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Stochastic Modelling of Lassa Fever Epidemic Disease

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Abstract: Evolutionary approaches have a critical role in different disciplines such as real-world problems, computer programming, machine learning, biological sciences, and many more. The design of the stochastic model is based on transition probabilities and non-parametric techniques. Positivity, boundedness, and equilibria are investigated in deterministic and stochastic senses. An essential tool, Euler–Maruyama, is studied for the solution of said model. Standard and nonstandard evolutionary approaches are presented for the stochastic model in terms of efficiency and low-cost approximations. The standard evolutionary procedures like stochastic Euler–Maruyama and stochastic Runge–Kutta fail to restore the essential features of biological problems. On the other hand, the proposed method is efficient, of meager cost, and adopts all the desired feasible properties. At the end of this paper the comparison section is presented to support efficient analysis.

Keywords: Lassa fever disease; stochastic epidemic model; stochastic evolutionary approaches; stability analysis

MSC: 34A34; 34K50; 37H05; 37H10; 65C30

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1. Literature Review

In 2020, Onah et al. formulated a dynamical system for the Lassa fever model depending on socio-economic class. The effect of the disease on the economy was studied worldwide [1]. In 2020, Peter et al. modified a fundamental disease model with the optimal control strategies [2]. In 2020, Bakare et al. worked on the transmission dynamics of the disease and derived a nonlinear ordinary differential equation by introducing the seasonal parameters. Although stochastic methods are suitable for quantitative study via a mathematical model, they play an important role in data analysis, such as in environmental cases [3–5], finance [6], energy [7], and epidemiology [8]. For this purpose, some steps are recommended, such as preventive measures, educational campaigns, community hygiene, and isolation of infected humans [9]. In 2021, Collins et al. formulated a mathematical model for control measures of Lassa fever. According to epidemiologists, the population was divided into higher and lower socio-economic classes, and control measures such as treatments, an educational campaign, community hygiene, and rodent

safety were discussed [10]. Caraballo et al. [11] studied the epidemic model through the comparative analysis of random and stochastic techniques. Jesus et al. [12] investigated the random perturbations of an ecoepidemiological model. In 2017, Obabiyi et al. developed a mathematical model for Lassa fever and divided the population into two parts: humans and rodents. They suggested early-stage treatment, working in a hygienic environment, and controlling the rodent population, affecting the human race [13]. In 2020, Dachollom et al. addressed the epidemic with a broader double-dimensional approach, such as the natural and medical sciences. They developed a mathematical model to control Lassa fever infection [14]. In 2013, Bawa et al. derived a deterministic model for a disease-free state and obtained R_O to control disease dynamics [15]. In 2020, Sattler et al. developed an animal model of Lassa fever from 2018 to 2019. The Nigerian fatality rate was 25.4%, and the socio-economic burden occurred massively in the endemic region [16]. In 2020, Okolo et al. developed a mathematical model for controlling Lassa fever by isolation and treatments. They proved that the disease-free equilibrium is locally asymptotically stable [17]. In 2019, Marien et al. derived a mathematical model based on field data for rodent control to fight Lassa fever. This model recommended continuous control or rodent vaccination as the only policy [18]. In 2019, Akhmetzhanov measured the periodic parameters of transmission for Lassa fever. This model is based on human infection and rodent population and also analyzed the infectious disease outbreaks in humans, animals, and plants [19]. In 2019, Nwasuka et al. analyzed the treatment as a control measure, formulated a mathematical model of Lassa fever with separation of the infected individual, and evaluated the transmission dynamics [20]. In 2019, Zhao et al. developed the association between disease reproduction number and local rainfall. The reproduction number was calculated from four different growth models: (1) Richards, (2) three-parameter logistics, (3) Gompertz, and (4) Weibull [21]. In 2020, Martins et al. developed a mathematical model to control the spreading Lassa fever and analyzed the existence and stability of a disease-free equilibrium [22]. In 2020, Abdulkarim et al. discussed the objective factors and death rates of the Bauchi state of Nigeria. The data was from 2015 to 2018, when the outbreaks and fatality rates increased, and death mainly occurred seven days later than the symptoms were shown [23]. In 2015, James et al. analyzed the transmission dynamics of Lassa fever. They derived that the zero-equilibrium state is stable when the population is less than the death rate or both are equal [24]. In 2017, Innocent et al. developed an understanding of Lassa fever and its control measures, developed a mathematical model for investigating the dynamics of the disease, and suggested avoiding contact with species that carried viruses and introducing vaccines for humans [25]. In 2018, Akinpelu et al. developed a mathematical model for sensitivity analysis of Lassa fever. This model is divided into five compartments of susceptible (S), latent (L), infected (I), isolated (I), and recovered (R) by using the next-generation method and obtained R_O, showing that disease-free equilibrium was locally and globally asymptotically stable [26]. In 2015, James et al. analyzed stability for Lassa fever and suggested quarantines and making strategies for permanent immunity [27]. In 2019, Obasi et al. derived the primary reproduction number of the Lassa fever epidemic. R_O is inversely proportional to the square of the inter-epidemic period of an infectious disease [28]. Aznar-Gimeno et al. [29] studied the stepwise algorithm for linearly combining biomarkers under index theory. The suitable way of mathematical modeling infectious diseases in all circumstances is stochastic. Many approaches are used to handle the stochastic models based on stochastic differential equations used in literature, such as Milstein, Euler–Maruyama, and many more. All existing techniques do not have symmetry with the fundamental structure of the continuous model, including positivity, boundedness, and dynamical consistency. The construction of a stochastic nonstandard finite difference method is needed to handle such issues and make symmetry with the continuous model. The strategy of our paper is as follows: in Section 2, the deterministic model and the analysis of the Lassa fever disease model are formulated. Section 3 is based on the invention of the disease model in the stochastic form. In Section 4 the computational

approaches to model the Lassa fever disease and their results are described. Finally, the conclusion and future framework are discussed.

2. Model Formulation

For any time t, the variables and constants of the model are as follows: $S_H(t)$: denoted as the susceptible class, $I_H\ (t)$: represented as the infectious class, $R_H(t)$: represented as the recovered class, characterized $S_R(t)$: represented as the susceptible rats, $I_R(t)$: represented as the infectious rats, $N_H(t)$: represented as whole humans' population. The dynamics of Lassa fever are presented in Figure 1 as follows [30]:

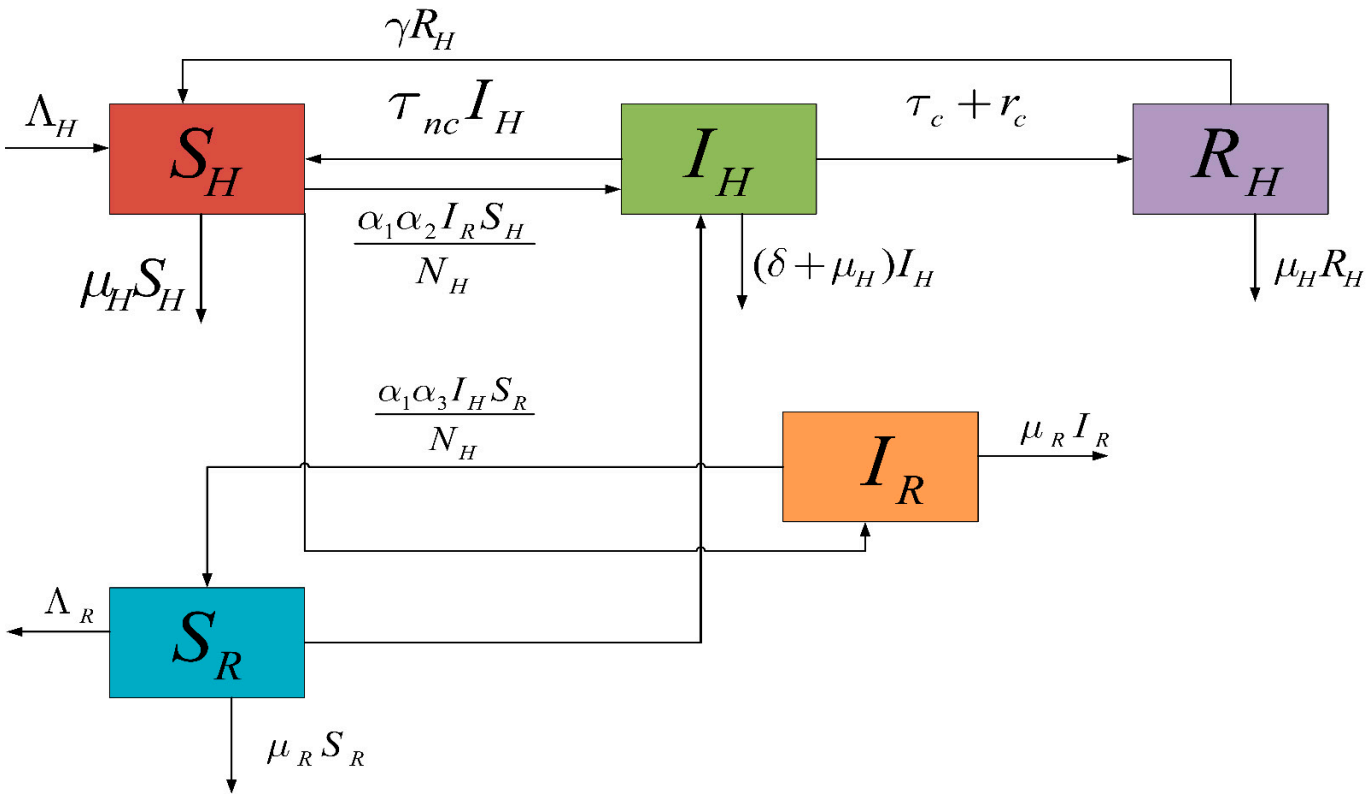

Figure 1. Flow chart of Lassa fever model.

