

Article

Optimum Design of Reinforced Concrete Folded Plate Structures to ACI 318-11 Using Soft Computing Algorithm

Sayed Yousif ¹, Mehmet Polat Saka ^{1,2}, Sanghun Kim ³  and Zong Woo Geem ^{4,*} ¹ Department of Civil Engineering, University of Bahrain, Manama 32038, Bahrain; syousif.mohammed@ewa.bh (S.Y.); mpsaka@metu.edu.tr (M.P.S.)² Department of Engineering Sciences, Middle East Technical University, Ankara 06800, Turkey³ Department of Civil and Environmental Engineering, Temple University, Philadelphia, PA 19122, USA; sanghun.kim@temple.edu⁴ Department of Smart City & Energy, Gachon University, Seongnam 13120, Korea

* Correspondence: geem@gachon.ac.kr

Abstract: In this paper, an optimum design algorithm is presented for reinforced concrete folded plate structures. The design provisions are implemented by ACI 318-11 and ACI 318.2-14, which are quite complex to apply. The design variables are divided into three classes. The first class refers to the variables involving the plates, which are the number of supports, thicknesses of the plates, configurations of longitudinal and transverse reinforcement, span length of each plate, and angle of inclination of the inclined plates. The second class consists of the variables involving the auxiliary members' (beams and diaphragms) depth and breadth and the configurations of longitudinal and shear reinforcement. The third class of variables can be the supporting columns, which involve the dimensions of the column along each axis and the configurations of longitudinal and shear reinforcement. The objective function is considered as the total cost of the folded plate structure, which consists of the cost of concrete, reinforcement, and formwork that is required to construct the building. With such formulation, the design problem becomes a discrete nonlinear programming problem. Its solution is obtained by using three different soft computing techniques, which are artificial bee colony, differential evolution, and enhanced beetle antennae search. The enhancement suggested makes use of the population of beetles instead of one, as is the case in the standard algorithm. With this novel improvement, the beetle antennae search algorithm became very efficient. Two folded plate structures are designed by the proposed optimum design algorithm. It is observed that the differential evolution algorithm performed better than the other two metaheuristics and achieved the cheapest solution.



Citation: Yousif, S.; Saka, M.P.; Kim, S.; Geem, Z.W. Optimum Design of Reinforced Concrete Folded Plate Structures to ACI 318-11 Using Soft Computing Algorithm. *Mathematics* **2022**, *10*, 1668. <https://doi.org/10.3390/math10101668>

Academic Editor: Frank Werner

Received: 15 April 2022

Accepted: 9 May 2022

Published: 12 May 2022

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Keywords: folded plate; supporting members; minimum cost design; structural optimization; meta-heuristic algorithms; beetle antennae search algorithm; artificial bee colony algorithm; differential evolution algorithm

MSC: 65K10

Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Folded plate structures are assemblies of multiple numbers of plates rigidly connected in a folding pattern. The concept of folds also exists in nature, as can be found on various types of tree leaves, insect wings, and seashells. This concept has been established in many other fields, such as in solar panels, nanochips, and medical devices. Folded plate structures carry loads without requiring supporting beams along longitudinal edges. The reason for the effectiveness of reinforced concrete folded plate systems is that they transfer the applied loads to the supporting members through both bending and membrane action. The bending is resisted by both the reinforcement and concrete, while the tensile and compressive membrane forces are resisted by reinforcement and concrete. The folded

plate structures can provide very light structures to cover large areas without columns. They have more inherent rigidity and high load-carrying capacity than other structures. Therefore, they are generally preferred when there is a need for construction without internal columns, such as exhibition halls, theater buildings, and assembly halls. It is interesting to notice that there are books and articles on the analysis and design of folded plate constructions [1–4], but there are not many studies on the optimum design of these structures [5,6].

The structural weight of folded plate structures is minimized in Kostem [6]. The azimuthal angles and the width and thickness of the individual panels are considered design variables. The target of the study is to find the optimum geometry of the folded plate structure starting from an arbitrary initial geometry. The flexibility method is used to formulate the mathematical model, and the standard Lagrangian approach is used to obtain the optimum solution. The technique determines the optimal shape of the folded plate but does not consider any displacement and strength constraints. It is stated by Sarma and Adeli [5] that the design of concrete structures should be based on cost rather than weight minimization. This is because the construction of concrete structures is different from steel structures. It involves three different materials such as concrete, steel, and formwork. In this paper, a review of papers on cost optimization of concrete structures is carried out. The review covers reinforced concrete beams, slabs, columns, frame structures, shear walls, water tanks, folded plates, and tensile members. It can be noticed that the review is 24 years old, which does not contain recent information on the subject. The literature survey carried out on the subject has shown that there is not even a single article on the optimum design of folded plate structures where the design code provisions are considered, and modern optimization algorithms are employed so that practicing structural designers can utilize the developed technique.

In this study, two optimization frameworks are developed for reinforced concrete folded plate structures and their auxiliary and supporting members under the provisions of ACI-318-11 [7] and ACI-318.2-14 [8]. In the development of the frameworks, the guidelines provided in the design of reinforced concrete shells and folded plates by Varhese [3] are followed. The first framework is used to optimize the “V-type” of folded plates and the second one is for the “three-segment” type of folded plates. Both types are linear, with a single angle of inclination for all the spans. The optimum design algorithm developed makes use of two software: MATLAB [9] and CSI-SAP2000 [10]. Application programming interface (API) is used to achieve communication between the two software. CSI-SAP2000 is used to carry out the finite element analysis. The optimum design problem and the solution techniques are coded in MATLAB. The objective function is to minimize the total cost of all materials used to construct the folded plate building according to the selected sizes. For the cost estimation of reinforcement, the development lengths, overlapping, and bar bending details for all the members are taken into consideration. The superimposed dead load, live load, and lateral wind load are assigned in accordance with ACSE 7-5 [11] and ACI-318.2-14. The dead loads are applied along the full length of the plates, the live load is on the horizontal plane area, and the wind load is on the vertical plane area of the respective elevation.

In the mathematical formulation of the optimum design problem of folded plate structures, the number of supports, the thicknesses of plates, reinforcement configuration along each direction, angle of inclination, length of each plate, and the sectional details of edge beams, internal beams, diaphragms, and columns are taken as design variables. A design pool is prepared for each design that contains values that are used in the practice. The objective function is taken as the overall cost of the folded plate structure. When the design provisions are implemented in accordance with ACI 318-11 and ACI 318.2-14 with these design variables, the design optimization problem turns out to be a discrete nonlinear programming problem. It is shown in the literature that soft computing techniques are quite effective in attaining the solution to discrete nonlinear programming problems by Kaveh [12]. Several soft computing techniques could be used to obtain the solution to

the design optimization problem. Among these, artificial bee colony (ABC) by Latif and Saka [13] and the improved form of the beetle antennae search algorithm (pbBAS) by Yousif and Saka [14] are selected due to their efficiency in providing optimum solutions for the real-size design optimization problems. In addition to these, differential evolution (DE) is also used due to its successful applications in structural optimization by Wang et al. [15].

The paper is arranged as follows. In the second section, the mathematical formulation of the design optimization problem is explained in detail for both V-shaped and three-segment folded plate structures. This explanation covers the design variables, the objective function, and the set of design constraints. Additionally, the soft computing techniques selected to obtain the solution to the design optimization problem are summarized. In the third section, the two design optimization problems are solved by each of these three algorithms. A comparison of the results is conducted, and the best-performing technique is identified. In the fourth section, the results are discussed, and the best performing algorithm is used to study the effect of different span sizes on the optimum designs. Finally, concluding remarks are stated in the last section.

2. Materials and Methods

2.1. Mathematical Modeling

2.1.1. Design Variables

The folded plate structure parameters listed in the first column of Table 1 are treated as design variables. Some of the design variables are grouped as mentioned in the table. The number of grouped design variables is different in the two types of folded plate structures. The V-type folded plates (design problem 1) have 11 variables, while the three-segment folded plates (design problem 2) have 13 variables. The two additional grouped design variables are the thickness and length of the horizontal plates, which are absent in the V-type folded plate structures. A plan view of a typical folded plate roofed building indicating the members being affected by each design variable is shown in Figure 1. Additionally, sections and elevations of the building and the members are displayed in Figures 2–6, indicating the grouped details listed under each design variable.

Table 1. Design variable classes.

Design Variables	Description	List of Details Grouped in the Design Variable	Number of Variables in Problem	
			1	2
V ₁	Number of internal intermediate supports	No grouping	1	1
V ₂ & V ₃	Plate thickness ^I	No grouping	1	2
V ₄ & V ₅	Plate's reinforcement configuration in each direction ^{II}	Two grouped details: 1. Diameter of reinforcement. 2. Center to center spacing between reinforcement.	2	2
V ₆	Length of horizontal plate ^{III}	No grouping	0	1
V ₇	Angle of plate's inclination ^{IV}	No grouping	1	1
V ₈ & V ₉	Beams sectional details ^V	Seven grouped details: 1. Width of beam. 2. Depth of beam. 3. Diameter of longitudinal rebars. 4. Number of longitudinal bars along beam width. 5. Number of longitudinal bars along beam depth. 6. Diameter of shear reinforcement. 7. Spacing between shear reinforcement.	2	2

Table 1. Cont.

Design Variables	Description	List of Details Grouped in the Design Variable	Number of Variables in Problem	
			1	2
V_{10}	Diaphragm sectional details ^{VI}	Seven grouped details: 1. Thickness of the diaphragm. 2. Depth of diaphragm below lower level of plates. 3. Diameter of horizontal rebars. 4. Number of horizontal bars along diaphragm width. 5. Spacing between horizontal bars along diaphragm depth. 6. Diameter of vertical reinforcement. 7. Spacing between vertical reinforcement.	1	1
V_{11} – V_{13}	Columns sectional details ^{VII}	Seven grouped details: 1. Size of column along x-direction. 2. Size of column along y-direction. 3. Diameter of longitudinal rebars. 4. Number of longitudinal bars along x-direction. 5. Number of longitudinal bars along y-direction. 6. Diameter of shear reinforcement. 7. Spacing between shear reinforcement.	3	3
Total			11	13

^I: two different plate thicknesses are allowed in the model for problem (2): the first is for the inclined plate, and the second is for the horizontal plate. ^{II}: two configurations of plate reinforcement: the first is for the longitudinal reinforcement, and the second is for the transverse reinforcement. ^{III}: for problem (2), one length is set for all the horizontal plates at each span. ^{IV}: one angle of inclination is set for all the inclined plates; the angle is measured in degrees from the horizontal axis. ^V: two different beam sections are modeled: edge beams and internal beams. ^{VI}: one diaphragm section is set on both sides of the building. ^{VII}: three different column sections are modeled: x-edge columns, y-edge columns, and corner columns.

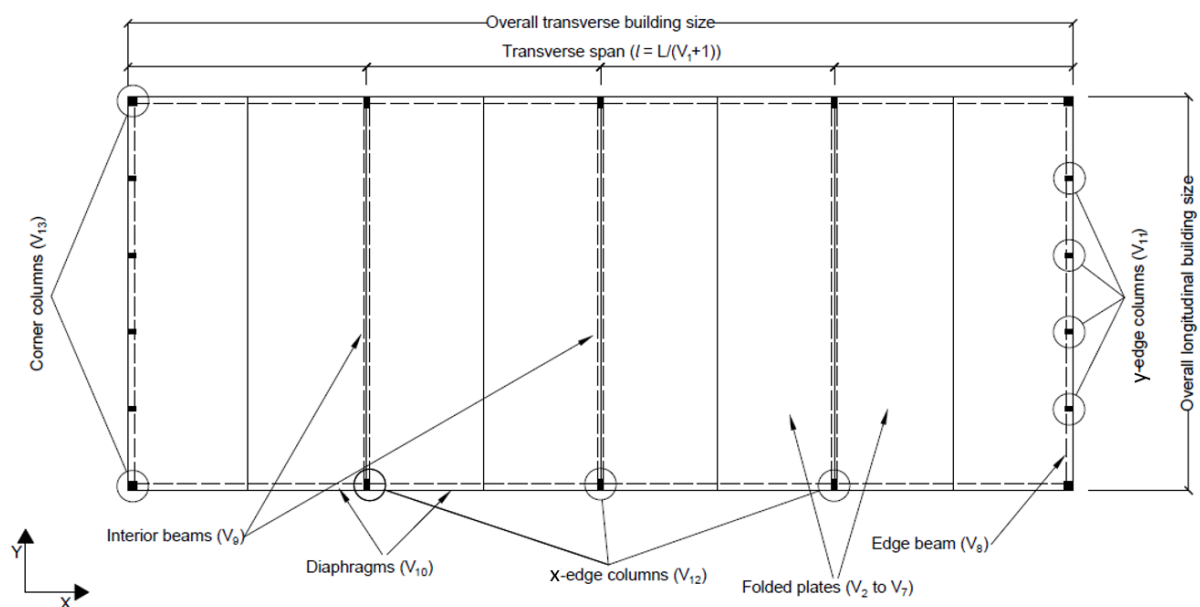


Figure 1. Typical folded plate building roof plan.

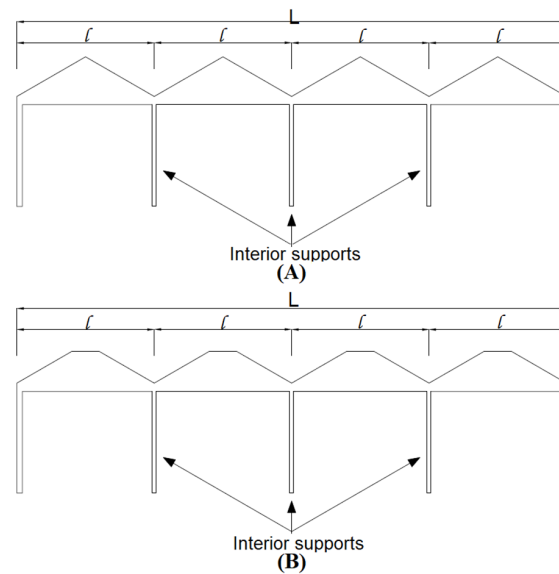


Figure 2. Folded plate roof number of internal supports design variable (V_1): (A) “V” type folded plate roof building elevation view; (B) Three-segment type folded plate roof building elevation view.

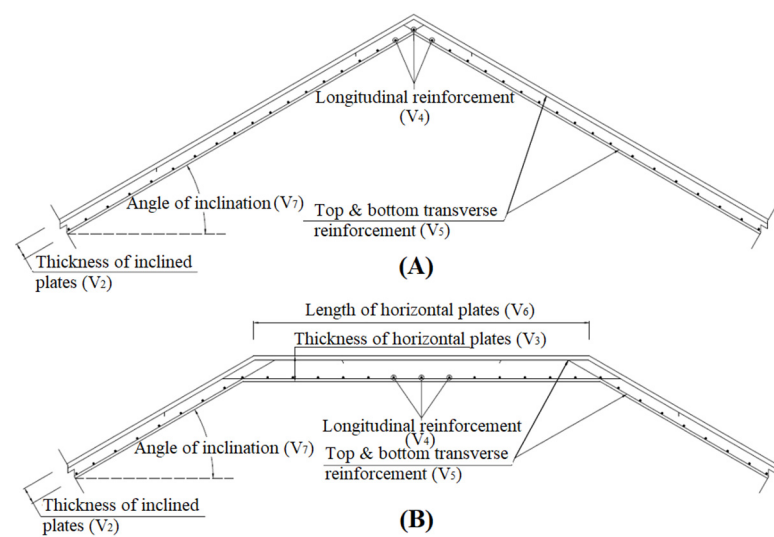


Figure 3. Folded plate roof section controlling design variables (V_2 through V_7): (A) “V” type folded plate roof section view; (B) Three-segment type folded plate roof section view.

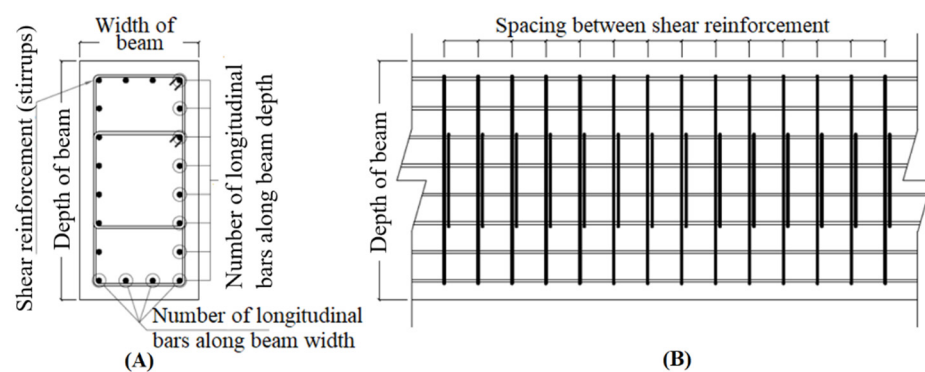


Figure 4. Beams controlling design variables (V_8 and V_9): (A) Beam section view; (B) Beam elevation view.

