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Equation for Egg Volume Calculation Based on Smart's Model

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Abstract: In the egg industry, it is necessary to estimate the egg volume accurately when estimating egg quality or freshness in a non-destructive method. Egg volume and weight could obtain egg density and could be used to determine egg freshness. Therefore, the egg geometric must be obtained first to establish a volume equation with a geometric shape. This research proposes an innovative idea to derive the mathematical model and volume equation of egg shape, calculate its volume, and verify the accuracy of the mathematical equation proposed using the volume displacement method. Using the proposed equation, the minimum error between the calculated egg volume) and actual egg volume is 0.01%. The maximum volume error does not exceed 2%. The egg shape equation can accurately draw the outer contour curve of the egg by the half-length of the maximum long axis and maximum breadth of the short axis, and the distance from the center point of the egg to the maximum breadth (x_m).

Keywords: egg shape equation; displacement of volume method; egg volume

MSC: 14-11

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1. Introduction

Egg geometry is often used in food research, agricultural engineering, biological sciences, mechanical engineering, architecture, and so on. This involves research on the classification and ecological morphology of poultry populations [1,2], predicting the weight relationship of chicks after egg hatching [3], and egg hatchability [4,5]. An egg can withstand external forces exerted during grading and transportation. Therefore, the mechanical properties of simulated eggshells are usually analyzed [6], and the best protection method is proposed. Eggshell shapes have also been used to design underwater installations, containers [7], and buildings [8]. Therefore, mathematical equations for egg shape, egg volume, and related parameters are required to perform related research and in applications.

Egg shape can be divided into oval, pyriform, circular, and elliptical forms [1,9]. The shape classification of eggs is mainly based on the ratio of the maximum breadth (B) to the maximum length (L) of the egg multiplied by 100, which is called the shape index (SI) [10,11]. The SI is used to judge whether the egg is approximately round or oval (i.e., degree of shape). Mathematically speaking, eggs are prolate spheroids [5], which approximate the volume of ellipsoids. Therefore, for egg volume calculation, the deformation equation with the volume of ellipsoids as the base is often used for calculation, as shown in Equation (1) [5,10].

$$V = k_v \frac{4}{3}\pi\left(\frac{L}{2}\right)\left(\frac{B}{2}\right)^2 \approx k_v L B^2. \tag{1}$$

k_v is the compensating coefficient, B is the maximum width equatorial, and L is the maximum egg length. Eggs shape of various breeds is different, and the k_v value of the volume calculation Equation (1) will also change accordingly [12]. The commonly used $k_v = 0.5236$ is a standard ellipsoid. To increase the accuracy of the egg volume calculation, Narushin (1997) set $k_v = 0.496$ [13]. A follow-up study by Narushin (2005) proposed a more accurate correction equation for egg volume, $k_v = (0.6057 - 0.0018\text{B})$, and the coefficient of determination reaching 0.958, as shown in Equation (2) [14]. It was subsequently proved that k_v is not a constant but a function of the linear parameters of the egg shape (that is, the maximum length L and maximum breadth B of the egg) [14–16]. Subsequently, Narushin (2021) again proposed that $k_v = 0.5136$ and Equation (2) can be more accurate and closer to the actual egg volume when calculations are performed [15].

$$V = \left(0.5202LB^2 - 0.4065\right). \tag{2}$$

In 1948, the German mathematician Fritz Hügelschäffer used two non-concentric circles to construct a deformed ellipse to form an egg-shaped curve [8,17]. Preston (1953) multiplied the basic ellipse equation by a cubic polynomial so that the deformed ellipse equation can describe a variety of egg shapes [18]. Smart (1969) found the tangent angle (taper angle) of the edge of the egg-shaped contour and introduced the short axis of the ellipse equation to describe the outer shape of the egg [19]. Narushin et al. (2020) applied Hügelschäffer's egg-shaped contour model to calculate the volume of an egg [20].

