

Supporting Information of “The Imprecision Issues of Four Powers and Eight Predictive Powers with Historical and Interim Data”

Abstract

Web Appendices, Figures, and Tables referenced in the main text are summarized here.

For clinical trials with interim data, we have

$$m_1 + m_2 = s,$$

and thus

$$m_1 \rightarrow s \text{ as } m_2 \rightarrow 0^+, \quad (\text{S1})$$

where s is the subtotal sample size of the early trial and the confirmatory trial of one arm.

We will consider 4 powers: Classical Power (CP), Classical Conditional Power (CCP), Bayesian Power (BP), and Bayesian Conditional Power (BCP).

There are 3 conclusions, that is, Control Superior (CS), Treatment Superior (TS), and Equivocal (E).

1. Web 1

1.1. The CP

The expressions of the three probabilities $P(\text{CS:CP})$, $P(\text{TS:CP})$, and $P(\text{E:CP})$ are given by

$$P(\text{CS:CP}) = P\left(S_{\alpha,0}^{C,d_2}|\delta\right) = \text{CP} = \Phi\left[\frac{\delta}{\sigma\sqrt{2}}\sqrt{m_2} - Z_\alpha\right], \quad (\text{S2})$$

$$P(\text{TS:CP}) = P\left(S_{\alpha,0}^{C-,d_2}|\delta\right) = \text{CP}^- = \Phi\left[\frac{-\delta}{\sigma\sqrt{2}}\sqrt{m_2} - Z_\alpha\right], \quad (\text{S3})$$

and

$$\begin{aligned}
P(\text{E:CP}) &= P\left(-Z_\alpha\sigma\sqrt{2/m_2} \leq d_2 \leq Z_\alpha\sigma\sqrt{2/m_2}|\delta\right) \\
&= 1 - \text{CP}^- - \text{CP} \\
&= \Phi\left[\frac{\delta}{\sigma\sqrt{2}}\sqrt{m_2} + Z_\alpha\right] - \Phi\left[\frac{\delta}{\sigma\sqrt{2}}\sqrt{m_2} - Z_\alpha\right].
\end{aligned} \tag{S4}$$

1.2. The 1st predictive power

The expressions of the three probabilities $P(\text{CS:I}_1)$, $P(\text{TS:I}_1)$, and $P(\text{E:I}_1)$ are given by

$$P(\text{CS:I}_1) = P\left(S_{\alpha,0}^{C,d_2}|d_0\right) = I_1 = \Phi\left[\left(\frac{d_0}{\sqrt{2/m_2}\sigma} - Z_\alpha\right)\sqrt{\frac{m_0}{m_0+m_2}}\right], \tag{S5}$$

$$P(\text{TS:I}_1) = P\left(S_{\alpha,0}^{C^-,d_2}|d_0\right) = I_1^- = \Phi\left[\left(-\frac{d_0}{\sqrt{2/m_2}\sigma} - Z_\alpha\right)\sqrt{\frac{m_0}{m_0+m_2}}\right], \tag{S6}$$

and

$$\begin{aligned}
P(\text{E:I}_1) &= P\left(-Z_\alpha\sigma\sqrt{2/m_2} \leq d_2 \leq Z_\alpha\sigma\sqrt{2/m_2}|d_0\right) = 1 - I_1^- - I_1 \\
&= \Phi\left[\left(\frac{d_0}{\sqrt{2/m_2}\sigma} + Z_\alpha\right)\sqrt{\frac{m_0}{m_0+m_2}}\right] - \Phi\left[\left(\frac{d_0}{\sqrt{2/m_2}\sigma} - Z_\alpha\right)\sqrt{\frac{m_0}{m_0+m_2}}\right].
\end{aligned} \tag{S7}$$

1.3. The 2nd predictive power

The expressions of the three probabilities $P(\text{CS:I}_2)$, $P(\text{TS:I}_2)$, and $P(\text{E:I}_2)$ are given by

$$\begin{aligned}
P(\text{CS:I}_2) &= P\left(S_{\alpha,0}^{C,d_2}|d_0, d_1\right) = I_2 \\
&= \Phi\left[\frac{m_0d_0 + m_1d_1 - Z_\alpha\sigma\sqrt{2/m_2}(m_0 + m_1)}{\sqrt{2/m_2}\sigma\sqrt{m_0 + m_1}\sqrt{m_0 + m_1 + m_2}}\right],
\end{aligned} \tag{S8}$$

$$\begin{aligned}
P(\text{TS:I}_2) &= P\left(S_{\alpha,0}^{C^-,d_2}|d_0, d_1\right) = I_2^- \\
&= \Phi\left[\frac{-m_0d_0 - m_1d_1 - Z_\alpha\sigma\sqrt{2/m_2}(m_0 + m_1)}{\sqrt{2/m_2}\sigma\sqrt{m_0 + m_1}\sqrt{m_0 + m_1 + m_2}}\right],
\end{aligned} \tag{S9}$$

and

$$\begin{aligned}
P(E:I_2) &= P\left(-Z_\alpha\sigma\sqrt{2/m_2} \leq d_2 \leq Z_\alpha\sigma\sqrt{2/m_2} | d_0, d_1\right) = 1 - I_2^- - I_2 \quad (\text{S10}) \\
&= \Phi\left[\frac{m_0d_0 + m_1d_1 + Z_\alpha\sigma\sqrt{2/m_2}(m_0 + m_1)}{\sqrt{2/m_2}\sigma\sqrt{m_0 + m_1}\sqrt{m_0 + m_1 + m_2}}\right] - \Phi\left[\frac{m_0d_0 + m_1d_1 - Z_\alpha\sigma\sqrt{2/m_2}(m_0 + m_1)}{\sqrt{2/m_2}\sigma\sqrt{m_0 + m_1}\sqrt{m_0 + m_1 + m_2}}\right].
\end{aligned}$$

Note that the length of the interval of d_2 for E for $S_{\alpha,0}^{C,d_2}$ and $S_{\alpha,0}^{C-,d_2}$ for (S4), (S7), and (S10) is $2Z_\alpha\sigma\sqrt{2/m_2}$, which is a decreasing function of m_2 . That is, when m_2 is small (imprecision), the length of the interval of d_2 for E for $S_{\alpha,0}^{C,d_2}$ and $S_{\alpha,0}^{C-,d_2}$ is large. Hence, it is probably that $P(E:CP)$, $P(E:I_1)$, and $P(E:I_2)$ will be large.

1.4. The CCP

The expressions of the three probabilities $P(\text{CS:CCP})$, $P(\text{TS:CCP})$, and $P(\text{E:CCP})$ are given by

$$\begin{aligned}
P(\text{CS:CCP}) &= P\left(S_{\alpha,0}^{C,d_1,d_2} | \delta, d_1\right) = \text{CCP} \quad (\text{S11}) \\
&= \Phi\left[\frac{\delta\sqrt{m_2} + \frac{m_1d_1}{\sqrt{m_2}} - Z_\alpha\sigma\sqrt{2\left(\frac{m_1}{m_2} + 1\right)}}{\sigma\sqrt{2}}\right],
\end{aligned}$$

$$\begin{aligned}
P(\text{TS:CCP}) &= P\left(S_{\alpha,0}^{C-,d_1,d_2} | \delta, d_1\right) = \text{CCP}^- \quad (\text{S12}) \\
&= \Phi\left[\frac{-\delta\sqrt{m_2} - \frac{m_1d_1}{\sqrt{m_2}} - Z_\alpha\sigma\sqrt{2\left(\frac{m_1}{m_2} + 1\right)}}{\sigma\sqrt{2}}\right],
\end{aligned}$$

and

$$\begin{aligned}
P(\text{E:CCP}) &\quad (\text{S13}) \\
&= P\left(\frac{-Z_\alpha\sigma\sqrt{2(m_1 + m_2)} - m_1d_1}{m_2} \leq d_2 \leq \frac{Z_\alpha\sigma\sqrt{2(m_1 + m_2)} - m_1d_1}{m_2} | \delta, d_1\right) \\
&= 1 - \text{CCP}^- - \text{CCP} \\
&= \Phi\left[\frac{\delta\sqrt{m_2} + \frac{m_1d_1}{\sqrt{m_2}} + Z_\alpha\sigma\sqrt{2\left(\frac{m_1}{m_2} + 1\right)}}{\sigma\sqrt{2}}\right] - \Phi\left[\frac{\delta\sqrt{m_2} + \frac{m_1d_1}{\sqrt{m_2}} - Z_\alpha\sigma\sqrt{2\left(\frac{m_1}{m_2} + 1\right)}}{\sigma\sqrt{2}}\right].
\end{aligned}$$

1.5. The 3rd predictive power

The expressions of the three probabilities $P(\text{CS}:I_3)$, $P(\text{TS}:I_3)$, and $P(\text{E}:I_3)$ are given by

$$\begin{aligned} P(\text{CS}:I_3) &= P\left(S_{\alpha,0}^{C,d_1,d_2}|d_0\right) = I_3 \\ &= \Phi\left[\frac{m_1d_1 + m_2d_0 - Z_\alpha\sigma\sqrt{2(m_1+m_2)}}{\sqrt{2m_2}\sigma}\sqrt{\frac{m_0}{m_0+m_2}}\right], \end{aligned} \quad (\text{S14})$$

$$\begin{aligned} P(\text{TS}:I_3) &= P\left(S_{\alpha,0}^{C-,d_1,d_2}|d_0\right) = I_3^- \\ &= \Phi\left[\frac{-m_1d_1 - m_2d_0 - Z_\alpha\sigma\sqrt{2(m_1+m_2)}}{\sqrt{2m_2}\sigma}\sqrt{\frac{m_0}{m_0+m_2}}\right], \end{aligned} \quad (\text{S15})$$

and

$$\begin{aligned} P(\text{E}:I_3) &= P\left(\frac{-Z_\alpha\sigma\sqrt{2(m_1+m_2)} - m_1d_1}{m_2} \leq d_2 \leq \frac{Z_\alpha\sigma\sqrt{2(m_1+m_2)} - m_1d_1}{m_2} | d_0\right) \\ &= 1 - I_3^- - I_3 \\ &= \Phi\left[\frac{m_1d_1 + m_2d_0 + Z_\alpha\sigma\sqrt{2(m_1+m_2)}}{\sqrt{2m_2}\sigma}\sqrt{\frac{m_0}{m_0+m_2}}\right] \\ &\quad - \Phi\left[\frac{m_1d_1 + m_2d_0 - Z_\alpha\sigma\sqrt{2(m_1+m_2)}}{\sqrt{2m_2}\sigma}\sqrt{\frac{m_0}{m_0+m_2}}\right]. \end{aligned} \quad (\text{S16})$$

1.6. The 4th predictive power

The expressions of the three probabilities $P(\text{CS}:I_4)$, $P(\text{TS}:I_4)$, and $P(\text{E}:I_4)$ are given by

$$\begin{aligned} P(\text{CS}:I_4) &= P\left(S_{\alpha,0}^{C,d_1,d_2}|d_0,d_1\right) = I_4 \\ &= \Phi\left[\frac{m_0m_2d_0 + m_1(m_0+m_1+m_2)d_1 - Z_\alpha\sigma(m_0+m_1)\sqrt{2(m_1+m_2)}}{\sqrt{2m_2}\sigma\sqrt{m_0+m_1}\sqrt{m_0+m_1+m_2}}\right], \end{aligned} \quad (\text{S17})$$

$$\begin{aligned}
P(\text{TS}:I_4) &= P\left(S_{\alpha,0}^{C-,d_1,d_2}|d_0,d_1\right) = I_4^- \quad (\text{S18}) \\
&= \Phi\left[\frac{-m_0m_2d_0 - m_1(m_0 + m_1 + m_2)d_1 - Z_\alpha\sigma(m_0 + m_1)\sqrt{2(m_1 + m_2)}}{\sqrt{2m_2}\sigma\sqrt{m_0 + m_1}\sqrt{m_0 + m_1 + m_2}}\right],
\end{aligned}$$

and

$$\begin{aligned}
P(\text{E}:I_4) & \quad (\text{S19}) \\
&= P\left(\frac{-Z_\alpha\sigma\sqrt{2(m_1 + m_2)} - m_1d_1}{m_2} \leq d_2 \leq \frac{Z_\alpha\sigma\sqrt{2(m_1 + m_2)} - m_1d_1}{m_2} | d_0, d_1\right) \\
&= 1 - I_4^- - I_4 \\
&= \Phi\left[\frac{m_0m_2d_0 + m_1(m_0 + m_1 + m_2)d_1 + Z_\alpha\sigma(m_0 + m_1)\sqrt{2(m_1 + m_2)}}{\sqrt{2m_2}\sigma\sqrt{m_0 + m_1}\sqrt{m_0 + m_1 + m_2}}\right] \\
&\quad - \Phi\left[\frac{m_0m_2d_0 + m_1(m_0 + m_1 + m_2)d_1 - Z_\alpha\sigma(m_0 + m_1)\sqrt{2(m_1 + m_2)}}{\sqrt{2m_2}\sigma\sqrt{m_0 + m_1}\sqrt{m_0 + m_1 + m_2}}\right].
\end{aligned}$$

