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Extended Tanh-Function Method and Its Applications in Nonlocal Complex mKdV Equations

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Abstract: In order to construct the multiple traveling wave solutions of the nonlocal modified Korteweg de Vires (mKdV) equation, the modified tanh-function approach for local soliton equations is extended to a nonlocal complex mKdV equation. The central idea of this method is to use the solution of the Riccati equation to replace the tanh function in the tanh function (THF) method. As an application, we investigate a new traveling wave solution for the nonlocal complex mKdV equation of Ablowitz and Musslimani. Moreover, some exciting diagrams show the underlying dynamics of some given solutions.

Keywords: nonlocal complex mKdV equation; extended tanh-funtion method; riccati equation; travelling wave solution; symbolic computation

MSC: 35C07; 35C08; 35Q55

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1. Introduction

It is noted that the exact solution of the soliton equation is one of the critical problems of soliton theory. These exact solutions usually predict the complex physical phenomena underlying these mathematical models [1–12]. For this field, the simple method is the tanh function method [3–7,10,13,14] and its more general form, namely, the extended tanh function approach, which is further advanced by Wazwaz [15,16] and Fan [17].

The Korteweg de Vries equation (KdV) equation and the mKdV equation are typical soliton equations that characterize the development of weak- and small-amplitude waves [18]. A complex version of the mKdV equation is

$$q_t + 6\varepsilon|q|^2 q_x + q_{xxx} = 0 \ (\varepsilon = \pm 1) \tag{1}$$

which appears in [19]. The initial problem and structural stability of Equation (1) are studied in [20,21]. Not long ago, Musslimani and Abowitz proposed the nonlocal complex mKdV equation in [22]

$$q_t(x,t) + 6\sigma q(x,t) q^*(-x,-t) q_x(x,t) + q_{xxx}(x,t) = 0 \ (\sigma = \pm 1) \tag{2}$$

Here, $q(x,t)$ is a complex-valued function of t and x; the complex conjugation is denoted by the symbol *. Some exact solutions of Equation (2) had been studied by using the Darboux transformation or inverse scattering transform method in [21,23]. Furthermore, asymptotic analysis of the mKdV equation was studied in [24]. These solutions show that the nonlocal mKdV Equation (2) has some new properties that are different from local mKdV Equation (1) in [25,26]. For example, in a physical application, the nonlocal mKdV equation has delayed time-reversal symmetry [27]. However, it was not found that the extended tanh function method (ETFM) can be used to solve the nonlocal complex mKdV equation. In this paper, the extended tanh function method of the local soliton equation is developed for the nonlocal complex mKdV equation, and exact solutions are obtained.

The structure of this paper is as follows. The first part briefly introduces the current study for the mKdV equation. See Section 2 for ETFM procedures. We apply ETFM to the nonlocal mKdV equation in Section 3. Section 4 is the conclusion.

2. Extended Tanh Function Method

For Nonlinear Evolution Equations system:

$$H(v, u, v_t, u_t, v_x, u_x, v_{xt}, u_{xt}, \cdots) = 0. \tag{3}$$

$$K(v, u, v_t, u_t, v_x, u_x, v_{xt}, u_{xt}, \cdots) = 0. \tag{4}$$

H and K are polynomials $v(x,t)$ and $u(x,t)$. The main steps are:

Step 1. Use the following converters,

$$u(x,t) = u(\xi), v(x,t) = v(\xi),\ \xi = x + ct. \tag{5}$$

C is a constant. The systems (3) and (4) are then converted to nonlinear ordinary differential equations.

$$H_1(v, u, v', u', v'', u'' \cdots) = 0. \tag{6}$$

$$K_1(v, u, v', u', v'', u'' \cdots) = 0. \tag{7}$$

where H_1 and K_1 are polynomials of the derivatives of $u(\xi)$ and $v(\xi)$, while $u' = \frac{du}{d\xi}$, $v' = \frac{dv}{d\xi}$.

