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Dynamic Scheduling of Intelligent Group Maintenance Planning under Usage Availability Constraint

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Abstract: Maintenance, particularly preventive maintenance, is a crucial measure to ensure the operational reliability, availability, and profitability of complex industrial systems such as nuclear asset, wind turbines, railway trains, etc. Powered by the continuous advancement of sensor technology, condition-based group maintenance has become available to enhance the execution efficiency and accuracy of maintenance plans. The majority of existing group maintenance plans are static, which require the prescheduling of maintenance sequences within fixed windows and, thus, cannot fully utilize real-time health information to ensure decision-making responsiveness. To address this problem, this paper proposes an intelligent group maintenance framework that is capable of dynamically and iteratively updating all component health information. A two-stage analytical maintenance model was formulated to capture the comprehensive impact of scheduled maintenance and opportunistic maintenance through failure analyses of both degradation and lifetime components. The penalty functions for advancing or postponing maintenance were calculated based on the real-time state and age information of each component in arbitrary groups, and the subsequent grouping of the time and sequence of components to be repaired were iteratively updated. A lifetime maintenance cost model was formulated and optimized under a usage availability constraint through the sequential dynamic programming of group sequences. Numerical experiments demonstrated the superior performance of the proposed approach in cost control and availability insurance compared with conventional static and periodic maintenance approaches.

Keywords: maintenance optimization; cluster decision making; replacement planning; cost benefit analysis; availability; dynamic programming

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1. Introduction

Complex, multicomponent systems or networks such as aircraft, high-speed railway trains, and smart grids share some common features, such as a huge amount of components, a relatively long lifetime cycle, and complex failure modes, posing a significant challenge for the joint scheduling of maintenance activities and the allocation of spare part resources [1]. On the other hand, the performance requirements of modern equipment are gradually increasing due to technical advancement, along with more complicated operational environments, which further increase the urgent demands for the timeliness, responsiveness, and robustness of maintenance control, scheduling, and management [2].

Most traditional group maintenance policies, such as postmaintenance and periodic maintenance, are arranged simply based on either operational age or degradation status, which are single-dimensional and lack the comprehensive thought of all the components, failure mechanisms, and operational patterns. Consequently, these measures usually face the challenge of low execution efficiency and inadequate information usage, making it difficult to satisfy service reliability and availability criteria. A feasible solution to achieve high availability and reduce maintenance cost is to combine the lifetime and failure characteristics of each component and, thereby, develop group maintenance scheduling approaches based on the temporal correlation of maintenance activities between components [3]. Particularly when the health information of a critical component is available, it is of great significance to seek a dynamic maintenance group approach based on the outcomes of fault prognosis and condition-based maintenance (CBM) optimization.

Condition-based maintenance and its newest evolution, predictive maintenance, have attracted a lot of attention in aviation equipment maintenance with the development of sensor-monitoring and live forecast technology [1,4]. Compared to age-based maintenance, its greatest benefit is that maintenance can be carried out according to the actual operation state of the equipment to avoid extra downtime losses caused by incorrect judgment based on prior information or a lack of understanding of the equipment health status [5,6]. Reliability analyses, as well as maintenance optimization approaches, in regard to performance deterioration have been widely reviewed in the past several years. For instance, Yang et al. [7] employed health status as the condition criterion for mission abort control. Zhang et al. [8] used the concepts of minimum maintenance and preventive maintenance to obtain an optimal replacement policy for aircraft structural corrosion. Aside from maintenance cost, usage availability, or operational readiness, is another critical optimization object of maintenance widely reviewed in the literature [9]. Stable availability is a critical metric for determining the effectiveness of critical equipment systems. Due to several mission requirements, the majority of military equipment yields availability constraints. As such, availability is equally, or even more important, than maintenance economy in asset operations and maintenance. As a result, the ability to appropriately assess system availability has a significant impact on maintenance policy. In related studies, Álvarez et al. [10] studied a condition-based maintenance model with check times as availability constraints. Shen et al. [11] devised an availability and optimal maintenance policy to deal with system degradation under a dynamic environment.

Notably, the dependence among components is the main motivation to schedule group maintenance [12–14]. In particular, the maintenance activities of multiple components can be coordinated to reduce maintenance costs and system outages since they share some maintenance resources or expenditures [15]. Group maintenance plans at the system level can be classified as preventive or opportunistic ones, depending on whether they are prescheduled in advance. The former entails a combination of multicomponent preventive maintenance and reprogramming. The latter schedules the preventive maintenance of other normal components when a failed component is correctively maintained. Rolling horizon is the common method for group maintenance scheduling, which makes the preventive maintenance interval of all the system component integer times the minimum preventive maintenance interval by adjusting the maintenance interval [16]. Fan et al. [17] proposed a group maintenance method for subsea Xmas trees with stochastic dependency. Barron [18] studied the group maintenance method of an R-out-of-N system with phase-type distribution. Li et al. [19] optimized a combination maintenance task based on an adaptive clustering search algorithm.

Through the review of the existing literature, current group maintenance policies are mainly globally or partially static, with a single degradation or reliability threshold combined with the minimum maintenance interval of multiple components as trigger conditions [20]. However, in practical maintenance projects, this technique has two disadvantages: (1) it is unable to completely utilize the real-time health information of each component, such as the remaining useful life, degradation degree, and operational age; and (2) it is impossible to provide sufficient buffer time for the preparation scheduling of maintenance resources. In other words, such maintenance policies cannot flexibly reflect the real-time health status and environmental impact variation of a system, resulting in premature maintenance, even if the component remains operational. Moreover, the lack of overall consideration for opportunistic maintenance delays maintenance, as a component needs to wait until the next maintenance cycle after failure, which eventually leads to an increase in downtime and maintenance costs, as well as a decrease in system availability. Moreover, current group maintenance models focus mainly on prescheduled preventive maintenance, where opportunistic maintenance that shares unscheduled downtime is rarely addressed, which may restrict the maintenance performance.

In order to address the foregoing problems, this paper innovatively devises an intelligent group maintenance policy for series-connected multicomponent systems under availability constraints. To the best of our knowledge, this is the first attempt to dynamically integrate scheduled maintenance and opportunistic maintenance into a unified framework through iteratively renewed system health information. A notable superiority of the proposed framework is that it is applicable to both lifetime cycle components with sudden failures and continuously degrading components, which are two common failure types [21]. To this end, a two-stage analytical maintenance model from the component level to the whole system is preliminarily established. The maintenance cost and stable availability of each component are analyzed, followed by the profitability increment calculation of grouping individual components. Finally, a backward dynamic-programming algorithm is proposed to solve the sequential grouping problem. Compared with traditional, static, group maintenance policies that presuppose the minimum maintenance interval, the proposed framework has the following advantages: (1) it combines the real-time health status information of each component to enable a more accurate availability evaluation; (2) it allows the dynamic scheduling of multiple components based on accurate assessments of availability to enhance the maintenance efficiency; and (3) it provides extra opportunities for unscheduled opportunistic group maintenance, which can make the most of system downtime to reduce loss.

To summarize, the contributions of this paper to system group maintenance are outlined below:

- Devising a dynamic group maintenance policy for complex systems that is capable of automatically and sequentially updating the health information of all the components;
- Integrating the scheduling of preventive maintenance and opportunistic maintenance into a unified group maintenance framework to enhance the maintenance performance;
- Developing a programming method for the sequential grouping of maintenance units and time;
- Analytically optimizing the maintenance model to jointly enhance the operational profitability and availability of the system.

The paper is structured as follows. Section 2 describes failure characteristics of systems and maintenance policies. Section 3 establishes a component-level maintenance cost model and an availability model. In Section 4, a dynamic group maintenance model at the system level is established. Section 5 verifies the applicability and effectiveness of the proposed approach through a numerical experiment.

2. Problem Statement

We considered a multicomponent system composed of n critical components connected in series, where the malfunction of each component led to an immediate system breakdown. As such, the operating states of these components posed a significant, indigenous impact on the overall reliability and availability of the system. Based on historical fault data and filed operational and maintenance records, these components were partitioned into the following two types [21]:

(1) Degradation component. The degradation mechanism of such a component is known, through structural health monitoring and other technologies, to acquire degradation data. As such, the residual lifetime of such a component can be predicted through either model-driven or data-driven approaches. Instances include gear bearing parts, hydraulic mechanical systems, and other mechanical components. We defined the collection of all the degradation components as N_{D}.

The degradation $D(t)$ of these components can be captured by the degradation data. The process of degradation establishment mainly includes monitoring data collection and health index construction [22,23]. First, measurements, such as vibration signals, are obtained from sensors to monitor the health of the component. Then, based on the measured data, signal-processing technology, artificial intelligence technology and other technologies are used to construct the component health indicators to indicate the health degradation of the component. After degradation establishment, the degradation process in the time domain can be captured according to the degradation tendency. Since the failure of a component is determined by the first hitting-time to the failure threshold, the lifetime distribution of a component can be obtained according to the degradation process, which includes the cumulative distribution function (CDF) $F(t)$ and probability density function (PDF) $f(t)$ of the component lifetime.

