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Optimized np Attribute Control Chart Using Triple Sampling

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Abstract: This paper studies an attribute control chart for monitoring the number of nonconforming items using a triple sampling (TS-np) which has not yet been applied to attribute control charts. The chart design and procedure for the decision about the state of the process are given. Mathematical expressions for the average run length (ARL) for in-control and out-of-control processes and the average sample number (ASN) are given. A bi-objective genetic algorithm that seeks to minimize the ASN and the probability of type 2 error is implemented in order to optimize the design of the TS-np control chart. A comparison between TS-np, single sampling np (SS-np), double sampling np (DS-np), and multiple dependent state repetitive sampling (MDSRS) control charts is carried out in terms of the out-of-control average run length (ARL_1). Tables of ARL_1 values for TS-np are presented in comparison with MDSRS and DS-np for various scenarios. The operation of the proposed control chart is shown through simulated data. Finally, it is concluded that the proposed TS-np chart has a better performance in terms of ARL_1 detecting small and moderate shifts in the process nonconforming rate in-control (p_0) compared with MDSRS and DS-np.

Keywords: control chart; attributes control chart; triple sampling; double sampling; average run length; average sample number; np control chart

MSC: 62P30

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1. Introduction

Attribute charts have important applications. They are particularly useful in service industries and in nonmanufacturing quality-improvement efforts, because so many of the quality characteristics found in these environments are not easily measured on a numerical scale [1]. Here, we study a control chart for monitoring the number of nonconforming items. The traditional Shewhart np control charts are the statistical control scheme most commonly used for monitoring the number of nonconforming items [1].

In a traditional Shewhart np control chart, a sample of size n is taken and inspected, and the number of nonconforming items, usually denoted by "d", is counted. If $LCL < d < UCL$, where LCL is the lower control limit and UCL is the upper control limit, the process is denoted to be in-control; otherwise, the process is assumed to be out-of-control, and corrective action should be taken.

Some research has proposed improvements to the traditional Shewhart np control chart in terms of ARL_1 and ASN. References [2,3] proposed a double sampling control chart for small fraction defectives in-control (p_0), while [4] designed an economic method of an attribute control chart. In [5] an algorithm is presented for optimizing the design of the np control chart with curtailment. Reference [6] proposed a double inspection, where the first inspection decides the process status according to the number of nonconforming units found in a sample, and the second inspection makes a decision based on the location of a particular nonconforming unit in the sample.

In [7], a new control chart for attributes was proposed based on two different previously proposed control charts one using repetitive sampling (RS) [8] and another one using multiple dependent state sampling (MDS) [9]. The control chart proposed by [7] joins these two control charts and it uses a multiple dependent state with repetitive sampling (MDSRS) which is more efficient in terms of ARL_1 compared with MDS and repetitive sampling. Given the above and the topic or said paper, we will present a comparison between the proposed control chart in this work and the MDSRS control chart in Section 3.

In other papers such as [10–19], more proposals for improving the np control chart can be found.

The new proposal in this article, named triple sampling np, or TS-np, consists of applying a triple sampling in every subgroup instead of taking a single sample to inspect as is done in the traditional Shewhart np control charts. At each time interval (t), which is fixed by the practitioners, a large total sample is taken, and it is divided into three subsamples n_1, n_2, and n_3, for inspection. Since the control chart presents decision rules that use the partial information of a subsample, it is not always necessary to inspect samples n_2 or n_3, as explained in Section 2.

Triple sampling has been introduced in variables control charts but not yet in attributes control charts. Reference [20] implemented a design of double and triple sampling X-bar control charts using genetic algorithms, and it is concluded that TS charts were more efficient than DS charts in terms of minimizing the ASN. More recent research focused on triple sampling control charts can be found in [21–25].

As TS-np is an extension of the DS-np [2], it is necessary to realize a comparison between the TS-np and the DS-np chart in Section 3.

In this work, we used the bi-objective genetic algorithm (which has commonly been used for control charts) explained in Section 2. In [26–43] genetics algorithms are employed as tools to find an optimal solution to the control chart studied.

The structure of this manuscript is as follows. Section 2 shows the procedure, $ARLs$ and ASN expressions, and the optimal design for the proposed TS-np control chart. In Section 3, a comparative study is conducted considering the TS-np control chart versus the MDSRS and DS-np control charts proposed by [7] and [2], respectively. Section 4 shows the functionality and operation of TS-np using/; simulated data, and finally, in Section 5 the conclusions of the investigation are presented.

2. Design of TS-np Control Chart

The TS-np chart is designed to only detect increases in the proportion of conforming units (p), the reason why being that the design of the chart only considers control and warning limits (WL and UCL) in the upper sense. Furthermore, as with Shewhart-type control charts, the TS-np chart also assumes that the sampling is random and that the quality variable does not have a temporal autocorrelation structure. This is a theoretical assumption that the user must verify.

The design and procedure of the TS-np control chart are detailed below based on the procedure made in [3]:

Step 1 Take a total sample of size $n_1 + n_2 + n_3$ items extracted from the process.

Step 2 Select a first subsample of n_1 items.

Step 3 Inspect the first subsample. Denote d_1, the number of nonconforming items found in this subsample.

- 3.1 If $d_1 > UCL_1$, the process is marked as out of control, the sample inspection stops, and corrective action should be taken.
- 3.2 If $d_1 < WL_1$, the process is considered in control, the sample inspection stops, and step 1 should be restarted at the next sampling.
- 3.3 If $WL_1 < d_1 < UCL_1$, go to step 4.

Step 4 Select a second subsample of n_2 items.

Step 5 Inspect the second subsample. Denote d_2 as the number of non-conforming items found in this subsample. Let $(d_1 + d_2)$ denote the cumulative number of nonconforming items found in the two subsamples (with a subsample size n_1 and n_2, respectively).

5.1 If $(d_1 + d_2) > UCL_2$, the process is marked as out of control, the sample inspection stops, and corrective action should be taken.

5.2 If $(d_1 + d_2) < WL_2$, the process is considered in control, the sample inspection stops, and step 1 should be restarted at the next sampling.

5.3 Otherwise, if $WL_2 < (d_1 + d_2) < UCL_2$, go to step 6.

Step 6 A third subsample is formed with the remaining n_3 items.

Step 7 Inspect the subsample of n_3 items. Denote d_3 as the number of nonconforming items found in this subsample. Let $(d_1 + d_2 + d_3)$ denote the cumulative number of nonconforming items found in the three subsamples (with a subsample size n_1, n_2, and n_3, respectively).

7.1 If $(d_1 + d_2 + d_3) > UCL_3$, the process is marked as out of control, and corrective action should be taken.

7.2 Otherwise, if $(d_1 + d_2 + d_3) < UCL_3$, the process is considered in control and step 1 should be restarted at the next sampling.

To facilitate an understanding of the operation of the TS-np chart, Figures 1 and 2, respectively, show a flowchart that summarizes the operation algorithm and the visual scheme of the proposed control chart with its decision rules.

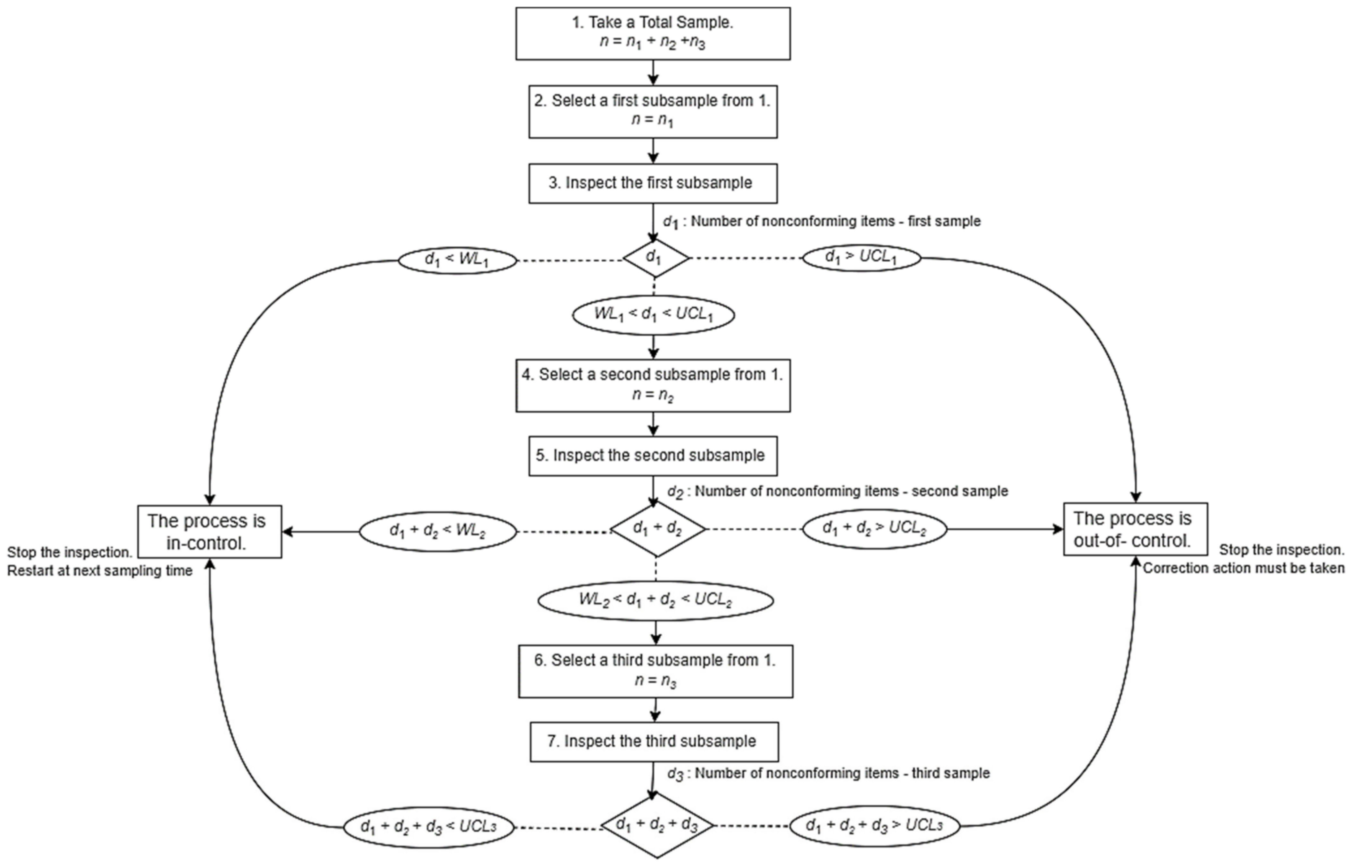

Figure 1. Flowchart—operation and decision rules of the TS-np chart.

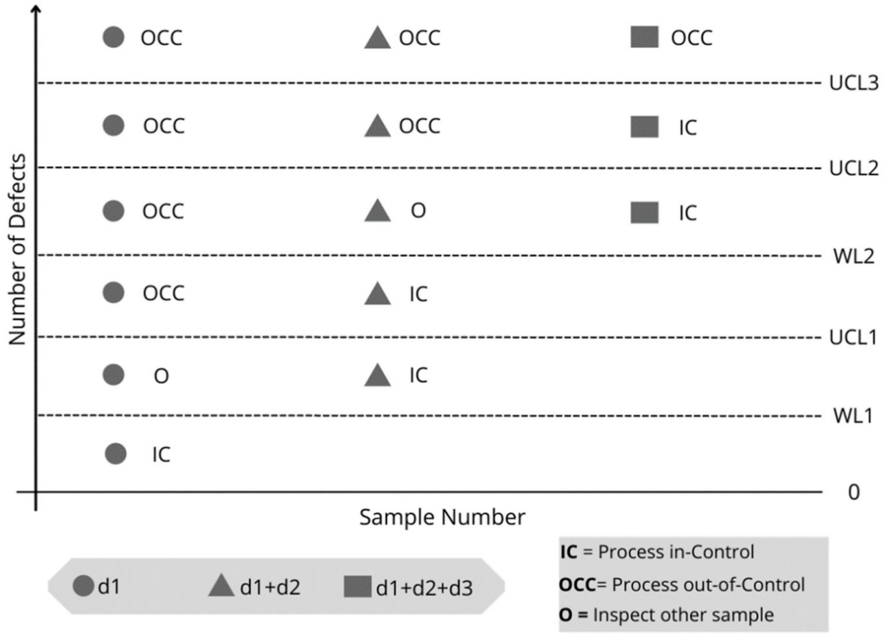

Figure 2. Visual scheme of the TS-np control chart.