Step 2. Suppose solutions of Equations (6) and (7),

$$u(\xi) = \sum_{i=0}^{m} a_i \phi^i \qquad v(\xi) = \sum_{i=0}^{n} b_i \phi^i. \tag{8}$$

where $a_i \neq 0, b_i \neq 0$ are undetermined constants, ϕ is suitable for the Riccati equation.

$$\phi' = d + \phi^2. \tag{9}$$

where $\phi' = \frac{d\phi}{d\xi}$, the solution of Equation (9) has three types according to the constant d:

Case 1. If $d < 0$, then

$$\phi = -\sqrt{-d}\tanh(\sqrt{-d}\xi),\ \text{or}\ \phi = -\sqrt{-d}\coth(\sqrt{-d}\xi). \tag{10}$$

Case 2. if $d = 0$, then

$$\phi = -\frac{1}{\xi}. \tag{11}$$

Case 3. if $d > 0$, then

$$\phi = \sqrt{d}\tan(\sqrt{d}\xi),\ \text{or}\ \phi = -\sqrt{d}\cot(\sqrt{d}\xi). \tag{12}$$

Step 3. Find the positive integer n, m in Equation (8). Make the highest derivative equal to the highest power of the nonlinear term in the Equations (6) and (7), m and n are determined.

Step 4. Substitute Equations (8) and (9) into Equations (6) and (7). The coefficients of $\phi^i (i = 1, 2, \cdots)$ are combined, and then the coefficient equals zero. To obtain the values of a_i and b_i, Mathematica is used to solve algebraic equations.

Step 5. Substitute b_i and a_i into Equation (8) to access the exact solutions of Equations (3) and (4).

3. Application to Nonlocal Complex mKdV Equation

Equation (2) is a nonlinear integrable system. By symmetry reduction

$$\sigma q^*(-x,-t) = r(x,t), \sigma = \pm 1, \tag{13}$$

Equation (2) converts to two classical equations

$$q_t(x,t) + q_{xxx}(x,t) - 6q(x,t)r(x,t)q_x(x,t) = 0. \tag{14}$$

$$r_t(x,t) + r_{xxx}(x,t) - 6q(x,t)r(x,t)r_x(x,t) = 0. \tag{15}$$

To obtain an exact solitary wave solution for mKdV Equation (2), transform Equations (14) and (15) by

$$r(x,t) = R(\xi), q(x,t) = Q(\xi), \xi = x + ct. \tag{16}$$

Then Equations (14) and (15) are reduced to two ordinary differential equations,

$$cQ' + Q''' - 6QRQ' = 0. \tag{17}$$

$$cR' + R''' - 6RQR' = 0. \tag{18}$$

According to the law of homogeneous equilibrium, the highest derivative term $q_{xxx}(x,t)$ is equal to the power of the nonlinear term $q(x,t)r(x,t)q_x(x,t)$ of the Equation (14), $n + 3 = 2n + 1 + m$, that is, $m = n = 1$. Then assume:

$$Q(\xi) = a_0 + a_1\phi, \tag{19}$$

$$R(\xi) = b_0 + b_1\phi. \tag{20}$$

where ϕ satisfies Riccati Equation (9). Then,

$$Q' = a_1\phi' = a_1(d+\phi^2), Q'' = 2a_1\phi\phi' = 2a_1\phi(d+\phi^2), Q''' = 2a_1d^2 + 8a_1d\phi^2 + 6a_1\phi^4 \tag{21}$$

$$R' = b_1\phi' = b_1(d+\phi^2), R'' = 2b_1\phi\phi' = 2b_1\phi(d+\phi^2), R''' = 2d^2b_1 + 8db_1\phi^2 + 6b_1\phi^4. \tag{22}$$

Substitute (19)–(21) for (17), and substitute (19), (20), (22) for (18). When the same power term of ϕ is combined and the coefficients ϕ^i is set to 0, the nonlinear equation of $a_0, a_1; b_0, b_1$ and c are obtained.