For example, the Wiener process, as used in our numerical experiment, can well-fit the degradation of such a component [24], and the lifetime distribution corresponding to this degradation process is an inverse Gaussian distribution, with the degradation and distribution parameters estimated with the degradation data. To be specific, we used the following linear Wiener process with Brownian motion to capture the degradation process:

$$D(t) = \nu t + \sigma B(t), \tag{1}$$

where ν is the drift coefficient, σ is the diffusion coefficient, and $B(t)$ represents the standard Brownian motion. Since the increment of the Wiener process obeys normal distribution, the CDF and PDF of degradation-based failure after specifying the failure threshold D_{F} are as follows:

$$\begin{aligned} F(x|\tau_i) &= P(D(x) - D(\tau_i) \geqslant D_{\mathrm{F}} - D(\tau_i)) = 1 - \Phi\left(\frac{D_{\mathrm{F}} - D(\tau_i) - \nu(x-\tau_i)}{\sigma\sqrt{x-\tau_i}}\right), \\ f(x|\tau_i) &= \frac{D_{\mathrm{F}} - D(\tau_i)}{\sqrt{2\pi\sigma^2(x-\tau_i)^3}} e^{\left(-\frac{(D_{\mathrm{F}} - D(\tau_i) - \nu(x-\tau_i))^2}{2\sigma^2(x-\tau_i)}\right)}, \end{aligned} \tag{2}$$

where represents the cumulative working time of the component, and $\Phi(\cdot)$ represents the CDF of the standard normal distribution.

(2) Lifetime component. Due to the complexity of specific spatial component and material characteristics, health-monitoring information is intractable for some types of components. Alternatively, the lifetime distribution of these components can be assessed using lifetime data, including failure data and right-centered operation data. Examples include high-voltage ignition wires, electronic systems, and other electromechanical equipment whose reliability is structured by operational age. We defined the collection of all the lifetime structures as N_{T}.

As there are no degradation data for these components, it is impossible to conduct degradation process modeling, so the lifetime distributions of the components, including the CDF $F(t)$ and PDF $f(t)$, can only be described by a specific distribution; this distribution is usually based on the distribution-fitting of failure records of components or on previous studies [25].

An example is the aircraft panel structure mentioned in our numerical experiment, whose lifetime was described using Gamma distribution, and the parameters of the distribution were estimated based on its historical failure data. In particular, the CDF and PDF of the component lifetime obeyed the Gamma distribution and could be expressed as follows:

$$\begin{aligned} F(x|\tau_i) &= P(T < x | T \geqslant \tau_i) = \frac{\int_{\tau_i}^{x} \frac{\beta^\alpha}{\Gamma(\alpha)} t^{\alpha-1} e^{-\beta t} dt}{\int_{\tau_i}^{+\infty} \frac{\beta^\alpha}{\Gamma(\alpha)} t^{\alpha-1} e^{-\beta t} dt}, \\ f(x|\tau_i) &= \frac{\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}}{\int_{\tau_i}^{+\infty} \frac{\beta^\alpha}{\Gamma(\alpha)} t^{\alpha-1} e^{-\beta t} dt}. \end{aligned} \tag{3}$$

where $\Gamma(\cdot)$ represents the Gamma function, α is the shape parameter, and β is the scale parameter.

2.1. Dynamic Group Maintenance

During system operation, once a component failure is located, a corrective maintenance is scheduled immediately. At the same time, preventive maintenance is essential for all the components to mitigate failure probabilities. Since there is spatial coupling between components, two types of maintenance times and costs are incurred [21]. At the same time, in order to facilitate component availability and maintenance cost modeling, we simplified these maintenance times and costs.

The first is the maintenance execution time t_{set} and cost C_{set} shared by the coupling component, which includes maintenance resource scheduling time, component disassembly time, labor cost, setting cost, and so on. The second is the maintenance execution time $t_{i,\text{pr/cr}}$ and cost $C_{i,\text{pr/cr}}$ of each component i, respectively. Note that the disassembly and commissioning times can be shared. Thus, the simultaneous maintenance of multiple components in a group structure can effectively save maintenance time and improve the availability of a system. Such a maintenance policy is called group maintenance.

2.2. Maintenance Level

In this study, the goal of group maintenance was to minimize the maintenance cost rate of an entire system under an availability constraint. To this end, the group maintenance policy was executed via the following two successive steps, from component-level static maintenance to system-level dynamic maintenance.

- **Component-level static maintenance**

At the component level, the maintenance interval time of each system component was optimized independently to obtain the optimal maintenance interval time of each component.

- **System-level dynamic maintenance**

As shown in Figure 1, maintenance planning at the system level was set according to the optimal maintenance interval of each component and whether there was component failure. According to whether such grouping was prescheduled or unscheduled, system-level group maintenance could be further partitioned into preventive group maintenance and opportunistic group maintenance.

(1) Preventive maintenance (PM) grouping

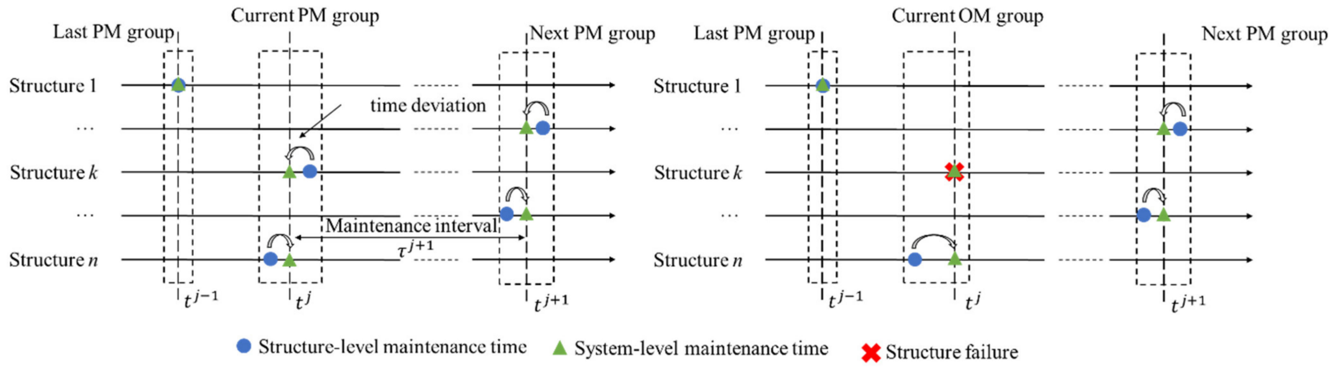

Figure 1. Illustrations of preventive grouping (**left**) and opportunistic grouping (**right**).

A component within a maintenance group can maintain normal working, but its failure probability is relatively high. Therefore, in order to prevent the occurrence of component faults and reduce maintenance costs, preventive maintenance planning needs to be carried out in advance according to the health status information of all the components. The contents of the planning include the component of the next group preventive maintenance and the time to start preventive maintenance.

(2) Opportunistic maintenance (OM) grouping

Component failure emerges in a maintenance group, which leads to corrective maintenance immediately. Such corrective maintenance provides additional opportunities for the preventive maintenance of other components [26]. Therefore, when a component fails, information is needed according to other components of health information, the maintenance cost of early or late maintenance, and part availability changes, as well as the system of the limitation of spare parts inventory and maintenance resources, for opportunistic maintenance planning, which is this article's key consideration in front of the opportunity maintenance factor, and the latter two are also important. However, this paper does not consider it, but takes it as the extended research direction in the future. The content of the plan includes the components of group opportunity maintenance and the time when opportunity maintenance begins.

Remark 1. *The prominent characteristics of this two-stage approach are that the group maintenance optimization of the whole machine can be carried out on the basis of the optimal maintenance interval of a single component, which can significantly reduce the maintenance downtime and the consumption of maintenance support resources. In practice, it is highly feasible and suitable for large-scale, multicomponent systems such as aircraft, railway trains, smart grids, etc.*

It should also be noted that the two-stage policy is different from the traditional cluster maintenance policy in that the service age, performance degradation degree, and maintenance sequence of each component are updated sequentially after each cluster maintenance task is completed, the execution calendar time and maintenance component sequence of the next maintenance group are arranged according to the updated component, and the dynamic iterative updating of the maintenance group is carried out continuously. We call this policy ***dynamic group maintenance with iterative information***.

