in completing command mission operations.

In order to realize the efficient operation of the command post system, the overall reliability of the command post network system was improved. This paper assumes that the four networks in the command post system, namely intelligence acquisition, intelligence processing, decision control and response execution, are separate mission stages, and the missions to be completed in each stage are different. It also introduces the indicators within the separate mission stages and analyzes the node load within the network of each mission stage, to establish the reliability model of the network in each stage to complete multiple missions, and then establishes the command post system multistage mission reliability model. Secondly, based on the characteristics of network nodes in each stage, the node criticality index for completing multiple missions is given, and the index is used to identify the key nodes in the four-stage network of the command post. Because this type of node has a great influence on the overall reliability of the network, this paper selects the hot backup system to determine the redundant structure of key nodes to improve the reliability of the command post system. Finally, in order to optimize the redundant structure of the key nodes of the command post system, this paper aims to maximize the reliability of the multistage mission of the command post system under the constraints of the commander's will, node location, node redundancy cost constraints and node load constraints. Taking the reliability as the objective function, a redundant reliability optimization model for multiple missions of the network at each stage is established.

Based on the existing relevant research combined with the actual situation, this paper proposes all the above models. These models can meet the actual operation of the command post system and can provide a certain reference for the practical application of system reliability optimization to a certain extent. However, in-depth research is still needed on

the load analysis of the multistage system and the redundancy strategy of cold backup and warm backup.

With the deepening of the interaction between complex systems and complex systems in real society, the multilayer network has gradually become a research hotspot. In reality, equipment support needs to complete the mission of maintenance and material support of combat equipment. An equipment support network can be divided into the kill layer, maintenance layer and storage supply layer. A multicoupling network of equipment support is formed through the mutual command relationship and support cooperation between layers. This paper does not consider coupling. In the future, we intend to study the influence of coupling between layers and coupling within the layers of the three-layer network of equipment support on the network synchronization and the control problem of the multilayer network, and define the corresponding topology structure for different practical problems and analyze its dynamic behavior. What is more, the network failure process is the superposition of all network node failure processes. Future work can address the impact of node failure on the reliability of multilayer networks to measure the importance of nodes. Redundancy design can improve system reliability. This paper uses the design method of parallel redundancy. Future work can study serial redundancy, modular redundancy and systematic redundancy. This paper mainly considers the cost constraints of nodes. We can consider the number, volume, quality and power consumption constraints of nodes in the future.

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References

1. Li, X.; Huang, H.; Li, F.Q. PRA based reliability analysis of complex space phased-mission system. *Syst. Eng. Electron.* **2019**, *41*, 2141–2147.
2. Li, X.Y.; Huang, H.Z.; Li, Y.F.; Xiong, X. A Markov regenerative process model for phased mission systems under internal degradation and external shocks. *Reliab. Eng. Syst. Saf.* **2021**, *215*, 107796. [[CrossRef](#)]
3. Xing, L. Reliability evaluation of phased-mission systems with imperfect fault coverage and common-cause failures. *IEEE Trans. Reliab.* **2007**, *56*, 58–68. [[CrossRef](#)]
4. Wu, X.; Yan, L. Numerical method for reliability analysis of phased-mission system using Markov chains. *Commun. Stat.-Theory Methods* **2012**, *41*, 3960–3973. [[CrossRef](#)]
5. Xing, L.; Amari, S.V.; Wang, C. Reliability of k-out-of-n systems with phased-mission requirements and imperfect fault coverage. *Reliab. Eng. Syst. Saf.* **2012**, *103*, 45–50. [[CrossRef](#)]
6. Li, X.; Tao, L.; Jia, M. A Bayesian networks approach for event tree time-dependency analysis on phased-mission system. *Eksplot. Niezawodn.-Maint. Reliab.* **2015**, *17*, 273–281. [[CrossRef](#)]
7. Yu, W.D.; Han, W.G. Reliability Analysis of Warship Combat System. *J. Ordnance Equip. Eng.* **2018**, *2018*, 66–68.
8. Chew, S.; Dunnett, S.; Andrews, J. Phased mission modeling of systems with maintenance free operating period using simulated Petri nets. *Reliab. Eng. Syst. Saf.* **2008**, *93*, 980–994. [[CrossRef](#)]