$$\begin{cases} 2a_1d^2 - 6a_0db_0a_1 + da_1c = 0 \\ a_1^2b_0d + a_0a_1db_1 = 0 \\ 8a_1d - 6a_0a_1b_0 - 6a_1^2b_1 + a_1c = 0 \\ a_1^2b_0 + a_0a_1b_1 = 0 \\ a_1 - a_1^2b_1 = 0 \end{cases} \tag{23}$$

$$\begin{cases} 2b_1d^2 - 6a_0db_0b_1 + db_1c = 0 \\ a_0db_1^2 + a_1db_0b_1 = 0 \\ 8db_1 - 6a_0b_0b_1 - 6a_1db_1^2 + b_1c = 0 \\ a_0b_1^2 + a_1b_0b_1 = 0 \\ b_1 - a_1b_1^2 = 0 \end{cases}. \tag{24}$$

Four groups of solutions of Equation (23) are obtained in Mathematica.

$$\{a_1 = 0\},$$

$$\left\{d = \frac{-6a_0^2 - a_1^2c}{2a_1^2}, b_0 = -\frac{a_0}{a_1^2}, b_1 = \frac{1}{a_1}\right\},$$

$$\left\{a_0 = -\frac{ia_1\sqrt{c}}{\sqrt{6}}, d = 0, b_0 = \frac{i\sqrt{c}}{\sqrt{6}a_1}, b_1 = \frac{1}{a_1}\right\},$$

$$\left\{a_0 = \frac{ia_1\sqrt{c}}{\sqrt{6}}, d = 0, b_0 = -\frac{i\sqrt{c}}{\sqrt{6}a_1}, b_1 = \frac{1}{a_1}\right\}.$$

Equation (24) is solved to obtain the other four solutions.

$$\{b_1 = 0\},$$

$$\left\{d = \frac{-6a_0^2 - a_1^2 c}{2a_1^2}, b_0 = -\frac{a_0}{a_1^2}, b_1 = \frac{1}{a_1}\right\},$$

$$\left\{a_0 = -\frac{ia_1\sqrt{c}}{\sqrt{6}}, d = 0, b_0 = \frac{i\sqrt{c}}{\sqrt{6}a_1}, b_1 = \frac{1}{a_1}\right\},$$

$$\left\{a_0 = \frac{ia_1\sqrt{c}}{\sqrt{6}}, d = 0, b_0 = -\frac{i\sqrt{c}}{\sqrt{6}a_1}, b_1 = \frac{1}{a_1}\right\}.$$

Riccati Equation (9) will have three types of general solutions according to Equations (10)–(12).

Case 1. According to Equation (10), When $d = \frac{-6a_0^2 - a_1^2 c}{2a_1^2}$, $c > -\frac{6a_0^2}{a_1^2}$, then $d < 0$, and $\phi = -\sqrt{\frac{6a_0^2 + a_1^2 c}{2a_1^2}} \tanh \sqrt{\frac{6a_0^2 + a_1^2 c}{2a_1^2}}\xi$, or $-\sqrt{\frac{6a_0^2 + a_1^2 c}{2a_1^2}} \coth \sqrt{\frac{6a_0^2 + a_1^2 c}{2a_1^2}}\xi$.