This study aims to derive a simplified equation that can quickly, directly, and accurately calculate the egg volume (V) based on the egg-shaped contour equation proposed by Smart (1969) and use the displacement of volume method to calculate volume. The egg volume calculated by the equation established in this paper is compared to the actual measured volume. This helps compare the theoretical equation and actual measurement error value as well as the accuracy of the theoretical equation. Further verifications were made by comparing the present results to the previously reported egg volume equations. The egg volume equation established in this study can be used as one of the reference methods for quick and accurate egg volume calculation. In addition, the egg-shaped contour equation was drawn and compared with the actual shooting shape, which further confirmed the reliability of the egg-shaped contour equation drawn in this research.

2. Materials and Methods

2.1. Egg Sample

The theoretical equation for egg volume proposed in this paper uses 47 eggs and measures the respective volume of each egg by the displacement of volume method. It is used to verify the difference between the egg volumes found using the theoretical equation and the actual experimental measurement.

2.2. Egg Dimension Parameter and Volume Measurement

For egg dimensions, a digital caliper is used to measure the maximum long (L) and maximum short (B) axes of eggs and the distance from the long axis half-length to the plane of the maximum breadth of eggs (x_m). The resolution of the digital caliper can reach 0.01 mm.

The actual volume of the egg is measured according to the volume displacement method, with the following practical guideline steps, as shown in Figure 1. Place the egg to be measured in a 500 mL graduated empty cylinder, and add water up to the 400 mL mark. Remove the egg from the measuring cylinder, reset the precision electronic balance (HG-2000, Shinko Denshi Co. Ltd., Tokyo, Japan) to zero, and then add the water in the measuring cylinder to 400 mL. As the density of water approaches 1 g/cm^3, the weight of the added water corresponds to the volume of the eggs. Removing the eggs from the graduated cylinder requires preventing the water in the graduated cylinder from overflowing. Therefore, let the water droplets adhering to the egg's surface remain in the measuring cylinder before removing the egg for higher precision. The water droplet adhering to the surface of the eggshell has a significant influence on the measurement of egg volume.

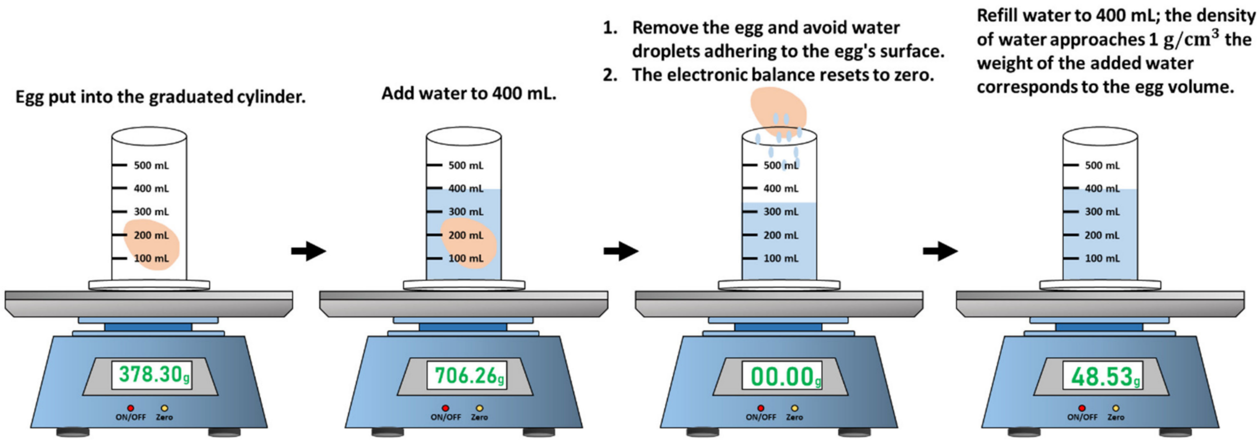

Figure 1. Flow chart of egg volume measurement.