9. Yang, X.; Wu, X.; Yang, X. Automated generation of mission reliability simulation model for space tracking, telemetry and control system by extensible markup language and extended object-oriented Petri net. *Proc. Inst. Mech. Eng. Part O J. Risk Reliab.* **2014**, *228*, 397–408. [\[CrossRef\]](#)
10. Li, M.; Yu, H.; Wu, X. Reliability modeling of phased mission system with phase backup by stochastic petri net. *J. Donghua Univ.* **2018**, *35*, 418–421. (In English)
11. Yu, H.; Wu, X.; Wu, X. An extended object-oriented petri net model for mission reliability evaluation of phased-mission system with time redundancy. *Reliab. Eng. Syst. Saf.* **2020**, *197*, 106786. [\[CrossRef\]](#)
12. Dui, H.; Li, S.; Xing, L.; Liu, H. System performance-based joint importance analysis guided maintenance for repairable systems. *Reliab. Eng. Syst. Saf.* **2019**, *186*, 162–175. [\[CrossRef\]](#)
13. Almoghathawi, Y.; Barker, K. Component importance measures for interdependent infrastructure network resilience. *Comput. Ind. Eng.* **2019**, *133*, 153–164. [\[CrossRef\]](#)
14. Miziula, P.; Navarro, J. Birnbaum importance measure for reliability systems with dependent components. *IEEE Trans. Reliab.* **2019**, *68*, 439–450. [\[CrossRef\]](#)
15. Cai, Z.; Si, S.; Sun, S.; Li, C. Optimization of linear consecutive-k-out-of-n system with a Birnbaum importance-based genetic algorithm. *Reliab. Eng. Syst. Saf.* **2016**, *152*, 248–258. [\[CrossRef\]](#)
16. Mao, X.; Lou, X.; Yuan, C.; Zhou, J. Resilience-Based Restoration Model for Supply Chain Networks. *Mathematics* **2020**, *8*, 163. [\[CrossRef\]](#)
17. Tan, Y.; Zhang, J.; Li, X. Importance evaluation of power grid nodes based on complex network theory. *Comput. Eng.* **2019**, *45*, 281–286.
18. Zhang, Y.; Zhang, C.; Wang, S.; Chen, R.; Tomovic, M.M. Performance Degradation Based on Importance Change and Application in Dissimilar Redundancy Actuation System. *Mathematics* **2022**, *10*, 843. [\[CrossRef\]](#)
19. Guo, W.; Fang, L.; Zhong, L. Node importance estimation in complex networks based on multi-index comprehension. *Comput. Eng. Des.* **2016**, *37*, 3146–3150.
20. Hui, W.U.; Peng, M.; Zhang, H.; Zhu, L.; Che, H.; Liu, Z. Node Vulnerability Assessment for Distribution Network Based on Complex Network Theory. *Complex Syst. Complex. Sci.* **2017**, *14*, 38–45.
21. Fan, X.; Xu, R.; Cao, L.; Song, Y. Learning Nonparametric Relational Models by Conjugately Incorporating Node Information in a Network. *IEEE Trans. Cybern.* **2017**, *47*, 589–599. [\[CrossRef\]](#) [\[PubMed\]](#)
22. Fan, W.; Hu, P.; Liu, Z. Multi-attribute node importance evaluation method based on Gini-coefficient in complex power grids. *IET Gener. Transm. Distrib.* **2016**, *10*, 2027–2034.
23. Kala, Z. New Importance Measures Based on Failure Probability in Global Sensitivity Analysis of Reliability. *Mathematics* **2021**, *9*, 2425. [\[CrossRef\]](#)
24. Sun, H.; Liu, B.; Cao, X. Research on reliability and security of vote redundancy system. *J. Electron. Meas. Instrum.* **2011**, *25*, 661–664. [\[CrossRef\]](#)
25. Ji, X.; Liang, X.; Zhou, Y.; Huo, Y.; Shi, X.; Yang, Y. A Fault Prediction Method Based on Load-capacity Model in the Communication Network. *IEICE Proc. Ser.* **2019**, *59*, 2–21.
