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Mathematics Teachers' Professional Competence Component Model and Practices in Teaching the Linear Functional Concept—An Experimental Study

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Abstract: Teachers' professional development is now widely regarded as a key determinant of school effectiveness and student achievement. However, Mathematics teachers in Vietnamese secondary schools face numerous challenges in their teaching practices, such as a lack of recourse assistance, a lack of innovational teaching policy, and difficulty in using real-life examples in teaching. This study aims to identify Mathematics teachers' professional competence components and organize a workshop to support Vietnamese teachers to enhance professional development through experiment research. A total of 40 postgraduate students on the master's course of Mathematics Education at Saigon University are involved in this study. To reach the study objectives, we conducted three research cycles of: (1) Reviews to identify the model of Mathematics teachers' professional competence for suitability in the Vietnamese context; (2) Developing materials and organizing the workshop on designing lessons that teach linear functional concepts in Grade 8 (13-year-old students); (3) Feedback analysis through a qualitative study with a group focus interview of 40 workshop participants. Data collection was recorded, and typed transcripts and theme development were employed in data analysis. The findings outline four criteria that serve as the cornerstone of our concept of professional competences for Mathematics teachers: I. Mathematical Knowledge; II. Interpretation of the Official Mathematics Curriculum's Intentions; III. Understanding of Students' Thoughts; and IV. Design of Teaching. This model was applied in materials design for the workshop and was highly appreciated and got a high level of satisfaction from participants. These findings are expected to bring benefits for stakeholders who are Mathematics teachers in schools or involved in Mathematics teacher education in higher education.

Keywords: mathematics teacher; professional competence model; linear functional concept

MSC: 97B50; 97C70; 97H10; 97I20; 97B10

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Citation: Nam, P.S.; Tuong, H.A.; Weinhandl, R.; Lavicza, Z. Mathematics Teachers' Professional Competence Component Model and Practices in Teaching the Linear Functional Concept—An Experimental Study. *Mathematics* **2022**, *10*, 4007. https://doi.org/10.3390/math10214007

Academic Editor: Jay Jahangiri

Received: 8 September 2022
Accepted: 24 October 2022
Published: 28 October 2022

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1. Introduction

Teachers' professional development is a key determinant of school effectiveness, as evidenced by empirical findings demonstrating links between teachers' professional knowledge and student achievement [1] (Schwarz & Kaiser, 2019). Competence-based approaches in professional teacher development include declarations about what teaching and learning should achieve, as well as guidelines for curriculum design, pedagogy, and assessment criteria [2–5] (Fauskanger, 2015; Fennema & Franke, 1992; Fullan, 2010; Floden et al., 1995).

The professional competencies of Mathematics teachers have sparked a slew of national and international research investigating the skills required to teach various courses at various levels of education [6–8] (Krauss et al., 2020; Katz and Raths, 1985; Wuttke & Seifried, 2017). This trend has increased the importance of empirical educational research, particularly research used to build courses to improve professional teacher competencies in schools and teacher education [9] (Zakharov et al., 2014).

In Vietnam, the main object of the Mathematics Education Program is to encourage students to develop problem-solving abilities with interdisciplinary integration between Mathematics and other subjects, such as Natural Science, Physics, Chemistry, Biology, Technology, and Informatics; Mathematics teachers have been requested to provide opportunities for students to experience and use mathematics in practice [10] (Mathematics General Education Program 2018, p. 6). Teachers have always been required in professional development, to have the ability to grasp the program's goals, understand students' thinking, and know how to plan their lessons in the direction of enhancing learners' competencies and abilities in practice. In this context, the Vietnamese Ministry of Education and Training issued a circular on professional standards for teachers, which also sets professional standards and criteria that teachers must achieve.

However, Mathematics teachers in Vietnamese secondary schools faced numerous challenges in their teaching practices, such as a weak understanding of basic mathematics knowledge and pedagogy in theories and practices, and a lack of experience in lesson design and teaching mathematics for student learning outcomes that are linked to competence development as the program goal. Vietnamese students have a reputation for being smart when it comes to the practice of mathematics exercises on paper and discussion in class but lack practical skills and flexibility when it comes to problem-solving, especially in real-world contexts [11] (Nguyễn T. Thủy, 2019). This could be due to the traditional pedagogy used in Vietnamese schools in which the majority of teachers simply follow textbook content and give lectures to students. In such learning settings, students were supposed to listen and there were few opportunities for students to actively participate in lessons or to discuss aspects of content taught in classrooms. Furthermore, the eastern educational culture impacts practices of learning evaluation in Vietnam that place too much emphasis on exam outcomes, while ignoring the process of knowledge formation, which leads to the form of test-oriented teaching and learning. Students were urged to recall content and prepare for practice, alongside a lack of providing expertise opportunities in exploring and constructively applying new knowledge and using it in real life.

Enhancing professional skills for Mathematics teachers is one of the important solutions for reducing the issues above in teaching and learning [12,13] (Schoenfeld, 2011; Silverman & Thompson, 2008). However, few formal programs provide suitable training for aspiring Mathematics instructors in professional competence development, let alone schoolteachers who were overworked in their teaching practice and faced numerous challenges. Within a professional development program for Mathematics teachers, our study explores the process of becoming a Mathematics teacher trainer. We examine the development of Mathematics teacher educators more precisely through their practices. This study aims to identify the factors involved in assessing Mathematics teachers' professional competencies and how these factors can be used in designing professional development workshops for teachers. The topic selected for teachers' practice in the workshop is "The linear functions", which is covered in Grade 8 in the Vietnamese Mathematics education program. This study starts with reviews of Mathematics teachers' professional competency models for conducting the workshop and explores the Vietnamese Mathematics teachers' experiences to explain the components of Mathematics teachers' professional competencies.

This study's objective is to bring benefits to stakeholders who are involved in developing and organizing the workshop for training Mathematics teachers by improving Mathematics teachers' professional competencies from theory to practice.

2. Literature Review

To identify factors of Vietnamese Mathematics teachers' professional competencies, we review the current body of knowledge concerning existing models of Mathematics teachers' professional competencies in literature reviews using two reputational databases, namely Scopus and Web of Sciences, and relevant documents in the Vietnamese language.

2.1. Models of Mathematics Teachers' Professional Competency

The author of [14] is the first scholar to identify general pedagogical content knowledge for expertizing teachers' understanding and practice, in which different content knowledge, a variety of pedagogical skills, including general pedagogical knowledge, knowledge of learner characteristics, knowledge of the teaching environment, knowledge of educational aims and values, and subject-specific content knowledge, are combined. In 1992, Fennema and Franke [3] pointed out the weakness of Shulman's approach [14] when not crucial for architectural quality and developed a model of teachers' knowledge development in context with an emphasis on teachers' beliefs (Figure 1). They suggest a teacher knowledge model that can be used to characterize what teachers require in the classroom, which consists of four components: knowledge of mathematics; pedagogical knowledge; knowledge of learners' cognition in mathematics; and the central component of context-specific knowledge [3,15] (Fennema and Franke, 1992). Ball et al. (2008) classified several sub-components from subject knowledge and pedagogy content knowledge (Figure 2). It left it open as to whether it falls under the knowledge of content and teaching, constitutes a new domain, or spans many domains. Teachers need to understand how mathematical themes are related to the Mathematics curriculum in teaching.