3. Component-Level Static Maintenance Model

To provide basic input for dynamic, system-level group maintenance scheduling, a component-level static maintenance model was formulated, with the optimal maintenance interval being optimized analytically for each component.

3.1. Availability Modeling

The stable availability of component i is calculated by the working time of component i in a renewal cycle divided by the length of the renewal cycle; the downtime caused by component failure and the time required for maintenance activities should not be ignored [27]. Then, the working time is equivalent to the length of the update cycle minus the downtime caused by maintenance. Therefore, the stable availability of component i can be established by calculating the downtime caused by maintenance. For the perspective of preventive maintenance, since component i does not fail during the cycle, the required time for maintenance is $t_{\text{set}} + t_{i,\text{pr}}$, and the component does not experience failure downtime. Until the next maintenance is completed, component i experiences the downtime of $t_{\text{set}} + t_{i,\text{pr}}$ in the unavailable state. Therefore, the average downtime caused by preventive maintenance is:

$$t_{i,\text{prdown}} = \left(t_{\text{set}} + t_{i,\text{pr}}\right) \int_{t-t_{\text{set}}-t_{i,\text{pr}}}^{+\infty} f(x_i|\tau_i) dx_i. \tag{4}$$

As for corrective maintenance, the corresponding failure downtime is $t - t_{\text{set}} - t_{i,\text{cr}} - x_i$ with the maintenance time as $t_{\text{set}} + t_{i,\text{cr}}$. Therefore, component i experiences the downtime of $t - x_i$ in the unavailable state until the next maintenance is completed. The average downtime caused by preventive maintenance is:

$$t_{i,\text{crdown}} = \int_{0}^{t-t_{\text{set}}-t_{i,\text{cr}}} f(x_i|\tau_i)(t - x_i) dx_i. \tag{5}$$

Therefore, the stable availability of component i is given by:

$$A_i(t) = 1 - \frac{\left(t_{\text{set}} + t_{i,\text{pr}}\right)\int_{t-t_{\text{set}}-t_{i,\text{pr}}}^{+\infty} f(x_i|\tau_i)dx_i + \int_0^{t-t_{\text{set}}-t_{i,\text{cr}}} f(x_i|\tau_i)(t-x_i)dx_i}{t}. \quad (6)$$

3.2. Maintenance Cost Modeling and Optimization

The maintenance cost and stable availability of each component are related to time. When a component changes from a completely healthy state to a need for maintenance, due to the maintenance repair of the component to as new as possible, it can be considered that the health state of the component after maintenance is not related to that prior to the maintenance, so the time interval of the component from each maintenance completion to the next maintenance completion can be regarded as the maintenance interval, and the maintenance process experienced can be regarded as the renewal process. According to the update-reward theory about the renewal process, different maintenance cycles are independent of each other, and the long-term impact of maintenance is the same as that of a complete maintenance cycle. Therefore, we can use the maintenance cost rate function of a component in a cycle to represent its long-term maintenance cost [28].

The maintenance cost incurred for each component in a renewal cycle consists of:

(1) Preventive maintenance cost C_{pr}, which is the cost of preventive maintenance for the component without failure in the renewal cycle;
(2) Corrective maintenance cost C_{cr} and downtime cost C_{down}: corrective maintenance cost refers to the cost required for corrective maintenance when component failure occurs within the renewal cycle, while downtime cost refers to the additional economic loss caused by system downtime due to component failure;
(3) Component set-up and labor cost C_{set}, which is the fixed cost generated when the component is maintained.

The maintenance cost rate is then calculated by calculating the probability of preventive and corrective maintenance within the maintenance interval of the component, as well as the downtime caused by failure if it occurs. On these bases, the probability of preventive maintenance of the component is equal to the probability that it does not fail during the update cycle:

$$P_{i,\text{pr}} = \int_{t-t_{\text{set}}-t_{i,\text{pr}}}^{+\infty} f(x_i|\tau_i)dx_i. \quad (7)$$

On the contrary, the probability of corrective maintenance is the probability that failure occurs during the cycle:

$$P_{i,\text{cr}} = \int_0^{t-t_{\text{set}}-t_{i,\text{cr}}} f(x_i|\tau_i)dx_i. \quad (8)$$

If a failure occurs at time x_i, the corresponding failure downtime is $t - t_{\text{set}} - t_{i,\text{cr}} - x_i$; hence, the average failure downtime in a cycle is:

$$t_{i,\text{failuredown}} = \int_0^{t-t_{\text{set}}-t_{i,\text{cr}}} f(x_i|\tau_i)(t - t_{\text{set}} - t_{i,\text{cr}} - x_i)dx_i. \quad (9)$$

In summary, the component-level maintenance cost rate is given by:

$$\overline{C}_i(t) = \frac{P_{i,\text{pr}} \cdot C_{i,\text{pr}} + P_{i,\text{cr}} \cdot C_{i,\text{cr}} + t_{i,\text{failuredown}} \cdot C_{\text{down}} + C_{\text{set}}}{t}. \quad (10)$$

Finally, the optimal update cycle t_i^*, i.e., the optimal maintenance interval of component i, can be obtained by minimizing the maintenance cost rate:

$$t_i^* = \text{argmin}\overline{C}_i(t). \quad (11)$$

Notably, the availability constraint is not addressed currently, because it is imposed at the system-level maintenance optimization stage in the following section.

4. System-Level Dynamic Group Maintenance Model

After obtaining the component-level maintenance cost and availability model in Section 3, this section jointly schedules a preventive maintenance group and an opportunistic maintenance group with the objective of minimizing the maintenance cost under the availability constraint. To this end, we considered the changes in maintenance cost and availability caused by advanced maintenance and postponed maintenance of each component and summed up the total changes.

Regarding group maintenance, in order to save the maintenance cost C_{set} and time t_{set} shared between components, it is necessary to coordinate the maintenance plans of all the components in the maintenance group, so the current optimal maintenance interval deviates. For component i, assuming the deviated time is t_i, the cost and stable availability changes caused by the deviation are:

$$\begin{cases} \Delta\overline{C}_i(t_i) = \overline{C}_i(t_i) - \overline{C}_i(t_i^*), \\ \Delta A_i(t_i) = A_i(t_i) - A_i(t_i^*). \end{cases} \tag{12}$$

We defined the change in maintenance cost $\Delta\overline{C}_i(t_i)$ and stable availability $\Delta A_i(t_i)$ caused by the deviation of the maintenance interval time of a component as the cost and availability penalty function of a component under group maintenance. Since the group maintenance completion time t_i deviated from the optimal component-level maintenance completion time t_i^*, the maintenance cost after deviation must be greater than the original cost, and the stable availability after deviation must be less than the original availability. Therefore, the cost penalty function for maintenance cost rate was positive, and that for stable availability was negative.

Then, we defined advanced maintenance and postponed maintenance and calculated their penalty functions based on the time sequence of a component's optimal maintenance time t_i^* and deviated maintenance time t_i within the maintenance interval.

4.1. Postponed Maintenance

Delayed maintenance occurred if the deviated maintenance time t_i was later than t_i^* ($t_i^* < t_i$). According to the relationship between the failure time x_i and the maintenance time t_i/t_i^* of component i, delayed maintenance could be divided into three cases, as shown in the left part of Figure 2.

- Failure occurs within $(0, t_i^*)$

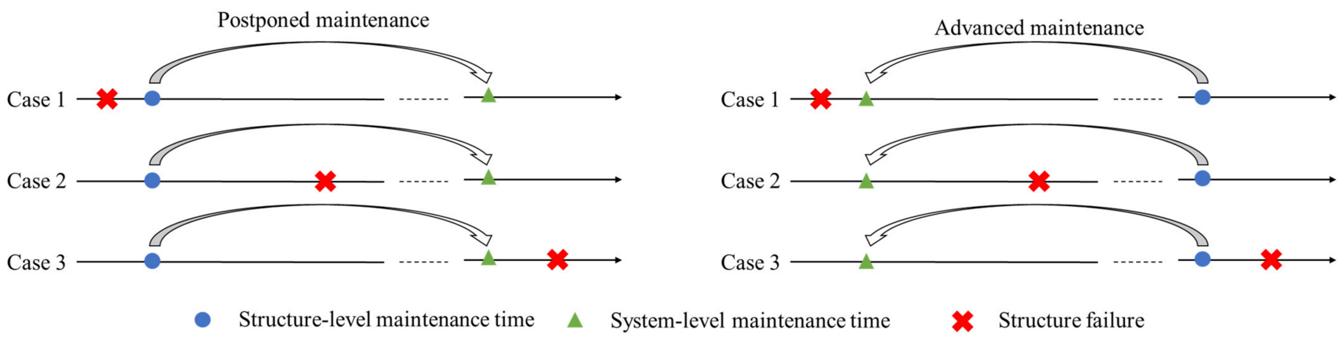

Figure 2. Illustrations of postponed maintenance (**left**) and advanced maintenance (**right**).