Table 1 is presented the physical relevance of the constants as follows:

Table 1. Physical applicability of the model.

Parameters	Descriptions	Values (Per Day)/[30]
N_H	Represented the human population.	≥ 0
α_1	Represented infection rate of vectors.	1.00166 (DFE) 3.00166 (EE)
α_2	Represented as the force of infection.	1.0004 (DFE) 3.0004 (EE)
α_3	Represented infection rate of humans interact.	0.1
τ_c	Represented the connection of humans with drugs.	0.7
τ_{nc}	Represented the rate at which humans do not have a relationship with drugs.	0.9
r_c	Represented the rate of awareness.	0.2
γ	Represented the loss of immunity	0.220
Λ_H	The birth rate of humans.	0.8 (Assumed)
μ_H	The death rate of humans.	0.8 (Assumed)
δ	Represented the rate of mortality of an infectious class.	0.133
Λ_R	The birth rate of vectors.	0.8 (Assumed)
μ_R	The death rate of vectors.	0.8 (Assumed)
σ_i: i = 1, 2, 3, 4, 5	Randomness of each sub-population.	$0 \leq i \leq 1$

The model with equations is as follows:

$$\frac{dS_H(t)}{dt} = \Lambda_H - \frac{\alpha_1\alpha_2 S_H(t)I_R(t)}{N_H} + \gamma R_H(t) + \tau_{nc} I_H(t) - \mu_H S_H(t),\ t \geq 0 \quad (1)$$

$$\frac{dI_H(t)}{dt} = \frac{\alpha_1\alpha_2 S_H(t)I_R(t)}{N_H} - \tau_c I_H(t) - r_c I_H(t) - \tau_{nc} I_H(t) - \delta I_H(t) - \mu_H I_H(t),\ t \geq 0 \quad (2)$$

$$\frac{dR_H(t)}{dt} = \tau_c I_H(t) + r_c I_H(t) - \gamma R_H(t) - \mu_H R_H(t),\ t \geq 0 \quad (3)$$

$$\frac{dS_R(t)}{dt} = \Lambda_R - \frac{\alpha_1\alpha_3 S_R(t)I_H(t)}{N_H} - \mu_R S_R(t),\ t \geq 0 \quad (4)$$

$$\frac{dI_R(t)}{dt} = \frac{\alpha_1\alpha_3 S_R(t)I_H(t)}{N_H} - \mu_R I_R(t), t \geq 0 \quad (5)$$

where $S_H(0) \geq 0,\ I_H(0) \geq 0,\ R_H(0) \geq 0,\ S_R(0) \geq 0,\ I_R(0) \geq 0.$

2.1. Positivity and Boundedness

For any time $t \geq 0$, the feasible region of the model is as follows:

$$\Pi = \left\{(S_H, I_H, R_H, S_R, I_R)\epsilon R_+^5 : S_H + I_H + R_H \leq \frac{\Lambda_H}{\mu_H},\ S_R + I_R \leq \frac{\Lambda_R}{\mu_R},\ S_H \geq 0, I_H \geq 0, R_H \geq 0, S_R \geq 0, I_R \geq 0\right\}$$

Lemma 1. *The solutions $(S_H,\ I_H, R_H,\ S_R,\ I_R)\epsilon R_+^5$ of the Equations (1)–(5) are positive at any time $t \geq 0$, with given non-negative initial conditions.*

Proof. It is clear from the Equations (1)–(5) that:

$$\left.\frac{dS_H}{dt}\right|_{S_H=0} = \Lambda_H + \gamma R_H + \tau_{nc} I_H \geq 0,\ \left.\frac{dI_H}{\mathrm{dt}}\right|_{I_H=0} = \frac{\alpha_1\alpha_2 S_H I_R}{N_H} \geq 0,\ \left.\frac{dR_H}{\mathrm{dt}}\right|_{R_H=0} = \tau_c I_H + r_c I_H \geq 0,$$

$$\left.\frac{dS_R}{\mathrm{dt}}\right|_{S_R=0} = \Lambda_R \geq 0,\ \left.\frac{dI_R}{\mathrm{dt}}\right|_{I_R=0} = \frac{\alpha_1\alpha_3 S_R I_H}{N_H} \geq 0,$$

as desired. □

Lemma 2. *The solutions $(S_H,\ I_H, R_H,\ S_R,\ I_R)\epsilon R_+^5$ of the Equations (1)–(5) are bounded at any time $t \geq 0$, to prove $\lim_{t\to\infty} Sup\ N_H(t) \leq \frac{\Lambda_H}{\mu_H}$, and $\lim_{t\to\infty} Sup\ N_R(t) \leq \frac{\Lambda_R}{\mu_R}$.*

Proof. By considering the function as follows:

$$N_H(\mathrm{t}) = S_H + I_H + R_H,$$
$$\frac{dN_H}{dt} = \frac{dS_H}{dt} + \frac{dI_H}{dt} + \frac{dR_H}{dt},$$
$$\frac{dN_H}{dt} = \Lambda_H - \mu_H N_H,$$
$$N_H(\mathrm{t}) = \mathrm{A} + \frac{\Lambda_H}{\mu_H},$$

By Gronwall's inequality, we get $N_H(\mathrm{t}) \leq N_H(0) + \frac{\Lambda_H}{\mu_H},\ \mathrm{t} \geq 0,$

$$\limsup_{\mathrm{t}\to\infty} N_R(\mathrm{t}) \leq \frac{\Lambda_H}{\mu_H}.$$

Let,

$$N_R(\mathrm{t}) = S_R + I_R,$$
$$\frac{dN_R}{dt} = \frac{dS_R}{dt} + \frac{dI_R}{dt},$$
$$\frac{dN_R}{dt} = \Lambda_R - \mu_R N_R$$
$$N_R(\mathrm{t}) = \mathrm{B} + \frac{\Lambda_R}{\mu_R},$$

By Gronwall's inequality, we get

$$N_R(\mathbf{t}) \leq N_R(\mathbf{0}) + \frac{\Lambda_R}{\mu_R},\ \mathbf{t} \geq \mathbf{0},$$
$$\lim_{\mathbf{t}\to\infty} \mathbf{Sup}\ N_R(\mathbf{t}) \leq \frac{\Lambda_R}{\mu_R},$$

as desired. □

2.2. Model Equilibria

There are two equilibria of Equation (1) to Equation (5), as follows: disease-free equilibrium (DFE) $= (S_H,\ I_H,\ R_H,\ S_R,\ I_R) = \left(\frac{\Lambda_H}{\mu_H},\ 0, 0,\ \frac{\Lambda_R}{\mu_R}, 0\right)$ and endemic equilibrium (EE) $= (S_H^*,\ I_H^*,\ R_H^*,\ S_R^*,\ I_R^*)$ where $S_H^* = \frac{\Lambda_H + \gamma A_1 I_H^* + A_2 I_H^*}{\mu_H}$, $I_H^* = \frac{\Lambda_H + A_4\mu_R}{A_4\alpha_1\alpha_3 + \gamma A_1 + A_2}$, $R_H^* = \frac{(\tau_c + r_c) I_H^*}{\gamma + \mu_H} = A_1 I_H^*$, $S_R^* = \frac{\Lambda_R}{\alpha_1\alpha_3 I_H^* + \mu_R}$, $I_R^* = \frac{\alpha_1\alpha_3 S_R^* I_H^*}{\mu_R}$. And, $A_1 = \frac{(\tau_c + r_c)}{\gamma + \mu_H}$, $A_2 = \tau_c + r_c + \delta + \mu_H$, $A_3 = \tau_c + r_c + \tau_{nc} + \delta + \mu_H$, $A_4 = \frac{A_3\mu_H\mu_R}{\alpha_1^2\alpha_2\alpha_3\Lambda_R}$.

The next-generation matrix method is used to calculate the reproduction number. The transmission matrix, denoted by F, and transition matrix, denoted by V, are obtained by considering the infected classes from the system (1)–(5) and adding the disease-free equilibrium. The reproduction number is represented as the largest eigenvalue of FV^{-1}.

$$F = \begin{bmatrix} \frac{\alpha_1\alpha_2 S_H}{N_H} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\alpha_1\alpha_3 S_R}{N_H} \end{bmatrix},\ V = \begin{bmatrix} \tau_c + r_c + \tau_{nc} + \delta + \mu_H & 0 & 0 \\ -\tau_c - r_c & \gamma + \mu_H & 0 \\ 0 & 0 & \mu_R \end{bmatrix}.\ \text{So,}$$

$$FV^{-1} = \frac{1}{(\tau_c + r_c + \tau_{nc} + \delta + \mu_H)(\gamma + \mu_H)\mu_R} \begin{bmatrix} \frac{\alpha_1\alpha_2\ \mu_R\Lambda_H(\gamma+\mu_H)}{\mu_H N_H} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\alpha_1\alpha_2\Lambda_R}{\mu_R N_H}(\tau_c + r_c + \tau_{nc} + \delta + \mu_H)(\gamma + \mu_H) \end{bmatrix}$$

where $R_0 = \frac{\alpha_1\alpha_2\Lambda_H}{\mu_H N_H(\tau_c + r_c + \tau_{nc} + \delta + \mu_H)}$, is called the reproduction number.