The parameter UCL_i $(i = 1,\ 2,\ 3)$ defines the location of the control limits for the three inspection stages and WL_i $(i = 1,\ 2)$ defines the location of the warning limits for stages 1 and 2. These parameters, in addition of the sample size n_i $(i = 1,\ 2,\ 3)$, must be defined by the user.

In order to prevent any number of nonconforming items from being positioned exactly at any of the control or warning limits and generating ambiguity in the decision rule, it is established that these parameters should not take integer values. To this end, we suggest adjusting its value according to the following mathematical expressions

$$\begin{aligned} WL_1 &= \lfloor WL_1 \rfloor + 0.5 \\ UCL_1 &= \lceil UCL_1 \rceil - 0.5 \\ WL_2 &= \lfloor WL_2 \rfloor + 0.5 \\ UCL_2 &= \lceil UCL_2 \rceil - 0.5 \\ UCL_3 &= \lfloor UCL_3 \rfloor + 0.5 \end{aligned}$$

Note that the control limits are covered by the symbols $\lfloor\rfloor$ and $\lceil\rceil$. $\lfloor\rfloor$ represents the smallest integer value (i.e., the floor function), and $\lceil\rceil$ represents the largest integer value for the limit in consideration (i.e., the ceiling function). It should not be mandatory to add or subtract 0.5 to the control limits. Any value between 0 and 1 being added or subtracted will make the control limits non-integers. In this case, it has been decided to add and subtract 0.5 in order to have a better visual representation of these limits on the schemes.

2.1. The ARLs Expression

To derive the performance indicators of the TS-np chart, we will denote p as the fraction of non-conforming units generated by the process. Particularly $p = p_0$ is the value of this fraction when the process remains in an in-control state and $p_1 = p_0(1 + c)$, $c > 0$, is the value of this fraction when the process goes into an out-of-control state. Under this second situation, with the process in the out-of-control state, the constant $c = \frac{p_1}{p_0} - 1$ indicates the relative magnitude of the increase in the fraction of nonconforming units.

In a general context, the probability that the TS-np chart declares an in-control state for the process is:

$$P_a = P_{a1} + P_{a2} + P_{a3}, \tag{1}$$

where P_{ai} $(i = 1,\ 2,\ 3)$ denotes the probability that the in-control state will be declared at the i^{th} inspection stage. Each probability P_{ai} depends on p and on the value of the design parameters of the TS-np chart.

Expressions for $P_{a1},\ P_{a2},$ and P_{a3} are developed below, supported by the fact that at each inspection stage $i = 1,\ 2,\ 3$, the random variable d_i follows a binomial distribution with parameters n_i and p, respectively.

$$P_{a1} = \Pr(d_1 < WL_1) = \sum_{d_1=0}^{\lfloor WL_1 \rfloor} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1}$$

$$P_{a2} = \Pr[(WL_1 < d_1 < UCL_1) \cap (d_1 + d_2 < WL_2)]$$

$$P_{a2} = \sum_{j=\lceil WL_1 \rceil}^{\lfloor UCL_1 \rfloor} \Pr(d_2 < WL_2 - j) \times \Pr(d_1 = j)$$

$$P_{a2} = \sum_{d_1=\lceil WL_1 \rceil}^{\lfloor UCL_1 \rfloor} \left\{ \left[\binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} \right] \times \left[\sum_{d_2=0}^{\lfloor WL_2 \rfloor - d_1} \binom{n_2}{d_2} p^{d_2}(1-p)^{n_2-d_2} \right] \right\}$$

$$P_{a3} = \Pr[(WL_1 < d_1 < UCL_1) \cap (WL_2 < d_1 + d_2 < UCL_2) \cap (d_1 + d_2 + d_3 < UCL_3)]$$

$$P_{a3} = \sum_{i=\lceil WL_1 \rceil}^{\lfloor UCL_1 \rfloor} \left\{ \Pr(d_1 = i) \times \sum_{h=\lceil WL_1 \rceil - i}^{\lfloor UCL_2 \rfloor - i} \left[Pr(d_2 = h) \times \sum_{k=0}^{\lfloor UCL_3 \rfloor - h - i} Pr(d_3 = k) \right] \right\}$$

$$\begin{aligned} P_{a3} = \sum_{d_1=\lceil WL_1 \rceil}^{\lfloor UCL_1 \rfloor} \Bigg\{ \Bigg[& \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} \Bigg] \\ & \times \Bigg[\sum_{d_2 \lceil = WL_1 \rceil - d_1}^{\lfloor UCL_2 \rfloor - d_1} \Bigg[\binom{n_2}{d_2} p^{d_2}(1-p)^{n_2-d_2} \\ & \times \sum_{d_3=0}^{\lfloor UCL_3 \rfloor - d_2 - d_1} \binom{n_3}{d_3} p^{d_3}(1-p)^{n_3-d_3} \Bigg] \Bigg] \Bigg\} \end{aligned}$$

The probability P_a is used to calculate "*the probability of type 1 error*", symbolized by α, which measures the probability of declaring an out-of-control state when the process is actually in-control.

$$\alpha = 1 - P_a(p = p_o). \tag{2}$$

In addition, P_a is used to calculate "*the probability of type 2 error*", symbolized by β, which measures the probability of declaring an in-control state when the process is actually out-of-control:

$$\beta = P_a(p = p_1). \tag{3}$$

Since the points that are drawn on the TS-np chart are independent between two consecutive samples, we can get the expression for the average run length when the process is in-control (ARL_0) and the average run length when the process is out of control (ARL_1) thus:

$$ARL_0 = \frac{1}{\alpha} \tag{4}$$

$$ARL_1 = \frac{1}{1-\beta} \tag{5}$$

2.2. Average Sample Number

The average sample size or average sample number (ASN) denotes, on average, how many units will be inspected in a subgroup. For the TS-np chart, the ASN is calculated as follows:

$$\begin{array}{l} ASN(p) = n_1P_1 + (n_1+n_2)P_2 + (n_1+n_2+n_3)P_3 \\ ASN(p) = n_1 + n_2(1-P_1) + n_3(1-P_1-P_2). \end{array} \tag{6}$$

Here, P_1 is the probability of deciding on the first sample:

$$P_1 = \Pr(d_1 < WL_1) + \Pr(d_1 > UCL_1)$$

$$P_1 = \sum_{d_1=0}^{\lfloor WL_1 \rfloor} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} + \sum_{d_1=\lceil UCL_1 \rceil}^{n_1} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1}$$

$$P_2 = (1-P_1) \times [\Pr(d_1+d_2 < WL_2) + \Pr(d_1+d_2 > UCL_2)]$$

$$\begin{array}{l} P_2 = (1-P_1) \times \left\{ \sum\limits_{d_1=\lceil WL_1 \rceil}^{\lfloor UCL_1 \rfloor} \left(\left[\binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} \right] \times \left[\sum\limits_{d_2=0}^{\lfloor WL_2 \rfloor - d_1} \binom{n_2}{d_2} p^{d_2}(1-p)^{n_2-d_2} \right] \right) + 1 \right. \\ \left. - \sum\limits_{d_1=\lceil WL_1 \rceil}^{\lfloor UCL_1 \rfloor} \left(\left[\binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} \right] \times \left[\sum\limits_{d_2=0}^{\lfloor UCL_2 \rfloor - d_1} \binom{n_2}{d_2} p^{d_2}(1-p)^{n_2-d_2} \right] \right) \right\} \end{array}$$

P_3 is the probability of deciding on the third sample.

$$P_3 = 1 - P_1 - P_2$$

One of the operational advantages of multi-stage inspection processes can be found right in the ASN metric. For example, in the TS-np chart, since it is considered to take a total sample for every time interval and then divide them into n_1, n_2, and n_3, therefore it is not always necessary to inspect the subsamples n_2 or n_3, since, as explained at the beginning of this section, the decision inspect a second sample of size n_2 or a third sample of size n_3 is subject to the number of defects and the position of the control limits (which are found using the G.A NSGA-II) in every sample. Therefore, for example, if a subsample n_1 is taken and the number of defects d_1 lies in the in-control zone, it is not necessary to inspect the other subsamples, thus reducing the average sample number.

As it happens, for the indicators P_a and ARL, the ASN can be evaluated for any input level p of the proportion of defective units. Evaluated for $p = p_0$, the ASN are obtained for the in-control state, and it is denoted by ASN_0. Otherwise, when evaluated for $p = p_1$, ASN_1 will be obtained, which characterizes the performance of the chart for the out-of-control state.

2.3. Optimal Design of TS-np Control Chart

To obtain $n_1,\ n_2,\ n_3, WL_1,\ UCL_1,\ WL_2,\ UCL_2,$ and UCL_3, we consider the following bi-objective optimization model:

Minimize

$$\beta = P_a(p = p_0(1+c)).$$

Minimize

$$ASN_0 = ASN(p = p_0).$$

Subject to:

$$ARL_o \geq ARL_{0min}.$$

$$ASN_0 \leq n_0.$$

$$WL_1 < UCL_1$$

$$WL_1 < WL_2$$
$$UCL_1 < UCL_2$$
$$WL_2 < UCL_2 < UCL_3$$
$$UCL_1 \leq n_1$$
$$UCL_2 \leq n_1 + n_2$$
$$UCL_3 \leq n_1 + n_2 + n_3$$

Here p_0 and c are input values that must be defined by the user while considering usual or critical levels for the fraction of nonconforming the unit (p). When fixing p_0, the user defines the usual or desired value of this fraction for the process in the in-control state. The value c is a relative magnitude that denotes the maximum percentage increase in this fraction from which a critical deterioration in the quality of the process is generated.

Additionally, the user must also define the thresholds for the performance metrics ARL_0 and ASN_0 (i.e., set ARL_{0min} and n_0). Through ARL_{0min}, the user establishes a mandatory minimum reference for the average number of samples that elapse between two false alarms. On the other hand, by setting n_0, the user requires the algorithm to search for a configuration that guarantees that, while the process is in-control, the sampling load will not exceed the equivalent of inspected n_0 units per sample, which contributes to controlling the costs of the inspection procedure.

To solve this bi-objective optimization problem, we have used a genetic algorithm (GA) called non-dominated sorting genetic algorithms II (NSGA-II), proposed by [44], which alleviates three difficulties: computational complexity, non-elitism approach, and the need for specifying a sharing parameter. According to [43], NSGA-II is an excellent option for multi-objective optimization, since it is classified as elitist because it incorporates a mechanism of preservation of the dominant solutions through several generations of a genetic algorithm. The NSGA-II algorithm is available in *R Statistical Software* through the package *mco*, developed by [45]. As in [43], this package was used as a tool to solve the bi-objective optimization problem of the design of the TS-np chart.

Since the development of the NSGA-II algorithm is beyond the scope of this study, it is recommended to consult [44] for a deeper understanding. On the other hand, in [45] explanations and examples can be consulted which clearly show how to implement the *mco* (NSGA-II algorithm) package in R.

The bi-objective algorithm provides a Pareto front which contains the non-dominated solutions found in the optimization process, that is, it is not a unique solution. The user will have the opportunity to evaluate the set of solutions in order to decide which of them meets their expectations.

3. Comparative Studies

In this section, we will analyze the performance of the TS-np control chart versus the proposed MDSRS and DS-np control charts in terms of ARL_1 values.

Comparisons will be made in two different subsections for MDSRS and DS-np, respectively.

Before presenting the comparisons, it is necessary to give a brief introduction of the control charts being compared.

3.1. MDSRS Control Chart

According to [7], the MDSRS is a mixture of both repetitive sampling and MDS sampling.

The operational procedure of the MDSRS control chart is given below, as shown in the original manuscript by [7]:

Step-1: A random sample of size n is selected from each subgroup in the production process and count the number of defectives D.

Step-2: The process is declared to be in control if $LCL_2 \leq D \leq UCL_2$. The process is declared to be out of control if $D \geq UCL_1$ or $D \leq LCL_1$. Otherwise, go to **Step-3**. Here, LCL_1, UCL_1, LCL_2, and UCL_2 are four control limits that should be constructed using the data when the process is in-control.

