

Internal Prices and Optimal Exploitation of Natural Resources

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Abstract: Within the framework of traditional fishery management, we propose an interpretation of natural resource prices. It leads to an economic taxation mechanism based on internal prices and reduces a complex problem of optimal long-term exploitation to a sequence of one-year optimization problems. Internal prices obey natural, economic patterns: the increase in resource amount diminishes taxes, and the rise in the number of “fishers” raises taxes. These taxes stimulate cooperative agent behavior. We consider new problems of optimal fishing, taking into account an adaptive migration of the fish population in two regions. To analyze these problems, we use evolutionary ecology models. We propose a paradoxical method to increase the catch yield through the so-called fish “luring” procedure. In this case, a kind of “giveaway” game occurs, where the region with under-fishing becomes more attractive for fish and for catches in the future.

Keywords: long-term optimization; dynamic programming; internal prices; taxes

MSC: 90C39; 91B64

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1. Introduction

A socially sound aim of the fishery is the sustainable provision of humankind with seafood. This subsumes the rational exploitation and preservation of the fish populations. A single responsible owner could ensure this. However, the catch is carried out by agents focused on their profit. Uncontrolled fishing by such agents can collapse, known as Tragedy of the Commons [1,2]. In the second half of the twentieth century, it was realized that the resources of the world ocean are finite, and overexploitation needs to be prevented [3]. In particular, this resulted in the 1982 UN Convention on the Law of the Sea and the Exclusive Economic Zone (EEZ) regime. This and other measures have not led to a solution to the problem of fish stock depletion [4,5]. It is estimated that one-third of the world's marine fish stocks are currently overfished [6].

Many management strategies aim to avoid the negative consequences of fishing and achieve the mentioned socially significant government goal [7–10]. Among them are taxes, setting TAC (Total Allowable Catch), granting exclusive ownership to a single firm, forming a fishers' cooperative, and establishing marine reserves [11]. Furthermore, if the TAC is fixed, it can be allocated between the fishers through a mechanism of quotas. The most common form is ITQ (Individual Transferable Quotas), which first were implemented in New Zealand in 1986 [12]. It would be naïve to assume that there is a simple tool that can solve a complex fishery problem. The main reason is that several factors are not included in any concrete model, and their impact can be quite substantial [13] where ITQs are discussed.

Since many agents on the level of states and firms are involved in fisheries, it is natural that game theory came into play [3,14]. Although game theory can usually provide only qualitative results, its impact on management is significant. In particular, the report

[15] on the results of the Expert Consultation on the Management of Shared Fish Stocks (FAO 2002) reviews the basic concepts of game theory and discusses its applications.

In this paper, we consider a catch management method based on taxes. This method is more related to optimization than to the game theory. However, it is also connected with the Stackelberg leader-follower game. Using taxes is often mentioned in the economic literature [16–19]. Nevertheless, past research [11] suggests that taxes have “seldom been attempted, at least not as the sole regulatory instrument.” In [3], it is mentioned that presumably, tax schemes cannot be designed to foster intra-industry cooperation among the fishers. The role of taxes is underestimated. Indeed, taxation has solid mathematical grounds in optimization, where the resource prices are related to Lagrange multipliers [20]. These prices can stimulate optimal cooperative behavior via a decentralized management scheme. This method is well known for flow control in communication networks [21].

We distinguish two levels of management: the leader (e.g., a government) and the follower (agent, resource use). As a rule, agents pursue short-term commercial goals and do not think about the environmental consequences of their activities. On the contrary, the government wants to economically force the agent to act optimally in light of a long-term perspective. It usually implements a “hard” control to satisfy the constraints. This traditional way often spawns corruption.

Another approach subsumes a significant reduction of the role of the government. It brings to the fore specific “soft” purely economic mechanisms (e.g., taxes, etc.), making them responsible for the constraint compliance. This paper outlines the basics of this approach and shows its capabilities. It became evident that assigning a tax per unit of fish production equal to the derivative of the value function (corresponding to the specified optimal yield problem) forces the fisher to adhere to the optimal catching strategy with respect to the infinite horizon. In this case, the population size stabilizes at the universal limiting value, not depending on the utility function. These ideas were expressed in [22].

Indeed, it is not easy to determine the correct taxes. First, they depend on the dynamics of the fish population, which can only be modelled approximately. Moreover, the real stock of fish may be unknown. Second, optimal taxes depend on the agent reward functions, which are even more difficult to model. Third, as already mentioned, there are many factors that cannot be taken into account within a single model. For these reasons, only qualitative investigations in fishery management prevail at this time. Even prior studies [23–25] using real data mainly result in qualitative conclusions. Thus, we are doubtful that the correct real-world tax can be obtained within the simple models under consideration. Furthermore, the traditional methods of fish stock evaluation based on random samples seem not to be entirely reliable. We believe that Artificial Intelligence will be able to make substantial progress in quantitative results. Comprehensive fisheries datasets have been collected. As is mentioned in [26], the dataset [27] contains information about 1433 stocks belonging to 387 unique species.

The aim of our research is to study the method of long-term optimal control of natural resources based on taxation. The proposed taxation mechanism is based on internal prices, settled by a responsible resource owner (government) to ensure sustainable development. The internal prices reduce a complex problem of optimal long-term exploitation to a sequence of one-year optimization problems and stimulate a cooperative behavior of competing agents. We also consider evolutionary and spatial aspects of this problem.

2. Methods

The concept of a price of a product (good) can be defined in different ways. In optimization theory, one first finds the maximum of a possible gain that this product can $P(x)$ bring. Here, x is the initial stock of a product, while $B(x)$ defines the gain from the stock. B is defined as an indirect utility. Then it is usually assumed [28] that the price c equals the marginal gain: $c(x) = B'(x)$. We call this price “internal.” In a two-dimensional case

the prices of goods are determined by partial derivatives: $a(x, y) = B_x(x, y)$, $b(x, y) = B_y(x, y)$.

As an example, consider the problem of optimal long-term fishing described by a simple discrete model

$$x_{t+1} = f(x_t - u_t) \quad (1)$$

where x_t and u_t are the current amount and annual catch of fish $u_t \in [0, x_t]$. We assume that the annual reward is given by a utility function $r(u)$. For an illustration, we consider the simplest class of nonlinear ecology models, where f, r are strictly increasing strictly concave smooth functions, satisfying the condition $f(0) = r(0) = 0$. For instance, $f(x) = 2x/(1+x)$ and $r(u) = \ln(1+u)$. The condition $f(0) = 0$ implies the representation $f(x) = x \cdot \varphi(x)$ for a smooth function φ . Moreover, the concavity of f implies that φ is decreasing. Sometimes φ is called a trophic function, which reflects a negative impact of the inter- and intra-specific competition. A useful collection of modern population growth functions and their discussion are presented in [29].