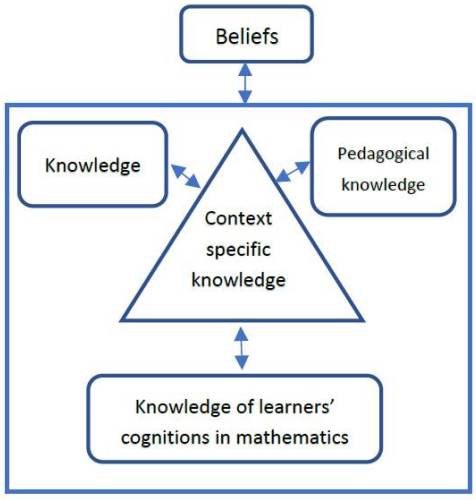


Figure 1. Fennema and Franke' Model (1992). Teachers' knowledge development in context. Source: Fennema and Franke, 1992.

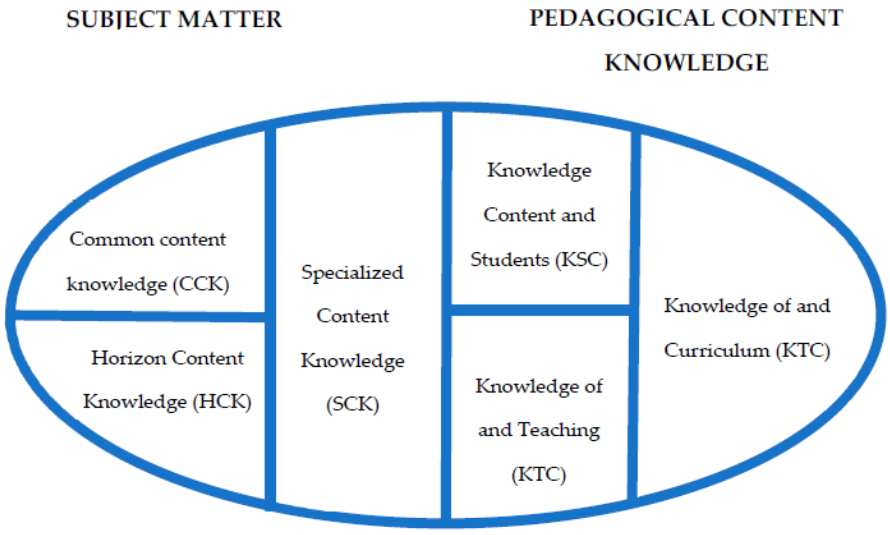


Figure 2. Ball et al. (2008)—Model. Domains of mathematical knowledge for teaching. Source: Ball et al., 2008.

Emphasizing the interest and motivation of learners, [16] Baumert and Kunter (2013) developed a model of teacher professional competence, including belief, motivational orientation, self-regulation, and professional knowledge. The domains of knowledge require the teacher's expertise in content, pedagogical content, pedagogical psychology, organizational knowledge, and counseling knowledge (Figure 3). [16] Baumert and Kunter (2013) indicated multi-facets of knowledge teachers need to be equipped with and apply in teaching effectively.

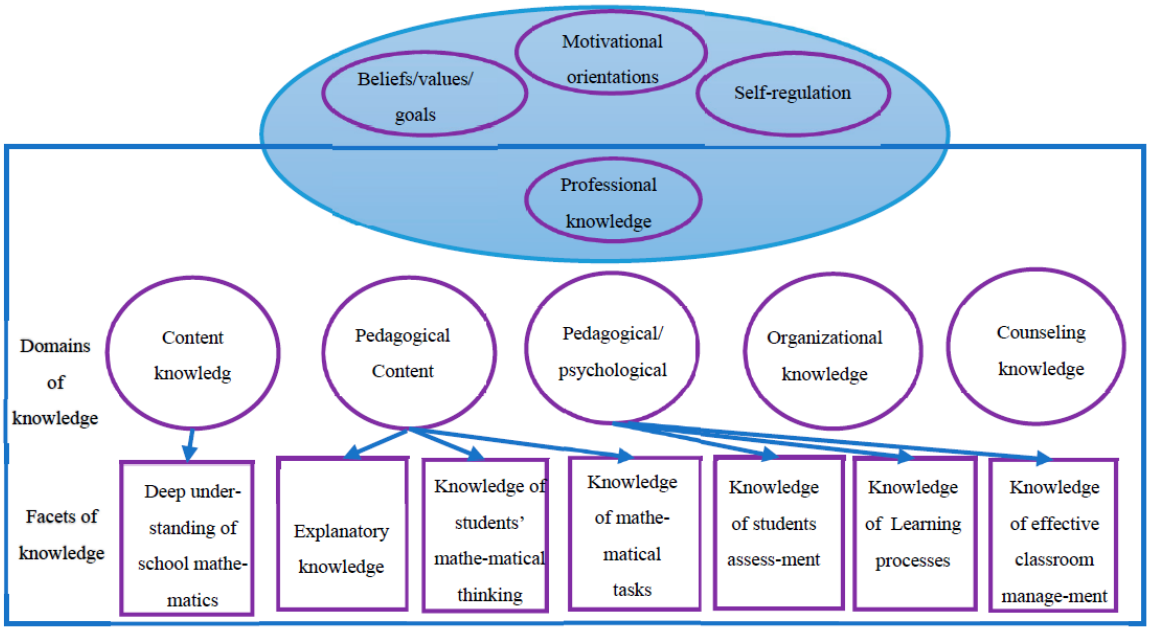


Figure 3. The COACTIV model of professional competence, with the aspect of professional knowledge specified for the context of teaching (Baumert and Kunter 2013, p. 29).

The author of [17] Döhrmann et al. (2012) developed the model of teacher competence based on synthetic multi-elements of previous studies and emphasized two main factors: cognitive abilities and affective-motivational characteristics (Figure 4).

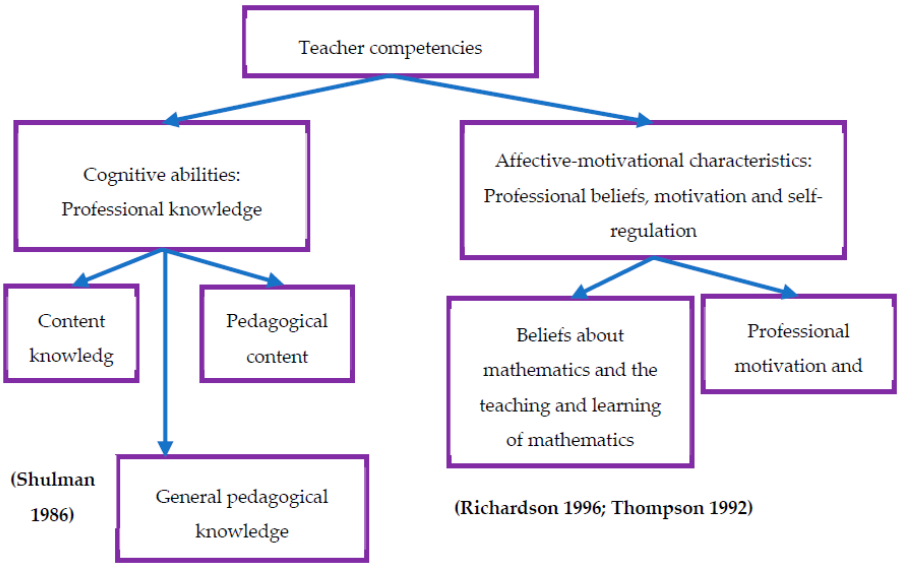


Figure 4. Conceptual model of teachers' professional competencies. Source: Döhrmann et al., 2012.