1.7. The BP

The expressions of the three probabilities $P(\text{CS:BP})$, $P(\text{TS:BP})$, and $P(\text{E:BP})$ are given by

$$\begin{aligned}
P(\text{CS:BP}) &= P\left(S_{\alpha,0}^{B,d_0,d_2}|\delta,d_0\right) = \text{BP} \quad (\text{S20}) \\
&= \Phi\left[\frac{\delta\sqrt{m_2} + \frac{m_0d_0}{\sqrt{m_2}} - Z_\alpha\sigma\sqrt{2\left(\frac{m_0}{m_2} + 1\right)}}{\sigma\sqrt{2}}\right],
\end{aligned}$$

$$\begin{aligned}
P(\text{TS:BP}) &= P\left(S_{\alpha,0}^{B-,d_0,d_2}|\delta,d_0\right) = \text{BP}^- \quad (\text{S21}) \\
&= \Phi\left[\frac{-\delta\sqrt{m_2} - \frac{m_0d_0}{\sqrt{m_2}} - Z_\alpha\sigma\sqrt{2\left(\frac{m_0}{m_2} + 1\right)}}{\sigma\sqrt{2}}\right],
\end{aligned}$$

and

$$\begin{aligned}
& P(\text{E:BP}) \tag{S22} \\
&= P\left(\frac{-Z_\alpha\sigma\sqrt{2(m_0+m_2)}-m_0d_0}{m_2} \leq d_2 \leq \frac{Z_\alpha\sigma\sqrt{2(m_0+m_2)}-m_0d_0}{m_2} \mid \delta, d_0\right) \\
&= 1 - \text{BP}^- - \text{BP} \\
&= \Phi\left[\frac{\delta\sqrt{m_2} + \frac{m_0d_0}{\sqrt{m_2}} + Z_\alpha\sigma\sqrt{2\left(\frac{m_0}{m_2}+1\right)}}{\sigma\sqrt{2}}\right] - \Phi\left[\frac{\delta\sqrt{m_2} + \frac{m_0d_0}{\sqrt{m_2}} - Z_\alpha\sigma\sqrt{2\left(\frac{m_0}{m_2}+1\right)}}{\sigma\sqrt{2}}\right].
\end{aligned}$$

1.8. The 5th predictive power

The expressions of the three probabilities $P(\text{CS}:I_5)$, $P(\text{TS}:I_5)$, and $P(\text{E}:I_5)$ are given by

$$\begin{aligned}
P(\text{CS}:I_5) &= P\left(S_{\alpha,0}^{B,d_0,d_2} \mid d_0\right) = I_5 \tag{S23} \\
&= \Phi\left[\frac{(m_0+m_2)d_0 - Z_\alpha\sigma\sqrt{2(m_0+m_2)}}{\sqrt{2m_2}\sigma} \sqrt{\frac{m_0}{m_0+m_2}}\right],
\end{aligned}$$

$$\begin{aligned}
P(\text{TS}:I_5) &= P\left(S_{\alpha,0}^{B-,d_0,d_2} \mid d_0\right) = I_5^- \tag{S24} \\
&= \Phi\left[\frac{-(m_0+m_2)d_0 - Z_\alpha\sigma\sqrt{2(m_0+m_2)}}{\sqrt{2m_2}\sigma} \sqrt{\frac{m_0}{m_0+m_2}}\right],
\end{aligned}$$

and

$$\begin{aligned}
& P(\text{E}:I_5) \tag{S25} \\
&= P\left(\frac{-Z_\alpha\sigma\sqrt{2(m_0+m_2)}-m_0d_0}{m_2} \leq d_2 \leq \frac{Z_\alpha\sigma\sqrt{2(m_0+m_2)}-m_0d_0}{m_2} \mid d_0\right) \\
&= 1 - I_5^- - I_5 \\
&= \Phi\left[\frac{(m_0+m_2)d_0 + Z_\alpha\sigma\sqrt{2(m_0+m_2)}}{\sqrt{2m_2}\sigma} \sqrt{\frac{m_0}{m_0+m_2}}\right] \\
&\quad - \Phi\left[\frac{(m_0+m_2)d_0 - Z_\alpha\sigma\sqrt{2(m_0+m_2)}}{\sqrt{2m_2}\sigma} \sqrt{\frac{m_0}{m_0+m_2}}\right].
\end{aligned}$$

1.9. The 6th predictive power

The expressions of the three probabilities $P(\text{CS}:I_6)$, $P(\text{TS}:I_6)$, and $P(\text{E}:I_6)$ are given by

$$\begin{aligned} P(\text{CS}:I_6) &= P\left(S_{\alpha,0}^{B,d_0,d_2}|d_0,d_1\right) = I_6 \\ &= \Phi\left[\frac{m_0(m_0+m_1+m_2)d_0+m_1m_2d_1-Z_\alpha\sigma(m_0+m_1)\sqrt{2(m_0+m_2)}}{\sqrt{2}m_2\sigma\sqrt{m_0+m_1}\sqrt{m_0+m_1+m_2}}\right], \end{aligned} \quad (\text{S26})$$

$$\begin{aligned} P(\text{TS}:I_6) &= P\left(S_{\alpha,0}^{B-,d_0,d_2}|d_0,d_1\right) = I_6^- \\ &= \Phi\left[\frac{-m_0(m_0+m_1+m_2)d_0-m_1m_2d_1-Z_\alpha\sigma(m_0+m_1)\sqrt{2(m_0+m_2)}}{\sqrt{2}m_2\sigma\sqrt{m_0+m_1}\sqrt{m_0+m_1+m_2}}\right], \end{aligned} \quad (\text{S27})$$

and

$$\begin{aligned} P(\text{E}:I_6) &= P\left(\frac{-Z_\alpha\sigma\sqrt{2(m_0+m_2)}-m_0d_0}{m_2} \leq d_2 \leq \frac{Z_\alpha\sigma\sqrt{2(m_0+m_2)}-m_0d_0}{m_2} | d_0, d_1\right) \\ &= 1 - I_6^- - I_6 \\ &= \Phi\left[\frac{m_0(m_0+m_1+m_2)d_0+m_1m_2d_1+Z_\alpha\sigma(m_0+m_1)\sqrt{2(m_0+m_2)}}{\sqrt{2}m_2\sigma\sqrt{m_0+m_1}\sqrt{m_0+m_1+m_2}}\right] \\ &\quad - \Phi\left[\frac{m_0(m_0+m_1+m_2)d_0+m_1m_2d_1-Z_\alpha\sigma(m_0+m_1)\sqrt{2(m_0+m_2)}}{\sqrt{2}m_2\sigma\sqrt{m_0+m_1}\sqrt{m_0+m_1+m_2}}\right]. \end{aligned} \quad (\text{S28})$$

1.10. The BCP

The expressions of the three probabilities $P(\text{CS:BCP})$, $P(\text{TS:BCP})$, and $P(\text{E:BCP})$ are given by

$$\begin{aligned} P(\text{CS:BCP}) &= P\left(S_{\alpha,0}^{B,d_0,d_1,d_2}|\delta,d_0,d_1\right) = \text{BCP} \\ &= \Phi\left[\frac{\delta\sqrt{m_2} + \frac{m_0d_0+m_1d_1}{\sqrt{m_2}} - Z_\alpha\sigma\sqrt{2\left(\frac{m_0+m_1}{m_2} + 1\right)}}{\sigma\sqrt{2}}\right], \end{aligned} \quad (\text{S29})$$

$$\begin{aligned}
P(\text{TS:BCP}) &= P\left(S_{\alpha,0}^{B-,d_0,d_1,d_2}|\delta,d_0,d_1\right) = \text{BCP}^- \quad (\text{S30}) \\
&= \Phi\left[\frac{-\delta\sqrt{m_2} - \frac{m_0d_0+m_1d_1}{\sqrt{m_2}} - Z_\alpha\sigma\sqrt{2\left(\frac{m_0+m_1}{m_2}+1\right)}}{\sigma\sqrt{2}}\right],
\end{aligned}$$

and

$$\begin{aligned}
P(\text{E:BCP}) & \quad (\text{S31}) \\
&= P\left(\frac{-Z_\alpha\sigma\sqrt{2(m_0+m_1+m_2)} - m_0d_0 - m_1d_1}{m_2} \leq d_2 \leq \frac{Z_\alpha\sigma\sqrt{2(m_0+m_1+m_2)} - m_0d_0 - m_1d_1}{m_2}|\delta,d_0,d_1\right) \\
&= 1 - \text{BCP}^- - \text{BCP} \\
&= \Phi\left[\frac{\delta\sqrt{m_2} + \frac{m_0d_0+m_1d_1}{\sqrt{m_2}} + Z_\alpha\sigma\sqrt{2\left(\frac{m_0+m_1}{m_2}+1\right)}}{\sigma\sqrt{2}}\right] \\
&\quad - \Phi\left[\frac{\delta\sqrt{m_2} + \frac{m_0d_0+m_1d_1}{\sqrt{m_2}} - Z_\alpha\sigma\sqrt{2\left(\frac{m_0+m_1}{m_2}+1\right)}}{\sigma\sqrt{2}}\right].
\end{aligned}$$

1.11. The 7th predictive power

The expressions of the three probabilities $P(\text{CS:}I_7)$, $P(\text{TS:}I_7)$, and $P(\text{E:}I_7)$ are given by

$$\begin{aligned}
P(\text{CS:}I_7) &= P\left(S_{\alpha,0}^{B,d_0,d_1,d_2}|d_0\right) = I_7 \quad (\text{S32}) \\
&= \Phi\left[\frac{(m_0+m_2)d_0 + m_1d_1 - Z_\alpha\sigma\sqrt{2(m_0+m_1+m_2)}}{\sqrt{2m_2}\sigma}\sqrt{\frac{m_0}{m_0+m_2}}\right],
\end{aligned}$$

$$\begin{aligned}
P(\text{TS:}I_7) &= P\left(S_{\alpha,0}^{B-,d_0,d_1,d_2}|d_0\right) = I_7^- \quad (\text{S33}) \\
&= \Phi\left[\frac{-(m_0+m_2)d_0 - m_1d_1 - Z_\alpha\sigma\sqrt{2(m_0+m_1+m_2)}}{\sqrt{2m_2}\sigma}\sqrt{\frac{m_0}{m_0+m_2}}\right],
\end{aligned}$$

and

$$\begin{aligned}
P(\text{E}:I_7) & \tag{S34} \\
&= P\left(\frac{-Z_\alpha\sigma\sqrt{2(m_0+m_1+m_2)}-m_0d_0-m_1d_1}{m_2} \leq d_2 \leq \frac{Z_\alpha\sigma\sqrt{2(m_0+m_1+m_2)}-m_0d_0-m_1d_1}{m_2} | d_0\right) \\
&= 1 - I_7^- - I_7 \\
&= \Phi\left[\frac{(m_0+m_2)d_0+m_1d_1+Z_\alpha\sigma\sqrt{2(m_0+m_1+m_2)}}{\sqrt{2m_2}\sigma}\sqrt{\frac{m_0}{m_0+m_2}}\right] \\
&\quad - \Phi\left[\frac{(m_0+m_2)d_0+m_1d_1-Z_\alpha\sigma\sqrt{2(m_0+m_1+m_2)}}{\sqrt{2m_2}\sigma}\sqrt{\frac{m_0}{m_0+m_2}}\right].
\end{aligned}$$