3. Stochastic Formulation Phase-I

A transition matrix technique, also known as a stochastic or probability matrix, is a square matrix representing the transition probabilities of a stochastic system. Let $\mathrm{C} = [S_H, I_H, R_H, S_R, I_R]^{\mathrm{T}}$ be a vector for the disease model. The changes in the disease model concerning time are calculated in Table 2.

$$\text{Expectation} = \mathrm{E}^*[\Delta\mathrm{C}] = \sum_{i=1}^{12} P_i(T)_i = \begin{bmatrix} \frac{\Lambda_H - \alpha_1\alpha_2 S_H I_R}{N_H} + \gamma R_H + \tau_{nc} I_H - \mu_H S_H \\ \frac{\alpha_1\alpha_2 S_H I_R}{N_H} - \tau_{nc} I_H - \tau_C I_H + r_C I_H - \delta I_H + \mu_H I_H \\ \Upsilon R_H + \tau_C I_H + r_c I_H - \mu_H R_H \\ \Lambda_R - \frac{\alpha_1\alpha_3 S_R I_H}{N_H} - \mu_R S_R \\ \frac{\alpha_1\alpha_3 S_R I_H}{N_H} - \mu_R I_R \end{bmatrix} \Delta\mathrm{t}.$$

$$\text{Variance} = \sum_{i=1}^{12} P_i T_i (T_i)^T.$$

$$= \begin{bmatrix} P_1 + P_2 + P_3 + P_4 + P_5 & -P_2 - P_4 & -P_3 & 0 & 0 \\ -P_2 - P_4 & P_2 + P_4 + P_6 + P_7 & -P_6 & 0 & 0 \\ -P_3 & -P_6 & P_3 + P_6 + P_3 & 0 & 0 \\ 0 & 0 & 0 & P_9 + P_{10} + P_{11} & -P_{10} \\ 0 & 0 & 0 & -P_{10} & P_{10} + P_{12} \end{bmatrix}.$$

$$\text{Drift} = \mathrm{G}(\mathrm{C},\mathrm{t}) = \frac{\mathrm{E}^*[\Delta \mathrm{C}]}{\Delta \mathrm{t}} = \begin{bmatrix} \frac{\Lambda_H - \alpha_1\alpha_2 S_H I_R}{N_H} + \gamma R_H + \tau_{nc} I_H - \mu_H S_H \\ \frac{\alpha_1\alpha_2 S_H I_R}{N_H} - \tau_{nc} I_H - \tau_C I_H + r_C I_H - \delta I_H + \mu_H I_H \\ YR_H + \tau_C I_H + r_c I_H - \mu_H R_H \\ \Lambda_R - \frac{\alpha_1\alpha_3 S_R I_H}{N_H} - \mu_R S_R \\ \frac{\alpha_1\alpha_3 S_R I_H}{N_H} - \mu_R I_R \end{bmatrix} \tag{6}$$

$$\text{Diffusion} = \mathrm{H}(\mathrm{C},\mathrm{t}) = \sqrt{\frac{\mathrm{E}^*\left[\Delta \mathrm{C}\ \Delta \mathrm{C}^{\mathrm{T}}\right]}{\Delta \mathrm{t}}} = \sqrt{\begin{bmatrix} P_1+P_2+P_3+P_4+P_5 & -P_2-P_4 & -P_3 & 0 & 0 \\ -P_2-P_4 & P_2+P_4+P_6+P_7 & -P_6 & 0 & 0 \\ -P_3 & -P_6 & P_3+P_6+P_3 & 0 & 0 \\ 0 & 0 & 0 & P_9+P_{10}+P_{11} & -P_{10} \\ 0 & 0 & 0 & -P_{10} & P_{10}+P_{12} \end{bmatrix}}. \tag{7}$$

Table 2. Possible changes in the process of the model.

Transition	Probabilities
$(\Delta \mathbf{C})_{\mathbf{1}} = [\mathbf{1}\ \ \mathbf{0}\ \ \mathbf{0}\ \ \mathbf{0}\ \ \mathbf{0}]^{\mathbf{T}}$	$\mathrm{P}_1 = (\wedge_H)\Delta \mathrm{t}$
$(\Delta \mathbf{C})_{\mathbf{2}} = [-\mathbf{1}\ \ \mathbf{1}\ \ \mathbf{0}\ \ \mathbf{0}\ \ \mathbf{0}]^{\mathbf{T}}$	$\mathrm{P}_2 = (\alpha_1\alpha_2 S_H I_R)\Delta \mathrm{t}$
$(\Delta \mathbf{C})_{\mathbf{3}} = [\mathbf{1}\ \ \mathbf{0}\ \ -\mathbf{1}\ \ \mathbf{0}\ \ \mathbf{0}]^{\mathbf{T}}$	$\mathrm{P}_3 = (YR_H)\Delta \mathrm{t}$
$(\Delta \mathbf{C})_{\mathbf{4}} = [\mathbf{1}\ \ -\mathbf{1}\ \ \mathbf{0}\ \ \mathbf{0}\ \ \mathbf{0}]^{\mathbf{T}}$	$\mathrm{P}_4 = (\tau_{nc} I_{H)}$
$(\Delta \mathbf{C})_{\mathbf{5}} = [-\mathbf{1}\ \ \mathbf{0}\ \ \mathbf{0}\ \ \mathbf{0}\ \ \mathbf{0}]^{\mathbf{T}}$	$\mathrm{P}_5 = (\mu_H S_H)\Delta \mathrm{t}$
$(\Delta \mathbf{C})_{\mathbf{6}} = [\mathbf{0}\ \ -\mathbf{1}\ \ \mathbf{1}\ \ \mathbf{0}\ \ \mathbf{0}]^{\mathbf{T}}$	$\mathrm{P}_6 = (\tau_C I_H + r_C I_H)\Delta \mathrm{t}$
$(\Delta \mathbf{C})_{\mathbf{7}} = [\mathbf{0}\ \ -\mathbf{1}\ \ \mathbf{0}\ \ \mathbf{0}\ \ \mathbf{0}]^{\mathbf{T}}$	$P_7 = (\delta I_H + \mu_H I_H)\Delta \mathrm{t}$
$(\Delta \mathbf{C})_{\mathbf{8}} = [\mathbf{0}\ \ \mathbf{0}\ \ -\mathbf{1}\ \ \mathbf{0}\ \ \mathbf{0}]^{\mathbf{T}}$	$P_8 = (\mu_H R_H)\Delta \mathrm{t}$
$(\Delta \mathbf{C})_{\mathbf{9}} = [\mathbf{0}\ \ \mathbf{0}\ \ \mathbf{0}\ \ \mathbf{1}\ \ \mathbf{0}]^{\mathbf{T}}$	$P_9 = (\wedge_R)\Delta \mathrm{t}$
$(\Delta \mathbf{C})_{\mathbf{10}} = [\mathbf{0}\ \ \mathbf{0}\ \ \mathbf{0}\ \ -\mathbf{1}\ \ \mathbf{1}]^{\mathbf{T}}$	$P_{10} = (\alpha_1\alpha_2 S_R I_H)\Delta \mathrm{t}$
$(\Delta \mathbf{C})_{\mathbf{11}} = [\mathbf{0}\ \ \mathbf{0}\ \ \mathbf{0}\ \ -\mathbf{1}\ \ \mathbf{0}]^{\mathbf{T}}$	$P_{11} = (\mu_R S_R)\Delta \mathrm{t}$
$(\Delta \mathbf{C})_{\mathbf{12}} = [\mathbf{0}\ \ \mathbf{0}\ \ \mathbf{0}\ \ \mathbf{0}\ \ -\mathbf{1}]^{\mathbf{T}}$	$P_{12} = (\mu_R I_R)\Delta \mathrm{t}$

$$\text{Thus, } \mathrm{dC}(\mathrm{t}) = \mathrm{G}(\mathrm{C},\mathrm{t})\mathrm{dt} + \mathrm{H}(\mathrm{C},\mathrm{t})\mathrm{dB}(\mathrm{t}).$$

$$\mathrm{d}\begin{bmatrix} S_H \\ I_H \\ R_H \\ S_R \\ I_R \end{bmatrix} = \begin{bmatrix} \frac{\Lambda_H - \alpha_1\alpha_2 S_H I_R}{N_H} + \gamma R_H + \tau_{nc} I_H - \mu_H S_H \\ \frac{\alpha_1\alpha_2 S_H I_R}{N_H} - \tau_{nc} I_H - \tau_C I_H + r_C I_H - \delta I_H + \mu_H I_H \\ YR_H + \tau_C I_H + r_c I_H - \mu_H R_H \\ \Lambda_R - \frac{\alpha_1\alpha_3 S_R I_H}{N_H} - \mu_R S_R \\ \frac{\alpha_1\alpha_3 S_R I_H}{N_H} - \mu_R I_R \end{bmatrix} \mathrm{dt} + \sqrt{\begin{bmatrix} P_1+P_2+P_3+P_4+P_5 & -P_2-P_4 & -P_3 & 0 & 0 \\ -P_2-P_4 & P_2+P_4+P_6+P_7 & -P_6 & 0 & 0 \\ -P_3 & -P_6 & P_3+P_6+P_3 & 0 & 0 \\ 0 & 0 & 0 & P_9+P_{10}+P_{11} & -P_{10} \\ 0 & 0 & 0 & -P_{10} & P_{10}+P_{12} \end{bmatrix}} \mathrm{dB}(\mathrm{t}). \tag{8}$$

