# Visual Poetry and Real Context Situations in Mathematical Problem Posing and Solving: A Study of the Affective Impact 

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#### Abstract

Affective aspects are key mediators in the learning process. Whereas some of them can be associated with a certain discipline, others are situational and connected with specific activities that trigger positive emotions. This study analyzes these affective aspects based on two ways of approaching mathematical problems: problem posing and problem solving. In both cases, the starting point will be situations presented in multimodal representation, but with three different mediating elements: a real situation close to the students' reality (text with data and image), a real situation far from the students' reality (text with data and image), and a visual poem (hybrid text with implicit mathematical content that generates critical reading and provokes an aesthetic emotion). The aim is to explore the extent to which the mediating elements have affective and performance implications. To this end, an investigation was designed with future primary school teachers. As will be shown, the results, both in terms of performance and affective factors, are different for problem posing and problem solving. Problem posing based on a visual poem is a stimulating challenge for future teachers. However, in problem solving, as this study shows, the problem posed in a remote real situation is more successful in both performance and affective aspects.


Keywords: affective impact; problem solving; problem posing; pre-service teacher; real context; visual poetry; affect

MSC: 97M80; 97C20

## 1. Introduction

The study of posing problems and, above all, solving them has a long tradition in the field of mathematics education. The work of Polya and Schoenfeld [1,2] constitutes in this respect the bases of what we understand by a mathematical problem as well as the detailed description of the resolution process. While some problems are categorized based on the context in which their statement is formulated, even distinguishing between intra-mathematical and verbal problems, it is also possible to establish two categories within the latter and differentiate those problems in which the context of the statement is important in the resolution process from those others in which the context is simply a disguise.

Numerous works focus on the importance of presenting students with problems based on real contexts. This allows them to develop mathematical competence, making sense of procedures within the framework of a real problem [3]. However, as Hartman, Krawitz, and Schukajlow claim [4], little research has been carried out focusing on posing problems based on real-world situations. In this regard, Cai and Leikin point out that it is important to analyze not only the cognitive aspects, but also the affective aspects, particularly when examining complex situations such as problem solving and problem posing [5]. In relation to problem solving, in recent years some progress has been made in the integrated study
of affective and cognitive aspects-see, for example, Rellesman and Schukajlow [6] and Mcleod and Adams [7].

Another key aspect in the process of posing and solving problems is the study of the effect of the situations in which the problems are formulated, especially in the representations used to describe such situations. In the present article, we are interested in investigating the effect of proposing situations that combine several modes of expression: text, numerical data, photographs, and visual poetry. Students will be given a multimodal proposal. Our aim is to contrast the effects on their performance and affective aspects of posing problematic situations with different characteristics: on one hand, real situations (close or not to the reality of the students) that are described verbally and accompanied by numerical data and an evocative image. On the other hand, situations whose starting point is a visual poem. As will be shown, poetry is a fundamental resource that helps arouse interest and motivation, providing a series of cognitive tools that enhance intellectual independence. Through poetry, a series of communicative, playful, social, and cultural dimensions are encouraged that are vital for whole-child development [8]. In visual poetry, relationships are established between iconic and verbal language. The verbal and the iconic converge in a syncretic art that gives preference to the plastic and non-discursive aspect. The visual poem incorporates elements from the visual arts (image, design, color, texture, three-dimensionality). Nowadays, elements such as movement, sound, light, etc., are also incorporated.

Accordingly, in this study we will focus on analyzing how future primary school teachers respond to an activity consisting of posing and solving problems based on differentiated stimuli. Our specific objectives are:

- Identify to what extent the context of the situation (close real, distant real, or evoked through a visual poem) affects performance when posing and solving problems.
- Study how the context of the situation affects different aspects linked to motivation in approaching and solving problems.


## 2. Theoretical Framework

### 2.1. Problem Posing and Problem Solving

In the development of scientific knowledge, as well as in the development of critical thinking, there is no doubt about the need to know how to solve problems. However, as Einstein and Infeld already pointed out, posing an interesting problem may be considered more important than knowing how to work it out [9]. Although problem solving has been common in mathematics teaching for decades and there is a tacit agreement between researchers and educators that problem solving is an essential part of the mathematics teaching and learning process, research in problem posing is considered, today, a lower rank [10].

