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Fuzzy Differential Subordination and Superordination Results Involving the q-Hypergeometric Function and Fractional Calculus Aspects

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Abstract: The concepts of fuzzy differential subordination and superordination were introduced in the geometric function theory as generalizations of the classical notions of differential subordination and superordination. Fractional calculus is combined in the present paper with quantum calculus aspects for obtaining new fuzzy differential subordinations and superordinations. For the investigated fuzzy differential subordinations and superordinations, fuzzy best subordinates and fuzzy best dominants were obtained, respectively. Furthermore, interesting corollaries emerge when using particular functions, frequently involved in research studies due to their geometric properties, as fuzzy best subordinates and fuzzy best dominants. The study is finalized by stating the sandwich-type results connecting the previously proven results.

Keywords: Riemann–Liouville fractional integral; q-hypergeometric function; fuzzy differential subordination; fuzzy differential superordination; fuzzy best dominant; fuzzy best subordinate

MSC: 30C45; 30A10; 33C15

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1. Introduction

Embedding the concept of the fuzzy set introduced by Lotfi A. Zadeh in 1965 [1] into already established mathematical theories was a constant preoccupation for researchers in different fields of mathematics. The review papers [2,3] present some aspects regarding different applications of this concept.

The concept of the fuzzy set was applied in the geometric function theory in 2011 [4] when the notion of fuzzy subordination was introduced; the theory of fuzzy differential subordination has its foundations in the paper published in 2012 [5] in which the first notions from the classical theory of differential subordination due to Miller and Mocanu [6] were adapted to fit the fuzzy context. From that point on, the theory was developed by many researchers who became interested in the merge between the fuzzy sets theory and geometric function theory. The dual notion of fuzzy differential superordination was introduced in 2017 [7]. A few steps in the development of fuzzy subordination and superordination theory are described in [8].

In recent years, the line of research involving different operators in the studies concerning fuzzy differential subordinations and superordinations developed nicely. Fuzzy differential subordinations were obtained using the Wanas operator [9,10], generalized Noor-Sălăgean operator [11], Sălăgean and Ruscheweyh operators [12] or a linear operator [13]; fuzzy differential subordinations were obtained for meromorphic functions in [14]; λ-pseudo starlike and λ-pseudo convex functions are associated with the studies of fuzzy differential subordinations in [15], the Mittag–Leffler-type Borel distribution is associated

with the studies regarding fuzzy differential subordination in [16] and the class of fuzzy α-convex functions is studied in [17].

Fractional calculus that is used in studies concerning fuzzy differential subordinations and superordinations shows excellent results. Fuzzy differential subordinations were obtained using fractional integral applied to the Mittag–Leffler function in [18], the Wanas operator is associated with fractional calculus for obtaining fuzzy differential subordinations in [10], the fractional derivative was used [19], the fractional integral of confluent hypergeometric function was applied [20,21], the Riemann–Liouville fractional integral of Ruscheweyh and Sălăgean operators were reviewed in [22], the Atangana–Baleanu fractional integral was used in [23], and the fractional integral of the Gaussian hypergeometric function was used [24].

In this paper, quantum calculus was added to the studies associated with fuzzy differential subordinations and superordinations for the first time using the operator introduced in [25] by combining the Riemann–Liouville fractional integral and q-hypergeometric function.

The main novelty of the paper consists of the new fuzzy subordinations and fuzzy superordinations obtained using the Riemann–Liouville fractional integral of q-hypergeometric function. As shown earlier, the Riemann–Liouville fractional integral was previously used in studies regarding fuzzy differential subordinations and fuzzy differential superordinations but not applied to the q-hypergeometric function. The theorems proved in this paper study fuzzy differential subordinations and fuzzy differential superordinations obtained using the Riemann–Liouville fractional integral of q-hypergeometric, for which the fuzzy best dominant and fuzzy best subordinate are given, respectively. Corollaries are established using particular functions, which are important for their geometric properties, as the fuzzy best dominant and fuzzy best subordinate. Examples are also constructed in order to prove the applicability of the results. The results proved for the two dual theories of fuzzy differential subordinations and fuzzy differential superordinations are connected by sandwich-type results.

2. Preliminary Results

The basic notations from the geometric function theory are first introduced.

Let $U = \{z \in \mathbb{C} : |z| < 1\}$ denote the unit disc of the complex plane. $\mathcal{H}(U)$ denotes the class of holomorphic functions in U.

The following classes are defined using holomorphic functions:

$$\mathcal{A}_n = \left\{ f \in \mathcal{H}(U) : f(z) = z + a_{n+1}z^{n+1} + \ldots, z \in U \right\},$$

with $\mathcal{A} = \mathcal{A}_1$, and

$$\mathcal{H}(a, n) = \left\{ f \in \mathcal{H}(U) : f(z) = a + a_n z^n + a_{n+1}z^{n+1} + \ldots, z \in U \right\},$$

when $a \in \mathbb{C}$, $n \in \mathbb{N}^*$.

The notions regarding fuzzy differential subordinations and superordinations used in this study are next presented.