So $Q(\xi) = q(x,t) = a_0 + a_1\phi = a_0 - a_1\sqrt{\frac{6a_0^2 + a_1^2 c}{2a_1^2}} \tanh \sqrt{\frac{6a_0^2 + a_1^2 c}{2a_1^2}}(x + ct)$ or

$$Q(\xi) = q(x,t) = a_0 + a_1\phi = a_0 - a_1\sqrt{\frac{6a_0^2 + a_1^2 c}{2a_1^2}} \coth \sqrt{\frac{6a_0^2 + a_1^2 c}{2a_1^2}}(x + ct).$$

Then

$$Q^*(-\xi) = q^*(-x,-t) = a_0 + a_1\sqrt{\frac{6a_0^2 + a_1^2 c}{2a_1^2}} \tanh \sqrt{\frac{6a_0^2 + a_1^2 c}{2a_1^2}}(x + ct)$$

or

$$Q^*(-\xi) = q^*(-x,-t) = a_0 + a_1\sqrt{\frac{6a_0^2 + a_1^2 c}{2a_1^2}} \coth \sqrt{\frac{6a_0^2 + a_1^2 c}{2a_1^2}}(x + ct).$$

$$R(\xi) = r(x,t) = b_0 + b_1\phi = -\frac{1}{a_1^2}\left(a_0 + a_1\sqrt{\frac{6a_0^2 + a_1^2 c}{2a_1^2}} \tanh \sqrt{\frac{6a_0^2 + a_1^2 c}{2a_1^2}}(x + ct)\right)$$

or

$$R(\xi) = r(x,t) = b_0 + b_1\phi = -\frac{1}{a_1^2}\left(a_0 + a_1\sqrt{\frac{6a_0^2 + a_1^2 c}{2a_1^2}} \coth \sqrt{\frac{6a_0^2 + a_1^2 c}{2a_1^2}}(x + ct)\right).$$

When $a_1 = \pm 1$, we have $r(x,t) = \sigma q^*(-x,-t), \sigma = -1$. According to the symmetric reduction condition, the system Equations (14) and (15) coupled to the nonlocal complex mKdV Equation (2). So, the common solutions of Equations (23) and (24) are

$$\left\{a_0 = \pm\sqrt{-\frac{2d + c}{6}}, b_0 = -a_0, b_1 = \pm 1, a_1 = \pm 1\right\}.$$

MKdV Equation (2) solution

$$Q(\xi) = q(x,t) = a_0 + a_1\phi = \pm\sqrt{-\frac{2d + c}{6}} - \left(\pm\sqrt{-d} \tanh \sqrt{-d}(x + ct)\right) \tag{25}$$

or

$$Q(\xi) = q(x,t) = a_0 + a_1\phi = \pm\sqrt{-\frac{2d+c}{6}} - \left(\pm\sqrt{-d}\coth\sqrt{-d}(x+ct)\right). \tag{26}$$

As a special case, when $c = 2, d = -2$, solution (25) simplified to

$$Q(\xi) = q(x,t) = \pm\left[\sqrt{\frac{1}{3}} - \sqrt{2}\tanh\sqrt{2}(x+2t)\right]. \tag{27}$$

Solution (27) shown in Figure 1.

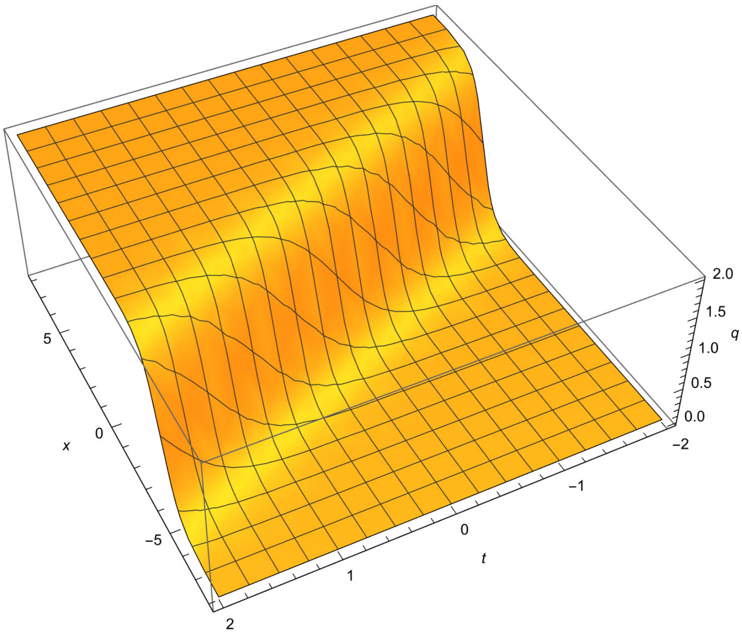

Figure 1. Graph of solution (27) taking the plus sign.