Let us first discuss the simplest model of population dynamics without fishing. On the one hand, to prevent the variables of this model from going to infinity, one should require a slow population growth, for instance, $\frac{f(x)}{x} < 1$, for all sufficiently large x (say, for $x > M$, where M is a constant). On the other hand, it is necessary to assume that $f'(0) > 1$, otherwise the population will “die by itself.” Under these conditions, there exists a unique positive equilibrium. For convenience, we also select a scale such that $r'(0) = 1$. For an initial population stock x_0 , the global profit is the supremum of the discounted sum of rewards over all admissible sequence of catches:

$$P(x_0) = \sup_{(u_t)} \sum_{t=0}^{\infty} r(u_t) \gamma^t, \quad \gamma \in (0, 1) \quad (2)$$

From an economic point of view, the discounting reflects the lesser significance of the future income compared to the past one. From a mathematical point of view, it ensures that the sum is finite.

We will consider the so-called “highly productive” populations with $F'(0) > \frac{1}{\gamma}$. In this case, the optimal catch satisfies the strict inequality $u(x) < x$ for all x [11]. For low-productive populations, the equality $u(x) = x$ is possible for large x . Computation of $B(x)$ is carried out according to well-known dynamic programming schemes [30]. Recall that the initial piece $B_n(x)$ of the optimal sum (2) from 0 to n satisfies the recursion:

$$B_{n+1}(x) = \max_{u \in [0, x]} [r(u) + \gamma B_n(f(x - u))] \quad (3)$$

with $B_0(x) = r(x)$. By induction in n , we easily obtain (see, e.g., [22,31]):

Property 1. Each $B_n(x)$ is a strictly increasing concave function satisfying the condition $B_n(0) = 0$.

It is natural to consider these functions as elements of the Banach space $C[0, M]$ of continuous functions with the sup-norm. The right-hand side of Equation (3) maps this space into itself.

Property 2. The mapping determined by the right-hand side of Equation (3) is a contraction with the coefficient γ .

Hence, the sequence of functions B_n converges uniformly to a monotone concave value function B . It was also shown that this function is continuously differentiable. We set the price of a fish unit equal to $c(x_0) = B'(x_0)$. From the strict concavity of at least one

of the functions f or r , it follows that B' is strictly decreasing. Of course, for other classes of models, the concavity property of $B(x)$ is rare, but its monotonicity often takes place. According to Lebesgue's theorem [32], a monotone function is differentiable almost everywhere. Therefore, in this case, we also can say something about the price of a fish (or a resource) for “almost all” x .

Let us mention some other useful characteristics of the fishery (Figure 1). Their justification is given in the monograph [19].

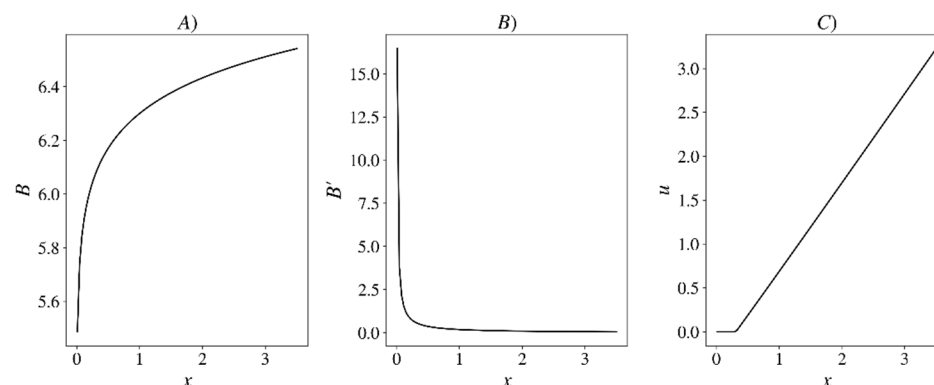


Figure 1. Fishery value function (A), internal price (B), and optimal catch (C) over the fish stock.

Property 3. Optimal fishing $u(x)$ is a continuous and non-decreasing function. For small x , we have $u = 0$, and for large x , it strictly increases.

This statement is, to some extent, consistent with the concept of minimum possible population size [33]. Indeed, in the infinite series (2), the optimal catch $u_t = u(x_t)$ depends on t only through the state variable x_t . According to Property 3, $u = 0$ for small x . Therefore $B(x) = \gamma B(f(x))$. Taking the derivative, we obtain $B'(+0) = [\gamma \cdot f'(0)] \cdot B'(+0)$. By assumption, the factor in square brackets is greater than 1. It follows that $B'(+0) = \infty$.

Using the next property, one can prove that the dynamical system (Equation (1)) with $u_t = u(x_t)$ has a unique positive globally stable equilibrium [22].

Property 4. The function $x - u(x)$ is strictly increasing.

Indeed, this property implies that the right side $h(x) = f(x - u(x))$ of Equation (1) is increasing.

In the absence of fishing, population dynamics obey the simplified version of Equation (1), namely $x_{t+1} = f(x_t)$. In some popular models, f can be a unimodal trophic function: recall Ricker's “humpy” function $f(x) = A x \exp(-x)$ [34]. Complex aperiodic regimes can arise in this model for a large constant A . Somewhat surprisingly, optimal fishing has a stabilizing effect here. The following result was proven in [19].

Property 5. For any parameter $A > 1$ in the Ricker model with fishing, each trajectory tends to some fixed equilibrium.

The next property is of key importance (see Appendix A for a proof).

Property 6. Let $u(x) > 0$ at some point x , then

$$B'(x) = r'(u) \quad \text{for } u = u(x) \quad (4)$$

In other words, the price generated by the “global utility function” must match the price from the “local utility function.” Note that Equation (4), the concavity of r and the inequality $u(x) < x$ implies that:

$$B'(x) > r'(x) \quad (5)$$

Now the complex search for the optimal harvest vector in the multi-year problem is reduced to a simple one-year optimization problem. Using Equation (4), it is easy to prove the following result (see Appendix A).

Property 7. *Each globally optimal catch $u_t = u(x_t)$ is an optimal solution to the elementary optimization problem*

$$r(u) - B'(x_t) \cdot u \rightarrow \max \quad \text{for } u \in [0, x_t] \quad (6)$$

Therefore, it is natural to interpret the internal price $c(x_t) = B'(x_t)$ as a tax per unit of fish caught.