Definition 1. *([4]) Fuzzy subset of X is a pair (A, F_A), with $F_A : X \to [0, 1]$ and $A = \{x \in X : 0 < F_A(x) \leq 1\}$. The support of the fuzzy set (A, F_A) is the set A and the membership function of (A, F_A) is F_A. It is denoted $A = \text{supp}(A, F_A)$.*

Definition 2. *([4]) Consider $D \subset \mathbb{C}$, the functions $f, g \in \mathcal{H}(D)$ and $z_0 \in D$ a fixed point. The function f is fuzzy subordinate to g, written $f \prec_{\mathcal{F}} g$, if the following conditions are satisfied:*

(1) $f(z_0) = g(z_0)$,

(2) $F_{f(D)}f(z) \leq F_{g(D)}g(z)$, $z \in D$.

Definition 3. *([5], Definition 2.2) Consider h a univalent function in U and $\psi : \mathbb{C}^3 \times U \to \mathbb{C}$, such that $h(0) = \psi(a, 0; 0) = a$. When p is analytic in U, such that $p(0) = a$ and the fuzzy differential subordination is satisfied*

$$F_{\psi(\mathbb{C}^3 \times U)} \psi(p(z), zp'(z), z^2 p''(z); z) \leq F_{h(U)} h(z), \quad z \in U, \tag{1}$$

then p is a fuzzy solution of the fuzzy differential subordination. The univalent function q is a fuzzy dominant of the fuzzy solutions of the fuzzy differential subordination, if $F_{p(U)} p(z) \leq F_{q(U)} q(z)$, $z \in U$, for all p satisfying (1). A fuzzy dominant $\tilde{q}$ that satisfies $F_{\tilde{q}(U)} \tilde{q}(z) \leq F_{q(U)} q(z)$, $z \in U$, for all fuzzy dominants q of (1) is the fuzzy best dominant of (1).

Definition 4. *([7]) Consider h an analytic function in U and $\varphi : \mathbb{C}^3 \times U \to \mathbb{C}$. When p and $\varphi(p(z), zp'(z), z^2 p''(z); z)$ are univalent functions in U and the fuzzy differential superordination is satisfied*

$$F_{h(U)} h(z) \leq F_{\varphi(\mathbb{C}^3 \times U)} \varphi(p(z), zp'(z), z^2 p''(z); z), \quad z \in U, \tag{2}$$

i.e.,

$$h(z) \prec_{\mathcal{F}} \varphi(p(z), zp'(z), z^2 p''(z); z), \quad z \in U,$$

then p is a fuzzy solution of the fuzzy differential superordination. An analytic function q is fuzzy-subordinate of the fuzzy differential superordination if

$$F_{q(U)} q(z) \leq F_{p(U)} p(z), \ z \in U,$$

for all p satisfying (2). A univalent fuzzy subordination $\tilde{q}$ that satisfies $F_{q(U)} q \leq F_{q(U)} \tilde{q}$ for all fuzzy-subordinate q of (2) is the fuzzy best subordinate of (2).

Definition 5. *([5]) Q represents the set of all analytic and injective functions f on $\overline{U} \backslash E(f)$, with $f'(\zeta) \neq 0$ for $\zeta \in \partial U \backslash E(f)$, where $E(f) = \{\zeta \in \partial U : \lim_{z \to \zeta} f(z) = \infty\}$.*

The Riemann–Liouville fractional integral is defined as it can be found in [26,27].

Definition 6. *([26,27]) The Riemann–Liouville fractional integral of order λ ($\lambda > 0$) for an analytic function f is defined by*

$$D_z^{-\lambda} f(z) = \frac{1}{\Gamma(\lambda)} \int_0^z \frac{f(t)}{(z-t)^{1-\lambda}} dt.$$

Applications of quantum calculus were first introduced by Jackson, who defined the q-derivative [28] and q-integral [29]. Although Ismail et al. [30] first used quantum calculus notions in the geometric function theory by introducing the class of q-starlike functions, the basic context for the applications of q-calculus in this theory was established in the book chapter written by Srivastava in 1989 [31], where the q-hypergeometric function was highlighted as a function, which can be particularly useful in studies. The recent review paper [32] follows the developments due to the use of q-calculus in the geometric function theory and also shows the numerous q-operators defined using fractional calculus.

Definition 7. *([33]) The q-hypergeometric function $\phi(m, n; q, z)$ is defined by*

$$\phi(m, n; q, z) = \sum_{k=0}^{\infty} \frac{(m, q)_k}{(n, q)_k (q, q)_k} z^k,$$

where

$$(m, q)_k = \begin{cases} 1, & k = 0, \\ (1-m)(1-mq)\left(1-mq^2\right) \ldots \left(1-mq^{k-1}\right), & k \in \mathbb{N}, \end{cases}$$

and $0 < q < 1$.

