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Reward-Penalty Mechanism or Subsidy Mechanism: A Closed-Loop Supply Chain Perspective

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Abstract: The government plays a crucial role in regulating the closed-loop supply chain (CLSC). We investigated the reward-penalty mechanism (RPM) for the manufacturer and the subsidy mechanism (SM) for the collector in CLSCs. The government's goal is to maximize social welfare. Based on the centralized and decentralized decision-making models without government intervention, we developed two CLSC models where the government rewards or penalizes the manufacturer and subsidizes the collector. Then, the impact of government input cost and environmental benefit coefficients on the decision variable, firm's profit and social welfare was analyzed. We found the following conclusions: (i) both RPM and SM increase the collection rate and the profit of CLSC partners, while the price of a new product decreases if the environmental benefit coefficient is moderate; (ii) social welfare and the profits of the manufacturer and retailer under RPM are higher than under SM, while a collector's profit under RPM is lower than that under SM; (iii) RPM is more feasible to implement in terms of the higher collection rate, buyback price and social welfare. These conclusions could provide several managerial implications for both the government and partners of the CLSC.

Keywords: reward-penalty mechanism; subsidy mechanism; closed-loop supply chain; social welfare

MSC: 90B06

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1. Introduction

One of the main ways for governments and manufacturers to promote the development of a low-carbon economy is to control carbon emissions by investing in low-carbon technology and collecting waste electrical and electronic equipment (WEEE) [1]. Driven by the objective requirements of environmental protection regulations and the benefits of collecting and remanufacturing, more and more firms tend to apply closed-loop supply chain (CLSC) management characterized by the collection and reuse of WEEEs in production practice. For instance, Suning Tesco's total recycling amount of WEEEs was nearly 7 billion in 2020 [2]. Governments also take various intervention mechanisms to participate in CLSC management activities. For instance, China has subsidizing recycling firms of WEEEs since 2012 and the EU adopted a new environmental design regulation in 2018. The regulations not only encourage the collection of WEEEs, but require manufacturers to modify the product design to extend the service life of products [3].

The government plays the role of "game rules" constitutor and supervisor in the operation of the CLSC to improve collection rates and promote recycling economy development [4]. The common modes of government intervention in a CLSC include legislation, rewards or penalties, taxes, subsidies and so on. Previous government intervention was mainly realized through the subsidy mechanism (SM) [5]. SM is a mechanism by which the government subsidizes the firms of the CLSC to improve the collection rate of WEEEs. The literature indicates that SM effectively improves the collection rate of WEEEs [6–8], by this

means, it also promotes the rapid development of a low-carbon economy [9,10]. In 2021, China issued the regulation on adjusting the subsidy standard of the fund for the disposal of WEEEs, which reduced the overall subsidy standard of the fund for "Four machines and one computer", whereas most developed countries in the East and West, such as Japan, US, Germany and so on, implemented the extended producer responsibility system for WEEE's collection. CLSCs that do not complete the recycling of WEEEs within the specified time will be punished. Motivated by the different directions for the recycling of WEEEs adopted by diverse countries, we discovered that several scholars have investigated the literature with regard to the implementation of the reward-penalty mechanism (RPM) for a CLSC. Interestingly, we found that RPM can promote the WEEE's recycling as well as SM [11,12].

To sum up, both mechanisms are effective for enhancing the collection of WEEEs, but whether they can increase the profits of the CLSC partners and improve social welfare needs to be explored. In addition, both are mostly discussed independently by scholars, i.e., they seldom consider their difference. Therefore, it is necessary to discover the advantages and disadvantages of two mechanisms. This research provides a scientific basis for the government to adopt an appropriate mechanism and guides firms to make optimal decisions under different government mechanisms.

In this paper, we developed a CLSC consisting of a manufacturer, a retailer, a collector, and consumers. After developing centralized and decentralized decision-making models without government intervention as the benchmark models, two CLSC models under RPM and SM were developed. Then, we analyzed the properties of equilibrium solutions of four models and further provided propositions. The effects of the fixed input cost and environmental benefit coefficients on the decision variable, the profit of CLSC partners and social welfare were compared using numerical experiments.

This paper answers the questions outlined below.

(i) What are the advantages and disadvantages of RPM for the manufacturer and SM for the collector?
(ii) Under the goal of maximizing social welfare, which mechanism is better for the government to guide partners of the CLSC to collect more WEEEs to remanufacture?

The remainder of this paper is arranged as follows. Section 2 reviews the related literature. The model assumptions, notation, and description are presented in Section 3. In Section 4, we further develop the benchmark models based on decentralized and centralized decision making and then model the RPM case and the SM case in the CLSC. The equilibrium solutions of key decision variables are compared and analyzed in Section 5. In Section 6, we conduct some simulations to further compare the difference between RPM and SM with other exogenous variables changing. The limitations, pivotal findings and research directions for the future are provided in Section 7.

2. Literature Review

This paper is related to two streams of literature. The first stream is the research about RPM in CLSC. Wang et al. proposed RPM to guide governments to improve the collection rate of WEEE's [11]. Later, based on consumers' environmental awareness, Chen and Akmalul'Ulya focused on the behavior of the supply chain under RPM from the perspective of the government. They found that RPM can facilitate the collection rate and green effort [12]. Considering collection rate and social welfare maximization, Wang et al. examined the influence of different implementation objects of RPM on decision making of partners of CLSC [8]. Li et al. further analyzed the influence of government RPM on optimal decision making and how CLSC partners choose their cooperative partners [13]. Moreover, Wang et al. developed an e-commerce closed-loop supply chain model consisting of a remanufacturer and a network recycling platform to compare the effects of RPM and the altruistic preference for decision-making recycling [14]. By constructing the tripartite evolutionary game model consisting of government, a manufacturer and an integrator, it was found that the government's implementation of reward and punishment strategy can guide the integrator to improve the recovery rate [15]. The above literature about RPM

mainly investigated the influence of implementing RPM on recycling WEEEs and a firm's decision-making.

The second stream identified in the literature is SM, which several scholars have conducted studies on in relation to CLSC. Hong et al. argued that subsidy and advanced collecting fees reduce the consumption of new products and encourage the collection of WEEEs [6]. Zhu et al. further found that larger values of government subsidies does not necessarily induce the collection of more used products [7]. Later, the different effects of replacement subsidies and government consumption subsidies in a dual-channel supply chain were considered [16]. Subsequently, Wang et al. examined the pricing decisions and effects of government subsidies in the reverse supply chain of WEEEs with two retailers, a remanufacturer and a collector [8]. For carbon emission reduction, it was proved that carbon emissions can be reduced without sacrificing manufacturers' profits by introducing government subsidy schemes [9]. Later, Zhu et al. discovered that a remanufacturing subsidy can always promote the increase in the consumption of remanufactured products, while government carbon regulation can always promote the decrease in the total carbon emissions [17]. Hu et al. investigated the government direct subsidy and mechanism bias in a CLSC. The results showed that the two strategies are both beneficial in terms of carbon emissions but found almost no difference in their impacts [18]. Yi et al. developed a new extended producer responsibility system for resource conservation, which includes a Stackelberg game that a government engages in with manufacturers and a collector, and a Cournot game between manufacturers. The results indicate that under different conditions, the government needs to adjust the joint tax SM to achieve pareto optimal results [10]. Considering the aim of maximizing social welfare, demand subsidy or production regulation-related policies were compared [19]. The result demonstrates that a demand subsidy generally improves social welfare, while production regulation mechanisms positively impact the environment. As mentioned, we found that studies on SM mainly focus on the influence of different SM objects on supply chain price decision and recycling decision. In addition, a small number of scholars compared the differences between subsidy policies and other policies in CLSC decision making.

The main differences between our study and those in the literature are as follows: most of the literature only considers RPM or SM, but a comparison between them is absent. In this paper, a comparative analysis of the two mechanisms and was conducted, and the marginal conditions of the advantages and disadvantages of the two policies were identified, which provides a scientific basis for the government to make policies. Secondly, the government was very rarely taken as a game player in previous studies. However, in our study, the government participated in the game, aiming at maximizing social welfare. Social welfare is the sum of environmental benefits, consumer surplus and profits of manufacturers, retailers and collectors. This paper is capable of providing theoretical support for the government and supply chain node firms in decision-making practices.

3. Problem Description and Model Assumptions

3.1. Problem Description

Figure 1 presents the CLSC's structure with the guidance of the government. This CLSC includes a manufacturer, a retailer, a collector, and consumers. The solid arrow shows the logistics direction, and the dotted arrow indicates the SM or RPM adopted by the government. The government implements RPM for the manufacturer or the SM for the collector to increase the social welfare as well as the collection rate of WEEEs. RPM refers to an incentive mechanism combining rewards with penalties. The target collection rate (τ_0) and the reward-penalty intensity (k) are the key parameters of RPM. If the actual collection rate of WEEEs is higher than the target collection rate, the government offers the manufacturer rewards. In contrast, if the CLSC's actual collection rate is lower than the target collection rate, the manufacturer faces fines from the government. The SM is a scheme in which the government subsidizes the collector to increase the collection rate in the CLSC (e.g., in 2015, Guangzhou province announced subsidies for a collecting company,

which increased its collection rate by nearly 10%. So far, more and more provinces such as Hubei, Sichuan and Guizhou have implemented SM to increase the collection rate of WEEEs).

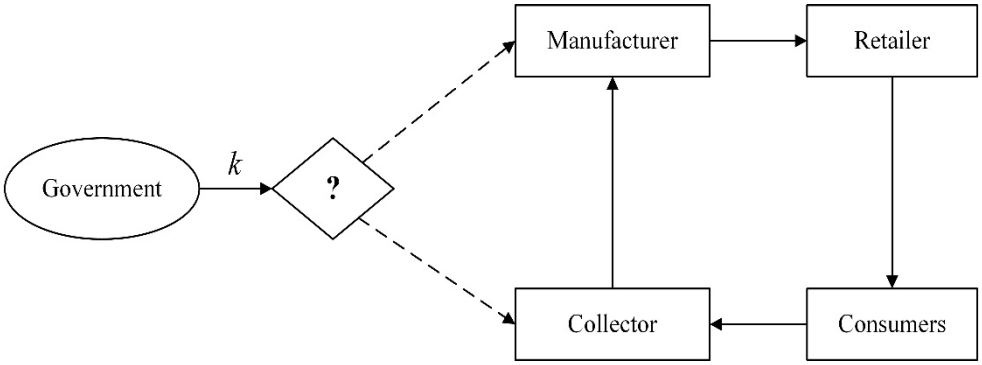


Figure 1. Close-loop supply chain (CLSC) structure under government guidance.

