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Designing of Optimal Reinsurance Indemnity

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Abstract: This paper contributes to relevant research in the area of optimal reinsurance indemnity and deals with the risk measures that are used in reinsurance. The research aims at finding optimal reinsurance contracts under different risk levels. The paper has demonstrated that the method of calculating the indemnity of the reinsurance contract discussed in the aforementioned article—the reduction of the square of excess of loss—can be generalised and is valid in all instances where $p \in (0; 1) \cup (1; +\infty)$. The results could be useful for the insurance companies calculating indemnities for different cases, as they could state the degrees that fit their needs most.

Keywords: reinsurance; risk measures; indemnity

MSC: 97M30

1. Introduction

There are many different reasons why insurance companies fail. According to Carter [1], some reasons are similar to those in other industries, such as inefficiencies or inappropriate pricing, but the most common failure among insurance companies is an indefinite increase in major expenditures. Therefore, the main role of reinsurance is to protect against potential contingent losses through risk diversification. Stashchuk et al. [2] emphasised that reinsurance not only ensures the stability of the insurers' development and their safe raising of funds but is also one of the most important measures to ensure effective protection against various natural, man-made or other risks, as well as a fair performance of their obligations as insurers. The need for reinsurance is also directly linked to the regulatory authority's requirement for insurance companies to control their risk by setting strict risk management policies.

Different optimisation criteria and conditions are calculated by various risk measurement tools. Bernard and Tian used value at risk (VaR) and the conditional tail expectation (CTE) to address the optimal reinsurance task [3]. Other authors have analysed VaR and conditional value at risk (CVaR) in their articles [4,5]. For instance, Zhang et al. [6] used VaR and tail value at risk (TVaR). In addition, Hu et al. [7] used CTE in their study.

The current paper is based on the article "Optimal Reinsurance Arrangements Under Tail Risk Measures" [3] which examines the optimal risk management strategy for an insurance company in light of regulatory constraints. The reduction of the square of large losses used in the article referred to is changed in this article to the p-th degree of losses, and by reducing the p-th degree of excess of loss (The reinsurance contract with an excess of loss provision indicates that the reinsurer is responsible for losses over a certain limit m [8].), the optimal indemnity (Indemnity is compensation to a party for a loss or damage that has already occurred) level of the reinsurance contract is found.

Despite the various risk measures used in reinsurance, this article focuses on and examines one of the methods of calculating reinsurance contract indemnity—the conditional tail—using the VaR margin condition and seeks to find the optimal level of indemnity that would reduce the insurance company's losses.

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The paper consists of three parts. First, the introduction describes the role of reinsurance, the different optimisation criteria and conditions used by the risk measurement tools, and the explanation of the basis on which this research was conducted. The second chapter is dedicated to the calculation of the indemnity of a reinsurance contract and seeks to find the optimal level of indemnity. The propositions and proofs are presented in this chapter. Finally, by comparing and discussing the results in relation to the relevant literature, the final chapter concludes the research.

2. Method

This article examines the methods of calculating the indemnity of a reinsurance contract and seeks the optimal indemnity that would reduce the losses of the insurance company. The study considers a risk measure that is based on the expected p-th degree excessive loss, and this risk is related to the variance tail measure [3]. Tail risk measures are useful as they summarise the extent of potential losses in the worst possible outcomes [9]. Tail risk measures refer to measures that are focused on the behaviour of the tails, such as value at risk. Thus, this work addresses the following problem: to find the optimal level of indemnity $I(X)$, which is solved by:

$$\min_{I(X)}\left\{\mathbb{E}\left[(W_0 - W - \nu)^{\mathrm{p}}\right]\mathbb{1}_{W_0 - W > \nu}\right\} \text{ s. t. } \begin{cases} 0 \le I(X) \le X, \\ \mathbb{E}[I(X)] = \widetilde{\Delta}, \end{cases} \tag{1}$$

where:

Δ—variation,

$\widetilde{\Delta} = \frac{\Delta}{1+\rho}$, $\rho > 0$,

W_0—initial assets of the insurance company (equity, premiums collected from insurance contracts sold),

W—the final assets of the insurance company, if reinsurance has been purchased,

$W = W_0 - P - X + I(X)$,

N—VaR quantity, i.e., the probability that the loss will exceed ν must be less than the confidence level α,

X—the total amount of indemnity payable at the end of the period (loss),

$I(X)$—reinsurance contract indemnity,

α—confidence level,

ρ—safety supplement,

P—premium for a reinsurance contract with indemnity $I(X)$.

This task aims to minimise the p-th degree of excessive losses.