The authors of [18] Blömeke et al. (2015) developed a model of teacher competency as a continuum that combined cognitive and contextual approaches (Figure 5). Performance is seen as a functionally related construct to competence, whereas competence is viewed as a multi-dimensional latent construct that manifests in teaching performance and contains all mental resources required to perform [19] (Klieme et al., 2008). Several empirical studies, particularly those by [20,21] Bruckmaier et al. (2016) and Knievel et al. (2016), can be linked to this perspective of viewing teacher knowledge and skills as integrated aspects of teacher competence underlying classroom performance in terms of instructional quality, which is hypothesized to affect student achievement, despite significant differences in construct conceptualization, labeling, and operationalization.

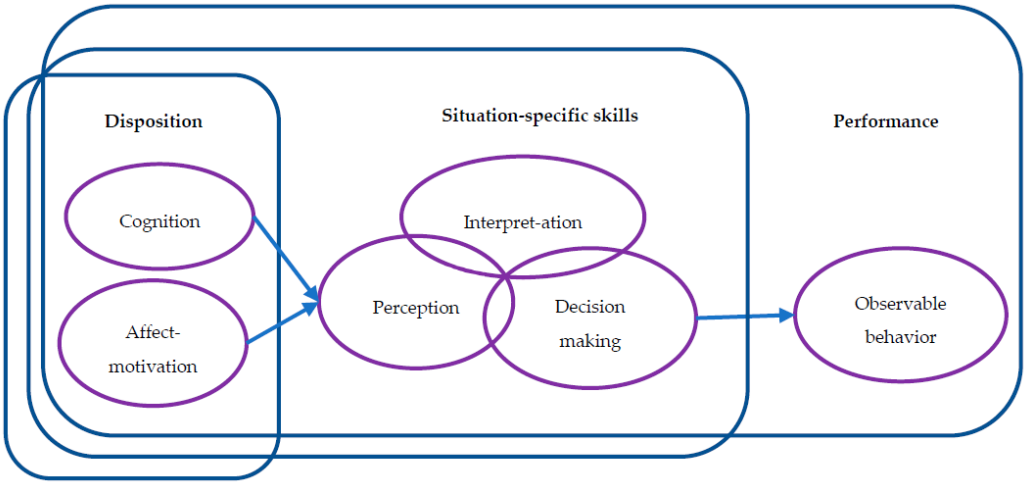


Figure 5. Modelling competence as a continuum. Source: Blömeke et al., 2015.

The authors of [22] Zhang Qinqiong and Max Stephens (2016) designed a model of Mathematics teachers' professional competence, and this model has been popularly employed in several nations due to its effectiveness in teaching and learning, as well as its suitability for Mathematics subjects. This model focused on the ability of the Mathematics teacher to plan lessons, and it found a strong correlation between three competence components: the teacher's understanding of mathematics, their interpretation of the official Mathematics curriculum, and their comprehension of students' mathematical thinking (Figure 6).

The author of [23] Koehler and Mishra (2009) developed the teacher knowledge model of technological pedagogical content knowledge, namely TPACK (technology, pedagogy, and content knowledge). This model emphasized pedagogical content knowledge (PCK) to include technology knowledge, which is a very important and indispensable skill in the teaching competencies structure of teachers in the IR4.0 period (Figure 7). The sorts of flexible knowledge required to successfully incorporate technology use into education are produced through the interaction of topic, pedagogy, and technology, both theoretically and practically.

In teaching Mathematics, teachers usually use technology in teaching students several fields or subjects, such as calculating, or software for drawing shapes in Geometry.

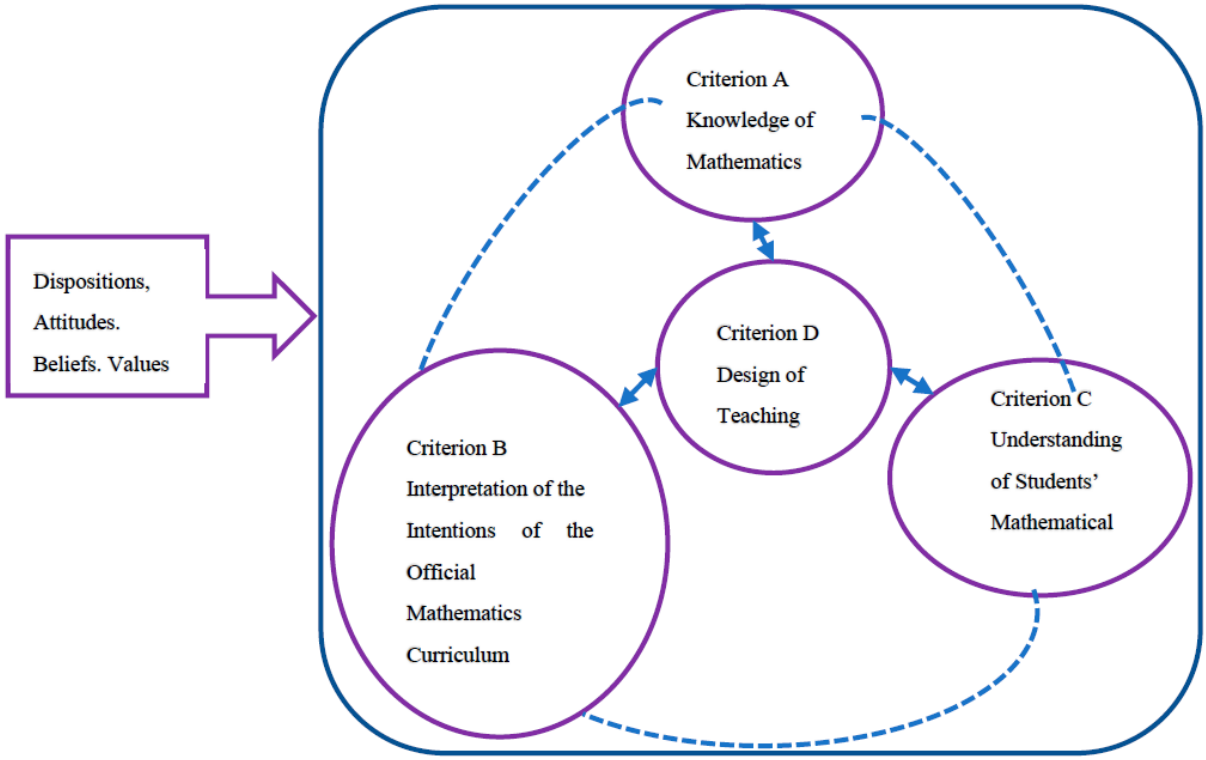


Figure 6. Model of teacher capacity. Source: Zhang and Stephens (2016).