26. Zhang, Z.; Song, Y.; Xia, L.; Li, Y.; Zhang, L.; Jiang, G. A Novel Load Capacity Model with a Tunable Proportion of Load Redistribution against Cascading Failures. *Secur. Commun. Netw.* **2018**, *2018*, 6254876. [\[CrossRef\]](#)
27. Guo, N.; Guo, P.; Dong, H. Modeling and analysis of cascading failures in projects: A complex network approach. *Comput. Ind. Eng.* **2019**, *127*, 1–7. [\[CrossRef\]](#)
28. Wang, N.; Gao, Y.; He, J.; Yang, J. Robustness evaluation of the air cargo network considering node importance and attack cost. *Reliab. Eng. Syst. Saf.* **2022**, *217*, 108026. [\[CrossRef\]](#)
29. Lin, Z.; Wen, F.; Wang, H.; Lin, G.; Mo, T.; Ye, X. CRITIC-Based Node Importance Evaluation in Skeleton-Network Reconfiguration of Power Grids. *IEEE Trans. Circuits Syst. II Express Briefs* **2018**, *65*, 206–210. [\[CrossRef\]](#)
30. Dui, H.; Xu, Z.; Chen, L.; Xing, L.; Liu, B. Data-driven maintenance priority and resilience evaluation on performance loss in main coolant system. *Mathematics* **2022**, *10*, 563. [\[CrossRef\]](#)
31. Xiao, Z.; Li, H.; Wang, R. Reliability Evaluation of Missile Test System Based on Different Types of Redundant Configuration. *J. Ordnance Equip. Eng.* **2020**, *41*, 94–98.
32. Ma, C.; Wang, Q.; Cai, Z.; Si, S.; Zhao, J. Component reassignment for reliability optimization of reconfigurable systems considering component degradation. *Reliab. Eng. Syst. Safety* **2021**, *215*, 107867. [\[CrossRef\]](#)
33. Ni, P.; Li, J.; Hao, H. Reliability analysis and design optimization of nonlinear structures. *Reliab. Eng. Syst. Saf.* **2020**, *198*, 106860. [\[CrossRef\]](#)
34. Ling, X.; Wei, Y.; Si, S. Reliability optimization of k-out-of-n system with random selection of allocative components. *Reliab. Eng. Syst. Saf.* **2019**, *186*, 186–193. [\[CrossRef\]](#)
35. Wu, X.; Wu, X. An importance based algorithm for reliability-redundancy allocation of phased mission systems. In Proceedings of the 2017 IEEE International Conference on Software Quality, Reliability and Security Companion, Prague, Czech Republic, 25–29 July 2017; pp. 152–159.
36. Lima, D.; Oliveira, G. Formal analysis in a cellular automata ant model using swarm intelligence in robotics foraging task. In Proceedings of the 2017 IEEE International Conference on Systems, Man, and Cybernetics (SMC), Banff, AB, Canada, 5–8 October 2017; Volume 10, pp. 1793–1798.

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37. Wu, X.Y.; Wu, X.Y.; Balakrishnan, N. Reliability allocation model and algorithm for phased mission systems with uncertain component parameters based on importance measure. *Reliab. Eng. Syst. Saf.* **2018**, *180*, 266–276. [[CrossRef](#)]
 38. Feng, Q.; Liu, M.; Dui, H.; Ren, Y.; Sun, B.; Yang, D.; Wang, Z. Importance measure-based phased mission reliability and UAV number optimization for swarm. *Reliab. Eng. Syst. Saf.* **2022**, *223*, 108478. [[CrossRef](#)]
 39. Wu, X.; Yu, H.; Balakrishnan, N. Modular model and algebraic phase algorithm for reliability modelling and evaluation of phased-mission systems with conflicting phase redundancy. *Reliab. Eng. Syst. Saf.* **2022**, *227*, 108735. [[CrossRef](#)]