Note: In the article of Bernard and Tian [3], the aim of this task was to minimise the square of excessive losses.

Proposition 1. *Assume that X has a continuous, strictly increasing distribution function, $X \in [0; +\infty)$ and $P \in S$, $S \in \left\{P : 0 \le P < \mathbb{E}\left[(X - \nu + P)^+ + C\left((X - \nu + P)^+\right)\right]\right\}$, where C is the cost function (The cost function is a formula used to predict the cost that will be experienced at a certain activity level.). Let d_P and l_P be deductibles (Deductible is the amount of money that the customer is responsible for paying toward an insured loss.), for which the corresponding premiums are P. In that case (1), the solution of the task is indemnity $(X - d_{P^*})^+$ in instances where $p \in (1; +\infty)$ and $(X - l_{P^*})^+$, and when $p \in (0;\ 1)$, for which P* is the solution to the problem of such minimisation:*

$$\min_{0 \le \mathrm{P} \le \widetilde{\Delta}} \mathbb{E}\left[(\mathrm{W}_0 - \mathrm{W}_\mathrm{P} - \nu)^{\mathrm{p}} \mathbb{1}_{\mathrm{W}_0 - \mathrm{W}_\mathrm{P} > \nu}\right]$$

with $\mathrm{W_P}$ being assets after relevant indemnity $(X - d_P)^+$ or $(X - l_{P^*})^+$ purchase.

Two auxiliary lemmas (Lemmas 1 and 2) are needed to find the solution.

Lemma 1. *Let $p \in (0; +\infty)$. Suppose Y^* satisfies the following conditions:*

(i) $0 \le Y^* \le X$,

(ii) $\mathbb{E}[Y^*] = \widetilde{\Delta}$,

(iii) There exists such a $\lambda > 0$, such that for each $\omega \in \Omega$, $Y^*(\omega)$ there is the following optimisation problem solution:

$$\min_{Y \in [0,\ X(\omega)]} \Big\{ (P + X(\omega) - Y - \nu)^p \mathbb{1}_{Y < P + X(\omega) - \nu} + \lambda Y \Big\}. \tag{2}$$

Then Y^* is the solution to the problem (2).

Proof of Lemma 1. Let I be the possible indemnity for a reinsurance contract that satisfies the limitations of the optimisation problem.

Let Y^* be the optimal indemnity for a reinsurance contract that satisfies all three conditions of the lemma.

Applying condition (iii), the result is that in all cases where $\omega \in \Omega$,

$$\begin{aligned}(P + X(\omega) - Y^*(\omega) - \nu)^p \, \mathbb{1}_{Y^*(\omega) < P + X(\omega) - \nu} + \lambda Y^*(\omega) \\ \le \ (P + X(\omega) - I(\omega) - \nu)^p \, \mathbb{1}_{I(\omega) < P + X(\omega) - \nu} + \lambda I(\omega).\end{aligned}$$

Thus, we derive that

$$\begin{aligned}(P + X(\omega) - Y^*(\omega) - \nu)^p \, \mathbb{1}_{Y^*(\omega) < P + X(\omega) - \nu} - (P + X(\omega) - I(\omega) - \nu)^p \, \mathbb{1}_{I(\omega) < P + X(\omega) - \nu} \\ \le \lambda (I(\omega) - Y^*(\omega)).\end{aligned}$$

Next, we calculate the expected values of both sides of the inequality, and based on condition (ii), we obtain

$$\begin{aligned}\mathbb{E}\Big[(P + X - Y^* - \nu)^p \, \mathbb{1}_{Y^* < P + X - \nu}\Big] \\ -\mathbb{E}\Big[(P + X - I - \nu)^p \, \mathbb{1}_{I < P + X - \nu}\Big] \le \lambda (\mathbb{E}[I(X)] - \widetilde{\Delta}) = 0\end{aligned}$$

This means that

$$\mathbb{E}\Big[(P + X - Y^*(\omega) - \nu)^p \, \mathbb{1}_{Y^* < P + X - \nu}\Big] \ \le \ \mathbb{E}\Big[(P + X - I - \nu)^p \, \mathbb{1}_{I < P + X - \nu}\Big].$$

The lemma is thus proved. □

Note: In the Bernard and Tian article [3], this lemma was proved, where $p = 1$ and 2, but it is correct in all instances where $p > 0$.