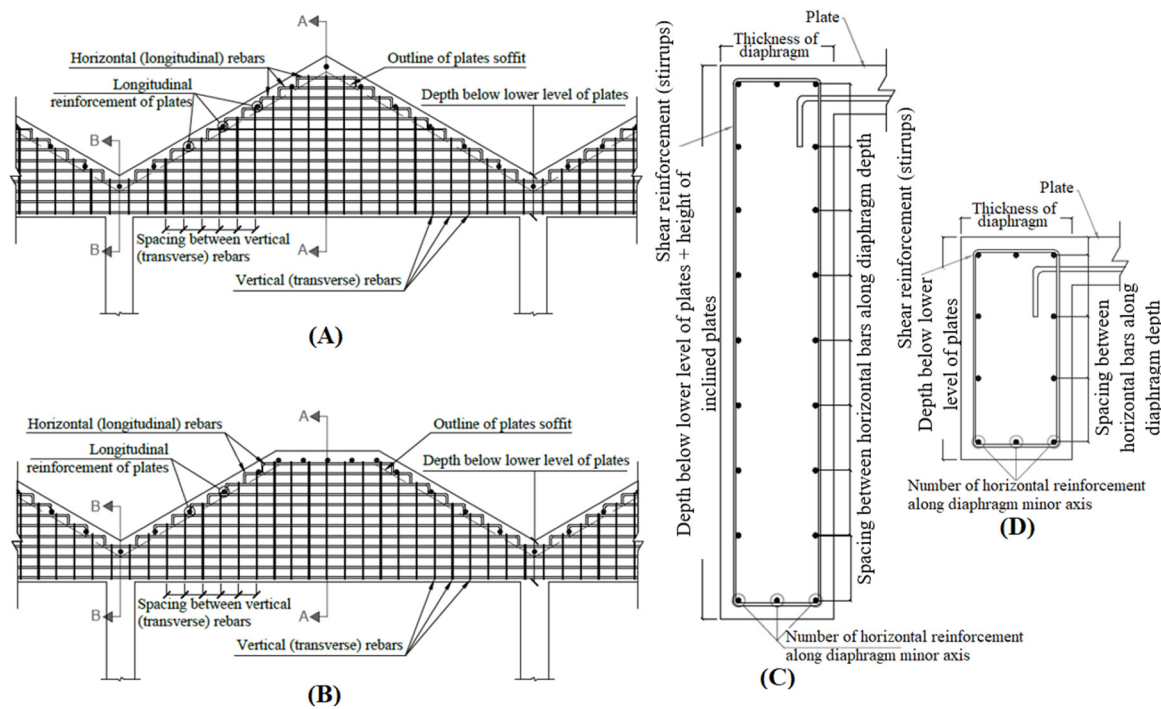


Figure 5. Diaphragms controlling design variables (V_{10}): (A) Elevation view of diaphragms in "V" type folded plates roofed building; (B) Elevation view of diaphragms in three-segment type folded plates roofed buildings; (C) Section A-A view; (D) Section B-B view.

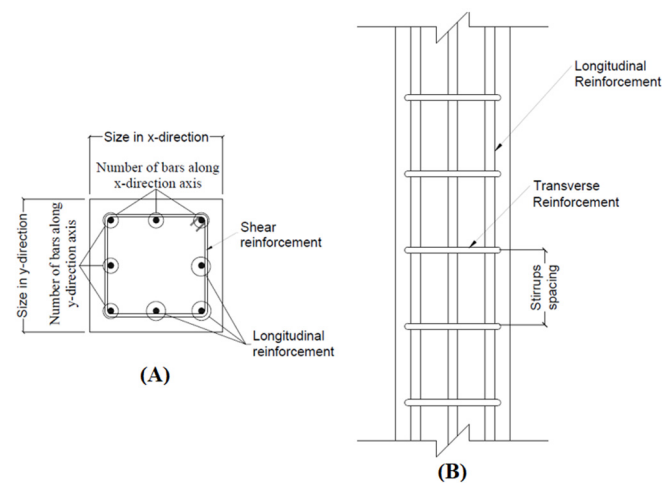


Figure 6. Columns controlling design variables (V_{11} , V_{12} , and V_{13}): (A) Columns section view; (B) Columns elevation view.

Folded Plate Variables

The first class of design variables refers to the folded plates. The first design variable controls the number of internal supports in each plot area, as listed in Table 2. The internal supports are represented by edge columns and internal beams connecting them. The second and third design variables involved are the thicknesses of the inclined and horizontal plates. It is assumed that all the inclined plates share one thickness, and the same applies to the horizontal plates. The fourth and fifth design variables are the configuration of the longitudinal and transverse reinforcement, respectively. The longitudinal reinforcement spans along the lengths of the plates are to resist membrane forces, and the transverse reinforcement spans along the folds are to resist bending moments. Both reinforcements are set the same along with all the plates, with overlapping occurring at plate folds or a maximum span of 6 m. The sixth variable controls the length of the horizontal plates, and

it only exists in the second problem. The seventh variable controls the angle of inclination of the folded plates, which is measured from the horizontal plane. The angle is kept the same for all inclined plates. The folded plates variables are shown in Figures 2 and 3.

Table 2. Design pools of folded plate variables.

Details			Pool
Number of interior supports (Nos.)			3:1:9
Thickness (mm)			70:10:200
Longitudinal and Transverse reinforcement	Diameter (mm)		{8, 10, 12, 16, 20}
	Spacing (mm)	For 8 mm diameter	{200, 250, 300}
		For 10 mm diameter	{200, 250}
		For 12 mm diameter	{150, 200, 250}
		For 16 mm diameter	{150, 200}
		For 20 mm diameter	{100, 150, 200}
Angle of plates inclination (for inclined plates measured from the horizontal axis in degrees)			5:1:40
Length of horizontal plates (mm)			500:500:5000

Auxiliary Members' Variables

The auxiliary members, which stiffen, strengthen, and support the folded plates, include the edge and internal beams and diaphragms. Each of the three members is represented by a single grouped variable that includes all the details necessary to model and check the member capacity. The first and second details are its dimensions (width/thickness and depth). Details 3 to 5 represent the longitudinal (horizontal) reinforcement of the member. The positions and diameters of each bar are set through these three details. The sixth and seventh details represent the configuration of the shear (vertical) reinforcement along the entire length of the member. The auxiliary members' design variables are shown in Figures 4 and 5.

Supporting Columns Variables

Similar to the auxiliary members, each column type is represented by a single design variable. In the model, there are three column sections: (1) x-edge columns, (2) y-edge columns, and (3) corner columns. A total of seven details are grouped in each design variable. The first and second details concern the dimensions of the column in the x and y directions. Details 3 to 5 represent the longitudinal reinforcement of the column. The positions and diameters of each bar are set through these three details. The sixth and seventh details represent the configuration of the shear (vertical) reinforcement along the entire length of the column. All the columns are detailed as ordinary moment-resisting frames. The column design variables are shown in Figure 6.

Variable Pool

The design pool from which the value of each design variable would be selected is listed in Tables 2–5. The values listed in the last column of the tables without brackets show the lower bound of the design variable, the second value is the increment in (mm or nos.), and the third value is the upper bound of that design variable. For example, values for plate thickness start from 70 mm goes up to 200 mm with an increment of 10 mm. The corresponding discrete values that can be taken by the variable are [70, 80, 90, 100, ..., 190, 200]. The numbers within the brackets indicate the possible values of the design variable. For example, the diameter of longitudinal and transverse reinforcement can be either 8 mm, 10 mm, 12 mm, 16 mm, or 20 mm. Considering the variable pools, the size of design variable groups is: 7, 14, 14, 13, 13, 10, 36, 484, 484, 315, 90, 90, and 72 for V_1 through V_{13} , respectively. The given sizes produce 2.5659×10^{19} and 3.5922×10^{21} possible solution combinations for problems 1 and 2, respectively.

Table 3. Design pools of edge and internal beam variables.

Details			Pool
Size	Depth (mm)		500:100:1200
	Width (mm)	For depth of 500 mm	{200}
		For depth of 600 mm	{200, 250}
		For depth of 700 mm	{250, 300}
		For depth of 800 mm	{300, 350}
		For depth of 900 mm	{350}
		For depth of 1000 mm	{350, 400}
Longitudinal reinforcement	Diameter (mm)	For depth of 1100 mm and 1200 mm	{400}
		For 200 × 500 beams	{12, 16}
		For 200 × 600 beams	{16, 20}
		For 250 × 600 beams	{20}
		For 250 × 700 beams	{16, 20}
		For 300 × 700 beams	{20}
		For 300 × 800 or larger beams	{20, 25}
	Minor axis	For 200 mm wide	{2}
		For 250 mm and 300 mm width	{3}
		For 350 mm and 400 mm width	{4}
	Major axis	Number of bars along (Nos.)	For 500 mm depth {5, 6}
			For 600 mm depth {5, 6, 7}
			For 700 mm depth {6, 7, 8}
			For 800 mm depth {7, 8, 9}
			For 900 mm depth {8, 9, 10}
			For 1000 mm depth {9, 10, 11}
Shear reinforcement	Diameter (mm)		For 1100 mm and 1200 mm depth {10, 11, 12}
			{8, 10, 12}
	Spacing (mm)	For 8 mm diameter	{250, 300}
		For 10 mm diameter	{200, 250}
		For 12 mm diameter	{100, 150, 200}

Table 4. Diaphragm variables design pools.

Details			Pool
Size	Depth below inclined plates bottom level (mm)		500:100:1000
	Thickness (mm)	For depth of 500 mm	{200, 250}
		For depth of 600 mm	{250, 300}
		For depth of 700 mm	{300, 350}
		For depth of 800 mm	{350}
		For depth of 900 mm	{350, 400}
		For depth of 1000 mm	{400}
Horizontal (longitudinal) reinforcement	Diameter (mm)	For 200 mm thickness	{12, 16}
		For 250 mm and 300 mm thickness	{16, 20}
		For 350 mm thickness	{20, 25}
		For 400 mm thickness	{25}
	Number of bars along minor axis (Nos.)	For 200 mm thickness	{2}
		For 250 mm thickness	{2, 3}
		For 300 mm thickness	{3}
		For 350 mm thickness	{4}
		For 400 mm thickness	{5}
	Spacing along Major axis (mm)		{100, 150, 200}

Table 4. *Cont.*

Details		Pool
Vertical (transverse) reinforcement	Diameter (mm)	{10, 12, 16}
	Spacing (mm)	{250, 300}
		{200, 250}
		{100, 150, 200}

Table 5. Column variables design pools.

Details			Pool
Size	x-edge columns	Along x-direction (mm)	400:100:700
		Along y-direction (mm)	For 400 mm and 500 mm depths {200}
			For 600 mm depths {250, 300}
	For 700 mm depths {300}		
	y-edge columns	Along y-direction (mm)	400:100:700
		Along x-direction (mm)	For 400 mm and 500 mm deep {200}
			For 600 mm depths {250, 300}
	For 700 mm depths {300}		
Corner columns	Both directions (mm)	{400, 450, 500}	
Longitudinal reinforcement	Diameter (mm)	x and y-edge columns	For 200 mm widths {12, 16}
			For 250 mm widths {16, 20}
			For 300 mm widths {20, 25}
		Corner columns	{16, 20}
	x and y-edge columns number of bars along	minor axis (Nos.)	For 200 mm and 250 mm widths {2}
			For 300 mm widths {3}
		Major axis (Nos.)	For 400 mm depths {4}
			For 500 mm depths {5}
			For 600 mm depths {6}
	For 700 mm depths {7}		
Corner columns number of bars along both directions (Nos.)	For 400 mm sizes {4}		
	For 450 mm and 500 mm sizes {4, 5}		
Shear reinforcement	Diameter (mm)	{8, 10, 12}	
	Spacing (mm)	For 8 mm diameter {200, 250, 300}	
		For 10 mm diameter {150, 200, 250}	
		For 12 mm diameter {100, 150, 200}	

2.1.2. Objective Function

The objective function considered for both optimization problems is the overall cost of materials required to construct the structural system. The cost is obtained by summing the costs of three materials: reinforcement, concrete, and the formwork system required to construct each member. The formwork system cost includes the cost of scaffolding, shoring, and plywood panels. The equations used to calculate the objective function, penalty, overall cost, and the cost of each element are summarized in Table 6. The objective function used in the solution techniques is the penalized one, which contains total constraint violations. This is carried out because soft computing techniques find the optimum solution to unconstrained optimization problems. Constrained optimization problems must be transformed into unconstrained optimization ones by the use of the penalty function method, as shown in Table 6.

Table 6. Expressions used to calculate the objective function, penalty, and costs of each element.

Element	Equation	Terms Definition
Penalized Objective function	$f(x) = C_{\text{total}}(1 + \text{Penalty})^\epsilon$	C_{total} : total cost of materials (actual objective function). Penalty: the value of constraints violation. ϵ : penalty exponent (taken as 1 for all algorithms)
Penalty	$\text{Penalty} = \sum_{i=1}^{nc} \max(g_i)$	nc : total number of constraints. g_i : i th constraint violation value.
Overall cost	$C_{\text{total}} = C_c + C_{\text{fw}} + C_{\text{rs}}$	C_c : total cost of concrete. C_{fw} : total cost of formwork. C_{rs} : total cost of reinforcing steel.
Concrete	$C_c = U_c(V_{\text{cp}} + V_{\text{cb}} + V_{\text{cd}} + V_{\text{cc}})$	U_c : unit cost of concrete. V_{cp} : volume of plates concrete. V_{cb} : volume of beams concrete. V_{cd} : volume of diaphragms concrete. V_{cc} : volume of columns concrete.
Formwork	$C_{\text{fw}} = U_{\text{fwh}}(A_{\text{fwhp}} + A_{\text{fwb}} + A_{\text{fwd}} + A_{\text{fwc}}) + U_{\text{fwi}}(A_{\text{fwip}})$	U_{fwh} : unit cost of horizontal and vertical formworks. U_{fwi} : unit cost of inclined formworks. A_{fwhp} : area of horizontal plates formworks. A_{fwip} : area of inclined plates formworks. A_{fwb} : area of beams formworks. A_{fwd} : area of diaphragms formworks. A_{fwc} : area of columns formworks.
Reinforcement	$C_{\text{rs}} = U_{\text{rs}}(W_{\text{rsp}} + W_{\text{rsb}} + W_{\text{rsd}} + W_{\text{rsc}})$	U_{rs} : unit cost of reinforcing steel. W_{rsp} : total weight of reinforcement used in plates. W_{rsb} : total weight of reinforcement used in beams. W_{rsd} : total weight of reinforcement used in diaphragms. W_{rsc} : total weight of reinforcement used in columns.

2.1.3. Design Constraints

To ensure that the proposed optimum design is safe and constructable, it should satisfy the limitations imposed by the design codes as well as constructability constraints. This can be achieved by introducing design constraints that cover all the design code provisions and practical application requirements. A total of 42 design constraints are set for both problems. The design constraints are divided into three parts; the first is for the plates, the second for the auxiliary members (beams and diaphragms), and the third is for the columns.

Plate Constraints

To ensure the safety and practical feasibility of the plates, their strength, service, and detailing requirements must be satisfied. The strength requirements comprise checking the tensile and compressive membrane strengths, bending moment capacity, and shear force capacity. As for the service requirements, the deflections at immediate and long-term stages shall be within the limits. Finally, for the detailing requirements, the reinforcement spacing should be less than the maximum limit, the thickness of the slab should be sufficient to allow the placement of all rebars, and the reinforcement area should be within the allowable range to prevent sudden brittle failure and satisfy the shrinkage and temperature limitations. The plate constraints are summarized in Table 7.

Table 7. Folded plates' constraints summary.