Due to failure occurring before the optimal maintenance time t_i^*, the maintenance type does not change, which retains corrective maintenance, so the cost and time required by the corrective maintenance are still $C_{i,\text{cr}}, C_{\text{down}}, C_{\text{set}}$ and $t_{\text{set}} + t_{i,\text{cr}}$, and the failure downtime increases as $t_i - t_i^*$. The corresponding penalty functions are:

$$\begin{aligned}\Delta\overline{C}_{i,1}(t_i) &= \frac{C_{i,\text{cr}}\cdot\int_0^{t_i^*-t_{\text{set}}-t_{i,\text{cr}}} f(x_i|\tau_i)dx_i + C_{\text{down}}\cdot\int_0^{t_i^*-t_{\text{set}}-t_{i,\text{cr}}} f(x_i|\tau_i)\left(t_i-t_{\text{set}}-t_{i,\text{cr}}-x_i\right)dx_i + C_{\text{set}}}{t_i} \\ &- \frac{C_{i,\text{cr}}\cdot\int_0^{t_i^*-t_{\text{set}}-t_{i,\text{cr}}} f(x_i|\tau_i)dx_i + C_{\text{down}}\cdot\int_0^{t_i^*-t_{\text{set}}-t_{i,\text{cr}}} f(x_i|\tau_i)\left(t_i-t_{\text{set}}-t_{i,\text{cr}}-x_i\right)dx_i + C_{\text{set}}}{t_i} \\ &= \left(\frac{1}{t_i}-\frac{1}{t_i^*}\right)\left(C_{i,\text{cr}}\cdot\int_0^{t_i^*-t_{\text{set}}-t_{i,\text{cr}}} f(x_i|\tau_i)dx_i - C_{\text{down}}\cdot\int_0^{t_i^*-t_{\text{set}}-t_{i,\text{cr}}} f(x_i|\tau_i)(t_{\text{set}}+t_{i,\text{cr}}+x_i)dx_i + C_{\text{set}}\right),\end{aligned} \tag{13}$$

and

$$\begin{aligned}\Delta A_{i,1}(t_i) &= \left(1-\frac{\int_0^{t_i^*-t_{\text{set}}-t_{i,\text{cr}}} f(x_i|\tau_i)(t_i-x_i)dx_i}{t_i}\right) - \left(1-\frac{\int_0^{t_i^*-t_{\text{set}}-t_{i,\text{cr}}} f(x_i|\tau_i)\left(t_i^*-x_i\right)dx_i}{t_i^*}\right) \\ &= \left(\frac{1}{t_i}-\frac{1}{t_i^*}\right)\int_0^{t_i^*-t_{\text{set}}-t_{i,\text{cr}}} x_i f(x_i|\tau_i)dx_i.\end{aligned} \tag{14}$$

- Failure occurs within $\left(t_i^*, t_i\right)$

Since the failure occurs between the optimal maintenance time t_i^* and the deviated maintenance time t_i, the maintenance type changes from preventive maintenance to corrective maintenance. Therefore, the cost and time required by corrective maintenance are increased from $C_{i,\text{pr}}, C_{\text{set}}$ and $t_{\text{set}} + t_{i,\text{pr}}$ to $C_{i,\text{cr}}, C_{\text{down}}, C_{\text{set}}$ and $t_{\text{set}} + t_{i,\text{cr}}$, respectively, and the failure downtime increases as $t_i - x_i$. The corresponding penalty functions are:

$$\begin{aligned}\Delta\overline{C}_{i,2}(t_i) &= \frac{C_{i,\text{cr}}\cdot\int_{t_i^*}^{t_i-t_{\text{set}}-t_{i,\text{cr}}} f(x_i|\tau_i)dx_i + C_{\text{down}}\cdot\int_{t_i^*}^{t_i-t_{\text{set}}-t_{i,\text{cr}}} f(x_i|\tau_i)\left(t_i-t_{\text{set}}-t_{i,\text{cr}}-x_i\right)dx_i + C_{\text{set}}}{t_i} \\ &- \frac{C_{i,\text{pr}}\cdot\int_{t_i^*}^{t_i-t_{\text{set}}-t_{i,\text{pr}}} f(x_i|\tau_i)dx_i + C_{\text{set}}}{t_i^*},\end{aligned} \tag{15}$$

$$\begin{aligned}\Delta A_{i,2}(t_i) &= \left(1-\frac{\int_{t_i^*}^{t_i-t_{\text{set}}-t_{i,\text{cr}}} f(x_i|\tau_i)(t_i-x_i)dx_i}{t_i}\right) - \left(1-\frac{\int_{t_i^*}^{t_i-t_{\text{set}}-t_{i,\text{pr}}} f(x_i|\tau_i)\left(t_{\text{set}}+t_{i,\text{pr}}\right)dx_i}{t_i^*}\right) \\ &= \frac{\int_{t_i^*}^{t_i-t_{\text{set}}-t_{i,\text{pr}}} f(x_i|\tau_i)\left(t_{\text{set}}+t_{i,\text{pr}}\right)dx_i}{t_i^*} - \frac{\int_{t_i^*}^{t_i-t_{\text{set}}-t_{i,\text{cr}}} f(x_i|\tau_i)(t_i-x_i)dx_i}{t_i}.\end{aligned} \tag{16}$$

- Failure occurs within $(t_i, +\infty)$

Due to failure occurring after the deviated maintenance time t_i, the maintenance type does not change, which retains preventive maintenance, so the cost and time required by preventive maintenance are still $C_{i,\text{pr}}$, C_{set} and $t_{\text{set}} + t_{i,\text{pr}}$, and failure downtime does not exist. The corresponding penalty functions are:

$$\begin{aligned}\Delta\overline{C}_{i,3}(t_i) &= \frac{C_{i,\text{pr}}\cdot\int_{t_i-t_{\text{set}}-t_{i,\text{pr}}}^{+\infty} f(x_i|\tau_i)dx_i + C_{\text{set}}}{t_i} - \frac{C_{i,\text{pr}}\cdot\int_{t_i-t_{\text{set}}-t_{i,\text{pr}}}^{+\infty} f(x_i|\tau_i)dx_i + C_{\text{set}}}{t_i^*} \\ &= \left(\frac{1}{t_i}-\frac{1}{t_i^*}\right)\left(C_{i,\text{pr}}\cdot\int_{t_i-t_{\text{set}}-t_{i,\text{pr}}}^{+\infty} f(x_i|\tau_i)dx_i + C_{\text{set}}\right),\end{aligned} \tag{17}$$

and

$$\begin{aligned}\Delta A_{i,3}(t_i) &= \left(1-\frac{\int_{t_i-t_{\text{set}}-t_{i,\text{pr}}}^{+\infty} f(x|\tau_i)\left(t_{\text{set}}+t_{i,\text{pr}}\right)dx}{t_i}\right) - \left(1-\frac{\int_{t_i-t_{\text{set}}-t_{i,\text{pr}}}^{+\infty} f(x|\tau_i)\left(t_{\text{set}}+t_{i,\text{pr}}\right)dx}{t_i^*}\right) \\ &= \left(\frac{1}{t_i}-\frac{1}{t_i^*}\right)\int_{t_i-t_{\text{set}}-t_{i,\text{pr}}}^{+\infty} f(x|\tau_i)\left(t_{\text{set}}+t_{i,\text{pr}}\right)dx.\end{aligned} \tag{18}$$

By adding the above three penalty functions, the penalty functions for postponed maintenance are:

$$\begin{cases}\Delta\overline{C}_{i,\text{postponed}}(t_i) = \sum\limits_{k=1}^{3}\Delta\overline{C}_{i,k}(t_i), \\ \Delta A_{i,\text{postponed}}(t_i) = \sum\limits_{k=1}^{3}\Delta A_{i,k}(t_i).\end{cases} \tag{19}$$

4.2. Advanced Maintenance

Advanced maintenance occurred if the deviated maintenance time t_i was earlier than t_i^* ($t_i < t_i^*$). Same as postponed maintenance, advanced maintenance could also be divided into three cases, as shown in the right part of Figure 2.