Problem solving has been a recurring theme in mathematics education research for more than 50 years. Actually, Polya's classic book on problem solving is now more than 75 years old [1]. In the field of research in mathematics education, the line dedicated to problem solving is extensive. Stanic and Kilpatrick summarize the research carried out up to that moment, distinguishing three central themes [11]. On one hand, problem solving as a cognitive activity; research focused on the descriptions or categorizations of student resolutions are included here. On the other hand, research focused on studying how to improve the development of problem-solving skills (problem solving as an object of study). Finally, the research that analyzes problem solving as a way to learn mathematics (it therefore develops skills and modifies beliefs).

Problem posing is a topic of great importance in Mathematics as a discipline. Just think of the 23 problems posed by David Hilbert at the beginning of the 20th century that have given rise to important advances in different areas of Mathematics. However, in the field of mathematics education, research focused on this aspect did not begin to be developed until the mid-1980s. In recent years, several reviews have been published that present the state of the art in relation to research in the mathematical approach of
problems. We could mention Cai et al. [12] and Osana and Pelczer [13], among others. Cai and Leikin [5] present a special issue in the journal Educational Studies in Mathematics that compiles various research on problem posing. In this article, the authors establish four categories of studies to classify research according to how problem posing is approached: problem posing as a tool for teaching mathematics; problem posing as a teaching objective; problem posing as a study or as a research tool whose object of study is exclusively the approach to problems. Our work falls into the first category, since we are going to establish comparisons between performance and affective impact depending on the context and also on the mathematical activity developed (solving or posing problems). In any case, although advances have been made in the field of research, problem posing is still perceived as an unimportant activity in classrooms. Indeed, as Crespo and Sinclair [14] indicate, the reality of classrooms "is that mathematics problems come from textbooks while the teacher's job is to assign them for students to solve" (p. 395). This is the case even in the initial training of mathematics teachers. In fact, few research works focus on the difficulties of future teachers when dealing with problem posing activities [14,15]. Thus, the present study aims to address an approach to problem posing that, to the best of our knowledge, has not yet been studied in depth in research: the study of affective and cognitive aspects in problem solving and posing.

### 2.2. Affect and Mathematical Activity

Although, in general, research in mathematics education in relation to problem solving and posing has focused on cognitive aspects and not so much on affective ones, the affective system is key in the development of cognitive processes [16]. When we refer to affective systems, it is convenient to distinguish global aspects, which include attitudes towards mathematics or beliefs about the nature of mathematics, from local aspects, such as satisfaction, fun, or interest that have derived from a specific mathematical activity [17]. The concept of affect is complex and, as Cai and Leikin point out, the numerous constructs involved in it make up the affective experience of student participation in mathematics, including emotions, attitudes, beliefs, values, and motivations [5].

In relation to research in problem solving, in the last 40 years numerous investigations have been carried out that focus on the importance of affective factors in teaching and learning. In his study on metacognition and problem solving, Silver analyzes the relationship between affective factors (such as confidence or persistence) and performance, intuiting that affective factors play an important role in the metacognitive processes of problem solvers [18]. Following this same idea, Schoenfeld [19] explores to what extent the solver's belief systems influence "control" during the process of solving a mathematical problem, that is, in the selection and implementation of resources to solve the problem.

In the field of research focused on problem posing, the analysis of the affective domain has been used, fundamentally, to examine and better understand the affective characteristics linked to mathematical thinking and learning, but also to analyze the aspects that promote or hinder problem posing [5].

In his studies on affectivity, McLeod identifies three key domains [20]: beliefs, attitudes, and emotions, ordered in a degree of increasing intensity and decreasing stability.

As DeBellis and Goldin [21] explain, mathematical competence, particularly in relation to problem solving (but also to problem posing), is based on different systems that interact with each other. These systems include both cognitive aspects (linked to the process of understanding, interpreting, mathematising, and solving mathematical activity) and affective aspects (emotions, attitudes, and beliefs). Indeed, as pointed out by Cai and Leikin [5], cognitive and affective fields are interrelated and support each other in the learning process (p. 287). In this article, we will focus on two components of attitudes: interest and value; and on two components of emotions: enjoyment and boredom.