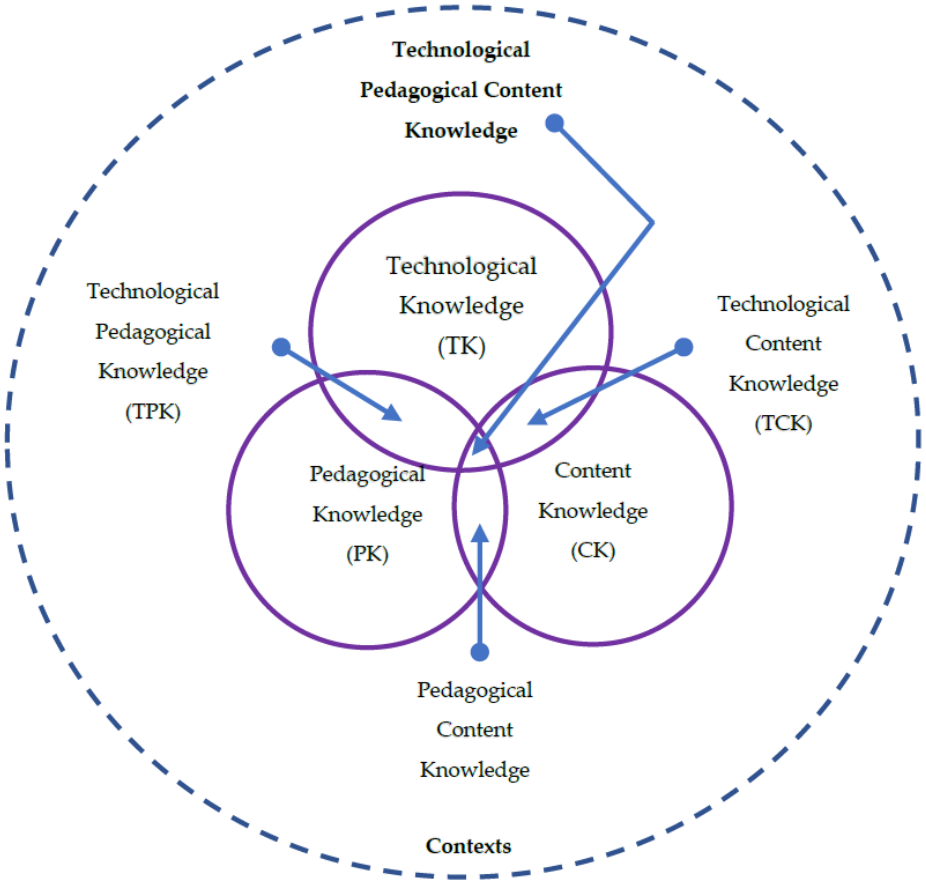


Figure 7. Model of TPACK. Source: Koehler & Mishra (2009).

Several studies in constructing models in reviews have focused on teacher professional competency (or unique subjects, such as mathematics). Every model appears to be a multi-dimensional construct that is useful for a successful teaching career. These general elements included a wide range of beliefs, topic knowledge, cognitive abilities, pedagogical abilities in teaching practice, knowledge transition and class management abilities, motivation, and engagement of students. These elements can be classified into two main groups: professional knowledge of basic mathematics, and pedagogy and professional affective motivation in teaching and learning practice. The first competency component is usually equipped and developed for teacher students during their studying in schools and teacher education courses; however, the second competency component improves during their teaching and learning application in the classroom. Therefore, when designing the professional competency program for Mathematics teachers who are teaching in schools, educators should be more concerned with professional affective motivation. Depending on the national education system, the Mathematics curriculum is set in different forms in each country, as well as the orientation of how students learn and assessment through multiple kinds of tests to ensure students achieve the syllabus or curriculum's objectives. That is the reason why the Mathematics curriculum has become the backbone for school managers and teachers to promote teacher professional competency development to enhance student learning performance. Based on the Mathematics curriculum, teachers revise mathematical knowledge, design lessons with learning activities that suit students as mentioned in the curriculum objectives and prepare the exercises or problem-solving tasks for students using mathematical knowledge they have learned in different situations to consolidate knowledge and skills. In teaching practice, teachers' motivating and engaging students in readiness to study plays an important role in the student learning process, occurring smoothly and effectively. Teachers use the curriculum content to identify the related knowledge fields that students already know and to predict students' difficulties when they learn the new knowledge in order to motivate or engage students effectively. To sum up, curriculum content is the nuclear content, which is the guidelines for teachers' lesson design to achieve the program outcomes, not only regarding knowledge or cognitive outcomes but also thinking skills, attitudes, and willingness of learning for development [22]. Based on this, the lesson design focuses on both lesson goals and program or curriculum content and outcomes that help teachers in schools improve their professional competencies.

2.2. Models Describing Mathematics Teacher Professional Competences in Vietnam

Despite existing models of Mathematics teacher professional competencies in the reviews, however, not all Vietnamese teachers could reach them for several reasons, such as lack of available materials in Vietnamese school libraries, language barriers, lower numbers of studies involving these topics, etc. Vietnamese teachers mainly use the professional teacher competences models which were established and guidelines implemented in Vietnam. These are summarized as below.

2.2.1. The Teacher Professional Competence Model

The model of professional teacher competencies has been introduced and used popularly in many courses of training and fostering for Vietnamese teachers [11] (Nguyen T. Thuy, 2019). This model is based on several German studies and has been used in professional teacher development courses in Vietnam for the last two decades [24] (Jakupec, Meier & Nguyen V. Cuong, 2006). This model constructs four capacity parts: (a) professional competency, (b) methodical competency, (c) social competency, and (d) individual competency [24] (Nguyen V Cuong, 2006). The central competence is created from the synthesis of component competencies and action competence plays the most important role in professional competencies formed only when associated with specific actions (Figure 8).

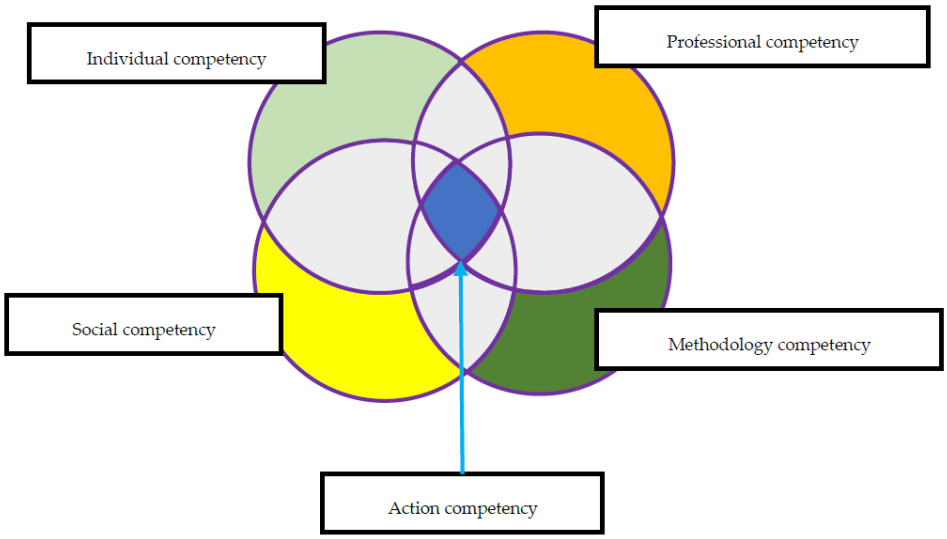


Figure 8. Model of teacher professional competence development. Source: SourJakupec Victor, Meier Bernd, Nguyen V. Cuong (2006).

To employ this model to improve Mathematics teachers' professional competence, teachers required discipline and practice of mathematics knowledge, pedagogy, communication and interaction with students, and personality development. However, this model generally applies to any subject that lacks the detailed components for mathematical teaching practice effectively [25,26] (Pham S. Nam & Stephens, 2013; Pham S. Nam, Stephens, 2014). When teachers apply this model in teaching, they need to add in the specific fields which connect with each subject's requirements in teaching and learning to ensure student learning outcomes reach the curriculum goals.