2.3. Egg Volume Equation

Preston (1953) multiplied the basic ellipse equation by a polynomial to modify the ellipse shape to achieve a variety of egg shape equations for describing the geometric shapes of various bird eggs [18]. The follow-up work by Smart (1969) also used the ellipse equation as the basis to measure the tangent angle of the outer edge of the egg and introduced the ellipse equation to describe the shape of the egg [19], as follows:

$$\frac{x^2}{a^2} + \frac{y^2}{(b + x\tan\theta)^2} = 1. \tag{3}$$

Köller [21] multiplied the y^2 term by $t(x)$ based on the ellipse Equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \cdot t(x) = 1, \tag{4}$$

where a and b are the long and short axes of the ellipse, respectively, as shown in Figure 2. $t(x) = 1 + kx$ is a simple function, which converts the ellipse into an egg-shaped curve. When the value of k is large, the y-axis of the ellipse is not symmetric. When the value of k is small, it will be closer to the standard ellipse shape, as shown in Figure 2b. $k = 0$ corresponds to the standard elliptic curve.

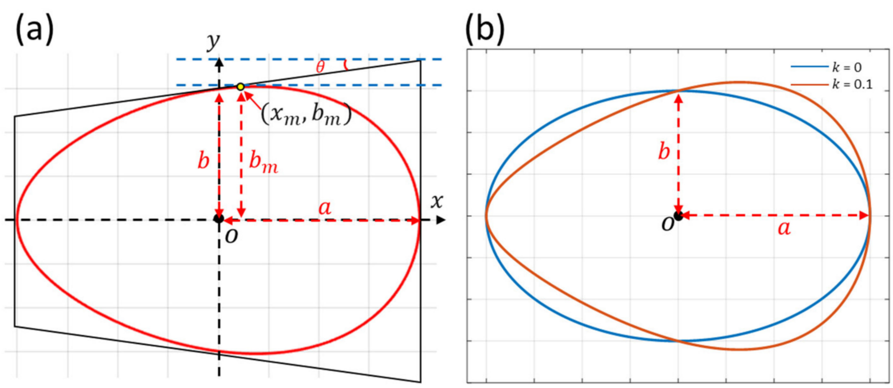

Figure 2. (**a**) Smart (1969) egg-shaped curve parameter location diagram. (**b**) Köller's egg-shaped curve diagram of the change of the function k value in Equation (4).

We have found that when $(b + x\tan\theta)^{-2}$ in Smart (1969) equation is converted by power series, $(1 + 2kx)$ can be obtained, and the conversion process is as follows:

Equation (5) can be obtained by expanding Equation (3) to power series as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\left[\left(1 - 2\frac{\tan\theta}{b}x\right) + \frac{-2(-3)}{2!}\left(\frac{\tan\theta}{b}x\right)^2 + \cdots\right] = 1. \tag{5}$$

When $x = x_m$, Equation (6) can be obtained by neglecting the higher-order terms in Equation (5) as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\left(1 - 2\frac{\tan\theta}{b}x\right) = 1 \tag{6}$$

and

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}(1 + kx) = 1, \tag{7}$$

where $k = -\frac{2\tan\theta}{b}$.

From the above process, it can be seen that the egg-shaped contour equations of Köller (2000) and Smart (1969) are consistent. To calculate the egg volume, we use the Smart (1969) egg contour equation and find the egg volume from the equation by rotating the integral around the long axis. The process is as follows:

$$V = \int_{-a}^{a} \pi y^2 dx \tag{8}$$

$$= \int_{-a}^{a} \pi\left(1 - \frac{x^2}{a^2}\right)(b + x\tan\theta)^2 dx \tag{9}$$

$$= \frac{\pi}{a^2}\left[\frac{4}{3}a^3b^2 + \frac{4}{15}a^5\tan^2\theta\right]. \tag{10}$$

Finally, the egg volume equation is obtained:

$$V = \pi(\frac{4}{3}ab^2 + \frac{4}{15}a^3\tan^2\theta), \tag{11}$$

where a is the half-length of the long axis of the egg, b is the half-length of the short-axis breadth of point O in the center of the egg, and θ is the egg taper angle, as shown in Figure 2.