Now, let us consider a slightly more general case when there are m agents (fishers). The dynamics x_t of a population, exploited by m agents are described by the Equation

$$x_{t+1} = f(x_t - u_1(x_t) - \dots - u_m(x_t)) \quad (7)$$

where $u_i \geq 0$ and $u(x) = u_1(x) + \dots + u_m(x) \leq x$. If the agents are far-sighted and act cooperatively, then their common objective function has the form

$$F(x; u) = \sum_{t=0}^{\infty} \gamma^t \sum_{i=1}^m r_i(u_i(x_t)) \rightarrow \max_u, \quad x_0 = x \quad (8)$$

Let $B_m(x)$ be the correspondent value function. To stimulate such cooperative behavior, the leader can settle the universal (the same for all agents) price $B'_m(x)$ per unit fish. The next result was proven in [31].

Theorem 1. *The solutions of the problems $r_i(u_i) - u_i B'_m(x) \rightarrow \max_{u_i \in [0, x]}$ for all $x > 0$ coincide with the far-sighted cooperative optimal strategies.*

This subsumes the assertion that the optimal myopic solutions $u_i^*(x)$ automatically satisfy the constraint $u_1^*(x) + \dots + u_m^*(x) \leq x$. Note that in the absence of a catch tax, each agent will strive to catch all the fish. In addition, when $m > 1$, a conflict situation arises between the agents, and they must agree on quotas. Theorem 1 shows that the tax $B'_m(x)$ eliminates these problems. The next result shows that the competition problem remains, although it takes a hidden form.

Theorem 2. *The tax increases with the number of agents: $B'_k(x) \geq B'_m(x)$ for $k > m$. Moreover, a strict inequality holds everywhere, except maybe a nowhere dense closed set.*

This implies that with an increase in the number of agents, the tax on catch increases and, accordingly, the optimal volumes of catch and profits of all agents decrease. Therefore, the fisher community will resist the entrance of new members. The reason for increasing the tax with an increase in the number of agents is economically clear. The continuous time case was considered in [35]. Technically, this case is more complicated, and the formulation of the results requires some modification. However, some analogs of Theorems 1 and 2 remain valid.

Above we considered traditional one-region population models. Now, we turn to the case of two regions to analyze harvesting in the case of spatial population dynamics using the ideas of an evolutionary ecology within some novel models. It is important to mention the ambiguity of the outcomes of the evolutionary process in the framework of the models

under consideration. In [36], when searching for migration routes (described by matrices) depending on the spatial distribution of food resources, it was found that the final matrix depends on the choice of the initial matrix. However, unexpectedly it turned out that all such final matrices have, in fact, the same positive eigenvector. This (Perron) vector turns out to be unique [37]. Each of its components is the relative time spent by the population in the i -th area. Therefore, any adaptive behavior is reduced to the restructuring of the sojourn times. Hence, the mentioned matrix is only one of many *forms* of a particular migration, and the true *content* of the migration is the Perron vector. The solution of the optimal harvesting problem can be supplemented with an economic filling based on the idea of internal prices of the fish population for different areas. This allows to set effective economic mechanisms: taxation, trade exchanges, etc.

Let the reservoir be divided, for example, into two regions (1 and 2), in each of which a catch is performed. It is assumed that the fish population migrates between these areas. Formally, let x_t and y_t be the current fish stocks of a given population in regions 1 and 2, respectively. Then its displacement per unit of time is given by a linear model:

$$\begin{pmatrix} x_{t+1} \\ y_{t+1} \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} x_t \\ y_t \end{pmatrix} \quad (9)$$

where $m_{ij} \geq 0$. Since the total population size does not change during “pure” migration, the sum of the elements in each column is equal to 1. Such matrices are called Markov. It is convenient to represent two-dimensional Markov matrices in the following form:

$$M = \begin{pmatrix} 1 - \alpha & \beta \\ \alpha & 1 - \beta \end{pmatrix} \quad (10)$$

where $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$. Off-diagonal elements α and β characterize the intensity of population movement. Thus, β is the fraction of the population that can move from region 2 to region 1. Similarly, α defines the fraction of the population moving from the first region to the second region. It is convenient to represent a matrix M as a point (α, β) in a plane. Together, they fill the square $Q = [0,1] \times [0,1]$ with the vertices $(0,0)$, $(0,1)$, $(1,1)$, $(1,0)$. Each point of Q is a convex combination of these vertices, which, in fact, constitute a “convex” basis in Q . Let us construct the corresponding “basis” matrices:

$$E_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, E_2 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, E_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, E_4 = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \quad (11)$$

Let us discuss the adaptation of the migration route from the point of view of evolutionary ecology. Assume that the original population with the migration matrix M_0 can generate “mutants” with routes μ close to M_0 . Namely:

$$\mu = (1 - \varepsilon)M_0 + \varepsilon E_i \quad (12)$$

where $\varepsilon > 0$ is small and E_i is one of the basis matrices. In the proposed modeling scheme, the route of the mutant will change from time to time. Therefore, to complete the picture, one should specify the order in which the basis matrices are generated. The number of mutants in the model is specified in advance. Of course, the smallest number of them is 1. It has been established that with an increase in the number of mutants, the result does not change much.

Here the process of competition of the initial population (numbered by 0) with its community of mutants (numbered by $1, \dots, m$) is of key importance. Assume that i -th population is described by the equation $z_{t+1}^i = z_t^i \varphi_i(z_t^i)$, $i = 0, 1, \dots, m$. If these populations are “similar” (for example, they are mutants of the original population), then their competition is given in the form [19]: $z_{t+1}^i = z_t^i \varphi_i(w_t)$, where $w_t = z_t^0 + z_t^1 + \dots + z_t^m$. We will use this approach for the construction of evolution mechanisms.

Consider the “basic example”, which describes the interaction of the initial population $I = \begin{pmatrix} x^i \\ y^i \end{pmatrix}$ with a single mutant $J = \begin{pmatrix} x^m \\ y^m \end{pmatrix}$. In the sequel for the specified population

vectors, we will use the lower symbol for the indication of the time moment. We assume that in the i -th region, the trophic function for each population is the same and equals to φ_i . The routes of their movement are specified given by the matrices:

$$N^i = \begin{pmatrix} 1 - \alpha & \beta \\ \alpha & 1 - \beta \end{pmatrix}, \quad N^m = \begin{pmatrix} 1 - a & b \\ a & 1 - b \end{pmatrix} \quad (13)$$

The model of joint competition and migration is a composition of two operators $K: I, J \rightarrow \tilde{I}, \tilde{J}$ and $M: \tilde{I}, \tilde{J} \rightarrow \hat{I}, \hat{J}$. As an illustration, we present formulas for the transformation of the vector I under K :

$$\tilde{x}^i = x^i \cdot \varphi_0(x^i + x^m), \quad \tilde{y}^i := y^i \cdot \varphi_1(y^i + y^m) \quad (14)$$

$$\tilde{x}^m = x^m \cdot \varphi_0(x^i + x^m), \quad \tilde{y}^m := y^m \cdot \varphi_1(y^i + y^m) \quad (15)$$

and formulas for its transformation under M :

$$\hat{x}^i = (1 - \alpha)\tilde{x}^i + \beta\tilde{y}^i, \quad \hat{y}^i = \alpha\tilde{x}^i + (1 - \beta)\tilde{y}^i \quad (16)$$

We assumed that the initial amount of population is about 1, while the initial mutant amounts are small. The computations were carried in several stages. Each stage corresponded to a 100-year forecast, which is a significant time in terms of evolution. The results were summarized at the end of each stage. We note right away that the competition outcome depends little on the choice of initial population sizes.