1.12. The 8th predictive power

The expressions of the three probabilities $P(\text{CS}:I_8)$, $P(\text{TS}:I_8)$, and $P(\text{E}:I_8)$ are given by

$$\begin{aligned}
P(\text{CS}:I_8) &= P\left(S_{\alpha,0}^{B,d_0,d_1,d_2} | d_0, d_1\right) = I_8 \tag{S35} \\
&= \Phi\left[\frac{(m_0+m_1+m_2)(m_0d_0+m_1d_1)-Z_\alpha\sigma(m_0+m_1)\sqrt{2(m_0+m_1+m_2)}}{\sqrt{2m_2}\sigma\sqrt{m_0+m_1}\sqrt{m_0+m_1+m_2}}\right],
\end{aligned}$$

$$\begin{aligned}
P(\text{TS}:I_8) &= P\left(S_{\alpha,0}^{B-,d_0,d_1,d_2} | d_0, d_1\right) = I_8^- \tag{S36} \\
&= \Phi\left[\frac{-(m_0+m_1+m_2)(m_0d_0+m_1d_1)-Z_\alpha\sigma(m_0+m_1)\sqrt{2(m_0+m_1+m_2)}}{\sqrt{2m_2}\sigma\sqrt{m_0+m_1}\sqrt{m_0+m_1+m_2}}\right],
\end{aligned}$$

and

$$\begin{aligned}
P(\text{E}:I_8) & \tag{S37} \\
&= P\left(\frac{-Z_\alpha\sigma\sqrt{2(m_0+m_1+m_2)}-m_0d_0-m_1d_1}{m_2} \leq d_2 \leq \frac{Z_\alpha\sigma\sqrt{2(m_0+m_1+m_2)}-m_0d_0-m_1d_1}{m_2} | d_0, d_1\right) \\
&= 1 - I_8^- - I_8 \\
&= \Phi\left[\frac{(m_0+m_1+m_2)(m_0d_0+m_1d_1)+Z_\alpha\sigma(m_0+m_1)\sqrt{2(m_0+m_1+m_2)}}{\sqrt{2m_2}\sigma\sqrt{m_0+m_1}\sqrt{m_0+m_1+m_2}}\right] \\
&\quad - \Phi\left[\frac{(m_0+m_1+m_2)(m_0d_0+m_1d_1)-Z_\alpha\sigma(m_0+m_1)\sqrt{2(m_0+m_1+m_2)}}{\sqrt{2m_2}\sigma\sqrt{m_0+m_1}\sqrt{m_0+m_1+m_2}}\right].
\end{aligned}$$

2. Web 2

2.1. The proof of Proposition 1

In this subsection, we will evaluate the limits at point 0 of the probabilities of CS, TS, and E of the CP and the 1st and 2nd predictive powers.

Now we evaluate the limits of the three probabilities (S2), (S3), and (S4) when $m_2 \rightarrow 0^+$. It is easy to show that

$$\begin{aligned} & \lim_{m_2 \rightarrow 0^+} P(\text{CS:CP}) \\ &= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{\delta}{\sigma\sqrt{2}} \sqrt{m_2} - Z_\alpha \right] \\ &= \Phi[-Z_\alpha] = P(Z \leq -Z_\alpha) = \alpha, \end{aligned}$$

$$\begin{aligned} & \lim_{m_2 \rightarrow 0^+} P(\text{TS:CP}) \\ &= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-\delta}{\sigma\sqrt{2}} \sqrt{m_2} - Z_\alpha \right] \\ &= \Phi[-Z_\alpha] = P(Z \leq -Z_\alpha) = \alpha, \end{aligned}$$

and

$$\begin{aligned} & \lim_{m_2 \rightarrow 0^+} P(\text{E:CP}) \\ &= \lim_{m_2 \rightarrow 0^+} [1 - P(\text{TS:CP}) - P(\text{CS:CP})] \\ &= 1 - \lim_{m_2 \rightarrow 0^+} P(\text{TS:CP}) - \lim_{m_2 \rightarrow 0^+} P(\text{CS:CP}) \\ &= 1 - \alpha - \alpha = 1 - 2\alpha. \end{aligned}$$

Now we evaluate the limits of the three probabilities (S5), (S6), and (S7) when $m_2 \rightarrow 0^+$. It is easy to show that

$$\begin{aligned} & \lim_{m_2 \rightarrow 0^+} P(\text{CS:I}_1) \\ &= \lim_{m_2 \rightarrow 0^+} \Phi \left[\left(\frac{d_0}{\sqrt{2/m_2}\sigma} - Z_\alpha \right) \sqrt{\frac{m_0}{m_0 + m_2}} \right] \\ &= \Phi[-Z_\alpha] = P(Z \leq -Z_\alpha) = \alpha, \end{aligned}$$

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_1) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\left(-\frac{d_0}{\sqrt{2/m_2}\sigma} - Z_\alpha \right) \sqrt{\frac{m_0}{m_0 + m_2}} \right] \\
&= \Phi[-Z_\alpha] = P(Z \leq -Z_\alpha) = \alpha,
\end{aligned}$$

and

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{E}:I_1) \\
&= \lim_{m_2 \rightarrow 0^+} [1 - P(\text{TS}:I_1) - P(\text{CS}:I_1)] \\
&= 1 - \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_1) - \lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_1) \\
&= 1 - \alpha - \alpha = 1 - 2\alpha.
\end{aligned}$$

Now we evaluate the limits of the three probabilities (S8), (S9), and (S10) when $m_2 \rightarrow 0^+$. By (S1), it is easy to show that

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_2) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{m_0 d_0 + m_1 d_1 - Z_\alpha \sigma \sqrt{2/m_2} (m_0 + m_1)}{\sqrt{2/m_2} \sigma \sqrt{m_0 + m_1} \sqrt{m_0 + m_1 + m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{m_0 d_0 + s d_1}{\sqrt{2/m_2} \sigma (m_0 + s)} - \frac{Z_\alpha \sigma \sqrt{2/m_2} (m_0 + s)}{\sqrt{2/m_2} \sigma (m_0 + s)} \right] \\
&= \Phi[-Z_\alpha] = P(Z \leq -Z_\alpha) = \alpha,
\end{aligned}$$

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_2) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-m_0 d_0 - m_1 d_1 - Z_\alpha \sigma \sqrt{2/m_2} (m_0 + m_1)}{\sqrt{2/m_2} \sigma \sqrt{m_0 + m_1} \sqrt{m_0 + m_1 + m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-m_0 d_0 - s d_1}{\sqrt{2/m_2} \sigma (m_0 + s)} - \frac{Z_\alpha \sigma \sqrt{2/m_2} (m_0 + s)}{\sqrt{2/m_2} \sigma (m_0 + s)} \right] \\
&= \Phi[-Z_\alpha] = P(Z \leq -Z_\alpha) = \alpha,
\end{aligned}$$

and

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(E:I_2) \\
&= \lim_{m_2 \rightarrow 0^+} [1 - P(TS:I_2) - P(CS:I_2)] \\
&= 1 - \lim_{m_2 \rightarrow 0^+} P(TS:I_2) - \lim_{m_2 \rightarrow 0^+} P(CS:I_2) \\
&= 1 - \alpha - \alpha = 1 - 2\alpha.
\end{aligned}$$

2.2. The proof of Proposition 2

In this subsection, we will evaluate the limits at point 0 of the probabilities of CS, TS, and E of the CCP and the 3rd and 4th predictive powers.

Now we evaluate the limits of the three probabilities (S11), (S12), and (S13) when $m_2 \rightarrow 0^+$. It is easy to show that

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(CS:CCP) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{\delta\sqrt{m_2} - Z_\alpha\sigma\sqrt{2\left(\frac{m_1}{m_2} + 1\right) + \frac{m_1 d_1}{\sqrt{m_2}}}}{\sigma\sqrt{2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha\sigma\sqrt{2\left(\frac{m_1}{m_2} + 1\right) + \frac{m_1 d_1}{\sqrt{m_2}}}}{\sigma\sqrt{2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha\sigma\sqrt{2}\sqrt{s} + s d_1}{\sigma\sqrt{2}\sqrt{m_2}} \right].
\end{aligned}$$

We have

$$\begin{aligned}
\lim_{m_2 \rightarrow 0^+} \frac{-Z_\alpha\sigma\sqrt{2}\sqrt{s} + s d_1}{\sigma\sqrt{2}\sqrt{m_2}} &= \begin{cases} \infty, & \text{if } -Z_\alpha\sigma\sqrt{2}\sqrt{s} + s d_1 > 0, \\ 0, & \text{if } -Z_\alpha\sigma\sqrt{2}\sqrt{s} + s d_1 = 0, \\ -\infty, & \text{if } -Z_\alpha\sigma\sqrt{2}\sqrt{s} + s d_1 < 0, \end{cases} \\
&= \begin{cases} \infty, & \text{if } d_1 > Z_\alpha\sigma\sqrt{2/s}, \\ 0, & \text{if } d_1 = Z_\alpha\sigma\sqrt{2/s}, \\ -\infty, & \text{if } d_1 < Z_\alpha\sigma\sqrt{2/s}. \end{cases}
\end{aligned}$$

Hence,

$$\begin{aligned} & \lim_{m_2 \rightarrow 0^+} P(\text{CS:CCP}) \\ &= \begin{cases} 1, & \text{if } d_1 > Z_\alpha \sigma \sqrt{2/s}, \\ 0.5, & \text{if } d_1 = Z_\alpha \sigma \sqrt{2/s}, \\ 0, & \text{if } d_1 < Z_\alpha \sigma \sqrt{2/s}. \end{cases} \end{aligned} \quad (\text{S38})$$

Similarly, it is easy to show that

$$\begin{aligned} & \lim_{m_2 \rightarrow 0^+} P(\text{TS:CCP}) \\ &= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2 \left(\frac{m_1}{m_2} + 1 \right)} - \frac{m_1 d_1}{\sqrt{m_2}} - \delta \sqrt{m_2}}{\sigma \sqrt{2}} \right] \\ &= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2 \left(\frac{m_1}{m_2} + 1 \right)} - \frac{m_1 d_1}{\sqrt{m_2}}}{\sigma \sqrt{2}} \right] \\ &= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2} \sqrt{s} - s d_1}{\sigma \sqrt{2} \sqrt{m_2}} \right]. \end{aligned}$$

We have

$$\begin{aligned} \lim_{m_2 \rightarrow 0^+} \frac{-Z_\alpha \sigma \sqrt{2} \sqrt{s} - s d_1}{\sigma \sqrt{2} \sqrt{m_2}} &= \begin{cases} \infty, & \text{if } -Z_\alpha \sigma \sqrt{2} \sqrt{s} - s d_1 > 0, \\ 0, & \text{if } -Z_\alpha \sigma \sqrt{2} \sqrt{s} - s d_1 = 0, \\ -\infty, & \text{if } -Z_\alpha \sigma \sqrt{2} \sqrt{s} - s d_1 < 0, \end{cases} \\ &= \begin{cases} \infty, & \text{if } d_1 < -Z_\alpha \sigma \sqrt{2/s}, \\ 0, & \text{if } d_1 = -Z_\alpha \sigma \sqrt{2/s}, \\ -\infty, & \text{if } d_1 > -Z_\alpha \sigma \sqrt{2/s}. \end{cases} \end{aligned}$$

Hence,

$$\begin{aligned} & \lim_{m_2 \rightarrow 0^+} P(\text{TS:CCP}) \\ &= \begin{cases} 1, & \text{if } d_1 < -Z_\alpha \sigma \sqrt{2/s}, \\ 0.5, & \text{if } d_1 = -Z_\alpha \sigma \sqrt{2/s}, \\ 0, & \text{if } d_1 > -Z_\alpha \sigma \sqrt{2/s}. \end{cases} \end{aligned} \quad (\text{S39})$$