The short-axis breadth of the egg center point of the long axis ($2b$) and edge taper angle (θ) are not easy to measure. In this study, the half-length of the long axis of the egg (a), maximum breadth of the short axis of the egg (b_m), and distance from the long axis half-length to the plane of maximum breadth (x_m) are known conditions. The correct egg shape can be obtained by calculating the theoretical values b and $\tan\theta$ of the egg shape parameters.

Equation (12) can be obtained by sorting out Equation (6) as follows:

$$y^2 = \left(1 - \frac{x^2}{a^2}\right)\frac{b^2}{(1 - \frac{2}{b}\cdot x\cdot\tan\theta)}. \tag{12}$$

Equation (12) is differentiated such that:

$$y\frac{dy}{dx} = \frac{-x}{a^2}\frac{b^2}{(1 - \frac{2}{b}\cdot x\cdot\tan\theta)} + \frac{1}{b}\tan\theta\cdot\left(1 - \frac{x^2}{a^2}\right)\cdot\frac{b^2}{(1 - \frac{2}{b}\cdot x\cdot\tan\theta)^2}. \tag{13}$$

When $\frac{dy}{dx} = 0$ is the horizontal tangent of the maximum width of the egg, $x = x_m$, the result is as follows:

$$-x_m + \frac{a^2}{b}\tan\left(1 - \left(\frac{x_m}{a}\right)^2\right) \cdot \frac{1}{1 - \frac{x_m}{b} \cdot \tan\theta} = 0, \tag{14}$$

According to the actual measurements in Table 1, putting the x_m and egg length of half-length ($\frac{L}{2}$) average into $1 - \left(\frac{x_m}{a}\right)^2 = 0.99$ can be obtained. The short-axis breadth half-length (b) and egg maximum breadth half-length ($\frac{B}{2}$) are close, so we use the maximum breadth of half-length directly for calculation; alos obtaining $1 - \frac{x_m}{b} \cdot \tan\theta = 0.99$.

Table 1. Egg shape geometry parameters.

	Minimum Value	Maximum Value	Average	Standard Deviation	Coefficient of Variation (%)
Egg length (L) (mm)	54.11	63.01	58.75	2.31	3.93
Egg maximum breadth (B) (mm)	41.50	47.13	44.63	1.19	2.66
Weight (g)	52.05	74.40	65.54	5.17	7.88
Shape index	70.97	83.81	76.04	2.46	3.24
x_m (mm)	0.9	3.31	2.17	0.51	23.53
Taper angle (degree)	0.53	7.44	3.88	1.46	37.56
Actual egg volume (cm^2)	48.32	69.42	60.53	4.91	8.11

Therefore assume when, $1 - \left(\frac{x_m}{a}\right)^2 \cong 1$ and $1 - \frac{x_m}{b} \cdot \tan\theta \cong 1$, then:

$$x_m \cong \frac{a^2}{b}\tan\theta. \tag{15}$$

From Figure 2a, the following can be obtained:

$$\frac{b_m - b}{x_m} \cong \tan\theta. \tag{16}$$

From Equations (15) and (16), the short-axis breadth half-length (b) of point O in the egg can be obtained as follows (see Figure 2a):

$$b = \frac{a^2 b_m}{(x_m^2 + a^2)}. \tag{17}$$

By substituting Equations (15) and (17) into Equation (11), the final egg volume can be obtained as:

$$V = \pi a^3 \left(\frac{b_m}{x_m^2 + a^2}\right)^2 \cdot \left(\frac{4}{3}a^2 + \frac{4}{15}x_m^2\right), \tag{18}$$

where $b_m = \frac{B}{2}$ and $a = \frac{L}{2}$.