Most importantly, at the end of a stage, a “strong” mutant can be revealed, whose amount can significantly exceed the amount of the original population (for example, by 10 or more times). If there are several strong mutants, then we choose the strongest. Furthermore, we assume that such a mutant sets the direction of natural selection. Namely, let μ^* be the migration matrix of a strong mutant. Then we deform the matrix of the original population as follows:

$$M_0 \rightarrow (1 - \xi)M_0 + \xi\mu^* \quad (17)$$

where the adaptation rate parameter $\xi > 0$ is small. If no strong mutants are found, then M_0 is saved.

After that comes the second stage. We act similarly since for each mutant a new route (specified in Equation (12)) is formed with another E_i . In the end, a stage comes where strong mutants are no longer formed. This means that the final matrix M_0 is evolutionarily stable (ES-matrix) since it is not displaced by its mutants with close migration routes. Of course, only ES migration matrices can be realized in nature. In the extreme version of this concept, the number of mutants can be arbitrary. We used only two mutants such that the enumeration of the basis matrices occurs in opposite directions (i.e., clockwise: $E_1 \rightarrow E_2 \rightarrow E_3 \rightarrow E_4 \rightarrow E_1$, and counterclockwise).

For example, consider an applied problem of changing the routes of fish migration depending on the ratio of region food resources under optimal fishing conditions. For example, assume that if u is the total harvest in the region 1, then its distribution over specific populations are uniform. Namely, the catch operator L is defined there as follows:

$$x^i \rightarrow x^i \cdot \left(1 - \frac{u}{x^i + x^m}\right), \quad x^m \rightarrow x^m \cdot \left(1 - \frac{u}{x^i + x^m}\right) \quad (18)$$

Similar formulas hold true for the components y^i, y^m , in the region 2. It is interesting to what extent do the obtained results depend on the number of generated mutants. Preliminary computations showed that they are preserved even for large number of mutants.

Next, we introduce the parameter $\lambda \in (0, 1)$, which characterizes the share of the 1-region in the reservoir. Denote by K the total amount of food in the reservoir. Then $K_1 = \lambda K$ is the amount of food in the first region, and $K_2 = (1 - \lambda)K$ is the amount of food in

region 2. We will discuss the simplest case, i.e., when food supplies are fixed and do not change in each region.

The optimization block of the model considers harvesting in both areas for a multi-year period. Denote by $B_T(x, y)$ the optimal discounted income for $T + 1$ years with initial fish stocks equal to x (in the first region) and y (in the second region). During the exploitation, the amount of fish changes. We denote the corresponding trajectory by (x_t, y_t) . By definition,

$$B_T(x, y) = \max \sum_{t=0}^T \gamma^t [p(u_t) + q(v_t)] \quad (19)$$

where the maximization is performed over all admissible catches $\{u_t\}$ and $\{v_t\}$ in both regions: $0 \leq u_t \leq x_t$, $0 \leq v_t \leq y_t$. As above, γ is the discount factor (put $\gamma = 0.9$). Furthermore, p, q are concave increasing utility functions with $p(0) = q(0) = 0$. We put $p(w) = q(w) = w/(1 + w)$. Note that $p'(0) = q'(0) = 1$.

We represent any z -population dynamics as $z_{t+1} = f(z_t, K)$. We will use the simplest variant of the non-linear growth, assuming that f is smooth, strictly increasing, and concave in the first argument. These assumptions are satisfied by the scheme from [38]:

$$f(z, K) = z \left(d + r \frac{K}{K + z} \right) \quad (20)$$

where $d = 0.25$ and $r = 3$ are the mortality and growth rates. Put $f_1(z) = f(z, K_1)$, $f_2(z) = f(z, K_2)$ for brevity. Now, taking into account the annual fishing: u_t (in the first region) and v_t (in the second region), the dynamics of one population takes the form:

$$x_{t+1} = (1 - \alpha)f_1(x_t - u_t) + \beta f_2(y_t - v_t) \quad (21)$$

$$y_{t+1} = \alpha f_1(x_t - u_t) + (1 - \beta)f_2(y_t - v_t) \quad (22)$$

Rewrite Equations (21) and (22) as $\tilde{x} = P(x - u, y - v)$, $\tilde{y} = Q(x - u, y - v)$. Note that P, Q are strictly increasing and concave in each variable. By the dynamic programming principle:

$$B_{T+1}(x, y) = \max [p(u) + q(v) + \gamma B_T(\tilde{x}, \tilde{y})] \quad (23)$$

where the maximum is taken over $0 \leq u \leq x$, $0 \leq v \leq y$. Evidently, $B_0(x, y) = p(x) + q(y)$. Each function B_T is continuous. Two-dimensional analogs of the previous properties still hold true.

Furthermore, B inherits the “good” properties of the utility and growth functions. It is strictly increasing and concave in each variable. Thus, the partial derivatives $B_x(x - 0, y)$, $B_x(x + 0, y)$ exist almost everywhere, and $B_x(x - 0, y) > B_x(x + 0, y)$. B can be regarded as an indirect utility function of the initial fish stocks by area equal to x and y . Its partial derivatives $B_x(x, y)$ and $B_y(x, y)$ can be interpreted as internal fish prices in the first and second regions, respectively.

Statement 1. Let the optimal catch $u = u(x, y)$ at (x, y) be positive, then there exists $B_x(x, y)$ and

$$B_x(x, y) = p'(u) \quad (24)$$

Statement 2. Let the optimal catch $v = v(x, y)$ at (x, y) be positive, then there exists $B_y(x, y)$ and $B_y(x, y) = q'(v)$.