Step-3: Declare the process as in control if i preceding subgroups have been declared as in-control. Otherwise, repeat **Step-1**. Here, the value of i may be specified by an engineer.

3.2. DS-np Control Chart

DS-np is an extension of the traditional SS-np. While in SS-np, a sample of size n is inspected, in DS-np two samples of size n_1 and n_2, respectively, are inspected.

The operational procedure of the DS-np control chart is also provided, as shown in the original manuscript by [2]: let WL, UCL_1, and UCL_2 represent, respectively, the warning and control limits for a DS-np chart. Then, the DS-np chart is defined by five parameters: n_1, n_2, WL, UCL_1, and UCL_2. Periodically, at fixed sampling intervals (say, every hour), a sample of size n_1 is drawn from the process. Let d_1 denote the number of non-conforming items found in this sample (of size n_1). If $d_1 < WL$, then the process is considered to be in control, and the control scheme continues operating with a sample size of n_1. Otherwise, if $d_1 > UCL_1$, the process is understood to be out of control, and an investigation should be initiated. However, if $WL < d_1 < UCL_1$, an additional sample of size n_2 items is immediately taken. Let d_2 denote the number of non-conforming items found in the sample of size n_2. In this case, the decision depends on $(d_1 + d_2)$, the information derived from the two samples. If $(d_1 + d_2) < UCL_2$, the process is considered to be in control. However, if $(d_1 + d_2) > UCL_2$, then the process is considered to be out of control, and an investigation should be initiated.

3.3. TS-np Versus MDSRS

In [7], it was concluded that the MDSRS control chart has the ability to detect an earlier shift in the process as compared to RS and MDS at all values of the given shift constants. Here p_1 is always greater than p_0 (except when c = 0, i.e., there is no change in p_0) as was stated in the previous section.

The comparisons of MDSRS vs. RS and MDSRS vs. MDS were made for only two cases in terms of ARL_{0min} (also named r_0), p_0, and n. In order to save space, the comparison of the proposed TS-np chart with RS, MDS, and simultaneously with MDSRS, represented in two graphics, will be made under the same conditions used by [7].

Figure 3 shows a graphic with values of ARL_1 for the TS-np control chart vs. the MDSRS and RS control charts under the following conditions: $ARL_{0min} = 370$, $p_0 = 0.1$, and $ASN_{max} = 82$.

Figure 4 shows a graphic with the values of ARL_1 for the TS-np control chart vs. the MDSRS and MDS control charts under the following conditions: $ARL_{0min} = 370$, $p_0 = 0.4$, and $ASN_{max} = 98$.

From Figures 3 and 4, it can be concluded that for small changes in p_0, TS-np is the fastest control chart to detect such changes when compared with MDSRS, RS, and MDS, i.e., for shift sizes (c) less than 0.5, the ARL_1 values in TS-np are considerably smaller. For example, from Figure 3 when $c = 0.01$, the ARL_1 value for TS-np is 303, whereas it is 338 for MDSRS and 387 for RS. From Figure 4, when $c = 0.07$, the value of ARL_1 for TS-np is 38, whereas it is 102 for MDSRS and 116 for MDS.

Moreover, Tables 1 and 2 are presented to compare the ARL_1 values between the TS-np control chart and MDSRS. The conditions used for ARL_{0min}, p_0, and ASN_{max} correspond to those used in [7] to generate ARL_1 values.

In Tables 1 and 2, the values for ASN and ARL_0 corresponding to TS-np are also shown. Tables 1 and 2 are presented for only two scenarios in order to avoid disrupting the flow of the manuscript. The rest of these tables can be found in Appendix A.

From Tables 1 and 2, it can be seen that, for small and moderate shift sizes, the proposed chart presents smaller ARL_1 values in comparison with MDSRS, complying with the conditions established for ASN_{max} and ARL_{0min}. The best values of ARL_1 are highlighted in bold.

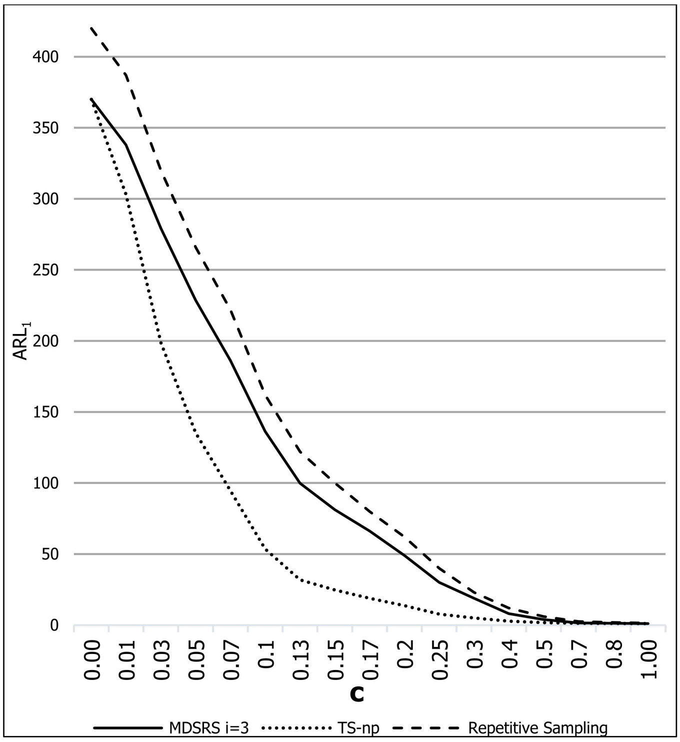

Figure 3. The ARL_1 values for the TS-np, MDSRS, and RS control charts.

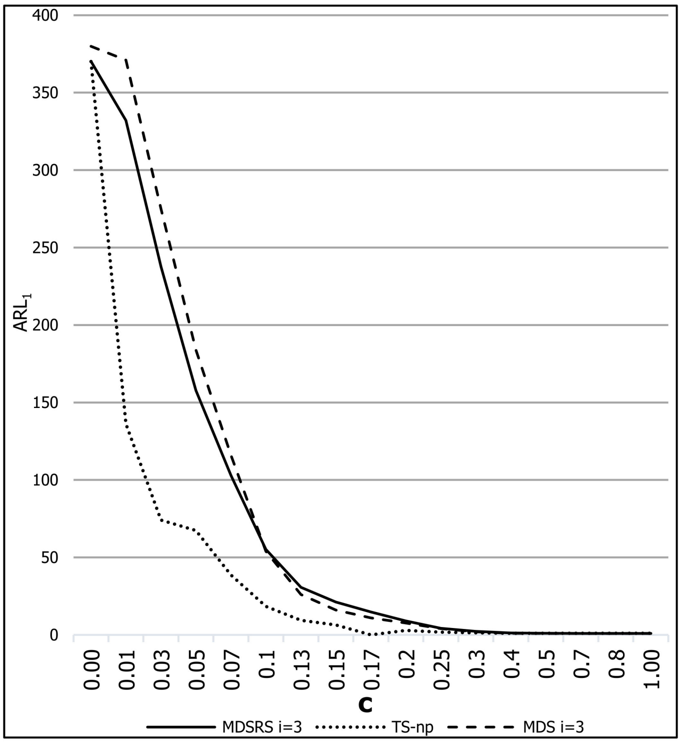

Figure 4. The ARL_1 values for TS-np, MDSRS and MDS control charts.

Table 1. The values of ARL_1 for the TS-np and MDSRS control charts when $ARL_{0min} = 300$.

			Shift Size (c)																
			0.00	**0.01**	**0.03**	**0.05**	**0.07**	**0.10**	**0.13**	**0.15**	**0.17**	**0.20**	**0.25**	**0.30**	**0.40**	**0.50**	**0.70**	**0.80**	**1.00**
$p_0 = 0.1$; ASNmáx = 90	MDSRS i = 3	ARL_1	300.29	271.10	221.59	181.93	150.11	113.58	86.93	73.19	61.91	48.57	33.10	23.10	11.98	6.71	2.70	1.97	1.33
	MDSRS i = 2		300.93	271.71	222.15	182.46	150.61	114.07	87.41	73.67	62.39	49.05	33.58	23.56	12.37	7.01	2.83	2.04	1.35
	TS-np	ASN	2.04	55.17	39.312	86.71	78.62	86.52	84.39	89.78	89.55	87.94	87.19	84.88	89.59	89.04	89.86	89.137	89.81
		ARL_0	300.00	300.33	300.04	305.08	300.71	304.40	300.06	307.14	303.41	300.36	303.08	301.52	301.34	302.51	305.01	347.26	306.12
		ARL_1	300.00	260.55	189.24	137.88	91.20	52.88	34.55	26.25	20.66	14.22	9.18	5.52	2.95	1.90	1.26	1.16	1.05
$p_0 = 0.2$; ASNmáx = 80	MDSRS i = 3	ARL_1	300.68	271.08	216.31	170.33	133.57	93.09	65.64	52.43	42.16	30.80	18.83	11.95	5.35	2.81	1.35	1.15	1.03
	MDSRS i = 2		300.99	271.37	216.59	170.60	133.84	93.37	65.93	52.73	42.47	31.12	19.16	12.25	5.57	2.93	1.36	1.16	1.03
	TS-np	ASN	6.34	44.02	78.43	40.59	49.49	78.19	64.10	76.76	76.911	63.07	79.38	78.31	77.59	78.74	79.61	77.40	79.94
		ARL_0	300.00	300.10	300.64	302.74	304.00	301.87	301.12	302.33	300.45	302.27	302.13	300.97	303.03	301.22	303.41	306.88	311.37
		ARL_1	300.00	246.30	166.68	121.32	78.05	45.28	29.88	20.88	15.99	11.26	6.40	4.07	2.12	1.41	1.05	1.01	1.00

Table 2. The values of ARL_1 for the TS-np and MDSRS control charts when $ARL_{0min} = 370$.

			Shift Size (c)																
			0.00	**0.01**	**0.03**	**0.05**	**0.07**	**0.10**	**0.13**	**0.15**	**0.17**	**0.20**	**0.25**	**0.30**	**0.40**	**0.50**	**0.70**	**0.80**	**1.00**
$p_0 = 0.1$; ASN máx = 82	MDSRS i = 3	ARL_1	370.01	338.02	279.29	228.63	186.21	136.3	99.77	81.17	66.17	48.93	30.03	18.82	8.00	3.89	1.56	1.26	1.06
	TS-np	ASN	2.03	48.89	81.89	70.66	73.94	70.39	77.86	81.04	73.16	75.82	81.45	81.95	79.60	81.55	81.92	81.67	81.81
		ARL_0	370.08	370.75	370.91	370.72	380.66	374.99	371.05	389.40	372.18	371.44	384.77	375.98	385.13	386.55	460.86	400.46	436.44
		ARL_1	370.08	303.10	198.75	135.28	94.57	53.53	31.78	24.72	18.92	13.67	7.74	4.98	2.71	1.79	1.28	1.17	1.07
$p_0 = 0.2$; ASN máx = 46	MDSRS i = 3	ARL_1	370.84	331.86	266.22	214.2	172.93	126.29	92.97	76.13	62.55	46.86	29.41	18.82	8.22	4.04	1.60	1.27	1.06
	TS-np	ASN	2.78	36.47	42.64	33.81	42.60	42.13	41.05	36.15	41.95	35.18	45.76	45.38	43.92	45.43	45.99	45.89	45.85
		ARL_0	370.01	370.17	370.31	374.27	371.08	374.16	375.74	371.37	375.11	370.46	371.42	370.08	372.41	382.86	486.54	515.74	421.21
		ARL_1	370.01	313.33	229.67	170.01	113.17	72.65	43.22	33.64	25.48	18.38	10.45	6.52	3.313	1.62	1.16	1.08	1.02

MDSRS outperforms TS-np only in specific cases of p_0, ASN_{max}, and c from Table 2, such as $p_0 = 0.1$; $ASN_{max} = 82; c = 0.00, 1.00$, $p_0 = 0.4$; $ASN_{max} = 98; c = 0.00$, $p_0 = 0.1$; $ASN_{max} = 83; c = 1.00$, $p_0 = 0.2$; $ASN_{max} = 84; c = 0.70$ and $p_0 = 0.5$; $ASN_{max} = 87; c = 0.17, 0.20, 0.25, 0.30$.

