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Application of Monte Carlo Simulation to Study the Probability of Confidence Level under the PFMEA's Action Priority

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Abstract: Failure mode and effects analysis (FMEA) is the most commonly used risk evaluation tool in industry and academia. After four revisions, the US Automotive Industry Action Groups (AIAG) and German Association of the Automotive Industry (VDA) issued the latest FMEA manual, called AIAG and VDA FMEA Handbook Edition 1, in June 2019. Risk priority number (RPN) in the old-edition FMEA is replaced with action priority (AP), where the numerical evaluation of severity (S), occurrence (O), and detection (D) are referred to in the AP form for judging high (H), medium (M), and low (L) priority in order to ensure appropriate actions for improving prevention or detection control. When evaluating design (D) or process (P) in FMEA, the FMEA team has to refer to the evaluation criteria for S, O, and D, so as to reduce the difference in the evaluation reference and fairness. Since the criteria evaluation form is the qualitative rating standard with semantic judgment, evaluation errors are likely to occur when the team judges S, O, and D. The FMEA cases in this study are preceded by the confidence level (CL) of the S, O, and D evaluation standards and the setting of a confidence interval (CI) for the actual evaluation events. With discrete nonuniform distribution as the simulation setting, Monte Carlo simulation is applied several times to evaluate the probability before and after the evaluation, which is compared with the AP form to confirm the probability values of high, medium, and low priority. It provides reference for the FMEA cross-functional team, improving the originally non-AP events. Finally, the AP calculated in the simulation is compared and analyzed with the RPN sequence to verify the judgment of better actions with AP.

Keywords: failure mode and effects analysis; action priority; Monte Carlo simulation; confidence interval; confidence level

MSC: 90B25; 90-08

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1. Introduction

Failure mode and effects analysis (FMEA), the most common risk evaluation tool in business [1], is a risk evaluation tool to increase reliability. It functions to determine potential failure modes, discuss the effect of failure on systems with proper evaluation, and adopt necessary correction actions and prevention strategies aiming at the failure of system reliability [2]. International Automotive Task Force (IATF) 16949: 2016 is a technical specification aimed at the development of a quality management system that provides for continual improvement, emphasizing defect prevention and the reduction in the variation and waste in the automotive industry supply chain and assembly process. Especially, when promoting IATF 16949: 2016 in the automotive supply chain, the technical regulations stress design, planning, and production, which are constrained on the strictest clauses. Further, each stage is analyzed according to the error type; the failure modes in design, process, and logistics; and the effects of design, process, and logistics with interdisciplinary technical regulations [3]. Razouk and Kern [4] pointed out the improvement of quality with FMEA, aiming to improve process quality in advance, provide good prevention in quality control, and maintain manufacturing technology to advance product quality. Therefore, FMEA is often applied to product design or the quality improvement of process and security analysis. To achieve customer satisfaction, FMEA also analyzes, identifies, confirms, and eliminates known failure, system potential problems or errors, and design, process, or service. FMEA aims to identify potential failure modes and evaluate the cause of failure and various failure effects [5].

FMEA has been broadly applied to eliminate known or potential failure of product design and process and enhance product or system reliability and security, but there are uncertainties after the numerical evaluation of severity (S), occurrence (O), and detection (D). Tong et al. [6] indicated that the data collection and parameter selection in FMEA used to depend too much on engineers' past experience and knowledge—the research method merely solved specific problems, appeared to be too restricted, and did not conform to real conditions. The proposed problems therefore might result in research results inconsistent with the real conditions. As a result, the evaluation of S, O, and D would change with cross-functional teams. The judgment of high (H), medium (M), and low (L) with action priority (AP) form in the new-edition AIAG (Automotive Industry Action Groups) and German Association of the Automotive Industry (German: Verband der Automobilindustrie, VDA) *FMEA Handbook* [7] might give different results. Moreover, the old-edition AIAG FMEA (2008) considered the evaluation of S, O, and D, and then judged the multiple result of S, O, and D; even the risk priority number (RPN) standards would be argued internally regarding an enterprise. The *AIAG and VDA FMEA Handbook* [7] also showed that the AP method provides 1000 possible combinations of S, O, and D. The method focuses first on severity, next on frequency, and finally on detection. The risk priority number (RPN) is the product of S x O x D and ranges from 1 to 1000. The RPN can provide some information about risk, but RPN alone does not determine whether more action is needed because RPN weights S, O, and D equally. Therefore, the RPN may have similar risk numbers for different combinations of S, O, and D. Managers or improving teams are unsure on how to prioritize failures. Thus, it is recommended to use other methods to prioritize similar RPN results. Tang et al. [8] mentioned that with the higher RPN (e.g., 100), immediate improvement actions should be adopted; when severity appeared 9 ~ 10, regardless of RPN, immediate actions were necessary. Yang [9] indicated that, regardless the calculation methods of RPN, using specific RPN as the improvement threshold was far from the intention of FMEA, which is to prevent deficiencies, and did not conform to the FMEA rule. Single systems of H, M, and L in AP were proposed in the 2019 new-edition FMEA to judge H, M, and L by referring to the AP form, so as to avoid RPN in the previous edition of FMEA. By adopting this improvement, enterprises enhanced RPN improvement standards.

Monte Carlo simulation is a decisive module with repeated evaluation. The input is a set of random numbers [10,11]. Such a method is often used for complicated evaluation, nonlinear or more than two uncertain parameters. For representativeness, a model can contain a simulation that is evaluated more 10,000 times. Monte Carlo simulation requires a large quantity of random numbers developed with random number generators, which is largely suitable for simulating a single event [12]. Monte Carlo simulation can be used to study random phenomena with large virtual experiments and computer-generated random numbers, explain the principles of random numbers with various probability distributions and the generated number, and simulate more complicated situations, e.g., the generation of a response surface and relevant random numbers [13]. Monte Carlo simulation can estimate quality and reliability, which can be used for calculating and estimating the S, O, and D evaluated by the FMEA cross-functional team, and then determines the corresponding possible probability of H, M, and L, referring to AP evaluation. When the FMEA cross-functional team evaluates S, O, and D in an event process, the evaluation is a qualitative rating that indicates the errors that may occur. Confidence level (CL) is therefore introduced for evaluating errors for the confirmation reference of probability. During the estimation in the study, an interval is used for setting the Monte Carlo simulation. This interval is the confidence interval (CI) and the actual probability within CI is called CL. Hsu and Lee [14] indicate that applying a 95% CL to the survey reveals that the error probability of the study is smaller than 5%.

Aiming at the method described in the *AIAG and VDA FMEA Handbook* [7], S, O, and D are evaluated with the setting of CL; each CI is set in the Monte Carlo simulation with a discrete nonuniform distribution as the simulation. The simulation numbers generated are evaluated with H, M, and L in AP form so that the probability of the research result can be actually applied to industries for reference. No scholars have proposed the idea of AP probability evaluation in FMEA that focus on the evaluation probability of CI for successive analyses. However, this would organize new research methods and statistical calculation for the application to AP probability evaluation in the new-edition FMEA. Furthermore, AP probability, with the combination of simulation calculation results, is compared and analyzed with the RPN sequence. These are initial studies.