Case 2. According to Equation (11), when $d = 0, c < 0$, then $\phi = -\frac{1}{\xi}$, we have

$$Q(\xi) = q(x,t) = a_0 - a_1\frac{1}{\xi} = a_0 - a_1\frac{1}{(x+ct)} = -\frac{ia_1\sqrt{c}}{\sqrt{6}} - a_1\frac{1}{(x+ct)}$$

$$Q^*(-\xi) = q^*(-x,-t) = -\frac{ia_1\sqrt{c}}{\sqrt{6}} + a_1\frac{1}{(x+ct)} = a_1\left(-\frac{i\sqrt{c}}{\sqrt{6}} + \frac{1}{(x+ct)}\right)$$

$$R(\xi) = r(x,t) = b_0 + b_1\phi = \frac{1}{a_1}\left(\frac{i\sqrt{c}}{\sqrt{6}} - \frac{1}{(x+ct)}\right).$$

when $a_1 = \pm 1$, we have $\sigma q^*(-x,-t) = r(x,t), \sigma = -1$. By this symmetric reduction condition, Equations (14) and (15) coupled to the nonlocal complex mKdV Equation (2), next, the common solutions of Equations (23) and (24) will be

$$\left\{a_0 = -(\pm\frac{i\sqrt{c}}{\sqrt{6}}), a_1 = \pm 1, d = 0, b_0 = \pm\frac{i\sqrt{c}}{\sqrt{6}}, b_1 = \pm 1\right\}.$$

Traveling wave solution of mKdV Equation (2) is obtained,

$$Q(\xi) = q(x,t) = a_0 - a_1\frac{1}{(x+ct)} = -\left[\pm\left(\frac{i\sqrt{c}}{\sqrt{6}} - \frac{1}{(x+ct)}\right)\right]. \tag{28}$$

Especially when $c = -4, b = 0$, the solution (28) simplified to

$$q(x,t) = \pm\left(\frac{2}{\sqrt{6}} + \frac{1}{(x - 4t)}\right). \tag{29}$$

See Figure 2.

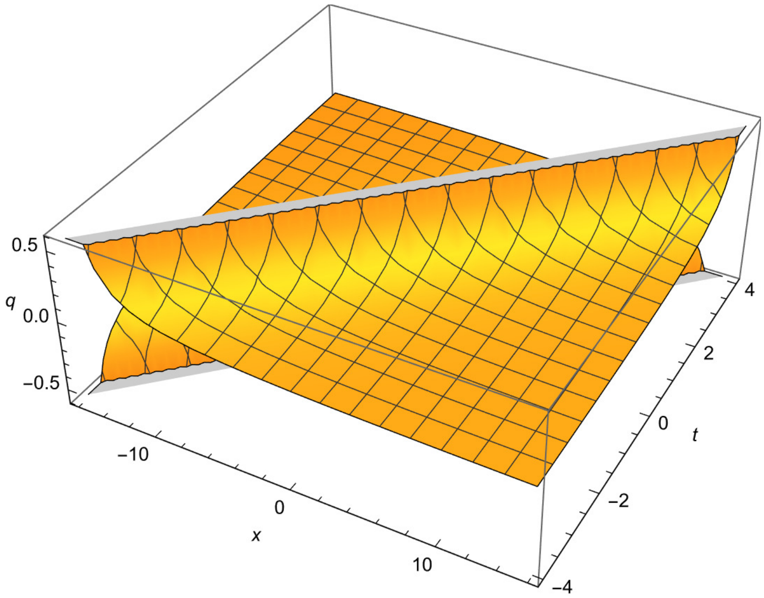

Figure 2. Graph of solution (29) taking the plus sign.