The Euler–Maruyama scheme is implemented on Equation (8) to find its stimulating results by using the scientific literature of the model. This is presented in Table 1 and is as follows:

$$C_{n+1} = C_n + G(C_n, t)\Delta t + H(C_n, t)dB.$$

$$\begin{bmatrix} {S_H}^{n+1} \\ {I_H}^{n+1} \\ {R_H}^{n+1} \\ {S_R}^{n+1} \\ {I_R}^{n+1} \end{bmatrix} = \begin{bmatrix} {S_H}^{n} \\ {I_H}^{n} \\ {R_H}^{n} \\ {S_R}^{n} \\ {I_R}^{n} \end{bmatrix} + \begin{bmatrix} \frac{\Lambda_H - \alpha_1\alpha_2 S_H I_R}{N_H} + \gamma R_H + \tau_{nc} I_H - \mu_H S_H \\ \frac{\alpha_1\alpha_2 S_H I_R}{N_H} - \tau_{nc} I_H - \tau_C I_H + r_C I_H - \delta I_H + \mu_H I_H \\ \Upsilon R_H + \tau_C I_H + r_c I_H - \mu_H R_H \\ \Lambda_R - \frac{\alpha_1\alpha_3 S_R I_H}{N_H} - \mu_R S_R \\ \frac{\alpha_1\alpha_3 S_R I_H}{N_H} - \mu_R I_R \end{bmatrix} \Delta t$$

$$+ \sqrt{\begin{bmatrix} \begin{bmatrix} P_1 + P_2 + P_3 + P_4 + P_5 & -P_2 - P_4 & -P_3 & 0 & 0 \\ -P_2 - P_4 & P_2 + P_4 + P_6 + P_7 & -P_6 & 0 & 0 \\ -P_3 & -P_6 & P_3 + P_6 + P_3 & 0 & 0 \\ 0 & 0 & 0 & P_9 + P_{10} + P_{11} & -P_{10} \\ 0 & 0 & 0 & -P_{10} & P_{10} + P_{12} \end{bmatrix} \Delta t \end{bmatrix} \Delta B_n}.$$

where Δt is the discretization parameter.

4. Formulation Phase-II

Considering the dynamical system (1)–(5), add uncertainty parameter with Brownian motion as follows [31]:

$$\frac{dS_H(t)}{dt} = \Lambda_H - \frac{\alpha_1\alpha_2 S_H(t) I_R(t)}{N_H} + \gamma R_H(t) + \tau_{nc} I_H(t) - \mu_H S_H(t) + \sigma_1 S_H(t)\frac{dB(t)}{dt},\ t \geq 0 \quad (9)$$

$$\frac{dI_H(t)}{dt} = \frac{\alpha_1\alpha_2 S_H(t) I_R(t)}{N_H} - \tau_c I_H(t) - r_c I_H(t) - \tau_{nc} I_H(t) - \delta I_H(t) - \mu_H I_H(t) + \sigma_2 I_H(t)\frac{dB(t)}{dt},\ t \geq 0 \quad (10)$$

$$\frac{dR_H(t)}{dt} = \tau_c I_H(t) + r_c I_H(t) - \gamma R_H(t) - \mu_H R_H(t) + \sigma_3 R_H(t)\frac{dB(t)}{dt},\ t \geq 0 \quad (11)$$

$$\frac{dS_R(t)}{dt} = \Lambda_R - \frac{\alpha_1\alpha_3 S_R(t) I_H(t)}{N_H} - \mu_R S_R(t) + \sigma_4 S_R(t)\frac{dB(t)}{dt},\ t \geq 0 \quad (12)$$

$$\frac{dI_R(t)}{dt} = \frac{\alpha_1\alpha_3 S_R(t) I_H(t)}{N_H} - \mu_R I_R(t) + \sigma_5 I_R(t)\frac{dB(t)}{dt}, t \geq 0 \quad (13)$$

where $\sigma_i,\ i = 1, 2, 3, 4, 5$ represents each compartment's randomness and $B(t)$ is the Brownian motion.

4.1. Fundamental Properties of the Stochastic Model

In this section, the positivity and boundedness of systems (9)–(13) are discussed. Let us consider the vectors as follows:

$$U(t) = (S_H(t),\ I_H(t), R_H(t)) \text{ and } V(t) = (S_R(t), I_R(t))$$

And the norms

$$|U(t)| = \sqrt{S_H^2(t) + I_H^2(t) + R_H^2(t)} \quad (14)$$

And

$$|V(t)| = \sqrt{S_R^2(t) + I_R^2(t)} \quad (15)$$

Also, denote $C_1^{2,1}\left(R^3 x(0,\infty) : R_+\right)$ *and* $C_2^{2,1}\left(R^2 x(0,\infty) : R_+\right)$ are the families of all non-negative functions $V_1(U,t)$ *and* $V_2(V,t)$ defined on $R^3 x(0,\infty)$, respectively. Also, twice differentiable in U and V and once in t.

We define the differential operators L_1 *and* L_2 associated with three dimensional SDEs:

$$dU(t) = H_1(U,t)dt + K_1(U,t)dB(t) \tag{16}$$

$$dV(t) = H_2(V,t)dt + K_2(V,t)dB(t) \tag{17}$$

as,

$$L_1 = \frac{\partial}{\partial t} + \sum_{i=1}^{3} H_{1_i}(U,t)\frac{\partial}{\partial U_i} + \frac{1}{2}\sum_{i,j=1}^{3}(K_1^T(U,t)K_1(U,t)_{i,j}\ x\ \frac{\partial^2}{\partial U_i \partial U_j}$$

and

$$L_2 = \frac{\partial}{\partial t} + \sum_{i=1}^{2} H_{2_i}(V,t)\frac{\partial}{\partial V_i} + \frac{1}{2}\sum_{i,j=1}^{2}(K_2^T(V,t)K_2(V,t)_{i,j}\ x\ \frac{\partial^2}{\partial V_i \partial V_j}.$$

If L_1 , L_2 acts on a function $U^*, V^* \in C^{2,1}\left(R^3 x(0,\infty) : R_+^3\right)$ then we denote

$$L_1U^*(U,t) = U_t^*(U,t) + U_U^*(U,t)H_1(U,t) + \tfrac{1}{2}Trace(K_1^T(U,t)U_{UU}^*(U,t)K_1(U,t).$$
$$L_2V^*(V,t) = V_t^*(V,t) + V_V^*(V,t)H_2(V,t) + \tfrac{1}{2}Trace(K_2^T(V,t)V_{VV}^*(V,t)K_2(V,t).$$

where T means Transportations.

Theorem 1. *For system (9)–(13) and any given initial conditions* $(S_H(0),\ I_H(0),\ R_H(0)) \in R_+^3$, *and* $(S_R(0),\ I_R(0)) \in R_+^2$, *there are unique solutions* $(S_H(t),\ I_H(t),\ R_H(t))$, *and* $(S_R(t),\ I_R(t))$ $t \geq 0$, *respectively, and it will remain in* R_+^5 *with probability one.*

Proof. Since the local Lipschitz constraints are satisfied with all model parameters. Therefore, by Ito's formula, the given model admits a positive solution in the sense of local on $[0, \tau_e]$, and explosion time is denoted by τ_e. To prove, the model has a global solution that is $\tau_e = \infty$.

Let $m_o = 0$ be sufficiently large for $S_H(0),\ I_H(0),\ R_H(0), S_R(0)$ *and* $I_R(0)$ lying with the interval $\left\{\frac{1}{m_o},\ m_o\right\}$. For each integer $m\ \geq\ 0$, define a sequence as follows:

$$\tau_m = inf\begin{Bmatrix} t \in [0,\tau_e] : (t) \in \left(\frac{1}{m},m\right) or\ E_H(t) \in \left(\frac{1}{m},m\right)\ or\ I_H(t) \in \left(\frac{1}{m},m\right)\ or \\ S_V(t) \in \left(\frac{1}{m},m\right)\ or\ E_V(t) \in \left(\frac{1}{m},m\right)\ or\ I_V(t) \in \left(\frac{1}{m},m\right) \end{Bmatrix} \tag{18}$$

where, we set $inf\varnothing = \infty$ ($\varnothing$ is empty set). Since τ_m is non-decreasing as m$\to\infty$,

$$\tau_\infty = \lim_{m\to\infty} \tau_m \tag{19}$$

Then $\tau_\infty\ \leq\ \tau_e$. Now, we wish to show $\tau_\infty = \infty$, as desired. If this statement is violated, then there exist $T > 0$ *and* $a_1 \in (0,1)$ such that

$$P(\tau_m \leq T) > a_1\ ,\ \forall\ m \geq m_1. \tag{20}$$

Define a c^3 functions $f : R_+^3 \to R_+$ by

$$f(S_H, I_H, R_H) = (S_H - 1 - \ln S_H) + (I_H - 1 - \ln I_H) + (R_H - 1 - \ln R_H). \tag{21}$$