- Failure occurs within $(0, t_i)$

Due to failure occurring before the deviated maintenance time t_i, the maintenance type does not change, which retains corrective maintenance, so the cost and time required by corrective maintenance are still $C_{i,\mathrm{cr}}$, C_{down}, C_{set} and $t_{\mathrm{set}} + t_{i,\mathrm{cr}}$, and the failure downtime reduces as $t_i^* - t_i$. The corresponding penalty functions are:

$$\begin{aligned}\Delta\overline{C}_{i,4}(t_i) &= \frac{C_{i,\mathrm{cr}}\cdot\int_0^{t_i-t_{\mathrm{set}}-t_{i,\mathrm{cr}}} f(x_i|\tau_i)dx_i + C_{\mathrm{down}}\cdot\int_0^{t_i-t_{\mathrm{set}}-t_{i,\mathrm{cr}}} f(x_i|\tau_i)\left(t_i-t_{\mathrm{set}}-t_{i,\mathrm{cr}}-x_i\right)dx_i + C_{\mathrm{set}}}{t_i} \\ &- \frac{C_{i,\mathrm{cr}}\cdot\int_0^{t_i-t_{\mathrm{set}}-t_{i,\mathrm{cr}}} f(x_i|\tau_i)dx_i + C_{\mathrm{down}}\cdot\int_0^{t_i-t_{\mathrm{set}}-t_{i,\mathrm{cr}}} f(x_i|\tau_i)\left(t_i^*-t_{\mathrm{set}}-t_{i,\mathrm{cr}}-x_i\right)dx_i + C_{\mathrm{set}}}{t_i^*} \\ &= \left(\frac{1}{t_i}-\frac{1}{t_i^*}\right)\left(C_{i,\mathrm{cr}}\cdot\int_0^{t_i-t_{\mathrm{set}}-t_{i,\mathrm{cr}}} f(x_i|\tau_i)dx_i + C_{\mathrm{down}}\cdot\int_0^{t_i-t_{\mathrm{set}}-t_{i,\mathrm{cr}}} f(x_i|\tau_i)(t_i-t_{\mathrm{set}}-t_{i,\mathrm{cr}}-x_i)dx_i + C_{\mathrm{set}}\right),\end{aligned} \tag{20}$$

and

$$\begin{aligned}\Delta A_{i,4}(t_i) &= \left(1-\frac{\int_0^{t_i-t_{\mathrm{set}}-t_{i,\mathrm{cr}}} f(x_i|\tau_i)(t_i-x_i)dx_i}{t_i}\right) - \left(1-\frac{\int_0^{t_i-t_{\mathrm{set}}-t_{i,\mathrm{cr}}} f(x_i|\tau_i)\left(t_i^*-x_i\right)dx_i}{t_i^*}\right) \\ &= \left(\frac{1}{t_i}-\frac{1}{t_i^*}\right)\int_0^{t_i-t_{\mathrm{set}}-t_{i,\mathrm{cr}}} f(x_i|\tau_i)(t_i-x_i)dx_i.\end{aligned} \tag{21}$$

- Failure occurs within $\left(t_i, t_i^*\right)$

Since the failure occurs between the deviated maintenance time t_i and the optimal maintenance time t_i^*, the maintenance type changes from corrective maintenance to preventive maintenance. Therefore, the cost and time required by the preventive maintenance are reduced from $C_{i,\mathrm{cr}}$, C_{down}, C_{set} and $t_{\mathrm{set}} + t_{i,\mathrm{cr}}$ to $C_{i,\mathrm{pr}}$, C_{set} and $t_{\mathrm{set}} + t_{i,\mathrm{pr}}$, respectively, and failure downtime does not exist. The corresponding penalty functions are:

$$\begin{aligned}\Delta\overline{C}_{i,4}(t_i) &= \frac{C_{i,\mathrm{pr}}\cdot\int_{t_i}^{t_i^*-t_{\mathrm{set}}-t_{i,\mathrm{pr}}} f(x_i|\tau_i)dx_i + C_{\mathrm{set}}}{t_i} \\ &- \frac{C_{i,\mathrm{cr}}\cdot\int_{t_i}^{t_i^*-t_{\mathrm{set}}-t_{i,\mathrm{cr}}} f(x_i|\tau_i)dx_i + C_{\mathrm{down}}\cdot\int_{t_i}^{t_i^*-t_{\mathrm{set}}-t_{i,\mathrm{cr}}} f(x_i|\tau_i)\left(t_i^*-t_{\mathrm{set}}-t_{i,\mathrm{cr}}-x_i\right)dx_i + C_{\mathrm{set}}}{t_i^*},\end{aligned} \tag{22}$$

$$\begin{aligned}\Delta A_{i,4}(t_i) &= \left(1-\frac{\int_{t_i}^{t_i^*-t_{\mathrm{set}}-t_{i,\mathrm{pr}}} f(x_i|\tau_i)\left(t_{\mathrm{set}}+t_{i,\mathrm{pr}}\right)dx_i}{t_i}\right) - \left(1-\frac{\int_{t_i}^{t_i^*-t_{\mathrm{set}}-t_{i,\mathrm{cr}}} f(x_i|\tau_i)\left(t_i^*-x_i\right)dx_i}{t_i^*}\right) \\ &= \frac{\int_{t_i}^{t_i^*-t_{\mathrm{set}}-t_{i,\mathrm{cr}}} f(x_i|\tau_i)\left(t_i^*-x_i\right)dx_i}{t_i^*} - \frac{\int_{t_i}^{t_i^*-t_{\mathrm{set}}-t_{i,\mathrm{pr}}} f(x_i|\tau_i)\left(t_{\mathrm{set}}+t_{i,\mathrm{pr}}\right)dx_i}{t_i}.\end{aligned} \tag{23}$$

- Failure occurs within $\left(t_i^*, +\infty\right)$

Due to failure occurring after the optimal maintenance time t_i^*, the maintenance type does not change, which retains preventive maintenance, so the cost and time required by preventive maintenance are still $C_{i,\mathrm{pr}}$, C_{set} and $t_{\mathrm{set}} + t_{i,\mathrm{pr}}$, and failure downtime does not exist. The corresponding penalty functions are:

$$\begin{aligned}\Delta\overline{C}_{i,6}(t_i) &= \frac{C_{i,\mathrm{pr}}\cdot\int_{t_i^*-t_{\mathrm{set}}-t_{i,\mathrm{pr}}}^{+\infty} f(x_i|\tau_i)dx_i + C_{\mathrm{set}}}{t_i} - \frac{C_{i,\mathrm{pr}}\cdot\int_{t_i^*-t_{\mathrm{set}}-t_{i,\mathrm{pr}}}^{+\infty} f(x_i|\tau_i)dx_i + C_{\mathrm{set}}}{t_i^*} \\ &= \left(\frac{1}{t_i}-\frac{1}{t_i^*}\right)\left(C_{i,\mathrm{pr}}\cdot\int_{t_i^*-t_{\mathrm{set}}-t_{i,\mathrm{pr}}}^{+\infty} f(x_i|\tau_i)dx_i + C_{\mathrm{set}}\right),\end{aligned} \tag{24}$$

and

$$\begin{aligned}\Delta A_{i,6}(t_i) &= \left(1 - \frac{\int_{t_i^*-t_{\text{set}}-t_{i,\text{pr}}}^{+\infty} f(x_i|\tau_i)\left(t_{\text{set}}+t_{i,\text{pr}}\right)dx_i}{t_i}\right) - \left(1 - \frac{\int_{t_i^*-t_{\text{set}}-t_{i,\text{pr}}}^{+\infty} f(x_i|\tau_i)\left(t_{\text{set}}+t_{i,\text{pr}}\right)dx_i}{t_i^*}\right) \\ &= \left(\frac{1}{t_i} - \frac{1}{t_i^*}\right)\int_{t_i^*-t_{\text{set}}-t_{i,\text{pr}}}^{+\infty} f(x_i|\tau_i)\left(t_{\text{set}} + t_{i,\text{pr}}\right)dx_i.\end{aligned} \tag{25}$$

By adding the above three penalty functions, the penalty functions for advanced maintenance are:

$$\begin{cases} \Delta\overline{C}_{i,\text{advanced}}(t_i) = \sum\limits_{k=4}^{6} \Delta\overline{C}_{i,k}(t_i), \\ \Delta A_{i,\text{advanced}}(t_i) = \sum\limits_{k=4}^{6} \Delta A_{i,k}(t_i). \end{cases} \tag{26}$$

4.3. Preventive Dynamic Grouping

Assuming that the system completes the j^{th} maintenance at time t^j for group G_i, that the components of the last group maintenance are defined as G_k, that the maintenance time of component i in the j^{th} group maintenance is t_i^j, and that the cumulative working time is τ_i^j, then:

(1) If component i is maintained at the $j-1^{\text{th}}$ group maintenance, the optimal maintenance completion time of component i in the j^{th} group maintenance is $t_i^j = t^{j-1} + t_i^*$, and the age of component i is approximately $\tau_i^j \approx t^j - t^{j-1}$;

(2) Otherwise, if component i is not maintained at the $j-1^{\text{th}}$ group maintenance, with the initial conditions of $t_i^1 = t_i^*$, $t^0 = 0$, and $\tau_i^0 = 0$, the optimal maintenance completion time of component i in the j^{th} group maintenance is $t_i^j = t_i^{j-1}$, and the age of component i is approximately $\tau_i^j \approx \tau_i^{j-1} + t^j - t^{j-1}$.