- Interest is defined as a psychological state that describes a relationship of positive affect between an individual and an object [22]. Research carried out by scholars such as Schiefele, Krapp, and Winteler [23] confirms that, especially in mathematics,
certain attitudinal aspects such as interest are important in educational processes. Rellensmann and Schukajlow [6] differentiate between two types of interest: individual interest and situational interest. Individual interest is relatively long-lasting, while situational interest is characteristic of a specific situation; it appears if a certain problem captures the student's attention. Because it is possible to move from situational interest to individual interest, it is crucial to study the interest generated by the different mathematical activities and, in particular, the degree of interest shown by students depending on the mathematical activity developed.
- Value characterizes the importance that we perceive is attached to objects, content, and actions [24]. The value attributed to a certain activity plays an important role in motivation. In this sense, the motivation of students to learn can be related to the importance they attribute to learning and its objects [25]. Thus, value can be related to the degree of importance that a student assigns to a certain activity. If a person does not value an activity, it is unlikely that he or she will make an effort to carry it out, even if they feel capable of doing it [26]. It is important to keep in mind that the characteristics of the activity, its relevance or perceived usefulness, may be the reasons why a student values or does not value such activity [27].
- Enjoyment is one of the most frequent positive emotions in the classroom. According to the control-value theory of achievement emotions [28], enjoyment is a positive emotion that can influence students to commit to the activity performed. It has been proven that student enjoyment is related to effort and performance [29]. In this way, enjoyment can not only accompany the development of interest, but can also positively influence it.
- Boredom, like enjoyment, is an emotion that can be related to learning. Indeed, it is one of the deactivating negative emotions that is reported more frequently along with anxiety, anger, frustration, hopelessness, and shame [30]. According to many studies, boredom is the result of lack of control over actions [28] and it has been found to be negatively related to performance in mathematics. Moreover, it is worth highlighting that the feeling of boredom is not simply the result of the lack of interest or enjoyment. If students are not interested in Math or do not enjoy Math classes, they can feel many different negative emotions such as anger or frustration, but not always boredom. Due to their different characteristics and consequences, enjoyment and boredom are distinct emotions that were found to be negatively correlated [31]. However, enjoyment and boredom are not opposites. As Pekrun et al. [32] point out, the lack of enjoyment does not necessarily imply the presence of boredom.


### 2.3. Multimodal Representation

Just as books are no longer printed only on paper, texts and mathematical problems are not only written by using words. In fact, a text can present many different modes of expression. The 21st century digitized world is characterized by the multiplicity of communication channels, which brings the need to reconsider the use of resources that focus both on words and on other modes of representation. Authors such as Kern [33], Serafini [34], Cope and Kalantzis [35], Paesani et al. [36], Warner and Dupuy [37], and Reyes-Torres et al. [38] concur that literacy development in our current multimodal society should embrace the ability to read and write, but also to speak, listen, view, and enable students to become meaning-makers. In fact, literacy has turned into multiliteracies-many literacies-and as a result, a multimodal approach should be incorporated in today's education [34,39]. Today learners face a multiliterate landscape in which they must be able to interpret and construct meaning not only from written texts, but also from pictures, songs, spaces, advertisement, and so forth. In this respect, multimodal resources constitute new types of texts and new forms of narration that differ significantly from conventional models $[39,40]$. As will be shown, they contribute both to the personal and social training of students and to the development of their ability to reflect, understand reality, and apply their linguistic knowledge to another knowledge base [41].

Significantly, nearly all texts that students encounter nowadays can be considered multimodal and include a significant number of signs and symbols to communicate information such as letters and words in varying fonts, drawings, pictures, color, videos, audio sounds, music, facial gestures, body language, etc. Therefore, the term "text" does not solely apply to a narrative discourse, but rather to a wide range of media products and modes of meaning-making: advertisements, picture books, music, images and visual poetry, among others [42-44]. In this respect, because modes are experienced in differing ways by each of the senses-usually visual, auditory, or tactile-learners' interaction can vary when working with a multimodal text and so do their meaning-making processes. Multimodal texts are therefore highly motivational and contribute to creating an effective learning experience. Indeed, the use of these resources in the Math class allows students to develop key aspects of literacy, reflect on the different elements that the text presents and, ultimately, think critically.