Lemma 2. *If $P \le \nu$, then each member of the family $\{Y_\lambda\}_{\lambda > 0}$ satisfies conditions (i) and (iii) of Lemma 2.*

Here:

$$Y_\lambda(\omega) = \begin{cases} 0, & if\ X(\omega) < \nu - P + \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}, \\ X(\omega) + P - \nu - \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}, & if\ \nu - P + \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}} \le X(\omega), \end{cases} \tag{3}$$

when $p \in (1; +\infty)$ and

$$Y_\lambda(\omega) = \begin{cases} 0, & \text{if} X(\omega) < \nu - P, \\ X(\omega) + P - \nu, & \text{if} \nu - P \le X(\omega) \le \nu - P + \left(\frac{1}{\lambda}\right)^{\frac{1}{1-p}}, \\ 0, & \text{if} X(\omega) > \nu - P + \left(\frac{1}{\lambda}\right)^{\frac{1}{1-p}}, \end{cases} \tag{4}$$

when $p \in (0;\ 1)$.

Proof of Lemma 2. Let us distinguish two instances:

I. When $p \in (1; +\infty)$;
II. When $p \in (0; 1)$.

I. When $p \in (1; +\infty)$.

1. We will demonstrate the inequalities of condition (i) of Lemma 1:

(a) If $X < \nu - P + \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}$, then $Y_\lambda = 0$, o $X \ge 0$.

(b) If $X \ge \nu - P + \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}$, then $0 \le Y_\lambda = X + P - \nu - \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}} \le X$.

From this, we deduce that $0 \le Y_\lambda \le X$ in both instances.

2. We will demonstrate property (iii) of Lemma 1.

Firstly, if $0 \le X < \nu - P$, then $P + X - \nu < 0$, and the function that needs to be minimised at interval $[0; X]$ equals λY (see (2)). The latter is increasing in the interval $[0; X]$, since $\lambda > 0$; therefore, the minimum is reached when $Y^* = 0$.

On the other hand, if $X \ge \nu - P$ and $P \le \nu$, then $0 \le P + X - \nu \le X$.

There are two instances:

(a) If $Y \in [0; P + X - \nu)$, then the function to be minimised is

$$\Phi_1(Y) = (\mathrm{P} + \mathrm{X} - \nu - \mathrm{Y})^p + \lambda Y.$$

We seek the minimum of the latter function:

$$\Phi'_1(Y) = -p(P + X - \nu - Y)^{p-1} + \lambda.$$

We find critical points in the interval $[0;\ P + X - \nu)$:

$$\begin{gathered} \Phi'_1(Y) = 0, \\ -p(P + X - \nu - Y)^{p-1} = -\lambda, \\ (P + X - \nu - Y)^{p-1} = \frac{\lambda}{p}, \\ P + X - \nu - Y = \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}. \end{gathered}$$

This point does not necessarily belong to the interval $[0;\ P + X - \nu)$, but $\Phi_1(Y)$ increases when $Y \in \left(P + X - \nu - \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}; +\infty\right)$ and decreases when $Y \in \left(-\infty;\ P + X - \nu - \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}\right)$.

At the critical point, the value of the function is $\Phi_1(Y)$, and at the left end of interval $[0; P + X - \nu)$:

$$\begin{aligned} \Phi_1\left(P + X - \nu - \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}\right) &= \left(P + X - \nu - P - X + \nu + \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}\right)^p + \lambda\left(P + X - \nu - \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}\right) \\ &= \lambda\left(P + X - \nu - \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}\right) + \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}. \end{aligned}$$

$$\Phi_1(0) = (P + X - \nu)^p$$

Since $0 \leq P + X - \nu \leq X$, then

$$(\Phi_1)_{min} = \Phi_1(Y_{min}), \text{ here}$$

$$Y_{min} = \max\left(0,\ X + P - \nu - \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}\right).$$

(b) If $Y \in [P + X - \nu,\ X]$, then the function to be minimised is $\Phi_2(Y) = \lambda Y$.

Its minimum is at the point $Y = P + X - \nu$, since the function $\Phi_2(Y)$ is increasing; hence, its minimum is at the left edge of the interval:

$$(\Phi_2)_{min} = \lambda(P + X - \nu).$$

Now, $(\Phi_1)_{min}$ needs to be compared with $(\Phi_2)_{min}$. We shall analyse two instances:

1. When $0 < X + P - \nu - \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}$, then

$$\begin{aligned}\Phi_1(Y) = \left(\tfrac{\lambda}{p}\right)^{\frac{p}{p-1}} + \lambda\left(P + X - \nu - \left(\tfrac{\lambda}{p}\right)^{\frac{1}{p-1}}\right) = \left(\tfrac{\lambda}{p}\right)^{\frac{p}{p-1}} + \lambda(P + X - \nu) - \lambda\left(\tfrac{\lambda}{p}\right)^{\frac{1}{p-1}} = \Phi_2(P + X - \nu) - \\ \left(\tfrac{\lambda}{p}\right)^{\frac{1}{p-1}}\left(\lambda - \tfrac{\lambda}{p}\right) = (\Phi_2)_{min} - \lambda\left(\tfrac{\lambda}{p}\right)^{\frac{1}{p-1}}\left(1 - \tfrac{1}{p}\right) < (\Phi_2)_{min}.\end{aligned} \tag{5}$$