3. Results and Discussion

This paper uses 47 eggs to verify the egg volume equation (Equations (11) and (18)) established based on Smart's equation (Equation (3)). The actual measured parameter results of 47 eggs are shown in Table 1. The most significant variation of egg shape parameters is the actual volume of the egg and the coefficient of variation of x_m, which are 23.53% and 8.11%, respectively. Egg maximum breadth has the least variability.

The egg volume Equation (11) is introduced by Equations (15) and (17) to obtain the final egg theoretical volume Equation (18). The angle of taper (θ) of the egg and half the breadth of the center point (b) of the egg can be described by Equations (15)–(17). To confirm the accuracy of the calculated taper angle (θ) and the short-axis breadth (b) of the egg center point, we selected the egg SIs as large: 81.08, medium: 76.11, small: 70.97. The profile of actual eggs of different specifications was compared with the calculated curve profile, as shown in Figure 3 The volume percent errors of the three eggs were 0.32%, −0.58%, and 0.33%, respectively, within ±1%. MATLAB was used to draw the egg-shaped contour curve of Equation (3). b and $\tan\theta$ in Equation (3) were calculated using Equations (15)–(17). Actual pictures of the eggs were added. This helps to check whether the theoretical equation egg contour, taper angle (θ) and half of the short axis width of the egg center point (b) are consistent with the actual egg contour. The results are shown in Figure 3. The calculated egg taper angle (θ) and half of the short-axis width of the center point (b) are almost consistent with the actual egg contour.

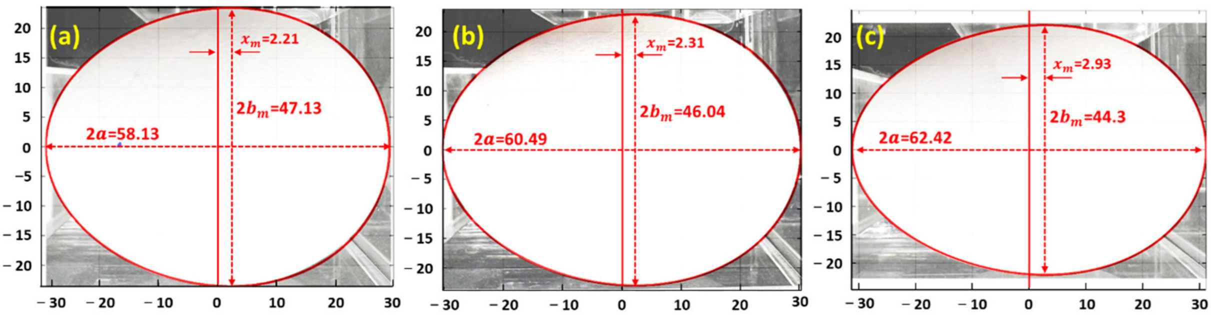

Figure 3. The actual egg shape and egg-shaped curve drawn by Equations (3) and (15)–(17). The SI (**a**) is 81.08; (**b**) 76.11; (**c**) 70.97.

The egg-shaped contour curve is converted into descriptions by a, b_m, and x_m from b and $\tan\theta$ in Equation (3). Therefore, the egg-shaped contour generated by the mathematical equation can approximate the real egg-shaped contour curve. However, according to the research and investigation in this study, there are still errors in the contour curves drawn by Equation (3) and Equations (15)–(17). In addition, because eggs are biological samples, they may be extruded deformed during production, resulting in an uneven shape.