### 2.4. Visual Poetry

The recent popularity of visual or experimental poetry has contributed to its expansion and to enhancing its creative and interdisciplinary possibilities in school contexts. Taking into account the changes in reading habits that have occurred in recent years due to social networks and the digital world, the use of experimental, multimodal, and interdisciplinary poems is claimed as an ideal way to work in the classroom. This is due to the fact that students not only appreciate poetry and reflect through it on the world around us, but also see it as innovative. Consequently, they enjoy reading, but also creating it [45] (p. 57). If in the 1980s calligrams were already part of the corpus of children's poetry, in the 21st century the presence of visual poems in poetic anthologies is no longer surprising [46].

In this line of thought, if we take into account that literacy and numeracy are the two most relevant codes of developed human societies, we can see that poetry and mathematics have in common the reflection on the limits of language that we can find in geometry, arithmetic, or the notion of infinity. Similarly, some experiences have been developed on the use of poetry in science teaching and it has been proven that the combination of poetry writing and illustration in a university course arouses the interest of students [47] and that the use of poetry in a chemistry class increases the interest of students [48]. In the particular case of the use of visual poetry in the teaching of mathematics, the Spanish group "Colectivo Frontera de Matemáticas" [49] directed by Antonio Ledesma, has shown the relationship between Visual Poetry and Mathematics based on experiences put into practice in secondary schools. They argue that if mathematical language has a place in the visual poem, visual poetry should be of interest to mathematicians and to mathematics teachers. For this reason, they use the so-called "mathematical poems", that the visual poet Toni Prat [50] defines as an artistic modality that links purely abstract numbers with the most emotional expressions in messages. The aim is to transcend the "coldness" of numbers and look for the poetic part. For Prat, numbers cease to exist to become an attempt at philosophical communication. An example of a mathematical visual poem by this author is the one that we use in this article: it represents a ladder through which a circle descends and ends up transformed into a hexagon (see Figure 3 Visual poem by Toni Prat [51]).

## 3. Research Questions and Expectations in the Present Study

As pointed out at the end of the introduction, this work focuses, on one hand, on two cognitive aspects-how students pose problems and how they solve them-and, on the other, on four affective aspects related positively or negatively to motivation-enjoyment, boredom, interest, and value. In short, we are interested in studying the influence on these cognitive and affective aspects of the contexts in which the information that allows for posing and solving mathematical problems is presented. In this study, we will focus on the approach and resolution of problems from extra-mathematical contexts. Thus, situations with three different characteristics will be proposed: (1) a situation that alludes to a real context and close to the reality of the study participants; (2) a situation that alludes to
a real context but far from the reality of the participants; and (3) a situation presented through a visual poetry. To this end, the problems raised by the students will be analyzed as well as the resolutions that they propose to the problems raised. In addition, in each case, evidence will be collected in order to analyze the affective and motivational impact generated by each of the activities that were carried out. With this data, we will try to answer the following questions:

1. How does the context of the initial situation influence the formulation of problems?
(a) Are students capable of posing mathematical problems from different contextualized situations?
(b) How and to what extent does the type of situation generated by posing a problem affect the four affective factors analyzed (enjoyment, boredom, value, and interest)?
2. How does the context of the initial situation influence problem solving?
(a) Is there an influence of the context of the situation on performance in the resolution process?
(b) How and to what extent does the type of situation generated by the resolution of a problem affect the four affective factors analyzed (enjoyment, boredom, value, and interest)?

## 4. Methodology

### 4.1. Sample

Since it is common practice for students to deal with problems or activities posed by the teacher or by the textbook, they often take for granted that someone else will always provide them with the math problems they must work on. In fact, although there are math teachers who create their own problems, many of them rely on external sources for the math problems that they bring to their classes [52]. This is why we consider it is essential that pre-service teachers of mathematics have the opportunity to pose their own mathematical problems in the formal school setting. Thus, this study was carried out with a convenience sample of $\mathrm{N}=55$ students that are in their second year of the Bachelor's Degree in Primary Education at the University of Valencia.

### 4.2. Experience Design

The experience, developed over 10 days during the first semester of the 2021-2022 academic year consists of two different parts that we describe below.