Due to the low manufacturing cost of using collected components and materials, manufacturers prefer to use WEEEs [20,21]. If WEEEs are exhausted, the manufacturer utilizes new materials. This assumption is reasonable since a rational manufacturer needs to consider the relevant regulations if the government enacts incentive mechanisms [8]. The large third-party collector not only collects the WEEEs but also undertakes dismantling, sorting, processing, etc. The government is responsible for actively guiding the participators in the CLSC to collect and remanufacture WEEEs.

The symbols and their description used in this paper are listed in Table 1.

3.2. Model Assumptions

The main assumptions are as follows.

(1) The remanufactured product's unit cost is lower than the newly manufactured product's unit cost, i.e., $c_n > c_r$, which indicates that the manufacturer saves on costs if they remanufacture WEEEs. For convenience, we define $\Delta = c_n - c_r$ [13]. The standard of remanufactured products is equal to that made by new materials; thus, consumers do not distinguish between them [24].

(2) τ is related to the investment of collection. Following the literature [22,25], we define the fixed collection cost as $I = m\tau^2/2$, where m is the scale parameter of collecting WEEEs. The cost of investment collection is a convex function of the collection rate, indicating that as the collection rate increases, the cost of investment collection increases sharply.

(3) All the WEEEs collected are utilized to produce new products as raw materials and components. This assumption avoids complicated mathematical calculations. As only a part of WEEEs can be used in remanufacturing, a parameter of the remanufacturing rate is required. However, this parameter has no effect on the development of managerial insights in this research [8].

(4) The government functions as the leader in the Stackelberg game while the manufacturer has enough channel power in CLSC to perform as the second leader [4,8].

Table 1. Notations.

Symbols	Descriptions
Decision variables	
w	Wholesale price of a new product.
b	Buyback price per unit of waste electrical and electronic equipment (WEEE) paid by the manufacturer to the collector.
p	Retail price of a new product.
τ	Collection rate, where $0 \leq \tau \leq 1$. Without loss of generality, it refers to the ratio of current generation products made from WEEE; i.e., the collected WEEE is utilized for remanufacturing. When $\tau = 1$, the manufacturer only uses collected components in production; when $\tau = 0$, the manufacturer only uses new components in production.
k	Subsidy intensity or reward-penalty intensity set by government for each unit of WEEE gathered deviating from the specified target. For the convenience of comparing the incentive mechanisms, we suppose the subsidy intensity and reward-penalty intensity to be equal.
Parameters	
c_n	Unit cost of a manufacturer using new materials and new components to produce new products.
c_r	Manufacturer's unit cost of using WEEE components to produce new products.
τ_0	The government's collection rate target. If $\tau \geq \tau_0$, the government will reward the manufacturer; if $\tau < \tau_0$, the government will penalize the manufacturer.
α	Fixed cost parameter that the government stipulates to conduct the reward-penalty mechanism (RPM) and the subsidy mechanism (SM). $\alpha > 0$ reflects the difficulty of the government implementing mechanisms. For the convenience of comparison, we assume that both mechanisms have the same fixed cost parameter. This assumption does not change the main findings of this research and avoids complex algebraic operations.
β	Environmental benefit parameter that indicates the degree of environmental benefit that a unit of returned WEEE brings to society. Here, $\beta > 0$.
m	Difficulty of gathering WEEEs.
c	Unit collection cost of the collector.
c_z	New product's unit production cost, which comprises newly manufactured and remanufactured products. Thus, we have $c_z = \tau c_r + (1-\tau)c_n = c_n - \tau(c_n - c_r) = c_n - \Delta\tau$, $(\Delta = c_n - c_r)$.
Derived functions	
$D(p)$	$D(p) = \phi - p$, which is the demand function of new products, where ϕ is the possible market size (according to the CLSC literature, such as Choi et al., 2013 [22], Zhong et al., 2020 [23], etc.).
π_m	Manufacturer's objective profit function.
π_r	Retailer's objective profit function.
π_c	Target profit function of the collector.
C_s	Consumer surplus.
E	Environmental benefits obtained by collecting the WEEEs.
π_g	Social welfare.
Indexes	
Superscript *, **	CLSC equilibrium solution under RPM for the manufacturer, CLSC equilibrium solution under SM for the collector. * and ** represent the equilibrium solutions under different cases.
Superscript I, 0	CLSC equilibrium solution with centralized decision-making without government intervention, CLSC equilibrium solution with decentralized decision-making without government intervention.
Subscript m, r, c, or g	Manufacturer's, Retailer's, Collector's, or Government's.

Notes: $c_n > c_r$, $0 \leq \tau \leq 1$, $0 \leq \tau_0 \leq 1$, $\Delta > b > c$ (explained in assumptions).

4. Models

4.1. Models without Government Intervention

(1) Centralized CLSC model (Benchmark Case1)

From the notations and assumptions provided (Section 3.1), the profit functions of the manufacturer, retailer, and collector without government intervention were obtained. The manufacturer decides the WEEE's buyback price and the new product's wholesale price. The collector depends on the collection rate to maximize profit. Maximizing profit through the retail price is the retailer's goal. Without government intervention, the manufacturer's profit is equal to the revenue from selling the product minus the cost of recycling and manufacturing the product; the retailer's profit is equal to the revenue from selling the

product minus the cost of the wholesale product; and the profit of the collector is equal to the revenue of the collector selling the recycled product to the manufacturer minus its own recycling cost. Consequently, the CLSC partner's goal functions are provided by

$$\max_{w,b} \pi_m = [w - c_n + (\Delta - b)\tau](\phi - p) \tag{1}$$

$$\max_{p} \pi_r = (p - w)(\phi - p) \tag{2}$$

$$\max_{\tau} \pi_c = (b - c)(\phi - p)\tau - m\tau^2/2 \tag{3}$$

Consumer surplus is the difference between the price consumers are willing to pay and the price they pay if they buy a given amount of goods. Hence, the total consumer surplus is denoted as the area of the triangle around the price axis, above the price line and under the demand curve. Then, the consumer surplus in the CLSC is denoted as:

$$C_s = (\phi - p)^2/2 \tag{4}$$

The environmental damage effect is reduced by the collection and remanufacturing of WEEEs. For convenience, we consider this effect as environmental benefits for the reason that environmental benefits are equivalent to less environmental damage. E represents the environmental benefits obtained by collecting and remanufacturing the WEEEs. Generally, the environmental benefits are in proportion to the return quantity, which is given as follows:

$$E = \beta\tau(\phi - p) \tag{5}$$

The social welfare is the sum of the consumer surplus, environmental benefits, and the profits of the collector, the retailer and the manufacturer. From Equations (1)–(5), we obtain the social welfare π_g as follows:

$$\pi_g = \frac{[\phi + p - 2c_n + 2(\Delta - c + \beta)\tau](\phi - p) - m\tau^2}{2} \tag{6}$$

According to Equations (1)–(3), the maximization profit function of the CLSC can be expressed as:

$$\max_{p,\tau} \pi = \pi_m + \pi_r + \pi_c = [p - c_n + (\Delta - c)\tau](\phi - p) - m\tau^2/2 \tag{7}$$

From Equation (7), we obtain $2m - (\Delta - c)^2 > 0$ from the equivalent relationship between the negative definition of the Hessian matrix and the quadratic function. The solution process is given in the Appendix A.1.

Then, the social welfare is derived as below:

$$\pi_g^I = \pi^I + C_s^I + E^I = \frac{m(\phi - c_n)^2[3m + 2\beta(\Delta - c) - (\Delta - c)^2]}{2[2m - (\Delta - c)^2]^2} \tag{8}$$

(2) Decentralized CLSC model (Benchmark Case 2)

In the decentralized benchmark Case 2, the collector, the retailer, and the manufacturer's profits are presented by Equations (1)–(3), respectively. The manufacturer is the CLSC Stackelberg leader without government guidance, then the game sequence is in the below.

In stage 1, the wholesale price w and the buyback price b are decided by the manufacturer;

In stage 2, a decision is made by the retailer regarding the decision result that the manufacturer makes: the retail price p is determined by the retailer.

In stage 3, the collector makes decisions about the decision result that the retailer makes: the collector determines the collection rate τ.

The backward induction is applied to solve these variables' equilibrium solutions. The detailed solving process is given in Appendix A.2.

Based on these solutions, the social welfare is calculated as follows:

$$\pi_g^0 = \pi_m^0 + \pi_r^0 + \pi_c^0 + C_s^0 + E^0 = \frac{m(\phi - c_n)^2[28m + 4\beta(\Delta - c) - (\Delta - c)^2]}{2[8m - (\Delta - c)^2]^2} \tag{9}$$

4.2. The Model under RPM for the Manufacturer (RPM Case)

The literature demonstrates that RPM is an effective incentive mechanism for the government to intervene in CLSC partners in collecting WEEEs (Wang et al., 2015 [11]). Different from the cases in Section 4.1, in this case, the government serves as the CLSC's Stackelberg leader, while the secondary leader is the manufacturer in the CLSC. The sequence of the event is as follows:

In stage 1, the reward-penalty intensity k is enacted by the government.

In stage 2, the buyback price b and the wholesale price w is set by the manufacturer simultaneously.

In stage 3, the retail price p is determined by the retailer.

In stage 4, the collector decides the collection rate τ.