Case 3. According to Equation (12), if

$$c < -\frac{6a_0^2}{a_1^2}.$$

then

$$d = \frac{-6a_0^2 - a_1^2 c}{2a_1^2} > 0.$$

then

$$\phi(\xi) = \sqrt{-\frac{6a_0^2 + a_1^2 c}{2a_1^2}} \tan \sqrt{-\frac{6a_0^2 + a_1^2 c}{2a_1^2}}\xi,$$

or

$$-\sqrt{-\frac{6a_0^2 + a_1^2 c}{2a_1^2}} \cot \sqrt{-\frac{6a_0^2 + a_1^2 c}{2a_1^2}}\xi.$$

So

$$Q(\xi) = q(x,t) = a_0 + a_1\phi = a_0 + a_1\sqrt{-\frac{6a_0^2 + a_1^2 c}{2a_1^2}} \tan \sqrt{-\frac{6a_0^2 + a_1^2 c}{2a_1^2}}(x + ct)$$

or

$$Q(\xi) = q(x,t) = a_0 + a_1\phi = a_0 - a_1\sqrt{-\frac{6a_0^2 + a_1^2 c}{2a_1^2}} \cot \sqrt{-\frac{6a_0^2 + a_1^2 c}{2a_1^2}}(x + ct).$$

And

$$Q^*(-\xi) = q^*(-x,-t) = a_0 - a_1\sqrt{-\frac{6a_0^2 + a_1^2 c}{2a_1^2}} \tan \sqrt{-\frac{6a_0^2 + a_1^2 c}{2a_1^2}}(x + ct),$$

or

$$Q^*(-\xi) = q^*(-x,-t) = a_0 + a_1\sqrt{-\frac{6a_0^2 + a_1^2 c}{2a_1^2}} \cot \sqrt{-\frac{6a_0^2 + a_1^2 c}{2a_1^2}}(x+ct).$$

And

$$R(\xi) = r(x,t) = b_0 + b_1\phi = -\frac{1}{a_1^2}\left(a_0 - a_1\sqrt{-\frac{6a_0^2 + a_1^2 c}{2a_1^2}} \tan \sqrt{-\frac{6a_0^2 + a_1^2 c}{2a_1^2}}(x+ct)\right),$$

or

$$R(\xi) = r(x,t) = b_0 + b_1\phi = -\frac{1}{a_1^2}\left(a_0 + a_1\sqrt{-\frac{6a_0^2 + a_1^2 c}{2a_1^2}} \cot \sqrt{-\frac{6a_0^2 + a_1^2 c}{2a_1^2}}(x+ct)\right).$$

When $a_1 = \pm 1$, we have $\sigma q^*(-x,-t) = r(x,t)$, $\sigma = -1$. Under this symmetric reduction condition, Equations (14) and (15) are coupled to a nonlocal complex mKdV Equation (2). By this condition, the solutions of Equations (23) and (24) is obtained.

$$\left\{a_0 = \pm\sqrt{-\frac{2d+c}{6}}, b_0 = -a_0, b_1 = \pm 1, a_1 = \pm 1\right\}.$$

Solution of the mKdV Equation (2)

$$Q(\xi) = q(x,t) = a_0 + a_1\phi = \pm\left[\sqrt{-\frac{2d+c}{6}} + \sqrt{d}\tan\sqrt{d}(x+ct)\right], \tag{30}$$

or

$$Q(\xi) = q(x,t) = a_0 + a_1\phi = \pm\left[\sqrt{-\frac{2d+c}{6}} - \sqrt{d}\tan\sqrt{d}(x+ct)\right]. \tag{31}$$

Especially when $c = -8, b = 1$, the solution (30) simplified to

$$Q(\xi) = q(x,t) = \pm[1 + \tan(x - 8t)]. \tag{32}$$

As is shown in Figure 3.