2. When $0 \geq X + P - \nu - \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}$, then

$$(\Phi_1)_{min} = (P + X - \nu)^p,$$

$$(\Phi_2)_{min} = \lambda(P + X - \nu),$$

$$P + X - \nu \leq \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}.$$

Inequality of power $(\Phi_1)_{min} \leq (\Phi_2)_{min}$, when, and only when, $0 \leq P + X - \nu \leq \lambda^{\frac{1}{p-1}}$, whereas in our case $P + X - \nu \leq \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}} < \lambda^{\frac{1}{p-1}}$, since $p > 1$, which means that $Y_{min} = Y_\lambda$ (see (3)), which satisfies condition (iii) of Lemma 1.

II. When $p \in (0; 1)$

The first part of the proof is analogical to parts (a) and (b) of the proof for Part I of Lemma 2, when $p \in (1;\ \infty)$, but in this instance, we obtain these values:

$$(\Phi_1)_{max} = \Phi_1(Y_{max}), \text{ here}$$

$$Y_{max} = \max\left(0,\ X + P - \nu - \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}\right).$$

At the critical point, a maximum is achieved, whereas the task is to minimise the purpose function, which therefore requires a minimum point which can be only at the extremes of interval $[0; P + X - \nu)$:

$$\Phi_1(0) = (P + X - \nu)^p,$$

$$\Phi_1(P + X - \nu) = \lim_{Y \to P+X-\nu} \Phi_1(Y) = \lambda(P + X - \nu).$$

Therefore,

$$(\Phi_1)_{min} = (P + X - \nu)^p, \text{ when}$$
$$(P + X - \nu)^p < \lambda(P + X - \nu),$$
$$X > \nu - P + \left(\frac{1}{\lambda}\right)^{\frac{1}{1-p}} \text{ and}$$
$$(\Phi_1)_{min} = \lambda(P + X - \nu), \text{ when}$$
$$(P + X - \nu)^p \geq \lambda(P + X - \nu),$$
$$X \leq \nu - P + \left(\frac{1}{\lambda}\right)^{\frac{1}{1-p}}, \text{ and}$$
$$(\Phi_2)_{min} = \lambda(P + X - \nu).$$

From this, it follows that

$$Y_{min} = \begin{cases} 0, & \text{when } X > \nu - P + \left(\frac{1}{\lambda}\right)^{\frac{1}{1-p}}, \\ P + X - \nu, & \text{when } \nu - P \leq X \leq \nu - P + \left(\frac{1}{\lambda}\right)^{\frac{1}{1-p}}, \\ 0, & \text{when } X < \nu - P, \end{cases}$$

which means that $Y_{min} = Y_\lambda$ (see (4)), and that satisfies condition (iii) of Lemma 1. □

Note. Y must not be a negative value; therefore, 0 is chosen when $X < \nu - P$.

Proof of Proposition 1. Two auxiliary theorems are needed to prove the proposition.

Lebesgue's dominated convergence theorem. If f_n $(n = 1, 2, \dots)$ are measurable functions, g is a non-negative integral function, $|f_n(\omega)| \leq g(\omega)(n = 1, 2, \dots)$ and $f_n \to f$, then

$$\lim_{n\to\infty} \int_I f_n(\omega)d\omega \to \int_I f(\omega)d\omega$$

Bolzano–Cauchy theorem. If the continuous function f in the segment $[a; b]$ acquires the uneven values $f(a) = A$ and $f(b) = B$ at the extremities of the segment, then every value C that satisfies condition $A < C < B$ is matched by some point c of the segment $[a; b]$, such that $f(c) = C$.

From Lemmas 1 and 2, it follows that we must prove the existence of $\lambda > 0$, such that Y_λ, which is defined in Lemma 2, satisfies condition (ii) of Lemma 1. First, the mean of Y_λ must be calculated. There are two instances:

1. $\varepsilon_\lambda := \mathbb{E}(Y_\lambda) = \mathbb{E}\left(\left(X + P - \nu - \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}\right)\mathbb{1}_{X \geq \nu - P + \left(\frac{\lambda}{p}\right)^{\frac{1}{p-1}}}\right)$ (see (3)),
2. $\delta_\lambda := \mathbb{E}(Y_\lambda) = \mathbb{E}\left((X + P - \nu)\mathbb{1}_{\nu - P \leq X \leq \nu - P + \left(\frac{1}{\lambda}\right)^{\frac{1}{1-p}}}\right)$ (see (4)).