When the system is scheduled for group maintenance, the penalty functions caused by deviation in the maintenance time lead to the shared maintenance cost C_{set} and time t_{set}. Therefore, when the system completes the j^{th} group maintenance of component group G_i, the maintenance cost rate and stable availability of the component group G_i change functions are:

$$\Delta\overline{C}_{G_i}\left(t^j\right) = \sum_{i\in G_i}\begin{cases} \Delta\overline{C}_{i,\text{advance}}\left(t^j\right), t^j < t_i^* \\ \Delta\overline{C}_{i,\text{postpone}}\left(t^j\right), t^j > t_i^* \end{cases} - \frac{(|G_i|-1)C_{\text{set}}}{t^j}, \tag{27}$$

and

$$\Delta A_{G_i}\left(t^j\right) = \prod_{k\notin G_i} A_k(t_k^*)\left(\prod_{i\in G_i}\left(A_i(t_i^*) + \begin{cases} \Delta A_{i,\text{advance}}\left(t^j\right), t^j < t_i^* \\ \Delta A_{i,\text{postpone}}\left(t^j\right), t^j > t_i^* \end{cases}\right) - \prod_{i\in G_i} A_i(t_i^*)\right) + \frac{(|G_i|-1)t_{\text{set}}}{t^j}, \tag{28}$$

where $|G_i|$ represents the component number of group G_i.

By minimizing the maintenance cost rate change function $\Delta\overline{C}_{G_i}\left(t^j\right)$ of group G_i under an availability constraint, the optimal time t^{j*} for completing the j^{th} group maintenance can be obtained:

$$\begin{aligned} &t^{j*} = \operatorname{argmin}\Delta\overline{C}_{G_i}\left(t^j\right), \\ &s.t.\Delta A_{G_i}\left(t^j\right) \geqslant \Delta A_{\text{control}}, \end{aligned} \tag{29}$$

where $\Delta A_{\text{control}}$ represents the **availability constraint**. For example, if the stable availability of group maintenance is required to be more than the stable availability of individual maintenance, the availability constraint is $\Delta A_{\text{control}} = 0$, and if the stable availability of the system is required to be greater than a certain value A_{L}, the availability constraint is $\Delta A_{\text{control}} = A_{\text{L}} - \prod\limits_{k\notin G_i} A_k\left(t_k^*\right) \prod\limits_{i\in G_i} A_i\left(t_i^*\right)$.

While determining the optimal time t^{j*} to complete group maintenance, we also needed to determine the optimal group G_i^* of maintenance. Here, we used a continuous-grouping structure for optimization. The advantage of this structure is that it can be optimized by dynamic programming, thereby reducing the complexity of optimization. In addition, previous studies have proved that dynamic programming grouping methods can not only optimize current maintenance benefits, but can also be the most suitable for long-term maintenance [29].

It is assumed that continuous grouping $GS = \{G_1, G_2\}$ is a full division of the critical components of a system, in which $G_1 \cap G_2 = \oslash$ and $G_1 \cup G_2 = N_{\mathrm{T}} \cup N_{\mathrm{D}}$, and each component is numbered sequentially $\{1, 2, \cdots, n\}$ according to the cumulative working time $\tau_1^j > \tau_2^j > \cdots > \tau_n^j$. Groups $G_1 = \{1, 2, \cdots, k\}$ and $G_2 = \{k, k+1, \cdots, n\}$ are continuous, numbered groups, and the optimal preventive maintenance group G_1^* for the j^{th} group maintenance is obtained by increasing the components in G_1 from no component to all components and minimizing the change in maintenance cost rate under an availability constraint:

$$\begin{aligned} &G_1^* = \operatorname{argmin} \Delta \overline{C}_{G_1}\left(t^{j*}\right), \\ &s.t. \Delta A_{G_1^*}\left(t^j\right) \geqslant \Delta A_{\text{control}}. \end{aligned} \tag{30}$$

If no failure occurred before the next group maintenance, G_1^* and t^{j*} were carried out as planned for the next preventive maintenance; otherwise, if a component failure occurred, it was necessary to replan the maintenance.

4.4. Opportunistic Dynamic Grouping

If a component failure occurs at time t_{f} before next preventive group maintenance, opportunistic group maintenance is performed immediately. Moreover, if the optimal component-level maintenance completion time t_i^* in group G_i^* is before t_{f} but the group maintenance completion time t^{j*} is after t_{f}, then component i is considered to be included in the opportunistic maintenance group G_{OM}. Assuming that the cumulative working time of these components is $\tau_i, i \in G_{\mathrm{OM}}$, the maintenance cost rate and stable availability of these components are:

$$\Delta \overline{C}_{G_{\mathrm{OM}}}(t_{\mathrm{f}}) = \sum_{i \in G_{\mathrm{OM}}} \begin{cases} \Delta \overline{C}_{i,\text{advance}}(t_{\mathrm{f}}), t_{\mathrm{f}} < t_i^* \\ \Delta \overline{C}_{i,\text{postpone}}(t_{\mathrm{f}}), t_{\mathrm{f}} > t_i^* \end{cases} - \frac{(|G_{\mathrm{OM}}| - 1)C_{\text{set}}}{t_{\mathrm{f}}}, \tag{31}$$

$$\Delta A_{G_{\mathrm{OM}}}(t_{\mathrm{f}}) = \prod_{k \notin G_{\mathrm{OM}}} A_k(t_k^*) \left(\prod_{i \in G_{\mathrm{OM}}} \left(A_i(t_i^*) + \begin{cases} \Delta A_{i,\text{advance}}(t_{\mathrm{f}}), t_{\mathrm{f}} < t_i^* \\ \Delta A_{i,\text{postpone}}(t_{\mathrm{f}}), t_{\mathrm{f}} > t_i^* \end{cases} \right) - \prod_{i \in G_{\mathrm{OM}}} A_i(t_i^*) \right). \tag{32}$$

According to the preventive maintenance dynamic-grouping method in Section 4.3, the optimal opportunity maintenance group G_{OM}^* is given by:

$$\begin{aligned} &G_{\mathrm{OM}}^* = \operatorname{argmin} \Delta \overline{C}_{G_{\mathrm{OM}}}(t_{\mathrm{f}}), \\ &s.t. \Delta A_{G_{\mathrm{OM}}}(t_{\mathrm{f}}) \geqslant \Delta A_{\text{control}}. \end{aligned} \tag{33}$$

We used this dynamic method for each maintenance group scheduling. If there was no component failure before the next group maintenance, the maintenance was carried out according to the planning content of the preventive maintenance dynamic grouping. Otherwise, if a component failure occurred, we could use the opportunistic maintenance dynamic-grouping method to replan the maintenance. After the completion of each maintenance, the health information of all the components was updated, and we replanned the component group and time of the next maintenance according to the updated component health information, which reflected the **dynamic feature** of our method.

5. Numerical Experiment

In this section, we verify the proposed maintenance strategy and model through an aircraft structural system maintenance project.

In the manufacturing of aircraft, aluminum alloy materials are mainly used to produce the fuselage, wings, tail, panels, and skin of aircraft. Taking an aircraft panel as an example, due to the influence of adverse factors, such as alternating stress, material aging, environmental corrosion, and manufacturing defects on the aircraft panel, structural damage inevitably occurs in the use of aircraft panels, among which fatigue cracks are the most serious. The fatigue cracks originate from fine cracks caused by manufacturing defects or external damage. The size of a crack continues to expand under cyclic loading. When a crack size expands to critical, the strength of the panel is insufficient to support the load, resulting in the rapid expansion of the crack size and fracture, as shown in the left part of Figure 3. Due to technical constraints, the crack length cannot be monitored online; particularly, the crack size of an aircraft panel can only be disassembled and measured when an aircraft is returned to the factory. Therefore, in the daily maintenance of aircraft, it is regarded as a lifetime structure.