In this case, $k(\tau - \tau_0) + \alpha k^2$ is the cost that the government needs to bear to implement RPM. The total cost is divided into fixed cost αk^2 and variable cost $k(\tau - \tau_0)$ depending on the collection rate. In this equation, α ($\alpha > 0$) expresses the fixed cost parameter for RPM implementation. Under RPM, the manufacturer's profit is made up of the sum of the revenue from sales and government bonuses minus the cost of recycling and production and government penalties. The profits of the retailer and collector under RPM is the same as in the scenario without government intervention. Thus, the objective function of firms and governments can be expresses as follows.

$$\max_{w,b} \pi_m = (\phi - p)(w - c_n + \tau\Delta - \tau b) + k(\tau - \tau_0) \tag{10}$$

$$\max_{p} \pi_r = (p - w)(\phi - p) \tag{11}$$

$$\max_{\tau} \pi_c = \tau(\phi - p)(b - c) - m\tau^2/2 \tag{12}$$

$$\max_{k} \pi_g = (\phi - p)(p - c_n + \tau\Delta - \tau c + \tau\beta) - m\tau^2/2 - (\phi - p)^2/2 - \alpha k^2 \tag{13}$$

Utilizing backward induction, where the solving sequence is inverse to the decision-making sequence of the event, we obtain the following equilibrium results:

$$k^* = \frac{8\Delta_2 + \Delta_3(-2c_n{\Delta_3}^2 - \beta\Delta_4\Delta_3 + 14c_n \cdot m - 6m\phi)}{5{\Delta_3}^2 + 8\beta\Delta_3 - 16m - 2\alpha(8m + {\Delta_3}^2)} \tag{14}$$

$$\tau^* = \frac{\Delta_4\Delta_3}{\Delta_1} + \frac{32\Delta_2 + 4\Delta_3(-2c_n{\Delta_3}^2 - \beta\Delta_4\Delta_3 + 14c_n \cdot m - 6m\phi)}{\Delta_1[5{\Delta_3}^2 + 8\beta\Delta_3 - 16m - 2\alpha(8m + {\Delta_3}^2)]} \tag{15}$$

$$p^* = \frac{2m(3\phi + c_n) - \phi{\Delta_3}^2}{\Delta_1} - \frac{8\Delta_2\Delta_3 + {\Delta_3}^2(-2c_n{\Delta_3}^2 - \beta\Delta_4\Delta_3 + 14c_n \cdot m - 6m\phi)}{\Delta_1[5{\Delta_3}^2 + 8\beta\Delta_3 - 16m - 2\alpha(8m + {\Delta_3}^2)]} \tag{16}$$

$$w^* = \frac{4m\Delta_4 - \phi{\Delta_3}^2}{\Delta_1} - \frac{16\Delta_2\Delta_3 + 2{\Delta_3}^2(-2c_n{\Delta_3}^2 - \beta\Delta_4\Delta_3 + 14c_n \cdot m - 6m\phi)}{\Delta_1[5{\Delta_3}^2 + 8\beta\Delta_3 - 16m - 2\alpha(8m + {\Delta_3}^2)]} \tag{17}$$

$$\begin{aligned} b^* = {} & \frac{m(\phi - c_n)(\Delta + c)[5{\Delta_3}^2 + 8\beta\Delta_3 - 16m - 2\alpha(8m + {\Delta_3}^2)]}{4m{\Delta_3}^2[\Delta_4 + \alpha(c_n - \phi)] + 8(\phi - c_n)[m\beta\Delta_3 - 4m^2(1 + \alpha)] - 2c_n{\Delta_3}^4 - \beta\Delta_4{\Delta_3}^3} \\ & + \frac{(4m + c\Delta_3)[8\Delta_2 + \Delta_3(-2c_n{\Delta_3}^2 - \beta\Delta_4\Delta_3 + 14c_n \cdot m - 6m\phi)]}{4m{\Delta_3}^2[\Delta_4 + \alpha(c_n - \phi)] + 8(\phi - c_n)[m\beta\Delta_3 - 4m^2(1 + \alpha)] - 2c_n{\Delta_3}^4 - \beta\Delta_4{\Delta_3}^3} \end{aligned} \tag{18}$$

In this paper, we set $\Delta_1 = 8m - (\Delta - c)^2$, $\Delta_2 = m\beta(c_n - \phi)$, $\Delta_3 = \Delta - c$ and $\Delta_4 = \phi + c_n$ for simplicity.

4.3. The Model under SM for the Collector (SM Case)

Section 4.2 provides the RPM case. In this section, we investigate a popular mechanism, SM, in which the government subsidizes the collector based on the CLSC's actual collection rate. The intensity of subsidies related to the actual collection rate was investigated. The decision-making order was the same as the RPM case.

In this scenario, $k\tau + \alpha k^2$ expresses the SM's whole cost, which includes the variable cost and the fixed cost. α represents the fixed cost parameter of the government implementing the SM. Under SM, the collector's profit consists of the sum of revenue of the collector selling the recycled product to manufacturer and government subsidy minus its own recycling cost. The profits of the manufacturer and retailer under SM is the same as the scenario without government intervention. Therefore, the CLSC partners' goal under SM are as below:

$$\max_{w,b} \pi_m = (\phi - p)(w - c_n + \tau\Delta - \tau b) \tag{19}$$

$$\max_{p} \pi_r = (p - w)(\phi - p) \tag{20}$$

$$\max_{\tau} \pi_c = \tau(\phi - p)(b - c) - m\tau^2/2 + k\tau \tag{21}$$

and the objective function of the government is $\max_{k} \pi_g = \pi_m + \pi_r + \pi_c + C_s + E - k\tau - \alpha k^2$. Through computation the following is obtained.

$$\max_{k} \pi_g = (\phi - p)(p - c_n + \tau\Delta - \tau c + \tau\beta) - m\tau^2/2 - (\phi - p)^2/2 - \alpha k^2 \tag{22}$$

According to backward induction, the following equilibrium results are obtained:

$$k^{**} = \frac{(\phi - c_n) \cdot (6m\Delta_3 + \beta{\Delta_3}^2) - 8\Delta_2}{2\alpha{\Delta_1}^2 - 5{\Delta_3}^2 - 8\Delta_3\beta + 16m} \tag{23}$$

$$\tau^{**} = \frac{\Delta_3(\phi - c_n)}{\Delta_1} + \frac{4[(\phi - c_n) \cdot (6m\Delta_3 + \beta{\Delta_3}^2) - 8\Delta_2]}{\Delta_1(2\alpha{\Delta_1}^2 - 5{\Delta_3}^2 - 8\Delta_3\beta + 16m)} \tag{24}$$

$$p^{**} = \frac{2m(3\phi + c_n) - \phi{\Delta_3}^2}{\Delta_1} - \frac{\Delta_3[(\phi - c_n) \cdot (6m\Delta_3 + \beta{\Delta_3}^2) - 8\Delta_2]}{\Delta_1(2\alpha{\Delta_1}^2 - 5{\Delta_3}^2 - 8\Delta_3\beta + 16m)} \tag{25}$$

$$w^{**} = \frac{4m(\phi + c_n) - \phi{\Delta_3}^2}{\Delta_1} - \frac{2(\phi - c_n)\Delta_3(6m\Delta_3 + \beta{\Delta_3}^2 + 8m\beta)}{\Delta_1(2\alpha{\Delta_1}^2 - 5{\Delta_3}^2 - 8\beta\Delta_3 + 16m)} \tag{26}$$

$$b^{**} = \frac{\Delta + c}{2} - \frac{\Delta_1[6m\Delta_3 + \beta{\Delta_3}^2 + 8m\beta]}{2\beta{\Delta_3}^3 + 8m\alpha{\Delta_1}^2 - 8m{\Delta_3}^2 - 16m\Delta_3\beta + 64m^2} \tag{27}$$

5. Properties of the Equilibrium Solution

In this section, we analyze the properties of equilibrium solutions of decision variables.

Proposition 1. *The collection rates under different mechanisms always hold:*

(1) $\tau^* > \tau^0$, *if* $\frac{1}{8}(\Delta - c)(3 + 10\alpha) \le \beta \le \frac{(\Delta - c)(10\alpha \cdot c_n + 6\phi - 7c_n)}{2(7c_n - 5\phi)}$;

(2) $\tau^{**} > \tau^0$, *if* $0 \le \beta \le \frac{9}{4}\alpha(\Delta - c)^3 + \frac{3}{8}(\Delta - c)$.

Proposition 1 suggests that if the environmental benefit coefficient is in a moderate range, RPM and SM can improve the collection rate of WEEEs. As for manufacturers, the implementation of RPM by the government will encourage the manufacturer to rise the buyback price to collect more WEEEs to remanufacture, which decreases the manufacturer's cost. Furthermore, if the actual collection rate of WEEEs meets or exceeds the goal collection rate regulated by the government, the manufacturer receives a bonus from the government. Therefore, the manufacturer is willing to collect more WEEEs. As for a collector, the

conduction of SM by the government will incentivize the collector to recycle more WEEEs and sell them to the manufacturer for remanufacturing, which increases the collector's profit. Similarly, if the actual collection rate of WEEEs meets or exceeds the goal collection rate regulated by government, the collector also receives a subsidy from the government. As a result, RPM and SM are both effective methods by which to enhance the collection of WEEEs if the environmental benefit coefficient is moderate.

Proposition 2. $\frac{\partial k^*}{\partial \alpha} < 0$, $\frac{\partial k^{**}}{\partial \alpha} < 0$, $\frac{\partial k^*}{\partial \beta} > 0$, $\frac{\partial k^{**}}{\partial \beta} > 0$.

From Proposition 2, we find that the intensity of RPM for the manufacturer decreases as is the case with SM for the collector with the increase in the fixed cost parameter of the government's input. This is consistent with our institutional experience, which shows that the more efficient the government is at investing, the less it needs to invest, which in turn indicates that the implementation of RPM and SM requires high investment efficiency rather than higher costs. Moreover, Proposition 2 indicates that the intensity of RPM for the manufacturer increases as is the case with SM for the collector, with increased environmental benefits, which underlines the importance of government guidance for CLSC partners to protect the environment. The reason may be that the government's guidance effectively promotes the recycling of WEEEs, thus, the government's implementation intensity also increases gradually with the improvement of the environmental efficiency coefficient.

Proposition 3. $\frac{\partial \tau^*}{\partial \alpha} < 0$, $\frac{\partial \tau^{**}}{\partial \alpha} < 0$, $\frac{\partial \tau^*}{\partial \beta} > 0$, $\frac{\partial \tau^{**}}{\partial \beta} > 0$.