We see that

$$\lim_{\lambda\to 0+} \varepsilon_\lambda = \mathbb{E}\Big((X + P - \nu)^+\Big) \text{ and } \lim_{\lambda\to+\infty} \varepsilon_\lambda = 0$$

That is to say, $P \in \Big(0;\ \mathbb{E}(X + P - \nu)^+\Big)$.

We will prove that ε_λ is a continuing function, i.e., $\lim_{\lambda\to\lambda_0} \varepsilon_\lambda = \varepsilon_{\lambda_0}$, here $\lambda_0 > 0$.

From the assumption that the distribution of X is continuous, it flows that the distribution of Y_λ is also continuous.

Based on Lebesgue's theorem of dominated convergence, we can assert that $|Y_\lambda| \leq |Y_0|$, where Y_0 is an integrable non-negative function. From this, it follows that

$$\lim_{\lambda\to\lambda_0} \varepsilon_\lambda = \lim_{\lambda\to\lambda_0} \mathbb{E}(Y_\lambda) = \mathbb{E}Y_0 = \varepsilon_{\lambda_0}.$$

Since ε_λ is a continuous function, and $0 < P < \mathbb{E}(X + P - \nu)^+$, based on the Bolzano–Cauchy theorem, it can be asserted that it fully completes the interval and that therefore there is a solution $\lambda^* \in \mathbb{R}_+^*$ satisfying the equation $\varepsilon_\lambda = \mathbb{E}[I(X)] = \widetilde{\Delta}$.

We see that

$$\lim_{\lambda \to 0+} \delta_\lambda = \mathbb{E}\Big((X + P - \nu)^+\Big) \text{ and}$$
$$\lim_{\lambda \to +\infty} \delta_\lambda = 0$$

From here on, the proof is analogical to that demonstrated in the first instance; i.e., there is a solution $\lambda^* \in \mathbb{R}_+^*$ satisfying the equation $\varepsilon_\lambda = \mathbb{E}[I(X)] = \widetilde{\Delta}$.

The proposition is proved. □

3. Discussion and Conclusions

Considering the results of the research in the research area of optimal reinsurance indemnity, the variety of approaches should be emphasised. The first research started in the 1960s when Borch [10] elucidated that reinsurance is optimal when the reinsurance premium is set according to the expected value premium principle, which is supported by Arrow [11]. Kaluszka [12] has derived optimal reinsurance under premium principles considering the variance of the retained loss as the function to be minimised. Later on, Kaluszka [13] calculated the reinsurance premium based on the expectation and variance of the total risk, and continued his research by applying many different convex premium principles, including TVaR [14]. The combination of VaR and CTE as risk measures was used by Cai and Tan [15]. Balbas et al. [16] analysed whether the classical reinsurance contracts could be optimal and proved that the optimal reinsurance problem could be solved by providing the optimal conditions for general risk measures. Chi [4] and Chi and Tan [5] asserted that the optimal layer reinsurance is always optimal under VaR and CVaR criteria. VaR and TVaR as criteria were applied by Zhang et al. [17] and Zhang et al. [6] concerning winning mutual benefits for both sides, insurer and reinsurer. CTE, as a condition in combination with Wang's premium principle, was analysed by Hu et al. [7]. To outline, it could be noticed that risk measures such as VaR, CTE and TVaR are used and analysed frequently under miscellaneous conditions. The current article contributes to the variety of approaches by adapting and expanding the research performed by Bernard and Tian [3], who found the optimal indemnity $I(X)$ by minimising the square of excess loss.

To conclude, this article's goal—to find the optimal indemnity $I(X)$, which minimises the p-th degree of excess loss—was achieved. One of the methods of calculating the reinsurance contract indemnity was considered: the conditional tail expectation using the VaR condition is different from the one used in Bernard and Tian's (2009) article on reducing the square of excessive losses. It can be argued that the optimal indemnity has been found, and the losses of the insurance company would be reduced. The results could be useful for insurance companies calculating indemnities for different cases, as they could state the degrees that fit their needs most.

However, solving the mentioned problems, the dynamism of the pension ecosystem and the uncertainty of economic growth cannot be ignored. In future research, such an analysis should consider other potential risk measures, premium principles and changing regulatory constraints.

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