2. Literature Review

2.1. Failure Mode and Effects Analysis (FMEA)

Failure mode and effects analysis (FMEA) can be a tool for prevention and reliability analysis [15]. Ebrahimipour et al. [16] indicated that the National Aeronautics and Space Administration (NASA) announced the reliability program "NPC 250-1" in 1963 for the promotion of space research, in which FMEA was used. It was then broadly applied to the evaluation of reliability and system security, and firms were required to apply FMEA to examine the design. Since national defense and the aerospace industry emphasize system security and reliability, three major motor companies in the US collaboratively established the FMEA standards with the US Automotive Industry Action Group and American Society for Quality in 1993. At that time, J1739 was initially announced as the first-edition FMEA. FMEA was then applied to the stages of product development and design and product process control to ensure the importance in the development process of the automotive industry [17].

Lin and Chung [18] and Rezaie et al. [19] indicated that the benefit of FMEA was affirmed and gradually changed for testing system security and reliability with the application from national defense and aerospace industry to general industries. The Automotive Industry Action Group (AIAG) and German Association of the Automotive Industry (VDA) revised the first edition FMEA [20] to its fourth edition *FMEA Handbook* [7], published in 2019. The FMEA development process is referred to in Figure 1.

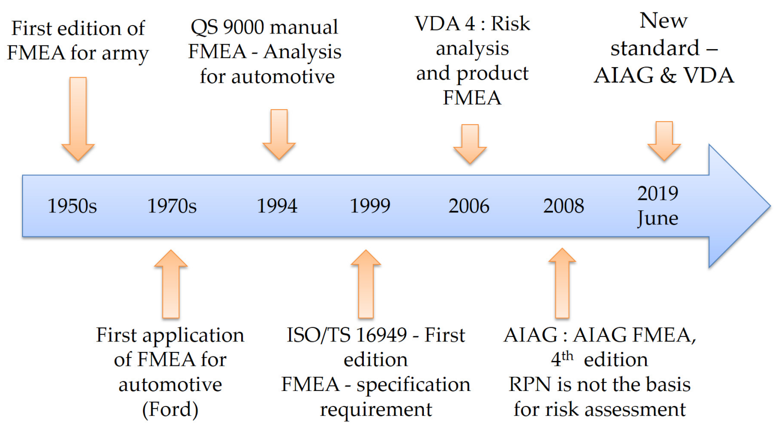

Figure 1. Evolution of the approach to FMEA. Data source: Dariusz et al. [21].

Regarding the risk evaluation in FMEA, RPN was the reference for evaluating risks in the old-edition FMEA before 2019; after several revisions, it was replaced by AP. The problems in RPN were pointed out in past literature. Xu et al. [22] considered that the traditional FMEA needed to evaluate product or system risks through RPN by calculating the multiple of S, O, and D, which did not have a definite linear relationship. Pillay and Wang [23] also pointed out the distinct importance of the three numbers of traditional RPN, which should not be evaluated the same numerically. From this literature, the calculation deficiency of RPN in the old-edition FMEA was discovered. To have the calculated numbers closer to the actual conditions, different methods were studied for verifying and improving the evaluation method. For instance, Mohamed and Aminah [24] analyzed RPN of FMEA using fuzzy and AHP methods. Warren [25] explained that it was necessary to collect a large amount of information for the execution of FMEA, and the information source was the past experience and accumulated knowledge of scholars. However, those were personal subjective opinions and traditional FMEA could not easily convert subjective opinions into definite quantitative data; the information acquired could not necessarily be used to respond to actual problems. Warren [25] pointed out certain errors in subjective judgment of S, O, and D; collecting a large amount of information would be needed to correct or evaluate errors.

Aiming at setting a CL with the evaluation of S, O, and D to evaluate probability, the evaluation of RPN was changed into AP evaluation after the revision in 2019. There was scant research on AP and its application. For example, Frunza et al. [26] mentioned the replacement of RPN by AP as an evaluation tool. Anackovski et al. [27] explained the current implementation situation of AP, particularly stressing the priority ordering of resources for reducing the most dangerous risks. AP prevented organizations and teams from solving RPN numerical problems. For example, RPN $\geqq$ 100 showed the need for improvement, regardless of the combined seriousness. Currently, the latest FMEA edition completely solves the trouble with RPN numbers by starting with improvement actions. The conversion table of S, O, D numbers in the new-edition design (D)/process (P) in FMEA and H, M, L in AP is shown in Table 1.

Table 1. D/P FMEA S, O, and D of AP.

S	O	D	AP	S	O	D	AP	S	O	D	AP	S	O	D	AP
9–10	8–10	7–10	H	7–8	8–10	7–10	H	4–6	8–10	7–10	H	2–3	8–10	7–10	M
		5–6	H			5–6	H			5–6	H			5–6	M
		2–4	H			2–4	H			2–4	M			2–4	L
		1	H			1	H			1	M			1	L
	6–7	7–10	H		6–7	7–10	H		6–7	7–10	M		6–7	7–10	L
		5–6	H			5–6	H			5–6	M			5–6	L
		2–4	H			2–4	H			2–4	M			2–4	L
		1	H			1	M			1	L			1	L
	4–5	7–10	H		4–5	7–10	H		4–5	7–10	M		4–5	7–10	L
		5–6	H			5–6	M			5–6	L			5–6	L
		2–4	H			2–4	M			2–4	L			2–4	L
		1	M			1	M			1	L			1	L
	2–3	7–10	H		2–3	7–10	M		2–3	7–10	L		2–3	7–10	L
		5–6	M			5–6	M			5–6	L			5–6	L
		2–4	L			2–4	L			2–4	L			2–4	L
		1	L			1	L			1	L			1	L
	1	1–10	L		1	1–10	L		1	1–10	L		1	1–10	L
												1	1–10	1–10	L

Data source: AIAG and VDA [7].

2.2. Monte Carlo Simulation

Monte Carlo simulation is a model that makes decisions with repeated evaluation, with the input being a set of random numbers [10]. Such a method is often used for comparatively complicated evaluation, nonlinearity or parameters with more than two uncertainties. For representativeness, a simulation model can contain more than 10 million simulation evaluations [11]. Monte Carlo simulation originated from "statistical sampling". Malvin and Paula [28] indicate that Stanislaw Ulam et al. invented Monte Carlo simulation for the nuclear weapon project at the Los Alamos National Laboratory in 1940s, when Stanislaw Ulam's uncle discovered a repeated and similar probability model in a famous casino in Monaco and named it after the capital of Monaco, Monte Carlo.

The calculation of random numbers and experiment can be traced back to Buffon's needle of Georges-Louis Leclerc and Comte deBuffon, in the early 18th century. After the invention of electronic computers, Monte Carlo simulation was then studied [29]. Monte Carlo simulation defined a probability density function (PDF) for all possible probabilities and accumulated the PDF to a cumulative probability function and adjusted the maximal value 1, termed normalization, to correctly respond to the probability characteristics of all events that appear, resulting in a total probability of 1. It also connected random number sampling and real problem simulation [30]. Yeh and Sun [12] mention that computer-generated random numbers evenly distributed in [0, 1] can simulate the possible reliability, tolerance, and CI through the input of a simulated probability distribution function, Figure 2.