The operator defined in [25] involves the notions from Definitions 6 and 7. The Riemann–Liouville fractional integral of the q-hypergeometric function is introduced as:

Definition 8. *([25]) Let m, n be complex numbers with $m \neq 0, -1, -2, \ldots$ and $\lambda > 0, 0 < q < 1$. We define the Riemann–Liouville fractional integral of q-hypergeometric function*

$$D_z^{-\lambda}\phi(m,n;q,z) = \frac{1}{\Gamma(\lambda)}\int_0^z \frac{\phi(m,n;q,t)}{(z-t)^{1-\lambda}}dt = \tag{3}$$

$$\frac{1}{\Gamma(\lambda)}\sum_{k=0}^{\infty}\frac{(m,q)_k}{(n,q)_k(q,q)_k}\int_0^z \frac{t^k}{(z-t)^{1-\lambda}}dt.$$

After a simple calculation, the Riemann–Liouville fractional integral of the q-hypergeometric function has the following form

$$D_z^{-\lambda}\phi(m,n;q,z) = \sum_{k=0}^{\infty}\frac{(m,q)_k}{(n,q)_k(q,q)_k(k+1)_\lambda}z^{\lambda+k}. \tag{4}$$

We note that $D_z^{-\lambda}\phi(m,n;q,z) \in \mathcal{H}[0,\lambda]$.

3. Proposed Method

The next two lemmas are tools in the proof of the new results presented in the next section.

Lemma 1. *[6] Consider g a univalent function in the unit disc U and φ, γ analytic functions in a domain $D \supset g(U)$, such that $\gamma(u) \neq 0$ for $u \in g(U)$. Let $G(z) = zg'(z)\gamma(g(z))$ and $h(z) = G(z) + \varphi(g(z))$, supposing G is starlike univalent in U and $Re\left(\frac{zh'(z)}{G(z)}\right) > 0$ for $z \in U$.*

When p is an analytic function, such that $p(0) = g(0)$, $p(U) \subseteq D$ and

$$F_{p(U)}\left(\varphi(p(z)) + zp'(z)\gamma(p(z))\right) \leq F_{h(U)}\left(\varphi(g(z)) + zg'(z)\gamma(g(z))\right),$$

then

$$F_{p(U)}p(z) \leq F_{g(U)}g(z)$$

and the fuzzy best dominant is g.

Lemma 2. *[34] Consider g a convex univalent function in U and φ, γ analytic functions in a domain $D \supset g(U)$. Assume that $Re\left(\frac{\varphi'(g(z))}{\gamma(g(z))}\right) > 0$ for $z \in U$ and $G(z) = zg'(z)\gamma(g(z))$ is a starlike univalent function in U. When $p(z) \in \mathcal{H}[g(0),1] \cap Q$, with $p(U) \subseteq D$ and $\varphi(p(z)) + zp'(z)\gamma(p(z))$ is a univalent function in U and*

$$F_{g(U)}\left(\varphi(g(z)) + zg'(z)\gamma(g(z))\right) \leq F_{p(U)}\left(\varphi(p(z)) + zp'(z)\gamma(p(z))\right),$$

then

$$F_{g(U)}g(z) \leq F_{p(U)}p(z)$$

and the fuzzy best subordinate is g.

The operator seen in (3) will be used for obtaining new fuzzy differential subordinations and superordinations for which the best fuzzy dominants and the best fuzzy subordinates are established in the theorems proved in the next section. Furthermore, using particular functions, well-known due to their geometric properties as fuzzy best dominants and fuzzy best subordinates, corollaries are established following each theorem. At the end of the study, the results obtained using the two dual theories are combined into sandwich-type results familiar to the geometric function theory.

4. Results

The first theorem and the two corollaries obtained for it are related to fuzzy differential subordination.

Theorem 1. *Consider g an analytic and univalent function in U, such that $g(z) \neq 0$, $z \in U$, and $\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha} \in \mathcal{H}(U)$, where m, n are complex numbers with $n \neq 0, -1, -2, \ldots$ and $\alpha, \lambda > 0$, $0 < q < 1$. Assuming $\frac{zg'(z)}{g(z)}$ is starlike univalent in U and*

$$Re\left(1 + \frac{\psi}{\delta}g(z) + \frac{2\varepsilon}{\delta}(g(z))^2 - \frac{zg'(z)}{g(z)} + \frac{zg''(z)}{g'(z)}\right) > 0, \tag{5}$$

for $\beta, \psi, \varepsilon, \delta \in \mathbb{C}$, $\delta \neq 0$, $z \in U$ and

$$\Psi_{\lambda}^{m,n,q}(\alpha, \beta, \psi, \varepsilon, \delta; z) := \beta + \psi\left[\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right]^{\alpha} + \tag{6}$$

$$\varepsilon\left[\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right]^{2\alpha} + \delta\alpha\left[\frac{z\left(D_z^{-\lambda}\phi(m,n;q,z)\right)'}{D_z^{-\lambda}\phi(m,n;q,z)} - 1\right].$$

If the fuzzy differential subordination is satisfied by g

$$F_{\Psi_{\lambda}^{m,n,q}(U)}\Psi_{\lambda}^{m,n,q}(\alpha, \beta, \psi, \varepsilon, \delta; z) \leq F_{q(U)}\left(\beta + \psi g(z) + \varepsilon(g(z))^2 + \delta\frac{zg'(z)}{g(z)}\right), \tag{7}$$

for $\beta, \psi, \varepsilon, \delta \in \mathbb{C}$, $\delta \neq 0$, then

$$F_{D_z^{-\lambda}\phi(U)}\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha} \leq F_{g(U)}g(z), \quad z \in U, \tag{8}$$

and the fuzzy best dominant is g.