Proposition 3 demonstrates a similar pattern to Proposition 2. In both mechanisms, the collection rate decreases with the mechanism input cost parameter and increases with the environmental benefit coefficient. If the cost of the government's input increases, the efficiency of the government's implementation mechanism decreases, so the government reduces the intensity of mechanisms' input. Without government incentives, the manufacturer and collector would be less eager to recycle more WEEEs. Therefore, a higher mechanism input cost parameter lowers WEEEs' collection rate. In addition, a higher environmental benefit coefficient indicates that unit WEEE recycling is more beneficial to the environment. Consequently, the government desires strengthening intervention mechanisms to promote waste-product recycling.

Proposition 4. $\frac{\partial b^{**}}{\partial \alpha} > 0$, $\frac{\partial b^{**}}{\partial \beta} < 0$.

Proposition 4 indicates that with the increase in the fixed cost parameter, the buyback price of the collector increases, which is conducive to improving the enthusiasm of collector in the case of SM. Additionally, the buyback price of the collector under SM decreases with the environmental benefit coefficient. At the same time, the enthusiasm of the collector decreases, which is harmful to environmental protection. Therefore, the government needs to increase subsidies to compensate the collector.

6. Numerical Experiments

The following section compares the decision results in four scenarios utilizing numerical examples. We investigated data from multiple retail and remanufacturing companies in mainland China and Taiwan. In combination with real situations in practice, we set the WEEE product's parameters as follows: $\phi = 27$, $c_n = 20$, $\Delta = 3.56$, $c = 2.2$, $\tau_0 = 0.01$, $\alpha = 0.01$, $\beta = 5$, $m = 20$. The solutions of the decision-making parameters and the firm's profits under four cases are given in Table 2 below.

Table 2. The four cases' decision-making outcomes and firm's profit.

	Benchmark Case 1	Benchmark Case 2	RPM Case	SM Case
w	–	23.46	23.27	23.30
p	23.33	25.23	25.13	25.15
b	–	2.88	5.90	0.45
τ	0.25	0.06	0.757	0.29
k	–	–	11.27	9.00
π_m	–	6.20	10.74	7.76
π_r	–	3.13	3.49	3.41
π_c	–	0.04	0.01	0.83
π	–	9.37	14.24	12.00
π_g	24.16	11.47	10.00	9.56

From Table 2, we get the observations as follows.

(i) Compared with the benchmark case, in RPM and SM, the wholesale price and retail price are lower; the collection rate and buyback price are higher; and the profits of the manufacturer, retailer, collector and CLSC increase. For the cost of two mechanisms, the government's benefit, i.e., social welfare, reduces.

(ii) Compared with the SM case, in the RPM case, the wholesale and retail price are lower; the collection rate, buyback price and incentive intensity are higher; the profits of the manufacturer and retailer are higher; the profit of the collector is lower; and the government benefit (social welfare) is higher.

To sum up, rewarding or penalizing the manufacturer enhances the CLSC's profit and social welfare as well as the collection rate of WEEEs. As the price of new products drops, the price advantage enhances the competitiveness of the manufacturer's products. The CLSC's profit increases, which improves the enthusiasm of all partners in the CLSC. However, the collection rate, CLSC's profit and social welfare under SM are all lower than those under RPM. Thus, RPM is preferred.

6.1. Decision Variables under RPM and SM vs. Fixed Cost Coefficient and Environmental Benefit Coefficient

We further examined the change in the main decision variables by assessing the fixed cost of mechanism input and environmental benefit under RPM and SM. Some observations are outlined below.

Observation 1. *As can be observed from the left of Figure 2, the intensity of both mechanisms decreased with the fixed cost of mechanism input. If* $0 < \alpha < 0.1$*, the intensity under SM in higher than RPM, while the intensity under SM is lower than RPM with the fixed input cost increasing. The right part of Figure 2 demonstrates that the subsidy intensity increases slowly with the increase in the environmental benefit coefficient, while the intensity of RPM increases rapidly.*

Observation 1 indicates that if $\alpha > 0.1$, RPM has a greater advantage in rewarding or penalizing the manufacturer, which indicates that the implementation of RPM is more efficient based on the same unit input cost than SM. Additionally, if the environmental benefit coefficient is small, the RPM intensity for the manufacturer is higher than the SM intensity for the collector. However, with the increase in the environmental benefit coefficient, the intensity of RPM increases sharply, while the modification of the subsidy intensity is small. This phenomenon shows that the government needs to increase rewards and penalties for manufacturers in order to increase environmental benefits, but there is no great requirement for subsidizing the collector. Therefore, from the perspective of mechanism intensity and long-term environmental benefits, the cost of the implementation of SM is lower than that of RPM. In other words, it is feasible to implement SM.

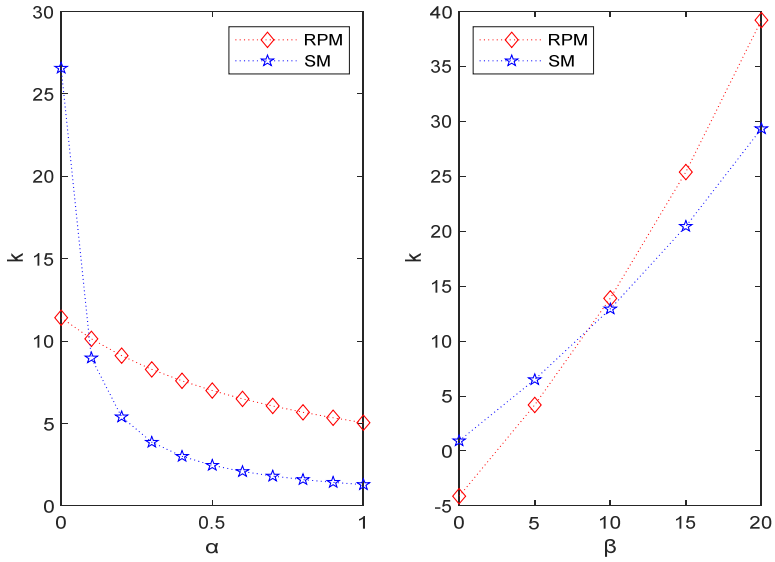


Figure 2. Intensity under RPM and SM vs. fixed cost coefficient (**left**) and environmental benefit coefficient (**right**).

Observation 2. *The collection rate decreases under both mechanisms as the fixed cost of the mechanism input, as shown in Figure 3. However, the collection rate under RPM is always higher than that under SM. Besides, with the increase in the environmental benefit coefficient, the collection rate increases under RPM and SM.*

From Observation 2, we discover that under the same unit mechanism investment fixed cost, the implementation of RPM can provide a higher collection rate than the conduction of SM, indicating that the implementation of RPM is efficient. From Figure 3, we also perceive that the increase in government input cost imparts a restraining effect on the collection rate of WEEEs. In addition, if the environmental benefit coefficient is smaller, the collection rate under RPM is lower than that under SM. However, with the increase in the environmental benefit coefficient, the difference in the collection rate between the two mechanisms becomes diminished. Thus, RPM is easier to implement from the perspective of the collection rate.

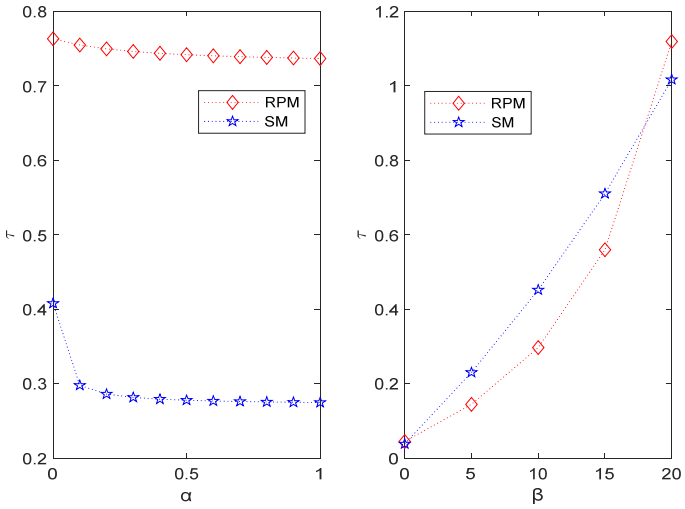


Figure 3. Collection rate under RPM and SM vs. fixed cost coefficient (**left**) and environmental benefit coefficient (**right**).

Observation 3. *As shown on the left of Figure 4, the retail price under both mechanisms increases as the fixed cost of the mechanism input increases. If* $0 < \alpha < 0.1$*, the retail price under RPM is higher than SM, while the retail price under RPM is lower than SM with the fixed input cost. As shown on the right of Figure 4, with the increase in the environmental benefit coefficient, the retail price decreases slowly under the SM, while the retail price decreases sharply under RPM.*

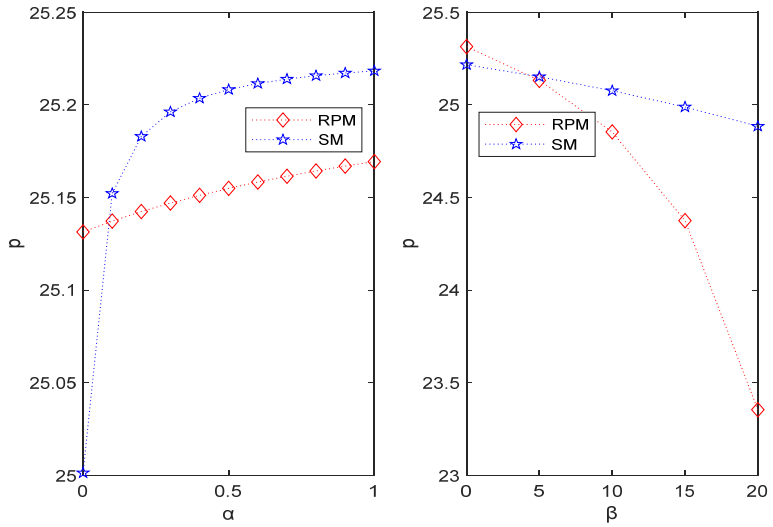


Figure 4. Retail price under RPM and SM vs. fixed cost coefficient (**left**) and environmental benefit coefficient (**right**).