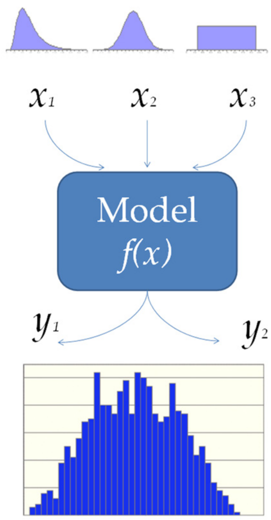

Figure 2. Outputting reliability, tolerance, and confidence interval with simulation. Data source: Wittwer [31].

Currently, the linear congruential method (LCG) is used broadly for generating random numbers. Park and Miller [32] mentioned that the congruential method was first proposed by Lehmer [33]. The basic principle of the linear congruential method can be referred to by Equation (1). The initial x of the random number generator requires a "seed" value. Calculating with (1) to be the new random number, a series of even random numbers in [0, 1] can be acquired. Monte Carlo simulation in Crystal Ball (CB) software is utilized in this study. According to Gonzalez et al. [34], there was not a "seed" value for CB software's random number generator in the beginning (Equation (2)), an iteration equation was regularly used for the multiplier congruential generator, the period length of the generator was 2,147,483,647, revealing that the numbers after billions of tests would repeat. Law and Kelton [35] explained the definition of the random number generator in detail. There few studies applying Monte Carlo simulation to evaluate RPN in FMEA. Eduardo et al. [36] and Seung and Kosuk [37] mentioned FMEA and applied Monte Carlo simulation to the evaluation. However, this application is different from the use of AP in this study.

$$x_{n+1} = (ax_n + b)\text{mod}c \tag{1}$$

where a, b, c are "magic numbers".

$$r \leftarrow (6208991 \times r)\text{mod}(2^{31} - 1) \tag{2}$$

2.3. Confidence Interval (CI) and Confidence Level (CL)

Neyman [38] first announced the idea of confidence interval (CI), which is a descriptive statistical scale for inferring potential population [39]. CI, an interval constructed under an established confidence level (CL), consists of an interval with an upper and lower limit defined by sample statistics and sampling error; a larger standard deviation (σ) shows a larger plus/minus value of point estimation [40]. CI also reveals the accuracy or reliability of sample statistics in general observation. The narrower CI interval shows the more reliable population estimation [41]. In comparison with point estimation, Brittany et al. [42] estimated population with sample statistics. CI also covers the information of estimation accuracy. Hazra [43] explained that CI of statistics can be regarded as a series of values calculated from sample observation, which might contain a true population with a certain degree of uncertainty. Although CI provides the estimate of unknown total parameters, the interval calculated from a specific sample does not necessarily contain true parameters. CL refers to CI covering the confidence, or reliability, in the population. CL is denoted by 1-α, where α stands for wrong probability, Figure 3. Furthermore, 95% CL refers to a 95% opportunity for the interval established under the CL to contain the true population; 90%, 95%, or 99% are often used for calculation [40]. Generally, a higher CL results in a wider CI, which indicates more accurate results.

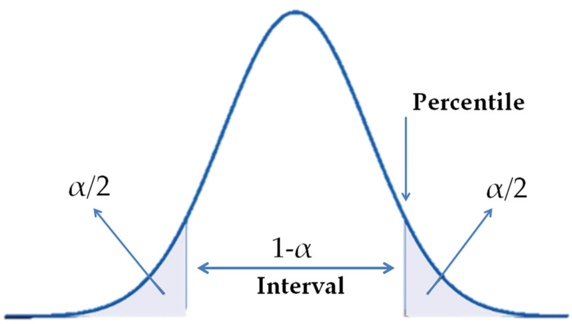

Figure 3. Construct of a CI percentile. Data source: Glen [44].

Values of 90%, 95%, or 99% are generally used as the CL indicator in normal distribution, and the corresponding Z distribution (with the difference of several σ) are 1.65, 1.96, and 2.58, Figure 4.

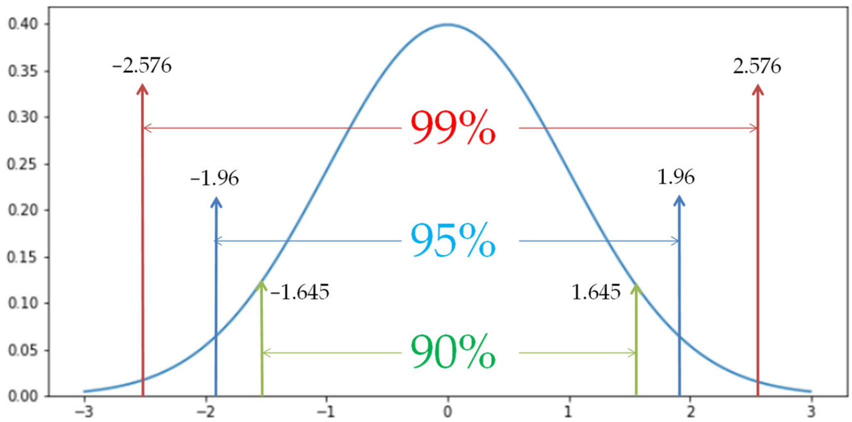

Figure 4. N (0,1) 90%, 95%, 99% cutoff values. Data source: Heumann and Schomaker [45].

CI refers to a researcher's confidence in inferring population according to the result; 95% CL or 99% CL is generally adopted in human science [46]. As shown in Equations (3) and (4), CL 95% is commonly used, but when calculating CI under other CL, e.g., 90%

or 99%, it is rare that CL would be 1. It means positive and 100% accuracy of the result; however, this does not exist in actual research.

$$95\%\ \text{CL},\ \text{CI} = \overline{X} \pm 1.96 * \left(\sigma / \sqrt{n}\right) \quad (3)$$

for *n* is number of samples

$$99\%\ \text{CL},\ \text{CI} = \overline{X} \pm 2.58 * \left(\sigma / \sqrt{n}\right) \quad (4)$$

for *n* is number of samples.

2.4. Discrete Nonuniform Distribution

In statistics and probability theory, discrete nonuniform distribution is a limit value with the same probability, although most situations in reality or physics are nonuniform distribution random numbers, e.g., radioactive decay. Experiments with distinct distribution generate nonuniform random number distributions with uniform random number generators [47], Figure 5. The probability mass function (PMF) of integral random numbers in [0, 1] shows discrete nonuniform distributions with 1 as the probability. Monte Carlo simulation is therefore applied in this study to generate random numbers, and the set probability number (95% CL or 1.96 CI) as the S, O, D probability generator in FMEA.

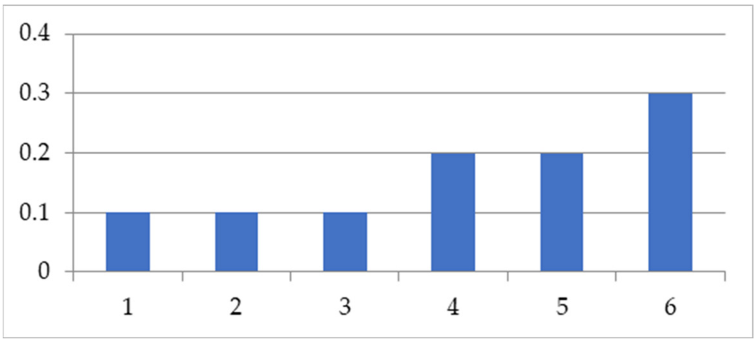

Figure 5. The histogram of PMF discrete nonuniform distribution. Data source: Cross Validated [48].