Proof. Define $p(z) := \left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha}$, $z \in U$, $z \neq 0$. Differentiating it we obtain $p'(z) = \alpha\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha-1}\left[\frac{\left(D_z^{-\lambda}\phi(m,n;q,z)\right)'}{z} - \frac{D_z^{-\lambda}\phi(m,n;q,z)}{z^2}\right] = \alpha\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha-1}\frac{\left(D_z^{-\lambda}\phi(m,n;q,z)\right)'}{z} - \frac{\alpha}{z}p(z)$. Then $\frac{zp'(z)}{p(z)} = \alpha\left[\frac{z\left(D_z^{-\lambda}\phi(m,n;q,z)\right)'}{D_z^{-\lambda}\phi(m,n;q,z)} - 1\right]$.

Set $\varphi(u) := \beta + \psi u + \varepsilon u^2$ and $\gamma(u) := \frac{\delta}{u}$, we can show easily that φ is analytic in $\mathbb{C}$, γ is analytic in $\mathbb{C}\backslash\{0\}$ and $\gamma(u) \neq 0$, $u \in \mathbb{C}\backslash\{0\}$.

Moreover, we set $G(z) = zg'(z)\gamma(g(z)) = \delta\frac{zg'(z)}{g(z)}$, which is starlike univalent in U, and $h(z) = G(z) + \varphi(g(z)) = \beta + \psi g(z) + \varepsilon(g(z))^2 + \delta\frac{zg'(z)}{g(z)}$.

Differentiating it, we obtain $h'(z) = \delta + g'(z) + 2\varepsilon g(z)g'(z) + \delta\frac{(g'(z)+zg''(z))g(z) - z(g'(z))^2}{(g(z))^2}$ and $\frac{zh'(z)}{G(z)} = \frac{zh'(z)}{\delta\frac{zg'(z)}{g(z)}} = 1 + \frac{\psi}{\delta}g(z) + \frac{2\varepsilon}{\delta}(g(z))^2 - \frac{zg'(z)}{g(z)} + \frac{zg''(z)}{g'(z)}$.

It yields that $Re\left(\frac{zh'(z)}{G(z)}\right) = Re\left(1 + \frac{\psi}{\delta}g(z) + \frac{2\varepsilon}{\delta}(g(z))^2 - \frac{zg'(z)}{g(z)} + \frac{zg''(z)}{g'(z)}\right) > 0$.

We obtain $\beta + \psi p(z) + \varepsilon(p(z))^2 + \delta\frac{zp'(z)}{p(z)} = \beta + \psi\left[\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right]^{\alpha} + \varepsilon\left[\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right]^{2\alpha} + \alpha\delta\left[\frac{z\left(D_z^{-\lambda}\phi(m,n;q,z)\right)'}{D_z^{-\lambda}\phi(m,n;q,z)} - 1\right]$.

By relation (7), we obtain

$F_{p(U)}\left(\beta + \psi p(z) + \varepsilon(p(z))^2 + \delta\frac{zp'(z)}{p(z)}\right) \leq F_{g(U)}\left(\beta + \psi g(z) + \varepsilon(g(z))^2 + \delta\frac{zg'(z)}{g(z)}\right)$.

Using Lemma 1 we obtain $F_{p(U)}p(z) \leq F_{g(U)}g(z)$, $z \in U$, i.e., $F_{D_z^{-\lambda}\phi(U)}\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha} \leq F_{g(U)}g(z)$, $z \in U$ and the fuzzy best dominant is g. □

Example 1. *Let* $\phi(3,2,q,z) = 1 + \frac{2}{1-q}z + \frac{2(1-3q)}{1-2q}z^2$ *for* $m = 3$, $n = 2$,$z \in U$.

Then $D_z^{-\lambda}\phi(3,2,q,z) = \frac{1}{\Gamma(\lambda)}\int_0^z \frac{\phi(3,2,q,t)}{(z-t)^{1-\lambda}}dt = \frac{1}{\Gamma(\lambda)}\int_0^z \left(1 + \frac{2}{1-q}z + \frac{2(1-3q)}{1-2q}z^2\right)(z-t)^{\lambda-1}dt$ *and after changing the variable* $z - t = u$ *and making a simple calculus, we obtain* $D_z^{-\lambda}\phi(3,2,q,z) = \frac{z^{\lambda-1}}{\Gamma(\lambda)}$, $z \in U$.

Consider $g(z) = \frac{1-z}{1+z}$, $z \in U$ *and differentiating it we obtain* $g'(z) = \frac{-2}{(1+z)^2}$ *and* $g''(z) = \frac{4}{(1+z)^3}$, $z \in U$.