It is also interesting to note that, for TS-np and MDSRS, as expected, the ARL_1 values are lower as c increases; this is because as p_1 increases, the change in p_0 can be detected more quickly.

3.4. TS-np Versus DS-np

In [2], comparisons are made with the single sampling np chart, variable sample size (VSS) np chart, CUSUM np, and EWMA np charts. The comparisons are carried out while considering the optimal design for each chart. Here, $ARL_{0min} = 200$ and $ARL_{0min} = 370.4$ are considered. p_1 is always greater than p_0, and it is calculated as follows:

$$p_1 = p_0 * \gamma$$

where $\gamma > 0$, and it shows the shift constant in the process.

To save space, the comparison of the proposed TS-np chart with DS-np, represented in seven tables, will be made under the same conditions used in Tables 1–7 by [2].

Tables 3–5 provide the ARL_0 and ARL_1 values as well as the optimal design for the SS-np, DS-np, and TS-np control charts in addition to the decision variables and the ASN for the latter. In particular, Table 5 is proposed by [2] to compare the SS-np chart with the DS-np chart in a scenario in which ARL_0 values are closer to 200 in SS-np. It is necessary to present this table in order to compare the SS-np chart with the DS-np and TS-np charts in a situation of equality, as in Tables 3 and 4 SS-np has high ARL_0 values with respect to ARL_{0min} = 200, thus placing it at a disadvantage.

Tables 6–9 exhibit the optimal design parameters of each np chart considered (SS-np, DS-np, TS-np, VSS, CUSUM, and EWMA) along with their ASN_0 and their $ARL1's$ for a range of values of γ besides the specific value (henceforth denoted by γ^*) for which the ARL_1 was minimized. As with Tables 1–4 and Tables 6–9 are presented only for some scenarios, and the rest of them are provided in Appendices B and C.

Table 3. Optimal design, ARL_1 and Pg values of the SS, DS, and TS-np charts, $ARL_{0min} = 200$.

			SS-np		DS-np							TS-np						
γ	p_0	n	ARL_0	ARL_1	ARL_0	ARL_1	WL_1	UCL_1	WL_2	UCL_2	UCL_3	n_1	n_2	n_3	ASN	ARL_0	ARL_1	Pg
1.5	0.005	100	597.63	142.60	200.52	36.97	0.5	3.5	1.5	6.5	11.5	49	116	982	97.75	200.03	17.50	52.67%
		200	282.05	55.14	200.47	21.58	0.5	4.5	1.5	9.5	19.5	54	249	1989	196.86	200.120	9.27	57.03%
		400	226.55	30.36	200.49	11.64	1.5	5.5	3.5	15.5	34.5	177	400	3999	398.23	235.00	5.09	56.24%
		800	362.20	23.81	200.42	5.91	2.5	9.5	4.5	23.5	59.5	335	484	7990	799.82	204.439	2.74	53.67%
2.0	0.005	100	597.63	54.42	200.52	13.14	0.5	3.5	1.5	6.5	13.5	39	200	1200	99.68	206.899	5.00	61.94%
		200	282.05	19.33	200.90	6.68	0.5	5.5	1.5	8.5	21.5	60	157	2400	199.35	200.029	2.97	55.53%
		400	226.55	9.12	223.75	3.38	1.5	6.5	2.5	26.5	31.5	189	196	3715	398.81	220.980	1.98	41.54%
		800	362.20	5.46	205.25	1.89	1.5	8.5	3.5	23.5	32.5	290	286	3583	798.35	207.814	1.46	22.95%
3.0	0.005	100	597.63	15.57	268.95	4.12	0.5	3.5	1.5	5.5	11.5	50	117	968	99.93	203.699	2.23	45.89%
		200	282.05	5.45	200.90	2.17	0.5	5.5	1.5	8.5	13.5	83	123	1139	199.22	211.151	1.58	27.32%
		400	226.55	2.54	216.97	1.40	1.5	9.5	2.5	16.5	17.5	240	135	1472	398.37	226.431	1.19	15.23%
		800	362.20	1.53	207.59	1.08	1.5	7.5	6.5	19.5	29.5	320	618	2394	790.96	611.322	1.06	2.02%

Table 4. Optimal design, ARL_1 and Pg values of the SS, DS, and TS-np charts, $ARL_{0min} = 370.4$.

			SS-np		DS-np							TS-np						
γ	p_0	n	ARL_0	ARL_1	ARL_0	ARL_1	WL_1	UCL_1	WL_2	UCL_2	UCL_3	n_1	n_2	n_3	ASN	ARL_0	ARL_1	Pg
1.5	0.005	100	597.63	142.60	372.43	55.45	0.5	5.5	1.5	8.5	14.5	42	161	1267	99.87	370.721	20.42	63.17%
		200	1773.23	234.08	370.48	29.74	0.5	4.5	1.5	8.5	24.5	48	273	2600	198.42	371.297	10.24	65.57%
		400	948.59	86.35	371.60	15.44	1.5	8.5	3.5	18.5	43.5	153	600	5200	399.03	373.941	5.06	67.22%
		800	1133.91	50.84	372.27	7.09	3.5	28.5	4.5	36.5	70.5	441	375	9598	795.37	386.512	2.82	60.17%
2.0	0.005	100	597.63	54.42	372.29	17.21	0.5	4.5	1.5	7.5	14.5	45	130	1297	99.50	370.580	5.52	67.91%
		200	1773.23	62.41	372.44	7.89	0.5	4.5	1.5	8.5	23.5	60	147	2551	199.48	377.263	3.15	60.09%
		400	948.59	19.91	396.07	3.84	1.5	8.5	2.5	16.5	31.5	188	221	3505	398.51	384.255	2.02	47.31%
		800	1133.91	9.02	371.44	2.00	3.5	17.5	6.5	36.5	45.5	497	501	5114	795.23	389.021	1.44	28.10%
3.0	0.005	100	597.63	15.57	372.47	4.65	0.5	3.5	1.5	5.5	13.5	49	111	1143	99.80	382.943	2.29	50.78%
		200	1773.23	12.14	416.74	2.34	0.5	4.5	1.5	7.5	14.5	84	115	1179	199.62	377.597	1.61	31.29%
		400	948.59	3.92	370.83	1.44	0.5	14.5	3.5	18.5	19.5	145	272	1595	396.30	481.993	1.25	12.98%
		800	1133.91	1.85	379.87	1.09	0.5	10.5	5.5	23.5	25.5	238	490	2089	796.70	523.691	1.06	2.38%

Table 5. Optimal design, ARL_1 and Pg values of the SS, DS, and TS-np charts, $ARL_{0min} \cong 200$.

			SS-np		DS-np							TS-np						
γ	p_0	n	ARL_0	ARL_1	ARL_0	ARL_1	WL_1	UCL_1	WL_2	UCL_2	UCL_3	n_1	n_2	n_3	ASN	ARL_0	ARL_1	Pg
1.5	0.01	68	204.16	51.84	200.36	33.90	0.5	3.5	1.5	7.5	17.5	22	109	884	67.92	202.19	11.33	66.58%
		109	202.04	40.60	202.04	21.90	0.5	4.5	1.5	8.5	24.5	27	108	1413	108.88	200.19	7.47	65.88%
		205	203.82	27.60	200.37	11.69	1.5	7.5	2.5	37.5	41.5	81	182	2665	204.99	200.05	4.24	63.72%
		434	203	15.17	200.97	5.54	2.5	13.5	3.5	18.5	54.5	173	199	3583	428.75	200.84	2.67	51.75%
2.0	0.01	68	204	21.04	200.36	11.26	0.5	3.5	1.5	7.5	17.5	22	109	884	67.93	202.19	3.80	66.29%
		109	202.04	14.62	214.45	6.31	0.5	12.5	1.5	13.5	22.5	31	80	1272	108.63	202.45	2.82	55.25%
		205	203.82	8.39	201.88	3.36	1.5	7.5	2.5	27.5	30.5	97	95	1789	204.46	205.12	1.93	42.47%
		434	203	3.92	200.76	1.79	2.5	11.5	4.5	33.5	38.5	204	177	2191	432.54	206.83	1.37	23.67%
3.0	0.01	68	204	6.78	201.17	3.32	0.5	4.5	1.5	5.5	12.5	32	56	535	59.98	204.83	1.87	43.62%
		109	202.04	4.35	202.47	2.10	0.5	5.5	1.5	6.5	13.5	46	55	571	108.99	204.09	1.51	28.14%
		205	203.82	2.39	210.12	1.37	0.5	6.5	3.5	10.5	15.5	74	156	556	204.11	219.89	1.21	11.35%
		434	203.33	1.33	207.95	1.06	1.5	8.5	4.5	13.5	21.5	195	176	828	432.84	201.94	1.03	2.74%

Table 6. Steady-state $ARL_1's$ of the optimal designs [1] for $ARL_{0min} = 200$ and p_0 = 0.005.

					Parameters											γ				
Scheme	n_1	n_2	n_3	WL_1	UCL_1	WL_2	UCL_2	UCL_3	λ	k	ASN_0	ARL_0	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**
SS	100				3.5						100.00	597.60	142.60	54.42	26.85	15.57	10.09	7.09	5.30	4.15
DS	81	283		1.5	3.5		5.5				98.50	200.50	36.97	13.14	6.65	4.16	2.99	2.35	1.97	1.73
TS	49	116	982	0.5	3.5	1.5	6.5	11.5			97.75	200.03	17.50	5.42	3.04	2.26	1.90	1.69	1.55	1.45
												ASN_1	145.13	203.40	265.35	323.99	373.69	410.76	433.55	442.18
VSS	27	457		0.5	3.5	2.5	5.5				99.50	205.40	25.78	8.97	5.36	4.08	3.47	3.10	2.87	2.70
												ASN1	153.00	182.00	185.00	181.00	179.00	181.00	185.00	190.00
EWMA	100				0.7				0.1		100.00	200.50	27.03	12.81	8.39	6.29	5.06	4.26	3.70	3.28
CUSUM	100				9.3					0.0	100.00	204.60	27.54	14.39	9.82	7.50	6.11	5.18	4.51	4.01
CUSUM (FIR)	100				6.0					0.1	100.00	204.50	28.63	13.07	8.47	6.32	5.08	4.27	3.70	3.28

[1] ARL_1 minimized for $\gamma^* = 1.5(p_1^* = 0.0075)$.

Table 7. Steady-state $ARL_1's$ of the optimal designs [1] for $ARL_{0min} = 370.4$ and p_0 = 0.005.

					PARAMETERS											γ				
Scheme	n_1	n_2	n_3	WL_1	UCL_1	WL_2	UCL_2	UCL_3	λ	k	ASN_0	ARL_0	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**
SS	100				3.5						100	597.6	142.6	54.42	26.85	15.57	10.09	7.09	5.3	4.15
DS	74	352		1.5	3.5		6.5				92.60	372.40	55.45	17.28	8.07	4.81	3.35	2.59	2.15	1.87
TS	42	161	1267	0.5	5.5	1.5	8.5	14.5			99.87	370.72	20.42	5.53	3.07	2.33	2.00	1.79	1.65	1.54
												ASN1	155.05	223.19	298.01	373.45	443.97	504.80	552.27	584.11
VSS	21	519		0.5	3.5	2.5	6.5				99.60	378.20	33.34	10.34	6.13	4.70	3.98	3.56	3.27	3.06
												ASN1	172.00	202.00	193.00	181.00	177.00	177.00	181.00	187.00
EWMA	100				0.7				0.0		100.00	391.50	35.74	16.47	10.71	7.99	6.40	5.37	4.64	4.10
CUSUM	100				7.1					0.1	100.00	371.60	34.82	15.47	9.95	7.39	5.92	4.96	4.29	3.79
CUSUM (FIR)	100				7.4					0.1	100.00	390.20	36.68	16.10	10.32	7.65	6.12	5.12	4.42	3.91

[1] ARL_1 minimized for $\gamma^* = 1.5(p_1^* = 0.0075)$.

Table 8. Zero-state $ARL_1's$ of the optimal designs[1] for $ARL_{0min} = 200$ and p_0 = 0.005.