3. Research Method

Regardless of design development or process, tools for problem prevention are extremely important for enterprises. FMEA is currently used for preventing problems or evaluating reliability in the automotive and electronic industry. In the evaluation process, S, O, and D evaluation ranking in the *FMEA Handbook* could be the reference, or a company could add real examples in the product line according to special products and processes; nevertheless, qualitative rating is definitely defined in the handbook, but error evaluation might appear in the FMEA evaluation process due to a qualitative rating. Cl with S, O, D evaluation ranking and the combinations are therefore added in this study to calculate the probability with Monte Carlo simulation and to acquire the H, M, L probability with the corresponding AP as the reference. By searching research on FMEA, there is no comparison between RPN in the old-edition FMEA and the new-edition AP. The comparison between RPN and AP is therefore discussed to prove and understand the schema that if the old RPN method is still used as the basis for improvement. It leads to an application that should be improved but not implemented, the possibility of monitoring unnecessary improvement becoming the primary improvement, and then reducing the problem of resource waste. In the new-edition AIAG FMEA, it shows that S, O, and D ratings are qualitative. Regardless of any professional's judgment of the S, O, and D value of FMEA, there must be deviation. This research is based on the assumption that the professional scoring results should fall within the 95% CL. For example, if 100 professional engineers score S, O, and D according to the ranking table, the judgment results cannot be 100% the same. Similar to a questionnaire, the results of answering questions are subject to a normal distribution and have their own CI. When there is a gap between the three scores, the AP results will be relatively different.

3.1. Establishing FMEA Evaluation Model

According to the new-edition *AIAG and VDA FMEA Handbook* [7], to evaluate the process in FMEA in this study, professionals in the cross-functional team evaluate each process and step based on S, O, D in FMEA. The relevant data are recorded in the new-edition FMEA form. The main structure and function analysis include process name, process step, and process task, and S of failure effect (FE), O of failure cause, and D of failure cause/failure mode (FM) are analyzed. After the evaluation, H, M, L are judged by referring to the AP form in PFMEA for the AP ordering.

3.2. Setting CL 95% of Each FMEA Rating

The *AIAG and VDA FMEA Handbook* [7] Subsection 3.5.5 shows that since each team's environment is unique, their respective individual ratings will be unique (e.g., the ratings are subjective). Additionally, a qualitative rating is marked in the *FMEA Handbook*, which is undoubtedly to evaluate the existence of a problem difference. There is always some uncertainty in ranking, such as the degree to which the questionnaire is distributed according to the normal distribution. Therefore, we use 1.96σ to estimate the level of uncertainty, which is included in the ±1 within the CL 95%. Thus, evaluation of S, O, D in FMEA with existing data analyses and execution is still based on a normal distribution. CL 95%, generally used in literature review, is therefore utilized for the statistical setting of S, O, D. Furthermore, factors in the evaluation referred in forms are evaluated, and the CL of S, O, D and the cutoff value are set to match the possibility of discrete S, O, D probability distribution not exceeding the evaluation range. The CI of CL 95 % is shown in Equation (5).

$$\left(\overline{X} - 1.96\sigma_{\overline{X}}\right) \sim \left(\overline{X} + 1.96\sigma_{\overline{X}}\right) \text{ or} \overline{X} \pm 1.96\sigma_{\overline{X}} \quad (5)$$

3.3. Setting the Output Model of the Monte Carlo Simulation

CL 95% is applied to set the Monte Carlo simulation in this study to the simulation distribution of S, O, D, which is generated by the 95% evaluation setting. The evaluation deviation ±1 of S, O, D appears as a 2.5% probability, Figure 6. The main probability, 95% of CI ±1.96σ, appears in the middle, and 2.5% each on both sides.

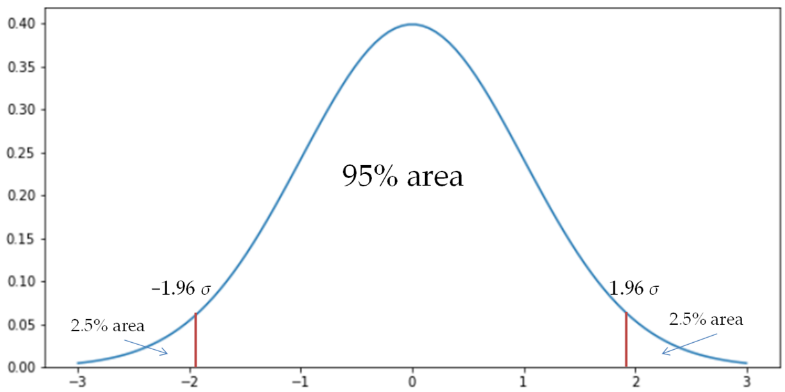

Figure 6. Graph showing ±1.96σ and 95% CI for the mean. Data source: Altman et al. [49].

After confirming the evaluation probability and error probability, CB software in the Monte Carlo simulation matched with probability theory is applied to define the probability of 1000-time individual evaluation of S, O, D. CB software is used as the random number generator to calculate the Monte Carlo simulation probability for the cutoff value = ±1, and the radius of the system CI error interval is set at 95%, as shown in Equation (6).

$$P(-z \leq Z \leq z) = 1 - \alpha = 0.95 \quad (6)$$

The custom assumptions of CB software are applied to set the discrete nonuniform probability distribution to conform to the research output. Regarding assumptions and probability distribution, CB software is selected to set the probability distribution required for the study, Figure 7. Additionally, the simulation model is set with S, O, D within 1~10.

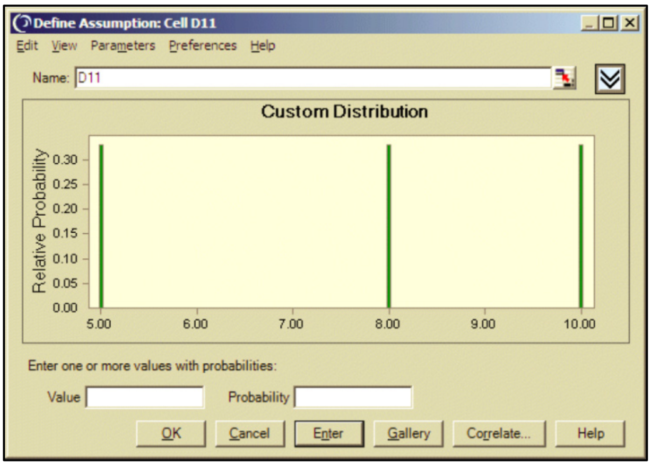

Figure 7. Custom assumptions of CB software for the Monte Carlo simulation.

3.4. AP Operation Description with the Combination of Monte Carlo Simulation Probability and the Comparison with RPN

With Monte Carlo simulation, the possible probability times for the 1000-time S, O, D evaluation are acquired. For instance, the evaluation rating of the original S is 4, after Monte Carlo simulation, the data show 3 for 27 times (2.7%), 4 for 949 times (94.9%), and 5 for 24 times (2.4%), Figure 8.