Therefore,

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{E:CCP}) \tag{S40} \\
&= 1 - \lim_{m_2 \rightarrow 0^+} P(\text{TS:CCP}) - \lim_{m_2 \rightarrow 0^+} P(\text{CS:CCP}) \\
&= \begin{cases} 1 - 1 - 0 = 0, & \text{if } d_1 < -Z_\alpha \sigma \sqrt{2/s}, \\ 1 - 0.5 - 0 = 0.5, & \text{if } d_1 = -Z_\alpha \sigma \sqrt{2/s}, \\ 1 - 0 - 0 = 1, & \text{if } -Z_\alpha \sigma \sqrt{2/s} < d_1 < Z_\alpha \sigma \sqrt{2/s}, \\ 1 - 0 - 0.5 = 0.5, & \text{if } d_1 = Z_\alpha \sigma \sqrt{2/s}, \\ 1 - 0 - 1 = 0, & \text{if } d_1 > Z_\alpha \sigma \sqrt{2/s}. \end{cases}
\end{aligned}$$

Now we evaluate the limits of the three probabilities (S14), (S15), and (S16) when $m_2 \rightarrow 0^+$. By (S1), it is easy to show that

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{CS:}I_3) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{m_1 d_1 + m_2 d_0 - Z_\alpha \sigma \sqrt{2(m_1 + m_2)}}{\sqrt{2m_2} \sigma} \sqrt{\frac{m_0}{m_0 + m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{m_1 d_1 - Z_\alpha \sigma \sqrt{2(m_1 + m_2)}}{\sigma \sqrt{2m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2} \sqrt{s} + s d_1}{\sigma \sqrt{2} \sqrt{m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} P(\text{CS:CCP}),
\end{aligned}$$

which is given by (S38). Similarly, by (S1), it is easy to show that

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{TS:}I_3) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-m_1 d_1 - m_2 d_0 - Z_\alpha \sigma \sqrt{2(m_1 + m_2)}}{\sqrt{2m_2} \sigma} \sqrt{\frac{m_0}{m_0 + m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-m_1 d_1 - Z_\alpha \sigma \sqrt{2(m_1 + m_2)}}{\sigma \sqrt{2m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2} \sqrt{s} - s d_1}{\sigma \sqrt{2} \sqrt{m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} P(\text{TS:CCP}),
\end{aligned}$$

which is given by (S39). Therefore,

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{E}:I_3) \\
&= 1 - \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_3) - \lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_3) \\
&= 1 - \lim_{m_2 \rightarrow 0^+} P(\text{TS:CCP}) - \lim_{m_2 \rightarrow 0^+} P(\text{CS:CCP}) \\
&= \lim_{m_2 \rightarrow 0^+} P(\text{E:CCP}),
\end{aligned} \tag{S41}$$

which is given by (S40).

Now we evaluate the limits of the three probabilities (S17), (S18), and (S19) when $m_2 \rightarrow 0^+$. By (S1), it is easy to show that

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_4) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{m_0 m_2 d_0 + m_1 (m_0 + m_1 + m_2) d_1 - Z_\alpha \sigma (m_0 + m_1) \sqrt{2(m_1 + m_2)}}{\sqrt{2} m_2 \sigma \sqrt{m_0 + m_1} \sqrt{m_0 + m_1 + m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{s(m_0 + s) d_1 - Z_\alpha \sigma (m_0 + s) \sqrt{2} \sqrt{s}}{\sigma \sqrt{2} m_2 (m_0 + s)} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2} \sqrt{s} + s d_1}{\sigma \sqrt{2} \sqrt{m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} P(\text{CS:CCP}),
\end{aligned}$$

which is given by (S38). Similarly, by (S1), it is easy to show that

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_4) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-m_0 m_2 d_0 - m_1 (m_0 + m_1 + m_2) d_1 - Z_\alpha \sigma (m_0 + m_1) \sqrt{2(m_1 + m_2)}}{\sqrt{2} m_2 \sigma \sqrt{m_0 + m_1} \sqrt{m_0 + m_1 + m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-s(m_0 + s) d_1 - Z_\alpha \sigma (m_0 + s) \sqrt{2} \sqrt{s}}{\sigma \sqrt{2} m_2 (m_0 + s)} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2} \sqrt{s} - s d_1}{\sigma \sqrt{2} \sqrt{m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} P(\text{TS:CCP}),
\end{aligned}$$

which is given by (S39). Therefore,

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{E}; I_4) \\
&= 1 - \lim_{m_2 \rightarrow 0^+} P(\text{TS}; I_4) - \lim_{m_2 \rightarrow 0^+} P(\text{CS}; I_4) \\
&= 1 - \lim_{m_2 \rightarrow 0^+} P(\text{TS}; \text{CCP}) - \lim_{m_2 \rightarrow 0^+} P(\text{CS}; \text{CCP}) \\
&= \lim_{m_2 \rightarrow 0^+} P(\text{E}; \text{CCP}),
\end{aligned} \tag{S42}$$

which is given by (S40).

2.3. The proof of Proposition 3

In this subsection, we will evaluate the limits at point 0 of the probabilities of CS, TS, and E of the BP and the 5th and 6th predictive powers.

Now we evaluate the limits of the three probabilities (S20), (S21), and (S22) when $m_2 \rightarrow 0^+$. It is easy to show that

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{CS}; \text{BP}) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{\delta \sqrt{m_2} - Z_\alpha \sigma \sqrt{2 \left(\frac{m_0}{m_2} + 1 \right) + \frac{m_0 d_0}{\sqrt{m_2}}}}{\sigma \sqrt{2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2 \left(\frac{m_0}{m_2} + 1 \right) + \frac{m_0 d_0}{\sqrt{m_2}}}}{\sigma \sqrt{2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2} \sqrt{m_0 + m_2} + m_0 d_0}{\sigma \sqrt{2} \sqrt{m_2}} \right].
\end{aligned}$$

We have

$$\begin{aligned}
\lim_{m_2 \rightarrow 0^+} \frac{-Z_\alpha \sigma \sqrt{2} \sqrt{m_0 + m_2} + m_0 d_0}{\sigma \sqrt{2} \sqrt{m_2}} &= \begin{cases} \infty, & \text{if } -Z_\alpha \sigma \sqrt{2} \sqrt{m_0} + m_0 d_0 > 0, \\ 0, & \text{if } -Z_\alpha \sigma \sqrt{2} \sqrt{m_0} + m_0 d_0 = 0, \\ -\infty, & \text{if } -Z_\alpha \sigma \sqrt{2} \sqrt{m_0} + m_0 d_0 < 0, \end{cases} \\
&= \begin{cases} \infty, & \text{if } d_0 > Z_\alpha \sigma \sqrt{2/m_0}, \\ 0, & \text{if } d_0 = Z_\alpha \sigma \sqrt{2/m_0}, \\ -\infty, & \text{if } d_0 < Z_\alpha \sigma \sqrt{2/m_0}, \end{cases}
\end{aligned}$$

by noting that

$$\begin{aligned} \lim_{m_2 \rightarrow 0^+} \frac{-Z_\alpha \sigma \sqrt{2} \sqrt{m_0 + m_2} + m_0 d_0}{\sigma \sqrt{2} \sqrt{m_2}} &= \lim_{m_2 \rightarrow 0^+} \frac{-Z_\alpha \sigma \sqrt{2} \frac{1}{2\sqrt{m_0 + m_2}}}{\sigma \sqrt{2} \frac{1}{2\sqrt{m_2}}} \\ &= \lim_{m_2 \rightarrow 0^+} -Z_\alpha \frac{\sqrt{m_2}}{\sqrt{m_0 + m_2}} = 0, \end{aligned}$$

if $-Z_\alpha \sigma \sqrt{2} \sqrt{m_0} + m_0 d_0 = 0$. Hence,

$$\begin{aligned} \lim_{m_2 \rightarrow 0^+} P(\text{CS:BP}) & \tag{S43} \\ &= \begin{cases} 1, & \text{if } d_0 > Z_\alpha \sigma \sqrt{2/m_0}, \\ 0.5, & \text{if } d_0 = Z_\alpha \sigma \sqrt{2/m_0}, \\ 0, & \text{if } d_0 < Z_\alpha \sigma \sqrt{2/m_0}. \end{cases} \end{aligned}$$

Similarly, it is easy to show that

$$\begin{aligned} \lim_{m_2 \rightarrow 0^+} P(\text{TS:BP}) &= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2 \left(\frac{m_0}{m_2} + 1 \right)} - \frac{m_0 d_0}{\sqrt{m_2}} - \delta \sqrt{m_2}}{\sigma \sqrt{2}} \right] \\ &= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2 \left(\frac{m_0}{m_2} + 1 \right)} - \frac{m_0 d_0}{\sqrt{m_2}}}{\sigma \sqrt{2}} \right] \\ &= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2} \sqrt{m_0 + m_2} - m_0 d_0}{\sigma \sqrt{2} \sqrt{m_2}} \right]. \end{aligned}$$

We have

$$\begin{aligned} \lim_{m_2 \rightarrow 0^+} \frac{-Z_\alpha \sigma \sqrt{2} \sqrt{m_0 + m_2} - m_0 d_0}{\sigma \sqrt{2} \sqrt{m_2}} &= \begin{cases} \infty, & \text{if } -Z_\alpha \sigma \sqrt{2} \sqrt{m_0} - m_0 d_0 > 0, \\ 0, & \text{if } -Z_\alpha \sigma \sqrt{2} \sqrt{m_0} - m_0 d_0 = 0, \\ -\infty, & \text{if } -Z_\alpha \sigma \sqrt{2} \sqrt{m_0} - m_0 d_0 < 0, \end{cases} \\ &= \begin{cases} \infty, & \text{if } d_0 < -Z_\alpha \sigma \sqrt{2/m_0}, \\ 0, & \text{if } d_0 = -Z_\alpha \sigma \sqrt{2/m_0}, \\ -\infty, & \text{if } d_0 > -Z_\alpha \sigma \sqrt{2/m_0}, \end{cases} \end{aligned}$$

by noting that

$$\begin{aligned} \lim_{m_2 \rightarrow 0^+} \frac{-Z_\alpha \sigma \sqrt{2} \sqrt{m_0 + m_2} - m_0 d_0}{\sigma \sqrt{2} \sqrt{m_2}} &= \lim_{m_2 \rightarrow 0^+} \frac{-Z_\alpha \sigma \sqrt{2} \frac{1}{2\sqrt{m_0 + m_2}}}{\sigma \sqrt{2} \frac{1}{2\sqrt{m_2}}} \\ &= \lim_{m_2 \rightarrow 0^+} -Z_\alpha \frac{\sqrt{m_2}}{\sqrt{m_0 + m_2}} = 0, \end{aligned}$$

if $-Z_\alpha \sigma \sqrt{2} \sqrt{m_0} - m_0 d_0 = 0$. Hence,

$$\begin{aligned} \lim_{m_2 \rightarrow 0^+} P(\text{TS:BP}) & \tag{S44} \\ &= \begin{cases} 1, & \text{if } d_0 < -Z_\alpha \sigma \sqrt{2/m_0}, \\ 0.5, & \text{if } d_0 = -Z_\alpha \sigma \sqrt{2/m_0}, \\ 0, & \text{if } d_0 > -Z_\alpha \sigma \sqrt{2/m_0}. \end{cases} \end{aligned}$$

Therefore,

$$\begin{aligned} \lim_{m_2 \rightarrow 0^+} P(\text{E:BP}) & \tag{S45} \\ &= 1 - \lim_{m_2 \rightarrow 0^+} P(\text{TS:BP}) - \lim_{m_2 \rightarrow 0^+} P(\text{CS:BP}) \\ &= \begin{cases} 1 - 1 - 0 = 0, & \text{if } d_0 < -Z_\alpha \sigma \sqrt{2/m_0}, \\ 1 - 0.5 - 0 = 0.5, & \text{if } d_0 = -Z_\alpha \sigma \sqrt{2/m_0}, \\ 1 - 0 - 0 = 1, & \text{if } -Z_\alpha \sigma \sqrt{2/m_0} < d_0 < Z_\alpha \sigma \sqrt{2/m_0}, \\ 1 - 0 - 0.5 = 0.5, & \text{if } d_0 = Z_\alpha \sigma \sqrt{2/m_0}, \\ 1 - 0 - 1 = 0, & \text{if } d_0 > Z_\alpha \sigma \sqrt{2/m_0}. \end{cases} \end{aligned}$$

Now we evaluate the limits of the three probabilities (S23), (S24), and (S25) when $m_2 \rightarrow 0^+$. It is easy to show that

$$\begin{aligned} \lim_{m_2 \rightarrow 0^+} P(\text{CS:}I_5) & \\ &= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{(m_0 + m_2) d_0 - Z_\alpha \sigma \sqrt{2(m_0 + m_2)}}{\sqrt{2m_2} \sigma} \sqrt{\frac{m_0}{m_0 + m_2}} \right] \\ &= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{m_0 d_0 - Z_\alpha \sigma \sqrt{2(m_0 + m_2)}}{\sigma \sqrt{2m_2}} \right] \\ &= \lim_{m_2 \rightarrow 0^+} P(\text{CS:BP}), \end{aligned}$$

which is given by (S43). Similarly, it is easy to show that

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_5) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-(m_0 + m_2) d_0 - Z_\alpha \sigma \sqrt{2(m_0 + m_2)}}{\sqrt{2m_2} \sigma} \sqrt{\frac{m_0}{m_0 + m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-m_0 d_0 - Z_\alpha \sigma \sqrt{2(m_0 + m_2)}}{\sigma \sqrt{2m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} P(\text{TS:BP}),
\end{aligned}$$

which is given by (S44). Therefore,

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{E}:I_5) \tag{S46} \\
&= 1 - \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_5) - \lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_5) \\
&= 1 - \lim_{m_2 \rightarrow 0^+} P(\text{TS:BP}) - \lim_{m_2 \rightarrow 0^+} P(\text{CS:BP}) \\
&= \lim_{m_2 \rightarrow 0^+} P(\text{E:BP}),
\end{aligned}$$

which is given by (S45).