Const.	Description	Constraint Equation	Term's Definition	Remarks
g ₁	Membrane compression	$\frac{f_{c(actual)}}{\phi f_{c(allowable)}} - 1 \leq 0$	$f_{c(actual)}$: plates ultimate compressive stress. $f_{c(allowable)}$: compression stress capacity. ϕ : compression strength reduction factor (0.65). f_{cu} : ultimate compressive strength of concrete.	$f_{c(allowable)} = 0.4 f_{cu}$
g ₂	Membrane tension	$\frac{f_{t(actual)}}{\phi f_{t(allowable)}} - 1 \leq 0$	$f_{t(actual)}$: plates ultimate tensile stress. $f_{t(allowable)}$: tension stress capacity. ϕ : tensile strength reduction factor (0.9). f_y : reinforcement yield strength. A_s : area of reinforcement.	$f_{t(allowable)} = A_s f_y$
g ₃	Transverse bending moment	$\frac{M_u}{\phi M_n} - 1 \leq 0$	M_u : ultimate transverse bending moment. M_n : nominal bending moment capacity. ϕ : bending moment capacity reduction factor (ranges between 0.65 to 0.9).	ACI318-11 Clause 18.7
g ₄	Transverse shear	$\frac{V_u}{\phi V_c} - 1 \leq 0$	V_u : ultimate transverse shear force at plates interface. V_c : shear capacity of plate section. ϕ : shear strength reduction factor (0.75).	ACI318-11 Clause 11.11
g ₅	Immediate deflection	$\frac{\delta_{i(actual)}}{\delta_{i(allowable)}} - 1 \leq 0$	$\delta_{i(actual)}$: actual immediate deflection. $\delta_{i(allowable)}$: immediate deflection limiting value. l : minimum span of plates.	$\delta_{i(allowable)} = l/360$
g ₆	Long-term deflection	$\frac{\delta_{u(actual)}}{\delta_{u(allowable)}} - 1 \leq 0$	$\delta_{u(actual)}$: actual long-term deflection. $\delta_{u(allowable)}$: long-term deflection limiting value.	$\delta_{im(allowable)} = l/240$
g ₇	Maximum reinforcement spacing	$\frac{S_{(provided)}}{S_{(max)}} - 1 \leq 0$	$S_{(provided)}$: provided reinforcement spacing. $S_{(max)}$: maximum allowed reinforcement spacing.	If $f_c \leq 0.3 \sqrt{f_{cu}}$: $S_{(max)} = \min(5h, 450)$ Otherwise: $S_{(max)} = \min(3h, 450)$
g ₈	Minimum thickness	$\frac{h_{(min)}}{h} - 1 \leq 0$	h : actual plates thickness. $h_{(min)}$: minimum allowed plate thickness. ϕ_t : diameter of transverse reinforcement. ϕ_l : diameter of longitudinal reinforcement. C_b : bottom face cover (15 mm). C_t : top face cover (25 mm).	$h_{(min)} = 2\phi_t + \phi_l + C_b + C_t$
g ₉	Minimum reinforcement area	$\frac{A_{s(min)}}{A_s} - 1 \leq 0$	A_s : provided area of reinforcement. $A_{s(min)}$: minimum allowable area of reinforcement. b : width of plate strip.	$A_{s(min)} = 0.0018 bh$
g ₁₀	Maximum reinforcement area	$\frac{A_s}{A_{s(max)}} - 1 \leq 0$	$A_{s(max)}$: maximum allowable area of reinforcement.	$A_{s(max)} = 0.04 bh$

Auxiliary Members' Constraints

The auxiliary members stiffen, strengthen, and support the plates. They comprise edge and internal beams and diaphragms. For auxiliary members, strength and detailing requirements must also be satisfied. Deflections are not checked for auxiliary members as their deflections are covered with the plates. For the strength requirements, the member should have sufficient flexural capacity to resist the combined bending moments and axial forces; this is checked by plotting the interaction diagram shown in Figure 7. Additionally, the shear capacity of the member should meet the applied forces along the whole length of the member. As for the detailing requirements, the spacing of the longitudinal reinforcement should be within the allowable range to prevent cracks and allow concrete to flow between the bars. The total area of the longitudinal reinforcement should be within the allowable range to prevent sudden brittle failure and satisfy the shrinkage and temperature requirements. The spacing of the shear reinforcement (stirrups) should be less than the allowable maximum limit, and the area of the provided shear reinforcement per unit length should be more than the minimum limit. The constraints related to auxiliary members are summarized in Table 8. Since the section of each of the three auxiliary members has its own grouped design variable, an independent constraint number is assigned to each of the auxiliary members. The constraints numbering is the following: through are for edge beams, through are for internal beams, and through are for diaphragms.

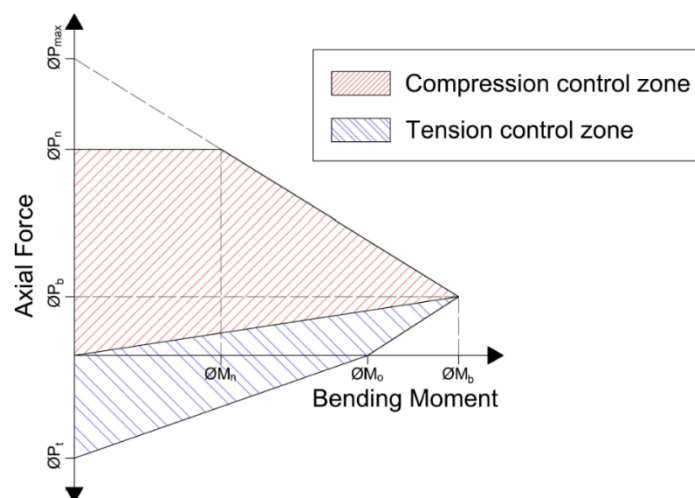


Figure 7. Interaction diagram linear approximation.

Column's Constraints

The columns are divided into three groups: x-edge, y-edge, and corner columns. Like auxiliary members, the column's strength and detailing requirements must be satisfied. For the strength requirements, the columns should have sufficient flexural capacity to resist the combined bending moments and axial forces; this is checked by plotting the interaction diagram shown in Figure 7 with the exclusion of the negative axial force part (tension part), as tension is not experienced in the column members. Additionally, the shear capacity of the column should meet the applied forces along with the whole height of the column. As for the detailing requirements, the spacing of the longitudinal reinforcement should be within the allowable range to prevent cracks and allow concrete to flow and passage of supported members' rebars between the longitudinal bars. The total area of the longitudinal reinforcement should be within the allowable range to prevent sudden brittle failure and reduce the creep effects. The spacing of the shear reinforcement (stirrups) should be less than the allowable maximum limit for ordinary moment-resisting frames, and the provided area of the shear reinforcement per unit length should be more than the minimum limit. The columns' constraints are summarized in Table 9. The three-column groups are checked under one constraint number for each of the above constraints.

Table 8. Auxiliary members' constraints summary.

Const.	Description	Constraint Equation	Terms Definition	Remarks
g_{11} , g_{19} , and g_{27}	Flexural strength	(1) Axial capacity: $\frac{P_u}{\phi P_n} - 1 \leq 0$ (2) Moment capacity: $\frac{M_u}{\phi M_n} - 1 \leq 0$ (3) Combined flexural capacity: $\frac{\sqrt{\left(\frac{M_u}{bh^2}\right)^2 + \left(\frac{P_u}{bh}\right)^2}}{\sqrt{\left(\frac{\phi M_n}{bh^2}\right)^2 + \left(\frac{\phi P_n}{bh}\right)^2}} - 1 \leq 0$	P_u : ultimate axial load acting on the section. M_u : ultimate bending moment acting on the section. P_n : axial force capacity of the section under the given ultimate loads. M_n : bending moment capacity of the section under the given ultimate loads. ϕ : capacity reduction factor (ranges between 0.65 to 0.9). b : perpendicular section dimension to the bending moment. h : parallel section dimension to the bending moment.	According to the interaction diagram shown in Figure 7.
g_{12} , g_{20} , and g_{28}	Shear strength	$\frac{V_u}{\phi V_c} - 1 \leq 0$	V_u : ultimate shear force acting on the section. V_c : shear capacity of the section. ϕ : shear strength reduction factor (0.75).	ACI318-11 Clause 11.11
g_{13} , g_{21} , and g_{29}	Minimum reinforcement spacing	$\frac{S_{(min)}}{S_{(provided)}} - 1 \leq 0$	$S_{(provided)}$: provided reinforcement spacing. $S_{(min)}$: minimum allowed reinforcement spacing for the section.	$S_{(min)} = 40$ mm
g_{14} , g_{22} , and g_{30}	Maximum reinforcement spacing	$\frac{S_{(provided)}}{S_{(max)}} - 1 \leq 0$	$S_{(max)}$: maximum allowed reinforcement spacing for the section.	$S_{(max)} = 300$ mm
g_{15} , g_{23} , and g_{31}	Minimum reinforcement area	$\frac{A_{s(min)}}{A_s} - 1 \leq 0$	A_s : provided reinforcement area in the section. $A_{s(min)}$: minimum allowed reinforcement area for the section. d_t : effective depth of furthest layer of reinforcement in tensile face.	$A_{s(min)} = \frac{\max(0.25\sqrt{f_{cu}}, 1.4)}{f_y} bd_t$
g_{16} , g_{24} , and g_{32}	Maximum reinforcement area	$\frac{A_s}{A_{s(max)}} - 1 \leq 0$	$A_{s(max)}$: maximum allowed reinforcement area for the section.	$A_{s(max)} = 0.04 bh$
g_{17} , g_{25} , and g_{33}	Maximum shear reinforcement spacing	$\frac{S_v}{S_{v(max)}} - 1 \leq 0$	S_v : provided shear reinforcement spacing along the member length. $S_{v(max)}$: maximum allowed shear reinforcement spacing for the section. V_s : required shear reinforcement capacity to safely allow the section capacity to reach the ultimate shear force.	If $\phi V_s \leq 0.33\sqrt{f_{cu}}bd$: $S_{v(max)} = \min(d_t/2, 600)$ Otherwise: $S_{v(max)} = \min(d_t/4, 300)$
g_{18} , g_{26} , and g_{34}	Minimum shear reinforcement area	$\frac{A_{v(min)}}{A_v} - 1 \leq 0$	A_v : provided shear reinforcement area per unit length. $A_{v(min)}$: minimum allowed shear reinforcement area for the section.	$A_{v(min)} = \frac{\max(0.062\sqrt{f_{cu}}, 0.35)}{f_y} h$

Table 9. Column's constraints summary.

Const.	Description	Constraint Equation	Terms Definition	Remarks
g35	Flexural strength	(1) Axial capacity: $\frac{P_u}{\phi P_n} - 1 \leq 0$ (2) Moment capacity: $\frac{M_u}{\phi M_n} - 1 \leq 0$ (3) Combined flexural capacity: $\frac{\sqrt{\left(\frac{M_u}{bh^2}\right)^2 + \left(\frac{P_u}{bh}\right)^2}}{\sqrt{\left(\frac{\phi M_n}{bh^2}\right)^2 + \left(\frac{\phi P_n}{bh}\right)^2}} - 1 \leq 0$	P_u : ultimate axial load acting on the column. M_u : ultimate bending moment acting on the column. P_n : axial force capacity of the column under the given ultimate loads. M_n : bending moment capacity of the column under the given ultimate loads. ϕ : capacity reduction factor (ranges between 0.65 to 0.9). b : perpendicular section dimension to the bending moment. h : parallel section dimension to the bending moment.	According to the interaction diagram shown in Figure 7 with the exclusion of the tension part (the part below the x-axis).
g36	Shear strength	$\frac{V_u}{\phi V_c} - 1 \leq 0$	V_u : ultimate shear force acting on the column. V_c : shear capacity of the column section. ϕ : shear strength reduction factor (0.75).	ACI318-11 Clause 11.11
g37	Minimum reinforcement spacing	$\frac{S_{(min)}}{S_{(provided)}} - 1 \leq 0$	$S_{(provided)}$: provided reinforcement spacing. $S_{(min)}$: minimum allowed reinforcement spacing for the column section.	$S_{(min)} = 50$ mm
g38	Maximum reinforcement spacing	$\frac{S_{(provided)}}{S_{(max)}} - 1 \leq 0$	$S_{(max)}$: maximum allowed reinforcement spacing for the column section.	$S_{(max)} = 300$ mm
g39	Minimum reinforcement area	$\frac{A_{s(min)}}{A_s} - 1 \leq 0$	A_s : provided reinforcement area in the column section. $A_{s(min)}$: minimum allowed reinforcement area for the column section.	$A_{s(max)} = 0.01 bh$
g40	Maximum reinforcement area	$\frac{A_s}{A_{s(max)}} - 1 \leq 0$	$A_{s(max)}$: maximum allowed reinforcement area for the column section.	$A_{s(max)} = 0.08 bh$
g41	Maximum shear reinforcement spacing	$\frac{S_v}{S_{v(max)}} - 1 \leq 0$	S_v : provided shear reinforcement spacing along the column length. $S_{v(max)}$: maximum allowed shear reinforcement spacing for the column section. ϕ_l : diameter of longitudinal reinforcement. ϕ_v : diameter of shear reinforcement.	$S_{v(max)} = \min(b, 16\phi_l, 48\phi_v)$
g42	Minimum shear reinforcement area	$\frac{A_{v(min)}}{A_v} - 1 \leq 0$	A_v : provided shear reinforcement area per unit length. $A_{v(min)}$: minimum allowed shear reinforcement area for the column section.	$A_{v(min)} = \frac{\max(0.062\sqrt{f_{cu}}, 0.35)}{f_y} h$

2.2. Soft Computing Techniques

The design optimization problem of V-type and three-segment folded plate structures has 11 and 13 design variables, respectively, which take discrete values, defined in Tables 2–5. The objective function of the design problem is given in Table 6, while the constraints are summarized in Tables 7–9. In order to attain the optimum solution to this design problem, it is necessary to select the appropriate values of design variables defined in Tables 2–5 such that the objective function becomes the minimum while all 42 constraints described in Tables 7–9 are satisfied. This is a problem of combinatorial optimization with discrete design variables. Certainly, it is possible to produce a very large number of combinations of possible solutions randomly or otherwise where the design constraints are satisfied. However, this does not guarantee that among these solutions, the one which gives the objective function its minimum value may exist. Therefore, it is not an easy task to find the optimum solution to such optimum design problems. Using gradient-based mathematical programming techniques to determine the optimum solution is not an option due to the fact that the design variables are non-continuous, and constraints are non-differentiable to Saka and Geem [16]. The use of the branch and bound method or other techniques of integer programming is impractical and computationally quite cumbersome to apply to determine the solution of this class of problems presented by Saka [17]. On the other hand, there are other techniques in mathematical programming that do not require gradient information of the objective function and design constraints. These are called direct search methods. The success of these techniques depends very much on the number of design variables in the combinatorial optimum design problem as well as the size of discrete pools from which the values of design variables are required to be selected in Saka et al. [18].

Soft computing techniques (metaheuristic algorithms) do not suffer from the above-mentioned limitations. These techniques are suitable for achieving global optimum or near-global optimum solutions [19,20]. These techniques are non-traditional stochastic search and optimization methods without the need to use gradient information of the objective function and constraints. They find the optimum solution by moving within a design domain randomly utilizing intelligent heuristics to guide the random search. The strategies that direct the search process are inspired by natural phenomena, social culture, biology, or laws of physics. There are several review papers and books available in the literature that comprehensively explain the use of metaheuristic algorithms in structural optimization [16–20]. An improved form of the recently developed technique, the beetle antennae search (BAS) algorithm [21,22], is used in this study to determine the optimum solution to the design problem of two folded plate structures. Two other well-established algorithms, the artificial bee colony (ABC) algorithm [23–26] and the differential evolution (DE) algorithm by Storn and Price [27], are widely applied in structural optimization. The standard forms of both algorithms have also been used to find the optimum solution to the design problems. A summary of the working steps of both the standard forms of the three algorithms and the modified form of the beetle antennae search algorithm is in the following.

2.2.1. Artificial Bee Colony (ABC) Algorithm

The artificial bee colony algorithm originated by Karaboga [23]. Its steps are based on the foraging behavior of a honeybee colony. In the working steps of the algorithm, the bees in the colony are expected to carry out three different types of tasks, and they are named according to these tasks. The bees that locate the food source, evaluate its amount of nectar, and keep its location in their memory are called employed bees. These bees fly back to the hive after finding a new food source and share this information with other bees by dancing in the dancing area. This dance is called the waggle dance. It consists of two loops, one on the left and one on the right. The line which separates these two loops shows the direction of the food source. The dancing time represents the amount of nectar in the food source. The other group that observes the waggle dance and makes a decision as to whether it is worthwhile to fly to that food source or not are called onlooker bees. It is apparent that if the

dancing time is long, this means the food source is rich, which is expected to attract more onlooker bees. The third group is called scout bees which explore new food sources near the hive randomly. Therefore, the task of scout bees is to carry out the exploration, and the task of the other group of bees is exploitation. In the implementation of the above tasks into a numerical optimization algorithm, each food source represents a possible solution for the optimization problem. The nectar amount of a food source is considered as the quality of the solution, which is identified by its fitness value. There are four stages in the application of the artificial bee colony algorithm. These stages are defined as the initialization phase, employed bees' phase, onlooker bees' phase, and scout bees' phase by Karaboga [23].

1. Initialization phase: In this phase, the population of food sources is initialized, $(x_p, p = 1, \dots, np)$ by using (1) where np is the population size (total number of artificial bees). Each food source consists of n variables $(x_{pi}, i = 1, \dots, n)$ which is a potential solution to the optimization problem.

$$x_{pi} = x_{li} + rand(0, 1)(x_{ui} - x_{li}) \quad (1)$$

where x_{li} and x_{ui} are lower and upper bound on x_i . $rand(0, 1)$ is a random number between 0 and 1.

2. Employed bees' phase: In this phase, new food sources are searched by employed bees by using (2).

$$v_{pi} = x_{pi} + \phi_{pi}(x_{pi} - x_{ki}) \quad (2)$$

where $k \neq i$ is a randomly selected food source, and ϕ_{pi} is a random number within range $[-1, 1]$. After producing the new food source, its fitness is calculated. If the fitness is better than x_{pi} , the new food source replaces the previous one. The fitness value of the food sources is calculated according to (3)

$$fitness(x_p) = \begin{cases} \text{and } \frac{1}{1+|f(x_p)|} & \text{if } f(x_p) \geq 0 \\ \text{and } 1 + abs(f(x_p)) & \text{if } f(x_p) < 0 \end{cases} \quad (3)$$

3. Onlooker bees' phase: There are two groups of unemployed bees which are onlooker bees and scouts. Employed bees share their food source information with onlooker bees. Onlooker bees choose their food source with the probability value P_p which is calculated using the fitness values of each food source in the population as shown in (4).

$$P_p = \frac{fitness(x_p)}{\sum_{p=1}^{np} fitness(x_p)} \quad (4)$$

When an onlooker bee selects a food source x_{pi} probabilistically, a neighborhood source is determined by using Equation (2), and its fitness value is computed using (3).