Observation 3 shows that if $\alpha > 0.1$, under the same unit fixed cost input, the implementation of SM has a higher retail price than the implementation of RPM, which indicates that the implementation of SM is not only conducive to encouraging the collector to collect WEEEs, but can also stimulate the enthusiasm of retailers. Furthermore, if the environmental benefit coefficient is small, the retail price under SM is lower than that under RPM, but if the environmental benefit coefficient increases by more than 5, the retail price under SM will be higher than that under RPM. Thus, RPM is more effective than SM in terms of its higher environmental benefit and lower retail price.

In Figure 5, it can be seen that the wholesale price alters with the two coefficients in a similar mode to the retail price, thus, we will not explain it here.

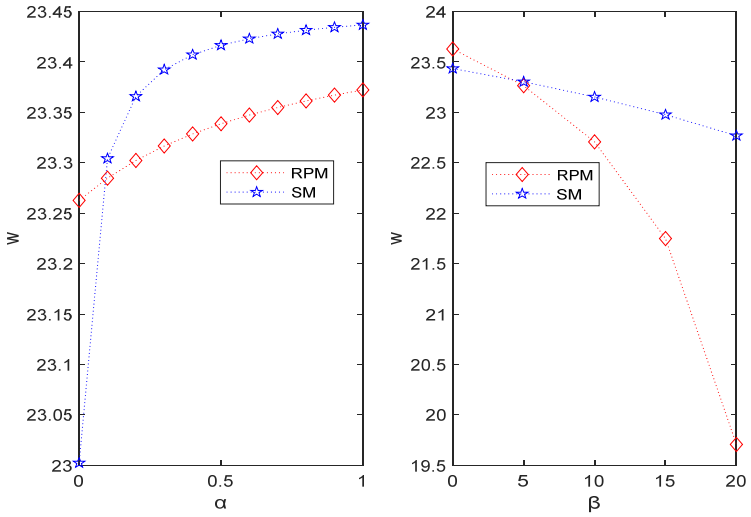


Figure 5. Wholesale price under RPM and SM vs. fixed cost coefficient (**left**) and environmental benefit coefficient (**right**).

Observation 4. *As shown on the left of Figure 6, with the increase in the fixed cost of mechanism input, the buyback price under RPM decreases while it increases under SM, narrowing the gap between the two gradually. However, the buyback price under RPM is still higher than that under SM. As shown on the right of Figure 6, with the increase in the environmental benefit coefficient, the buyback price under SM decreases slowly, while the buyback price under RPM increases sharply.*

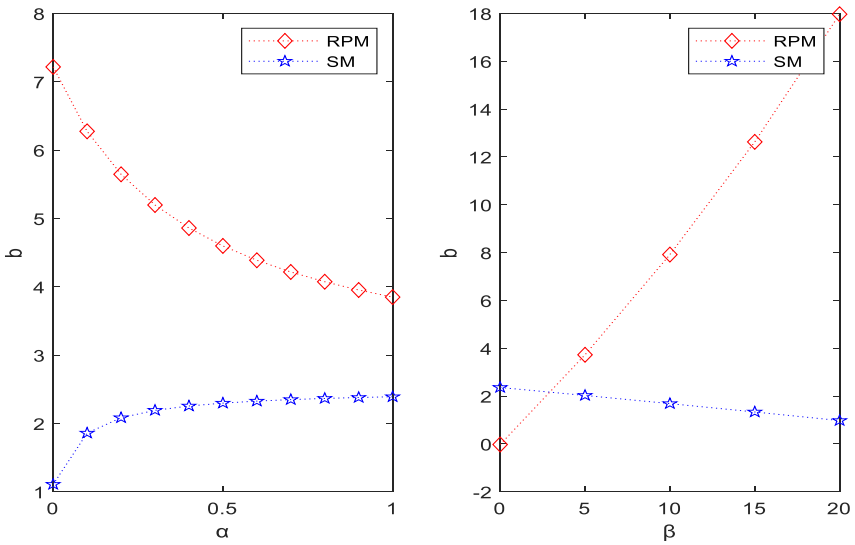


Figure 6. Buyback price under RPM and SM vs. fixed cost coefficient (**left**) and environmental benefit coefficient (**right**).

Observation 4 reveals that under the same unit fixed cost input, the implementation of RPM leads to a higher buyback price than the implementation of SM, which increases the incentive of the manufacturer to encourage downstream firms to collect and remanufacture WEEEs. In addition, the buyback price under SM is slightly affected by the change in the environmental benefit coefficient, while the buyback price under RPM is greatly affected by the change in the environment benefit coefficient. If the unit WEEE recycling environmental benefits are large and if the government enhances the intensity under RPM for the manufacturer, the manufacturer will increase the buyback price to encourage the collector to recycle more WEEEs; however, if the government raises the subsidies under SM for the collector, the manufacturer's interests are not raised; therefore, the buyback price under SM changes only slightly. This result is consistent with the conclusion of Proposition 4.

6.2. Profit of CLSC Partners and Social Welfare vs. Fixed Cost Coefficient and Environmental Benefit Coefficient under RPM and SM

The above observations take into account the influence of the two coefficients on the decision-making variables of the two mechanisms. In the following, the joint influence of the two coefficients on the profit of supply chain partners and social welfare is further compared and analyzed.

Observation 5. *Figures 7–10 illustrate that RPM leads to greater social welfare and increases profits for the manufacturer, while the collectors' profit under SM is greater. RPM and SM have no great difference with respect to the retailer's profit.*

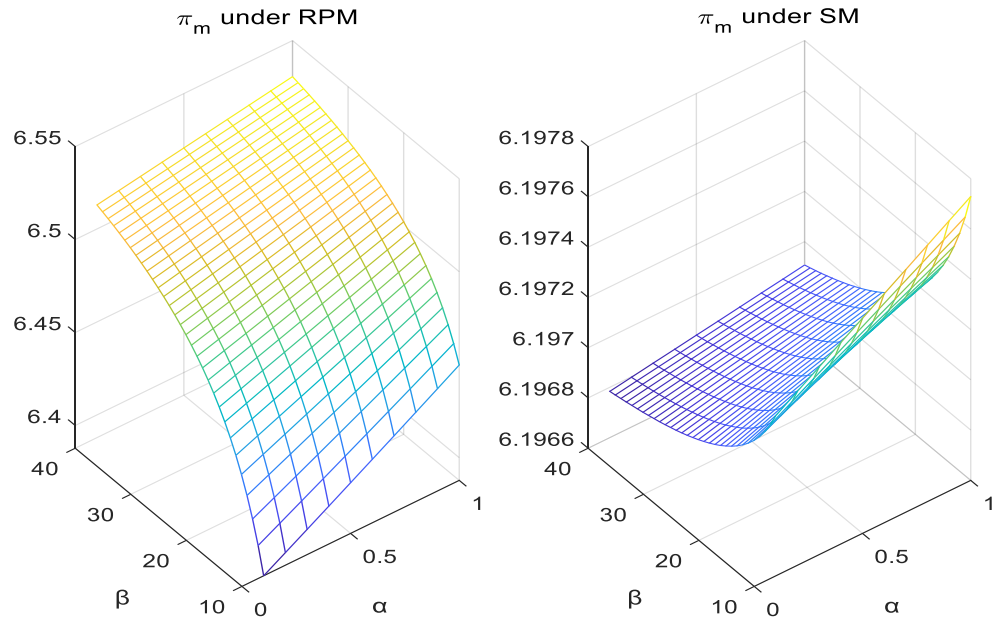


Figure 7. Manufacturer's profit vs. fixed cost coefficient and environmental benefit coefficient under RPM (**left**) and SM (**right**).

From Figure 7, under both mechanisms, it can be seen that the manufacturer's profit increases with the fixed cost input coefficient, but varies with the environmental benefit coefficient. The manufacturer's profit under RPM increases with the environmental coefficient, but decreases under SM. This may be because the government implements rewards and fines for the manufacturer to incentivize them to collect more WEEEs under RPM, resulting in an increase in collection rates and environmental benefits. However, in SM the collector is subsidized, which does not motivate the manufacturer to collect and remanufacture, so the manufacturer's profit decreases with the increase in environmental benefits. Additionally, under the same unit fixed cost input and environmental benefit coefficient, the manufacturer's profit under RPM is higher than that under SM. Therefore, from the perspective of manufacturer's profit and collection rate, RPM is more effective.

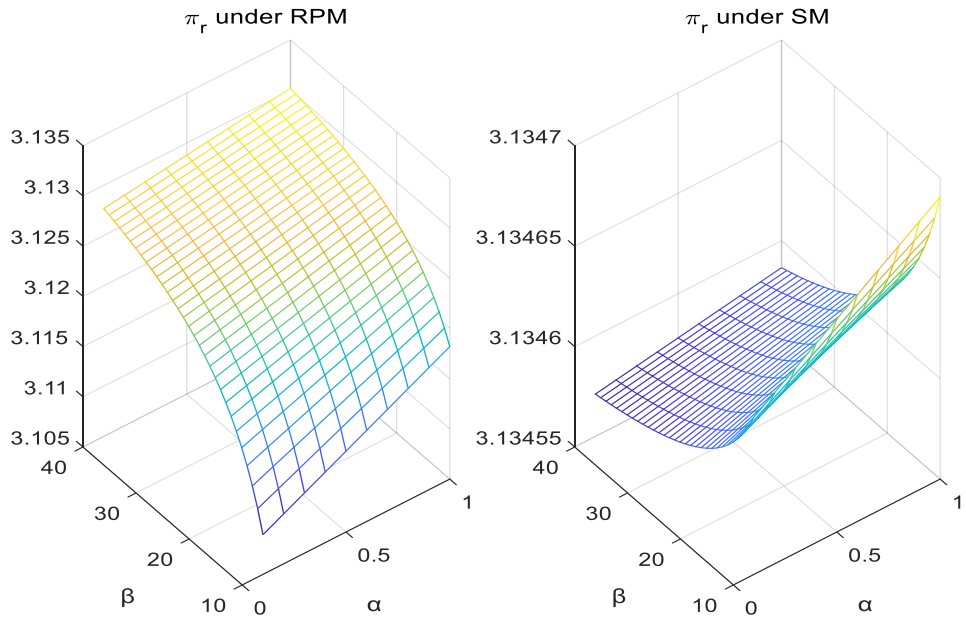


Figure 8. Retailer's profit vs. fixed cost coefficient and environmental benefit coefficient under RPM (**left**) and SM (**right**).