The quantity, e.g., $c = B_x(x, y)$ can be used as a tax per unit of fish caught in the first region. Let us discuss the solution of the following simple optimization problem for the fisher of the first region:

$$p(u) - B_x(x, y) \cdot u \rightarrow \max \quad \text{over } u \in [0, x] \quad (25)$$

To find out how much the controls differ in Equations (6) and (25), let us consider two cases for Equation (25). Let $c = B_x(x, y) \geq p'(0) = 1$, then the objective function $H(u) = p(u) - c \cdot u$ is strictly decreasing since its derivative is negative: $H'(u) = p'(u) - c < 0$. It follows that $u^* = 0$ is optimal. Now assume that $c < 1$. Then by the strict concavity of H , the desired control satisfies the Equation $H'(u^*) = 0$, that is $c = p'(u^*)$. Clearly, this relation coincides with Equation (24). Similarly, $B_y(x, y)$ can be used as an effective catch tax in the second region. Thus, the solution of the multi-step optimization problem (Equation (12)) reduces to the solution of two one-step optimization problems in the form of Equation (25).

3. Results

We will use the following global characteristics $\bar{c}_1 = B_x(\bar{x}, \bar{y})$, $\bar{c}_2 = B_y(\bar{x}, \bar{y})$, where $(\bar{x}, \bar{y})$ is the average asymptotic value of a sufficiently representative bundle of trajectories of the model given by Equations (21) and (22). The search for an optimal harvest strategy and the construction of the accompanying ES migration matrices includes two successive stages (blocks). The first stage (block) is optimization. For a given migration matrix M a search for optimal catches (functions u, v) is performed using the dynamic programming method. This corresponds to a numerically implemented mapping $M \rightarrow \langle u(x, y), v(x, y) \rangle$. The second stage (block) is evolution. For fixed catch functions $u(x, y)$, $v(x, y)$ a competitive interaction of the initial population (with the migration matrix M) with several mutants whose matrices are close to M , is realized (Equation (12)). When a strong mutant is detected as a result of selection, we perform some deformation of M (Equation (17)). Formally, a mapping $\langle u(x, y), v(x, y) \rangle \rightarrow M_1$ is implemented numerically.

If, after multiple repetitions of stages 1 and 2, this process converges, then a stable evolutionary matrix M^* and optimal catches u^*, v^* are determined for a given value of food in areas 1 and 2. In each case, the corresponding computations were carried out for initial values varying on a certain grid. Let K be the total supply of food in the reservoir, and the amounts of food $K_1 = \lambda \cdot K$ and $K_2 = (1 - \lambda) \cdot K$ for each of the two regions are given by a parameter λ from the segment $[0.1, 0.9]$. The dynamics of the internal prices for various values of λ are presented in Figure 2.

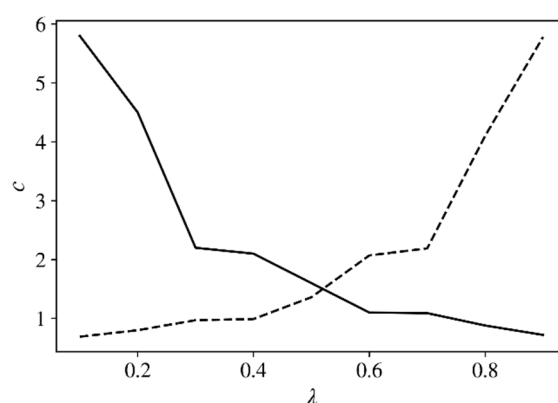


Figure 2. The dependence of internal prices $\bar{c}_1$ (solid line) and $\bar{c}_2$ (dashed line) on the fraction λ of total food supply in the first region.

In the first area, as expected, it turned out that the average number and catch of fish increased with an increase in the parameter λ due to the growth of food there. The opposite changes occur in the second region. Of course, the smaller the amount of fish, the

higher its price. According to obtained results, as λ increases, the Perron vector π runs through the values:

$$\begin{pmatrix} 0.06 \\ 0.94 \end{pmatrix}, \begin{pmatrix} 0.06 \\ 0.94 \end{pmatrix}, \begin{pmatrix} 0.21 \\ 0.79 \end{pmatrix}, \begin{pmatrix} 0.41 \\ 0.59 \end{pmatrix}, \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}, \begin{pmatrix} 0.59 \\ 0.41 \end{pmatrix}, \begin{pmatrix} 0.78 \\ 0.22 \end{pmatrix}, \begin{pmatrix} 0.89 \\ 0.11 \end{pmatrix}, \begin{pmatrix} 0.94 \\ 0.06 \end{pmatrix}$$

Finally, if, for example $\bar{c}_1 > \bar{c}_2$, then an additional benefit is possible from the sale of cheap fish from the second region to the owner of the first region. Harvesting can be considered an evolutionary game of “giveaway.” The influence of harvesting on the evolution of the fish migration route can be used to optimize the fishing of one population in two neighboring states A (region 1) and B (region 2) within the same reservoir. In fact, these countries compete for fish resources. What are ecological and economic mechanisms effective here? One of the paradoxical approaches is covered in [39]. Below, we present this in more detail. We will proceed in the spirit of hierarchical management, and at the same time, we will somewhat specify the previous agreements on fishing strategies. Namely, we assume that the Center, based on the solution of the problem given in Equation (19), issues recommendations to these states on the catch quota, which should not exceed the optimal values u and v , and depending on (x, y) . Of course, in some years, it is possible to catch even less (liberal fishing strategy). Assume that the initial migration matrix is of the form:

$$M_0 = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$$

and the food resources of the water areas of these countries are the same. It is convenient to use chess terminology.

First, consider an equilibrium “innocuous” situation. If the countries follow the Leader's recommendations (taking into account M_0 and the solution of Equation (12)), then the migration matrix is preserved, and the following average values arise: $\bar{u} = \bar{v} = 1.85$; $\bar{x} = \bar{y} = 3.89$; $\bar{c}_1 = \bar{c}_2 = 1.28$. Clearly, for M_0 we have

$$\pi = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

In the second stage, A loses material but gains a positional advantage. Let country A harvest 75% of the recommended amount, and country B harvest 100%. Then the A-water area becomes “less lethal” and, therefore, more attractive for fish. There is a “luring” of the fish population into the A-water area. As a result of evolution, the migration matrix is transformed to:

$$M_1 = \begin{pmatrix} 0.54 & 0.54 \\ 0.46 & 0.46 \end{pmatrix}$$

The correspondent average values: $\bar{u} = 1.48$, $\bar{v} = 2.12$; $\bar{x} = 4.68$, $\bar{y} = 3.96$; $\bar{c}_1 \approx \bar{c}_2 = 1.04$. Clearly, for M_1 we have:

$$\pi = \begin{pmatrix} 0.54 \\ 0.46 \end{pmatrix}$$

In fact, A played a “giveaway” with B here, but gained a positional advantage by constructing a more profitable migration matrix.