### 4.2.1. Phase 1: Problem Posing

To begin, the participants faced a problem posing activity. To do this, they were provided with three situations with differentiated characteristics that we will now describe.

Situation 1: Real context and close to the participants. In this situation, the starting point is a park located in the city of Valencia that is well known by all the inhabitants. Given that in recent months the park has been closed for rehabilitation works, students were presented with a text with information on the construction of the park and the current restoration process. Numerical information is included in the text and is accompanied by two images, a photograph of the construction process and an aerial image to scale extracted from GoogleMaps. We can see the translation of the problem provided to participants in Figure 1 below.

## Situation 1: Gulliver

Surely you all know the Gulliver Park in the Turia Park, just before reaching the area of the City of Arts and Sciences, but maybe you don't know some interesting aspects... The creator of the original idea was the municipal architect Rafael Rivera, but no sculptor dared to materialise it. The fallero artist Manolo Martín was immediately excited by the idea and the project began to take shape. A comic artist, Sento Llobell, joined the project and made a first proposal.


All the knowledge of Fallas art served the architecture, the figure of Gulliver was built in Manolo Martín's atelier with the collaboration of numerous Fallas artists. He worked with a 1:35 scale model divided into 68 slices.

The park is currently closed due to comprehensive renovation work and will reopen at the end of this year.

Figure 1. Situation 1, real context near to participants reality.
Situation 2: Real context and far from the participants' reality. For this situation, we used a real context extracted from the work described in Schukajlow and Krug [29]. It is the Salt Mountain situation, see Figure 2. The reason for its selection is that Hartmann et al. [53] also used it in an experience similar to the one we have developed in the present study and, therefore, it may be useful to compare the results in the future.

## Situation 2: Salt mountain

In the Middle Ages, salt was obtained from the evaporation of seawater. Today it is mainly obtained by mining. The salt ( 1.2 t per $\mathrm{m}^{3}$ ) is piled up in salt mountains using 1.2 m wide conveyor belts.

In the figure you can see a salt mountain. The length of the slope is $c=20 \mathrm{~m}$ and the diameter is $\mathrm{d}=30 \mathrm{~m}$. The salt mountain weighs about 3740 tonnes.

Technical data of the transport truck:


- Truck loading platform: $5.11 \mathrm{~m} \times 2.3 \mathrm{~m} \times 1.8 \mathrm{~m}$
- Power: 125 kw
- Volume capacity: $5249 \mathrm{~cm}^{3}$
- Maximum load: 26.8 tonnes

Figure 2. Situation 2, real context far from participants' reality, extracted from [30].
Situation 3: Visual poetry. For this situation, we included a visual poem by Toni Prat [51] that, as can be seen in Figure 3, shows an important mathematical component at first glance.

## Situation 3: Visual poem



Here I show you a visual poem by the poet Toni Prat. In a visual poem there is, in most cases, an iconic element through which the poem is defined.

Look closely at the poem which, as you can see, has an important mathematical component.

Figure 3. Situation 3, visual poem.
In addition, after each of the situations, we asked the students to indicate their level of agreement with different statements by using a 1-5 Likert scale (where 1 is strongly disagree and 5 is strongly agree). The objective of these items is to collect information on the affective aspects in the part of their experience that has to do with posing problems, focusing on 4 factors: enjoyment, boredom, interest, and value. Thus, for the writing of the 5 items, we rewrote them by using a previous study based on the work of the research team of Prof. Schukajlow [6,54]. Each of these items aims to assess a different affective dimension.

- To measure the degree of "enjoyment" they are asked to rate the level according to the statement: "I enjoyed posing a problem based on this situation" (Cronbach's Alpha 0.755).
- To obtain information on the degree of boredom, they are asked to rate the degree according to the statement: "I found it boring to try to pose a problem from this situation" (Cronbach's Alpha 0.831).
- To obtain information on the interest generated by each of the three activities consisting of posing a problem, they are asked on the degree of agreement with the statement: "I found it interesting to think of a problem statement based on this situation" (Cronbach's Alpha 0.821).
- Finally, we are interested in knowing to what extent they value the importance of posing problems from a given context. For this, we ask them the degree of agreement with the statement: "I think it is important to be able to pose problems from situations like this one" (Cronbach's Alpha 0.86).
The participants completed this activity individually and without a time limit. They were provided with the situations and affectivity items in a printed document and had 7 days to submit it. Since it is an activity carried out in the classroom context as a part of the daily work, the data is not anonymous and the students were aware that, once finished, the results would be shared.