Choosing $\alpha = 1$, $\beta = 1$, $\delta = 1$, $\psi = 1$, $\varepsilon = 1$, *we obtain after a long calculus that* $\Psi_\lambda^{3,2,q}(1,1,1,1,1;z) = \lambda - 1 + \frac{z^{\lambda-2}}{\Gamma(\lambda)} + \left(\frac{z^{\lambda-2}}{\Gamma(\lambda)}\right)^2$ *and* $Re\left(1 + \frac{\psi}{\delta}g(z) + \frac{2\varepsilon}{\delta}(g(z))^2 - \frac{zg'(z)}{g(z)} + \frac{zg''(z)}{g'(z)}\right) = Re\left(1 + \frac{1-z}{1+z} + 2\left(\frac{1-z}{1+z}\right)^2 + \frac{2z}{1-z^2} - \frac{2}{1+z}\right) = Re\frac{2-4z+8z^2-2z^3}{(1+z)^2(1-z)}$.

We have also that $\beta + \psi g(z) + \varepsilon(g(z))^2 + \delta\frac{zg'(z)}{g(z)} = 1 + \frac{1-z}{1+z} + \left(\frac{1-z}{1+z}\right)^2 - \frac{2z}{1-z^2} = \frac{3-5z-z^2-z^3}{(1+z)^2(1-z)}$.

Using Theorem 1, when $Re\frac{2-4z+8z^2-2z^3}{(1+z)^2(1-z)} > 0$ *and* $\frac{zg'(z)}{g(z)} = \frac{-2z}{1-z^2}$ *is starlike univalent in* U, *we obtain* $F_U\left(\lambda - 1 + \frac{z^{\lambda-2}}{\Gamma(\lambda)} + \left(\frac{z^{\lambda-2}}{\Gamma(\lambda)}\right)^2\right) \leq F_U\left(\frac{3-5z-z^2-z^3}{(1+z)^2(1-z)}\right)$, $z \in U$, *induce* $F_U\left(\frac{z^{\lambda-2}}{\Gamma(\lambda)}\right) \leq F_U\left(\frac{1-z}{1+z}\right)$.

Corollary 1. *Consider* m, n *complex numbers with* $n \neq 0, -1, -2, \ldots; \lambda, \alpha > 0$, $0 < q < 1$ *and suppose that relation (5) holds. When*

$$F_{\Psi_\lambda^{m,n,q}(U)}\Psi_\lambda^{m,n,q}(\alpha,\beta,\psi,\varepsilon,\delta;z) \leq F_{g(U)}\left(\beta + \psi\frac{Mz+1}{Nz+1} + \varepsilon\left(\frac{Mz+1}{Nz+1}\right)^2 + \delta\frac{(M-N)z}{(Mz+1)(Nz+1)}\right),$$

with $\beta, \psi, \varepsilon, \delta \in \mathbb{C}$, $\delta \neq 0$, $-1 \leq N < M \leq 1$, *and* $\Psi_\lambda^{m,n,q}$ *defined by relation (6), then*

$$F_{D_z^{-\lambda}\phi(U)}\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha} \leq F_{g(U)}\left(\frac{Mz+1}{Nz+1}\right), \quad z \in U,$$

and the fuzzy best dominant is $\frac{Mz+1}{Nz+1}$.

Proof. Put $g(z) = \frac{Mz+1}{Nz+1}$, $-1 \leq N < M \leq 1$ in Theorem 1 and we obtain the corollary. □

Corollary 2. *Consider* m, n *complex numbers with* $n \neq 0, -1, -2, ..; \lambda, \alpha > 0$, $0 < q < 1$ *and suppose that relation (5) holds. When*

$$F_{\Psi_\lambda^{m,n,q}(U)}\Psi_\lambda^{m,n,q}(\alpha,\beta,\psi,\varepsilon,\delta;z) \leq F_{g(U)}\left(\beta + \psi\left(\frac{z+1}{1-z}\right)^{\kappa} + \varepsilon\left(\frac{z+1}{1-z}\right)^{2\kappa} + \delta\frac{2\kappa z}{1-z^2}\right),$$

with $\beta, \psi, \varepsilon, \delta \in \mathbb{C}$, $0 < \kappa \leq 1$, $\delta \neq 0$, *and* $\Psi_\lambda^{m,n,q}$ *defined by relation (6), then*

$$F_{D_z^{-\lambda}\phi(U)}\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha} \leq F_{g(U)}\left(\frac{z+1}{1-z}\right)^{\kappa}, \quad z \in U,$$

and the fuzzy best dominant is $\left(\frac{z+1}{1-z}\right)^{\kappa}$.

Proof. For $g(z) = \left(\frac{z+1}{1-z}\right)^{\kappa}$, $0 < \kappa \leq 1$, in Theorem 1, it follows the corollary. □

The next theorem and the corollaries associated involve fuzzy differential superordination aspects.