In the third stage, A realizes positional advantage and obtains a material advantage. Assume that considering the matrix, a coordinating center issues new recommendations for the optimal catch, which are now 100% implemented by both participants. This results in $\bar{u} = 2.03$, $\bar{v} = 1.7$; $\bar{x} = 4.21$, $\bar{y} = 3.56$; $\bar{c}_1 = 1.14$, $\bar{c}_2 = 1.45$. It is remarkable that if now both countries A and B adhere to this catch, then the migration matrix is no longer “spoiled.” Therefore, it is evolutionarily stable under the indicated controls. Most importantly, after the “luring” procedure, the catch in country A will always be greater than in country B. Since $\bar{c}_1 < \bar{c}_2$, A has the opportunity for additional trade benefits by selling its “cheap” fish to B. To restore the status quo, country B must likely apply the “lure” procedure in turn. An endless series of A-lures and B-lures occurs.

4. Discussion

In this paper, we used taxes as a sole instrument for fishery management. Some authors [40–42] argued that taxes could work better than ITQs under some types of uncertainty. Certainly, such results are model dependent and do not mean that the regulators should confine themselves to a single economic instrument. There are also many papers considering the influence of taxes on optimal harvesting [43–45]. These papers use traditional optimal control methods. Our approach differs in the problem formulation. We look for a tax, stimulating (socially) optimal behavior of myopic agents. We claim that, in general, it is the marginal value function. This tax automatically takes into account the fish stock and agent preferences. Since these quantities are not usually perfectly known, it would be interesting to evaluate the influence of this uncertainty on the agent reactions and dynamics of fish populations.

The influence of Darwinian evolution on optimal harvesting was the subject of many studies (see [46–49]). Usually, economically motivated and far-sighted agents apply strategies to ensure sustainable resource exploitation. Our results show that, in principle, such optimal behavior of myopic agents can be stimulated by taxes. The concept of internal prices can be introduced for any structure that brings economic profitability. Thus, for purely water objects, the following problems are relevant.

Consider a reservoir. Assume that the elements of water balance (inflow, precipitation, evaporation) are repeated annually, and the resulting volumes of river flow satisfy the condition: spring > autumn > winter > summer. What is the price of water for individual seasons? Here, the corresponding value function depends not only on the volume of water but also on the season number [50]. After construction of the corresponding value function and computing its partial derivatives, it was found that the greater the amount of water, the lower its price. Therefore, the reverse order of prices takes place: summer > winter > autumn > spring.

Furthermore, consider a river. For simplicity, assume that the river consists of a cascade of two sections (upper and lower). Which water has a higher price? The water from the lower section is probably more expensive. The lower section receives residual water consumption from the upper section. In this case, it is beneficial to buy cheap water from the upper section. Thus, the internal prices can also be useful for the optimal exploitation of water resources. Of course, internal and market prices do not quantitatively coincide, but their trends are qualitatively similar. Therefore, the preliminary construction of the landscape of internal prices can predict the direction of profitable flows in an ecological-economic system.

Finally, we mention that taxes are rarely used in practice due to technical and social problems [8]. However, there exist some successful experiments. Specifically, we indicate Iceland's rent taxation scheme [51], where quotas (ITQs) and taxes are combined. More precisely, taxes were added to ITQs after the latter provided profitability and sustainability to the fishing industry. The role of taxes here is mainly in the replenishment of the state budget and a fair distribution of income. This differs from our approach, where taxes are used to ensure optimal agent behavior.

5. Conclusions

Internal prices can be used as a tax per unit of fish caught. In this case, the problem of long-term optimization is reduced to solving problems of maximizing one-year catch. An appealing feature of this approach is that it ensures the cooperative behavior of competitive agents. This approach can be applied in various situations. In this paper, we presented a paradoxical strategy of fishing competition between neighboring countries ("giveaway game"), when, along with optimization of the agent's own catch, the evolutionary process of "luring" the fish population into the corresponding area is implicitly used. Let us mention some other interesting problems.

For example, it is possible to develop algorithms for learning internal prices. Such algorithms can be based on online learning [52] and reinforcement learning [53] theories. Note that the neural networks can give an analytical approximation of the value function, which is convenient for computing of the internal price by the automatic differentiation. In connection with this, there is a big demand for realistic data-driven dynamic models of fish populations and models of agent behavior. These models can be in the form of data generators since this is enough for reinforcement learning algorithms.

Furthermore, we considered the case where, roughly speaking, a leader's aim is to make myopic agents far-sighted. However, the leaders can also have their own economic goals. This assumption changes the problem substantially. A stimulation problem of this type was considered in [54]; however, with the use of nonlinear incentive functions. It is unclear to what extent a linear pricing system can be used in this case.

As for the spatial distribution of the exploited population, one can raise further questions. For example, is it true that in the model considered in our results section, the largest amount of fish is located in areas with the lowest internal price. Furthermore, in our research, it was assumed that the amount of food in the areas of fishing was constant. However, in a real situation, food is eaten, and this negatively affects the size of the fish population. Therefore, the luring procedure probably becomes less effective. But maybe it is still sufficient to beat a greedy opponent? This requires an appropriate modeling study in the future. These questions are relevant to better understand the distribution of the fish shared between neighboring countries. Finally, in the spirit of robust control theory, it would be interesting to study if the closeness of the utility and reproduction functions imply the closeness of the internal prices.

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Appendix A

Proof of Property 6. We give a justification without the assumption that the value function B is differentiable. Let $u = u(x) > 0$ at some point x . By virtue of Property 4 we have $u < x$. For this control, from the Bellman Equation we have

$$B(x) = r(u) + \gamma B(f(x - u)) \quad (A1)$$

Take a number ε , which is sufficiently small (in absolute value). At a close point $x + \varepsilon$ the catch $u + \varepsilon$ is admissible but may not be optimal. Therefore,

$$B(x + \varepsilon) \geq r(u + \varepsilon) + \gamma B(f(x - u)) \quad (A2)$$

By subtracting (A1) from (A2), we obtain

$$B(x + \varepsilon) - B(x) \geq r(u + \varepsilon) - r(u) \quad (A3)$$

Next, consider two different options for selecting the sign of ε :

(1) $\varepsilon > 0$. Letting in (A3) $\varepsilon \rightarrow 0$, we immediately obtain

$$B'(x + 0) \geq r'(u) \quad (\text{A4})$$

(2) $\varepsilon < 0$. Similarly, letting in (A3) $\varepsilon \rightarrow 0$, we obtain

$$B'(x - 0) \leq r'(u) \quad (\text{A5})$$

Finally, by the concavity of the value function,

$$B'(x - 0) \geq B'(x + 0)$$

Thus, $B'(x) = r'(u)$. $\square$

Proof of Property 7. Consider an auxiliary function $h(u) = r(u) - c \cdot u$ on $[0, x]$. It is concave and $h(0) = 0$. For $c \geq 1$ this function is strictly decreasing and attains its maximum at $u = 0$. In other words, with a high tax, fishing is not profitable for the entire period of fishing.