Define a c^2 functions $f : R_+^2 \to R_+$ by

$$g(S_R, I_R) = (S_R - 1 - \ln S_R) + (I_R - 1 - \ln I_R) \tag{22}$$

By Ito's formula on (21), we have

$$df(S_H, I_H, R_H) = \left(1 - \tfrac{1}{S_H}\right)dS_H + \left(1 - \tfrac{1}{I_H}\right)dI_H + \left(1 - \tfrac{1}{R_H}\right)dR_H + \tfrac{\sigma_1^2+\sigma_2^2+\sigma_3^2}{2}dt$$

$$df(S_H, I_H, R_H) = \left(1 - \tfrac{1}{S_H}\right)\left[\left(\Lambda_H - \tfrac{\alpha_1\alpha_2 S_H I_R}{N_H} + \gamma R_H + \tau_{nc} I_H - \mu_H S_H\right)dt + \sigma_1 S_H dB(t)\right] + \left(1 - \tfrac{1}{I_H}\right)\left[\left(\tfrac{\alpha_1\alpha_2 S_H I_R}{N_H} - \tau_c I_H - r_c I_H - \tau_{nc} I_H - \delta I_H - \mu_H I_H\right)dt + \sigma_2 I_H dB(t)\right] + \left(1 - \tfrac{1}{R_H}\right)[(\tau_c I_H + r_c I_H - \gamma R_H - \mu_H R_H)dt + \sigma_3 R_H dB(t)].$$

$$df(S_H, I_H, R_H) \leq \left[\Lambda_H + \mu_H + \delta + \frac{\sigma_1^2+\sigma_2^2+\sigma_3^2}{2}\right]dt + \sigma_1 S_H(t)dB(t) + \sigma_2 I_H(t)dB(t) + \sigma_3 r_H(t)dB(t) \tag{23}$$

For simply, we let $N_1 = \Lambda_H + \mu_H + \delta + \frac{\sigma_1^2+\sigma_2^2+\sigma_3^2}{2}$, then Equation (23) will be written as

$$df(S_H, I_H, R_H) \leq N_1 dt + [\sigma_1 S_H(t) + \sigma_2 I_H(t) + \sigma_3 R_H(t)]dB(t). \tag{24}$$

where, N_1 is positive constant. By integrating Equation (24) from 0 to $\tau_m \Lambda\tau$.

$$\int_0^{\tau_m\Lambda\tau} df(S_H(s), I_H(s), R_H(s)) \leq \int_0^{\tau_m\Lambda\tau} N_1 ds + \int_0^{\tau_m\Lambda\tau} (\sigma_1 S_H(s) + \sigma_2 I_H(s) + \sigma_3 R_H(s))dB(s) \tag{25}$$

where $\tau_m\Lambda\tau = \min(\tau_m, T)$, then Expectation will be

$$EU^*(S_H(\tau_m\Lambda\tau), I_H(\tau_m\Lambda\tau), R_H(\tau_m\Lambda\tau) \leq U^*(S_H(0), I_H(0), R_H(0)) + N_1 T. \tag{26}$$

Set $\Omega_m = \{\tau_m \leq T\}$ *for* $m > m_1$ and from Equation (14), we have P $(\Omega_m \geq a_1\}$. For every $r_1 \in \Omega_m$ there are some i such that $U_i(\tau_m, v_1)$ equals either m or $\frac{1}{m}$ for I = 1,2,3. Hence, $U^*(S_H(\tau_m, v_1), I_H(\tau_m, v_2), R_H(\tau_m, v_3))$ is less than $\min\left\{m - 1 - lnm, \frac{1}{m} - 1 - ln\frac{1}{m}\right\}$.

Then we obtain

$$U^*(S_H(0), I_H(0), R_H(0)) + N_1 T \geq E(I_{\Omega_m(v_1)} U^*(S_H(\tau_m), I_H(\tau_m), R_H(\tau_m)) \geq \left\{\min\left\{m - 1 - lnm, \frac{1}{m} - 1 - ln\frac{1}{m}\right\}\right\} \tag{27}$$

The indicator function is represented by $I_{\Omega_m(v_1)}$ of Ω_m. Letting $m \to \infty$ leads to the contradiction.

$$\infty = U^*(S_H(0), I_H(0), R_H(0)) + N_1 T < \infty.$$

as desired.

Again, by applying Ito's formula on Equation (22), we have

$$dg(S_R, I_R) = \left(1 - \frac{1}{S_R}\right)dS_R + \left(1 - \frac{1}{I_R}\right)dI_R + \frac{\sigma_4^2 + \sigma_5^2}{2}\,dt$$

$$dg(S_R, I_R) = \left(1 - \frac{1}{S_R}\right)\left[\left(\Lambda_R - \frac{\alpha_1\alpha_3 S_R I_H}{N_H} - \mu_R S_R\right)dt + \sigma_4 S_R(t)dB(t)\right] + \left(1 - \frac{1}{I_R}\right)\left[\left(\frac{\alpha_1\alpha_3 S_R I_H}{N_H} - \mu_R I_R\right)dt + \sigma_5 I_R(t)dB(t)\right]$$

$$dg(S_R, I_R) \leq \left[\Lambda_R + \mu_R + \frac{\sigma_4^2 + \sigma_5^2}{2}\right]dt + (\sigma_4 S_R(t) + \sigma_5 I_R(t))dB(t). \tag{28}$$

To simplify, we let $N_2 = \Lambda_R + \mu_R + \frac{\sigma_4^2+\sigma_5^2}{2}$, then Equation (28) as

$$dg(S_R, I_R) \leq N_2 dt + [\sigma_4 S_R(t) + \sigma_5 I_R(t)]dB(t). \tag{29}$$

where, N_2 is a positive constant by integrating Equation (29) from 0 *to* $\tau_m\ \Lambda\tau$.

$$\int_0^{\tau_m\ \Lambda\tau} dg(S_R(s), I_R(s)) \leq \int_0^{\tau_m\ \Lambda\tau} N_2 ds + \int_0^{\tau_m\ \Lambda\tau} [\sigma_4 S_R(s) + \sigma_5 I_R(s)]dB(s) \tag{30}$$

where $\tau_m\ \Lambda\tau = \min(\tau_m, \tau)$, then expectation will be

$$EV^*(S_R(\tau_m\Lambda\tau), I_R(\tau_m\Lambda\tau)) \leq V^*(S_R(0), I_R(0)) + N_2 T \tag{31}$$

Hence, $V^*(S_R(\tau_m, V_1), I_R(\tau_m, V_1))$ is less than $\min\left\{m - 1 - \ln m, \frac{1}{m} - 1 - \ln \frac{1}{m}\right\}$. Then we obtain,

$$V^*(S_R(0), I_R(0)) + N_2 T \geq E\left(I_{\Omega_m(v_1)} V^*(S_R(\tau_m), I_R(\tau_m))\right) \geq \left\{ \min\left\{m - 1 - \ln m, \frac{1}{m} - 1 - \ln \frac{1}{m}\right\}\right\}. \quad (32)$$

$$\infty = V^*(S_R(0), I_R(0)) + N_2 T < \infty.$$

as desired. □

4.2. Stochastic Euler Approach

The discretization of the system (9)–(13) under the rules of the stochastic Euler approach is as follows:

$$S_H^{n+1} = S_H^n + h\left[\Lambda_H - \frac{\alpha_1 \alpha_2 s_H^n I_R^n}{N_H} + \gamma R_H^n + \tau_{nc} I_H^n - \mu_H S_H^n + \sigma_1 S_H^n \Delta B_n\right] \quad (33)$$

$$I_H^{n+1} = I_H^n + h\left[\frac{\alpha_1 \alpha_2 S_H^n I_R^n}{N_H} - \tau_c I_H^n - \Delta_c I_H^n - \tau_{nc} - \delta I_H^n - \mu_H I_H^n + \sigma_2 I_H^n \Delta B_n\right] \quad (34)$$

$$R_H^{n+1} = R_H^n + h\left[\tau_c I_H^n + \Delta_c I_H^n - \gamma R_H^n - \mu_H R_H^n + \sigma_3 R_H^n \Delta B_n\right] \quad (35)$$

$$\mathrm{S_R^{n+1}} = \mathrm{S_R^n} + h\left[\Lambda_R - \frac{\alpha_1 \alpha_3 \mathrm{S_R^n I_H^n}}{\mathrm{N_H}} - \mu_R \mathrm{S_R^n} + \sigma_4 \mathrm{S_R^n} \Delta \mathrm{B_n}\right] \quad (36)$$

$$I_R^{n+1} = I_R^n + h\left[\frac{\alpha_1 \alpha_3 S_R^n I_H^n}{N_H} - \mu_R I_R^n + \sigma_5 I_R^n \Delta B_n\right] \quad (37)$$

where h is any discretization parameter and $n \geq 0$.