Now we evaluate the limits of the three probabilities (S26), (S27), and (S28) when $m_2 \rightarrow 0^+$. By (S1), it is easy to show that

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_6) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{m_0(m_0 + m_1 + m_2) d_0 + m_1 m_2 d_1 - Z_\alpha \sigma (m_0 + m_1) \sqrt{2(m_0 + m_2)}}{\sqrt{2m_2} \sigma \sqrt{m_0 + m_1} \sqrt{m_0 + m_1 + m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{m_0(m_0 + s) d_0 - Z_\alpha \sigma (m_0 + s) \sqrt{2(m_0 + m_2)}}{\sigma \sqrt{2m_2} (m_0 + s)} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2} \sqrt{m_0 + m_2} + m_0 d_0}{\sigma \sqrt{2} \sqrt{m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} P(\text{CS:BP}),
\end{aligned}$$

which is given by (S43). Similarly, by (S1), it is easy to show that

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_6) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-m_0(m_0 + m_1 + m_2)d_0 - m_1m_2d_1 - Z_\alpha\sigma(m_0 + m_1)\sqrt{2(m_0 + m_2)}}{\sqrt{2m_2}\sigma\sqrt{m_0 + m_1}\sqrt{m_0 + m_1 + m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-m_0(m_0 + s)d_0 - Z_\alpha\sigma(m_0 + s)\sqrt{2(m_0 + m_2)}}{\sigma\sqrt{2m_2}(m_0 + s)} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha\sigma\sqrt{2}\sqrt{m_0 + m_2} - m_0d_0}{\sigma\sqrt{2}\sqrt{m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} P(\text{TS:BP}),
\end{aligned}$$

which is given by (S44). Therefore,

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{E}:I_6) \tag{S47} \\
&= 1 - \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_6) - \lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_6) \\
&= 1 - \lim_{m_2 \rightarrow 0^+} P(\text{TS:BP}) - \lim_{m_2 \rightarrow 0^+} P(\text{CS:BP}) \\
&= \lim_{m_2 \rightarrow 0^+} P(\text{E:BP}),
\end{aligned}$$

which is given by (S45).

2.4. The proof of Proposition 4

In this subsection, we will evaluate the limits at point 0 of the probabilities of CS, TS, and E of the BCP and the 7th and 8th predictive powers.

Now we evaluate the limits of the three probabilities (S29), (S30), and (S31)

when $m_2 \rightarrow 0^+$. It is easy to show that

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{CS:BCP}) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{\delta \sqrt{m_2} - Z_\alpha \sigma \sqrt{2 \left(\frac{m_0 + m_1}{m_2} + 1 \right) + \frac{m_0 d_0 + m_1 d_1}{\sqrt{m_2}}}}{\sigma \sqrt{2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2} \frac{\sqrt{m_0 + s}}{\sqrt{m_2}} + \frac{m_0 d_0 + s d_1}{\sqrt{m_2}}}{\sigma \sqrt{2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2} \sqrt{m_0 + s} + m_0 d_0 + s d_1}{\sigma \sqrt{2} \sqrt{m_2}} \right].
\end{aligned}$$

We have

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} \frac{-Z_\alpha \sigma \sqrt{2} \sqrt{m_0 + s} + m_0 d_0 + s d_1}{\sigma \sqrt{2} \sqrt{m_2}} \\
&= \begin{cases} \infty, & \text{if } -Z_\alpha \sigma \sqrt{2} \sqrt{m_0 + s} + m_0 d_0 + s d_1 > 0, \\ 0, & \text{if } -Z_\alpha \sigma \sqrt{2} \sqrt{m_0 + s} + m_0 d_0 + s d_1 = 0, \\ -\infty, & \text{if } -Z_\alpha \sigma \sqrt{2} \sqrt{m_0 + s} + m_0 d_0 + s d_1 < 0, \end{cases} \\
&= \begin{cases} \infty, & \text{if } m_0 d_0 + s d_1 > Z_\alpha \sigma \sqrt{2} (m_0 + s), \\ 0, & \text{if } m_0 d_0 + s d_1 = Z_\alpha \sigma \sqrt{2} (m_0 + s), \\ -\infty, & \text{if } m_0 d_0 + s d_1 < Z_\alpha \sigma \sqrt{2} (m_0 + s). \end{cases}
\end{aligned}$$

Hence,

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{CS:BCP}) \tag{S48} \\
&= \begin{cases} 1, & \text{if } m_0 d_0 + s d_1 > Z_\alpha \sigma \sqrt{2} (m_0 + s), \\ 0.5, & \text{if } m_0 d_0 + s d_1 = Z_\alpha \sigma \sqrt{2} (m_0 + s), \\ 0, & \text{if } m_0 d_0 + s d_1 < Z_\alpha \sigma \sqrt{2} (m_0 + s). \end{cases}
\end{aligned}$$

Similarly, it is easy to show that

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{TS:BCP}) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2 \left(\frac{m_0+m_1}{m_2} + 1 \right)} - \frac{m_0 d_0 + m_1 d_1}{\sqrt{m_2}} - \delta \sqrt{m_2}}{\sigma \sqrt{2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2} \frac{\sqrt{m_0+s}}{\sqrt{m_2}} - \frac{m_0 d_0 + s d_1}{\sqrt{m_2}}}{\sigma \sqrt{2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2} \sqrt{m_0+s} - (m_0 d_0 + s d_1)}{\sigma \sqrt{2} \sqrt{m_2}} \right].
\end{aligned}$$

We have

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} \frac{-Z_\alpha \sigma \sqrt{2} \sqrt{m_0+s} - (m_0 d_0 + s d_1)}{\sigma \sqrt{2} \sqrt{m_2}} \\
&= \begin{cases} \infty, & \text{if } -Z_\alpha \sigma \sqrt{2} \sqrt{m_0+s} - (m_0 d_0 + s d_1) > 0, \\ 0, & \text{if } -Z_\alpha \sigma \sqrt{2} \sqrt{m_0+s} - (m_0 d_0 + s d_1) = 0, \\ -\infty, & \text{if } -Z_\alpha \sigma \sqrt{2} \sqrt{m_0+s} - (m_0 d_0 + s d_1) < 0, \end{cases} \\
&= \begin{cases} \infty, & \text{if } m_0 d_0 + s d_1 < -Z_\alpha \sigma \sqrt{2} (m_0 + s), \\ 0, & \text{if } m_0 d_0 + s d_1 = -Z_\alpha \sigma \sqrt{2} (m_0 + s), \\ -\infty, & \text{if } m_0 d_0 + s d_1 > -Z_\alpha \sigma \sqrt{2} (m_0 + s). \end{cases}
\end{aligned}$$

Hence,

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{TS:BCP}) \tag{S49} \\
&= \begin{cases} 1, & \text{if } m_0 d_0 + s d_1 < -Z_\alpha \sigma \sqrt{2} (m_0 + s), \\ 0.5, & \text{if } m_0 d_0 + s d_1 = -Z_\alpha \sigma \sqrt{2} (m_0 + s), \\ 0, & \text{if } m_0 d_0 + s d_1 > -Z_\alpha \sigma \sqrt{2} (m_0 + s). \end{cases}
\end{aligned}$$

Therefore,

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{E:BCP}) \tag{S50} \\
&= 1 - \lim_{m_2 \rightarrow 0^+} P(\text{TS:BCP}) - \lim_{m_2 \rightarrow 0^+} P(\text{CS:BCP}) \\
&= \begin{cases} 1 - 1 - 0 = 0, & \text{if } m_0 d_0 + s d_1 < -Z_\alpha \sigma \sqrt{2(m_0 + s)}, \\ 1 - 0.5 - 0 = 0.5, & \text{if } m_0 d_0 + s d_1 = -Z_\alpha \sigma \sqrt{2(m_0 + s)}, \\ 1 - 0 - 0 = 1, & \text{if } -Z_\alpha \sigma \sqrt{2(m_0 + s)} < m_0 d_0 + s d_1 < Z_\alpha \sigma \sqrt{2(m_0 + s)}, \\ 1 - 0 - 0.5 = 0.5, & \text{if } m_0 d_0 + s d_1 = Z_\alpha \sigma \sqrt{2(m_0 + s)}, \\ 1 - 0 - 1 = 0, & \text{if } m_0 d_0 + s d_1 > Z_\alpha \sigma \sqrt{2(m_0 + s)}. \end{cases}
\end{aligned}$$

Now we evaluate the limits of the three probabilities (S32), (S33), and (S34) when $m_2 \rightarrow 0^+$. By (S1), it is easy to show that

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{CS:}I_7) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{(m_0 + m_2) d_0 + m_1 d_1 - Z_\alpha \sigma \sqrt{2(m_0 + m_1 + m_2)}}{\sqrt{2} m_2 \sigma} \sqrt{\frac{m_0}{m_0 + m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2} \sqrt{m_0 + s} + m_0 d_0 + s d_1}{\sigma \sqrt{2} \sqrt{m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} P(\text{CS:BCP}),
\end{aligned}$$

which is given by (S48). Similarly, by (S1), it is easy to show that

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{TS:}I_7) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-(m_0 + m_2) d_0 - m_1 d_1 - Z_\alpha \sigma \sqrt{2(m_0 + m_1 + m_2)}}{\sqrt{2} m_2 \sigma} \sqrt{\frac{m_0}{m_0 + m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2} \sqrt{m_0 + s} - (m_0 d_0 + s d_1)}{\sigma \sqrt{2} \sqrt{m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} P(\text{TS:BCP}),
\end{aligned}$$

which is given by (S49). Therefore,

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{E}:I_7) \\
&= 1 - \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_7) - \lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_7) \\
&= 1 - \lim_{m_2 \rightarrow 0^+} P(\text{TS:BCP}) - \lim_{m_2 \rightarrow 0^+} P(\text{CS:BCP}) \\
&= \lim_{m_2 \rightarrow 0^+} P(\text{E:BCP}),
\end{aligned} \tag{S51}$$

which is given by (S50).