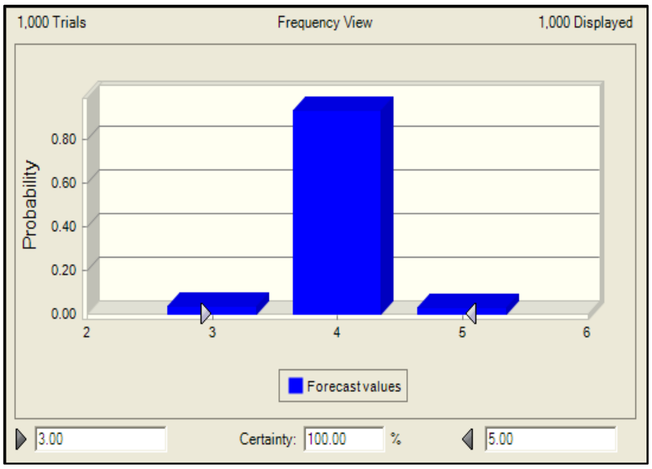

Figure 8. Forecast values of the Monte Carlo simulation with 1000 trials according to research definition in CB software.

According to three S, O, D evaluation outputs, each evaluation shows three probability distributions to generate 27 combinations (3 × 3). In this case, the probability of the combinations is multiplied. For example, the original S, O, and D of 4, 8, and 5 are calculated, giving probabilities of 4 ± 1, 8 ± 1, and 5 ± 1 after the simulation (Equation (7)) to acquire a new evaluation probability.

$$S\% \times O\% \times D\% = A\ new\ \%\ of\ the\ PFMEA\ process \quad (7)$$

When referring the 27 output S, O, D probability combinations to the PFMEA AP form in the *FMEA Handbook* [7], the AP H, M, L of each combination is acquired. The possibility of event FMEA differentiating the H, M, L probability is then calculated. Finally, the overall event probability is verified to ensure the sum of H, M, L being 100% (Equation (8)), revealing the accuracy of the entire simulation results.

$$\sum_{i=1}^{27} SOD\%_i = 100\% \quad (8)$$

Using the sequence of RPN in FMEA for the reference of improvement priority has been eliminated. For the previous criterion, enterprises normally used to set the improvement sequence of FMEA as S $\geq$ 9. It means failure to meet safety issues. Enterprises must take actions to prevent the effect. In contrast, when RPN $\geq$ 100, normally enterprises should take actions to start the improvement sequence to prevent the effect. The new edition changes AP as a single system to simply judge the level of H, M, L risk and improve the priority. The H, M, L probability is acquired through the simulation in this study, and the difference analysis of AP probability and RPN for adopting improvement actions is further studied.

4. Research Results

4.1. Data Explanation and Application

To actually discuss and simulate the research method proposed in this study, the content of an enterprise studying a semiconductor plant is modeled and converted into the format required in the new-edition *AIAG and VDA FMEA Handbook*. The AP number is judged according the AP form, Table 2. After analyzing the FMEA evaluation content, AP results and RPN numbers are particularly selected for judging the FMEA process with an opposite result from the improvement actions. Different actions to define AP in the handbook follow:

H— Highest priority for review and action. The team needs to either identify an appropriate action to improve prevention and/or detection controls or justify and document why current controls are adequate.

M—Medium priority for review and action. The team should identify appropriate actions to improve prevention and/or detection controls, or, at the discretion of the company, justify and document why controls are adequate.

L— Low priority for review and action. The team could identify actions to improve prevention or detection controls.

RPN $\geqq$ 100 and AP being H or RPN <100 and AP being M or L is first excluded. The remaining PFMEA is discovered along with the relative results of AP and RPN—in other words, RPN $\geqq$ 100 and AP being L, or RPN<100 and AP being H. After deciding to research the extreme numbers of RPN and AP, according to PFMEA (Table 2) abnormal 1 and 5 in FMEA are selected as the research objects for evaluating H, M, L, and then analyzing the difference in FMEA before/after actions. The selected research objects are then marked in red.

According to the selected PFMEA, FE and FM are delivered to customers, and customers' complaints are about many problems with circuit breaks, resulting in 20% yield loss and an abnormal computer-etching-time calculation of the equipment, as in case 1. The S, O, D appears as 8, 6, and 2; after the CB software setting, S appears as 9, 8, 7, with the probability 2.5%, 95%, and 2.5%. O is set as 7, 6, 5, with the probability 2.5%, 95%, and 2.5%, and D is set as 3, 2, 1, with the probability 2.5%, 95%, and 2.5%. Another analyzed PFMEA: FE and FM indicate the case of possible harm for the equipment or maintainer and the leak of NH_4. The S, O, D shows 9, 3, 4; after the CB software setting, S is set as 10, 9, 8, with the probability 2.5%, 95%, and 2.5% (Figure 9). O is set as 4, 3, 2, with the probability 2.5%, 95%, and 2.5% (Figure 10), and D is set as 5, 4, 3, with the probability 2.5%, 95%, and 2.5% (Figure 11).

Table 2. Failure mode and effects analysis of equipment in a semiconductor plant.

				Equipment Failure Mode and Effects Analysis					
			Structure Analysis				FMEA No.: 0001		
	1. Process Item		**2. Process Step**		**3. Process Work Equipment**		Key Date: 20XX/3/21		
	Semiconductor equipment.		Repairing & Abnormal		Troubleshooting, preventive Maintenance		FMEA Start Date: 20XX/4/1		
			Failure Analysis				Cross-Functional Team: Process & Equipment Dept.		
	2. Function of Process Item		**3. Process Step**		**4. Process Work Equipment**		Process Responsibility: Equipmant Dept.		
	Key Equipment Assessment		Repairer & Maintenance		Accord to OCAP & recover				
	Failure Effects(FE)	**S**	**Failure Mode(FM)**	**Failure cause(FC)**	**Prevention Controls(PC) of FC**	**O**	**Detection Control(DC) of FC or FM**	**D**	**AP**
1	Damage equipment or operator.	9	NH4 gas leak	Equipment pipe broken	APC System	3	To receive and detect equipment message.	4	L
2	Product reworks 10 pcs.	4	Equipment down	Equipment PCB broken	APC system	5	Receive and detect equipment message, but cannot auto hold equipment when PCB broken.	6	M
3	Effect production, and lead to shipping.	3	Equipment down	Unclear equipment issue	APC system	4	100% receive and detect equipment message, auto hold equipment when abnormal happen.	2	L
4	Customer complains, the issue lead to Yield loss 30%.	8	Implant dose abnormal	Implanter broken ,and did not notice	APC system	2	It cannot detect until product happen abnormal.	9	M
5	Shipping to customer lead to customer complains, and Yield loss 20%.	8	Product over ETCH	Equipment Etch time abnormal	APC System	6	Receive and detect equipment message, define control Spec., but cannot auto hold equipment when abnormal.	2	H
6	Slight impact on production line.	4	Equipment Arm down	Device arm screw loose	APC system	5	Receive device signal value to detect device status.	6	L
7	Customer complains, the issue lead to scrap 10 lots.	8	Film thickness insufficient	Target position abnormal	APC system	4	Unable to detect and only find the device when the product is abnormal	7	H

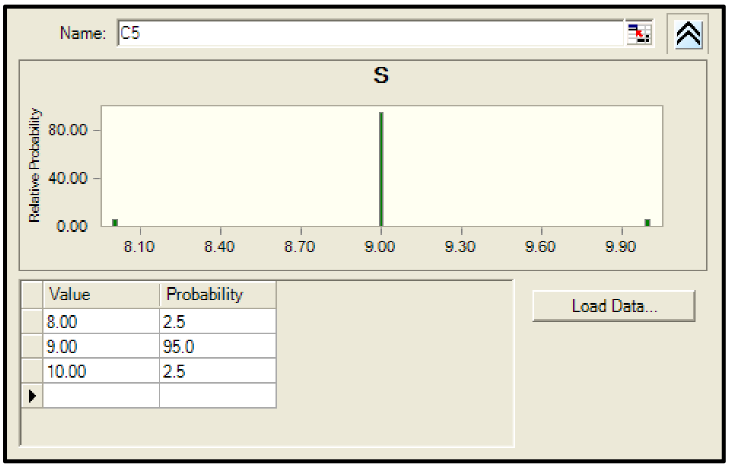

Figure 9. Case 1: Discrete nonuniform distribution of S in CB software.