For egg volume calculation, Equation (1) was used in the past. The k_v value was mainly set to 0.5236 for the elliptical volume equation. To make it more accurate, Narushin (2021) found that for $k_v = 0.5163$, calculation error variation can be reduced to a range of 0–3.7%, with an average of 1.1% [15]. Additionally, from the volume of the Hügelschäffer egg-shaped body and ellipsoids volume Equation (1) $k_v = 0.5236$, another Equation (2) for calculating the egg-shaped volume is obtained [15]. Equations (2) and (19) calculate the egg-shaped volume, with no significant difference between the two values obtained [15].

$$V = 0.5163LB^2. \tag{19}$$

In this study, the actual volume of eggs was measured by the volume displacement method. The shape parameters of 47 eggs were substituted into the theoretical egg volume Equation (18), ellipse Equation (1), and egg volume Equations (2) and (19) proposed by Narushin (2021). Errors between the egg volume calculated by the four equations and the actual measured egg volume were compared. The root mean square (rms) error values obtained were 0.47, 0.96, 0.43, and 0.63, respectively. This indicates that our theoretical equation and that proposed by Narushin (2021) ($k_v = 0.5163$) can calculate relatively accurate egg volumes. With $k_v = 0.5236$ ellipse volume equation, the rms error of egg volume calculated is only 0.96. However, because eggs are not in a standard ellipse shape, the error between the egg volume calculated by the ellipse volume Equation (1) ($k_v = 0.5236$) and the actual measured volume will be more significant.

We use the actual measured volume of the egg and the calculated volume of the equation to express the percentage error, which is a standard used to measure the accuracy of the measurement results. It is calculated as the estimate minus the actual value divided by the actual value, multiplied by 100%. It is used to express the accuracy of the volume converted by its equation.

Figure 4a shows that from Narushin (2021) Equation (19), and theoretical calculation Equation (18) in this study, the calculated volume percentage errors are mainly distributed within ±2%. Therefore, from the volume obtained by the theoretical volume equation in this study and that measured by the volume of displacement method, the volume percentage errors of ±0.5%, ±1%, ±1.5%, and ±2% correspond to 22 eggs, 16 eggs, 7 eggs, and 2 eggs, respectively. Therefore, the standard deviation is 0.47%. The volume percentage errors calculated by Equation (19) of Narushin (2021) are ±0.5%, ±1%, ±1.5%, and 2%, corresponding to 22 eggs, 17 eggs, 7 eggs, and 1 egg, respectively. The standard deviation is 0.4%. For the theoretical volume equation (i.e., Equation (18)) in this study, the maximum volume error of the 47 eggs is 1.91%, minimum volume error is 0.01%. Narushin's equation (i.e., Equation (19)) maximum error is 1.59%, and the minimum volume error is 0.05%. The resulting data point distribution and volume calculation accuracy are roughly the same for our theoretical volume Equation (18) and Narushin's Equation (19). However, the maximum volume percentage error of Equation (18) proposed in this paper is 1.91%, which may be caused by the fact that Equation (5) is expanded by the power series method, and the higher-order terms of Equation (6) are omitted for the convenience of calculation. Another reason may be that $\tan\theta$ and b are approximate values obtained by converting Equations (15)–(17) after measuring a, b_m, and x_m, resulting in a slight error in using the theoretical volume Equation (18) in this study. The percentage of error in the volume calculation of the ellipse volume equation is mostly greater than 1.5%, the maximum volume error can reach 3%, and the standard deviation is 0.74%, while the egg volume calculated by Narushin's Equation (2) is mostly evenly distributed between 0 and 3%, as shown in Figure 4b. Figure 4a shows that the egg volume obtained by this theoretical Equation (18) and Narushin's (2021) Equation (19) is quite accurate. Therefore, both the theoretical equation and Equation (19) can accurately and quickly estimate the egg volume.