4. Scout bees' phase: Scout bees choose their food sources randomly. Employed bees become scout bees when their food sources cannot be improved anymore after a predetermined number of trials. This causes abandonment of these solutions. These scouts' bees start to search for new solutions.

5. Phases 2–4 are repeated until termination criteria is satisfied.

2.2.2. Differential Evolution (DE) Algorithm

One other metaheuristic algorithm which is widely applied in structural optimization is the differential evolution technique, developed by Storn and Price [27]. This technique belongs to the evolutionary optimization algorithms group, which is also population based. The differential evolution algorithm initiates the search for an optimum solution by first setting up an initial population. The initial population consists of randomly generated m individuals that are expected to cover the entire design space. Uniform probability distribution is used for all random decisions. An individual in a population represents a candidate solution to the optimization problem. This is the same as the chromosomes

or genomes of a genetic algorithm. However, the differential evolution algorithm does not use binary representation for the design variables, but it makes use of real numbered representation. The individual is called an agent, and the objective function is called a fitness function. New solution vectors are generated by adding the weighted difference between two population vectors to a third vector. This operation is called a mutation. The mutated vectors are then mixed with the parameters of another predetermined vector, the target vector, to yield the trial vector. This is referred to as crossover. If the trial vector produces a lower cost function value than the target vector, the trial vector replaces the target vector in the next generation. This operation is called selection. Each population vector has to serve once as a target vector so that N competition takes place in one generation. Generations are continued until a predetermined maximum number of generations is reached. The steps of the algorithm are summarized in the following as given by Storn and Price [27].

1. Initial population is generated randomly in the search space which consists of m number of agents x , each of which comprises n design variables.
2. For each agent x_j where $j = 1, \dots, n$ the following is carried out:
 - Three agents x_a , x_b , and x_c which are distinctly different from each other and that of x_j are selected randomly from the population.
 - Index k , which is between 1 to n is selected randomly.
 - The agent's trial vector x_t is computed by iterating over each $i \in \{1, \dots, m\}$ as follows:
 - ❖ Select a random number $r_i \sim U(0, 1)$
 - ❖ Compute the trial vector as $x_t = x_a + F(x_b - x_c)$ if $i = k$ or $r_i \leq CR$ otherwise $x_t = x_j$ where CR is the crossover rate and F is the scaling (weighting) factor defined by users.
 - The trial vector is updated considering the lower and upper bound vectors as $x_t = x_L$ if $x_t < x_L$, $x_t = x_u$ if $x_t > x_u$. If $W(x_t) < W(x_j)$ then x_t is replaced by the agent x_j .
3. The agent x_o from the population having the lowest fitness $W(x_o)$ is the best-found solution within this generation.
4. Continue the generation until stopping criteria is satisfied.

It is reported in the literature that control variables m , F , and CR of the differential evolution algorithm are not difficult to choose in order to obtain good results. It is suggested that the selection of the total value of the initial population between 5 and 10 times the number of parameters in the optimization problem is reasonable for a good performance of the algorithm. It is advised by Storn and Price [27] to select the initial values for F and CR as 0.5 and 0.1, respectively, to attain stable convergence.

2.2.3. Beetle Antennae Search (BAS) Algorithm

The beetle antennae search algorithm originated by Jiang [21]. Longhorn beetles have very long antennas. These antennas, in some case even longer than the beetle's length, have receptor cells that serve to receive odors of prey or any other pheromones. Beetles move each antenna waveringly from side to side to receive an odor when it searches for food or mates. When the antenna on one side detects a higher concentration of an odor, the beetle moves in that direction; otherwise, it would move in the other direction. It means the beetle explores nearby areas randomly using both antennas. The beetle antennae search algorithm imitates this searching behavior. The steps of the algorithm can be collected in five stages: initialization of beetle position, randomization of movement direction, estimation of right-hand and left-hand side movements, movement in the best direction, and update of sensing length and step size.

1. Initialization of beetle position: the beetle position $x = \{x_1, x_2, \dots, x_n\}$ where n is the total number of variables is initialized. The value of the objective function $f(x)$ is computed

for this initial position. The parameters d^0 and δ^0 are initialized. The initial position of the beetle is estimated using the below equation:

$$x_i = x_{li} + rand(0, 1) \times (x_{ui} - x_{li}) \quad (5)$$

where x_{li} and x_{ui} are the lower and upper bounds for variable i , respectively.

2. Randomization of movement direction: the value of movement direction b is randomly determined through the equation below:

$$b_i = \frac{rands_i}{||rands(n, 1)||} \quad (6)$$

where $rands_i$ is a random number ranging from -1 to 1 for variable i , and $||rands(n, 1)||$ is the norm of the randomized values of all the variables.

3. Estimation of right-and left-hand side movements: the right- and left-hand sides movement, $x_{(right)}$ and $x_{(left)}$ are estimated for a given sensing antennas length (d). The right- and left-hand side positions are determined from Equations (7) and (8). These are used to evaluate the objective function value in order to determine the best movement direction.

$$x_{i(right)} = x_i + d^t \times b_i \quad (7)$$

$$x_{i(left)} = x_i - d^t \times b_i \quad (8)$$

where d^t is the sensing length of antenna at t th iteration.

4. Movement in the best direction: after evaluating the left- and right-hand side movements, the one with better objective function value is set to be the direction of movement for the beetle. The beetle makes use of the given step size (δ) to move in the better direction. Furthermore, the objective value is also determined for the new position regardless of whether its value is better or worse than the current solution.

$$x_i^{t+1} = x_i^t - \delta^t \times b_i \times \text{sign}\left(f\left(x_{(right)}\right) - f\left(x_{(left)}\right)\right) \quad (9)$$

where δ^t is the step size at t th iteration.

5. Update of sensing length and step size: when the beetle position is updated, the sensing length d and step size δ are also updated. (Jiang 2017) recommended the following update rules.

$$d^{t+1} = 0.95d^t + 0.01 \quad (10)$$

$$\delta^{t+1} = 0.95\delta^t \quad (11)$$

where d^{t+1} and δ^{t+1} are the sensing length and step size for the next iteration.

6. Steps 2 to 5 are repeated until the termination criterion is satisfied.

2.2.4. Population Based Beetle Antenna Search (pbBAS) Algorithm

Yousif and Saka [22] have suggested that the performance of the original BAS algorithm can be developed by performing greedy selection along with introducing a population of beetles instead of a single beetle. This enhancement in the algorithm provides a better capability of avoiding being stuck at the local optimal solutions. Furthermore, the procedure of updating the beetle position has also been modified in this study. The algorithm parameters are listed in Table 10. The flowchart of pbBAS algorithm is shown in Figure 8.

Table 10. pbBAS algorithm parameters description.

Parameters	Description
Number of Working Beetles	The working population size of beetles
Maximum Number of Evaluations	The maximum number of objective function evaluations
P	Probability value for beetle parameter changes (ranges between zero and one)

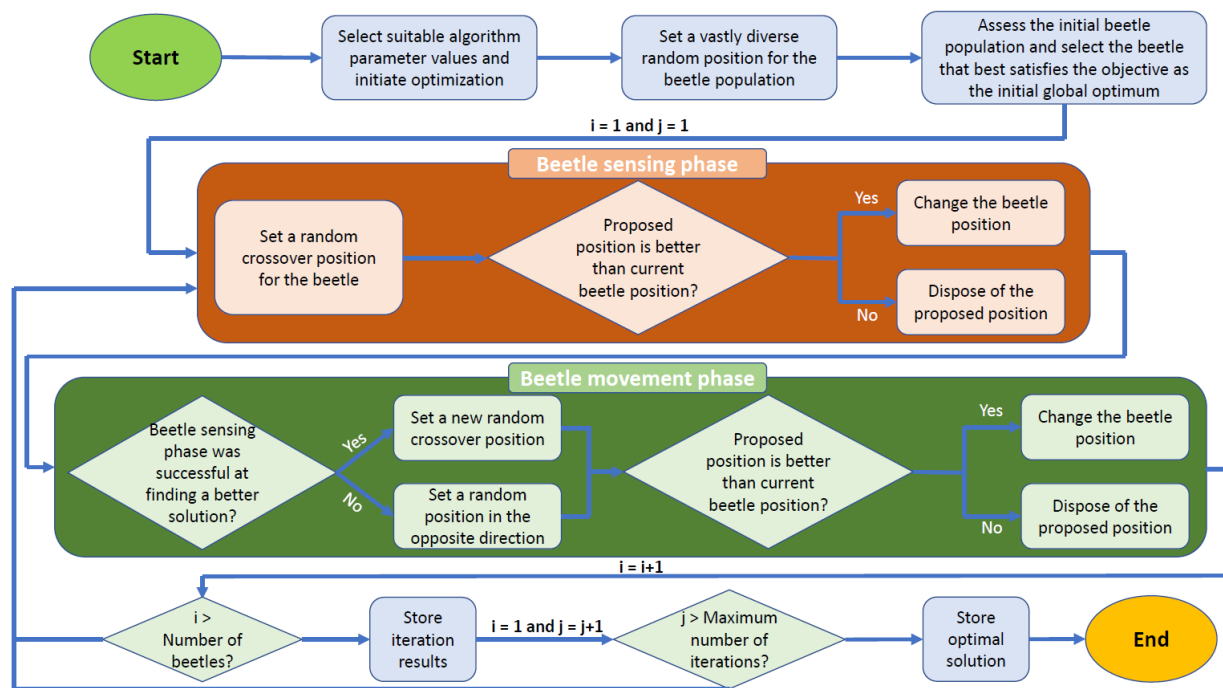


Figure 8. Flow chart of pbBAS algorithm.

1. Initialization of beetles' positions: After deciding the population size, the number of beetle groups is introduced in such a way that the selection covers the design domain. The number of beetle groups is taken as the number of design variables in the optimization problem times the selected population size. The initial position of each beetle in the group is randomly estimated through the equation below:

$$x_{ji} = x_l + rand(1, n) \times (x_u - x_l) \quad (12)$$

where x_{ji} is the i th beetle position of the j th group, x_l and x_u are the row values of the lower and upper bounds for the variables, respectively, and n is the number of design variables.

The standard deviation value is calculated for each variable in each group after calculating the position of each beetle in all the groups randomly. The initial population is then set to be a combination of the largest deviated variables within the randomized groups. Once the population is established, the objective function value for each beetle is computed.

2. Iteration: this stage consists of two phases; the beetle sensing phase and the beetle movement phase. The two phases are conducted for each beetle in a row before proceeding to the next beetle.

(a) Beetle sensing phase:

At this phase, two random beetles distinct from each other are selected, x_a and x_b . It is possible that one of the beetles is the same as the beetle being considered for the update x_i . A new beetle position S is computed using the equation below:

$$S = x_i + \beta(x_a - x_b) \quad (13)$$

where β is a random number ranging between -1 to 1 .

After generating the new beetle position, it is set to crossover with the considered beetle's position depending on the probability (p) value. The crossover is carried out for the variables that satisfy: $j = j_o$ or $rand \leq P$. Where j is the variable number, j_o is a randomly selected variable, $rand$ is a random number generated for each variable which ranges from 0 to 1, and p is the probability of variable changes parameter.

The proposed position is then evaluated using the objective function. If its fitness value is larger than the considered beetle's position, the beetle position is updated.

(b) Beetle movement phase:

There are two possible outcome scenarios depending on the result of the previous phase:

(b-1) Sensing phase succeeds at finding a better solution: proceed with another crossover using the same procedure discussed in the beetle sensing phase.

(b-2) Sensing phase fails at finding a better solution: move the beetle in the opposite direction. The new beetle position is calculated using the following equation:

$$S = x_i - \beta(x_a - x_b) \quad (14)$$

It should be noted that if the second condition applies, the values of β , x_a , and x_b are set as the exact same values as in the previous phase. Moreover, both j_o and $rand$ that are used to select the variables to be crossed over are the same as in the previous phase.

The proposed new position is then evaluated using the objective function regardless of the scenario outcome. If its fitness value is larger than the considered beetle position, the beetle position is updated.

3. Results

Two folded plate structures are designed by using three optimum design algorithms developed. The first design example is a building of 60 m by 25 m dimensions with a V-type folded plate roof that has a single angle of inclination for all the spans. The second design example is a three-segment folded plate roof building. Both gravity and lateral loads are considered in the design problems. The gravity loads acting on the roof structure are dead loads (DL), including the superimposed loads and live loads (LL), which act on the planer area of the building. Only the wind loads (WL) acting on the elevation projection area are considered lateral loads. Using these loads, 77 load combinations are generated in accordance with ACI-318-11; 2 of which are service load combinations, and the remaining 75 are all ultimate load combinations. The load combinations used to design each member are summarized in Table 11. The values mentioned in the table represent the weights assigned to each load case under each load combination. Because wind loads can act in any direction, they are represented by 12 load cases in accordance with ASCE 7-05. The long-term deflection load combination considers 25% of the live loads as sustained loads.

Table 11. Load combinations summary.

Load Combination		DL	LL	WL	Members Checked against Combination
Type	Name				
Ultimate	1	1.4	-	-	All members
	2	1.2	0.5	-	All members
	3	1.2	1.6	-	All members
	4~27	1.2	1.6	±0.5	Columns only
	28~51	1.2	0.5	±1.0	Columns only
	52~75	0.9	-	±1.0	Columns only
Service	Immediate deflection	-	1.0	-	Plates only
	Long-term deflection	1.835	1.20875	-	Plates only

Because the metaheuristic algorithms generate random results, each algorithm is run 10 times using 10 different seed values in each design problem. The results achieved by each algorithm are used to conduct a statistical study. For the statistical analysis number of runs was selected as 10. This number is large enough to form a population for conducting a statistical study and, in the meantime, small enough for an affordable computational time. The random seed values were selected from zero to nine in sequential order.

The optimum design framework was developed based on two software: MATLAB version R2014a which is a product from MathWorks located in Galway, Ireland and CSI-SAP2000 version 20 produced by Computers & Structures, inc. in Berkeley, CA, USA. The metaheuristic algorithms are coded in a MATLAB environment. These codes generate and update solutions and evaluate the fitness of each solution by estimating its cost and calculating constraints' violations, if there are any, to penalize the cost. It is apparent that structural analysis is needed to identify the values of constraints' violations. The structural analysis is carried out in CSI-SAP200 by the finite element method, and the response of the structure is transferred to the MATLAB environment by utilizing the API coding language. A flow chart representing each of the two involved MATLAB codes is shown in Figures 9 and 10.

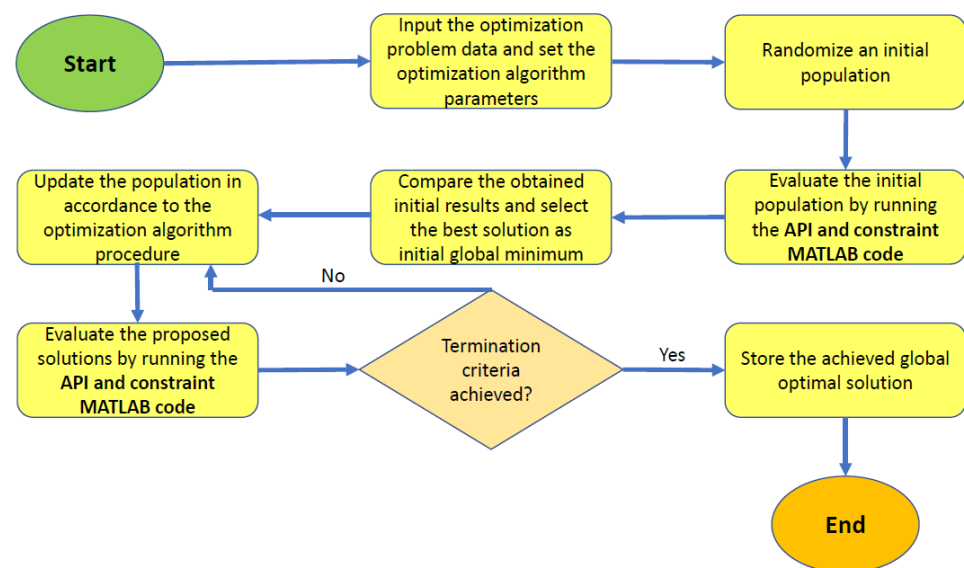


Figure 9. Flow chart for general metaheuristic algorithm in MATLAB.