					PARAMETERS											γ				
Scheme	n_1	n_2	n_3	WL_1	UCL_1	WL_2	UCL_2	UCL_3	λ	k	ASN_0	ARL_0	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**
SS	100				3.5						100.00	597.60	142.60	54.42	26.85	15.57	10.09	7.09	5.30	4.15
DS	74	352		1.5	3.5		6.5				92.60	372.40	55.45	17.28	8.07	4.81	3.35	2.59	2.15	1.87
TS	49	116	982	0.5	3.5	1.5	6.5	11.5			97.75	200.03	17.50	5.42	3.04	2.26	1.90	1.69	1.55	1.45
VSS	21	519		0.5	3.5	2.5	6.5				99.60	378.20	33.34	10.34	6.13	4.70	3.98	3.56	3.27	3.06
EWMA	100				0.7				0.0		100.00	391.50	35.74	16.47	10.71	7.99	6.40	5.37	4.64	4.10
CUSUM	100				7.1					0.1	100.00	371.60	34.82	15.47	9.95	7.39	5.92	4.96	4.29	3.79
CUSUM (FIR)	100				7.4					0.1	100.00	390.20	36.68	16.10	10.32	7.65	6.12	5.12	4.42	3.91

[1] ARL_1 minimized for $\gamma^* = 1.5$ $(p_1^* = 0.0075)$.

Table 9. Zero-state $ARL_1's$ of the optimal designs [1] for $ARL_{0min} = 370.4$ and p_0 = 0.005.

					PARAMETERS											y				
Scheme	n_1	n_2	n_3	WL_1	UCL_1	WL_2	UCL_2	UCL_3	λ	k	ASN_0	ARL_0	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**
SS	100				3.5						100.00	597.60	142.60	54.42	26.85	15.57	10.09	7.09	5.30	4.15
DS	74	352		1.5	3.5		6.5				92.60	372.40	55.45	17.28	8.07	4.81	3.35	2.59	2.15	1.87
TS	42	161	1267	0.5	5.5	1.5	8.5	14.5			99.87	370.72	20.42	5.53	3.07	2.33	2.00	1.79	1.65	1.54
VSS	21	519		0.5	3.5	2.5	6.5				99.60	378.20	33.34	10.34	6.13	4.70	3.98	3.56	3.27	3.06
EWMA	100				0.7				0.0		100.00	391.50	35.74	16.47	10.71	7.99	6.40	5.37	4.64	4.10
CUSUM	100				7.1					0.1	100.00	371.60	34.82	15.47	9.95	7.39	5.92	4.96	4.29	3.79
CUSUM (FIR)	100				7.4					0.1	100.00	390.20	36.68	16.10	10.32	7.65	6.12	5.12	4.42	3.91

[1] ARL_1 minimized for $\gamma^* = 1.5$ $(p_1^* = 0.0075)$.

As is mentioned in [2], there are two contexts under which it is of interest to calculate the ARL_1, and for CUSUM and EWMA schemes (which can be modelled as Markov chains), these lead to different ARL_1 values: the zero-state, which is relevant when the process monitoring starts with the control statistic at its expected value under control; and the steady-state, which considers the idea that when the change in the process occurs, there is no certainty about the location of the control statistic, so the effect of the initial value of the CUSUM or EWMA statistic has dissipated.

Tables 6 and 7 give the steady-state optimal designs (the designs that minimize the steady-state ARL_1) and Tables 8 and 9 give the zero-state optimal designs. For the SS, DS, and TS schemes, in which the probability of a signal in a sample is not conditional on the values of previous samples, there is no difference between the zero-state $ARL's$ and steady-state $ARL's$ in the case of shifts occurring between sampling times.

For more details with respect to the steady-state and zero-state concepts, see [2] (p. 98). As in [2], from Tables 3–5, the following conclusions can be made:

- Overall, the TS-np chart presents betters values of ARL_1 for $\gamma = 1.5$, 2.0, and 3.0. With these values for γ, the percentage gain (Pg) when the optimal TS-np chart is used instead of the optimal DS-np chart ranges from approximately 2.02% to 67.91%. For $\gamma = 3.0$, the TS-np chart presents lower values for Pg than for $\gamma = 1.5$ and 2.0. Nevertheless, these percentages range between 2.02% and 50.78%, so it is still a good option to use TS-np for large changes in p_0.
- On average, the TS-np chart presents lower values of ARL_1 for $ARL_{0\min} = 370.4$ than for $ARL_{0\min} = 200$.
- For shifts of magnitude $\gamma = 3.0$ and $nP_0 = 4$ (with n assuming one of the values 200, 400, or 800), the absolute values of the ARL_1′s of the DS-np and TS-np charts are close (always between 1.0 and 2.0).
- As P_0 decreases and n (which is the ASN_{max} for TS-np) increases, n_1, n_2, and n_3 increase.
- For different values of n and p_0, as long as nP_0 is constant, the optimal n_1, n_2, and n_3 are proportional to n.
- The optimal WL, UCL_1, and WL_2 values change very little as long as nP_0 and γ are constants. The ARL_1 values, too, change very little for different values of n and p_0 as long as nP_0 and γ are constants.

From Tables 6–9, the following conclusions can be made:

- Overall, the TS-np chart presents better ARL_1 values for $n = 100,\ 200,\ 400$, and 800, and $\gamma = 1.5,\ 2.0,\ 2.5,\ 3.0$, and 3.5—that is, for small and moderate changes in the P_0. For large changes in P_0, it is recommended to use the DS-np chart.
- On average, the TS-np chart presents lower values of ARL_1 for $ARL_{0\min} = 370.4$ than for $ARL_{0\min} = 200$.

4. Control Scheme TS-np with Simulated Data

In this section, we will show the functionality of TS-np through simulated data for two cases.

In the first case, we generate the data with the same conditions used in Section IV for [7], where $ARL_{0min} = 370,\ p_0 = 0.2,\ ASN_{max} = 46$, and $c = 0.5$.

Here, 40 subgroups are taken, and it is assumed that in the first 10 subgroups, the process is in control. In the following subgroups, the fraction defective has shifted to $p_1 = 0.3$.

Figure 5 shows the control scheme with $WL_1 = 6.5$, $UCL_1 = 14.5$, $WL_2 = 9.5$, $UCL_2 = 50.5$, and $UCL_3 = 59.5$.

From Figure 5, it can be seen that TS-np detects the shift at the twelfth subgroup (with $d_1 = 8,\ d_2 = 6$, and $d_3 = 49$), which is detected before that in MDSRS. Only in this subgroup was it necessary to inspect $n_1,\ n_2$, and n_3, where $n_1 = 27,\ n_2 = 21$, and $n_3 = 168$. In the previous subgroup, only n_1 and n_2 were inspected, thus reducing the ASN. It is interesting to note that in Figure 5, $WL_2 < UCL_1$; in fact, analyzing the optimization model presented in Section 2, it is not strictly necessary for WL_2 to be greater than UCL_1.

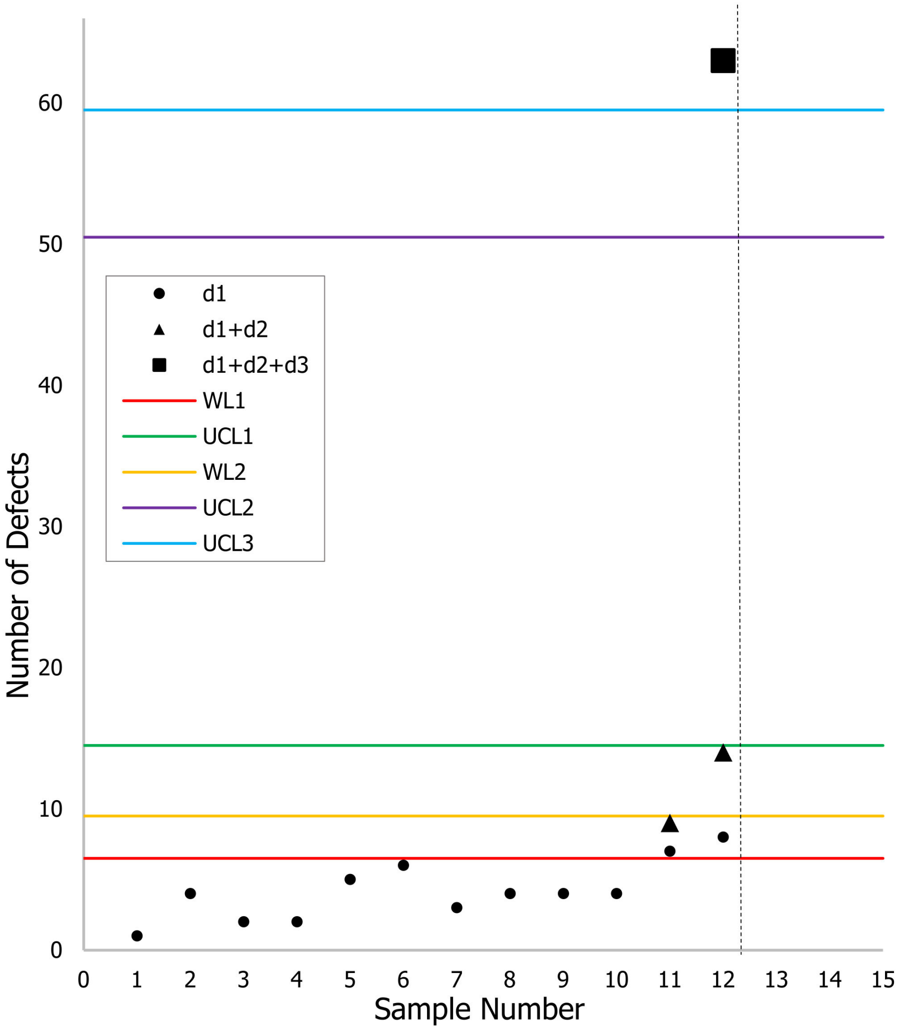

Figure 5. The TS-np control scheme using simulated data when $ARL_{0min} = 370$, $p_0 = 0.2$, $ASN_{max} = 46$, and $c = 0.5$.

The vertical line in Figures 5 and 6 represents the fact that when detecting the change in the process, the chart stops, and corrective action must be taken.

In the second case, we generated the data with the following conditions: $ARL_{0min} = 370$, $p_0 = 0.4$, $ASN_{max} = 98$, and $c = 0.25$. Here, we can also take 40 subgroups, and it is assumed that in the first 10 subgroups, the process is in-control. In the following subgroups, the fraction defective has shifted to $p_1 = 0.5$.

Figure 6 shows the control scheme with $WL_1 = 27.5$, $UCL_1 = 42.5$, $WL_2 = 51.5$, $UCL_2 = 68.5$, and $UCL_3 = 107.5$.

From Figure 6, it can be seen that TS-np detects the shift at the twelfth subgroup (with $d_1 = 33$, $d_2 = 26$, and $d_3 = 50$). Only in this subgroup was it necessary to inspect n_1, n_2, and n_3, where $n_1 = 67$, $n_2 = 51$, and $n_3 = 100$.

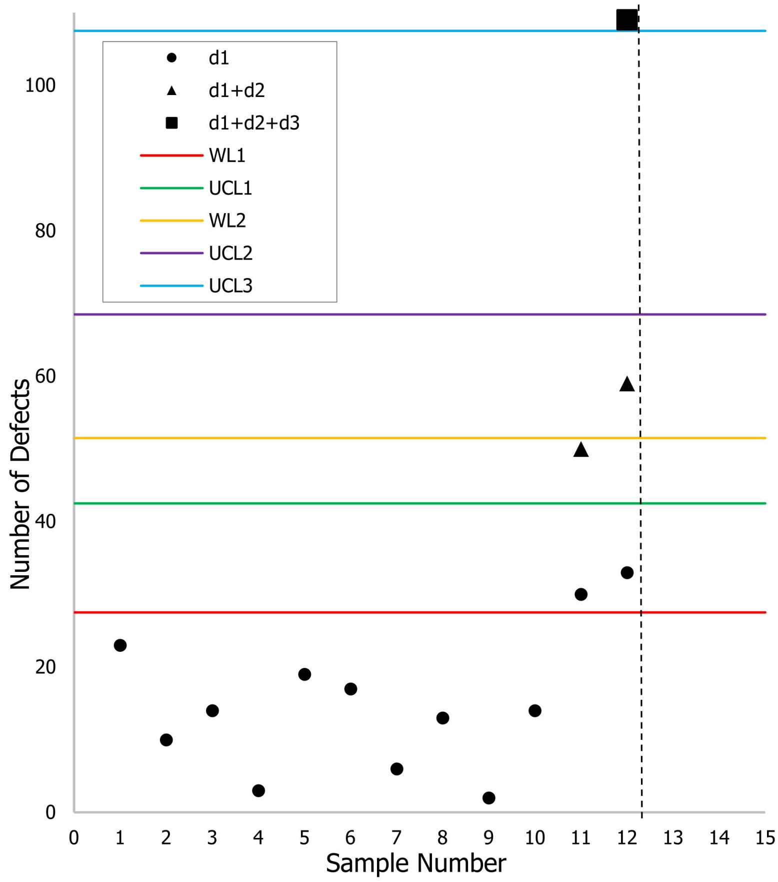

Figure 6. The TS-np control scheme for simulated data when $ARL_{0min} = 370$, $p_0 = 0.4$, $ASN_{max} = 98$, and $c = 0.25$.