2.2.2. The Ministry of Education and Training of Vietnam

The Ministry of Education and Training of Vietnam promulgate an official document on professional standards for teachers [27]. Criterion 1 is on the characteristics of teachers; criterion 2 is on professional development; the profession defines the requirements: professional mastery and professional proficiency; regularly update and improve professional and professional capacity to meet educational innovation requirements with the following criteria: Criterion 3. Personal professional development. Criterion 4. Create lesson ideas and educational strategies that help students become better and more capable people. Criterion 5. Use instructional and teaching strategies to help students develop their skills and talents. Criterion 6. Examination and assessment develop students' qualities and abilities. Criterion 7. Student counseling and support.

Although the list of criteria regarding professional teacher competencies was sent to teachers in schools to enhance awareness of and improve teaching skills, different subjects have different specific pedagogies to teach students to reach learning goals. Moreover, the competencies model is often not a single skill, but a cohesive and integrated set of skills in both content understanding and soft skills, both in the whole curriculum and each lesson.

The Circular has stated the *pass*, *good*, and *excellent* levels for each criterion. As a result, the Ministry of Education and Training has emphasized professional growth, while defining standards and criteria for teachers' professional standards. This norm applies to all topics. Furthermore, the Circular only specifies one standard and addresses relatively general topics, but it is essential to have more detailed standards when teaching Mathematics.

Criterion 5, "Use teaching and educational methods to develop students' qualities and abilities", and Criterion 6, "Examination and assessment develop students' qualities and abilities", are both applicable. Students' competencies, on the other hand, must focus on the aspects that have not been demonstrated to create attributes. However, when counseling

and support are needed to advise and support students to acquire knowledge, it is not demonstrated in Criterion 7, "Counseling and supporting students".

In Vietnam, the system of criteria as mentioned above reflect the general teacher's professional competence. This model has a positive impact on teacher training and teacher assessment. However, when organizing a workshop on the strengthening of Mathematics teachers' professional competence, if the trainers only simply apply for the general programs and follow the system of criteria, trainees are usually dissatisfied and the workshops fail to reach the trainees' needs [25] (Pham S. Nam, Max S. 2014). Teachers who are teaching in schools need a deep discussion of pedagogy in theories connected with the curriculum and lessons they feel that their students find hard to understand and practice when they teach. This issue has raised questions for educators who organize workshops regarding enhancing the professional competency of Vietnamese teachers in finding effective solutions and methods to promote teachers and strengthen their ability in their career development. On the other hand, this collection of criteria does not clearly define categories of knowledge or cover the key parts that a teacher needs to properly teach. It is, therefore, necessary to design a course or workshop for training and fostering Mathematics teachers in enhancing professional competence in Vietnam.

3. Research Methodology

To achieve the research objectives, we decided to use qualitative research approaches with a case study in which the techniques were supported by utilizing the basic principles of design-based research (DBR). The design-based research outline covered three phases, including those in Table 1.

Table 1. Three phases of the design-based research outline

Phase 1	Phase 2	Phase 3
To identify the criteria components when evaluating Mathematics teachers' professional competencies, the review methodology was discussed and employed by three experts to choose the model which suits mathematics education policy in the Vietnamese context and produce workshop materials for enhancing Mathematics teachers' professional competencies.	An experimental study was conducted with a workshop organized for enhancing Mathematics teachers' professional competence by using materials from phase 1. 40 postgraduate students on the Masters of Mathematics Pedagogy course 20.1, Saigon University, were involved in this study. They were Mathematics teachers in schools before studying master's courses at Saigon University. The workshop took place over four weeks (November 2021). The main goal of the workshop helped postgraduate students' awareness of the components of Mathematics teachers' professional competence and how to apply lesson design effectively, using a case study in the lesson titled "the linear function" for Grade 8 in the Vietnamese Mathematics Education program in Vietnam.	A qualitative study was conducted to explore the Mathematics teachers' feedback on the workshop. Group focus interviews were employed for data collection; four groups created of 40 postgraduate students participated in the workshop. The interview occurred for around one hour in groups, then each group reported both group opinions and individual ideas. These reports and interview recordings contribute to data collection and data analysis. The analysis aims to understand better competence growth through the practice of participating in the workshop, and, second, we aimed to explore the feedback of the theoretical model by applying it to professional development for Mathematics teachers in Vietnam.

The rationale for choosing the case study is based on clearly defined systems of actual persons who experienced particular interventions in actual scenarios, according to [28] Cohen et al. (2007). Participants in this study included 40 postgraduate Saigon University master's degree students who worked as Mathematics teachers in schools [28–30] (Anderson & Shattuck, 2012; Barab & Squire, 2004; Cohen et al., 2007). According to [31] Reinmann (2005), improving the professional competencies of Mathematics teachers in our case involved too many steps, including identifying the model's components, having participants practice lessons on "The Linear Functions" for Grade 8, and conducting a

qualitative study to examine the current problems and potential solutions. For this reason, we used DBR principles.

The complex procedure of this study is suitable for design-based research, supported by several previous studies, such as: (a) a study conducted on the exploration of knowledge regarding the practice that can inform practitioners; (b) a study often conducted in a single setting over a long time; and (c) a collaboration between practitioners and researchers [32,33] (Wang & Hannafin, 2005; Weinhandl et al., 2021).

The instrument used in data collection in this study was self-developed and checked by an expert review process and a pilot study with three participants. The interview session was conducted after the workshop was completed. The conversation was recorded and transcribed using the N-Vivo software for data management and analysis.

4. Findings

4.1. Develop Materials for The Workshops on Enhancing Professional Competence Development for Mathematics Teachers

As mentioned in the review, there are existing models regarding Mathematics teachers' professional competence development. Every model appears to be a multi-dimensional construct that can help Mathematics teachers succeed in career development. A wide range of beliefs, topic knowledge, cognitive abilities, pedagogical abilities in teaching practice, knowledge transition and class management abilities, motivation, involvement of students, and so on were among the general elements.

To the best of our knowledge, for workshop materials adapted to the Vietnamese Mathematics curriculum context, we self-developed documents that are specifically designed for two main factors: components of curriculum knowledge and lesson design abilities that are compatible with Vietnamese mathematics education policy and context.

The summaries of the model of Mathematics teachers' professional competencies include:

Criterion 1—The application of mathematical knowledge to activities that students have already done or are being given is the goal. The main mathematical concepts needed to teach a particular subject are intended to be captured by knowledge of mathematics.
Criterion 2—Interpretation of the Intentions of the Official Mathematics Curriculum is concerned with how teachers connect the requirements or suggestions in China's and Australia's official curriculum documents to the material that pupils are expected to acquire. In contrast to MKT [15] (Ball et al., 2008), this criterion places more importance on official curricular papers and teachers' willingness to use them when developing lesson plans.
Criterion 3—Understanding of Students' Mathematical Thinking, or Criterion 3, is specifically concerned with teachers' ability to interpret, differentiate, and foresee what students will likely do. It suggests that teachers are able to identify the typical mistakes that students make and the mathematical assumptions that led to these mistakes.
Criterion 4—As a result, the design of teaching clearly emphasizes the ability of teachers to create lessons that advance students' thinking and to react to particular examples of students' thinking in the context of official curriculum materials. The purpose of Criterion 4 is to place more emphasis on how educators use their professional expertise to develop relevant instruction on particular subjects.