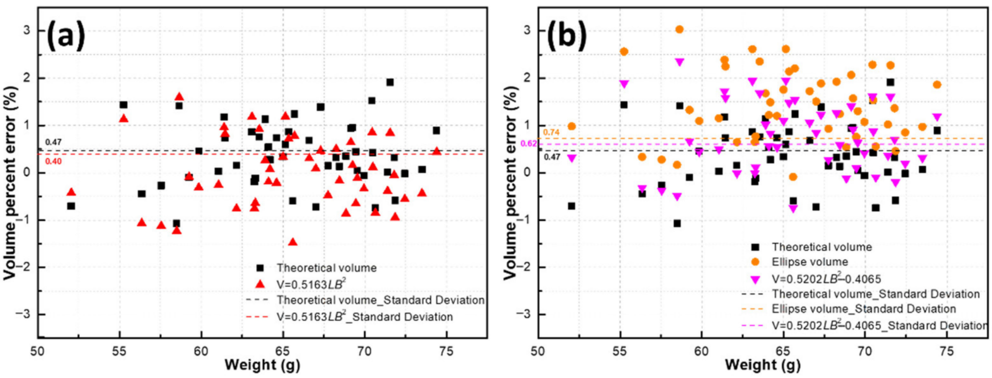

Figure 4. The volume percentage error corresponds to the actual weight of the egg. (**a**) theoretical volume Equations (18) and (19); (**b**) theoretical volume Equation (18), elliptical volume equation, Equation (2).

The theoretical egg volume Equation (18) established in this study is derived from Smart's egg-shaped curve Equation (3), with the parameters shown in Figure 2a including the taper angle (θ) and short-axis width (b) of the egg's geometric center point. The egg volume equation was verified using 47 eggs, and the egg volume could be accurately estimated when the SI of the egg ranged from 83.81 to 70.97. The research equation in this paper can be applied to analytical software such as the finite element method in the future to model eggshell or egg-shaped structures. Furthermore, it helps in improving the accuracy of the analysis in engineering or agriculture for analyzing parameters such as the mechanical strength of egg-shaped structures.

4. Conclusions

This study used the egg-shaped curve equation proposed by Smart (1969) [19] to extend the calculation equation of egg volume as:

$$V = \pi a^3 \left(\frac{b_m}{x_m^2 + a^2} \right)^2 \cdot \left(\frac{4}{3} a^2 + \frac{4}{15} x_m^2 \right). \tag{20}$$

The egg volume equation proposed in this study only requires the measurement of the half maximum length of the long axis (a), half maximum breadth of the short axis (b_m), and distance from the center of the egg to the maximum breadth of the short axis (x_m). Thus, it can quickly estimate the volume of the egg. It was verified that the egg volume with a SI of 70.97 to 83.81, the error range of the calculated theoretical volume and actual volume is within $\pm 2\%$.

Another contribution of this study is that we do not need to measure the egg's outer taper angle (θ) and breadth (b) of the center point. Instead, it can be converted from x_m, b_m, and a using Equations (15)–(17) established in this work. By obtaining the length distance between θ and b, Smart's (1969) egg-shaped curve Equation (3) can be directly used to draw the egg-shaped curve that is almost the same as the actual egg shape. In addition, the equation proposed in this paper can be applied to the fields of agriculture or food to analyze and simulate the curve modeling of eggs, which will be of great help and can effectively improve the simulation accuracy.