5. Conclusions

In this work, we have developed an optimal design for an attribute control chart using triple sampling for monitoring the number of nonconforming items. The proposed control chart has better performance in terms of ARL_1, detecting small or moderate shifts in the nonconforming rate p_0 and maintaining a lower ASN as compared with MDSRS and DS-np. The MDSRS control chart proposed by [7] presents lower ARL_1 values than the TS-np only for some conditions. The DS-np control chart proposed by [2] presents lower ARL_1 values than TS-np only in some cases when the shift constant in the process $\gamma \geq 4.0$.

On the other hand, a bi-objective optimization model is presented, which seeks to minimize the ASN and the probability of committing a type 2 error (β). The triple sampling procedure and optimization model used in this work can be used in future research applied to other attribute control charts, such as p, c, or u control charts.

As evidenced by [2], the movement from a simple sampling scheme to a double sampling scheme contributes to improving the statistical performance of the np control chart. In our work, we show that this performance improvement remains substantial for small changes and is not negligible for changes of medium magnitude when a third sampling stage is added to the decision rule. Naturally, assuming a greater number of sampling stages also leads to assuming greater complexity in the operational work of the control chart. At this point, the user must evaluate his or her particular situation and decide whether this additional effort is compensated for by the gain in the ability to quickly detect changes to the process.

This trend in performance improvement when moving from single to double and double to triple sampling, naturally leads one to wonder whether the performance would continue to improve with the inclusion of a new sampling stage (a 4th sampling, for example). We do not currently have the answer to this quandary, and precisely this is a research opportunity.

Finally, this research would not have been possible without the R software, which was of great help in obtaining the optimal solutions presented in this paper.

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Appendix A

Table A1. The values of ARL_1 for TS-np and MDSRS control charts when $ARL_{0min} = 300$. Continued.

			Shift Size (c)																
			0.00	**0.01**	**0.03**	**0.05**	**0.07**	**0.10**	**0.13**	**0.15**	**0.17**	**0.20**	**0.25**	**0.30**	**0.40**	**0.50**	**0.70**	**0.80**	**1.00**
$p_0 = 0.3$; ASNmáx = 81	MDSRS i = 3	ARL_1	300.19	280.71	228.63	175.19	130.49	82.90	53.82	40.13	30.54	20.66	11.29	6.53	2.67	1.53	1.05	1.01	1.00
	MDSRS i = 2	ARL_1	300.40	280.90	228.82	175.38	130.68	83.11	53.51	40.38	30.80	20.93	11.55	6.75	2.77	1.55	1.05	1.01	1.00
	TS-np	ASN	14.67	58.24	65.01	49.13	70.84	73.19	79.20	74.12	77.99	65.89	77.43	80.52	80.21	80.82	78.42	78.07	80.62
		ARL_0	14.67	300.01	300.19	300.08	302.65	302.19	300.08	300.15	301.25	301.47	304.19	312.49	311.88	316.25	301.12	302.78	356.25
		ARL_1	300.01	244.88	158.66	112.33	73.67	39.69	24.00	17.40	13.06	8.56	4.88	3.08	1.64	1.19	1.00	1.00	1.00
$p_0 = 0.4$; ASNmáx = 83	MDSRS i = 3	ARL_1	300.11	292.96	243.21	178.35	123.82	70.32	40.77	28.86	20.73	12.96	6.35	3.44	1.52	1.10	1.00	1.00	1.00
	MDSRS i = 2	ARL_1	300.26	293.11	243.36	178.50	123.99	70.52	41.00	29.10	20.99	13.21	6.56	3.57	1.54	1.10	1.00	1.00	1.00
	TS-np	ASN	4.05	26.83	47.14	45.16	72.80	82.98	67.92	64.71	77.12	74.73	82.96	68.94	76.36	81.25	82.00	82.97	69.14
		ARL_0	300.07	300.17	300.41	300.59	300.21	300.93	300.60	300.19	300.07	300.20	300.20	300.06	300.09	300.01	314.05	313.15	310.75
		ARL_1	300.07	237.17	142.32	89.89	52.67	28.14	16.02	11.40	8.36	5.50	3.12	1.87	1.18	1.02	1.00	1.00	1.00
$p_0 = 0.5$; ASNmáx = 31	MDSRS i = 3	ARL_1	300.51	296.32	266.21	220.00	172.62	114.90	75.69	57.59	44.12	30.04	16.48	9.50	3.64	1.79	1.04	1.00	1.00
	MDSRS i = 2	ARL_1	300.13	295.93	265.82	219.61	172.21	114.44	75.17	57.03	43.53	29.39	15.79	8.81	3.13	1.52	1.01	1.00	1.00
	TS-np	ASN	7.87	9.46	24.87	30.34	18.96	29.60	28.60	30.73	30.40	27.59	30.34	30.75	30.91	29.64	30.91	30.03	2.59
		ARL_0	300.45	303.02	300.62	300.57	305.55	300.91	302.00	304.8	302.48	304.14	300.23	303.72	306.38	303.95	303.30	303.28	1024
		ARL_1	300.45	247.51	163.65	110.54	82.80	45.41	28.27	20.86	15.66	10.56	5.82	3.57	1.76	1.19	1.00	1.00	1.00

Table A2. The values of ARL_1 for TS-np and MDSRS control charts when $ARL_{0min} = 370$. Continued.

			Shift Size (c)																
			0.00	**0.01**	**0.03**	**0.05**	**0.07**	**0.10**	**0.13**	**0.15**	**0.17**	**0.20**	**0.25**	**0.30**	**0.40**	**0.50**	**0.70**	**0.80**	**1.00**
$p_0 = 0.3$; ASN máx = 89	MDSRS i = 3	ARL_1	370.1	327.75	244.35	176.03	125.58	76.11	46.97	34.43	25.47	16.46	8.33	4.53	1.86	1.22	1.01	1.00	1.00
	TS-np	ASN	9.17	80.46	81.12	67.36	69.34	66.47	88.82	87.19	87.53	83.04	87.98	87.54	88.49	87.08	88.89	9.40	87.38
		ARL_0	370.00	372.09	383.30	372.29	371.13	370.93	377.75	371.29	376.92	370.90	374.20	377.13	408.34	385.14	410.97	961.92	639.96
		ARL_1	370.00	271.18	157.90	87.30	51.97	25.21	13.48	9.27	6.37	3.94	2.45	1.63	1.15	1.03	1.00	1.00	1.00
$p_0 = 0.4$; ASN máx = 98	MDSRS i = 3	ARL_1	370.3	332.11	237.67	157.77	102.98	55.20	30.67	21.16	14.82	8.95	4.21	2.32	1.22	1.03	1.00	1.00	1.00
	MDSRS i = 2	ARL_1	370.11	331.83	237.26	157.28	102.46	54.67	30.17	20.69	14.39	8.58	3.95	2.14	1.16	1.02	1.00	1.00	1.00
	TS-np	ASN	5.52	67.91	87.68	88.71	83.51	79.70	97.39	97.34	79.15	96.28	96.63	97.65	96.61	94.90	97.85	97.85	93.98
		ARL_0	370.12	377.19	389.89	370.07	374.22	370.35	371.87	371.46	372.01	371.55	382.01	394.22	388.02	372.86	433.47	394.11	470.57
		ARL_1	370.12	136.22	74.15	67.37	38.70	18.38	9.40	6.37	4.50	2.94	1.76	1.30	1.03	1.00	1.00	1.00	1.00
$p_0 = 0.5$; ASN máx = 50	MDSRS i = 3	ARL_1	370.17	361.2	301.25	222.12	154.11	85.81	47.55	32.21	21.92	12.46	5.18	2.52	1.19	1.02	1.00	1.00	1.00
	TS-np	ASN	16.52	31.80	47.62	49.99	31.36	45.65	48.87	48.95	49.20	49.84	49.71	49.91	47.33	49.70	49.70	49.69	3.63
		ARL_0	370.11	371.98	378.86	375.73	370.64	374.27	384.13	373.05	394.23	377.95	382.38	397.69	403.22	494.18	483.44	531.03	512.00
		ARL_1	370.11	270.32	146.59	81.28	49.60	18.17	8.86	8.31	4.38	2.75	1.59	1.27	1.03	1.00	1.00	1.00	1.00
$p_0 = 0.1$; ASN máx = 83	MDSRS i = 2	ARL_1	370.07	336.32	275.66	224.48	182.19	132.97	97.21	79.06	64.43	47.64	29.21	18.27	7.70	3.71	1.50	1.22	1.05
	TS-np	ASN	2.03	42.21	82.67	72.32	79.21	82.89	79.88	82.23	78.43	82.29	81.99	82.80	82.21	82.54	82.46	82.96	82.78
		ARL_0	370.01	370.50	370.19	370.02	370.11	370.68	372.25	375.22	370.96	373.97	370.15	370.12	376.39	375.61	457.30	370.13	441.27
		ARL_1	370.01	310.34	202.19	140.62	98.00	53.00	33.98	25.48	20.69	13.38	7.94	5.09	2.81	1.92	1.30	1.17	1.06
$p_0 = 0.2$; ASN máx = 84	MDSRS i = 2	ARL_1	370.66	318.55	233.78	170.98	125.07	78.57	49.73	36.85	27.43	17.80	8.98	4.84	1.92	1.23	1.02	1.00	1.00
	TS-np	ASN	5.34	61.90	50.42	68.87	83.44	82.96	83.96	81.04	83.82	83.63	82.10	83.43	83.85	81.92	83.79	83.62	83.93
		ARL_0	370.00	370.05	371.62	374.72	371.24	372.82	370.49	370.59	400.91	370.23	374.86	373.84	383.54	370.56	392.08	562.99	414.23
		ARL_1	370.00	295.38	188.52	110.40	66.53	35.55	20.40	11.99	8.88	5.92	3.34	2.32	1.50	1.19	1.03	1.00	1.00
$p_0 = 0.3$; ASN máx = 78	MDSRS i = 2	ARL_1	370.67	345.23	273.65	200.45	141.07	80.94	46.14	31.82	22.05	12.89	5.61	2.80	1.28	1.04	1.00	1.00	1.00
	TS-np	ASN	7.30	42.71	77.55	72.86	77.29	76.39	72.77	77.55	77.26	77.69	77.41	77.29	77.55	77.53	77.89	77.39	77.28
		ARL_0	370.01	371.51	370.34	377.35	371.67	379.94	370.07	375.17	379.86	372.07	381.66	374.51	377.44	379.63	562.52	910.47	420.59
		ARL_1	370.01	292.07	175.50	98.22	60.74	32.29	13.81	9.39	6.91	4.15	2.47	1.70	1.16	1.04	1.00	1.00	1.00
$p_0 = 0.5$; ASN máx = 87	MDSRS i = 2	ARL_1	370.03	349.01	233.02	126.97	64.73	23.03	8.46	4.60	2.72	1.56	1.08	1.01	1.00	1.00	1.00	1.00	1.00
	TS-np	ASN	48.10	72.43	86.15	86.52	86.99	84.21	81.02	85.24	85.65	86.77	82.68	86.77	86.17	86.78	86.80	86.99	4.50
		ARL_0	370.00	370.50	370.32	370.83	372.88	373.72	372.46	374.95	371.21	394.63	383.23	373.51	380.35	373.47	417.75	616.89	512.00
		ARL_1	370.00	270.07	126.79	66.83	32.25	14.24	6.86	4.36	2.90	2.00	1.35	1.14	1.00	1.00	1.00	1.00	1.00

Appendix B

Table A3. Optimal design, ARL_1, and Pg values of the SS, DS, and TS-np charts, $ARL_{0min} = 200$. Continued.