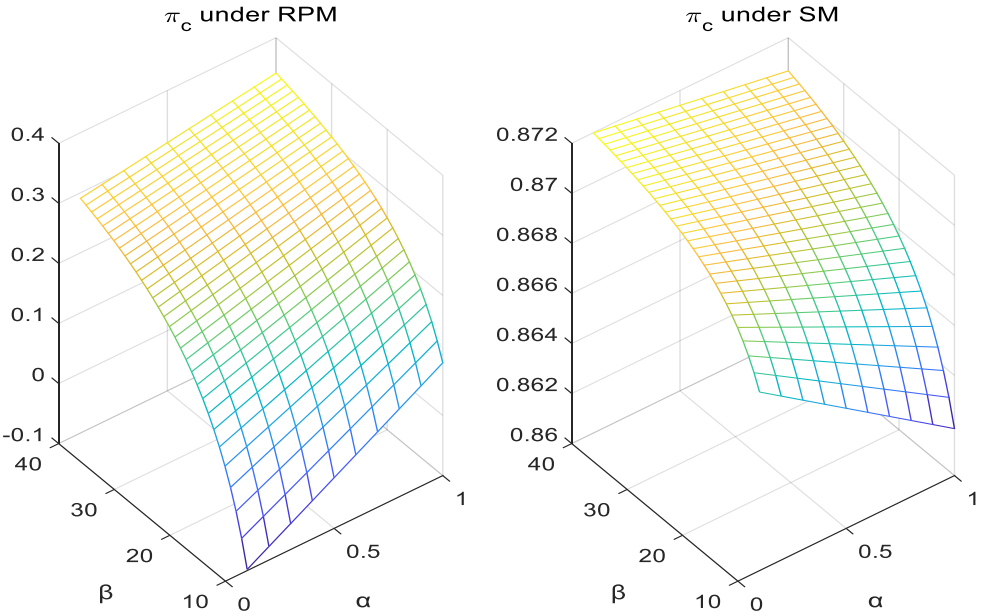


Figure 9. Collector's profit vs. fixed cost coefficient and environmental benefit coefficient under RPM (**left**) and SM (**right**).

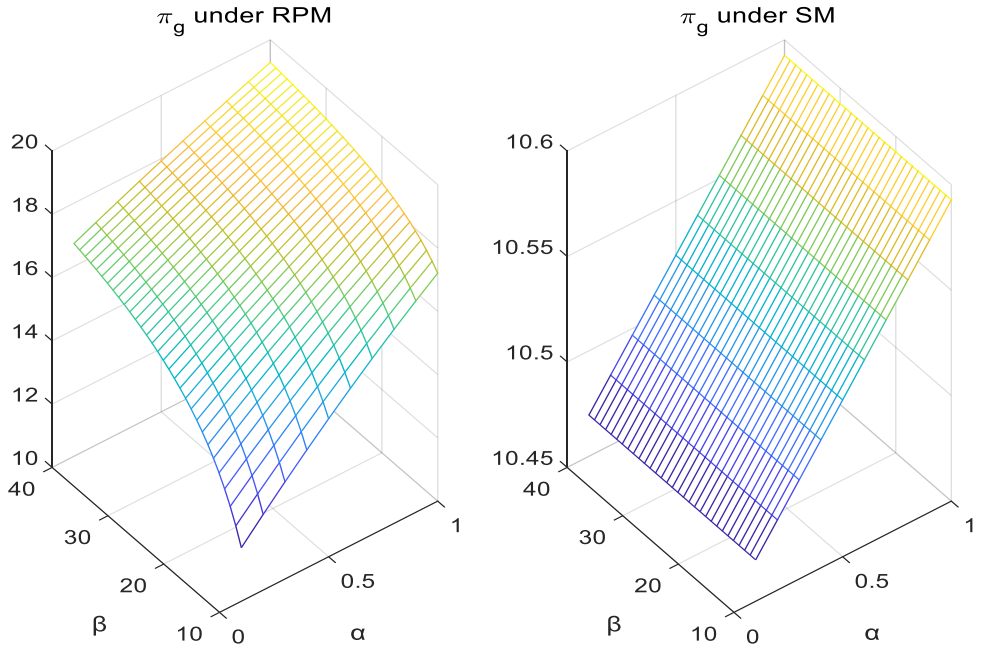


Figure 10. Social welfare vs. fixed cost coefficient and environmental benefit coefficient under RPM (**left**) and SM (**right**).

Figure 8 suggests that the trend in the retailer's profit is similar to that of the manufacturer, which is not repeated here.

In Figure 9, it can be seen that the collector's profit increases with the environmental benefit coefficients under both mechanisms. The reason may be that the larger the environmental benefit coefficient is, the more WEEEs are collected, and the higher the collector's profits will be. However, the alteration is different with the increase in the fixed cost input. Under RPM, the profit of the collector increases with the fixed cost input, while it decreases under SM, which demonstrates that government input is costly, but the low efficiency leads to profit reduction for the collector. The high efficiency under RPM not only increases the profit of the manufacturer, but also contributes to increasing the profit of the collector. However, under the same unit fixed cost input and environmental benefit coefficient, the

profit of the collector under RPM is lower than that under SM. Therefore, in terms of the collector's profit, the SM can more effectively promote the collection of WEEEs.

As shown in Figure 10, social welfare increases with the fixed cost input coefficient but varies with the increased environmental benefit coefficient under both mechanisms. Social welfare increases with the environmental benefit coefficient under RPM while it does not change under SM. Additionally, under the same unit fixed cost input and environmental benefit coefficient, the social welfare under RPM is higher than that under SM. Thus, considering social welfare, RPM is more effective than SM.

7. Conclusions

Comparing the effects of RPM and the SM on the collection rate, the retail price, the wholesale price, the buyback price, firm's profit and social welfare, this paper investigated the advantages and disadvantages of two mechanisms to provide theoretical evidence for ways the government can induce CLSCs to collect and remanufacture WEEEs. The main conclusions are as follows.

(i) Both RPM and the SM are effective in improving the collection rate. However, the higher the fixed cost the government invests under these two mechanisms, the lower the intensity of the mechanisms and the collection rate.
(ii) SM has the advantage that the profit of the collector is higher than that under RPM, which can improve the enthusiasm of the collector. RPM has the advantage that the buyback price improves, which helps the government to induce the CLSC to collect more WEEEs to remanufacture.
(iii) RPM for the manufacturer is more effective than SM for the collector. Compared with SM for the collector, RPM for the manufacturer not only improves the collection rate, but also leads to increased social welfare and reduces the price of new products. Besides, the profits of all the partners of the CLSC except the collector increase, which promotes the enthusiasm of these partners. Furthermore, RPM optimizes the allocation of resources and maximizes the benefits of the CLSC. Therefore, RPM is feasible to implement for the manufacturer.

Previous studies indicated that RPM and SM both promote the recycling of WEEEs effectively, which is consistent with our conclusion. However, this paper further examined the difference between RPM and SM and considered the government as a game player. We found the social welfare and manufacturer and retailer's profits are higher under RPM than SM. Besides, RPM improves the buyback price and decreases the retail price to motivate the sale of WEEEs and their collection. In conclusion, RPM has more advantages in implementation.

We proposed the government as the leader for the Stackelberg game, whose decision-making objective is to maximize social welfare. Considering the government not only can reward or penalize the manufacturer, but can subsidize the collector, this paper contributes to the literature by highlighting the advantages of RPM for manufacturers and SM for collectors in theory. Additionally, our research also provides some managerial insights for future policy-making decisions by governments and decision making by partners of CLSCs in practice.

(i) The government, using RPM, can encourage the manufacturer to acquire a higher WEEE's collection rate and increase social welfare to a greater extent compared to the effects of SM for the collector. Under the same unit cost input, the intensity under RPM is greater than that under SM. Thus, the government should strengthen the reward and penalty behavior for the manufacturer to promote the manufacturer to increase the buyback price. The higher buyback price will stimulate the collector to collect more WEEEs.
(ii) For the manufacturer, the implementation of RPM improves their own profit while it damages the collector's profit. Therefore, the manufacturer should enhance the

buyback price or share their profits with the collector to stimulate the recycling of WEEEs.

(iii) For the retailer, RPM will reduce the wholesale price of new products for the manufacturer to increase the retailer's profits. Therefore, the retailer agrees with rewards and penalties for the manufacturer.

(iv) For the collector, although the SM implemented by the government increases his own profit, it reduces the profits of other partners, which is not conducive to the coordinated development of the whole CLSC. As a result, if the government implements SM, the collector should work closely with other partners and share some profits to avoid supply chain disruption.

This paper does not explore the case with RPM for the collector. RPM is a less effective way by which to subsidize the collector since RPM is uncommon in developing countries such as China and India, where collecting WEEEs is in the pilot phase. The majority of collectors operate at a loss. Therefore, providing them with subsidies is realistic. This case can be explored in future research.