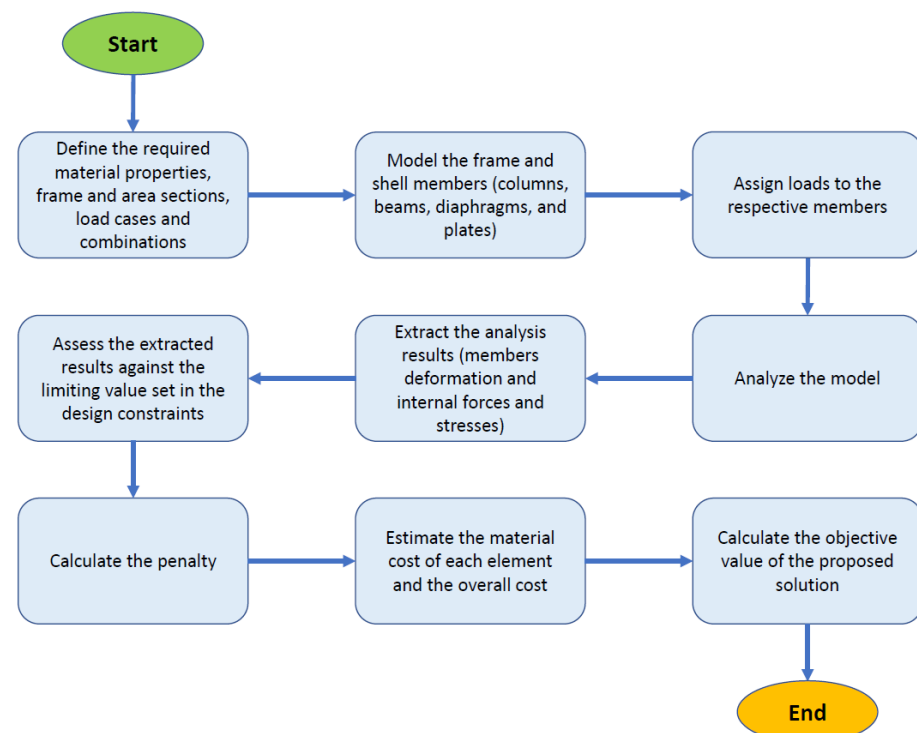


Figure 10. Flow chart for API and constraints in MATLAB.

3.1. V-Type Folded Plate Roof Problem

The first design problem is for a V-type folded plate roof building with a size of 60 m \times 25 m. The internal supports may vary from three to nine with an increment of one. Figure 11 shows the building with three internal supports. The total number of ungrouped design variables in this problem is 49, which are grouped into 11 design variables. The 11 design variables are internal supports, plate thickness, longitudinal and transverse reinforcement configurations, angle of inclination of plates, edge beam sectional details, internal beam sectional details, diaphragm sectional details, x-edge columns sectional details, y-edge columns sectional details, and corner columns sectional details. Input details of this design problem and the selected controlling parameter setting of each optimization algorithm are summarized in Tables 12 and 13, respectively. The maximum number of permissible structural analyses is set to be 9000 for each of the three algorithms. The optimum results obtained by the 10 runs, their minimum (best), first quartile, second quartile (median), third quartile, average, and maximum (worst), are listed in Table 14. In Figures 12–14, the design histories of each algorithm are plotted for the best, average, and worst optimum results achieved, respectively. Additionally, the box plots of the algorithm results are plotted in Figure 15. The best overall optimum solution corresponding to the cost of \$224,177 was achieved by the DE algorithm. As opposed to the solution obtained by traditional design, the best overall optimum solution produces a cost savings of 21%, as shown in Table 14.

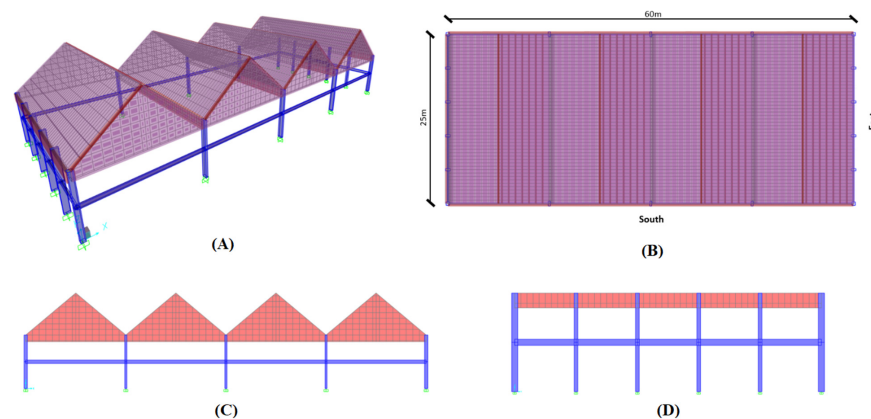


Figure 11. V-type folded plate roof problem: (A) 3D view; (B) Plan view; (C) South elevation view; (D) East elevation view.

Table 12. Input data for the V-type folded plate roof building.

Detail			Value	Unit
Material Properties	Concrete ultimate cylindrical strength		30	MPa
	Reinforcement	Yield stress	420 (Grade 60)	MPa
		Modulus of elasticity	200	GPa
Model	Building overall dimensions	Along x-direction	25,000	mm
		Along y-direction	60,000	mm
	Meshing size	Along x-direction	1000	mm
		Along y-direction	500	mm
	Number of column lines	Along x-direction	2 + V_1 (variable)	Nos.
		Along y-direction	6	Nos.
	Total number of columns		12 + $2V_1$ (variable)	Nos.
	Transverse span of plates (l)		$60,000/(1 + V_1)$	mm
	Longitudinal span of plates		25,000	mm
	Building height	To the bottom level of inclined plates	8000	mm
		To the top level of inclined plates	$8000 + \tan(V_5) \times l/2$	mm

Table 12. Cont.

Detail		Value	Unit
Load Assignments	Plates superimposed dead load	1.5	kPa
	Plates live load (acting on the planer area)	0.75	kPa
	Wind load	Basic gust wind speed	160 kmh
		Exposure Category	"C"
		Importance factor	1
		Topography factor	1
		Directional factor	0.85
		Gust factor	0.85
		Pressure coefficient	Windward face +0.8
			Leeward face −0.5
Unit Cost *	Concrete	125	\$/m ³
	Reinforcement	0.95	\$/kg
	Formwork	Vertical and horizontal	40 \$/m ²
		Inclined	50 \$/m ²

*: all unit cost rates are obtained from the material suppliers in Bahrain local market.

Table 13. Adopted internal parameter values for each metaheuristic algorithm in the V-type folded plate roof building.

Algorithm	Parameter	Value
ABC	Number of bees (colony size)	30
	Number of food sources	15
	Maximum number of iterations	300
	Trial limit	300
DE	Population size	20
	Crossover probability	0.2
	Maximum number of iterations	450
	Scaling factor	Lower bound 0
		Upper bound 1
pbBAS	Number of beetles	10
	Probability of beetle movement	0.1
	Maximum number of iterations	450

Table 14. Optimum results achieved in the V-type folded plate roof building by each metaheuristic algorithm.

		Metaheuristic Algorithms			Traditional Design (\$)
		ABC (\$)	pbBAS (\$)	DE (\$)	
The optimum results in ascending order	1	225,826.80	227,366.24	224,177.14	283,817.05
	2	227,186.97	228,635.54	224,196.82	
	3	227,980.34	228,706.35	224,462.70	
	4	228,697.83	228,756.96	224,556.83	
	5	229,019.23	229,861.84	225,736.41	
	6	229,376.60	230,242.71	226,510.54	
	7	229,552.96	232,715.97	226,538.59	
	8	229,653.36	233,123.04	226,690.03	
	9	230,138.80	233,725.44	228,018.30	
	10	231,598.06	234,348.75	229,249.29	

Table 14. Cont.

	Metaheuristic Algorithms			Traditional Design (\$)
	ABC (\$)	pbBAS (\$)	DE (\$)	
Minimum	225,826.80	227,366.24	224,177.14	
Q1	227,980.34	228,706.35	224,462.70	
Q2 (Median)	229,197.91	230,052.27	226,123.48	
Average	228,903.09	230,748.28	226,013.66	
Q3	229,653.36	233,123.04	226,690.03	
Maximum	231,598.06	234,348.75	229,249.29	
Standard dev.	1609.5193	2503.6906	1723.8909	

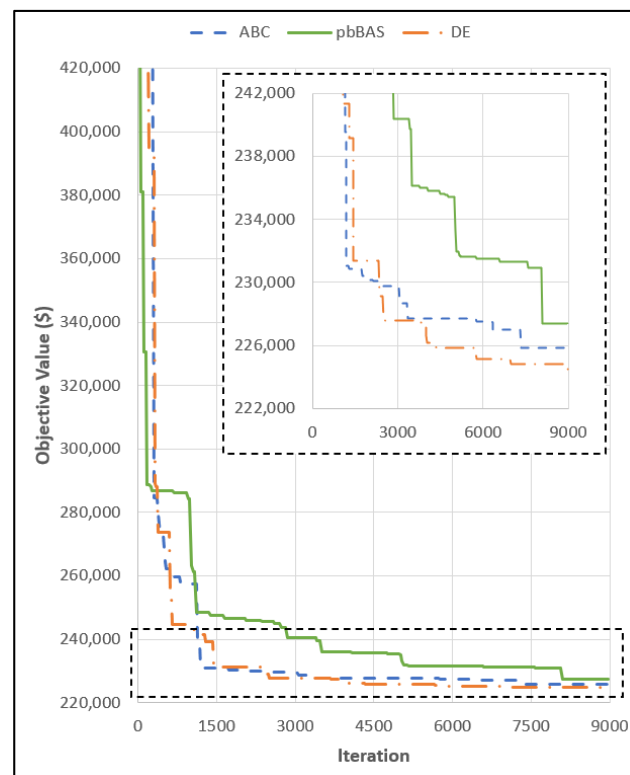


Figure 12. Design history of the best run achieved by each metaheuristic algorithm for the V-type folded plate roof problem.

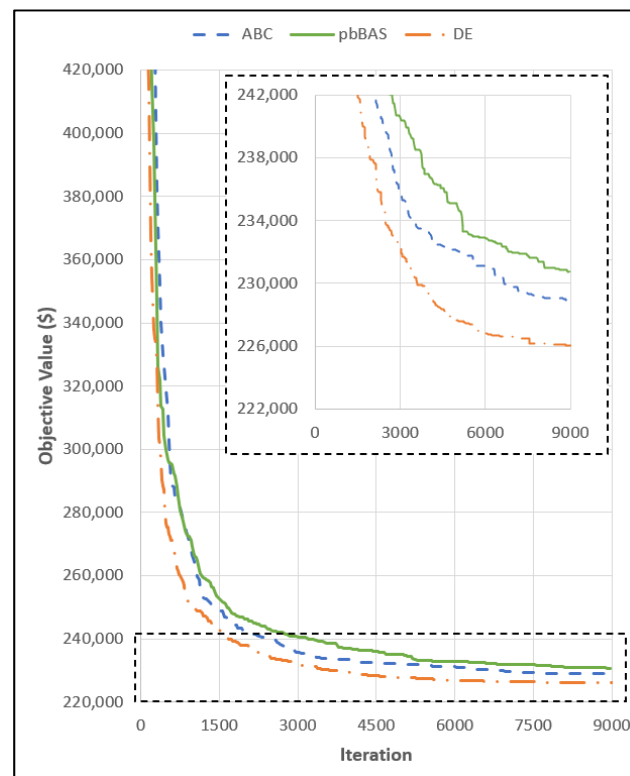


Figure 13. Design history of the sample average of the 10 runs achieved by each metaheuristic algorithm for the V-type folded plate roof problem.

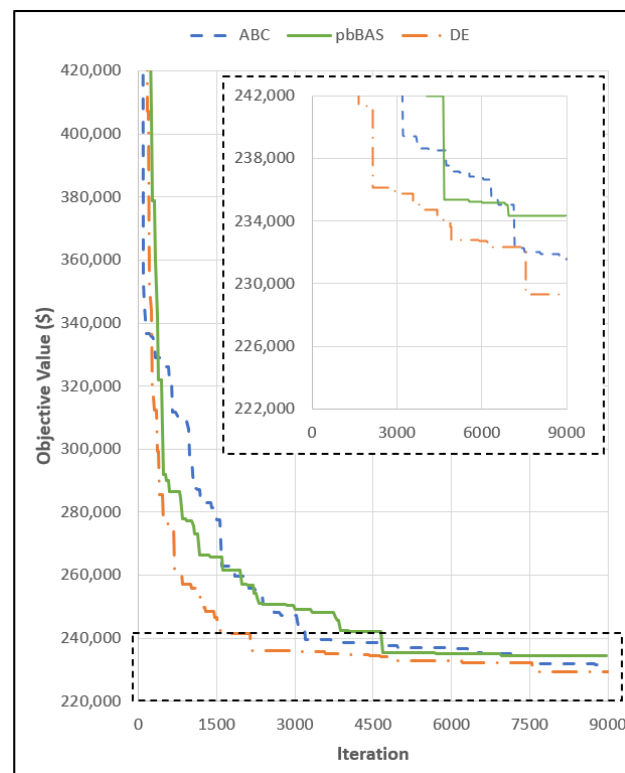


Figure 14. Design history of the worst run achieved by each metaheuristic algorithm for the V-type folded plate roof problem.

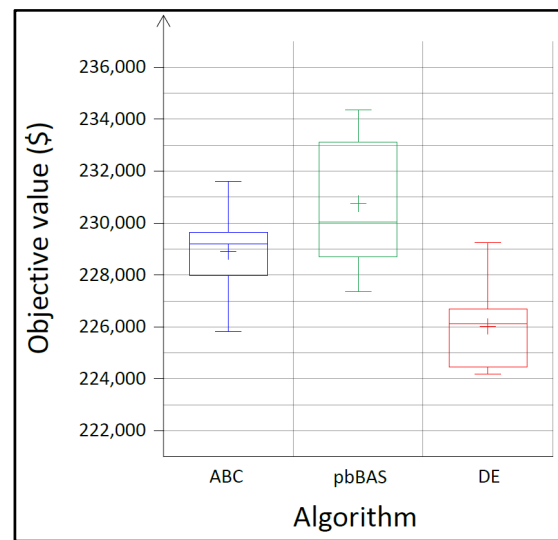


Figure 15. Box plot of the results achieved by each metaheuristic algorithm after 10 runs for the V-type folded plate roof problem.

The details of the best optimum result achieved by each algorithm are summarized in Tables 15–17. Additionally, the maximum values of demand to capacity ratios for that design are listed in Tables 18–20. The cost consumption by each material of each member for the overall best optimum design is summarized in Table 21 and plotted as percentile contribution in Figure 16.

Table 15. The optimum values of folded plates variables obtained by each metaheuristic algorithm for the V-type folded plate roof building.

Design Variable	Detail	Best Optimum Designs			Traditional Design
		ABC	pbBAS	DE	
V ₁	Number of interior supports	6 Nos. (<i>l</i> = 8.571 m)	6 Nos. (<i>l</i> = 8.571 m)	6 Nos. (<i>l</i> = 8.571 m)	4 Nos. (<i>l</i> = 12 m)
V ₂	Thickness of inclined plates	90 mm	80 mm	80 mm	110 mm
V ₄	Longitudinal reinforcement configuration	T12@250 mm	T10@200 mm	T10@250 mm	T12@200 mm
V ₅	Transverse reinforcement configuration	T12@250 mm	T12@200 mm	T12@250 mm	T12@150 mm
V ₇	Angle of inclination	30°	30°	29°	40°

Table 16. The optimum values of auxiliary members variables obtained by each metaheuristic algorithm for the V-type folded plate roof building.

Design Variable	Member	Detail	Best Optimum Designs			Traditional Design
			ABC	pbBAS	DE	
V ₈	Edge beams	Width	300 mm	350 mm	350 mm	300 mm
		Depth	800 mm	900 mm	900 mm	700 mm
		Diameter of longitudinal reinforcement	20 mm	20 mm	20 mm	20 mm
		Number of longitudinal rebars along minor axis	3 Nos.	4 Nos.	4 Nos.	3 Nos.
		Number of longitudinal rebars along major axis	7 Nos.	8 Nos.	8 Nos.	6 Nos.
		Shear reinforcement configuration	T8@300 mm	T10@250 mm	T8@250 mm	T10@250 mm

Table 16. Cont.

Design Variable	Member	Detail	Best Optimum Designs			Traditional Design
			ABC	pbBAS	DE	
V ₉	Interior beams	Width	350 mm	350 mm	400 mm	400 mm
		Depth	1000 mm	1000 mm	1100 mm	1200 mm
		Diameter of longitudinal reinforcement	25 mm	25 mm	25 mm	20 mm
		Number of longitudinal rebars along minor axis	4 Nos.	4 Nos.	4 Nos.	4 Nos.
		Number of longitudinal rebars along major axis	11 Nos.	11 Nos.	11 Nos.	10 Nos.
		Shear reinforcement configuration	T10@200 mm	T10@250 mm	T10@250 mm	T10@250 mm
V ₁₀	Diaphragms	Thickness	250 mm	250 mm	250 mm	350 mm
		Depth of diaphragm below lower level of plates	500 mm	500 mm	500 mm	800 mm
		Diameter of horizontal reinforcement	16 mm	16 mm	16 mm	20 mm
		Number of horizontal rebars along minor axis	3 Nos.	3 Nos.	3 Nos.	4 Nos.
		Spacing between horizontal rebars along major axis	200 mm	200 mm	200 mm	200 mm
		Vertical reinforcement configuration	T16@200 mm	T16@200 mm	T16@200 mm	T16@150 mm

Table 17. Optimum values of column's variables obtained by each metaheuristic algorithm for the V-type folded plate roof building.