For $c < 1$ by the inequality (5) implies that $h'(x) = r'(x) - c < 0$. Hence, the “humpy” function h attains its maximum at an internal point of the interval $[0, x]$. At this point u^* the derivative of h equals 0. Thus, $r'(u^*) = B'(x)$, and u^* is an optimal catch at x^* according to Equation (4). $\square$

References

- Hardin, G. The tragedy of the commons. *Science* **1968**, *162*, 1243–1248. <https://doi.org/10.1126/science.162.3859.1243>.
- Stavins, R.N. The problem of the commons: Still unsettled after 100 years. *Am. Econ. Rev.* **2011**, *101*, 81–108. <https://doi.org/10.1257/aer.101.1.81>.
- Grønbaek, L.; Lindroos, M.; Munro, G.; Pintassilgo, P. *Game Theory and Fisheries Management: Theory and Applications*; Springer Nature: Cham, Switzerland, 2020.
- Butchart, S.H.M.; Walpole, M.; Colen, B.; van Strien, A.; Scharlemann, J.P.W.; Almond, R.E.A.; Baillie, J.E.M.; Bomhard, B.; Brown, C.; Bruno, J.; et al. Global biodiversity: Indicators of recent declines. *Science* **2010**, *328*, 1164–1168. <https://doi.org/10.1126/science.1187512>.
- Roberson, L.A.; Watson, R.A.; Klein, C.J. Over 90 endangered fish and invertebrates are caught in industrial fisheries. *Nat. Commun.* **2020**, *11*, 4764. <https://doi.org/10.1038/s41467-020-18505-6>.
- The State of World Fisheries and Aquaculture 2018—Meeting the Sustainable Development Goals*; Fisheries and Aquaculture Department, Food and Agriculture Organization of the United Nations: Rome, Italy, 2018.
- Clark, C.W. *Mathematical Bioeconomics. The Mathematics of Conservation*, 3rd ed.; John Wiley and Sons: Hoboken, NJ, USA, 2010.
- Arnason, R. Fisheries management and operations research. *Eur. J. Oper. Res.* **2009**, *193*, 741–751.
- Abakumov, A.I. *Mathematical Ecology*; DalGU: Vladivostok, Russia, 1994. (In Russian)
- Skaletskaya, E.I.; Frisman, E.Y.; Shapiro, A.P. *Discrete Models of Population Dynamics and Optimization of Harvesting*; Nauka: Moscow, Russia, 1979. (In Russian)
- Clark, C.W. *The Worldwide Crisis in Fisheries: Economic Models and Human Behavior*; Cambridge University Press: Cambridge, UK, 2006.
- Costello, C.; Lynham, J.; Lester, S.E.; Gaines, S.D. Economic incentives and global fisheries sustainability. *Annu. Rev. Resour. Econ.* **2010**, *2*, 299–318. <https://doi.org/10.1146/annurev.resource.012809.103923>.
- Young, O.R.; Webster, D.G.; Cox, M.E.; Raakjær, J.; Blaxekjær, L.Ø.; Einarsson, N.; Virginia, R.A.; Acheson, J.; Bromley, D.; Cardwell, E.; et al. Moving beyond panaceas in fisheries governance. *Proc. Natl. Acad. Sci. USA* **2018**, *115*, 9065–9073. <https://doi.org/10.1073/pnas.1716545115>.
- Long, N.V. Dynamic Games in the Economics of Natural Resources: A Survey. *Dyn. Games Appl.* **2011**, *1*, 115–148. <https://doi.org/10.1007/s13235-010-0003-2>.
- Munro, G.R.; Van Houtte, A.; Willmann, R. *The Conservation and Management of Shared Fish Stocks: Legal and Economic Aspects*; FAO Fisheries Technical Paper No. 465; FAO: Rome, Italy, 2004.
- Clark, C.W. Towards a predictive model for the economic regulation of commercial fisheries. *Can. J. Fish. Aquat. Sci.* **1980**, *37*, 1111–1129. <https://doi.org/10.1139/f80-144>.
- McKelvey, R. Common Property and the Conservation of Natural Resources. In *Applied Mathematical Ecology*; Levin, S.A., Hallam, T.G., Gross, L.J., Eds.; Biomathematics Volume 18; Springer: Berlin/Heidelberg, Germany, 1989. https://doi.org/10.1007/978-3-642-61317-3_3.
- Hanley, N.; Shorgen, J.F.; White, B. *Environmental Economics in Theory and Practice*; Macmillan Education: London, UK, 1997.
- Il'ichev, V.G. *Stability, Adaptation and Management in Ecological Systems*; Fizmatlit: Moscow, Russia, 2009. (In Russian)