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Appendix A

Appendix A.1. Centralized Decision-Making Model

The CLSC is an idealized super organization in this case, from Equation (1) to Equation (3), the maximization profit function of the CLSC is represented as

$$\max_{p,\tau} \pi = \pi_m + \pi_r + \pi_c = [p - c_n + (\Delta - c)\tau](\phi - p) - m\tau^2/2 \tag{A1}$$

To ensure Equation (A1) is concave and to obtain the maximum, the Hessian matrix $\begin{pmatrix} -2 & \Delta - c \\ \Delta - c & -m \end{pmatrix}$ should be negative definite. The discrimination matrix's first order principal minor sequence is negative $-2 < 0$ and the discrimination matrix's second order principal minor sequence is positive $2m - (\Delta - c)^2 > 0$. Then, the function of the maximization of profit is a concave function and has a maximum. According to $\partial\pi/\partial p = 0$ and $\partial\pi/\partial\tau = 0$, we gain p and τ by the first order conditions outlined below.

$$p^I = \frac{m(\phi + c_n) - \phi(\Delta - c)^2}{2m - (\Delta - c)^2} \tag{A2}$$

$$\tau^I = \frac{(\phi - c_n)(\Delta - c)}{2m - (\Delta - c)^2} \tag{A3}$$

By substituting Equations (A2) and (A3) into Equations (4) and (5) and (A1), we obtain

$${C_s}^I = \frac{m^2(\phi - c_n)^2}{2[2m - (\Delta - c)^2]^2} \tag{A4}$$

$$E^I = \frac{m\beta(\phi - c_n)^2(\Delta - c)}{\left[2m - (\Delta - c)^2\right]^2} \tag{A5}$$

$$\pi^I = \frac{m(\phi - c_n)^2}{2[2m - (\Delta - c)^2]} \tag{A6}$$

By substituting Equation (A4) to Equation (A6) into Equation (A1), we obtain

$$\pi_g^I = \pi^I + C_s^I + E^I = \frac{m(\phi - c_n)^2[3m + 2\beta(\Delta - c) - (\Delta - c)^2]}{2\left[2m - (\Delta - c)^2\right]^2} \tag{A7}$$

Appendix A.2. Decentralized Decision-Making Model

The concrete solving process is divided into two stages which are inverse to the decision-making sequences as follows.

Firstly, from the first-order conditions $\partial\pi_r/\partial p = 0$ and $\partial\pi_c/\partial\tau = 0$, we obtain

$$p = \frac{\phi + w}{2} \text{ and } \tau = \frac{(b - c)(\phi - p)}{m}$$

Substituting p and τ into Equation (1), we obtain

$$\pi_m = [w - c_n + (\Delta - b)\frac{(b - c)(\phi - w)}{2m}] \cdot \frac{(\phi - w)}{2} \tag{A8}$$

From $\partial\pi_m/\partial b = 0$, we thus obtain

$$b^0 = (\Delta + c)/2 \tag{A9}$$

Substituting b^0, p and τ into Equation (1), we obtain

$$\pi_m = [w - c_n + \frac{(\Delta - c)^2(\phi - w)}{8m}]\frac{(\phi - w)}{2} \tag{A10}$$

Then, through the first order condition $d\pi_m/dw = 0$, we find

$$w^0 = \frac{4m(\phi + c_n) - \phi(\Delta - c)^2}{8m - (\Delta - c)^2} \tag{A11}$$

Substituting w^0 into the formula of p, we have

$$p^0 = \phi - \frac{2m(\phi - c_n)}{8m - (\Delta - c)^2} \tag{A12}$$

Substituting b^0 and p^0 into τ, we obtain

$$\tau^0 = \frac{(\phi - c_n)(\Delta - c)}{8m - (\Delta - c)^2} \tag{A13}$$

Substituting Equations (A9) and (A11) to (A13) into Equations (1)–(6), we find the social welfare as below.

$$\pi_g^0 = \pi_m^0 + \pi_r^0 + \pi_c^0 + C_s^0 + E^0 = \frac{m(\phi - c_n)^2[28m + 4\beta(\Delta - c) - (\Delta - c)^2]}{2\left[8m - (\Delta - c)^2\right]^2} \tag{A14}$$

Appendix A.3. The Model of RPM for the Manufacturer

From the first order conditions $\partial\pi_r/\partial p = 0$ and $\partial\pi_c/\partial\tau = 0$, we have

$$p = \frac{\phi + w}{2} \text{ and } \tau = \frac{(b-c)(\phi - p)}{m}.$$

By substituting these response functions into π_m, from the first order conditions $\partial\pi_m/\partial w = 0$ and $\partial\pi_m/\partial b = 0$, we get

$$b = \frac{m(\phi - c_n)(\Delta + c) + 4mk + c(\Delta - c)k}{2m(\phi - c_n) + (\Delta - c)k} \text{ and } w = \frac{4m(\phi + c_n) - \phi(\Delta - c)^2 - 2(\Delta - c)k}{8m - (\Delta - c)^2}$$

Substituting b and w into p and τ, we find

$$p = \frac{2m(3\phi + c_n) - \phi(\Delta - c)^2 - (\Delta - c)k}{8m - (\Delta - c)^2} \text{ and } \tau = \frac{(\phi + c_n)(\Delta - c) + 4k}{8m - (\Delta - c)^2}.$$

Then, again, by substituting p and τ into the objective function of the government, i.e., Equation (11), and solving the first derivative of k, i.e., $d\pi_g/dk = 0$, we obtain

$$k^* = \frac{8\Delta_2 + \Delta_3(-2c_n{\Delta_3}^2 - \beta\Delta_4\Delta_3 + 14c_n \cdot m - 6m\phi)}{5{\Delta_3}^2 + 8\beta\Delta_3 - 16m - 2\alpha(8m + {\Delta_3}^2)} \tag{A15}$$

Substituting k^* into the other decision variables above, we can obtain their equilibrium results, respectively.

$$\tau^* = \frac{\Delta_4\Delta_3}{\Delta_1} + \frac{32\Delta_2 + 4\Delta_3(-2c_n{\Delta_3}^2 - \beta\Delta_4\Delta_3 + 14c_n \cdot m - 6m\phi)}{\Delta_1[5{\Delta_3}^2 + 8\beta\Delta_3 - 16m - 2\alpha(8m + {\Delta_3}^2)]} \tag{A16}$$

$$p^* = \frac{2m(3\phi + c_n) - \phi{\Delta_3}^2}{\Delta_1} - \frac{8\Delta_2\Delta_3 + {\Delta_3}^2(-2c_n{\Delta_3}^2 - \beta\Delta_4\Delta_3 + 14c_n \cdot m - 6m\phi)}{\Delta_1[5{\Delta_3}^2 + 8\beta\Delta_3 - 16m - 2\alpha(8m + {\Delta_3}^2)]} \tag{A17}$$

$$w^* = \frac{4m\Delta_4 - \phi{\Delta_3}^2}{\Delta_1} - \frac{16\Delta_2\Delta_3 + 2{\Delta_3}^2(-2c_n{\Delta_3}^2 - \beta\Delta_4\Delta_3 + 14c_n \cdot m - 6m\phi)}{\Delta_1[5{\Delta_3}^2 + 8\beta\Delta_3 - 16m - 2\alpha(8m + {\Delta_3}^2)]} \tag{A18}$$

$$\begin{aligned} b^* = {} & \frac{m(\phi - c_n)(\Delta + c)[5{\Delta_3}^2 + 8\beta\Delta_3 - 16m - 2\alpha(8m + {\Delta_3}^2)]}{4m{\Delta_3}^2[\Delta_4 + \alpha(c_n - \phi)] + 8(\phi - c_n)[m\beta\Delta_3 - 4m^2(1+\alpha)] - 2c_n{\Delta_3}^4 - \beta\Delta_4{\Delta_3}^3} \\ & + \frac{(4m + c\Delta_3)[8\Delta_2 + \Delta_3(-2c_n{\Delta_3}^2 - \beta\Delta_4\Delta_3 + 14c_n \cdot m - 6m\phi)]}{4m{\Delta_3}^2[\Delta_4 + \alpha(c_n - \phi)] + 8(\phi - c_n)[m\beta\Delta_3 - 4m^2(1+\alpha)] - 2c_n{\Delta_3}^4 - \beta\Delta_4{\Delta_3}^3} \end{aligned} \tag{A19}$$

Appendix A.4. Proof of Proposition 1

Proof of Proposition 1. If $x = \Delta - c$. (1) Extract the common factor $2/(8m - x^2)$ of $\tau^* - \tau^0$. Since this common factor is positive, the sign of $\tau^* - \tau^0$ depends on $\frac{c_n \cdot x^3 - 2\beta\phi \cdot x^2 - 12m\phi \cdot x + 6c_n \cdot \beta \cdot x^2 + 12c_n \cdot m \cdot x - 16m \cdot \alpha \cdot c_n \cdot x - 2\alpha \cdot c_n \cdot x^3 + 16m\beta(c_n - \phi)}{5x^2 + 8\beta x - 16m - 2\alpha(8m + x^2)}$ We know the denominator of this formula is $5x^2 + 8\beta x - 16m - 2\alpha(8m + x^2) > 5x^2 + 8\beta x - 8x^2 - 2\alpha(4x^2 + x^2) = 8\beta x - x^2(3 + 10\alpha)$, so when $8\beta x - x^2(3 + 10\alpha) \geq 0$ that is $\beta \geq (3 + 10\alpha)/8$, the denominator of this formula is positive. We also know the numerator of this formula $c_n \cdot x^3 - 2\beta\phi \cdot x^2 - 12m\phi \cdot x + 6c_n \cdot \beta \cdot x^2 + 12c_n \cdot m \cdot x - 16m \cdot \alpha \cdot c_n \cdot x - 2\alpha \cdot c_n \cdot x^3 + 16m\beta(c_n - \phi) > c_n \cdot x^3 - 2\beta\phi \cdot x^2 - 6x^2 \cdot \phi \cdot x + 6c_n \cdot \beta \cdot x^2 + 6c_n \times x^2 \cdot x - 8x^2 \cdot \alpha \cdot c_n \cdot x - 2\alpha \cdot c_n \cdot x^3 + 8x^2 \cdot \beta(c_n - \phi) = x^2[x \cdot (7c_n - 10\alpha \cdot c_n - 6\phi) + 2\beta(7c_n - 5\phi)]$, so $x^2[x \cdot (7c_n - 10\alpha \cdot c_n - 6\phi) + 2\beta(7c_n - 5\phi)] \geq 0$, the numerator of this formula is positive. $\phi \gg c_n$, so when $\beta < \frac{x(10\alpha \cdot c_n + 6\phi - 7c_n)}{2(7c_n - 5\phi)}$, the numerator is positive. Finally, we obtain $\tau^* > \tau^0$ if $\frac{1}{8}(\Delta - c)(3 + 10\alpha) \leq \beta \leq \frac{(\Delta - c)(10\alpha \cdot c_n + 6\phi - 7c_n)}{2(7c_n - 5\phi)}$.