Design Variable	Member	Detail	Best Optimum Designs			Traditional Design
			ABC	pbBAS	DE	
V ₁₁	x-edge columns	Size in x-direction	400 mm	400 mm	400 mm	400 mm
		Size in y-direction	200 mm	200 mm	200 mm	200 mm
		Diameter of longitudinal reinforcement	12 mm	16 mm	16 mm	16 mm
		Number of longitudinal rebars along x-direction	4 Nos.	4 Nos.	4 Nos.	4 Nos.
		Number of longitudinal rebars along y-direction	2 Nos.	2 Nos.	2 Nos.	2 Nos.
		Shear reinforcement configuration	T10@150 mm	T10@150 mm	T8@200 mm	T10@200 mm
V ₁₂	y-edge columns	Size in x-direction	250 mm	250 mm	250 mm	300 mm
		Size in y-direction	600 mm	600 mm	600 mm	700 mm
		Diameter of longitudinal reinforcement	16 mm	16 mm	16 mm	20 mm
		Number of longitudinal rebars along x-direction	2 Nos.	2 Nos.	2 Nos.	3 Nos.
		Number of longitudinal rebars along y-direction	6 Nos.	6 Nos.	6 Nos.	7 Nos.
		Shear reinforcement configuration	T8@200 mm	T8@200 mm	T8@200 mm	T10@250 mm
V ₁₃	Corner columns	Size in x-direction	450 mm	400 mm	450 mm	450 mm
		Size in y-direction	450 mm	400 mm	450 mm	450 mm
		Diameter of longitudinal reinforcement	16 mm	20 mm	16 mm	16 mm
		Number of longitudinal rebars along x-direction	5 Nos.	4 Nos.	5 Nos.	5 Nos.
		Number of longitudinal rebars along y-direction	5 Nos.	4 Nos.	5 Nos.	5 Nos.
		Shear reinforcement configuration	T8@250 mm	T8@200 mm	T8@250 mm	T12@150 mm

Table 18. Folded plates constraints the maximum demand capacity ratios in the optimum designs achieved by each metaheuristic algorithm for the V-type folded plate roof building.

Const. No.	Description	Best Optimum Designs			Traditional Design
		ABC	pbBAS	DE	
R ₁	Membrane compression	0.788	0.822	0.808	0.437
R ₂	Membrane tension	0.971	0.917	0.942	0.916
R ₃	Transverse bending moment	0.964	0.804	0.988	0.929
R ₄	Transverse shear force	0.860	0.940	0.942	0.923
R ₅	Immediate deflection	0.084	0.099	0.095	0.083
R ₆	Long-term deflection	0.748	0.825	0.805	0.843
R ₇	Maximum reinforcement spacing	0.926	0.833	0.625	0.606
R ₈	Minimum plates thickness	0.844	0.925	0.925	0.691
R ₉	Minimum reinforcement area	0.358	0.367	0.458	0.350
R ₁₀	Maximum reinforcement area	0.126	0.123	0.098	0.129

Table 19. Auxiliary members' constraints the maximum demand capacity ratio in the optimum designs achieved by each metaheuristic algorithm for the V-type folded plate roof building.

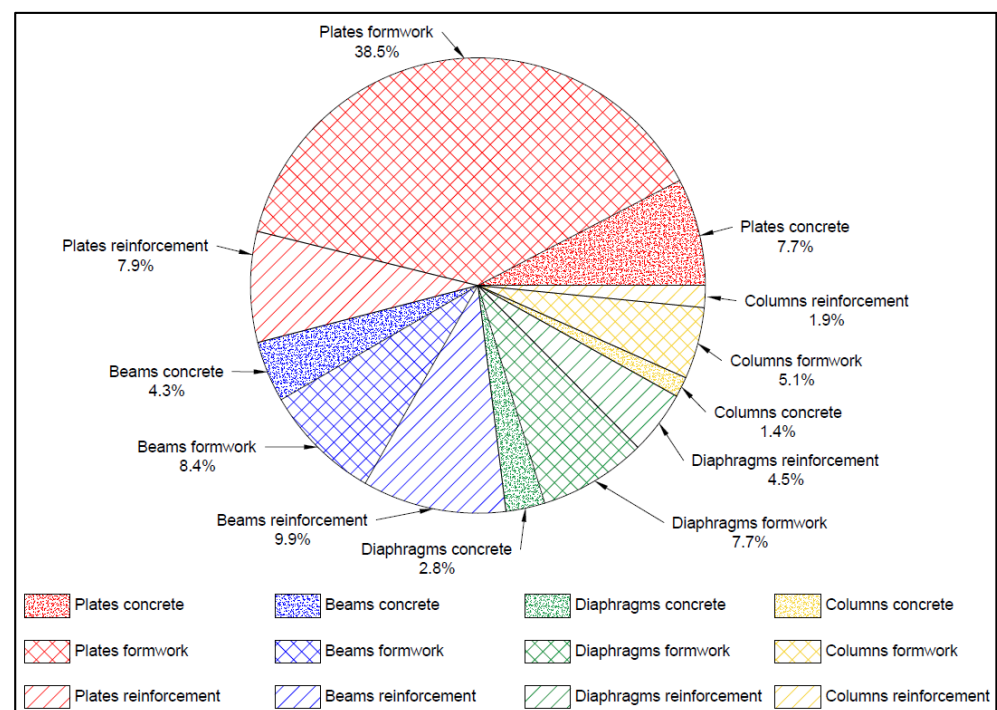
Member	Const. No.	Description	Best Optimum Designs			Traditional Design
			ABC	pbBAS	DE	
Edge beams	R ₁₁	Flexural strength	0.685	0.446	0.430	0.850
	R ₁₂	Shear strength	0.336	0.190	0.255	0.362
	R ₁₃	Minimum longitudinal reinforcement spacing	0.556	0.706	0.690	0.571
	R ₁₄	Maximum longitudinal reinforcement spacing	0.313	0.305	0.307	0.320
	R ₁₅	Minimum longitudinal reinforcement area	0.148	0.156	0.156	0.146
	R ₁₆	Maximum longitudinal reinforcement area	0.524	0.499	0.499	0.524
	R ₁₇	Maximum shear reinforcement spacing	0.809	0.595	0.594	0.781
	R ₁₈	Minimum shear reinforcement area	0.995	0.597	0.933	0.464
Interior beams	R ₁₉	Flexural strength	0.970	0.969	0.972	0.968
	R ₂₀	Shear strength	0.623	0.704	0.601	0.519
	R ₂₁	Minimum longitudinal reinforcement spacing	0.800	0.800	0.600	0.545
	R ₂₂	Maximum longitudinal reinforcement spacing	0.208	0.208	0.242	0.333
	R ₂₃	Minimum longitudinal reinforcement area	0.086	0.086	0.108	0.202
	R ₂₄	Maximum longitudinal reinforcement area	0.912	0.912	0.725	0.393
	R ₂₅	Maximum shear reinforcement spacing	0.427	0.533	0.482	0.439
	R ₂₆	Minimum shear reinforcement area	0.531	0.663	0.729	0.796
Diaph-ragms	R ₂₇	Flexural strength	0.995	0.977	0.991	0.790
	R ₂₈	Shear strength	0.330	0.321	0.321	0.266
	R ₂₉	Minimum horizontal reinforcement spacing	0.889	0.889	0.889	0.759
	R ₃₀	Maximum horizontal reinforcement spacing	0.603	0.603	0.575	0.589
	R ₃₁	Minimum horizontal reinforcement area	0.335	0.335	0.343	0.327
	R ₃₂	Maximum vertical reinforcement area	0.402	0.402	0.402	0.393
	R ₃₃	Maximum vertical reinforcement spacing	0.917	0.917	0.917	0.409
	R ₃₄	Minimum shear reinforcement area	0.616	0.616	0.596	0.907

Table 20. Column's constraints the maximum demand capacity ratio in the optimum designs achieved by each metaheuristic algorithm for the V-type folded plate roof building.

Const. No.	Description	Best Optimum Designs			Traditional Design
		ABC	pbBAS	DE	
R ₃₅	Flexural strength	0.999	0.999	0.991	0.886
R ₃₆	Shear strength	0.421	0.434	0.424	0.388
R ₃₇	Minimum longitudinal reinforcement spacing	0.730	0.735	0.730	0.752
R ₃₈	Maximum longitudinal reinforcement spacing	0.407	0.407	0.407	0.262
R ₃₉	Minimum longitudinal reinforcement area	0.884	0.622	0.629	0.629
R ₄₀	Maximum longitudinal reinforcement area	0.201	0.295	0.251	0.299
R ₄₁	Maximum shear reinforcement spacing	0.977	0.800	1.000	1.000
R ₄₂	Minimum shear reinforcement area	0.995	0.995	0.995	0.928

Table 21. Distribution of the optimum cost values among different materials and structural parts for the V-type folded plate roof building.

Member	Material			Sum	
	Concrete	Formwork	Reinforcement	Amount	Percentage
Plates	17,151.45	86,356.35	17,645.97	121,153.77	54%
Beams	9688.88	18,858.80	22,141.53	50,689.20	23%
Diaphragms	6254.99	17,164.85	10,108.75	33,528.59	15%
Columns	3250.00	11,380.00	4175.58	18,805.58	8%
Sum	Amount	36,345.32	133,760.00	54,071.83	224,177.14
	Percentage	16%	60%	24%	100%

**Figure 16.** Pie chart of the cost components for the optimum design achieved in the V-type folded plate roof problem.

3.2. Three-Segment Type Folded Plate Roof Problem

The three-segment type folded plate roof design problem is sized as 60 m \times 25 m. The internal supports may vary from three to nine with an increment of one. Figure 17 shows the building with three internal supports. The total number of ungrouped design variables to be considered in this problem is 51, which are grouped into 13 design variables. The 13 design variables are several internal supports, inclined plates thickness, horizontal plates thickness, longitudinal and transverse reinforcement configurations, lengths of horizontal plates at each span, angle of inclination of plates, edge beam sectional details, internal beam sectional details, diaphragm sectional details, x-edge columns sectional details, y-edge columns sectional details, and corner columns sectional details. Input details of this design problem are summarized in Table 22. The selected values of controlling parameter settings of the optimization algorithms are the same as the first problem, summarized in Table 13. The maximum number of permissible structural analyses is set to be 9000 for each of the three algorithms. The optimum results obtained by the 10 runs, their minimum (best), first quartile, second quartile (median), third quartile, average, and maximum (worst), are listed in Table 23. In Figures 18–20, the design histories of each algorithm are plotted for the best, average, and worst optimum results achieved, respectively. Additionally, the box plots of the algorithm results are plotted in Figure 21. The best overall optimum solution corresponding to the cost of \$207,220 was achieved by the DE algorithm. As opposed to the solution obtained by traditional means of design, the best overall optimum solution produces a cost saving of 18.5%, as shown in Table 23.

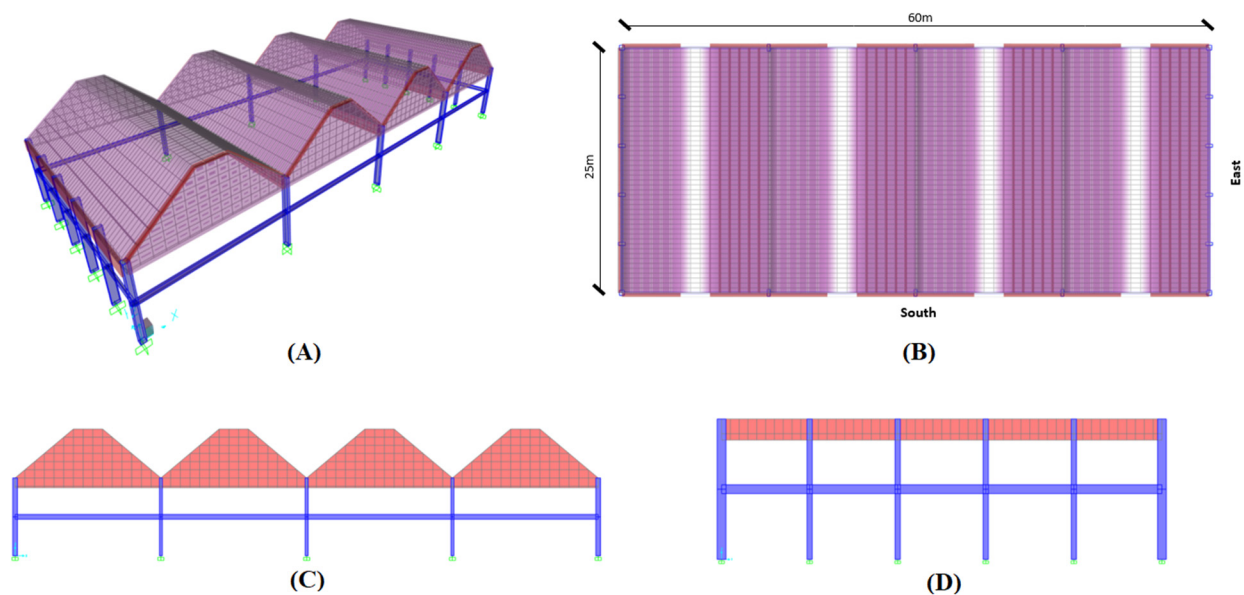


Figure 17. Three-segment type folded plate roof problem: (A) 3D view; (B) Plan view; (C) South elevation view; (D) East elevation view.

Table 22. Input data for the three-segment type folded plate roof building.

Details			Value	Unit	
Material Properties	Concrete ultimate cylindrical strength		30	MPa	
	Reinforcement	Yield stress	420 (Grade 60)	MPa	
		Modulus of elasticity	200	GPa	
Model	Building overall dimensions	Along x-direction	25,000	mm	
		Along y-direction	60,000	mm	
	Meshing size	Along x-direction	1000	mm	
		Along y-direction	500	mm	
	Number of supporting columns	Along x-direction	2 + V ₁ (variable)	Nos.	
		Along y-direction	6	Nos.	
	Total number of columns		12 + 2V ₁ (variable)	Nos.	
	Transverse span of plates (<i>l</i>)		60,000/(1 + V ₁)	mm	
	Longitudinal span of plates		25,000	mm	
	Length of inclined plates (<i>l_{inc}</i>)		(<i>l</i> − V ₅)/2	mm	
	Building height	To the bottom level of inclined plates	8000	mm	
		To the top level of inclined plates	8000 + tan(V ₆) × <i>l_{inc}</i> /2	mm	
Load Assignments	Plates superimposed dead load		1.5	kPa	
	Plates live load (acting on the planer area)		0.75	kPa	
	Wind load	Basic gust wind speed		160	kmh
		Exposure Category		“C”	-
		Importance factor		1	-
		Topography factor		1	-
		Directional factor		0.85	-
		Gust factor		0.85	-
		Pressure coefficient	Windward face	+0.8	-
			Leeward face	-0.5	-
Unit Cost *	Concrete		125	\$/m ³	
	Reinforcement		0.95	\$/kg	
	Formwork	Vertical and horizontal	40	\$/m ²	
		Inclined	50	\$/m ²	

*: all unit cost rates are obtained from the material suppliers in Bahrain local market.

Table 23. Optimum results achieved in the three-segment type folded plate roof building by each metaheuristic algorithm.

		Metaheuristic Algorithms			Traditional Design (\$)
		ABC (\$)	pbBAS (\$)	DE (\$)	
The optimum results in ascending order	1	210,656.56	208,478.29	207,220.00	283,817.05
	2	210,707.69	209,755.98	207,622.50	
	3	210,872.94	211,992.57	207,751.13	
	4	214,388.27	212,312.85	208,035.78	
	5	214,733.15	212,497.28	208,800.37	
	6	215,135.04	212,676.13	209,294.18	
	7	215,235.31	213,042.65	209,564.31	
	8	215,235.71	214,883.12	210,200.58	
	9	216,562.97	215,122.33	210,463.83	
	10	217,985.42	217,211.59	213,178.85	

Table 23. Cont.

	Metaheuristic Algorithms			Traditional Design (\$)
	ABC (\$)	pbBAS (\$)	DE (\$)	
Minimum	210,656.56	208,478.29	207,220.00	
Q1	210,872.94	211,992.57	207,751.13	
Q2 (Median)	214,934.09	212,586.71	209,047.28	
Average	214,151.31	212,797.28	209,213.15	
Q3	215,235.71	214,883.12	210,200.58	
Maximum	217,985.42	217,211.59	213,178.85	
Standard dev.	2561.4537	2542.8093	1782.4083	

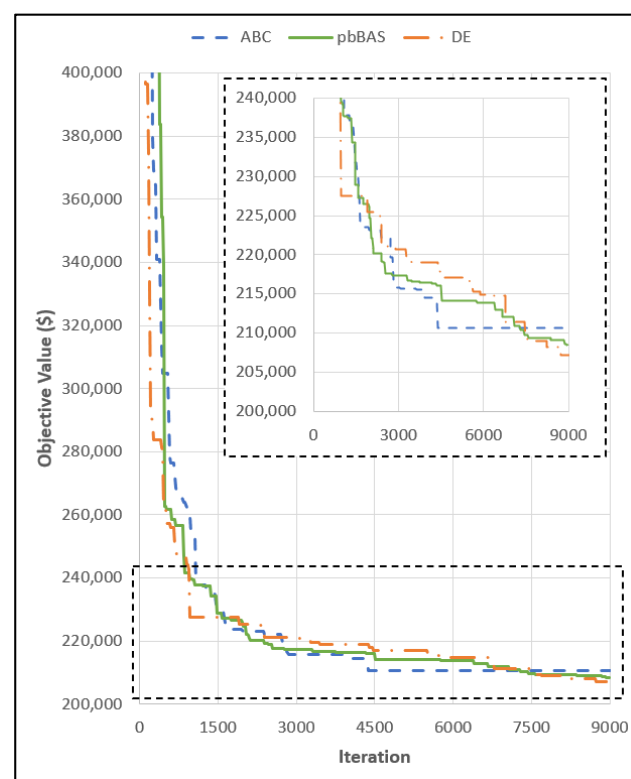


Figure 18. Design history of the best run achieved by each metaheuristic algorithm for the three-segment type folded plate roof problem.