20. Nesterov, Y.; Shikhman, V. Dual subgradient method with averaging for optimal resource allocation. *Eur. J. Oper. Res.* **2018**, *270*, 907–916.
21. Low, S.H.; Lapsley, D.E. Optimization flow control. I. Basic algorithm and convergence. *IEEE/ACM Trans. Netw.* **1999**, *7*, 861–874.
22. Il'ichev, V.G.; Rokhlin, D.B.; Ugol'nitskii, G.A. On Economic Control Mechanisms for Bioresources. *J. Comput. Syst. Sci. Int.* **2000**, *39*, 585–591.
23. Salenius, F. International management of North Atlantic pelagic fisheries—The role of competing species and exploiters. *Fish. Res.* **2018**, *203*, 12–21. <https://doi.org/10.1016/j.fishres.2017.08.001>.
24. Engström, G.; Gren, Å.; Li, C.-Z.; Krishnamurthy, C.K.B. Valuing biodiversity and resilience: An application to pollinator diversity in the Stockholm region. *Spat. Econ. Anal.* **2020**, *15*, 238–261. <https://doi.org/10.1080/17421772.2020.1784988>.
25. Ekerhovd, N.-A.; Flåm, S.D.; Steinshamn, S.I. On shared use of renewable stocks. *Eur. J. Oper. Res.* **2021**, *290*, 1125–1135. <https://doi.org/10.1016/j.ejor.2020.08.052>.
26. Miqueleiz, I.; Miranda, R.; Ariño, A.H.; Ojea, E. Conservation-status gaps for marine top-fished commercial species. *Fishes* **2022**, *7*, 2. <https://doi.org/10.3390/fishes7010002>.
27. RAM Legacy Stock Assessment Database. RAM Legacy Stock Assessment Database v4.495. Available online: <https://zenodo.org/record/4824192> (accessed on 26 April 2022).
28. Zangwill, W.I. *Nonlinear Programming: A Unified Approach*; Prentice-Hall: Englewood Cliffs, NJ, USA, 1969.
29. Tyutyunov, Y.V.; Titova, L.I. From Lotka–Volterra to Arditi–Ginzburg: 90 Years of Evolving Trophic Functions. *Biol. Bull. Rev.* **2018**, *10*, 167–185. <https://doi.org/10.1134/S207908642003007X>.
30. Bellman, R. *Dynamic Programming*; Rand Corp.: Santa Monica, CA, USA, 1956.
31. Rokhlin, D.B. The derivative of the solution of the functional Bellman equation and the value of bioresources. *Sib. Z. Ind. Mat.* **2000**, *3*, 169–181. (In Russian)
32. Rudin, W. *Real and Complex Analysis*, 3rd ed.; International Series in Pure and Applied Mathematics; McGraw–Hill: New York, NY, USA, 1987.
33. Traill, L.W.; Bradshaw, C.J.A.; Brook, B.W. Minimum viable population size: A meta-analysis of 30 years of published estimates. *Biol. Conserv.* **2007**, *139*, 159–166. <https://doi.org/10.1016/j.biocon.2007.06.011>.
34. Ricker, W.E. Stock and recruitment. *J. Fish. Res. Board Can.* **1954**, *11*, 559–623. <https://doi.org/10.1139/f54-039>.
35. Rokhlin, D.B.; Usov, A. Rational taxation in an open access fishery model. *Arch. Control Sci.* **2017**, *27*, 5–27. <https://doi.org/10.1515/acsc-2017-0001>.
36. Il'ichev, V.G.; Il'icheva, O.A. Spatial adaptation of populations in models of ecology. *Biophysics* **2018**, *63*, 373–381. (In Russian)
37. Roberts, F.S. *Discrete Mathematical Models, with Applications to Social, Biological, and Environmental Problems*; Prentice-Hall: Hoboken, NJ, USA, 1976.
38. Contois, D.E. Kinetics of bacterial growth relationship between population density and specific growth rate of continuous culture. *J. Gen. Microbiol.* **1959**, *21*, 40–50. <https://doi.org/10.1099/00221287-21-1-40>.
39. Il'ichev, V.G. Economical and evolutionary aspects of optimal fishery. *Econ. Math. Methods* **2019**, *55*, 86–99. <https://doi.org/10.31857/S042473880005778-4>. (In Russian)
40. Weitzman, M.L. Landing fees vs harvest quotas with uncertain fish stocks. *J. Environ. Econ. Manag.* **2002**, *43*, 325–338. <https://doi.org/10.1006/jeem.2000.1181>.
41. Jensen, F.; Vestergaard, N. Prices versus quantities in fisheries models. *Land Econ.* **2003**, *79*, 415–425. <https://doi.org/10.2307/3147026>.
42. Hansen, L.G.; Jensen, F.; Russell, C. The Choice of Regulatory Instrument when There Is Uncertainty About Compliance with Fisheries Regulations. *Am. J. Agric. Econ.* **2008**, *90*, 1130–1142. <https://doi.org/10.1111/j.1467-8276.2008.01169.x>.
43. Dubey, B.; Chandra, P.; Sinha, P. A model for fishery resource with reserve area. *Nonlinear Anal. Real World Appl.* **2003**, *4*, 625–637. [https://doi.org/10.1016/S1468-1218\(02\)00082-2](https://doi.org/10.1016/S1468-1218(02)00082-2).
44. Kar, T.K. Conservation of a fishery through optimal taxation: A dynamic reaction model. *Commun. Nonlinear Sci. Numer. Simul.* **2005**, *10*, 121–131. [https://doi.org/10.1016/S1007-5704\(03\)00101-1](https://doi.org/10.1016/S1007-5704(03)00101-1).
45. Huo, H.-F.; Jiang, H.-M.; Meng, X.-Y. A dynamic model for fishery resource with reserve area and taxation. *J. Appl. Math.* **2012**, *2012*, 794719. <https://doi.org/10.1155/2012/794719>.
46. Jørgensen, C.; Enberg, K.; Dunlop, E.S.; Arlinghaus, R.; Boukal, D.S.; Brander, K.; Ernande, B.; Gårdmark, A.G.; Johnston, F.; Matsumura, S.; et al. Ecology: Managing evolving fish stocks. *Science* **2008**, *318*, 1247–1248. <https://doi.org/10.1126/science.1148089>.
47. Guttormsen, A.G.; Kristofersson, D.; Nævdal, E. Optimal management of renewable resources with Darwinian selection induced by harvesting. *J. Environ. Econ. Manag.* **2008**, *56*, 167–179. <https://doi.org/10.1016/j.jeem.2007.11.005>.
48. Heino, M.; Baulier, L.; Boukal, D.S.; Ernande, B.; Johnston, F.D.; Mollet, F.M.; Pardoe, H.; Therkildsen, N.O.; Uusi-Heikkilä, S.; Vainikka, A.; et al. Can fisheries-induced evolution shift reference points for fisheries management? *ICES J. Mar. Sci.* **2013**, *70*, 707–721. <https://doi.org/10.1093/icesjms/fst077>.
49. Salvio, M.; Dubbeldam, J.; Staňková, K.; Brown, J.S. Fisheries management as a Stackelberg evolutionary game: Finding an evolutionarily enlightened strategy. *PLoS ONE* **2021**, *16*, e0245255. <https://doi.org/10.1371/journal.pone.0245255>.

-
50. Matishov, G.G.; Il'ichev, V.G. Optimal utilization of water resources: The concept of internal prices. *Dokl. Earth Sci.* **2006**, *406*, 86–88. <https://doi.org/10.1134/S1028334X06010211>.
 51. Gunnlaugsson, S.B.; Kristofersson, D.; Agnarsson, S. Fishing for a fee: Resource rent taxation in Iceland's fisheries. *Ocean Coast. Manag.* **2018**, *163*, 141–150. <https://doi.org/10.1016/j.ocecoaman.2018.06.001>.
 52. Hoi, S.C.H.; Sahoo, D.; Lu, J.; Zhao, P. Online learning: A comprehensive survey. *Neurocomputing* **2021**, *459*, 249–289. <https://doi.org/10.1016/j.neucom.2021.04.112>.
 53. Sutton, R.S.; Barto, A.G. *Reinforcement Learning: An Introduction*, 2nd ed.; MIT Press: Cambridge, MA, USA, 2018.
 54. Rokhlin, D.B.; Ougolnitsky, G.A. Stackelberg equilibrium in a dynamic stimulation model with complete information. *Autom. Remote Control* **2008**, *79*, 701–712. <https://doi.org/10.1134/S0005117918040112>.