Theorem 2. *Consider g an analytic and univalent function in U with the properties $g(z) \neq 0$ and $\frac{zg'(z)}{g(z)}$ is starlike univalent in U. Suppose that*

$$Re\left(\frac{2\varepsilon}{\delta}(g(z))^2 + \frac{\psi}{\delta}g(z)\right) > 0, \text{ for } \psi, \varepsilon, \delta \in \mathbb{C},\ \delta \neq 0. \tag{9}$$

For m, n complex numbers with $n \neq 0, -1, -2, \ldots$ and $\lambda, \alpha > 0$, $0 < q < 1$, when $\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha} \in \mathcal{H}[0,(\lambda-1)\alpha] \cap Q$ and $\Psi_{\lambda}^{m,n,q}(\alpha,\beta,\psi,\varepsilon,\delta;z)$ defined by (6) is univalent in U, then

$$F_{g(U)}\left(\beta + \psi g(z) + \varepsilon(g(z))^2 + \delta\frac{zg'(z)}{g(z)}\right) \leq F_{\Psi_{\lambda}^{m,n,q}(U)}\Psi_{\lambda}^{m,n,q}(\alpha,\beta,\psi,\varepsilon,\delta;z) \tag{10}$$

implies

$$F_{g(U)}g(z) \leq F_{D_z^{-\lambda}\phi(U)}\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha},\ z \in U, \tag{11}$$

and the fuzzy best subordinate is g.

Proof. Define $p(z) := \left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha}$, $z \in U$, $z \neq 0$.

Set $\varphi(u) := \beta + \psi u + \varepsilon u^2$ and $\gamma(u) := \frac{\delta}{u}$ and it is easy to show that φ is analytic in $\mathbb{C}$, γ is analytic in $\mathbb{C}\backslash\{0\}$ with $\gamma(u) \neq 0$, $u \in \mathbb{C}\backslash\{0\}$.

We can write $\frac{\varphi'(g(z))}{\gamma(g(z))} = \frac{g'(z)[\psi+2\varepsilon g(z)]g(z)}{\delta}$, and $Re\left(\frac{\varphi'(g(z))}{\gamma(g(z))}\right) = Re\left(\frac{\psi}{\delta}g(z) + \frac{2\varepsilon}{\delta}(g(z))^2\right) > 0$, for $\varepsilon, \psi, \delta \in \mathbb{C}$, $\delta \neq 0$.

We obtain

$$F_{g(U)}\left(\beta + \psi g(z) + \varepsilon(g(z))^2 + \delta\frac{zg'(z)}{g(z)}\right) \leq F_{p(U)}\left(\beta + \psi p(z) + \varepsilon(p(z))^2 + \delta\frac{zp'(z)}{p(z)}\right).$$

Applying Lemma 2, we obtain

$$F_{g(U)}g(z) \leq F_{D_z^{-\lambda}\phi(U)}\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha},\quad z \in U,$$

and the fuzzy best subordinate is g. □

Example 2. *Taking the same functions as in Example 1, $\phi(3,2,q,z) = 1 + \frac{2}{1-q}z + \frac{2(1-3q)}{1-2q}z^2$ for $m = 3$, $n = 2$,$z \in U$, with $D_z^{-\lambda}\phi(3,2,q,z) = \frac{z^{\lambda-1}}{\Gamma(\lambda)}$, $z \in U$, and $g(z) = \frac{1-z}{1+z}$, $z \in U$ with $g'(z) = \frac{-2}{(1+z)^2}$ and $g''(z) = \frac{4}{(1+z)^3}$, $z \in U$.*

Choosing $\alpha = 1$, $\beta = 1$, $\delta = 1$, $\psi = 1$, $\varepsilon = 1$, we obtain $\Psi_{\lambda}^{3,2,q}(1,1,1,1,1;z) = \Psi_{\lambda}^{3,2,q}(1,1,1,1,1;z) = \lambda - 1 + \frac{z^{\lambda-2}}{\Gamma(\lambda)} + \left(\frac{z^{\lambda-2}}{\Gamma(\lambda)}\right)^2$ and $Re\left(\frac{2\varepsilon}{\delta}(g(z))^2 + \frac{\psi}{\delta}g(z)\right) = Re\left(2\left(\frac{1-z}{1+z}\right)^2 + \frac{1-z}{1+z}\right) = Re\frac{z^2-4z+3}{(1+z)^2}$.

We have also that $\beta + \psi g(z) + \varepsilon(g(z))^2 + \delta\frac{zg'(z)}{g(z)} = 1 + \frac{1-z}{1+z} + \left(\frac{1-z}{1+z}\right)^2 - \frac{2z}{1-z^2} = \frac{3-5z-z^2-z^3}{(1+z)^2(1-z)}$.

Using Theorem 2, when $Re\frac{z^2-4z+3}{(1+z)^2} > 0$ and $\frac{zg'(z)}{g(z)} = \frac{-2z}{1-z^2}$ is starlike univalent in U, we obtain $F_U\left(\frac{3-5z-z^2-z^3}{(1+z)^2(1-z)}\right) \leq F_U\left(\lambda - 1 + \frac{z^{\lambda-2}}{\Gamma(\lambda)} + \left(\frac{z^{\lambda-2}}{\Gamma(\lambda)}\right)^2\right)$, $z \in U$, induce $F_U\left(\frac{1-z}{1+z}\right) \leq F_U\left(\frac{z^{\lambda-2}}{\Gamma(\lambda)}\right)$.