Now we evaluate the limits of the three probabilities (S35), (S36), and (S37) when $m_2 \rightarrow 0^+$. By (S1), it is easy to show that

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_8) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{(m_0 + m_1 + m_2)(m_0 d_0 + m_1 d_1) - Z_\alpha \sigma (m_0 + m_1) \sqrt{2(m_0 + m_1 + m_2)}}{\sqrt{2} m_2 \sigma \sqrt{m_0 + m_1} \sqrt{m_0 + m_1 + m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{(m_0 + s)(m_0 d_0 + s d_1) - Z_\alpha \sigma (m_0 + s) \sqrt{2(m_0 + s)}}{\sigma \sqrt{2} m_2 (m_0 + s)} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2} \sqrt{m_0 + s} + m_0 d_0 + s d_1}{\sigma \sqrt{2} \sqrt{m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} P(\text{CS:BCP}),
\end{aligned}$$

which is given by (S48). Similarly, by (S1), it is easy to show that

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_8) \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-(m_0 + m_1 + m_2)(m_0 d_0 + m_1 d_1) - Z_\alpha \sigma (m_0 + m_1) \sqrt{2(m_0 + m_1 + m_2)}}{\sqrt{2} m_2 \sigma \sqrt{m_0 + m_1} \sqrt{m_0 + m_1 + m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-(m_0 + s)(m_0 d_0 + s d_1) - Z_\alpha \sigma (m_0 + s) \sqrt{2(m_0 + s)}}{\sigma \sqrt{2} m_2 (m_0 + s)} \right] \\
&= \lim_{m_2 \rightarrow 0^+} \Phi \left[\frac{-Z_\alpha \sigma \sqrt{2} \sqrt{m_0 + s} - (m_0 d_0 + s d_1)}{\sigma \sqrt{2} \sqrt{m_2}} \right] \\
&= \lim_{m_2 \rightarrow 0^+} P(\text{TS:BCP}),
\end{aligned}$$

which is given by (S49). Therefore,

$$\begin{aligned}
& \lim_{m_2 \rightarrow 0^+} P(E:I_8) \\
&= 1 - \lim_{m_2 \rightarrow 0^+} P(TS:I_8) - \lim_{m_2 \rightarrow 0^+} P(CS:I_8) \\
&= 1 - \lim_{m_2 \rightarrow 0^+} P(TS:BCP) - \lim_{m_2 \rightarrow 0^+} P(CS:BCP) \\
&= \lim_{m_2 \rightarrow 0^+} P(E:BCP),
\end{aligned} \tag{S52}$$

which is given by (S50).

3. Web Figure S1

The graph of the three limits (10), (11), and (12) by one plot is depicted in Web Figure S1.

[Web Figure S1 about here.]

4. Web Figure S2

The probabilities of CS, TS, and E for the 8 predictive powers as functions of m_2 under the sceptical prior are plotted in Web Figure S2. From the figure, we observe the following facts.

(1) For I_7 and I_8 , we observe

$$\begin{aligned}
\lim_{m_2 \rightarrow 0^+} P(CS:I_7.s) &= \lim_{m_2 \rightarrow 0^+} P(CS:I_8.s) = \lim_{m_2 \rightarrow 0^+} P(CS:BCP.s) = 1, \\
\lim_{m_2 \rightarrow 0^+} P(TS:I_7.s) &= \lim_{m_2 \rightarrow 0^+} P(TS:I_8.s) = \lim_{m_2 \rightarrow 0^+} P(TS:BCP.s) = 0, \\
\lim_{m_2 \rightarrow 0^+} P(E:I_7.s) &= \lim_{m_2 \rightarrow 0^+} P(E:I_8.s) = \lim_{m_2 \rightarrow 0^+} P(E:BCP.s) = 0.
\end{aligned}$$

(2) The probabilities of TS for all the 8 predictive powers are increasing functions of m_2 . The probabilities of CS are increasing functions of m_2 for the 1st and 5th predictive powers, they are decreasing functions of m_2 for the 3rd, 4th, 7th, and 8th predictive powers, and they are first increasing and then decreasing functions of m_2 for the 2nd and 6th predictive

powers. The probabilities of E are decreasing functions of m_2 for the 1st, 2nd, 5th, and 6th predictive powers, and they are first increasing and then decreasing functions of m_2 for the 3rd, 4th, 7th, and 8th predictive powers.

- (3) When m_2 is small (imprecision), the probabilities of E are large for the 1st, 2nd, 5th, and 6th predictive powers, and thus it is hard to discriminate between CS and TS. That is, the 1st, 2nd, 5th, and 6th predictive powers have the imprecision issues. However, when m_2 is small, the probabilities of E are small for the 3rd, 4th, 7th, and 8th predictive powers, and now they will predict CS.

[Web Figure S2 about here.]

5. Web Figure S3

The probabilities of CS, TS, and E for the 8 predictive powers as functions of m_2 under the optimistic prior are plotted in Web Figure S3. Similar to Web Figure S2, from Web Figure S3, we observe the following facts.

- (1) For I_7 and I_8 , we observe

$$\begin{aligned}\lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_7\text{-o}) &= \lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_8\text{-o}) = \lim_{m_2 \rightarrow 0^+} P(\text{CS:BCP-o}) = 0, \\ \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_7\text{-o}) &= \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_8\text{-o}) = \lim_{m_2 \rightarrow 0^+} P(\text{TS:BCP-o}) = 0, \\ \lim_{m_2 \rightarrow 0^+} P(\text{E}:I_7\text{-o}) &= \lim_{m_2 \rightarrow 0^+} P(\text{E}:I_8\text{-o}) = \lim_{m_2 \rightarrow 0^+} P(\text{E:BCP-o}) = 1.\end{aligned}$$

- (2) The probabilities of TS for all the 8 predictive powers are increasing functions of m_2 . The probabilities of CS are decreasing functions of m_2 for the 1st, 3rd, and 4th predictive powers, they are first increasing and then decreasing functions of m_2 for the 2nd and 8th predictive powers, and they are almost 0 as functions of m_2 for the 5th, 6th, and 7th predictive powers. The probabilities of E are decreasing functions of m_2 for the 1st, 2nd, 5th, 6th, 7th, and 8th predictive powers, and they are first increasing and then decreasing functions of m_2 for the 3rd and 4th predictive powers.

- (3) When m_2 is small (imprecision), the probabilities of E are large for the 1st, 2nd, 5th, 6th, 7th, and 8th predictive powers, and thus it is hard to discriminate between CS and TS. That is, the 1st, 2nd, 5th, 6th, 7th, and 8th predictive powers have the imprecision issues. However, when m_2 is small, the probabilities of E are small for the 3rd and 4th predictive powers, and now they will predict CS.

[Web Figure S3 about here.]

Comparing Web Figure S2 and Web Figure S3, we observe the following facts.

- (1) The three probabilities (probabilities of CS, TS, and E) sum to 1 for all the 8 predictive powers.
- (2) For I_1 and I_2 , we observe

$$\begin{aligned}\lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_1\text{-s}) &= \lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_2\text{-s}) = \lim_{m_2 \rightarrow 0^+} P(\text{CS:CP}) \\ &= \lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_1\text{-o}) = \lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_2\text{-o}) = \alpha,\end{aligned}$$

$$\begin{aligned}\lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_1\text{-s}) &= \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_2\text{-s}) = \lim_{m_2 \rightarrow 0^+} P(\text{TS:CP}) \\ &= \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_1\text{-o}) = \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_2\text{-o}) = \alpha,\end{aligned}$$

$$\begin{aligned}\lim_{m_2 \rightarrow 0^+} P(\text{E}:I_1\text{-s}) &= \lim_{m_2 \rightarrow 0^+} P(\text{E}:I_2\text{-s}) = \lim_{m_2 \rightarrow 0^+} P(\text{E:CP}) \\ &= \lim_{m_2 \rightarrow 0^+} P(\text{E}:I_1\text{-o}) = \lim_{m_2 \rightarrow 0^+} P(\text{E}:I_2\text{-o}) = 1 - 2\alpha.\end{aligned}$$

For I_3 and I_4 , we observe

$$\begin{aligned}\lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_3\text{-s}) &= \lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_4\text{-s}) = \lim_{m_2 \rightarrow 0^+} P(\text{CS:CCP}) \\ &= \lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_3\text{-o}) = \lim_{m_2 \rightarrow 0^+} P(\text{CS}:I_4\text{-o}) = 1,\end{aligned}$$

$$\begin{aligned}\lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_3\text{-s}) &= \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_4\text{-s}) = \lim_{m_2 \rightarrow 0^+} P(\text{TS:CCP}) \\ &= \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_3\text{-o}) = \lim_{m_2 \rightarrow 0^+} P(\text{TS}:I_4\text{-o}) = 0,\end{aligned}$$

$$\begin{aligned}\lim_{m_2 \rightarrow 0^+} P(E:I_3.s) &= \lim_{m_2 \rightarrow 0^+} P(E:I_4.s) = \lim_{m_2 \rightarrow 0^+} P(E:CCP) \\ &= \lim_{m_2 \rightarrow 0^+} P(E:I_3.o) = \lim_{m_2 \rightarrow 0^+} P(E:I_4.o) = 0.\end{aligned}$$

For I_5 and I_6 , we observe

$$\begin{aligned}\lim_{m_2 \rightarrow 0^+} P(CS:I_5.s) &= \lim_{m_2 \rightarrow 0^+} P(CS:I_6.s) = \lim_{m_2 \rightarrow 0^+} P(CS:BP.s) = \lim_{m_2 \rightarrow 0^+} P(CS:BP.o) \\ &= \lim_{m_2 \rightarrow 0^+} P(CS:I_5.o) = \lim_{m_2 \rightarrow 0^+} P(CS:I_6.o) = 0,\end{aligned}$$

$$\begin{aligned}\lim_{m_2 \rightarrow 0^+} P(TS:I_5.s) &= \lim_{m_2 \rightarrow 0^+} P(TS:I_6.s) = \lim_{m_2 \rightarrow 0^+} P(TS:BP.s) = \lim_{m_2 \rightarrow 0^+} P(TS:BP.o) \\ &= \lim_{m_2 \rightarrow 0^+} P(TS:I_5.o) = \lim_{m_2 \rightarrow 0^+} P(TS:I_6.o) = 0,\end{aligned}$$

$$\begin{aligned}\lim_{m_2 \rightarrow 0^+} P(E:I_5.s) &= \lim_{m_2 \rightarrow 0^+} P(E:I_6.s) = \lim_{m_2 \rightarrow 0^+} P(E:BP.s) = \lim_{m_2 \rightarrow 0^+} P(E:BP.o) \\ &= \lim_{m_2 \rightarrow 0^+} P(E:I_5.o) = \lim_{m_2 \rightarrow 0^+} P(E:I_6.o) = 1.\end{aligned}$$

- (3) The 1st and 2nd (3rd and 4th, 5th and 6th, and 7th and 8th) predictive powers display similar patterns of increasing and decreasing characteristics.
- (4) For the 1st and 5th predictive powers, they only utilize the historical data, and thus the range of m_2 is $[0, 200]$ with $m_2 = s = 115$ being marked in the plot (\circ , \triangle , and $+$ for CS, TS, and E, respectively). For the other 6 predictive powers, they utilize both the historical data and the interim data, and thus the range of m_2 is $[0, s] = [0, 115]$ with $m_2 = m_2^r = 69$ being marked in the plot (\circ , \triangle , and $+$ for CS, TS, and E, respectively).
- (5) The sceptical prior favors control, and thus $P(CS:I_i.s)$ is larger than $P(CS:I_i.o)$ for $i = 1, \dots, 8$. The optimistic prior favors treatment, and thus $P(TS:I_i.o)$ is larger than $P(TS:I_i.s)$ for $i = 1, \dots, 8$.
- (6) The $P(E:I_i.s)$ and $P(E:I_i.o)$ display similar patterns of increasing and decreasing characteristics as functions of m_2 for $i = 1, 2, 3, 4, 5, 6$. However, the $P(E:I_i.s)$ and $P(E:I_i.o)$ display different patterns of increasing and decreasing characteristics as functions of m_2 for $i = 7, 8$.

- (7) When m_2 is small (imprecision), the $P(E:I_i.s)$ and $P(E:I_i.o)$ are similar for $i = 1, 2, 3, 4, 5, 6$. However, the $P(E:I_i.s)$ and $P(E:I_i.o)$ are quite different for $i = 7, 8$.

6. Web Table S1

The probabilities of CS, TS, and E for CP, CCP, BP, and BCP as functions of m_2 when $\delta = 1$ which favors control are summarized in Web Table S1. This table reports the numerical values of the probabilities in Figure 3. There are some NA values in the table for CCP and BCP, because they utilize the interim data and the range of m_2 is $[0, s] = [0, 115]$.

[Web Table S1 about here.]

7. Web Table S2

The probabilities of CS, TS, and E for CP, CCP, BP, and BCP as functions of m_2 when $\delta = -1$ which favors treatment are summarized in Web Table S2. This table reports the numerical values of the probabilities in Figure 4. There are some NA values in the table for CCP and BCP, because they utilize the interim data and the range of m_2 is $[0, s] = [0, 115]$.