4.3. Stochastic Runge—Kutta Approach

The discretization of the system (9)–(13) under the rules of the stochastic Runge–Kutta approach is as follows:

First Stage

$$K_1 = h\left[\wedge_H - \frac{\alpha_1 \alpha_2 S_H^n I_R^n}{N_H} + \Upsilon R_H^n + \tau_{nc} I_H^n - \mu_H S_H^n + \sigma_1 S_H^n \Delta\, B_n\right]$$

$$L_1 = h\left[\frac{\alpha_1 \alpha_2 S_H^n I_R^n}{N_H} - \tau_c I_H^n - \Delta_c I_H^n - \tau_{nc} I_H^n - \delta I_H^n - \mu_H I_H^n + \sigma_2 I_H^n \Delta\, B_n\right]$$

$$M_1 = h\left[\tau_c I_H^n + r_c I_H^n - \gamma R_H^n - \mu_H R_H^n + \sigma_3 R_H^n \Delta\, B_n\right]$$

$$N_1 = h\left[\Lambda_R - \frac{\alpha_1 \alpha_3 S_R^n I_H^n}{N_H} - \mu_R S_R^n + \sigma_4 S_R^n \Delta\, B_n\right]$$

$$O_1 = h\left[\frac{\alpha_1 \alpha_3 S_R^n I_H^n}{N_H} - \mu_R I_R^n + \sigma_5 I_R^n \Delta\, B_n\right]$$

Second Stage

$$K_2 = h\left[\Lambda_H - \frac{\alpha_1 \alpha_2 \left(S_H^n + \frac{K_1}{2}\right)\left(I_R^n + \frac{O_1}{2}\right)}{N_H} + \gamma\left(R_H^n + \frac{M_1}{2}\right) + \tau_{nc}\left(I_H^n + \frac{L_1}{2}\right) - \mu_H\left(S_H^n + \frac{K_1}{2}\right) + \sigma_1\left(S_H^n + \frac{K_1}{2}\right)\Delta B_n\right]$$

$$L_2 = h\left[\frac{\alpha_1 \alpha_2 \left(S_H^n + \frac{K_1}{2}\right)\left(I_R^n + \frac{O_1}{2}\right)}{N_H} - \tau_c\left(I_H^n + \frac{L_1}{2}\right) - r_c\left(I_H^n + \frac{L_1}{2}\right) - \tau_{nc}\left(I_H^n + \frac{L_1}{2}\right) - \delta\left(I_H^n + \frac{L_1}{2}\right) - \mu_H\left(I_H^n + \frac{L_1}{2}\right) + \sigma_2\left(I_H^n + \frac{L_1}{2}\right)\Delta B_n\right]$$

$$M_2 = h\left[\tau_c\left(I_H^n + \frac{L_1}{2}\right) + r_c\left(I_H^n + \frac{L_1}{2}\right) - \gamma\left(R_H^n + \frac{M_1}{2}\right) - \mu_H\left(R_H^n + \frac{M_1}{2}\right) + \sigma_3\left(R_H^n + \frac{M_1}{2}\right)\Delta B_n\right]$$

$$N_2 = h\left[\Lambda_R - \frac{\alpha_1 \alpha_3 \left(S_R^n + \frac{N_1}{2}\right)\left(I_H^n + \frac{L_1}{2}\right)}{N_H} - \mu_R\left(S_R^n + \frac{N_1}{2}\right) + \sigma_4\left(S_R^n + \frac{N_1}{2}\right)\Delta B_n\right]$$

$$O_2 = h\left[\frac{\alpha_1 \alpha_3 \left(S_R^n + \frac{N_1}{2}\right)\left(I_H^n + \frac{L_1}{2}\right)}{N_H} - \mu_R\left(I_R^n + \frac{O_1}{2}\right) + \sigma_5\left(I_R^n + \frac{O_1}{2}\right)\Delta B_n\right]$$

Third Stage

$$K_3 = h\left[\Lambda_H - \frac{\alpha_1\alpha_2\left(S_H^n+\frac{K_2}{2}\right)\left(I_R^n+\frac{O_2}{2}\right)}{N_H} + \gamma\left(R_H^n+\frac{M_2}{2}\right) + \tau_{nc}\left(I_H^n+\frac{L_2}{2}\right) - \mu_H\left(S_H^n+\frac{K_2}{2}\right) + \sigma_1\left(s_H^n+\frac{K_2}{2}\right)\Delta B_n\right]$$

$$L_3 = h\left[\frac{\alpha_1\alpha_2\left(S_H^n+\frac{K_2}{2}\right)\left(I_R^n+\frac{O_2}{2}\right)}{N_H} - \tau_c\left(I_H^n+\frac{L_2}{2}\right) - r_c\left(I_H^n+\frac{L_2}{2}\right) - \tau_{nc}\left(I_H^n+\frac{L_2}{2}\right) - \delta\left(I_H^n+\frac{L_2}{2}\right) - \mu_H\left(I_H^n+\frac{L_2}{2}\right) + \sigma_2\left(I_H^n+\frac{L_2}{2}\right)\Delta B_n\right]$$

$$M_3 = h\left[\tau_c\left(I_H^n+\frac{L_2}{2}\right) + r_c\left(I_H^n+\frac{L_2}{2}\right) - \gamma\left(R_H^n+\frac{M_2}{2}\right) - \mu_H\left(R_H^n+\frac{M_2}{2}\right) + \sigma_3\left(R_H^n+\frac{M_2}{2}\right)\Delta B_n\right]$$

$$N_3 = h\left[\Lambda_R - \frac{\alpha_1\alpha_3\left(S_R^n+\frac{N_2}{2}\right)\left(I_H^n+\frac{L_2}{2}\right)}{N_H} - \mu_R\left(S_R^n+\frac{N_2}{2}\right) + \sigma_4\left(S_R^n+\frac{N_2}{2}\right)\Delta B_n\right]$$

$$O_3 = h\left[\frac{\alpha_1\alpha_3\left(S_R^n+\frac{N_2}{2}\right)\left(I_H^n+\frac{L_2}{2}\right)}{N_H} - \mu_R\left(I_R^n+\frac{O_2}{2}\right) + \sigma_5\left(I_R^n+\frac{O_2}{2}\right)\Delta B_n\right]$$

Fourth Stage

$$K_4 = h\left[\Lambda_H - \frac{\alpha_1\alpha_2(S_H^n+K_3)(I_R^n+O_3)}{N_H} + \gamma(R_H^n+M_3) + \tau_{nc}(I_H^n+L_3) - \mu_H\,(S_H^n+K_3)\sigma_1(S_H^n+K_3)\Delta B_n\right]$$

$$L_4 = h\left[\frac{\alpha_1\alpha_2(S_H^n+K_3)(I_R^n+o_3)}{N_H} - \tau_c(I_H^n+L_3) - r_c(I_H^n+L_3) - \tau_{nc}(I_H^n+L_3) - \delta(I_H^n+L_3) - \mu_H(I_H^n+L_3) + \sigma_2(I_H^n+L_3)\Delta B_n\right]$$

$$M_4 = h\left[\tau_c(I_H^n+L_3) + r_c(I_H^n+L_3) - \gamma(R_H^n+M_3) - \mu_H(R_H^n+M_3) + \sigma_3(R_H^n)\Delta B_n\right]$$

$$N_4 = h\left[\Lambda_R - \frac{\alpha_1\alpha_3(S_R^n+N_3)(I_H^n+L_3)}{N_H} - \mu_R(S_R^n+N_3) + \sigma_4(S_R^n+N_3)\Delta B_n\right]$$

$$O_4 = h\left[\frac{\alpha_1\alpha_3(S_R^n+N_3)(I_H^n+L_3)}{N_H} - \mu_R(I_R^n+O_3) + \sigma_5(I_R^n+O_3)\Delta B_n\right]$$

Final Stage

$$S_H{}^{n+1} = S_H^n + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4] \tag{38}$$

$$I_H{}^{n+1} = I_H^n + \frac{1}{6}[L_1 + 2L_2 + 2L_3 + L_4] \tag{39}$$

$$R_H{}^{n+1} = R_H^n + \frac{1}{6}[M_1 + 2M_2 + 2M_3 + M_4] \tag{40}$$

$$S_R{}^{n+1} = S_R^n + \frac{1}{6}[O_1 + 2O_2 + 2O_3 + O_4] \tag{41}$$

$$I_R{}^{n+1} = I_R^n + \frac{1}{6}[P_1 + 2P_2 + 2P_3 + P_4] \tag{42}$$

where h is any discretization parameter and $n \geq 0$.

4.4. Stochastic Nonstandard Finite Difference Approach

The stochastic non-standard finite difference scheme for Equations (9)–(13) can be defined as follows [32,33]:

$$S_H^{n+1} = \frac{S_H^n + h\Lambda_H + \gamma h R_H^n + h\tau_{nc} I_H^n + h\sigma_1 S_H^n \Delta B_n}{1 + \frac{h\alpha_1\alpha_2 I_R^n}{N_H} + \mu_H h} \tag{43}$$

$$I_H^{n+1} = \frac{I_H^n + \frac{h\alpha_1\alpha_2 S_H^n I_R^n}{N_H} + h\sigma_2 I_H^n \Delta B_n}{1 + h\tau_c + r_c h + h\tau_{nc} + \delta h + \mu_H h} \tag{44}$$

$$R_H^{n+1} = \frac{R_H^n + h\tau_c I_H^n + \Delta_c I_H^n + h\sigma_3 R_H^n \Delta B_n}{1 + \gamma h + \mu_H h} \tag{45}$$

$$S_R^{n+1} = \frac{S_R^n + h\Lambda_R + h\sigma_4 S_R^n \Delta B_n}{1 + \frac{h\alpha_1\alpha_3 I_H^n}{N_H} + \mu_R h} \tag{46}$$

$$I_R^{n+1} = \frac{I_R^n + \frac{h\alpha_1\alpha_3 S_R^n I_H^n}{N_H} + h\sigma_5 I_R^n \Delta B_n}{1 + \mu_R h} \tag{47}$$

where h is any discretization parameter and $n \geq 0$.