4.2. Learning Product in the Workshop with the Linear Function Concept Lesson Design

The workshops are designed in two parts, theoretical and practical, and implemented by merging with each other. In the workshop, participants firstly warm up by discussing the general themes, such as what and how important the Mathematics teacher's professional competencies development is. Why do we need to know about the models of professional competencies? How can we enhance awareness of the application of models of professional competencies in teaching and learning practices? After reading the documents and listening to an overview of [22] Zhang and Stephens's (2016) theoretical model, participants present a summary of the models in their own words.

The practical part implemented in the design of "The Linear Function" lesson for Grade 8 is based on teamwork organization. A total of 40 participants were divided into four groups. The outcomes or learning products connect with the components of Mathematics teacher's professional competence. The results are summarized in the following Table 2:

Table 2. Learning products connected with criteria of Mathematics teachers' professional competence model.

ITEMS IN PROGRAM	LEARNING PRODUCTS
Criterion 1—Knowledge of Mathematics	
(1) Understand the role and position of the linear function concept in the program and understand the relationship of the linear function concept with other knowledge units.	Two groups stated, "The notion of a linear function appeared in ancient and medieval times as a tool to solve difficulties of measurement, motion, and astronomy", while one group stated, " linear function was explored after the end of ancient time, like a tool to address problems of measurement, motion, and astronomy". Mathematicians investigate things and occurrences in motion rather than stillness, in interconnectedness rather than isolation.
(2) Recognize the connection between the concept of linear functions and real-life situations and other subjects (if any).	The application of the "linear function" in actual life was recognized by all groups. Group 1 displays "a linear function that calculates the amount y must pay in terms of minutes x for a package phone charge...", while group 2 shows "a linear function that shows the sale and purchases, the relationship between the number of kilograms of oranges and the amount to be paid...". The third group depicts linear functions as uniform motions in Physics.
(3) Understand the meaning of terms in the concept of linear functions.	Regarding the word "linear", which is defined as the highest exponent of x equals one by all groups, "most" also means "one" in Vietnamese.
(4) Identify the differences between related knowledge	The groups demonstrated that the linear function is a particular case of the function; for the general function $y = f(x)$, y is a function of x, but x is not always a function of y; nonetheless, x is a function of y for linear functions.
Criterion 2—Understanding of formal curriculum goals	
(1) Problem solving should be used to measure understanding of linear functions in teaching and learning.	All four groups feel that it is important to deduce from a real model that leads to a linear function for students to understand the concept of linear functions, and that experience is a favorable condition for the construction of the concept.
(2) Assist students in grasping the notion of linear functions to understand and support curriculum goals.	The criterion "Recognize real-life models leading to the idea of function" was accepted by all groups as important for grasping the concept of a function. The program's prerequisites for "Applying linear functions to address various practical issues (for example, the problem of uniform motion in Physics)" provide the opportunity for students to observe the concept in action.
(3) Consider the concept of a linear function by connecting natural events in life.	The groups identified ways to use the concept of linear functions in everyday life, such as in purchases when prices are fixed, uniform motion in physics, tax calculation, and taxi fares...

Table 2. *Cont.*

ITEMS IN PROGRAM	LEARNING PRODUCTS
(4) Point out the concept of a linear function that is necessary for students' continued learning and future life.	The process of presenting knowledge related to linear functions in the current Mathematics program in high school is shown: first presenting the function $y = ax$, then the function $y = ax + b$, and the function given by many formulas related to linear functions, such as $y = \|x\|$, functions on each interval, construct a function t representing the amount to be paid when calculating electricity bills, postage deposit, taxi fare, tax, etc.
Criterion 3—Understanding students' thinking	
(1) Anticipate common student difficulties and misconceptions about the concept of linear functions.	Students may struggle to find the missing linear function b while identifying linear functions, or they may focus just on the highest exponent of x in the function expression as one and ignore the system condition. The value of x must be greater than zero, or it must be expressed as a polynomial rather than a fraction.
(2) Give students precise and logical reasons for their poor replies.	The groups discuss how to interpret the students' incorrect answers according to the following situations: If students believe that a function that lacks b, for example, $y = 6x$ is not a linear function, the teacher can ask "can it be written in the form $y = 6x + b$?" to realize that it is possible to write $y = 6x + 0$ and come to the conclusion that $y = 6x$ is a linear function. If the students think that $y = 0x + b$ is a linear function, the teacher asks the question "What is it equal to in this case?", "Does $a = 0$ satisfy the definition of a linear function?" If the students believe that the function $y = a\frac{1}{x} + b$ is a linear function, the teacher can ask the question "in the form of $y = ax + b$, a times x, in $y = a\frac{1}{x} + b$, is a multiplied by x?".
(3) Based on the students' replies, distinguish between the students' levels of understanding of the idea of linear functions, particularly the difference between the students' wrong answers.	Regarding the word "linear", which is defined as the highest exponent of x equals one by all groups, "most" also means "one" in Vietnamese.
(4) Recognize the importance of using situations, helping students understand situations that contain the concept of linear functions.	Building cognitive circumstances is essential for children to completely comprehend activities. As a result, these scenarios should concentrate on common mistakes made by students. Realistic models that lead to the concept of linear functions are vital in helping students understand the concept of linear functions and how to use it in real-life situations. Students connect the concept of linear functions to practice and explore how it may be applied to real-world challenges. Students may struggle to connect real-life scenarios to the arithmetic curriculum, or they may misunderstand real-life situations in real-life projects. Students, for example, have trouble determining how to solve problems that demand consideration of which of the two forms is more ideal. The teacher helped at the time by instructing students to "compute the entire cost of each form" and "compare which technique has more money".
Criterion 4—Instructional Lesson Design	
We propose the process of teaching concepts through four activities: Experiencing; Analyzing–discovering–Constructive new knowledge; Confirming; Application.	

Table 2. *Cont.*

ITEMS IN PROGRAM	LEARNING PRODUCTS
Experiencing	The key signals of topics are approached by students through visual depiction or hands-on experience. The teacher creates unique scenarios for pupils to experience the existence or effect of the object to be defined. This practice is designed to assist students in activating prior knowledge; the end product is a symbol of new information to be learned.
Analyze–Explore–Constructive new knowledge.	Students analyze and explore from there, generalizing to understand. Activity Analysis–Discovery–Drawing new knowledge is conducted through the results of experience, and students analyze and explore from there to generalize to understand.
Confirm (Consistency)	To help pupils confirm and consistently understand the concepts they have learned, consolidation activities are undertaken with two-component activities: identification and expression.
Application	Application activities are designed to allow students to apply concepts from mathematics to real-life situations.

The product of lessons designed as presented above was considered the creative result of this study, giving an example in a real class with a special context to support the development of Mathematics teachres' professional competence. This is evidence of lesson design following the structure of the Mathematics professional competence model. The findings emphasize that teachers should not start to prepare the activities in the classroom first, as many teachers do today. To improve Mathematics professional competence, first of all, teachers need to constantly understand the basic knowledge of mathematics lessons, understand the curriculum structure and outcome requirements, and be concerned with the ability and level of the students, as well as to what extent and how students learn in the class, and with these, teachers are ready to design lessons with detailed activities for student learning to achieve learning outcomes.