[Web Table S2 about here.]

8. Web Table S3

The probabilities of CS, TS, and E for CP, CCP, BP, and BCP as functions of m_2 when $\delta = 0$ which favors equivocal are summarized in Web Table S3. This table reports the numerical values of the probabilities in Figure 5. There are some NA values in the table for CCP and BCP, because they utilize the interim data and the range of m_2 is $[0, s] = [0, 115]$.

[Web Table S3 about here.]

9. Web Table S4

The probabilities of CS, TS, and E for the 8 predictive powers as functions of m_2 under the sceptical prior are summarized in Web Table S4. This table reports the numerical values of the probabilities in Web Figure S2. There are some NA values in the table for the 2nd, 3rd, 4th, 6th, 7th, and 8th predictive powers, because they utilize the interim data and the range of m_2 is $[0, s] = [0, 115]$.

[Web Table S4 about here.]

10. Web Table S5

The probabilities of CS, TS, and E for the 8 predictive powers as functions of m_2 under the optimistic prior are summarized in Web Table S5. This table reports the numerical values of the probabilities in Web Figure S3. There are some NA values in the table for the 2nd, 3rd, 4th, 6th, 7th, and 8th predictive powers, because they utilize the interim data and the range of m_2 is $[0, s] = [0, 115]$.

[Web Table S5 about here.]

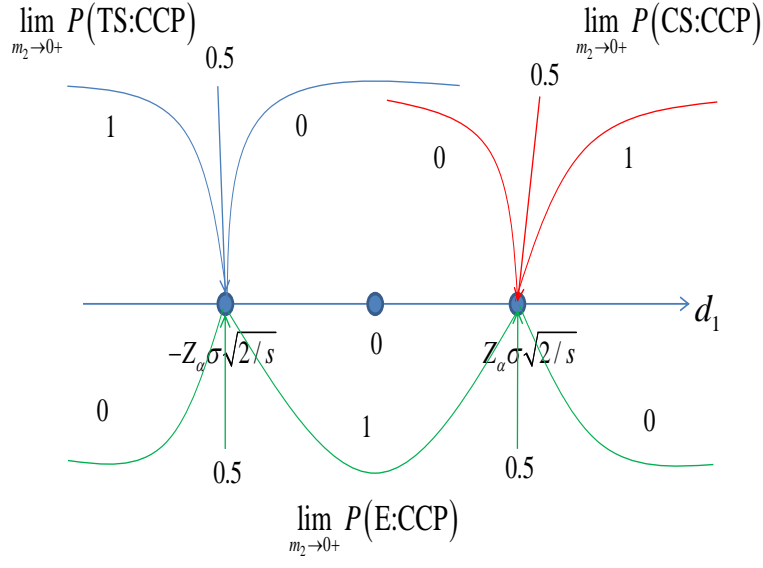


Figure S1: (Web) The graph of the three limits (10), (11), and (12) by one plot.

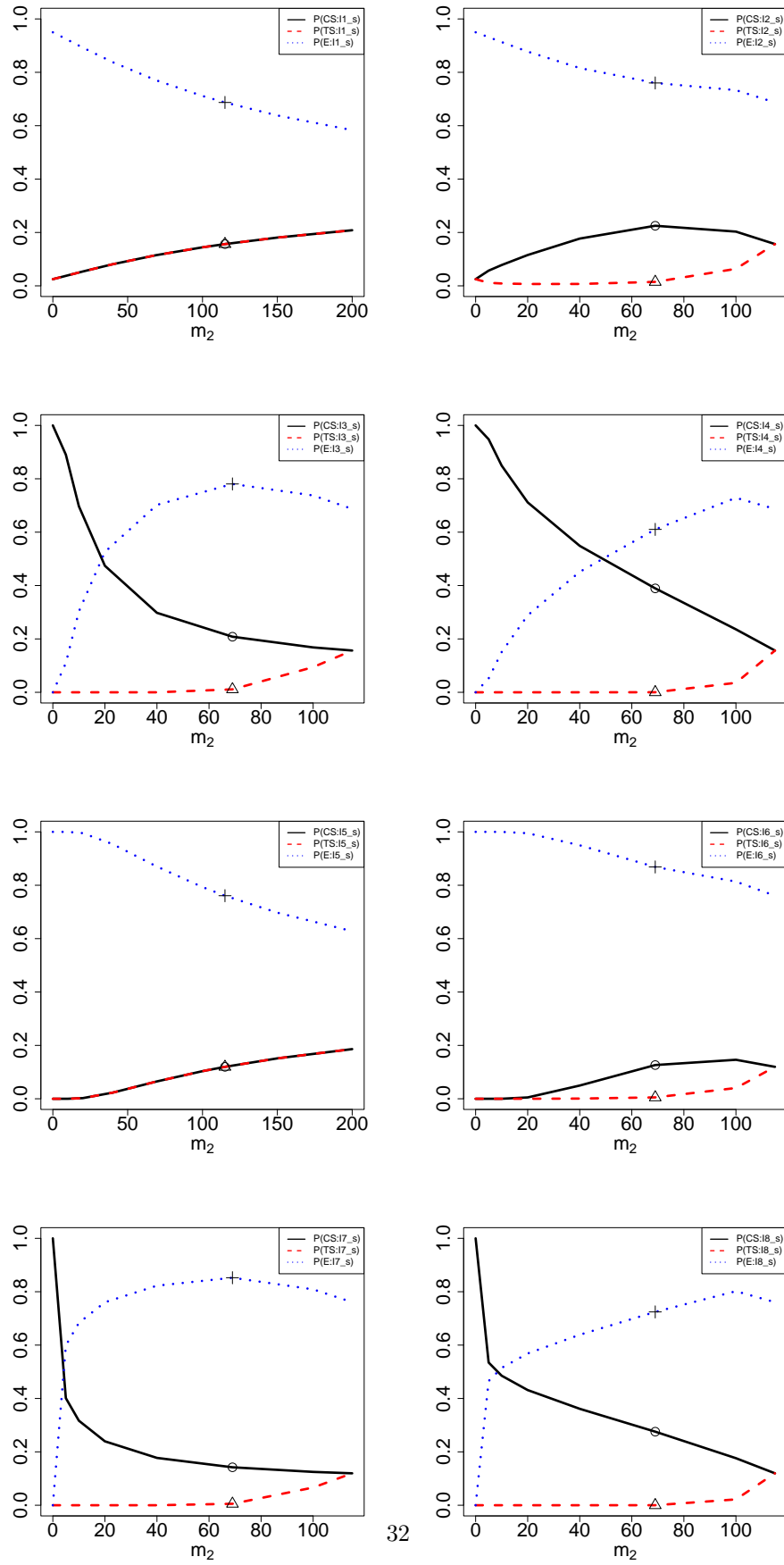


Figure S2: (Web) The probabilities of CS, TS, and E for the 8 predictive powers as functions of m_2 under the sceptical prior.

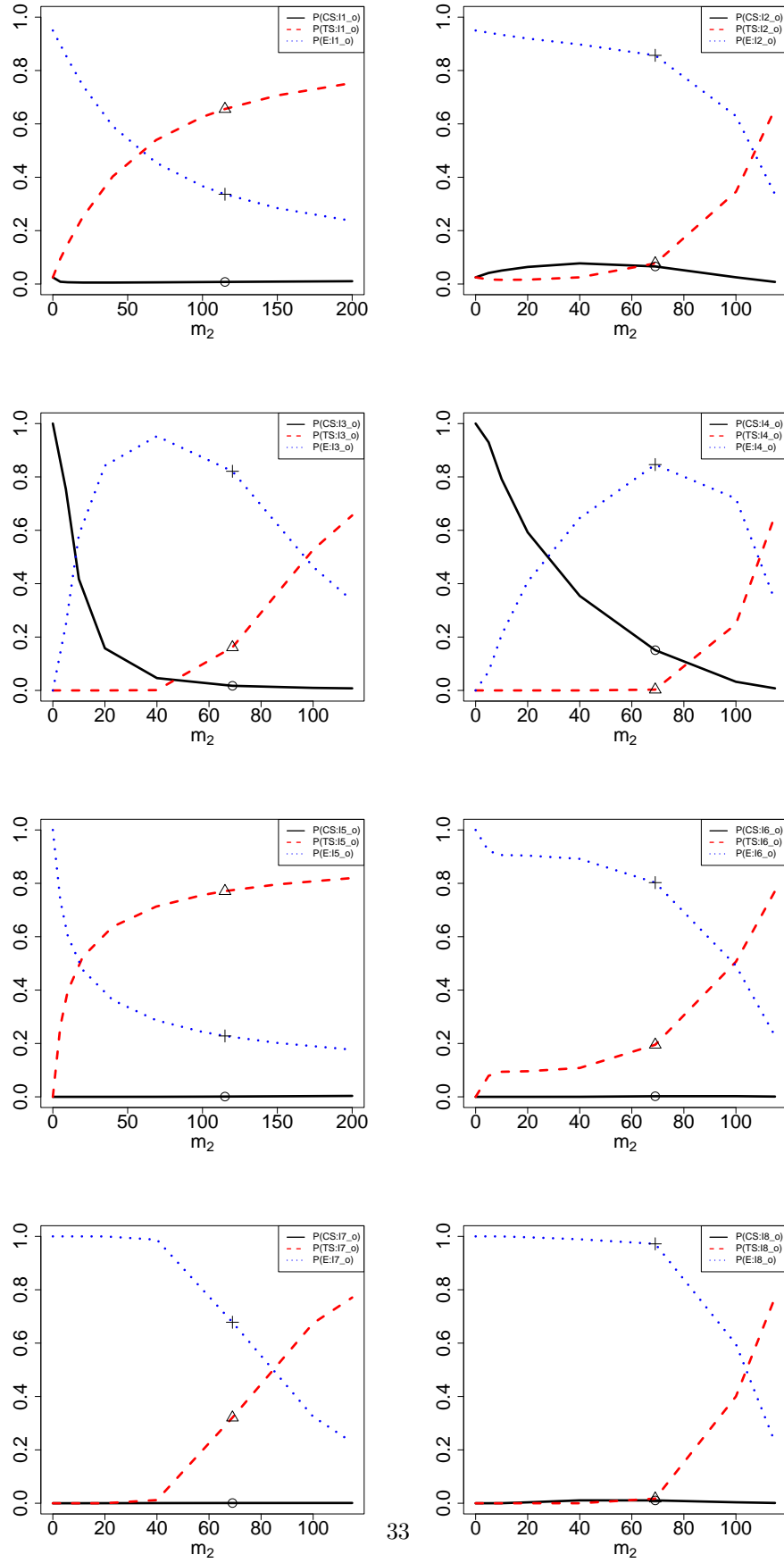


Figure S3: (Web) The probabilities of CS, TS, and E for the 8 predictive powers as functions of m_2 under the optimistic prior.