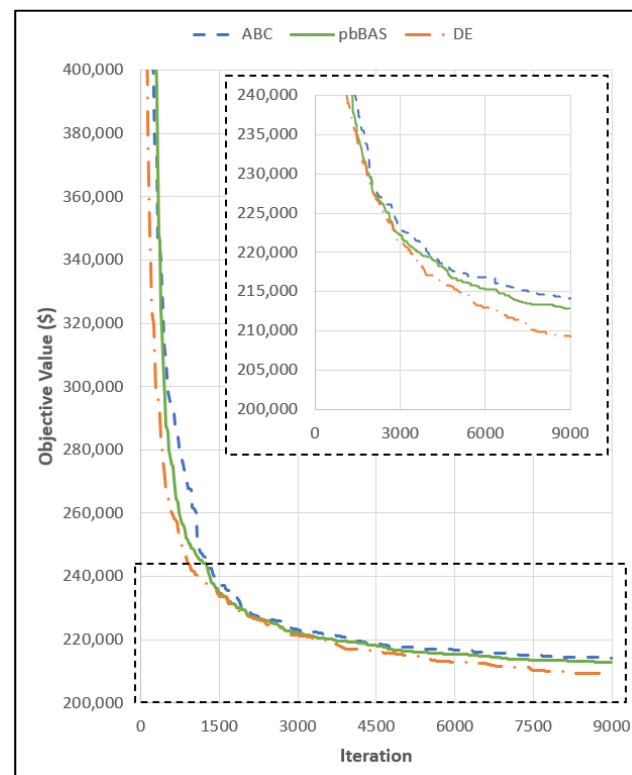


Figure 19. Design history of the sample average of the 10 runs achieved by each metaheuristic algorithm for the three-segment type folded plate roof problem.

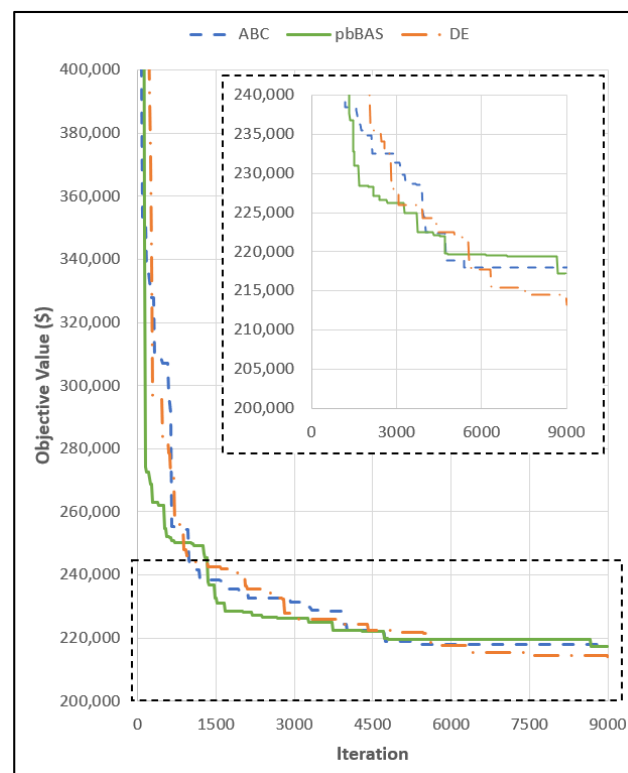


Figure 20. Design history of the worst run achieved by each metaheuristic algorithm for the three-segment type folded plate roof problem.

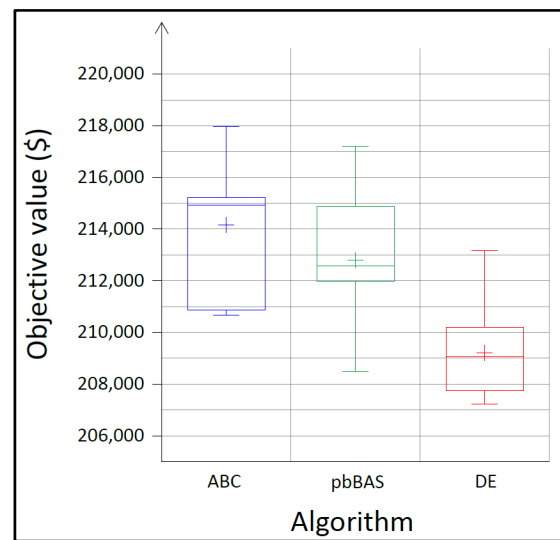


Figure 21. Box plot of the results achieved by each metaheuristic algorithm after 10 runs for the three-segment type folded plate roof problem.

The details of the best optimum result achieved by each algorithm are illustrated in Tables 24–26. The maximum values of demand to capacity ratios for that design are also listed in Tables 27–29. The cost values of each member for the overall best optimum design are summarized in Table 30 and plotted according to their percentile contribution in Figure 22.

Table 24. The best optimum values of folded plate variables obtained by each metaheuristic algorithm for the three-segment type folded plate roof building.

Design Variable	Detail	Optimum Designs			Traditional Design
		ABC	pbBAS	DE	
V ₁	Number of interior supports	5 Nos. (l = 10 m)	5 Nos. (l = 10 m)	5 Nos. (l = 10 m)	4 Nos. (l = 12.5 m)
V ₂	Thickness of inclined plates	70 mm	70 mm	70 mm	100 mm
V ₃	Thickness of horizontal plates	70 mm	70 mm	70 mm	160 mm
V ₄	Longitudinal reinforcement configuration	T10@200 mm	T8@200 mm	T8@200 mm	T16@200 mm
V ₅	Transverse reinforcement configuration	T10@200 mm	T10@250 mm	T10@250 mm	T12@200 mm
V ₆	Length of horizontal plates	3500 mm	3500 mm	3500 mm	1500 mm
V ₇	Angle of inclination	31°	31°	31°	37°

Table 25. The best optimum values of auxiliary members variables obtained by each metaheuristic algorithm for the three-segment type folded plate roof building.

Design Variable	Member	Detail	Best Optimum Designs			Traditional Design
			ABC	pbBAS	DE	
V ₈	Edge beams	Width	300 mm	400 mm	400 mm	300 mm
		Depth	800 mm	1000 mm	1000 mm	700 mm
		Diameter of longitudinal reinforcement	20 mm	20 mm	20 mm	20 mm
		Number of longitudinal rebars along minor axis	3 Nos.	4 Nos.	4 Nos.	3 Nos.
		Number of longitudinal rebars along major axis	8 Nos.	9 Nos.	9 Nos.	6 Nos.
		Shear reinforcement configuration	T8@300 mm	T12@200 mm	T10@200 mm	T8@300 mm

Table 25. Cont.

Design Variable	Member	Detail	Best Optimum Designs			Traditional Design
			ABC	pbBAS	DE	
V ₉	Interior beams	Width	400 mm	400 mm	400 mm	250 mm
		Depth	1200 mm	1200 mm	1200 mm	700 mm
		Diameter of longitudinal reinforcement	25 mm	25 mm	25 mm	20 mm
		Number of longitudinal rebars along minor axis	4 Nos.	4 Nos.	4 Nos.	3 Nos.
		Number of longitudinal rebars along major axis	11 Nos.	12 Nos.	11 Nos.	8 Nos.
		Shear reinforcement configuration	T10@200 mm	T10@200 mm	T10@250 mm	T10@200 mm
V ₁₀	Diaphragms	Thickness	250 mm	250 mm	250 mm	350 mm
		Depth of diaphragm below lower level of plates	600 mm	500 mm	600 mm	700 mm
		Diameter of horizontal reinforcement	20 mm	16 mm	20 mm	20 mm
		Number of horizontal rebars along minor axis	2 Nos.	3 Nos.	2 Nos.	4 Nos.
		Spacing between horizontal rebars along major axis	200 mm	100 mm	200 mm	150 mm
		Vertical reinforcement configuration	T12@200 mm	T12@200 mm	T12@250 mm	T16@200 mm

Table 26. The best optimum values of columns variables obtained by each metaheuristic algorithm for the three-segment type folded plate roof building.

Design Variable	Member	Detail	Best Optimum Designs			Traditional Design
			ABC	pbBAS	DE	
V ₁₁	x-edge columns	Size in x-direction	600 mm	400 mm	400 mm	400 mm
		Size in y-direction	250 mm	200 mm	200 mm	200 mm
		Diameter of longitudinal reinforcement	16 mm	16 mm	16 mm	16 mm
		Number of longitudinal rebars along x-direction	6 Nos.	4 Nos.	4 Nos.	4 Nos.
		Number of longitudinal rebars along y-direction	2 Nos.	2 Nos.	2 Nos.	2 Nos.
		Shear reinforcement configuration	T8@200 mm	T8@200 mm	T8@200 mm	T10@200 mm
V ₁₂	y-edge columns	Size in x-direction	250 mm	250 mm	250 mm	300 mm
		Size in y-direction	600 mm	600 mm	600 mm	600 mm
		Diameter of longitudinal reinforcement	16 mm	20 mm	20 mm	20 mm
		Number of longitudinal rebars along x-direction	2 Nos.	2 Nos.	2 Nos.	3 Nos.
		Number of longitudinal rebars along y-direction	6 Nos.	6 Nos.	6 Nos.	6 Nos.
		Shear reinforcement configuration	T10@250 mm	T10@200 mm	T8@200 mm	T10@250 mm
V ₁₃	Corner columns	Size in x-direction	450 mm	400 mm	400 mm	450 mm
		Size in y-direction	450 mm	400 mm	400 mm	450 mm
		Diameter of longitudinal reinforcement	16 mm	16 mm	20 mm	16 mm
		Number of longitudinal rebars along x-direction	5 Nos.	4 Nos.	4 Nos.	5 Nos.
		Number of longitudinal rebars along y-direction	5 Nos.	4 Nos.	4 Nos.	5 Nos.
		Shear reinforcement configuration	T8@250 mm	T8@250 mm	T8@300 mm	T10@250 mm

Table 27. The values of maximum demand capacity ratios of folded plate constraints in the optimum design achieved by each metaheuristic algorithm for the three-segment type folded plate roof building.

Const. No.	Description	Best Optimum Designs			Traditional Design
		ABC	pbBAS	DE	
R ₁	Membrane compression	0.618	0.612	0.611	0.584
R ₂	Membrane tension	0.921	0.998	0.998	0.860
R ₃	Transverse bending moment	0.833	0.948	0.944	0.995
R ₄	Transverse shear force	0.801	0.801	0.801	0.861
R ₅	Immediate deflection	0.098	0.096	0.096	0.087
R ₆	Long-term deflection	0.718	0.712	0.710	0.800
R ₇	Maximum reinforcement spacing	0.952	0.714	0.714	0.667
R ₈	Minimum plates thickness	1.000	0.971	0.971	0.800
R ₉	Minimum reinforcement area	0.321	0.501	0.501	0.509
R ₁₀	Maximum reinforcement area	0.140	0.090	0.090	0.141

Table 28. The maximum demand capacity ratio of auxiliary members' constraints in the optimum design achieved by each metaheuristic algorithm for the three-segment type folded plate roof building.

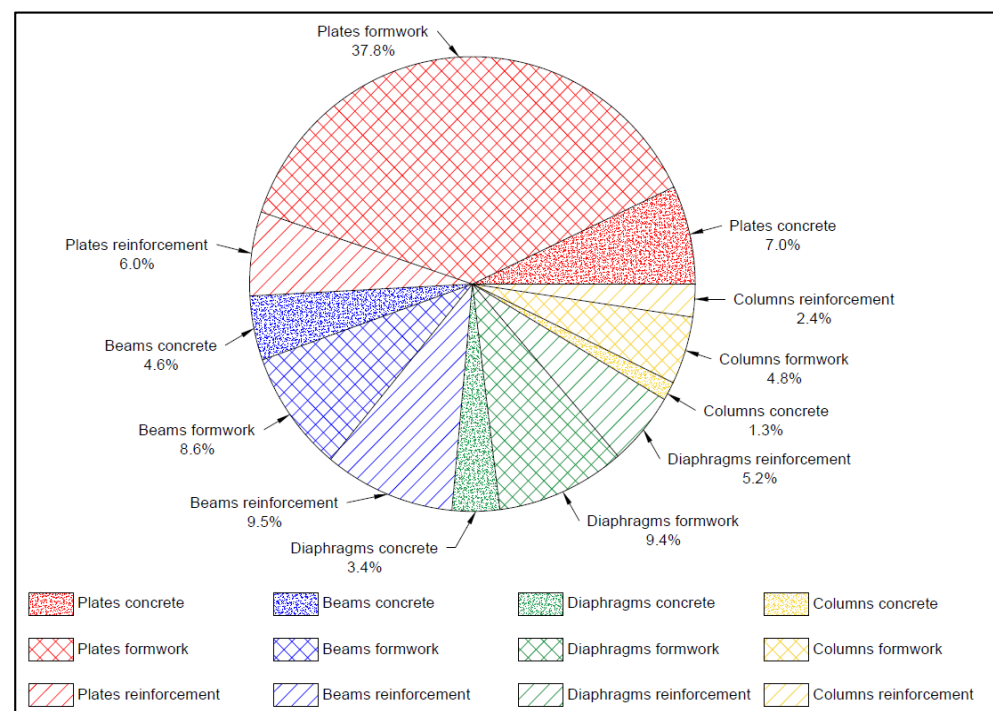
Member	Const. No.	Description	Best Optimum Designs			Traditional Design
			ABC	pbBAS	DE	
Edge beams	R ₁₁	Flexural strength	0.705	0.412	0.411	0.827
	R ₁₂	Shear strength	0.346	0.125	0.160	0.541
	R ₁₃	Minimum longitudinal reinforcement spacing	0.556	0.556	0.545	0.556
	R ₁₄	Maximum longitudinal reinforcement spacing	0.259	0.298	0.300	0.323
	R ₁₅	Minimum longitudinal reinforcement area	0.131	0.181	0.181	0.146
	R ₁₆	Maximum longitudinal reinforcement area	0.589	0.432	0.432	0.524
	R ₁₇	Maximum shear reinforcement spacing	0.809	0.426	0.426	0.935
	R ₁₈	Minimum shear reinforcement area	0.995	0.368	0.531	0.870
Interior beams	R ₁₉	Flexural strength	0.994	0.934	0.992	0.982
	R ₂₀	Shear strength	0.484	0.485	0.551	0.882
	R ₂₁	Minimum longitudinal reinforcement spacing	0.600	0.600	0.600	0.889
	R ₂₂	Maximum longitudinal reinforcement spacing	0.275	0.242	0.275	0.210
	R ₂₃	Minimum longitudinal reinforcement area	0.119	0.110	0.119	0.094
	R ₂₄	Maximum longitudinal reinforcement area	0.665	0.716	0.665	0.808
	R ₂₅	Maximum shear reinforcement spacing	0.352	0.352	0.440	0.625
	R ₂₆	Minimum shear reinforcement area	0.637	0.637	0.796	0.371
Diaphragms	R ₂₇	Flexural strength	0.903	1.000	0.937	0.818
	R ₂₈	Shear strength	0.388	0.496	0.395	0.397
	R ₂₉	Minimum horizontal reinforcement spacing	0.377	0.816	0.377	0.759
	R ₃₀	Maximum horizontal reinforcement spacing	0.547	0.270	0.547	0.426
	R ₃₁	Minimum horizontal reinforcement area	0.231	0.186	0.231	0.248
	R ₃₂	Maximum vertical reinforcement area	0.419	0.563	0.419	0.513
	R ₃₃	Maximum vertical reinforcement spacing	0.743	0.909	0.743	0.631
	R ₃₄	Minimum shear reinforcement area	0.940	0.904	0.940	0.965

Table 29. The maximum demand capacity ratios of columns constraints in the optimum design achieved by each metaheuristic algorithm for the three-segment type folded plate roof building.

Const. No.	Description	Best Optimum Designs			Traditional Design
		ABC	pbBAS	DE	
R ₃₅	Flexural strength	1.000	0.970	0.965	0.941
R ₃₆	Shear strength	0.428	0.383	0.464	0.370
R ₃₇	Minimum longitudinal reinforcement spacing	0.730	0.694	0.694	0.741
R ₃₈	Maximum longitudinal reinforcement spacing	0.407	0.367	0.380	0.262
R ₃₉	Minimum longitudinal reinforcement area	0.629	0.663	0.497	0.629
R ₄₀	Maximum longitudinal reinforcement area	0.201	0.314	0.314	0.305
R ₄₁	Maximum shear reinforcement spacing	0.977	1.000	1.000	1.000
R ₄₂	Minimum shear reinforcement area	0.995	0.829	0.995	0.796

Table 30. Distribution of the optimum cost values among different materials and structural parts for the three-segment type folded plate roof building.