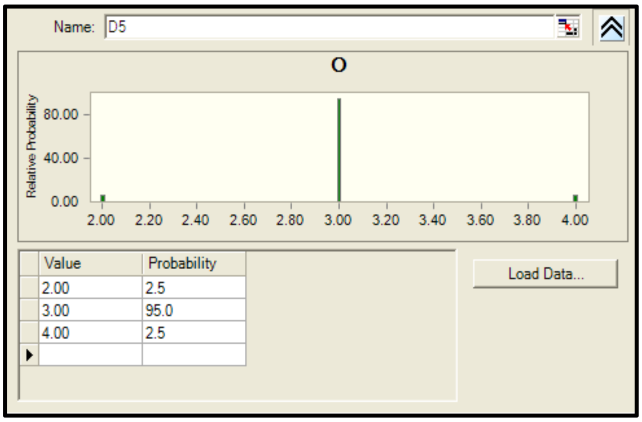

Figure 10. Case 1: Discrete nonuniform distribution of O in CB software.

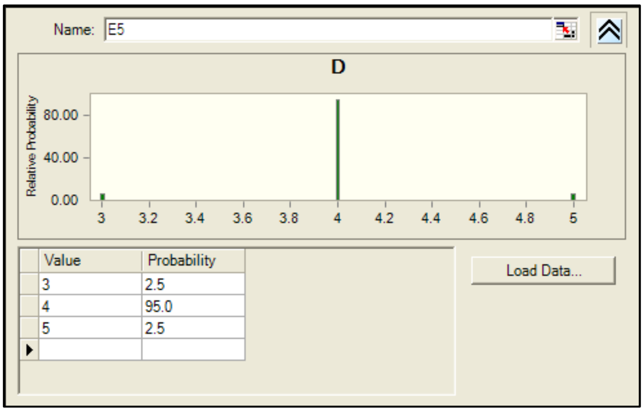

Figure 11. Case 1: Discrete nonuniform distribution of D in CB software.

4.2. Simulation Data for Case 1

Case 1: Monte Carlo simulation through CB software outputs S, O, D probability distribution and times, as shown in Figures 12–14.

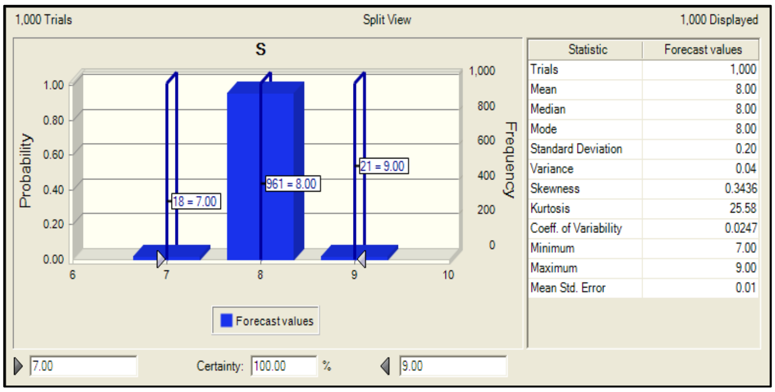

Figure 12. Case 1: Monte Carlo simulation of S in CB software.

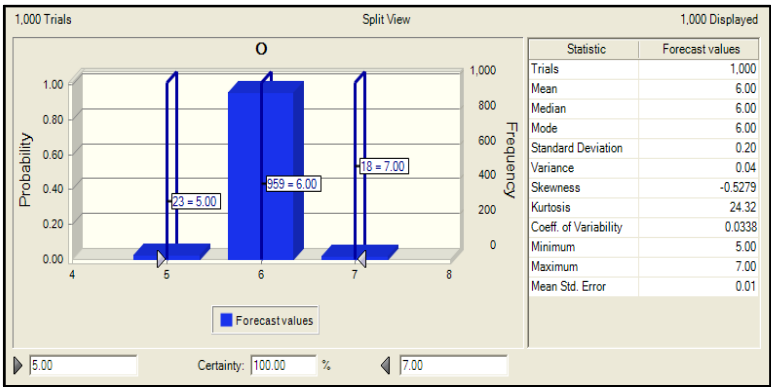

Figure 13. Case 1: Monte Carlo simulation of O in CB software.

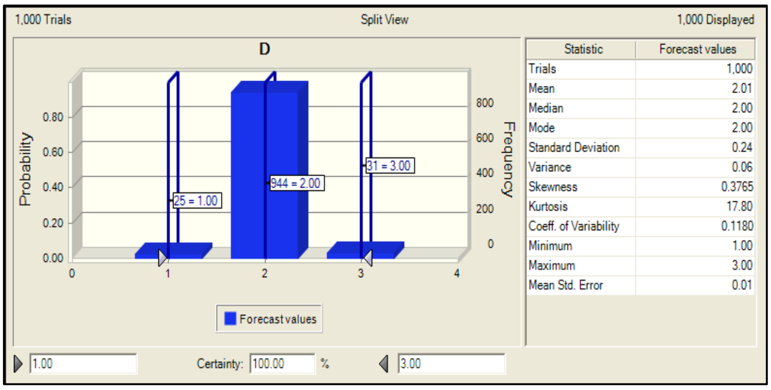

Figure 14. Case 1: Monte Carlo simulation of D in CB software.

By integrating the simulation results, S, O, D probability and times are organized and shown in Table 3.

Table 3. Case 1: Probability and times of S, O, and D.

Severity	Times	Probability	Occurrence	Times	Probability	Detection	Times	Probability
9	18	1.800%	7	23	2.300%	3	25	2.500%
8	961	96.100%	6	959	95.900%	2	944	94.400%
7	21	2.100%	5	18	1.800%	1	31	3.100%

4.3. Analysis Results for Case 1

According to the probability outputs in Table 3, the 27 combinations of S, O, and D in the study are ordered. AP is judged, where H, M, or L in the AP column is referred to S, O, D in Table 1. The PFMEA probability of this study is output by referring to (7). According to Equation (8), the probability sum of the entire simulation evaluation is used as the verification column to ensure the probability sum of the 27 combination results being 100%. Finally, the old-edition RPN is calculated for successive comparison and study, Table 4.

Table 4. Case 1: Order, probability, AP, and RPN for 27 combinations of S, O, and D.