Table S1: (Web) The probabilities of CS, TS, and E for CP, CCP, BP, and BCP as functions of m_2 when $\delta = 1$ which favors control.

m_2	0	5	10	20	40	69	100	115	150	200
$P(\text{CS:CP})$	0.025	0.200	0.352	0.609	0.885	0.986	0.999	1.000	1.000	1.000
$P(\text{TS:CP})$	0.025	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$P(\text{E:CP})$	0.950	0.799	0.647	0.391	0.115	0.014	0.001	0.000	0.000	0.000
$P(\text{CS:CCP})$	1.000	0.992	0.984	0.984	0.992	0.998	0.999	1.000	NA	NA
$P(\text{TS:CCP})$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	NA	NA
$P(\text{E:CCP})$	0.000	0.008	0.016	0.016	0.008	0.002	0.001	0.000	NA	NA
$P(\text{CS:BP}_s)$	0.000	0.000	0.002	0.115	0.642	0.953	0.996	0.999	1.000	1.000
$P(\text{TS:BP}_s)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$P(\text{E:BP}_s)$	1.000	1.000	0.998	0.885	0.358	0.047	0.004	0.001	0.000	0.000
$P(\text{CS:BP}_o)$	0.000	0.000	0.000	0.000	0.095	0.655	0.946	0.982	0.999	1.000
$P(\text{TS:BP}_o)$	0.000	0.009	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$P(\text{E:BP}_o)$	1.000	0.991	0.996	0.999	0.905	0.345	0.054	0.018	0.001	0.000
$P(\text{CS:BCP}_s)$	1.000	0.803	0.853	0.915	0.969	0.992	0.998	0.999	NA	NA
$P(\text{TS:BCP}_s)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	NA	NA
$P(\text{E:BCP}_s)$	0.000	0.197	0.147	0.085	0.031	0.008	0.002	0.001	NA	NA
$P(\text{CS:BCP}_o)$	0.000	0.000	0.011	0.160	0.575	0.871	0.965	0.982	NA	NA
$P(\text{TS:BCP}_o)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	NA	NA
$P(\text{E:BCP}_o)$	1.000	1.000	0.989	0.840	0.425	0.129	0.035	0.018	NA	NA

Table S2: (Web) The probabilities of CS, TS, and E for CP, CCP, BP, and BCP as functions of m_2 when $\delta = -1$ which favors treatment.

m_2	0	5	10	20	40	69	100	115	150	200
$P(\text{CS:CP})$	0.025	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$P(\text{TS:CP})$	0.025	0.200	0.352	0.609	0.885	0.986	0.999	1.000	1.000	1.000
$P(\text{E:CP})$	0.950	0.799	0.647	0.391	0.115	0.014	0.001	0.000	0.000	0.000
$P(\text{CS:CCP})$	1.000	0.572	0.157	0.010	0.000	0.000	0.000	0.000	NA	NA
$P(\text{TS:CCP})$	0.000	0.000	0.000	0.000	0.003	0.662	0.995	1.000	NA	NA
$P(\text{E:CCP})$	0.000	0.428	0.843	0.990	0.997	0.338	0.005	0.000	NA	NA
$P(\text{CS:BP}_{\text{S}})$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$P(\text{TS:BP}_{\text{S}})$	0.000	0.000	0.002	0.115	0.642	0.953	0.996	0.999	1.000	1.000
$P(\text{E:BP}_{\text{S}})$	1.000	1.000	0.998	0.885	0.358	0.047	0.004	0.001	0.000	0.000
$P(\text{CS:BP}_{\text{O}})$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$P(\text{TS:BP}_{\text{O}})$	0.000	0.452	0.686	0.879	0.979	0.998	1.000	1.000	1.000	1.000
$P(\text{E:BP}_{\text{O}})$	1.000	0.548	0.314	0.121	0.021	0.002	0.000	0.000	0.000	0.000
$P(\text{CS:BCP}_{\text{S}})$	1.000	0.083	0.017	0.001	0.000	0.000	0.000	0.000	NA	NA
$P(\text{TS:BCP}_{\text{S}})$	0.000	0.000	0.000	0.000	0.000	0.499	0.987	0.999	NA	NA
$P(\text{E:BCP}_{\text{S}})$	0.000	0.917	0.983	0.999	1.000	0.501	0.013	0.001	NA	NA
$P(\text{CS:BCP}_{\text{O}})$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	NA	NA
$P(\text{TS:BCP}_{\text{O}})$	0.000	0.000	0.000	0.000	0.053	0.898	0.999	1.000	NA	NA
$P(\text{E:BCP}_{\text{O}})$	1.000	1.000	1.000	1.000	0.947	0.102	0.001	0.000	NA	NA

Table S3: (Web) The probabilities of CS, TS, and E for CP, CCP, BP, and BCP as functions of m_2 when $\delta = 0$ which favors equivocal.

m_2	0	5	10	20	40	69	100	115	150	200
$P(\text{CS:CP})$	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
$P(\text{TS:CP})$	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
$P(\text{E:CP})$	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950
$P(\text{CS:CCP})$	1.000	0.903	0.717	0.468	0.228	0.092	0.038	0.025	NA	NA
$P(\text{TS:CCP})$	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.025	NA	NA
$P(\text{E:CCP})$	0.000	0.097	0.283	0.532	0.772	0.907	0.955	0.950	NA	NA
$P(\text{CS:BP}_s)$	0.000	0.000	0.000	0.000	0.003	0.007	0.010	0.011	0.013	0.016
$P(\text{TS:BP}_s)$	0.000	0.000	0.000	0.000	0.003	0.007	0.010	0.011	0.013	0.016
$P(\text{E:BP}_s)$	1.000	1.000	1.000	0.999	0.995	0.987	0.980	0.978	0.973	0.969
$P(\text{CS:BP}_o)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.002
$P(\text{TS:BP}_o)$	0.000	0.108	0.136	0.143	0.131	0.114	0.102	0.097	0.089	0.080
$P(\text{E:BP}_o)$	1.000	0.892	0.864	0.857	0.869	0.886	0.898	0.902	0.910	0.918
$P(\text{CS:BCP}_s)$	1.000	0.396	0.298	0.194	0.097	0.040	0.017	0.011	NA	NA
$P(\text{TS:BCP}_s)$	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.011	NA	NA
$P(\text{E:BCP}_s)$	0.000	0.604	0.702	0.806	0.903	0.960	0.980	0.978	NA	NA
$P(\text{CS:BCP}_o)$	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	NA	NA
$P(\text{TS:BCP}_o)$	0.000	0.000	0.000	0.000	0.000	0.002	0.043	0.097	NA	NA
$P(\text{E:BCP}_o)$	1.000	1.000	1.000	0.999	0.999	0.997	0.956	0.902	NA	NA

Table S4: (Web) The probabilities of CS, TS, and E for the 8 predictive powers as functions of m_2 under the sceptical prior.

m_2	0	5	10	20	40	69	100	115	150	200
$P(\text{CS}:I_{1-s})$	0.025	0.032	0.039	0.054	0.081	0.115	0.144	0.156	0.181	0.208
$P(\text{TS}:I_{1-s})$	0.025	0.032	0.039	0.054	0.081	0.115	0.144	0.156	0.181	0.208
$P(\text{E}:I_{1-s})$	0.950	0.936	0.921	0.893	0.838	0.770	0.711	0.687	0.638	0.583
$P(\text{CS}:I_{2-s})$	0.025	0.057	0.078	0.115	0.177	0.225	0.203	0.156	NA	NA
$P(\text{TS}:I_{2-s})$	0.025	0.011	0.009	0.007	0.007	0.015	0.064	0.156	NA	NA
$P(\text{E}:I_{2-s})$	0.950	0.932	0.913	0.878	0.816	0.760	0.733	0.687	NA	NA
$P(\text{CS}:I_{3-s})$	1.000	0.890	0.697	0.474	0.298	0.208	0.168	0.156	NA	NA
$P(\text{TS}:I_{3-s})$	0.000	0.000	0.000	0.000	0.000	0.011	0.094	0.156	NA	NA
$P(\text{E}:I_{3-s})$	0.000	0.110	0.303	0.526	0.702	0.781	0.738	0.687	NA	NA
$P(\text{CS}:I_{4-s})$	1.000	0.948	0.849	0.712	0.549	0.389	0.236	0.156	NA	NA
$P(\text{TS}:I_{4-s})$	0.000	0.000	0.000	0.000	0.000	0.000	0.035	0.156	NA	NA
$P(\text{E}:I_{4-s})$	0.000	0.052	0.151	0.288	0.451	0.610	0.729	0.687	NA	NA
$P(\text{CS}:I_{5-s})$	0.000	0.000	0.000	0.002	0.023	0.064	0.103	0.120	0.151	0.186
$P(\text{TS}:I_{5-s})$	0.000	0.000	0.000	0.002	0.023	0.064	0.103	0.120	0.151	0.186
$P(\text{E}:I_{5-s})$	1.000	1.000	1.000	0.995	0.954	0.871	0.793	0.761	0.697	0.628
$P(\text{CS}:I_{6-s})$	0.000	0.000	0.000	0.005	0.050	0.126	0.146	0.120	NA	NA
$P(\text{TS}:I_{6-s})$	0.000	0.000	0.000	0.000	0.001	0.005	0.040	0.120	NA	NA
$P(\text{E}:I_{6-s})$	1.000	1.000	1.000	0.995	0.950	0.869	0.814	0.761	NA	NA
$P(\text{CS}:I_{7-s})$	1.000	0.401	0.317	0.239	0.177	0.142	0.125	0.120	NA	NA
$P(\text{TS}:I_{7-s})$	0.000	0.000	0.000	0.000	0.000	0.005	0.066	0.120	NA	NA
$P(\text{E}:I_{7-s})$	0.000	0.599	0.683	0.761	0.823	0.852	0.809	0.761	NA	NA
$P(\text{CS}:I_{8-s})$	1.000	0.535	0.485	0.431	0.361	0.276	0.176	0.120	NA	NA
$P(\text{TS}:I_{8-s})$	0.000	0.000	0.000	0.000	0.000	0.000	0.022	0.120	NA	NA
$P(\text{E}:I_{8-s})$	0.000	0.465	0.515	0.569	0.639	0.724	0.802	0.761	NA	NA

Table S5: (Web) The probabilities of CS, TS, and E for the 8 predictive powers as functions of m_2 under the optimistic prior.

m_2	0	5	10	20	40	69	100	115	150	200
$P(\text{CS}:I_{1-o})$	0.025	0.008	0.006	0.005	0.005	0.006	0.007	0.008	0.009	0.010
$P(\text{TS}:I_{1-o})$	0.025	0.095	0.150	0.251	0.403	0.539	0.626	0.656	0.707	0.753
$P(\text{E}:I_{1-o})$	0.950	0.897	0.843	0.744	0.592	0.454	0.367	0.336	0.284	0.236
$P(\text{CS}:I_{2-o})$	0.025	0.041	0.050	0.064	0.077	0.066	0.025	0.008	NA	NA
$P(\text{TS}:I_{2-o})$	0.025	0.017	0.016	0.016	0.025	0.077	0.345	0.656	NA	NA
$P(\text{E}:I_{2-o})$	0.950	0.942	0.934	0.920	0.898	0.857	0.629	0.336	NA	NA
$P(\text{CS}:I_{3-o})$	1.000	0.754	0.417	0.158	0.046	0.017	0.010	0.008	NA	NA
$P(\text{TS}:I_{3-o})$	0.000	0.000	0.000	0.000	0.001	0.161	0.527	0.656	NA	NA
$P(\text{E}:I_{3-o})$	0.000	0.246	0.583	0.842	0.953	0.821	0.463	0.336	NA	NA
$P(\text{CS}:I_{4-o})$	1.000	0.930	0.792	0.592	0.354	0.151	0.032	0.008	NA	NA
$P(\text{TS}:I_{4-o})$	0.000	0.000	0.000	0.000	0.000	0.003	0.249	0.656	NA	NA
$P(\text{E}:I_{4-o})$	0.000	0.070	0.208	0.408	0.646	0.846	0.719	0.336	NA	NA
$P(\text{CS}:I_{5-o})$	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.003
$P(\text{TS}:I_{5-o})$	0.000	0.264	0.398	0.524	0.637	0.713	0.756	0.771	0.796	0.820
$P(\text{E}:I_{5-o})$	1.000	0.736	0.602	0.476	0.363	0.287	0.243	0.228	0.202	0.177
$P(\text{CS}:I_{6-o})$	0.000	0.000	0.000	0.000	0.000	0.002	0.002	0.001	NA	NA
$P(\text{TS}:I_{6-o})$	0.000	0.079	0.094	0.096	0.108	0.195	0.506	0.771	NA	NA
$P(\text{E}:I_{6-o})$	1.000	0.921	0.906	0.904	0.892	0.803	0.491	0.228	NA	NA
$P(\text{CS}:I_{7-o})$	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	NA	NA
$P(\text{TS}:I_{7-o})$	0.000	0.000	0.000	0.000	0.012	0.321	0.674	0.771	NA	NA
$P(\text{E}:I_{7-o})$	1.000	1.000	1.000	1.000	0.988	0.678	0.325	0.228	NA	NA
$P(\text{CS}:I_{8-o})$	0.000	0.000	0.000	0.003	0.011	0.011	0.004	0.001	NA	NA
$P(\text{TS}:I_{8-o})$	0.000	0.000	0.000	0.000	0.000	0.017	0.400	0.771	NA	NA
$P(\text{E}:I_{8-o})$	1.000	1.000	1.000	0.997	0.989	0.972	0.596	0.228	NA	NA