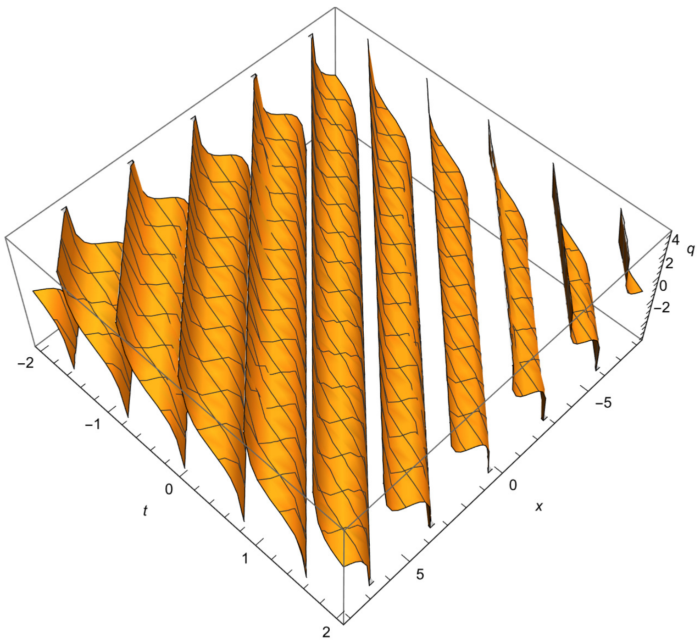

Figure 3. Graph of solution (32) taking the plus sign.

4. Conclusions

The tanh function approach for the local soliton equation is extended to the nonlocal complex mKdV equation using the solution of the Riccati equation, replacing the tanh function in the tanh function method. As an application, several multiple traveling wave solutions for the nonlocal complex mKdV equation are obtained. Furthermore, compared with the local soliton equation, the extended tanh function method for the nonlocal complex mKdV equation contains more restrictions on corresponding parameters.