4.3. Explore the Feedback of Participants after Participating in the Workshop

Overview

Our combined expertise lies in experiment research for participants joining in active learning and applying in a lesson design practice, namely on "the linear function". We designed and facilitated a support program as a workshop for 40 postgraduate students at Saigon University. They were Mathematics teachers in schools and had experienced existing challenges and issues in teaching mathematics to secondary students. The workshop intends to maximize their understanding of the Mathematics teachers' professional competence model and to provide experience of applying the lesson design of the linear function for Grade 8 to self-improve professional competencies and contribute to innovation in teaching and learning mathematics in Vietnam.

After the workshop, the feedback from participants was collected and analyzed.

Theme 1: *What are the existing challenges or problems in the innovative teaching of mathematics in Vietnamese secondary schools?*

- ***1.1.*** *Transmissive instruction and exam-oriented teaching culture are popular in Vietnamese schools*

In Vietnam, traditional mathematics education was impacted by the "Confucianism concept", which regarded teachers as respected authorities and students usually requested repeated exercises as one of the most effective ways to solidify student knowledge and skills. Teachers used direct transmissive instruction and spent a lot of time presenting regularly regulated content in typical mathematical lessons. Students have less time for

the experience of discussing the situation in the real world, as most of the time is spent on exercise practice due to an exam-oriented teaching culture.

> *Following the existing instruction books in teaching mathematics, when teaching a new mathematics concept, teachers usually carry out the following steps: Conceptual guidance; concept formation; consolidate; application [34] (Nguyen Ba Kim, 2009). The author usually chooses an example in mathematical theory or formulates the step of "conceptual guidance" to introduce students to observing and being aware of the new knowledge. Then, most of the time is spent on exercise practice. [P 5]*

> *When families and society pay too much attention to student exam results, this culture impacts teaching orientation. Teachers organize mathematics lessons in class with much time spent on exercise practice and test preparation, and less time for students to construct new knowledge.*

- ***1.2.*** *Lack of instructional materials and facility support for mathematic teachers*

Although the Ministry of Education and school administrators demand that instructors engage in innovative teaching and learning based on a student-centered approach, instructors' awareness of innovative teaching to produce student learning outcomes that are adapted to new era that demands critical thinking and flexible problem-solving skills is hampered by a shortage of instructional materials or papers.

In Vietnam, innovative teaching and learning are still a part of the national policy. However, teachers frequently construct lessons on their own in the classroom. Most teachers reported significant difficulties in locating real-world situations for students' learning and a lack of assistance.

> *Before attending this class, I was hesitant to self-design a real-life situation for students to learn a new concept of Mathematics to ensure both knowledge and practice. There is a lack of book assistants who can do this task in-depth and with precision. In particular, when it comes to student learning in the local context, there is a lot of confusion about whether it is a typical sample or not.*

Teachers, on the other hand, claim that finding real-world circumstances for these two steps is more challenging than finding pure mathematics tasks.

Theme 2: *How the workshop can help you overcome the existing challenges or issues in teaching mathematics.*

- ***2.1.*** *Change from knowledge transmission to constructive knowledge in teaching and learning*

The workshop organized active learning activities for participants to understand the new way of teaching students, formatting new knowledge by the constructive method to help students develop their mathematical competencies; participants in the workshop were required to give their examples to distinguish between traditional teaching by transmission and innovative teaching by constructive knowledge.

> *I usually request teaching with the student-centered approach in innovational teaching and learning. However, when participating in this workshop, I realized the core of this matter is that teachers need to know the ways students learn for formatting knowledge, then we can design learning activities in the class.*

> *This workshop helps me open my mind to distinguish between traditional teaching and innovative teaching. Transmission knowledge will be used less in time and students' experience is a priority in the learning process.*

The workshop renews the main steps in the teaching process in the class to orient teachers to active teaching of mathematics. The modern ways of teaching mathematical concepts can be visualized as follows, starting with the process of teaching mathematics according to the approach of capacity development: Experience; Analyze–Discover–Draw New Knowledge; Consolidate; Application.

These processes are nearly identical to the existing design process. Experiential activities, for example, are encouraged to come from reality, from experiences with genuine contexts that students are familiar with, but they must be guided to the information that needs to be formed. We emphasize transferring it to real life in the application step, in addition to applying it in mathematics.

Experiential activities based on real-life situations benefit students by increasing access, stimulating learning, and through a reduction in anxiety or difficulties for the student when learning natural mathematical knowledge. Students can also understand the value of information in their daily lives thanks to practical application.

Theme 3: *How does the model of Mathematics teachers' professional competencies components impact participants' career development?*

The workshop organized participants into groups with the main task of teaching lesson design. During the workshops, participants requested deep discussion and that learning products follow the criteria of the teacher capacity model (Table 1). This model emphasizes the importance of understanding the curriculum knowledge and interpreting the curriculum objectives, proposing student learning outcomes that orient learning activities in lesson design. Successful lesson design guides the teacher's organization of the classroom effectively in teaching and learning.

> *This model expressed in the workshop was useful in the orientation of the pathways for us to discuss and learn. When seeing learning products from all groups, we can understand deeply the content of the lesson and how to organize teaching activities effectively.*
>
> *This model is related to our teaching culture and school managers' requirement in designing a lesson for teaching. The competence components were valuable and helped the teacher pay attention and understand deeply "Knowledge Mathematics" to find suitable real situations for students to experience and learn new knowledge.*
>
> *Before that, I rarely looked at the curriculum content and I automatically taught each lesson one by one. This model helps me to overview all Mathematics education curricula and understand and interpret the main goals of each lesson.*
>
> *Normally, when teaching in class, I can understand students' thinking. With this workshop, I self-improved my ability to predict how students think, what errors they may make, and design multiple options to adapt to diverse students.*
>
> *For example: For some arising situations, for example, if students think that the function is missing "b", such as $y = 6x$ is not a linear function, the teacher can ask "can it be written in the form $y = 6x + b$?" to realize that it is possible to write $y = 6x + 0$ and come to the conclusion that $y = 6x$ is a linear function.*
>
> *If the students think that $y = 0x + b$ is a linear function, the teacher asks the question "What is it equal to in this case?", "Does a = 0 satisfy the definition of a linear function?"*
>
> *If the students think that the function $y = a\frac{1}{x} + b$ is a linear function, the teacher can ask the question "in the form of $y = ax + b$, times x, in $y = a\frac{1}{x} + b$, is multiplied by x?".*
>
> *In another example, in Vietnamese, "most" also implies "one", as indicated by all groups as the highest exponent of x is one. Students were able to connect the language of ordinary life to the language of mathematics by explanation. This aids in the comprehension of abstract terms by students.*

Theme 4: *How does the Workshop Impact Participants in Emotional Career Development?*

When asked about your pleasure and excitement when participating in the "The Mathematics Teacher Professional Competence Development" workshop, the results received 100% "very excited" and "excited". All of them said they were happy with their achievement after joining this workshop. Some reasons for getting pleasure after the workshop are expressed here:

> *"The design focuses on real-life situations, creating excitement for students when learning, that I was so interested", "helping students see the meaning of the knowledge learned, reducing boredom in learning natural math", "design is a series of steps suitable for students' cognitive progress, which makes me feel confident in teaching" "help students easily form and deepen knowledge", "the criteria of understanding students' thinking are applied in the reinforcement phase, helping teachers to pay attention to students' thinking, predict students' error, etc.*
>
> *I am feeling more confident in designing lessons that connect with the Mathematics teachers' professional competence development. This achievement helps me gain more love for my teaching career and put more effort into enhancing the quality of teaching and learning of mathematics in secondary school.*
>
> *I think that this workshop not only helps us in lesson design toward innovational teaching methods for professional competence development but also helps us be confident in organizing and managing class activities for students to learn effectively.*
>
> *. . . When teachers understand deeply the mathematical knowledge of lesson content and find the real situation for students' experiences in exploring and constructing knowledge, they know about curriculum goals and requirements; in particular, we prepare some situations based on the prediction of students' thinking or errors. We can teach Mathematics in secondary school successfully and be satisfied with our contribution.*

However, a few devoted teachers acknowledged that this program presented some difficulties for them. For example, designing lesson activities takes a long time the first time; however, after teachers practice several times, they will be comfortable with the lesson design and its effectiveness when used in teaching, which will engage and motivate teachers who are self-directed in their career development and connect with the model of professional competence development.