No.	Severity	Probability	Occurrence	Probability	Detection	Probability	AP	Probability (S * O * D)	RPN
1	9	1.800%	7	2.300%	3	2.500%	H	0.001%	189
2	9	1.800%	7	2.300%	2	94.400%	H	0.039%	126
3	9	1.800%	7	2.300%	1	3.100%	H	0.001%	63
4	9	1.800%	6	95.900%	3	2.500%	H	0.043%	162
5	9	1.800%	6	95.900%	2	94.400%	H	1.630%	108
6	9	1.800%	6	95.900%	1	3.100%	H	0.054%	54
7	9	1.800%	5	1.800%	3	2.500%	H	0.001%	135
8	9	1.800%	5	1.800%	2	94.400%	H	0.031%	90
9	9	1.800%	5	1.800%	1	3.100%	H	0.001%	45
10	8	96.100%	7	2.300%	3	2.500%	H	0.055%	168
11	8	96.100%	7	2.300%	2	94.400%	H	2.087%	112
12	8	96.100%	7	2.300%	1	3.100%	M	0.069%	56
13	8	96.100%	6	95.900%	3	2.500%	H	2.304%	144
14	8	96.100%	6	95.900%	2	94.400%	H	86.999%	96
15	8	96.100%	6	95.900%	1	3.100%	M	2.857%	48
16	8	96.100%	5	1.800%	3	2.500%	M	0.043%	120
17	8	96.100%	5	1.800%	2	94.400%	M	1.633%	80
18	8	96.100%	5	1.800%	1	3.100%	M	0.054%	40
19	7	2.100%	7	2.300%	3	2.500%	H	0.001%	147
20	7	2.100%	7	2.300%	2	94.400%	H	0.046%	98
21	7	2.100%	7	2.300%	1	3.100%	M	0.001%	49
22	7	2.100%	6	95.900%	3	2.500%	H	0.050%	126
23	7	2.100%	6	95.900%	2	94.400%	H	1.901%	84
24	7	2.100%	6	95.900%	1	3.100%	M	0.062%	42
25	7	2.100%	5	1.800%	3	2.500%	M	0.001%	105
26	7	2.100%	5	1.800%	2	94.400%	M	0.036%	70
27	7	2.100%	5	1.800%	1	3.100%	M	0.001%	35
							SUM	100%	

After the simulation and operation, the H, M, L probability of AP is 95.243%, 4.757%, and 0%, respectively, Table 5. The results confirm that M probability in PFMEA AP still presents 4.757%, indicating that the team should ensure proper actions for improving prevention or detection control. Furthermore, the L probability of AP is 0% while RPN in the old-edition FMEA reveals 96. A low risk is judged in the FMEA process for general enterprises with RPN<100; thus, improvement actions are not necessary. However, the judgment in this case is completely opposite, AP is H, but L probability is 0%. Furthermore, for the general RPN criterion that is 100, the probability of RPN appears as 44.444% and 55.556% in Table 6. It can be seen that the AP probability is completely different from the RPN concept. In this case, AP judgment could obviously benefit the error-proof for actions judgment.

Table 5. Case 1: Probability statistics of AP.

AP	Probability
H	95.243%
M	4.757%
L	0.000%

Table 6. Case 1: Probability of RPN (criterion: 100).

RPN	No.	Probability
≧ 100	12	44.444%
<100	15	55.556%

4.4. Simulation Data for Case 2

Case 2: Monte Carlo simulation through CB software outputs S, O, D probability distribution and times, as shown in Figures 15–17.

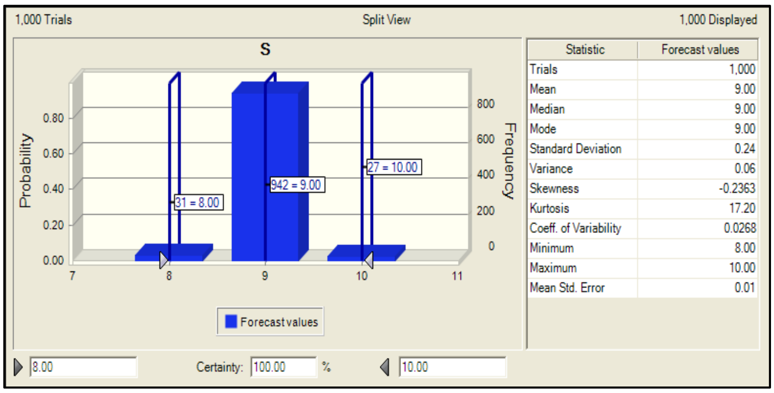

Figure 15. Case 2: Monte Carlo simulation of S in CB software.

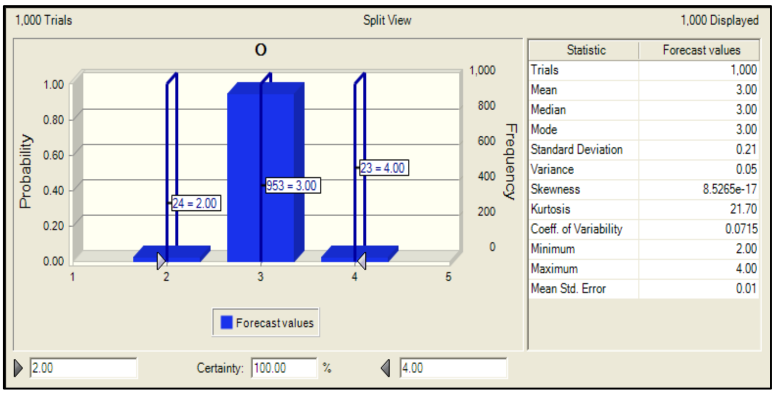

Figure 16. Case 2: Monte Carlo simulation of O in CB software.

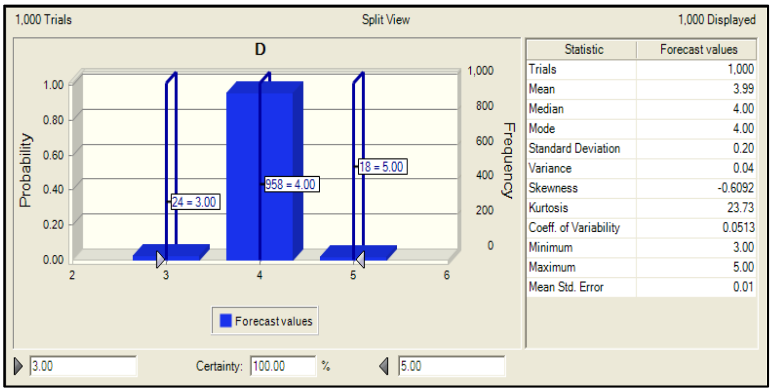

Figure 17. Case 2: Monte Carlo simulation of D in CB software.

By integrating the simulation results, S, O, D probability and times are shown in Table 7.

Table 7. Case 2: Probability and times of S, O, and D.

Severity	Times	Probability	Occurrence	Times	Probability	Detection	Times	Probability
10	31	3.100%	4	24	2.400%	5	24	2.400%
9	942	94.200%	3	953	95.300%	4	958	95.800%
8	27	2.700%	2	23	2.300%	3	18	1.800%

4.5. Analysis Results for Case 2

According to the probability output in Table 7, 27 combinations of S, O, D in the research are ordered. Based on the PFMEA AP form, AP, H, M, or L in the AP column are judged by referring to S, O, D in Table 1. The PFMEA probability is output by referring to (7). The probability sum of the entire simulation evaluation results, based on (8), is regarded as the verification column to ensure the probability sum of the 27 combinations is 100%. Finally, the old-edition FMEA RPN is calculated for successive comparison and study, Table 8.