			SS-np		DS-np		TS-np											
γ	p_0	n	ARL_0	ARL_1	ARL_0	ARL_1	WL_1	UCL_1	WL_2	UCL_2	UCL_3	n_1	n_2	n_3	ASN	ARL_0	ARL_1	Pg
1.5	0.01	50	626.50	148.47	201.34	36.83	0.5	5.5	1.5	8.5	11.5	24	62	490	49.06	200.003	17.21	53.26%
		100	291.35	56.52	200.48	21.49	0.5	4.5	1.5	8.5	19.5	28	115	1000	99.88	204.435	9.24	57.02%
		200	232.80	30.89	201.00	11.60	1.5	9.5	2.5	12.5	29.5	91	147	1666	199.15	200.097	5.17	55.42%
		400	372.71	24.17	200.05	5.88	2.5	9.5	3.5	18.5	49.5	181	102	3246	397.07	201.888	2.86	51.42%
	0.02	25	691.62	161.66	202.04	36.86	0.5	2.5	1.5	5.5	12.5	10	50	250	24.59	202.160	18.36	50.19%
		50	311.55	59.49	201.40	21.23	0.5	3.5	1.5	7.5	19.5	14	57	500	49.96	204.931	9.23	56.52%
		100	246.18	32.02	202.11	11.52	1.5	6.5	2.5	20.5	38.5	40	101	1199	98.42	200.715	4.45	61.33%
		200	395.16	24.92	200.05	5.85	2.5	12.5	3.5	60.5	67.5	80	92	2391	199.81	202.907	2.64	54.87%
2.0	0.01	50	626.50	56.31	201.34	13.02	0.5	4.5	1.5	7.5	11.5	23	73	478	49.69	200.363	5.32	59.18%
		100	291.35	19.67	201.41	6.69	0.5	3.5	1.5	12.5	19.5	28	115	999	99.75	200.339	3.05	54.44%
		200	232.80	9.21	201.00	3.40	1.5	10.5	2.5	25.5	28.5	96	107	1617	198.91	200.691	1.95	42.55%
		400	372.71	5.49	202.74	1.88	3.5	19.5	4.5	25.5	35.5	261	100	1972	399.45	202.084	1.38	26.56%
	0.02	25	691.62	60.53	205.27	13.04	0.5	3.5	1.5	6.5	11.5	12	31	245	24.60	202.270	5.28	59.51%
		50	311.55	20.42	205.56	6.60	0.5	4.5	1.5	7.5	18.5	16	40	479	49.65	200.279	3.01	54.47%
		100	246.18	9.40	201.73	3.34	1.5	7.5	2.5	18.5	28.5	48	54	809	99.70	203.880	1.94	42.05%
		200	395.16	5.55	200.48	1.86	3.5	12.5	4.5	34.5	38.5	126	70	1080	199.59	248.298	1.39	25.48%
3.0	0.01	50	626.50	15.93	272.18	4.09	0.5	3.5	1.5	5.5	11.5	25	58	488	49.95	200.506	2.21	45.91%
		100	291.35	5.49	201.04	2.15	0.5	4.5	1.5	10.5	12.5	44	57	501	99.49	205.844	1.57	27.14%
		200	232.80	2.54	204.42	1.39	1.5	6.5	2.5	12.5	16.5	122	70	660	199.70	213.563	1.18	15.12%
		400	372.71	1.52	200.01	1.08	2.5	14.5	5.5	15.5	23.5	245	164	913	399.17	277.521	1.03	4.32%
	0.02	25	691.62	16.73	205.27	4.09	0.5	3.5	1.5	4.5	11.5	13	27	235	24.99	216.526	2.19	46.56%
		50	311.55	5.57	205.56	2.14	0.5	4.5	1.5	8.5	13.5	21	29	288	49.90	212.936	1.55	27.52%
		100	246.18	2.54	205.46	1.38	1.5	7.5	2.5	8.5	16.5	62	31	329	99.82	246.482	1.17	14.93%
		200	395.16	1.52	211.64	1.07	2.5	11.5	3.5	16.5	21.5	120	31	434	192.97	284.267	1.03	3.32%

Table A4. Optimal design, ARL_1, and Pg values of the SS, DS, and TS-np charts, $ARL_{0min} = 370.4$. Continued.

			SS-np		DS-np		TS-np											
γ	p_0	n	ARL_0	ARL_1	ARL_0	ARL_1	WL_1	UCL_1	WL_2	UCL_2	UCL_3	n_1	n_2	n_3	ASN	ARL_0	ARL_1	Pg
1.5	0.01	50	626.50	148.47	372.23	54.92	0.5	4.5	1.5	7.5	13.5	22	69	574	48.13	370.884	21.99	59.97%
		100	1870.79	244.46	372.40	29.60	0.5	4.5	1.5	8.5	24.5	24	138	1300	99.98	372.190	10.15	65.70%
		200	987.60	88.81	371.44	15.35	1.5	6.5	2.5	20.5	42.5	79	206	2600	199.98	374.022	4.98	67.59%
		400	372.71	24.17	370.94	7.04	2.5	9.5	3.5	52.5	73.5	161	139	5168	399.62	387.918	2.80	60.29%
	0.02	25	691.62	161.66	371.88	53.92	0.5	4.5	1.5	7.5	13.5	10	41	284	23.22	372.565	21.97	59.26%
		50	2091.10	267.64	377.19	29.77	0.5	4.5	1.5	9.5	21.5	14	52	548	49.90	370.761	11.15	62.54%
		100	1073.03	94.13	372.96	15.22	0.5	5.5	2.5	10.5	29.5	20	85	804	98.78	371.553	6.75	55.65%
		200	395.16	24.92	371.63	7.00	2.5	9.5	5.5	42.5	49.5	91	136	1477	199.06	371.021	3.24	53.71%
2.0	0.01	50	626.50	56.31	372.53	16.97	0.5	4.5	1.5	7.5	13.5	24	57	583	49.57	371.997	5.81	65.76%
		100	1870.79	64.58	372.40	7.84	0.5	4.5	1.5	9.5	23.5	30	73	1280	99.86	373.159	3.12	60.15%
		200	987.60	20.27	371.44	3.77	1.5	11.5	2.5	17.5	33.5	93	104	1932	199.83	373.185	2.01	46.76%
		400	372.71	5.49	371.01	1.99	2.5	11.5	5.5	26.5	45.5	190	249	2599	399.61	419.353	1.45	27.31%
	0.02	25	691.62	60.53	371.88	16.55	0.5	4.5	1.5	6.5	13.5	11	36	286	24.57	370.621	5.72	65.45%
		50	2091.10	69.39	377.19	7.87	0.5	4.5	1.5	8.5	21.5	15	42	557	49.95	371.162	3.15	60.00%
		100	1073.03	21.05	372.96	3.73	1.5	13.5	2.5	20.5	29.5	49	47	808	99.75	373.574	2.00	46.48%
		200	395.16	5.55	375.82	1.97	2.5	10.5	3.5	35.5	39.5	97	54	1128	199.29	375.157	1.42	28.07%
3.0	0.01	50	626.50	15.93	373.00	4.58	0.5	4.5	1.5	6.5	13.5	24	58	581	49.87	372.152	2.27	50.47%
		100	1870.79	12.37	395.27	2.30	0.5	6.5	1.5	13.5	14.5	43	51	601	99.28	374.032	1.60	30.32%
		200	987.60	3.94	401.13	1.44	0.5	5.5	3.5	13.5	18.5	76	132	724	199.07	467.328	1.24	13.61%
		400	372.71	1.52	379.46	1.09	1.5	21.5	6.5	22.5	30.5	177	256	1262	399.71	1107.967	1.05	3.68%
	0.02	25	691.62	16.73	385.70	4.51	0.5	4.5	1.5	5.5	13.5	12	29	289	24.97	382.267	2.24	50.36%
		50	2091.10	12.88	379.00	2.26	0.5	4.5	1.5	8.5	14.5	21	28	298	49.89	383.513	1.58	30.06%
		100	1073.03	3.97	377.81	1.42	1.5	11.5	2.5	15.5	18.5	58	43	369	99.84	417.858	1.19	16.46%
		200	395.16	1.52	376.04	1.08	1.5	14.5	6.5	21.5	26.5	90	132	504	199.73	696.832	1.04	3.50%

Appendix C

Table A5. Steady-state $ARL_1's$ of the optimal designs [1] for $ARL_{0min} = 200$ and p_0 = 0.005. Continued.

	Parameters												γ							
Scheme	n_1	n_2	n_3	WL_1	UCL_1	WL_2	UCL_2	UCL_3	λ	k	ASN_0	ARL_0	1.5	2	2.5	3	3.5	4	4.5	5
SS	200				4.5						200.00	282.10	55.14	19.33	9.30	5.45	3.65	2.69	2.13	1.78
DS	162	736		2.5	5.5		9.5				197.50	200.50	21.58	6.73	3.52	2.40	1.88	1.59	1.41	1.29
TS	54	249	1989	0.5	4.4	1.9	9.8	19.2			196.86	200.12	9.27	3.10	2.14	1.82	1.64	1.51	1.42	1.34
												ASN1	331.03	482.53	631.82	761.34	856.34	907.60	913.38	879.26
VSS	89	707		1.5	5.5	2.5	7.5				199.90	200.50	15.91	6.39	4.23	3.33	2.84	2.55	2.36	2.22
												ASN1	323.00	324.00	310.00	311.00	322.00	335.00	348.00	360.00
EWMA	200				1.5				0.1		200.00	206.60	18.56	8.11	5.21	3.89	3.14	2.66	2.33	2.08
CUSUM	200				8.8					0.1	200.00	202.00	18.09	8.71	5.83	4.45	3.64	3.10	2.72	2.43
CUSUM (FIR)	200				9.4					0.1	200.00	202.00	18.82	9.01	6.00	4.54	3.68	3.13	2.74	2.45
SS	400				6.5						400.00	226.60	30.36	9.12	4.22	2.54	1.81	1.45	1.26	1.15
DS	304	1404		3.5	8.5		15.5				399.00	200.50	11.64	3.41	1.97	1.50	1.28	1.16	1.10	1.06
TS	177	400	3999	1.5	5.5	3.5	15.5	34.5			398.23	235.00	5.09	2.04	1.57	1.35	1.23	1.15	1.10	1.07
												ASN1	831.22	1409.30	1951.55	2313.08	2414.17	2254.80	1910.99	1495.90
VSS	120	1542		1.5	5.5	7.5	13.5				391.60	200.10	8.98	3.93	2.92	2.50	2.28	2.13	2.01	1.92
												ASN1	699.00	653.00	649.00	690.00	729.00	761.00	786.00	800.00
EWMA	400				2.7				0.1		400.00	206.80	11.84	5.35	3.54	2.70	2.22	1.91	1.69	1.53
CUSUM	400				8.1					0.4	400.00	205.70	11.80	5.19	3.40	2.58	2.12	1.82	1.61	1.46
CUSUM (FIR)	400				8.2					0.4	400.00	200.80	11.98	5.24	3.44	2.63	2.16	1.86	1.64	1.48
SS	800				10.5						800.00	362.20	23.81	5.46	2.40	1.53	1.21	1.08	1.03	1.01
DS	586	2787		5.5	14.5		26.5				799.30	200.40	5.91	1.97	1.35	1.15	1.06	1.02	1.01	1.00
TS	335	484	7990	2.5	9.5	4.5	23.5	59.5			799.82	204.44	2.74	1.56	1.27	1.14	1.07	1.04	1.02	1.01
												ASN1	2136.09	3961.01	5544.66	6468.95	6601.22	5992.39	4856.59	3530.01
VSS	256	4002		2.5	7.5	23.5	28.5				797.10	200.90	5.13	2.84	2.35	2.11	1.95	1.83	1.72	1.62
												ASN1	1473.00	1602.00	1816.00	1947.00	2012.00	2018.00	1988.00	1923.00
EWMA	800				5.4				0.2		800.00	200.80	7.43	3.37	2.28	1.78	1.50	1.31	1.19	1.11
CUSUM	800				9.0					0.8	800.00	204.40	7.40	3.26	2.17	1.68	1.39	1.21	1.11	1.05
CUSUM (FIR)	800				9.2					0.8	800.00	201.10	7.49	3.27	2.19	1.71	1.44	1.25	1.13	1.06

[1] ARL_1 minimized for $\gamma^* = 1.5$ ($p_1^* = 0.0075$).