(2) Extract the common factor $\frac{4(\phi-c_n)[6mx+\beta x^2+8m\beta]}{8m-x^2}$ of $\tau^{**}>\tau^0$. Since this common factor is positive, the sign of $\tau^{**}>\tau^0$ depends on $2\alpha[8m-x^2]^2-5x^2-8x\beta+16m$. We know the formula $2\alpha[8m-x^2]^2-5x^2-8x\beta+16m > 2\alpha\left[8\times\frac{x^2}{2}-x^2\right]^2-5x^2-8x\beta+8x^2 = 18\alpha x^4+3x^2-8x\beta$, so, when $18\alpha x^4+3x^2-8x\beta\geq 0$ that is $\beta\leq\frac{9}{4}\alpha x^3+\frac{3}{8}x$, the formula is positive. We thus obtain $\tau^{**}>\tau^0$. □

Appendix A.5. Proof of Proposition 2

Proof of Proposition 2. If $x=\Delta-c$. (1) Extract the common factor $\frac{2(8m+x^2)}{[5x^2+8\beta x-16m-2\alpha\cdot(8m+x^2)]^2}$ of $\frac{\partial k^*}{\partial\alpha}$. Since this common factor is positive, the sign of $\frac{\partial k^*}{\partial\alpha}$ depends on $8m\beta(c_n-\phi)+x\cdot[-2c_n\cdot x^2-\beta(\phi+c_n)\cdot x+14c_n\cdot m-6m\phi]$. We know that the following formula is negative $8m\beta(c_n-\phi)+x\cdot[-2c_n\cdot x^2-\beta(\phi+c_n)\cdot x+14c_n\cdot m-6m\phi] > x^2\cdot[(3c_n-5\phi)\beta+(5c_n-3\phi)\cdot x]$. For $\phi\gg c_n$, $x^2\cdot[(3c_n-5\phi)\beta+(5c_n-3\phi)\cdot x]$. Thus, we obtain $\frac{\partial k^*}{\partial\alpha}<0$.

(2) $\frac{\partial k^{**}}{\partial\alpha}=-\frac{2(8m-x^2)^2\cdot(\phi-c_n)\cdot(6mx+\beta x^2+8m\beta)}{[2\alpha(8m-x^2)^2-5x^2-8x\beta+16m]^2}<0$ always holds, so we obtain $\frac{\partial k^{**}}{\partial\alpha}<0$.

(3) $\frac{\partial k^*}{\partial\beta} = \frac{[8m(c_n-\phi)-(\phi+c_n)\cdot x^2]\cdot[5x^2+8\beta x-16m-2\alpha(8m+x^2)]}{[5x^2+8\beta x-16m-2\alpha(8m+x^2)]^2} - \frac{8x\cdot\{8m\beta(c_n-\phi)+x\cdot[-2c_n\cdot x^2-\beta(\phi+c_n)x+14c_n\cdot m-6m\phi]\}}{[5x^2+8\beta x-16m-2\alpha(8m+x^2)]^2}$. The denominator of the formula is clearly greater than 0, so the sign of $\frac{\partial k^*}{\partial\beta}$ depends on $[8m(c_n-\phi)-(\phi+c_n)\cdot x^2]\cdot[5x^2+8\beta x-16m-2\alpha(8m+x^2)]-8x\cdot\{8m\beta(c_n-\phi)\ +x\cdot[-2c_n\cdot x^2-\beta(\phi+c_n)x+14c_n\cdot m-6m\phi]\}$ We know it is greater than $39(\phi-c_n)\cdot x^4+30\alpha(\phi-c_n)\cdot x^4+10\times(2\alpha\cdot\phi-c_n)\cdot x^4>0$, so we obtain $\frac{\partial k^*}{\partial\beta}>0$.

(4) $\frac{\partial k^{**}}{\partial\beta}=\frac{(\phi-c_n)\cdot(x^2+8m)\cdot[2\alpha(8m-x^2)^2-5x^2-8x\beta+16m]}{[2\alpha(8m-x^2)^2-5x^2-8x\beta+16m]^2}+\frac{8x(\phi-c_n)\cdot(6mx+\beta x^2+8m\beta)}{[2\alpha(8m-x^2)^2-5x^2-8x\beta+16m]^2}$. The denominator of the formula is markedly greater than 0, so the sign of $\frac{\partial k^{**}}{\partial\beta}$ depends on $(\phi-c_n)\cdot(x^2+8m)\cdot[2\alpha(8m-x^2)^2-5x^2-8x\beta+16m]+8x(\phi-c_n)\cdot(6mx+\beta x^2+8m\beta)$. We know it is greater than $39(\phi-c_n)\cdot x^4+90\alpha(\phi-c_n)\cdot x^6>0$, so we obtain $\frac{\partial k^{**}}{\partial\beta}>0$. □

Appendix A.6. Proof of Proposition 3

Proof Proposition 3. If $x=\Delta-c$. (1) Extract the common factor $\frac{2(64m^2-x^4)}{\{(8m-x^2)\cdot[5x^2+8\beta x-16m-2\alpha(8m+x^2)]\}^2}$ of $\frac{\partial\tau^*}{\partial\alpha}$. Since this common factor is positive, the sign of $\frac{\partial\tau^*}{\partial\alpha}$ depends on $32m\beta(c_n-\phi)+4x\cdot[-2c_nx^2-\beta(\phi+c_n)\cdot x+14c_n\cdot m-6m\phi]$. We know the section of this formula $32m\beta(c_n-\phi)+4x\cdot[-2c_nx^2-\beta(\phi+c_n)\cdot x+14c_n\cdot m-6m\phi]=-\{32m\beta(\phi-c_n)+4x\cdot[2c_nx^2+\beta(\phi+c_n)\cdot x-14c_n\cdot m+6m\phi]\}$ and $32m\beta(\phi-c_n)+4x\cdot[2c_nx^2+\beta(\phi+c_n)\cdot x-14c_n\cdot m+6m\phi] > (20\phi-12c_n)\beta x^2+4(3\phi-5c_n)x^3>0$. Thus, we find $32m\beta(c_n-\phi)+4x\cdot[-2c_nx^2-\beta(\phi+c_n)\cdot x+14c_n\cdot m-6m\phi]<0$ and further obtain $\frac{\partial\tau^*}{\partial\alpha}<0$.

(2) $\frac{\partial\tau^{**}}{\partial\alpha}=-\frac{8(\phi-c_n)\cdot(8m-x^2)^2\cdot[6mx+\beta x^2+8m\beta]}{(8m-x^2)\cdot[2\alpha(8m-x^2)^2-5x^2-8x\beta+16m]^2}<0$, thus, we gain $\frac{\partial\tau^{**}}{\partial\alpha}<0$

(3) Extract the common factor $\frac{8m-x^2}{\{(8m-x^2)\cdot[5x^2+8\beta x-16m-2\alpha(8m+x^2)]\}^2}$ of $\frac{\partial\tau^*}{\partial\beta}$. Since this common factor is positive, the sign of $\frac{\partial\tau^*}{\partial\beta}$ depends on $[32m(c_n-\phi)-4(\phi+c_n)\cdot x^2]\cdot[5x^2+8\beta x-16m-2\alpha\cdot(8m+x^2)]-8x\{32m\beta(c_n-\phi)+\ 4x[-2c_n\cdot x^2-\beta(\phi+c_n)\cdot x+14\cdot c_n\cdot m-6m\phi]\}$. We know the formula $[32m(c_n-\phi)-4(\phi+c_n)\cdot x^2]\cdot[5x^2+8\beta x-16m-2\alpha\cdot(8m+x^2)]-8x\{32m\beta(c_n-\phi)+4x[-2c_n\cdot x^2-\beta(\phi+c_n)\cdot x\ +14\cdot c_n\cdot m-6m\phi]\} > 4(37\phi-47c_n)\cdot x^4+40\alpha(5\phi-3c_n)\cdot x^4>0$. We thus obtain $\frac{\partial\tau^*}{\partial\beta}>0$.

(4) Extract the common factor $\frac{4}{(8m-x^2)\cdot[2\alpha(8m-x^2)^2-5x^2-8x\beta+16m]^2}$ of $\frac{\partial\tau^{**}}{\partial\beta}$. Since this common factor is positive, the sign of $\frac{\partial\tau^{**}}{\partial\beta}$ depends on $[8m(\phi-c_n)+(\phi-c_n)\cdot x^2]\cdot[2\alpha\cdot(8m-x^2)^2-5x^2-8\beta x+16m]+8x\{6m(\phi-c_n)x+\beta(\phi-c_n)x^2+8m(\phi-c_n)\beta\}$. We know the formula $[8m(\phi-c_n)+(\phi-c_n)\cdot x^2]\cdot[2\alpha\cdot(8m-x^2)^2-5x^2-8\beta x+16m]+8x\{6m(\phi-c_n)x+\beta(\phi-c_n)x^2+8m(\phi-c_n)\beta\}>39(\phi-c_n)x^4+90\alpha(\phi-c_n)x^6>0$.

We thus obtain $\frac{\partial\tau^{**}}{\partial\beta}>0$. □

Appendix A.7. Proof of Proposition 4

Proof of Proposition 4. If $x=\Delta-c$. (1) $\frac{\partial b^{**}}{\partial\alpha}=\frac{8m\cdot(8m-x^2)^3\cdot[6mx+\beta x^2+8m\beta]}{[2\beta x^3+8m\alpha(8m-x^2)^2-8m\cdot x^2-16m\beta x+64m^2]^2}>0$.

(2) Extract the common factor $-\frac{1}{[2\beta x^3+8m\alpha(8m-x^2)^2-8m\cdot x^2-16m\beta x+64m^2]^2}$ of $\frac{\partial b^{**}}{\partial\beta}$. Since this common factor is negative, the sign of $\frac{\partial b^{**}}{\partial\beta}$ depends on $(64m^2-x^4)\cdot[2\beta x^3+8m\alpha(8m-x^2)^2-8mx^2-16m\beta x+64m^2]-(8m-x^2)\cdot(6mx+\beta x^2+8m\beta)\cdot(-16mx+2x^3)$. We know the section is greater than $234x^8+540\alpha x^{10}>0$, so we obtain $\frac{\partial b^{**}}{\partial\beta}<0$. □

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