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References

1. Hereman, W. Exact solitary wave solutions of coupled nonlinear evolution equations using MACSYMAf. *Comput. Phys. Commun.* **1991**, *65*, 143–150. [CrossRef]
2. Hereman, W.; Takaoka, M. Solitary wave solutions of nonlinear evolution and wave equations using a direct method and MACSYMA. *J. Phys. Math. Gen.* **1990**, *23*, 4805. [CrossRef]
3. Li, Z.B; Wang, M.L. Travelling wave solutions to the two-dimensional KdV-Burgers equation. *J. Phys. Math. Gen.* **1993**, *26*, 6027.
4. Zhang, G.; Li, Z.; Duan, Y. Exact solitary wave solutions of nonlinear wave equations. *Sci. China Ser. A-Math.* **2001**, *44*, 396–401. [CrossRef]
5. Parkes, E.J.; Duffy, B.R. Travelling solitary wave solutions to a compound KdV-Burgers equation. *Phys. Lett.* **1997**, *229*, 217–220. [CrossRef]
6. Duffy, B.R.; Parkes, E.J. Travelling solitary wave solutions to a seventh-order generalized KdV equation. *Phys. Lett.* **1996**, *214*, 271–272. [CrossRef]
7. Parkes, E.J.; Duffy, B.R. An automated tanh-function method for finding solitary wave solutions to non-linear evolution equations. *Comp. Phys. Commun.* **1996**, *98*, 288–300. [CrossRef]
8. Lou, S.Y.; Ruan, H.Y. Revisitation of the localized excitations of the (2+ 1)-dimensional KdV equation. *J. Phys. A Math. Gen.* **2001**, *35*, 305–316. [CrossRef]
9. Gao, Y.T.; Tian, B. Generalized tanh method with symbolic computation and generalized shallow water wave equation. *Comput. Math. Appl.* **1997**, *33*, 115–118. [CrossRef]
10. Parkes, E.J. Exact solutions to the two-dimensional Korteweg-de Vries-Burgers equation. *J. Phys. Math. Gen.* **1994**, *27*, L497. [CrossRef]
11. Adem, A.R.; Muatjetjeja, B. Conservation laws and exact solutions for a 2D Zakharov–Kuznetsov equation. *Appl. Math. Lett.* **2015**, *48*, 109–117. [CrossRef]
12. Salas, A.H. Solving nonlinear partial differential equations by the sn-ns method. *Abstr. Appl. Anal.* **2012**, *25*, 340824 [CrossRef]
13. Malfliet, W. Solitary wave solutions of nonlinear wave equations. *Am. J. Phys.* **1992**, *60*, 650. [CrossRef]
14. Ganji, D.D.; Sadeghi, E.M.; Rahmat, M.G. Modified Camassa–Holm and Degasperis–Procesi Equations Solved by Adomian's Decomposition Method and Comparison with HPM and Exact Solutions. *Acta Appl. Math.* **2008**, *104*, 303–311. [CrossRef]
15. Wazwaz, A.M. The extended tanh method for new solitons solutions for many forms of the fifth-order KdV equations. *Appl. Math. Comput.* **2007**, *184*, 1002–1014. [CrossRef]
16. Wazwaz, A.M. New solitary wave solutions to the modified forms of Degasperis–Procesi and Camassa–Holm equations. *Appl. Math. Comput.* **2007**, *186*, 130–141. [CrossRef]
17. Fan, E. Extended tanh-function method and its applications to nonlinear equations. *Phys. Lett.* **2000**, *277*, 212–218. [CrossRef]
18. Ma, L.Y.; Shen, S.F.; Zhu, Z.N. Soliton solution and gauge equivalence for an integrable nonlocal complex modified Korteweg-de Vries equation. *J. Math. Phys.* **2017**, *58*, 103501. [CrossRef]

19. Ablowitz, M.J.; Clarkson, P.A. *Soliton, Nonlinear Evolution Equations, Inverse Scattering*; Cambridge University Press: New York, NY, USA, 1991.
20. Fornberg, B.; Whitham, G.B. A numerical and theoretical study of certain nonlinear wave phenomena. *Philos. Trans. R. Soc. Lond. Ser. Math. Phys. Sci.* **1978**, *289*, 373–404.
21. Makhankov, V. Computer experiments in soliton theory. *Comput. Phys. Commun.* **1980**, *21*, 1–49. [CrossRef]
22. Ablowitz, M.J.; Musslimani, Z.H. Inverse scattering transform for the integrable nonlocal nonlinear Schrödinger equation. *Nonlinearity* **2016**, *29*, 915–946. [CrossRef]
23. Ablowitz, M.J.; Musslimani, Z.H. Integrable nonlocal nonlinear equations. *Stud. Appl. Math.* **2016**, *139*, 7–59. [CrossRef]
24. He, F.J.; Fan, E.G.; Xu, J. Long-Time asymptotics for the nonlocal MKdV equation. *Commun. Theor. Phys.* **2019**, *71*, 475–488. [CrossRef]
25. Ji, J.L.; Zhu, Z.N. On a nonlocal modified Korteweg-de Vries equation: Integrability, Darboux transformation and soliton solutions. *Commun. Nonlinear Sci. Numer. Simul.* **2017**, *42*, 699–708. [CrossRef]
26. Ji, J.L.; Zhu, Z.N. Soliton solutions of an integrable nonlocal modified Korteweg–de Vries equation through inverse scattering transform. *J. Math. Anal. Appl.* **2017**, *453*, 973–984. [CrossRef]
27. Lou, S.Y.; Huang, F. Alice-Bob physics: Coherent solutions of nonlocal KdV systems. *Sci. Rep.* **2017**, *7*, 869. [CrossRef]