Corollary 3. *For m, n complex numbers with $n \neq 0, -1, -2, \ldots$ and $\lambda, \alpha > 0$, $0 < q < 1$, suppose that relation (9) holds. When $\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha} \in \mathcal{H}[0,(\lambda-1)\alpha] \cap Q$ and*

$$F_{g(U)}\left(\beta + \psi\frac{Mz+1}{Nz+1} + \varepsilon\left(\frac{Mz+1}{Nz+1}\right)^2 + \delta\frac{(M-N)z}{(Mz+1)(Nz+1)}\right) \leq F_{\Psi_\lambda^{m,n,q}(U)}\Psi_\lambda^{m,n,q}(\alpha,\beta,\psi,\varepsilon,\delta;z),$$

where $\beta, \psi, \varepsilon, \delta \in \mathbb{C}$, $\delta \neq 0$, $-1 \leq N < M \leq 1$, and $\Psi_\lambda^{m,n,q}$ is defined by relation (6), then

$$F_{g(U)}\left(\frac{Mz+1}{Nz+1}\right) \leq F_{D_z^{-\lambda}\phi(U)}\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha}, \quad z \in U,$$

and the fuzzy best subordinate is $\frac{Mz+1}{Nz+1}$.

Proof. The corollary yields for $g(z) = \frac{Mz+1}{Nz+1}$, $-1 \leq N < M \leq 1$, in Theorem 2. □

Corollary 4. *For m, n complex numbers with $n \neq 0, -1, -2, \ldots$; $\lambda, \alpha > 0$, $0 < q < 1$, suppose that relation (9) holds. When $\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha} \in \mathcal{H}[0,(\lambda-1)\alpha] \cap Q$ and*

$$F_{g(U)}\left(\beta + \psi\left(\frac{z+1}{1-z}\right)^{\kappa} + \varepsilon\left(\frac{z+1}{1-z}\right)^{2\kappa} + \delta\frac{2\kappa z}{1-z^2}\right) \leq F_{\Psi_\lambda^{m,n,q}(U)}\Psi_\lambda^{m,n,q}(\alpha,\beta,\psi,\varepsilon,\delta;z),$$

where $\beta, \psi, \varepsilon, \delta \in \mathbb{C} \in \mathbb{C}$, $0 < \kappa \leq 1$, $\delta \neq 0$, and $\Psi_\lambda^{m,n,q}$ is defined by relation (6), then

$$F_{g(U)}\left(\frac{z+1}{1-z}\right)^{\kappa} \leq F_{D_z^{-\lambda}\phi(U)}\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha}, \quad z \in U,$$

and the fuzzy best subordinate is $\left(\frac{z+1}{1-z}\right)^{\kappa}$.

Proof. Corollary yields by using Theorem 2 when $g(z) = \left(\frac{z+1}{1-z}\right)^{\kappa}$, $0 < \kappa \leq 1$. □

Combining the results obtained in Theorems 1 and 2, the following sandwich-type result can be established.

Theorem 3. *Consider g_1, g_2 analytic and univalent functions in U with the properties $g_1(z) \neq 0$, $g_2(z) \neq 0$, for all $z \in U$, and $\frac{zg_1'(z)}{g_1(z)}$, $\frac{zg_2'(z)}{g_2(z)}$ are starlike univalent functions. Assume that g_1 satisfies relation (5) and g_2 satisfies relation (9). For m, n complex numbers with $n \neq 0, -1, -2, \ldots$ and $\lambda, \alpha > 0$, $0 < q < 1$, when $\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha} \in \mathcal{H}[0,(\lambda-1)\alpha] \cap Q$ and $\Psi_\lambda^{m,n,q}(\alpha,\beta,\psi,\varepsilon,\delta;z)$ defined by (6) is univalent in U, then*

$$F_{g_1(U)}\left(\beta + \psi g_1(z) + \varepsilon(g_1(z))^2 + \delta\frac{zg_1'(z)}{g_1(z)}\right) \leq F_{\Psi_\lambda^{m,n,q}(U)}\Psi_\lambda^{m,n,q}(\alpha,\beta,\psi,\varepsilon,\delta;z)$$

$$\leq F_{g_2(U)}\left(\beta + \psi g_2(z) + \varepsilon(g_2(z))^2 + \delta\frac{zg_2'(z)}{g_2(z)}\right),$$

for $\beta, \psi, \varepsilon, \delta \in \mathbb{C}$, $\delta \neq 0$, implies

$$F_{g_1(U)g_1(z)} \leq F_{D_z^{-\lambda}\phi(U)}\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha} \leq F_{g_2(U)g_2(z)}, \quad z \in U,$$

and the fuzzy best subordinate is g_1 and the fuzzy best dominant is g_2.

Considering $g_1(z) = \frac{M_1z+1}{N_1z+1}$ and $g_2(z) = \frac{M_2z+1}{N_2z+1}$, with $-1 \leq N_2 < N_1 < M_1 < M_2 \leq 1$, the following corollary yields.