Another difficulty is building experiential circumstances from reality, and applying them to reality takes a long time, and teachers encounter numerous challenges. Teachers have some difficulties predicting students' errors in developing appropriate reinforcement scenarios in reinforcement exercises.

5. Discussion

This experimental research provided opportunities for Mathematics teachers in Vietnam to participate in a workshop, active learning, and discussion in designing lessons on linear functions to help them exhibit their professional competency. The following competencies have been demonstrated: knowledge of mathematics; comprehension of formal curriculum goals; comprehension of students' thinking; and teaching lesson design.

The emerging theme in this study is that "curriculum knowledge" and curriculum objectives have become the crucial elements in Mathematics teachers' professional competence for teachers who are teaching in schools to improve their lesson design skills and apply them effectively in teaching and learning. This finding requests teachers design lessons that curriculum connect with the previous lessons in the curriculum, connect with real life, and prepare students' knowledge and skills to solve the problem in the next lesson. This professional competency component should be practiced in the workshops, where participants come from different schools with different experiences in teaching, to further identify difficulties or challenges for students when they learn this topic and find effective solutions. In this way, the issues of the curriculum are also explored and reflected in innovation and improvement year by year. Regarding the knowledge of relevant mathematics and comprehension of the formal curriculum's goals, the importance of defined functions is "one of the most significant ideas in school algebra—function" [35] (Arcavi, Drijvers, and Stacey, 2017, p. 69). The study of linear functions is crucial because it gives students their first taste of recognizing and analyzing the relationship between two dependent variables. Many students' first experience dealing with two linked variables comes in the form of linear functions, which marks a critical turning point in their mathematical

growth [36] (Hill et al., 2008). It helps teachers think about and comprehend the value of the notion of linear functions by allowing them to demonstrate their competence through experiments based on standards and criteria. When teachers realize its importance, they teach effectively in class.

Many previous studies are interested in studying the roles of teaching. "The author of [37] Sfard (1991) claimed that most mathematical things begin as processes and evolve into object-like characters through a series of steps, the culminating stage being dubbed reification. When mathematical entities become objects, they can be acted on to create new processes and finally new objects, but not before. [35] (Arcavi, Drijvers, and Stacey, 2017, p. 69); learning about functions entails much more than transitions between process and object [35] (Arcavi, Drijvers, and Stacey, 2017, p. 72).

"Developing the concept of a function as a single entity rather than a collection of calculations is a crucial basis for future mathematics that will take time to fully develop" [35] (Arcavi, Drijvers, and Stacey, 2017, p. 124), connecting representations in the process of deploying ideas and concepts, operating on them, revealing nuances, and producing new knowledge about mathematical material (e.g., [38] DiSessa, Hammer & Sherin, 1991). The ability to understand a topic and generate a solution to a problem may be influenced by translating between and across representations. The process perspective of functions is that "a function converts an input into an output" [35] (Arcavi, Drijvers, and Stacey, 2017, p. 70).

An entity with various representations and properties, which may be subject to higher-order processes such as differentiation, a function is a mathematical object with a variety of aspects and interpretations, such as instructions for processing numbers, an input–output device, a dependency relationship in which changes in the independent variable cause determined changes in the dependent variable, and an entity with these various aspects and interpretations [35] (Arcavi, Drijvers, and Stacey, 2017, p. 124).

In terms of the concept's practical usefulness, research reveals that it is necessary to connect mathematics to real life. The teaching was almost entirely based on real-world problems that the students were familiar with, an approach backed up by [39] Freudenthal (1991), who claims that mathematics begins with common sense and that students' mathematical ideas evolve by beginning with such experientially real situations. It has created a requirement for teachers to strive to present significant scenarios in life by being interested in the application of linear functions in practice.

The workshop training aimed to improve the professional capacity of Vietnamese Mathematics instructors. In this study, the model of professional competency components for Mathematics teachers was carefully chosen based on Constructivism theories of learner knowledge and Vietnam's strategy of innovative teaching and learning mathematics. Using the linear function lesson design as an example for developing professional capacity for Mathematics teachers in Vietnam. Participants in this study had experience with lesson planning and were familiar with modern teaching methods. They needed to move from transitional to constructive methods for student mathematical competency development. This method allows teachers to give pupils more time to use their mathematical skills in a problem-solving context.

In this study, understanding student thinking as a clear example of using a student-centered approach in teaching and teachers' interpretation of curriculum goals interacted in each session to develop successful techniques for students' achievement of mathematics education curriculum expectations. Previous studies, such as [40–42] (Wang et al., 2022; Do D. Thai et al., 2019; Wilkie, 2014), have strongly supported this study by emphasizing the importance of innovative teaching and learning mathematics successfully when moving from transition to constructive methodologies and suggesting multiple resources and pathways to assis teachers during their development. That is why understanding student thinking is an important condition for teachers to design learning tasks that adapt to student needs.

6. Conclusions

The Mathematics teachers' professional competence model plays a critical role in the establishment of materials and organizing the training and fostering workshops for teacher development of mathematical competence. The findings describe four criteria that form the basis of our construction of Mathematics teachers' professional competencies, including: i. Mathematical Knowledge; ii. Interpretation of the Official Mathematics Curriculum's Intentions; iii. Understanding of Students' Thoughts; and iv. Design of Teaching. As a result, identifying teacher competency standards or criteria is important in teaching practice to improve professional competence for improving student learning achievement. This study intended to provide frameworks for Mathematics instructors' professional competence in training or fostering and evaluating their practice.

The teaching experience of each teacher is enhanced by experimentation; proof of this is that the predictions of student errors were diverse based on student thinking and culture. Based on common student errors, the case study identifies the concept of a meaningful linear function, which has aided in the design of learning scenarios. These factors play a crucial part in teaching, since they aid students in gaining a better understanding of subjects. The discussion in the exchange of the members gave the expression of the real situation, which is a useful point in the products of these groups. The real situations related to linear functions are quite diverse, and the discussion of the members made the expression of the real situation more reasonable, providing a wide range of meaningful and practical scenarios to resolve real-life difficulties.

Author Contributions: Literature review, P.S.N. and H.A.T.; research methodology, P.S.N. and H.A.T.; writing—original draft preparation, P.S.N. and H.A.T.; writing—review and editing, P.S.N., H.A.T., R.W. and Z.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by Sai Gon University, grant number TĐ 2020-43.

Data Availability Statement: Not applicable.

Acknowledgments: The authors would like to acknowledge Nguyen Thuy Van, Nguyen Huu Hau, and Branko Andjic who gave some comments.

Conflicts of Interest: The authors declare no conflict of interest.

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