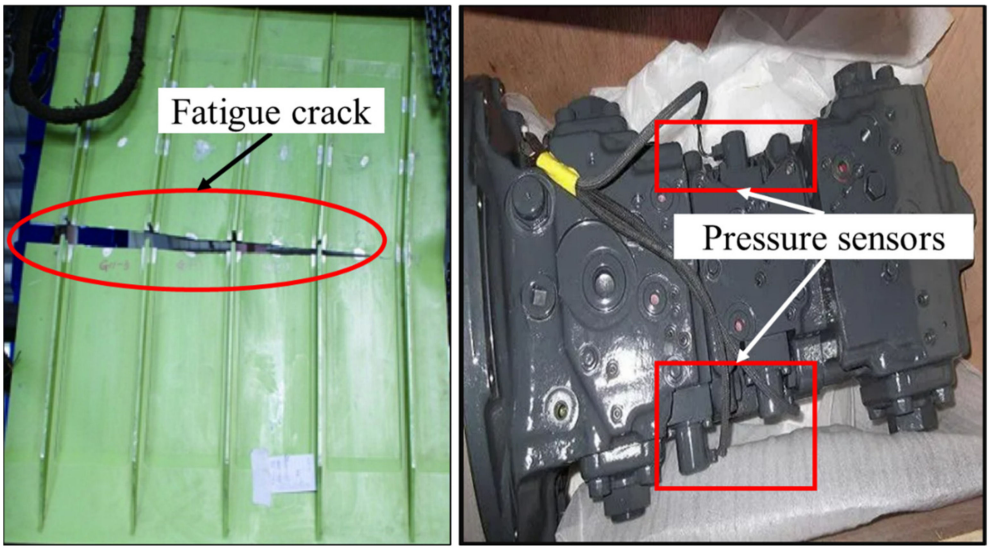

Figure 3. Crack in aircraft panel (**left**) and pressure sensor of hydraulic pump (**right**).

For a panel, since the load in each flight causes the fatigue crack to expand and the expansion size of the fatigue crack cannot be known in daily maintenance, the load in each flight can be regarded as a Poisson shock. This shock leads to the expansion of the crack size, and its accumulation leads to the fracture of the panel. Therefore, the life distribution of the panel can be regarded as the Gamma distribution accumulated by multiple Poisson shocks.

On the other hand, a hydraulic pump is an important component in a hydraulic system and an important guarantee for the normal operation of the entire hydraulic system. In the actual work of the hydraulic pump, with the increase in the number of operations of the components, the components are subjected to the extrusion of liquid, and friction occurs with the contact surface, resulting in large, nonlinear deformation. At this time, the size and shape of the sealing gap also change, and the sealing of the hydraulic pump gradually decreases, resulting in liquid leakage and even more serious consequences. Hydraulic pump failure is usually caused by the pressure drop caused by hydraulic oil leakage, which can be monitored online by pressure sensors, as shown in the right part of Figure 3. By setting a critical threshold of pressure, we can take the pressure value in the hydraulic pump as the performance degradation value that characterizes the degradation of the hydraulic pump. Therefore, we can regard the hydraulic pump as a degradation structure.

Since the pressure value of a hydraulic pump has a small range of fluctuation while it is degraded, a linear Wiener process with Brownian drift motion can well-fit this degradation process. Therefore, we selected this as the failure and degradation models used in our paper, and these two models have also been adopted in previous studies [30,31].

In this study, we considered a structure group consisting of five airborne hydraulic pumps and three aircraft panels. According to the data provided by the operation and maintenance department of the airline, the former obeyed a linear Wiener process with Brownian motions, and the latter obeyed a Gamma distribution. Here we used #1–3 to represent the airborne hydraulic pumps and #4–8 to represent the aircraft panels. The degradation or failure, as well as the maintenance parameters, of each structure are shown in Table 1. The failure or degradation and cost data we used are all from the records (including fault time, running deadline, degradation monitoring, maintenance records, etc.) of the operation and maintenance department of the airline, and the failure and degradation parameters were obtained using maximum likelihood estimation. Some adjustments were made to these parameters to increase the difference between the structures' degradation and failure, which did not affect whether the proposed framework was correct and effective.

Table 1. Degradation and failure parameters and maintenance cost of all structures.

<table>
<tr><th>Structure</th><th>D_F</th><th>ν</th><th>σ</th><th>α</th><th>β</th><th>t_{pr} (Hour)</th><th>t_{cr} (Hour)</th><th>t_{set} (Hour)</th><th>C_{pr} (CNY)</th><th>C_{cr} (CNY)</th><th>C_{down} (CNY/Hour)</th><th>C_{set} (CNY)</th></tr>
<tr><td>1</td><td>25</td><td>0.481</td><td>0.407</td><td>-</td><td>-</td><td>1</td><td>6</td><td rowspan="8">3</td><td>50</td><td>1000</td><td rowspan="8">100</td><td rowspan="8">200</td></tr>
<tr><td>2</td><td>20</td><td>0.573</td><td>0.412</td><td>-</td><td>-</td><td>1.5</td><td>12</td><td>56</td><td>1120</td></tr>
<tr><td>3</td><td>15</td><td>0.632</td><td>0.428</td><td>-</td><td>-</td><td>2</td><td>15</td><td>100</td><td>2000</td></tr>
<tr><td>4</td><td></td><td>-</td><td>-</td><td>1.321</td><td>0.00162</td><td>1.5</td><td>12</td><td>80</td><td>1600</td></tr>
<tr><td>5</td><td></td><td>-</td><td>-</td><td>1.682</td><td>0.00454</td><td>2</td><td>15</td><td>70</td><td>1400</td></tr>
<tr><td>6</td><td></td><td>-</td><td>-</td><td>1.052</td><td>0.00357</td><td>1</td><td>9</td><td>70</td><td>1400</td></tr>
<tr><td>7</td><td></td><td>-</td><td>-</td><td>0.883</td><td>0.00275</td><td>1.5</td><td>12</td><td>40</td><td>800</td></tr>
<tr><td>8</td><td></td><td>-</td><td>-</td><td>0.902</td><td>0.00121</td><td>2</td><td>15</td><td>60</td><td>1200</td></tr>
</table>

5.1. Component-Level Maintenance Performance

Based on the data in Table 1 combined with the component-level static maintenance policy proposed in Section 3, the variations in the maintenance cost rate and stable availability of these structures with the maintenance interval are shown in Figure 4, and the optimal preventive maintenance intervals for the minimum maintenance cost rate and maximum stable availability are shown in Table 2.

Table 2. Optimal maintenance intervals for minimum maintenance cost rate and maximum availability.

Structure	Optimal Preventive Maintenance Interval for Minimum Maintenance Cost Rate (Hour)	Minimum Maintenance Cost Rate (CNY/Hour)	Optimal Preventive Maintenance Interval for Maximum Stable Availability (Hour)	Maximum Stable Availability
1	49	5.313	53	0.931
2	42	5.597	44	0.965
3	35	6.231	36	0.989
4	86	5.705	108	0.928
5	66	6.205	84	0.909
6	42	12.306	52	0.862
7	38	10.929	52	0.838
8	58	8.180	83	0.883

It can be seen from Figure 4 and Table 2 that, for the degradation-based failure structures (#1–3), the optimal preventive maintenance intervals corresponding to the minimum maintenance cost rate and the maximum stable availability were almost equal. However, for the age-based failure structures (#4–8), the optimal preventive maintenance intervals corresponding to the minimum maintenance cost rate and the maximum stable availability were quite different, but the change in the preventive maintenance interval from the latter to the former did not lead to a large reduction in the stable availability, so the optimal preventive maintenance interval at this time was acceptable.