4.5. Stability Analysis

Considering the functions A, B, C, D, and E for the system (43)–(47) by assuming the $\Delta B_n = 0$ as follows:

$$A = \frac{S_H+h\Lambda_H+\gamma hR_H+h\tau_{nc}I_H}{1+\frac{h\alpha_1\alpha_2 I_R}{N_H}+\mu_H h}, B = \frac{I_H+\frac{h\alpha_1\alpha_2 S_H I_R}{N_H}}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h}, C = \frac{R_H+h\tau_c I_H+r_c I_H}{1+\gamma h+\mu_H h}, D = \frac{S_R+h\Lambda_R}{1+\frac{h\alpha_1\alpha_3 I_H}{N_H}+\mu_R h}.$$
$$E = \frac{I_R+\frac{h\alpha_1\alpha_3 S_R I_H}{N_H}}{1+\mu_R h}.$$

The elements of Jacobean matrix as follows:

$$\frac{\partial A}{\partial S_H} = \frac{1}{1+\mu_H h}, \frac{\partial A}{\partial I_H} = \frac{h\tau_{nc}}{1++\mu_H h}, \frac{\partial A}{\partial R_H} = \frac{\gamma h}{1+\mu_H h}, \frac{\partial A}{\partial S_R} = 0, \frac{\partial A}{\partial I_R} = -\frac{(S_H+h\Lambda_H+\gamma hR_H+h\tau_{nc}I_H)\left(\frac{h\alpha_1\alpha_2}{N_H}\right)}{\left(1+\frac{h\alpha_1\alpha_2 I_R}{N_H}+\mu_H h\right)^2},$$
$$\frac{\partial B}{\partial S_H} = \frac{\frac{h\alpha_1\alpha_2 I_R}{N_H}}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h}, \frac{\partial B}{\partial I_H} = \frac{1}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h} \frac{\partial B}{\partial R_H} = 0, \frac{\partial B}{\partial S_R} = 0,$$
$$\frac{\partial B}{\partial I_R} = \frac{\frac{h\alpha_1\alpha_2 S_H}{N_H}}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h}, \frac{\partial C}{\partial S_H} = 0, \frac{\partial C}{\partial I_H} = \frac{h\tau_c+r_c}{1+\gamma h+\mu_H h}, \frac{\partial C}{\partial R_H} = \frac{1}{1+\gamma h+\mu_H h}, \frac{\partial C}{\partial S_R} = 0,$$
$$\frac{\partial C}{\partial I_R} = 0, \frac{\partial D}{\partial S_H} = 0, \frac{\partial D}{\partial I_H} = -\frac{(S_R+h\Lambda_R)\left(\frac{h\alpha_1\alpha_3}{N_H}\right)}{\left(1+\frac{h\alpha_1\alpha_3 I_H}{N_H}+\mu_R h\right)^2}, \frac{\partial D}{\partial R_H} = 0, \frac{\partial D}{\partial S_R} = \frac{1}{1+\mu_R h}, \frac{\partial D}{\partial I_R} = 0, \frac{\partial E}{\partial S_H} = 0,$$
$$\frac{\partial E}{\partial I_H} = \frac{\frac{h\alpha_1\alpha_3 S_R}{N_H}}{1+\mu_R h}, \frac{\partial E}{\partial R_H} = 0, \frac{\partial E}{\partial S_R} = \frac{\frac{h\alpha_1\alpha_3 I_H}{N_H}}{1+\mu_R h}, \frac{\partial E}{\partial I_R} = \frac{1}{1+\mu_R h}.$$

Theorem 2. *For $n \geq 0$, the eigenvalues of the Jacobian matrix at the disease-free equilibrium for the system (43)–(47) lie in the unit circle if $R_0 < 1$.*

Proof. The Jacobean matrix at disease-free equilibrium (DFE-E_0) = $\left(\frac{\Lambda_H}{\mu_H}, 0, 0, \frac{\Lambda_R}{\mu_R}, 0\right)$ is as follows:

$$J(E_0) = \begin{bmatrix} \frac{1}{1+\mu_H h} & \frac{h\tau_{nc}}{1++\mu_H h} & \frac{\gamma h}{1+\mu_H h} & 0 & -\frac{(S_H+h\Lambda_H+\gamma hR_H+h\tau_{nc}I_H)\left(\frac{h\alpha_1\alpha_2}{N_H}\right)}{\left(1+\frac{h\alpha_1\alpha_2 I_R}{N_H}+\mu_H h\right)^2} \\ \frac{\frac{h\alpha_1\alpha_2 I_R}{N_H}}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h} & \frac{1}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h} & 0 & 0 & \frac{\frac{h\alpha_1\alpha_2 S_H}{N_H}}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h} \\ 0 & \frac{h\tau_c+r_c}{1+\gamma h+\mu_H h} & \frac{1}{1+\gamma h+\mu_H h} & 0 & 0 \\ 0 & -\frac{(S_R+h\Lambda_R)\left(\frac{h\alpha_1\alpha_3}{N_H}\right)}{\left(1+\frac{h\alpha_1\alpha_3 I_H}{N_H}+\mu_R h\right)^2} & 0 & \frac{1}{1+\mu_R h} & 0 \\ 0 & \frac{\frac{h\alpha_1\alpha_3 S_R}{N_H}}{1+\mu_R h} & 0 & \frac{\frac{h\alpha_1\alpha_3 I_H}{N_H}}{1+\mu_R h} & \frac{1}{1+\mu_R h} \end{bmatrix}$$

$$|\mathrm{J}(E_0)-\lambda| = \begin{vmatrix} \frac{1}{1+\mu_R h}-\lambda & \frac{h\tau_{nc}}{1+\mu_H h} & \frac{\gamma h}{1+\mu_H h} & 0 & -\frac{\left(\frac{\Lambda_H}{\mu_H}+h\Lambda_H\right)\left(\frac{h\alpha_1\alpha_2}{N_H}\right)}{(1+\mu_H h)^2} \\ 0 & \frac{1}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h}-\lambda & 0 & 0 & \frac{\frac{h\alpha_1\alpha_2\frac{\Lambda_H}{\mu_H}}{N_H}}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h} \\ 0 & \frac{h\tau_c+r_c}{1+\gamma h+\mu_H h} & \frac{1}{1+\gamma h+\mu_H h}-\lambda & 0 & 0 \\ 0 & -\frac{\left(\frac{\Lambda_R}{\mu_R}+h\Lambda_R\right)\left(\frac{h\alpha_1\alpha_2}{N_H}\right)}{(1+\mu_R h)^2} & 0 & \frac{1}{1+\mu_R h}-\lambda & 0 \\ 0 & \frac{\frac{h\alpha_1\alpha_3\frac{\Lambda_R}{\mu_R}}{N_H}}{1+\mu_R h} & 0 & 0 & \frac{1}{1+\mu_R h}-\lambda \end{vmatrix} = 0.$$

$$\lambda_1 = \left|\frac{1}{1+\mu_H h}\right| < 1,\ \lambda_2 = \left|\frac{1}{1+\gamma h+\mu_H h}\right| < 1,\ \lambda_3 = \left|\frac{1}{1+h\mu_R}\right| < 1.$$

$$|\mathrm{J}(E_0)-\lambda| = \begin{vmatrix} \left(\frac{1}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h}\right)-\lambda & \frac{\frac{h\alpha_1\alpha_2\frac{\Lambda_H}{\mu_H}}{N_H}}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h} \\ \frac{\frac{h\alpha_1\alpha_3\frac{\Lambda_R}{\mu_R}}{N_H}}{1+\mu_R h} & \frac{1}{1+\mu_R h}-\lambda \end{vmatrix} = 0.$$

$$P_1 = Trace\ of\ J = \left(\frac{1}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h}\right)+\frac{1}{1+\mu_R h}$$

$$P_2 = Determinant\ of\ J = \left(\left(\frac{1}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h}\right)\left(\frac{1}{1+\mu_R h}\right)\right)-\left(\frac{\frac{h\alpha_1\alpha_3\frac{\Lambda_R}{\mu_R}}{N_H}}{1+\mu_R h}\right)\left(\frac{\frac{h\alpha_1\alpha_2\frac{\Lambda_H}{\mu_H}}{N_H}}{1+h\tau_c+r_c h+h\tau_{nc}+\delta h+\mu_H h}\right).$$

□

Lemma 3. *For the quadratic equation* $\lambda^2 - P_1\lambda + P_2 = 0$, $|\lambda_i| < 1,\ i = 1,\ 2,3$, *if and only if the following conditions are satisfied:*

(i) $1 + \mathrm{P}_1 + \mathrm{P}_2 > 0.$
(ii) $1 - \mathrm{P}_1 + \mathrm{P}_2 > 0.$
(iii) $\mathrm{P}_2 < 1.$

Proof. The proof is straightforward. □

4.6. Comparison Section

This section compares the behavior of the graphs of infected humans of Euler Maruyama, stochastic Euler, and sto-chastic Runge Kutta schemes with the NSFD scheme for different step sizes.

5. Concluding Remarks

Table 3 predicts the efficacy of the existing methods with the proposed technique. All methods are consistent in small time steps (the disease behavior for a short period). After taking an increase in time, the existing methods are not compatible with the solution of the continuous model, even violating the properties, such as positivity, boundedness, and dynamical consistency. This means the current techniques are unsuitable for predicting the disease's behavior for a long time. The graphical behavior of Euler–Maruyama, stochastic Euler, stochastic Runge–Kutta, and SNSFD schemes are given in Figures 2 and 3. Figure 4 presents the schematic map of the Lassa fever model. The analysis predicts that mouse-

to-human transmission rate and mouse death rate are among the most critical parameters. Hence, the abundance of mice is the most crucial driver of Lassa fever transmission. The following steps could be adequate to control the disease by decreasing rodent-to-human communication, e.g., using rodent-safe food containers, collecting garbage far from the houses, and reducing human-to-human transmission. Our simulations suggest modifying control parameters corresponding to such measures might mitigate the epidemic, but they seem insufficient to drive it to extinction. In the future, we shall extend this idea to annealing genetic GAN for imbalanced web data learning as presented in [34].