### 4.2.2. Phase 2: Problem Solving

The second part of the experience took place during a regular class session. In this phase, students had to solve 3 problems, all formulated in a particular context. Its design and structure is symmetrical to the one followed in phase 1 since the problems are posed based on 3 situations: a real situation close to the reality of the students, a real situation
far from the students' reality, and a situation based on a visual poem. The three problems put into play similar mathematical procedures: all of them are based on the use of the Pythagorean theorem in context. However, due to the characteristics of each of them, the resolution process differs.

Problem 1. Real context close to the participants. In this case, the starting situation is a mobilization of students that has affected the participating students. From an explanatory text and an aerial image with a scale, the students must obtain an answer based on an estimate. This problem, in addition to putting the Pythagorean theorem into play, requires the solver to make estimates and assumptions, being able to register in what is called a Fermi problem [54]. Figure 4 below presents the translation of the statement.

Problem 1: Human chain against port expansion
As you know, the Valencia City Council has planned an expansion of the port which will allow a growth in the import and export of goods but which, according to various reports, will also lead to serious ecological damage in the mid and long term. The students of the Facultat de Magisteri, organised through the student syndicates, have organised a mobilisation to draw the attention of the public to the risks of the extension of the port of Valencia.

To this end, a human chain will link the Facultat de Magisteri with the northern part of
 the port via two busy avenues: Manuel Candela and Avenida del Puerto. Thus, the human chain will cover the sides AB and BC of the triangle that you can see in the following image.

Knowing that in the Facultat de Magisteri there is a total of 3511 students enrolled in one of the two degrees, that the distance of the route in Manuel Candela is 1.65 km and that the distance in a straight line from the Facultat de Magisteri (A) to the port (C) is 2.5 km , try to find out if it will be possible to form a human chain with the students participating in the mobilisation.

Figure 4. Problem 1, second part of the experience, real context near to participants' reality.
Problem 2. Real context away from the participants. In this case, a problem extracted from [55] was presented and the geographical context was modified: a mountain known by the students was chosen, but since it is on the Canary Islands is far from their reality (even if any of the students have been there, it is not easy for them to keep in mind the real dimensions or the precise details of the place). Figure 5 below presents the translation of the statement.

Problem 3. Problem based on a visual poem. Since the aim is to pose problems based on mathematical procedures linked to the Pythagorean theorem, it was decided to choose a simple unpublished visual poem signed by the author Antonio Ledesma. The poem presents the image of a circle accompanied by the expression "truncated square". Figure 6 shows the poem and the statement of the problem.

## Problem 2: Teide cable car

The cable of the Teide cable car needs to be replaced. Each metre of cable costs $8 €$. What is the approximate cost of replacing the current cable? Explain your solution in detail.

The Teide cable car is located in the Teide National Park in Tenerife (Canary Islands) and is the highest cable car in Spain. It ascends from
 the base station, located at 2356 metres above sea level on the slopes of Mount Teide, to the La Rambleta station at 3555 metres above sea level and only 163 m from the summit of the volcano, overcoming a drop of 1199 m . The journey takes between eight and ten minutes. The ride lasts between eight and ten minutes at a maximum speed of $8 \mathrm{~m} / \mathrm{sec}$. The cabins can accommodate 44 people.

Figure 5. Problem 2, second part of the experience.

## Problem 3: Visual poem

Here is an image of a visual poem. Look at it carefully and answer the questions below.

From the dimensions of the picture that appears in the poem, try to find the dimensions of:
(a) The smallest square that contains the red circle.

b) The largest square that is included in the red circle.

Cuadrado truncado
Figure 6. Visual poem problem.
The participants had a 90 min session to solve these 3 problems plus the 3 problems proposed in the first phase of the experience by another participant. In addition, after each of the six resolutions, they had to assess the five items of affective aspects on a $1-5$ Likert scale. This time the statements were formulated in the same way as in the work of Schukajlow et al. [25].