Corollary 5. *For m, n complex numbers with $n \neq 0, -1, -2, \ldots; \lambda, \alpha > 0$, $0 < q < 1$, suppose that relations (5) and (9) hold. When $\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha} \in \mathcal{H}[0,(\lambda-1)\alpha] \cap Q$ and*

$$F_{g_1(U)}\left(\beta + \psi\frac{M_1z+1}{N_1z+1} + \varepsilon\left(\frac{M_1z+1}{N_1z+1}\right)^2 + \delta\frac{(M_1-N_1)z}{(M_1z+1)(N_1z+1)}\right)$$

$$\leq F_{\Psi_\lambda^{m,n,q}(U)}\Psi_\lambda^{m,n,q}(\alpha,\beta,\psi,\varepsilon,\delta;z) \leq$$

$$F_{g_2(U)}\left(\beta + \psi\frac{M_2z+1}{N_2z+1} + \varepsilon\left(\frac{M_2z+1}{N_2z+1}\right)^2 + \delta\frac{(M_2-N_2)z}{(M_2z+1)(N_2z+1)}\right),$$

with $\beta, \psi, \varepsilon, \delta \in \mathbb{C}$, $\delta \neq 0$, $-1 \leq N_2 \leq N_1 < M_1 \leq M_2 \leq 1$, and $\Psi_\lambda^{m,n,q}$ defined by relation (6), then

$$F_{g_1(U)}\left(\frac{M_1z+1}{N_1z+1}\right) \leq F_{D_z^{-\lambda}\phi(U)}\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha} \leq F_{g_2(U)}\left(\frac{M_2z+1}{N_2z+1}\right),$$

the fuzzy best subordinate is $\frac{M_1z+1}{N_1z+1}$ and the fuzzy best dominant is $\frac{M_2z+1}{N_2z+1}$, respectively.

Considering $g_1(z) = \left(\frac{z+1}{1-z}\right)^{\kappa_1}$ and $g_2(z) = \left(\frac{z+1}{1-z}\right)^{\kappa_2}$, with $0 < \kappa_1, \kappa_2 \leq 1$, the following corollary yields.

Corollary 6. *For m, n complex numbers with $n \neq 0, -1, -2, \ldots$ and $\lambda, \alpha > 0$, $0 < q < 1$, suppose that relations (5) and (9) hold. When $\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha} \in \mathcal{H}[0,(\lambda-1)\alpha] \cap Q$ and*

$$F_{g_1(U)}\left(\beta + \psi\left(\frac{z+1}{1-z}\right)^{\kappa_1} + \varepsilon\left(\frac{z+1}{1-z}\right)^{2\kappa_1} + \delta\frac{2\kappa_1 z}{1-z^2}\right) \leq F_{\Psi_\lambda^{m,n,q}(U)}\Psi_\lambda^{m,n,q}(\alpha,\beta,\psi,\varepsilon,\delta;z)$$

$$\leq F_{g_2(U)}\left(\beta + \psi\left(\frac{z+1}{1-z}\right)^{\kappa_2} + \varepsilon\left(\frac{z+1}{1-z}\right)^{2\kappa_2} + \delta\frac{2\kappa_2 z}{1-z^2}\right),$$

with $\beta, \psi, \varepsilon, \delta \in \mathbb{C}$, $0 < \kappa_1, \kappa_2 \leq 1$, $\delta \neq 0$, and $\Psi_\lambda^{m,n,q}$ defined by relation (6), then

$$F_{g_1(U)}\left(\frac{z+1}{1-z}\right)^{\kappa_1} \leq F_{D_z^{-\lambda}\phi(U)}\left(\frac{D_z^{-\lambda}\phi(m,n;q,z)}{z}\right)^{\alpha} \leq F_{g_2(U)}\left(\frac{z+1}{1-z}\right)^{\kappa_2},$$

the fuzzy best subordinate is $\left(\frac{z+1}{1-z}\right)^{\kappa_1}$ and the fuzzy best dominant is $\left(\frac{z+1}{1-z}\right)^{\kappa_2}$, respectively.

5. Discussion

Using the previously defined operator given by (3), a new fuzzy differential subordination is studied in the first theorem proved and the best fuzzy dominant is given. Using two particular functions well-known in the geometric function theory due to their geometric properties as fuzzy best dominant, two corollaries are stated in connection to the first theorem. In the second theorem, a new fuzzy differential superordination is considered for which the fuzzy best subordinate is established. Using this result and particular functions used in the fuzzy subordination case, two corollaries derive from Theorem 2. Combining the results presented in Theorems 1 and 2, a sandwich theorem is given in Theorem 3 and two corollaries follow naturally by combining the previously obtained corollaries of Theorems 1 and 2.

6. Conclusions

As part of future work, the theorems proved for the Riemann–Liouville fractional integral of the q-hypergeometric function suggest that this operator could be used for introducing fuzzy classes, which could be investigated regarding different aspects, such as starlike and convexity geometrical properties, or for obtaining coefficient estimates. Using as inspiration the results obtained here, Riemann–Liouville fractional integral can be applied to other q-calculus functions and its $(p;q)$-variation. Moreover, considering the numerous applications in real life of the fuzzy sets theory and fractional calculus, hopefully, some interdisciplinary results could be obtained in the future involving the operator and the fuzzy best dominant and fuzzy best subordinate provided by this study.

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