Table 3. Comparison of numerical techniques for different step sizes '*h*.

h	*Euler–Maruyama*	*Stochastic Euler*	*Stochastic Runge Kutta*	*Stochastic NSFD*
0.01	Convergence	Convergence	Convergence	Convergence
0.5	Divergence	Divergence	Divergence	Convergence
1	Divergence	Divergence	Divergence	Convergence
2	Divergence	Divergence	Divergence	Convergence

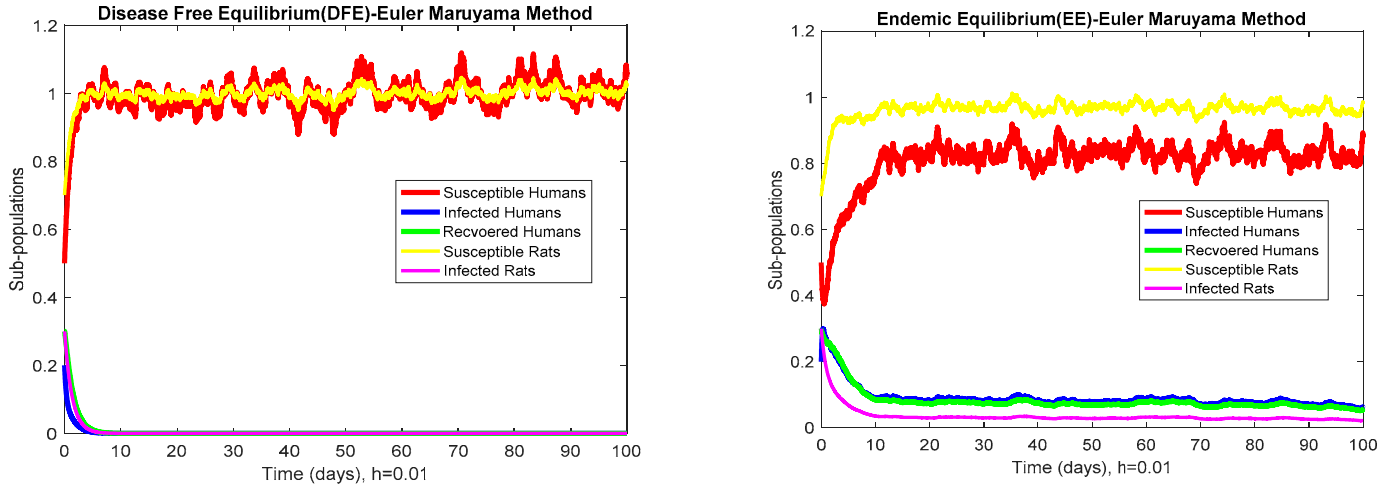

Figure 2. Depicts the graphical behavior of the system (8) through the Euler–Maruyama method with the help of stochastic differential equations (SDEs) package. (**a**) The behavior of each subpopulation for disease-free equilibrium at $h = 0.01$. (**b**) The behavior of each subpopulation for endemic equilibrium at $h = 0.01$.

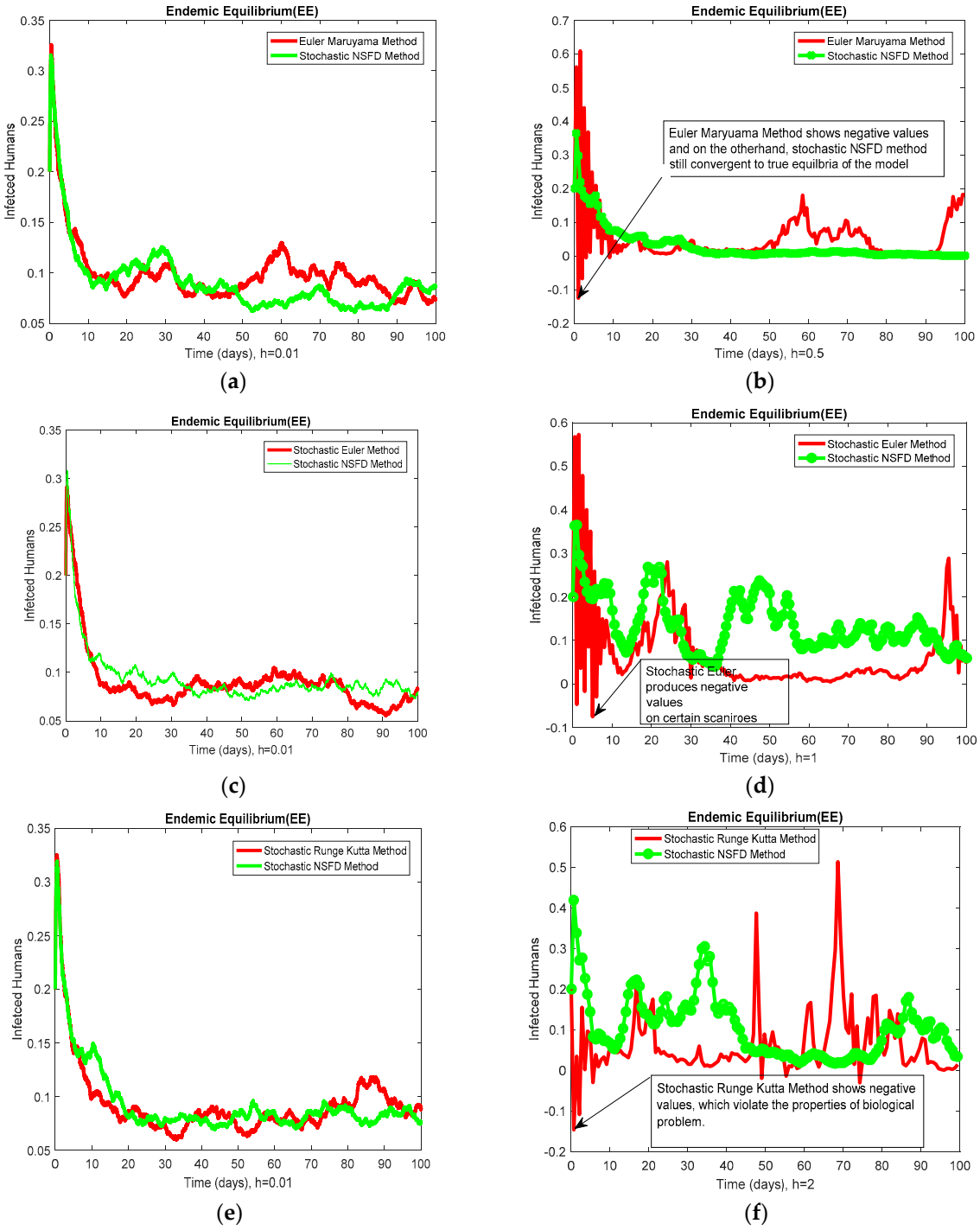

Figure 3. Combined graphical behaviors of NSFD with Euler-Maruyama, stochastic Euler, and stochastic Runge-Kutta methods at different time-step sizes. (**a**) The behavior of infected humans through both methods converge to the proper equilibrium at $h = 0.01$. (**b**) Euler-Maruyama method diverges and even produces negative values after taking the long-term behavior, but the proposed method is still convergent. (**c**) The stochastic Euler method depicts the exact behavior of the disease, like the stochastic nonstandard finite difference method. (**d**) However, the stochastic Euler method fails to restore the dynamical properties at $h = 1$. (**e**) The stochastic Runge-Kutta methods converges like stochastic NSFD at $h = 0.01$ (**f**) The stochastic Runge-Kutta method diverges when we take $h = 2$ days, but the proposed method is convergent and restores the dynamical properties of the model.

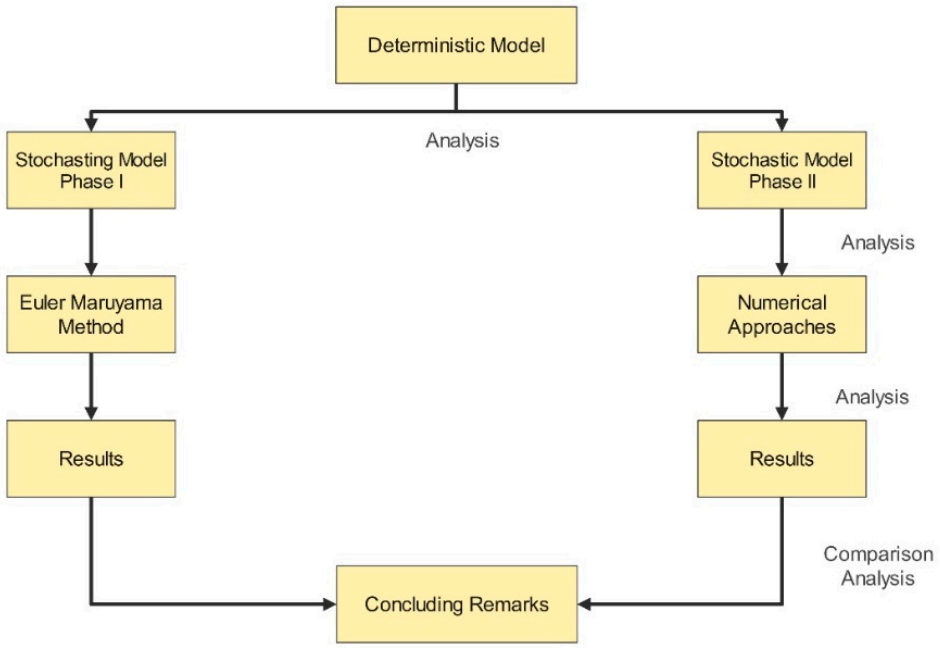

Figure 4. A schematic diagram of the Lassa fever model.

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