Table A6. Steady-state $ARL_1's$ of the optimal designs [1] for $ARL_{0min} = 370.4$ and p_0 = 0.005. Continued.

	Parameters												γ							
Scheme	n_1	n_2	n_3	WL_1	UCL_1	WL_2	UCL_2	UCL_3	λ	k	ASN_0	ARL_0	1.5	2	2.5	3	3.5	4	4.5	5
SS	200				5.5						200.00	1773.00	234.08	62.41	24.41	12.14	7.11	4.69	3.38	2.60
DS	161	772		2.5	6.5		10.5				197.80	370.50	29.74	7.91	3.80	2.48	1.91	1.61	1.42	1.30
TS	48	273	2600	0.5	4.5	1.5	8.5	24.5			198.42	371.30	10.24	3.21	2.29	1.96	1.76	1.61	1.51	1.42
												ASN1	342.45	501.58	647.53	756.19	813.43	817.89	779.03	712.05
VSS	81	749		1.5	4.5	2.5	8.5				199.70	382.20	20.91	7.49	4.76	3.64	3.05	2.70	2.46	2.29
												ASN1	357.00	345.00	315.00	309.00	316.00	327.00	341.00	353.00
EWMA	200				1.4				0.1		200.00	400.60	23.12	10.32	6.70	5.02	4.05	3.42	2.97	2.65
CUSUM	200				11.1					0.1	200.00	372.70	22.83	10.79	7.13	5.38	4.36	3.69	3.22	2.87
CUSUM (FIR)	200				8.6					0.2	200.00	384.20	23.29	10.01	6.43	4.80	3.87	3.28	2.86	2.55
SS	400				7.5						400.00	948.60	86.35	19.91	7.57	3.92	2.49	1.82	1.47	1.28
DS	304	1407		3.5	9.5		16.5				399.30	371.60	15.44	3.79	2.03	1.51	1.28	1.17	1.10	1.06
TS	153	600	5200	1.5	8.5	3.5	18.5	43.5			399.03	373.94	5.06	2.25	1.75	1.49	1.33	1.23	1.16	1.11
												ASN1	849.18	1479.41	2167.66	2762.78	3093.51	3053.67	2686.59	2156.59
VSS	120	1549		1.5	5.5	7.5	14.5				399.30	385.10	11.11	4.15	2.96	2.51	2.27	2.12	2.01	1.92
												ASN1	783.00	707.00	671.00	696.00	736.00	768.00	789.00	804.00
EWMA	400				2.8				0.1		400.00	377.60	14.27	6.10	3.96	3.00	2.45	2.09	1.85	1.66
CUSUM	400				9.6					0.4	400.00	383.40	14.23	6.09	3.95	2.98	2.44	2.09	1.85	1.67
CUSUM (FIR)	400				9.8					0.4	400.00	381.20	14.44	6.16	4.00	3.03	2.47	2.12	1.87	1.68

Table A6. *Cont.*

					Parameters												γ				
Scheme	n_1	n_2	n_3	WL_1	UCL_1	WL_2	UCL_2	UCL_3	λ	k	ASN_0	ARL_0	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**
SS	800				11.5						800.00	1134.00	50.84	9.02	3.31	1.85	1.35	1.14	1.06	1.02
DS	582	2910		5.5	14.5		28.5				799.20	372.30	7.09	2.04	1.36	1.15	1.06	1.03	1.01	1.00
TS	441	375	9598	3.5	28.5	4.5	36.5	70.5			795.37	386.51	2.82	1.56	1.25	1.11	1.05	1.02	1.01	1.00
												ASN1	2260.82	4629.65	6896.50	8507.06	9462.13	9966.43	10211.56	10317.54
VSS	256	3952		2.5	9.5	23.5	29.5				788.20	371.00	5.69	2.87	2.38	2.15	2.02	1.93	1.86	1.80
												ASN1	1560.00	1610.00	1807.00	1964.00	2056.00	2107.00	2122.00	2104.00
EWMA	800				5.5				0.2		800.00	375.00	8.77	3.77	2.51	1.94	1.61	1.40	1.25	1.15
CUSUM	800				9.8					0.9	800.00	373.70	8.69	3.63	2.38	1.83	1.51	1.30	1.16	1.08
CUSUM (FIR)	800				10.8					0.8	800.00	373.40	8.76	3.77	2.51	1.93	1.60	1.37	1.21	1.11

[1] ARL_1 minimized for $\gamma^* = 1.5$ ($p_1^* = 0.0075$).

Table A7. Zero-state $ARL_1's$ of the optimal designs [1] for $ARL_{0min} = 200$ and p_0 = 0.005. Continued.

					Parameters												γ				
Scheme	n_1	n_2	n_3	WL_1	UCL_1	WL_2	UCL_2	UCL_3	λ	k	ASN_0	ARL_0	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**
SS	200				5.5						200.00	1773.00	234.08	62.41	24.41	12.14	7.11	4.69	3.38	2.60
DS	161	772		2.5	6.5		10.5				197.80	370.50	29.74	7.91	3.80	2.48	1.91	1.61	1.42	1.30
TS	54	249	1989	0.5	4.4	1.9	9.8	19.2			196.86	200.12	9.27	3.10	2.14	1.82	1.64	1.51	1.42	1.34
VSS	81	749		1.5	4.5	2.5	8.5				199.70	382.20	20.91	7.49	4.76	3.64	3.05	2.70	2.46	2.29
EWMA	200				1.4				0.1		200.00	400.60	23.12	10.32	6.70	5.02	4.05	3.42	2.97	2.65
CUSUM	200				11.1					0.1	200.00	372.70	22.83	10.79	7.13	5.38	4.36	3.69	3.22	2.87
CUSUM (FIR)	200				8.6					0.2	200.00	384.20	23.29	10.01	6.43	4.80	3.87	3.28	2.86	2.55
SS	400				7.5						400.00	948.60	86.35	19.91	7.57	3.92	2.49	1.82	1.47	1.28
DS	304	1407		3.5	9.5		16.5				399.30	371.60	15.44	3.79	2.03	1.51	1.28	1.17	1.10	1.06
TS	177	400	3999	1.5	5.5	3.5	15.5	34.5			398.23	235.00	5.09	2.04	1.57	1.35	1.23	1.15	1.10	1.07
VSS	120	1549		1.5	5.5	7.5	14.5				399.30	385.10	11.11	4.15	2.96	2.51	2.27	2.12	2.01	1.92
EWMA	400				2.8				0.1		400.00	377.60	14.27	6.10	3.96	3.00	2.45	2.09	1.85	1.66
CUSUM	400				9.6					0.4	400.00	383.40	14.23	6.09	3.95	2.98	2.44	2.09	1.85	1.67
CUSUM (FIR)	400				9.8					0.4	400.00	381.20	14.44	6.16	4.00	3.03	2.47	2.12	1.87	1.68
SS	800				11.5						800.00	1134.00	50.84	9.02	3.31	1.85	1.35	1.14	1.06	1.02
DS	582	2910		5.5	14.5		28.5				799.20	372.30	7.09	2.04	1.36	1.15	1.06	1.03	1.01	1.00
TS	335	484	7990	2.5	9.5	4.5	23.5	59.5			799.82	204.44	2.74	1.56	1.27	1.14	1.07	1.04	1.02	1.01
VSS	256	3952		2.5	9.5	23.5	29.5				788.20	371.00	5.69	2.87	2.38	2.15	2.02	1.93	1.86	1.80
EWMA	800				5.5				0.2		800.00	375.00	8.77	3.77	2.51	1.94	1.61	1.40	1.25	1.15
CUSUM	800				9.8					0.9	800.00	373.70	8.69	3.63	2.38	1.83	1.51	1.30	1.16	1.08
CUSUM (FIR)	800				10.8					0.8	800.00	373.40	8.76	3.77	2.51	1.93	1.60	1.37	1.21	1.11

[1] ARL_1 minimized for $\gamma^* = 1.5$ ($p_1^* = 0.0075$).

Table A8. Zero-state $ARL_1's$ of the optimal designs [1] for $ARL_{0min} = 370.4$ and p_0 = 0.005. Continued.

					Parameters												y				
Scheme	n_1	n_2	n_3	WL_1	UCL_1	WL_2	UCL_2	UCL_3	λ	k	ASN_0	ARL_0	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**
SS	200				5.5						200.00	1773.00	234.08	62.41	24.41	12.14	7.11	4.69	3.38	2.60
DS	161	772		2.5	6.5		10.5				197.80	370.50	29.74	7.91	3.80	2.48	1.91	1.61	1.42	1.30
TS	48	273	2600	0.5	4.5	1.5	8.5	24.5			198.42	371.30	10.24	3.21	2.29	1.96	1.76	1.61	1.51	1.42
VSS	81	749		1.5	4.5	2.5	8.5				199.70	382.20	20.91	7.49	4.76	3.64	3.05	2.70	2.46	2.29
EWMA	200				1.4				0.1		200.00	400.60	23.12	10.32	6.70	5.02	4.05	3.42	2.97	2.65
CUSUM	200				11.1					0.1	200.00	372.70	22.83	10.79	7.13	5.38	4.36	3.69	3.22	2.87
CUSUM (FIR)	200				8.6					0.2	200.00	384.20	23.29	10.01	6.43	4.80	3.87	3.28	2.86	2.55
SS	400				7.5						400.00	948.60	86.35	19.91	7.57	3.92	2.49	1.82	1.47	1.28
DS	304	1407		3.5	9.5		16.5				399.30	371.60	15.44	3.79	2.03	1.51	1.28	1.17	1.10	1.06
TS	153	600	5200	1.5	8.5	3.5	18.5	43.5			399.03	373.94	5.06	2.25	1.75	1.49	1.33	1.23	1.16	1.11
VSS	120	1549		1.5	5.5	7.5	14.5				399.30	385.10	11.11	4.15	2.96	2.51	2.27	2.12	2.01	1.92
EWMA	400				2.8				0.1		400.00	377.60	14.27	6.10	3.96	3.00	2.45	2.09	1.85	1.66

Table A8. *Cont.*

	Parameters												γ							
Scheme	n_1	n_2	n_3	WL_1	UCL_1	WL_2	UCL_2	UCL_3	λ	k	ASN_0	ARL_0	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**
CUSUM	400				9.6					0.4	400.00	383.40	14.23	6.09	3.95	2.98	2.44	2.09	1.85	1.67
CUSUM (FIR)	400				9.8					0.4	400.00	381.20	14.44	6.16	4.00	3.03	2.47	2.12	1.87	1.68
SS	800				11.5						800.00	1134.00	50.84	9.02	3.31	1.85	1.35	1.14	1.06	1.02
DS	582	2910		5.5	14.5		28.5				799.20	372.30	7.09	2.04	1.36	1.15	1.06	1.03	1.01	1.00
TS	441	375	9598	3.5	28.5	4.5	36.5	70.5			795.37	386.51	2.82	1.56	1.25	1.11	1.05	1.02	1.01	1.00
VSS	256	3952		2.5	9.5	23.5	29.5				788.20	371.00	5.69	2.87	2.38	2.15	2.02	1.93	1.86	1.80
EWMA	800				5.5				0.2		800.00	375.00	8.77	3.77	2.51	1.94	1.61	1.40	1.25	1.15
CUSUM	800				9.8					0.9	800.00	373.70	8.69	3.63	2.38	1.83	1.51	1.30	1.16	1.08
CUSUM (FIR)	800				10.8					0.8	800.00	373.40	8.76	3.77	2.51	1.93	1.60	1.37	1.21	1.11

[1] ARL_1 minimized for $\gamma^* = 1.5$ ($p_1^* = 0.0075$).

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