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References

1. Narushin, V.G.; Romanov, M.N.; Griffin, D.K. Egg and math: Introducing a universal formula for egg shape. *Ann. N. Y. Acad. Sci.* **2021**, *1505*, 169–177. [CrossRef] [PubMed]
2. Stoddard, M.C.; Yong, E.H.; Akkaynak, D.; Sheard, C.; Tobias, J.A.; Mahadevan, L. Avian egg shape: Form, function, and evolution. *Science* **2017**, *356*, 1249–1254. [CrossRef] [PubMed]
3. Narushin, V.G.; Romanov, M.N.; Bogatyr, V.P. Ap–animal production technology: Relationship between pre-incubation egg parameters and chick weight after hatching in layer breeds. *Biosyst. Eng.* **2002**, *83*, 373–381. [CrossRef]
4. Narushin, V.; Romanov, M. Physical characteristics of chicken eggs in relation to their hatchability and chick weight. In Proceedings of the ASAE Annual Meeting American Society of Agricultural and Biological Engineers, Chicago, IL, USA, 28–31 July 2002. [CrossRef]
5. Maclaury, D.W.; Insko, W.M.; Begin, J.J.; Johnson, T.H. Shape index versus hatchability of fertile eggs of Japanese quail (*Coturnix coturnix japonica*). *Poult. Sci.* **1973**, *52*, 558–562. [CrossRef]
6. Haiyan, S.; Fang, W.; Jianguo, Z.; Yinong, Z.; Shugang, Y. Finite element analysis of mechanical characteristics of dropped eggs based on fluid-solid coupling theory. *Shock. Vib.* **2017**, *2017*, 4512497. [CrossRef]
7. Zhang, J.; Wang, M.; Wang, W.; Tang, W. Buckling of egg-shaped shells subjected to external pressure. *Thin Walled Struct.* **2017**, *113*, 122–128. [CrossRef]
8. Petrovic, M.; Obradovic, M.; Mijailovic, R. Suitability analysis of Hügelschäffer's egg curve application in architectural structures' geometry. In Proceedings of the International Conference on Engineering Graphics and Design, ICEGD JASSY, Iaşi, Romania, 1 June 2011.
9. Nishiyama, Y. The mathematics of egg shape. *Int. J. Pure Appl. Math.* **2012**, *78*, 679–689.
10. Romanoff, A.L.; Romanoff, A.J. *The Avian Egg*; John Wiley & Sons: New York, NY, USA, 1949.
11. Brunson, C.C.; Godfrey, G.F. The relationship of egg shape, egg weight, specific gravity and 21-day incubation weight-loss to hatchability of broad-breasted bronze turkey eggs. *Poult. Sci.* **1953**, *32*, 846–849. [CrossRef]
12. Hoyt, D.F. Practical methods of estimating volume and fresh weight of bird eggs. *Auk* **1979**, *96*, 73–77. [CrossRef]
13. Narushin, V.G. The Avian Egg: Geometrical Description and Calculation of Parameters. *J. Agric. Eng. Res.* **1997**, *68*, 201–205. [CrossRef]
14. Narushin, V.G. Egg geometry calculation using the measurements of length and breadth. *Poult. Sci.* **2005**, *84*, 482–484. [CrossRef] [PubMed]
15. Narushin, V.G.; Romanov, M.N.; Griffin, D.K. Non-destructive measurement of chicken egg characteristics: Improved formulae for calculating egg volume and surface area. *Biosyst. Eng.* **2021**, *201*, 42–49. [CrossRef]

16. Narushin, V.G.; Lu, G.; Cugley, J.; Romanov, M.N.; Griffin, D.K. A 2-D imaging-assisted geometrical transformation method for non-destructive evaluation of the volume and surface area of avian eggs. *Food Control* **2020**, *112*, 107112. [CrossRef]
17. Petrovic, M.; Obradovic, M. The Complement of the Hügelschäffer's construction of the Egg Curve. In *BILTEN, Proceedings of 25th National and 2nd International Scientific Conference moNGeometrija, Beograd, Serbia, 24–27 June 2010*; Faculty of Architecture in Belgrade: Belgrade, Serbia; Serbian Society for Geometry and Graphics (SUGIG): Belgrade, Serbia, 2010; pp. 520–530.
18. Preston, F. The shapes of birds' eggs. *Auk* **1953**, *70*, 160–182. [CrossRef]
19. Smart, I.H. The method of transformed co-ordinates applied to the deformations produced by the walls of a tubular viscus on a contained body: The avian egg as a model system. *J. Anat.* **1969**, *104 (Pt 3)*, 507–518.
20. Narushin, V.G.; Romanov, M.N.; Lu, G.; Cugley, J.; Griffin, D.K. Digital imaging assisted geometry of chicken eggs using Hügelschäffer's model. *Biosyst. Eng.* **2020**, *197*, 45–55. [CrossRef]
21. Köller, J. Egg Curves and Ovals. 2000. Available online: http://www.mathematische-basteleien.de/eggcurves.htm (accessed on 13 April 2022).