Table 8. Case 2: Order, probability, AP, and RPN of 27 combinations of S, O, D.

No.	Severity	Probability	Occurrence	Probability	Detection	Probability	AP	Probability (S * O * D)	RPN
1	10	3.100%	4	2.400%	5	2.400%	H	0.002%	200
2	10	3.100%	4	2.400%	4	95.800%	H	0.071%	160
3	10	3.100%	4	2.400%	3	1.800%	H	0.001%	120
4	10	3.100%	3	95.300%	5	2.400%	M	0.071%	150
5	10	3.100%	3	95.300%	4	95.800%	M	2.830%	120
6	10	3.100%	3	95.300%	3	1.800%	M	0.053%	90
7	10	3.100%	2	2.300%	5	2.400%	M	0.002%	100
8	10	3.100%	2	2.300%	4	95.800%	L	0.068%	80
9	10	3.100%	2	2.300%	3	1.800%	L	0.001%	60
10	9	94.200%	4	2.400%	5	2.400%	H	0.054%	180
11	9	94.200%	4	2.400%	4	95.800%	H	2.166%	144
12	9	94.200%	4	2.400%	3	1.800%	H	0.041%	108
13	9	94.200%	3	95.300%	5	2.400%	M	2.155%	135
14	9	94.200%	3	95.300%	4	95.800%	L	86.002%	108
15	9	94.200%	3	95.300%	3	1.800%	L	1.616%	81
16	9	94.200%	2	2.300%	5	2.400%	M	0.052%	90
17	9	94.200%	2	2.300%	4	95.800%	L	2.076%	72
18	9	94.200%	2	2.300%	3	1.800%	L	0.039%	54

Table 8. *Cont.*

No.	Severity	Probability	Occurrence	Probability	Detection	Probability	AP	Probability (S * O * D)	RPN
19	8	2.700%	4	2.400%	5	2.400%	M	0.002%	160
20	8	2.700%	4	2.400%	4	95.800%	M	0.062%	128
21	8	2.700%	4	2.400%	3	1.800%	M	0.001%	96
22	8	2.700%	3	95.300%	5	2.400%	M	0.062%	120
23	8	2.700%	3	95.300%	4	95.800%	L	2.465%	96
24	8	2.700%	3	95.300%	3	1.800%	L	0.046%	72
25	8	2.700%	2	2.300%	5	2.400%	M	0.001%	80
26	8	2.700%	2	2.300%	4	95.800%	L	0.059%	64
27	8	2.700%	2	2.300%	3	1.800%	L	0.001%	48
							SUM	100%	

After the simulation and operation, H, M, L probability of AP shows 2.335%, 5.291%, and 92.374%, respectively, Table 9. The results confirm M probability 5.291% and H probability 2.335% of PFMEA AP. It therefore suggests that the team needs to or should take proper actions to improve prevention or detection control. Moreover, L probability in AP is 92.374%, while the old-edition FMEA RPN is 108. In the FMEA judgment, RPN ≧ 100 for general enterprises indicates high risk and requires improvement actions. However, the judgment result in this case is completely opposite. Nevertheless, the old-edition FMEA suggests adopting actions when S > 9, while the new-edition FMEA mentions potential severity for 9~10. Action priority is the failure factor in H and M that is suggested to have the management level review all suggested and adopted actions. In fact, AP L in case 2 does not mention actions. It would need discussions for decisions. The AP L probability in this case is 92.374%, while RPN reveals the priority for improvement of actions. Furthermore, the RPN criterion is 100 for the general case, and the probability of RPN appears as 51.851% and 48.149% in Table 10. It can be seen that the AP probability is completely different from the RPN concept. AP judgment is obviously beneficial to error-proof actions judgment.

Table 9. Case 2: Probability of AP.

AP	Probability
H	2.335%
M	5.291%
L	92.374%

Table 10. Case 2: Probability of RPN (criterion: 100).

RPN	No.	Probability
≧100	14	51.851%
<100	13	48.149%

4.6. Discussion

The RPN distribution can provide some information about failures, but the RPN alone is insufficient to determine if more operations are needed for the failures, since RPN in AP assigns equal weights to S, O, and D. Monte Carlo simulation was combined to prioritize similar RPN results.

The RPN of the new-edition and the old-edition AIAG FMEA are different. The new-edition FMEA clearly defines the use of Table 1 to determine AP. RPN is the number multiplied by S, O, and D; S, O, and D are the same weight. There is no difference among S, O, and D. For example, when S = 6, O = 2, and D = 2, they are equivalent to S = 2, O = 6, and D = 2, and RPN is 24 for both. For another example, when S = 9, O = 6, and D = 1, then the AP is H and RPN is 54, but if the old-edition AIAG FMEA S = 9, then there is a security issue and an improvement must be proposed, even if RPN = 54. The new-edition AIAG FMEA refers to Table 1 when S = 9, O = 2, and D = 2. The AP is still L. This proves that consideration for measures taken differ between the new-edition and old-edition AIAG FMEA.

Based on case 1 and 2, using Monte Carlo simulation to evaluate S, O, and D, makes a rating possible; there are different AP judgment results for reference. According to the results, managers or improvement teams take proper actions to improve prevention or detection control.

5. Conclusions

Since the announcement of AIAG and VDA FMEA in 2019, the new-edition content is used for evaluating D/PFMEA in businesses. The judgment criteria of structure, table, S, O, and D are required for coping with risk evaluation in the current situation, and further executing risk analysis, thought control, systemization, and bullet points to perfect FMEA. Especially, the objective of FMEA has undergone a major adjustment by replacing the old-edition RPN with AP as the reference for the priority of actions. Currently, there is scant analysis and research on AP evaluation; some research on revised FMEA even retains the old concepts, using RPN as the reference for the priority of actions. Once the cross-functional team has completed an initial identification of failure modes and effects, and causes and controls, including a rating of S, O, and D, then cross-functional improvement teams need to decide whether further efforts are required to reduce the risk, according to the product of S, O, and D. They must prioritize these actions due to resources, time, and technology constraints. Aiming at the new-edition AP analysis being the reference of revised FMEA, there are possibly 1000 combinations of S, O, and D, while the actual AP judgment merely appears in three action priority levels. Using a qualitative rating for judgment when independently evaluating S, O, and D, errors may appear in the evaluation results. Additionally, AP, H, M, L are judged as a single result. The idea of error evaluation is therefore included in this study to evaluate AP probability with a 95% CL. Monte Carlo simulation and the comparison with the old-edition RPN present the value of this study. The possible H, M, L probability acquired could provide the original AP evaluation for enterprises adopting preventive risk mitigation actions for improving prevention or detection control, or proving the control of current actions. The old-edition RPN and the new-edition AP are also preceded difference analysis in the cases. In case 1, the judgment value of the previous RPN is 96, with the risk evaluation indicating no need for improvement. However, AP judgment shows that H probability exceeding 95% and an L probability of 0% prove that an improvement action is reasonable and appropriate. Accordingly, it proves that AP is obviously beneficial to error-proof. Even the control of D is effective (D = 2), AP is regarded as H when S and D are above M, which indicates a need for improvement actions.

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