Member		Material			Sum	
		Concrete	Formwork	Reinforcement	Amount	Percentage
Plates		14,546.59	78,385.77	12,370.22	105,302.59	50.8%
Beams		9480.00	17,820.80	19,669.39	46,970.19	22.7%
Diaphragms		7116.14	19,376.32	10,770.65	37,263.12	18.0%
Columns		2780.00	9976.00	4928.10	17,684.10	8.5%
Sum	Amount	33,922.73	125,558.90	47,738.37	207,220.00	100.0%
	Percentage	16.4%	60.6%	23.0%	100.0%	

**Figure 22.** Pie chart of the cost components for the optimum design achieved in the three-segment type folded plate roof problem.

4. Discussion

4.1. V-Type Folded Plate Roof Problem

Table 14 reveals the fact that the DE algorithm had the best performance in its 10 trial runs, achieving the best statistical minimum, first quartile, average, median, and maximum solution. The second-best performed algorithm is ABC; it was slightly behind the DE algorithm with the smallest interquartile range and standard deviation. This indicates the consistency of the results achieved by each trial run of the ABC algorithm. As for the pbBAS algorithm, its performance was the worst among the algorithms in this problem mainly due to the difficulty of finding solutions that satisfy the set of constraints which impacts the algorithm's searching technique. As shown in Table 15, the best optimum solution achieved by all the algorithms had a span of 8.571 m (6 internal supports). Table 21 and Figure 16 show that in the best overall optimum solution found, more than half the material cost is generated from the plates, about one quarter is from the beams, and less than one quarter is distributed between the diaphragms and columns with a ratio of 2:1, respectively.

After finding that the DE algorithm performed the best among the three metaheuristic algorithms, it is used to find the optimum design of the V folded roof type under different transverse span values. This is achieved by fixing the number of internal supports in the building at each run. Dividing the overall building length into equal transverse spans and selecting nine through three internal supports produces transverse span values of 6 m, 6.667 m, 7.5 m, 8.571 m, 10 m, 12 m, and 15 m, respectively. It is apparent that in this optimum design problem, the number of design variables is reduced to 10 as the total number of internal supports is no longer a design variable. The internal controlling parameter values of the DE algorithm are shown in Table 13. The amounts representing the cost of each element for the optimum solution at each transverse span are represented in Figure 23.

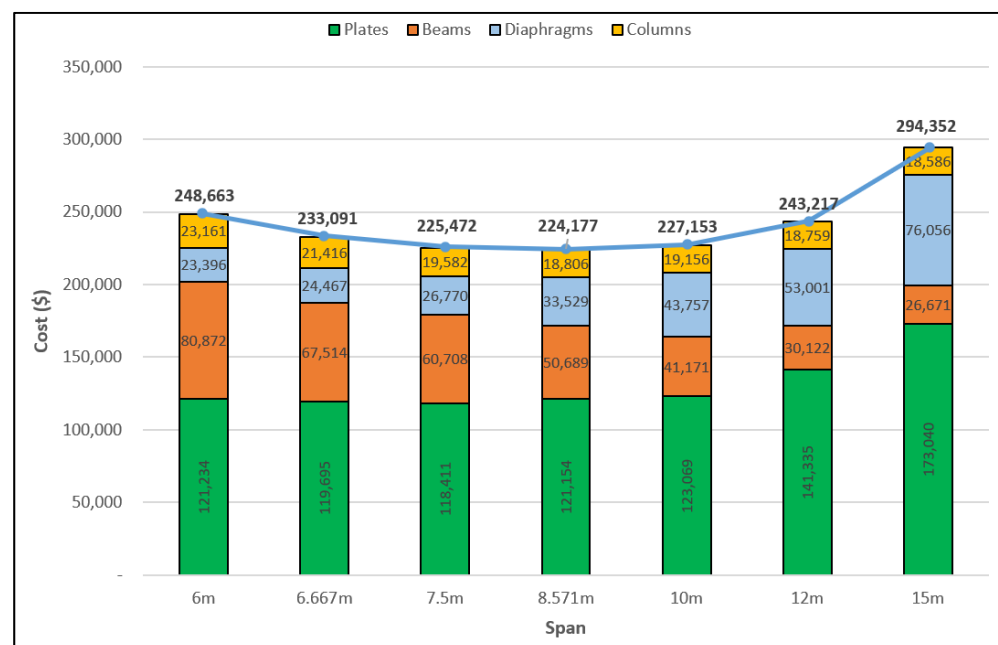


Figure 23. Cost components of the optimum designs achieved at different transverse spans for the V-type folded plate roof building.

As shown in Figure 23, the transverse span that generates the optimum solution for the V-type folded plate roofs is 8.571 m (a building having 6 internal supports). The solution is slightly better than the optimum solutions achieved for the 7.5 m and 10 m transverse spans generated when having seven and five internal supports, respectively. The cost of plates

remains approximately the same when the transverse span is 6 m to 10 m but changes dramatically when the transverse span starts to increase to 12 m and 15 m. This shows that it is not economical to have V-type folded plate roof buildings with a transverse span longer than 10 m. The cost of beams and diaphragms are inversely related; for shorter transverse spans, the cost of beams dominates and takes more than three times the cost of diaphragms representing a large portion of the entire cost, while in longer spans the opposite is true. This is further supported by the concept of load transfer in buildings; when the transverse span is shorter, the plates start to work in one direction, transferring almost the entire load to the internal and external beams, thus requiring them to have more capacities. Finally, the cost of the supporting columns is relatively fixed for all the spans, and it represents a small portion of the entire cost.

4.2. Three-Segment Type Folded Plate Roof Problem

Table 23 illustrates that the DE algorithm also had the best performance in its 10 trial runs, achieving the best statistical minimum, first quartile, average, median, and maximum solution. The second-best performed algorithm is pbBAS; it achieved slightly better results than the ABC algorithm with approximately the same standard deviation value. This large standard deviation value of both the pbBAS and the ABC algorithms indicates the inconsistency of the results achieved. The standard deviation value of all the algorithms is higher in this problem because of having a larger number of design variables compared to the first problem. However, the objective values of the results are lower. This shows that the three-segment type folded plates roofed buildings are less expensive than V-type folded plates roofed buildings but are more difficult to optimize. As shown in Table 24, the optimum solution achieved by all the algorithms had a span of 10 m (5 internal supports) and a horizontal plate's length of 3.5 m. Table 30 and Figure 22 show that the cost contribution of each member is approximately the same as the first problem, with the plates consuming almost half the cost of the materials.

After finding that the DE algorithm performed the best among the three metaheuristic algorithms, it is used to find the optimum design of the three-segment folded plate roof type under different transverse span settings. As done in with the V-type folded plate roof, the overall building length is divided into equal transverse spans of 6 m, 6.667 m, 7.5 m, 8.571 m, 10 m, 12 m, and 15 m for buildings with nine through three internal supports, respectively. The number of grouped design variables then becomes 12 for the three-segment type folded plate roof building. The internal controlling parameter values of the DE algorithm are presented in Table 13. The amounts representing the cost of each element for the optimum solution at each transverse span are represented in Figure 24.

As shown in Figure 24, the transverse span that generates the optimum solution for the three-segment type folded plate roofs is 10 m (a building having five internal supports). The optimum solution is close to the best solution found by the DE algorithm in its 10 trial runs, as discussed in Section 4.2. Unlike the V-type folded plate roofs, the cost of plates remained approximately the same for the entirety of the transverse spans studied. This shows that this type of folded plate is suitable even for 15 m long transverse spans. Like the V-type folded plates, the cost of beams and diaphragms are inversely related; for shorter transverse spans, the cost of beams dominates and takes more than three times the cost of diaphragms representing a large portion of the entire cost, while in longer spans the opposite is true. Finally, the cost of the supporting columns in this type of roof is also relatively fixed for all the spans, and it represents a small portion of the entire cost.

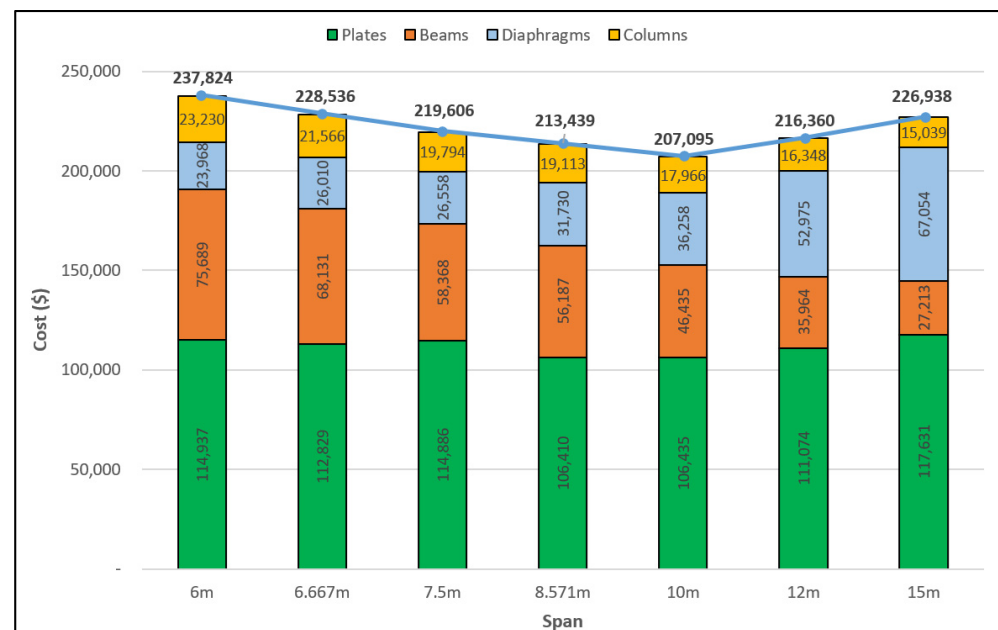


Figure 24. Cost components of the optimum designs achieved at different transverse spans for the three-segment type folded plate roof building.

5. Conclusions

It is shown that the optimum design of reinforced concrete V- and three-segment types of folded plate structures with their auxiliary and supporting members in accordance with ACI 318-11 can be achieved using different metaheuristic algorithms. The design problems consider the strength, serviceability, and applicability requirements set by the code. Additionally, all the detailing provisions, such as development and hook lengths of both longitudinal and transverse reinforcements, are considered while estimating the overall cost of reinforcement. In both roof types, the DE algorithm achieved the best optimization results proving its suitability for the optimization of such structures. Studying the effects of changing the transverse span while maintaining the overall dimensions of the building gave an indication of how the optimum cost varies with the percentages represented by each member. For the V-type folded plate roofs, the best optimum transverse span is 8.571 m. While for the three-segment type folded plate roofs, the best optimum transverse span is 10 m. These spans indicate that it is preferable to have transverse to longitudinal span ratios of 1:3 and 1:2.5 for V- and three-segment types of folded plate roofs, respectively. The cost that each member represents of the overall cost of building differs depending on the span being considered; for shorter transverse spans, it is mainly consumed by the plates and beams, while for the longer transverse spans, it is consumed by the plates and diaphragms. The cost of supporting columns remains almost the same for all transverse spans. It is found that the cost of formwork constitutes almost two-thirds of the overall material cost of the building, while the concrete and reinforcements together represent only one-third. This makes folded plate structures, compared to other types of structures, economical and environment-friendly construction. It should be noticed that the formwork can be removed and reused once the members develop their required strengths. This indicates the profitability of folded plate structures. Furthermore, it also implies less emission to the atmosphere due to the production of less amount of construction materials.

Author Contributions: Conceptualization, S.Y. and M.P.S.; methodology, S.Y.; software, S.Y.; validation, M.P.S.; writing—original draft preparation, S.Y.; writing—review and editing, S.Y., M.P.S., S.K. and Z.W.G.; supervision, M.P.S. and Z.W.G.; funding acquisition, Z.W.G. All authors have read and agreed to the published version of the manuscript.

Funding: This research was supported by the Energy Cloud R&D Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT (2019M3F2A1073164).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Bar-Yoseph, P.; Hersckovitz, I. Analysis of folded plate structures. *Thin-Walled Struct.* **1989**, *7*, 139–158. [CrossRef]
2. Wilby, C.B. *Concrete Folded Plate Roofs*; Elsevier-Butterworth-Heinemann: Oxford, UK, 2005; ISBN 0-340-66266-2.
3. Varghese, P.C. *Design of Reinforced Concrete Shells and Folded Plates*; PHI Learning Private Ltd.: New Delhi, India, 2010; ISBN 978-81-203-4111-1.
4. Gomez, R.A. Design of Folded Plates, PDH Online Course S275 (5PDH), PDH Center. 2013. Available online: <https://pdhonline.com/courses/s275/s275content.pdf> (accessed on 14 April 2022).
5. Sarma, K.C.; Adeli, H. Cost optimization of concrete structures. *J. Struct. Eng.* **1998**, *124*, 570–578. [CrossRef]
6. Kostem, C.N. Optimization of folded plate roofs. *Comput. Struct.* **1973**, *3*, 125–132. [CrossRef]
7. ACI 318-11; Building Code Requirements for Structural Concrete and Commentary. American Concrete Institute Committee: Farmington Hills, MI, USA, 2011; ISBN 978-0-87031-744-6.
8. ACI.318.2-14; Building Code Requirements for Concrete Thin Shells and Commentary. American Concrete Institute Committee: Farmington Hills, MI, USA, 2014; ISBN 978-0-87031-934-1.
9. Mathworks. Available online: <https://www.mathworks.com/products/matlab.html> (accessed on 25 April 2021).
10. Computers & Structures, Inc. Available online: <https://www.csiamerica.com/products/sap2000> (accessed on 25 April 2021).
11. ASCE/SEI 7-05; Minimum Design Loads for Buildings and Other Structures. American Society of Civil Engineers: Farmington Hills, MI, USA, 2005; ISBN 0-7844-0831-9.
12. Kaveh, A. *Advances in Metaheuristic Algorithms for Optimal Design of Structures*; Springer: Berlin/Heidelberg, Germany, 2017. [CrossRef]
13. Latif, M.A.; Saka, M.P. Optimum design of tied-arch bridges under code requirements using enhanced artificial bee colony algorithm. *Adv. Eng. Softw.* **2019**, *135*, 102685. [CrossRef]
14. Yousif, S.; Saka, M.P. Optimum design of post-tensioned flat slabs with its columns to ACI 318-11 using population-based beetle antenna search algorithm. *Comput. Struct. Int. J.* **2021**, *256*, 106520. [CrossRef]
15. Wang, Z.; Tang, H.; Li, P. Optimum design of truss structures based on differential evolution strategy. In Proceedings of the International Conference on Information Engineering and Computer Science, Wuhan, China, 1–5 December 2009. [CrossRef]
16. Saka, M.P.; Geem, Z.W. Mathematical and metaheuristic applications in design optimization of steel frame structures: An extensive review. *Math. Probl. Eng.* **2012**, *2013*, 271031. [CrossRef]
17. Saka, M.P. Design code optimization of steel structures using adaptive harmony search algorithm. In *Harmony Search Algorithms for Structural Design Optimization*; Series: Studies in Computational Intelligence; Geem, Z.W., Ed.; Springer: Berlin/Heidelberg, Germany, 2009; Volume 239. [CrossRef]
18. Saka, M.P.; Dogan, E.; Aydogdu, I. Review and analysis of swarm-intelligence based algorithms. In *Swarm Intelligence and Bio-Inspired Computation*; Yang, X.S., Cui, Z., Xiao, R., Gandomi, A.H., Karamanoglu, M., Eds.; Elsevier: Amsterdam, The Netherlands, 2013; pp. 25–47. [CrossRef]
19. Kaveh, A.; Ghazaan, M.A.I. *Metaheuristic Algorithms for Optimal Design of Real-Size Structures*; Springer: Berlin/Heidelberg, Germany, 2018. [CrossRef]
20. Kaveh, A.; Eslamlu, A.D. *Metaheuristic Optimization Algorithms in Civil Engineering: New Applications*; Springer: Berlin/Heidelberg, Germany, 2017. [CrossRef]
21. Jiang, X.; Li, S. BAS: Beetle Antennae Search Algorithm for Optimization Problems. *Int. J. Robot. Control.* **2018**, *1*, 1. [CrossRef]
22. Yousif, S.; Saka, M.P. Enhanced beetle antenna search: A swarm intelligence algorithm. *Asian J. Civ. Eng.* **2021**, *22*, 73–91. [CrossRef]
23. Karaboga, D. *An Idea Based on Honeybee Swarm for Numerical Optimization Technical Report-TR06*; Computer Engineering Department, Engineering Faculty, Erciyes University: Kayseri, Turkey, 2005.
24. Karaboga, D.; Basturk, B. A powerful and efficient algorithm for numerical function optimization: Artificial bee colony (ABC) algorithm. *J. Glob. Optim.* **2007**, *39*, 459–471. [CrossRef]
25. Karaboga, D.; Basturk, B. On the performance of artificial bee colony (ABC) algorithm. *Appl. Soft Comput.* **2008**, *8*, 687–697. [CrossRef]
26. Karaboga, D.; Akay, B. A modified artificial bee colony (ABC) algorithm for constrained optimization problems. *Appl. Soft Comput.* **2011**, *11*, 3021–3031. [CrossRef]
27. Storn, R.; Price, K. Differential evolution- A simple and efficient heuristic for global optimization over continuous spaces. *J. Glob. Optim.* **1997**, *11*, 341–359. [CrossRef]