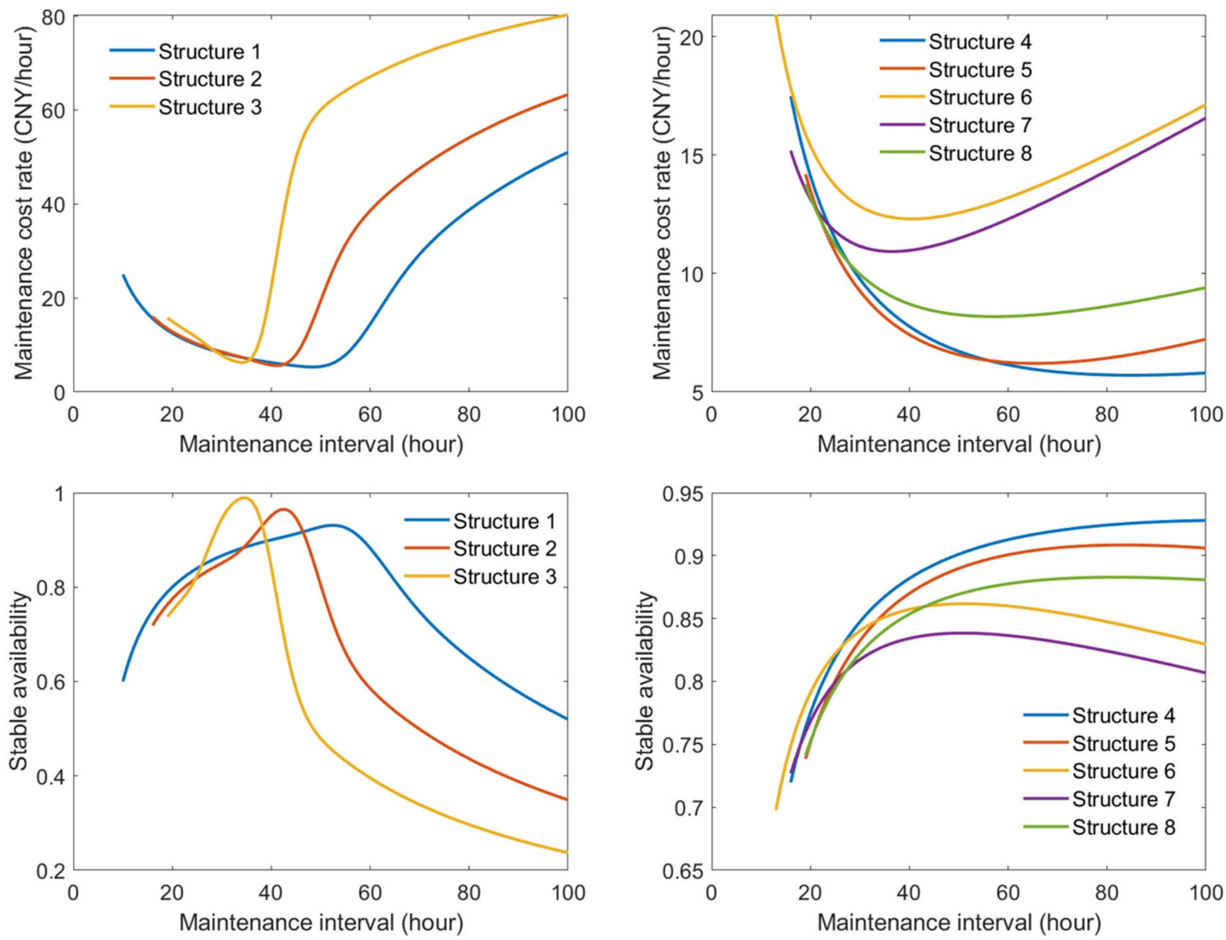

Figure 4. Variations in maintenance cost rate and stable availability with maintenance interval.

5.2. System-Level Dynamic Maintenance Schedule

According to the system-level dynamic preventive maintenance policy proposed in Section 4.3, we could obtain the optimal PM schedule in a time cycle (here, we set the time cycle to 100 h and the availability constraint was $\Delta A_{\mathrm{control}} = 0$) for each group, containing the completion of the maintenance structure, the group maintenance completed time, the maintenance cost rate reduction, and the stable availability improvement, as shown in Table 3 and the left part of Figure 5.

Table 3. Optimal system-level PM plans under $\Delta A_{\mathrm{control}} = 0$ in 100 h.

Maintenance Order	Maintenance Type	Maintenance Group	Maintenance Completion Time	Cost Rate Reduction (CNY/Hour)	Relative Proportion	Availability Improvement	Relative Proportion
1	PM	{3,7}	35	5.686	33%	0.083	4.5%
2	PM	{1,2,6}	42	9.727	42%	0.136	4.9%
3	PM	{8}	58	0	0	0	0
4	PM	{3,5,7}	70	5.661	24%	0.073	2.7%
5	PM	{2,4,6}	84	4.762	20%	0.058	2.1%
6	PM	{1}	92	0	0	0	0

If the structure failed during the cycle (here, we assumed that structure #3 failed at $t = 50$ h), the maintenance plan could be adjusted according to the opportunity maintenance policy in Section 4.4, and the adjusted PM+OM schedules are shown in Table 4 and the right part of Figure 5. From Tables 3 and 4, it shows that, after the dynamic group maintenance policy was applied to the critical structure of the aircraft, we could reasonably combine the maintenance activities of multiple structures to share the fixed maintenance cost and maintenance time. At the same time, maintenance work on multiple structures was performed concurrently, thereby reducing the maintenance cost rate and improving the stable availability of the critical structures in aircraft. The maintenance cost rate reduction based on the proposed policy was between 0 and 9.727 CNY/hour, and cost savings accounted for the original cost up to 42%. The availability improvement was between 0 and 0.136, which was up to 4.9% higher than the original availability. Moreover, the number of maintenance shutdowns was shortened from 11 times at the component level to 6 times at the system level, which proved the effectiveness of the proposed policy.

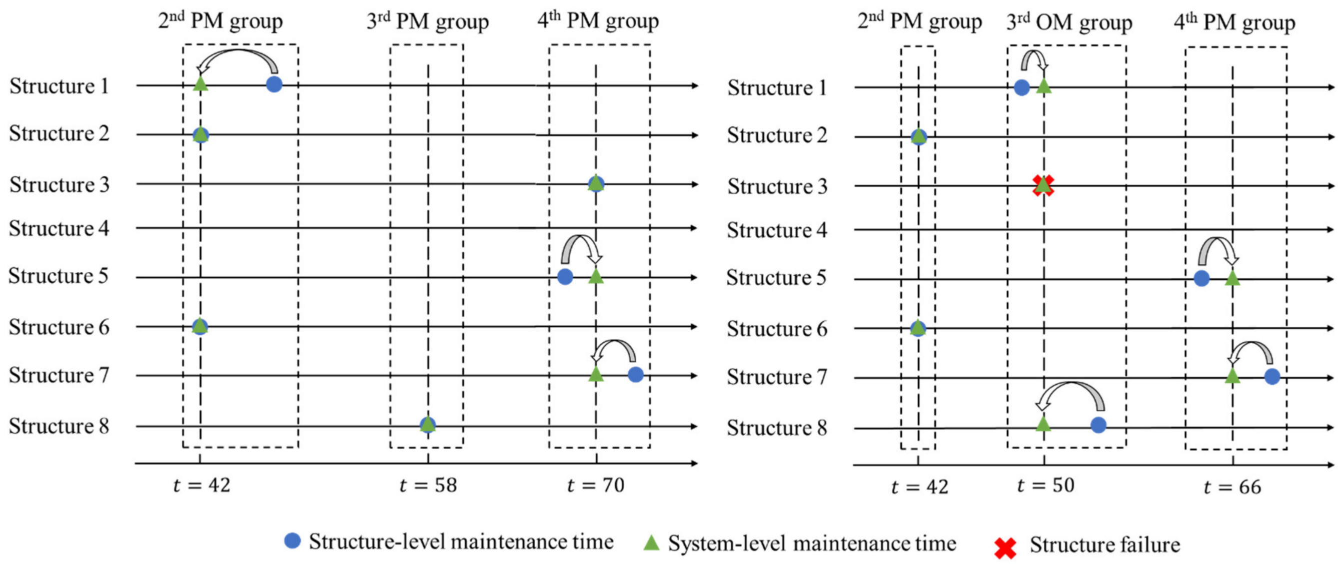

Figure 5. Illustrations of PM grouping (**left**) and PM+OM grouping (**right**).

Table 4. Adjusted system-level PM+OM plans under $\Delta A_{\text{control}} = 0$ in 100 h.

Maintenance Order	Maintenance Type	Maintenance Group	Maintenance Completion Time	Cost Rate Reduction (CNY/Hour)	Relative Proportion	Availability Improvement	Relative Proportion
1	PM	{3,7}	35	5.686	33%	0.083	4.5%
2	PM	{2,6}	42	4.762	27%	0.071	3.9%
3	OM	{1,3,8}	50	7.838	40%	0.115	4.1%
4	PM	{5,7}	66	3.030	18%	0.041	2.3%
5	PM	{2,3,4,6}	84	6.966	23%	0.100	2.7%
6	PM	{1,7}	99	3.981	25%	0.020	1.1%

6. Conclusions

This study proposed an intelligent group maintenance framework for series-connected multicomponent systems that was capable of dynamically and iteratively updating all the component health information. Compared to traditional static maintenance frameworks, our dynamic framework had three advantages. First, it was capable of automatically and sequentially updating the health information for all the components, which enabled the robustness of maintenance decision making. Second, both PM and OM scheduling were integrated in a unified analytical framework, which significantly reduced downtime and relative loss. Third, it could jointly ensure the operational profitability and availability of aircraft by sequentially optimizing the maintenance grouping approach. The superior performance of the proposed approach in cost control and availability insurance compared with conventional static and periodic maintenance approaches was demonstrated through numerical experiments.

Under the current maintenance framework, there are five possible extensions. First, complex systems with multiple dependent failure modes are worth investigation. Second, imperfect maintenance, including imperfect repair and imperfect inspection, can be incorporated into the current framework, which can enable more maintenance selections. Third, methods of modeling the availability of nonseries systems with redundant and parallel components need to be studied to extend this maintenance framework to such systems. Fourth, not only are the availability of the system and its maintenance cost critical factors, but also reliability and other maintenance resources such as the spare parts of a system are essential, so reliability and maintenance resources can be further considered in this framework. Fifth, the maintenance cost and time we currently used can be further divided and combined with other maintenance scenarios.

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