- To measure the degree of "enjoyment" they are asked to rate the level according to the statement "I enjoyed solving a problem based on this situation" (Cronbach's Alpha 0.703).
- To obtain information on the degree of boredom, they are asked to rate the degree according to the statement "I found it boring trying to solve a problem based on this situation" (Cronbach's Alpha 0.779).
- To obtain information on the interest generated by each of the three activities consisting of posing a problem, they are asked to rate the degree of agreement with the statement "I found it interesting to solve a problem based on this situation" (Cronbach's Alpha 0.831).
- Finally, we are interested in knowing to what extent they value the importance of posing problems from a given context. To do this, we ask for the degree of agreement with the statement "I think it is important to be able to solve problems based on situations like this one." The Cronbach's alpha value for this item is slightly below 0.7 (0.635). This is a value which, in Taber's study [56] on the use of this statistic in educational science work, is usually taken as satisfactory (p. 1279). However, as this author pointed out, it is convenient to reflect on the reason for this value: it is
certainly complex for the participants in this study (pre-service teachers) to assess the degree of importance of a certain activity related to teaching. Indeed, in the other three items, they are asked to value aspects that are directly related to them (enjoyment, boredom, or interest in solving a task), while in this last item they are asked to value an aspect that goes beyond this, and which requires an epistemological reflection: the importance, understood as educational value, of solving a specific type of mathematics problem.


### 4.3. Data Analysis

### 4.3.1. The Problems Students Posed

To analyze the problem proposals raised by the participants, a qualitative analysis of the content was carried out [57]. Thus, the first step was to carry out an exploratory analysis of the proposals of a random subsample of 30 participants with the aim of defining a category system. This exploratory analysis was carried out jointly by two researchers so that, by agreement, the variables to be categorized were agreed upon. First, it was identified whether or not the proposal could be considered a math problem (dichotomous variable). To discriminate the productions categorized as "mathematical problems" we discarded from this category those productions that posed a problem whose answer did not require any mathematical procedure. For example, a student, in relation to the context based on the visual poem, proposes the question: "Looking at the image, how many straight sides of the red figures are there in total?" Another student poses the following question: "What is the name of the figure at the bottom of the ladder?" To answer these questions, you do not have to complete any mathematical procedure (beyond a mere count). These questions were included in the category "Non-mathematical problem".

In those cases, in which the proposal was considered a problem, two categories were established: solvable mathematical problem and unsolvable mathematical problem. Indeed, during this exploratory analysis, problems particularly associated with the context of the visual poem were identified as unsolvable. This is the instance of the production shown in Figure 7 in which the translated statement is: "The area of the hexagon is $20 \mathrm{~cm}^{2}$. The radius of the circle is 3 cm , how many $\mathrm{cm}^{2}$ have been lost?" Since, as shown in Figure 3, the hexagon is generated as a regular polygon with six sides inscribed in the initial disk, it is not possible for the area of the hexagon to be equal to $20 \mathrm{~cm}^{2}$ if we assume that the radius of the circle it is 3 cm . In other words, it is a problem that apparently looks like a math problem, but the data provided makes it impossible to rely on it to give a correct answer.

Plantea aquí tu problema
El área del hexágono es de $20 \mathrm{~cm}^{2}$. Si el radio del cícula
son 3 cm , ¿cuánton $\mathrm{cm}^{2}$ se han perdido?

Figure 7. Problem proposed in relation to the visual poem that has been categorized as an unsolvable mathematical problem: "The area of the hexagon is $20 \mathrm{~cm}^{2}$. If the radius of the circle is 3 cm , how many $\mathrm{cm}^{2}$ has it been reduced?".

Once the exploratory analysis was completed and the categories were established, the analysis phase of all the productions began. To assess the reliability of the analysis, two evaluators independently analyzed $15 \%$ of the students' productions (a total of 24 statements), obtaining adequate reliability values, kappa with quadratic weighting equal to 0.8018 (standard error 0.133).

### 4.3.2. Students' Solution to Provided Problems

To analyze the resolutions of the problems, the process was simpler. The three problems raised in this phase were solved using the Pythagorean Theorem. In the first two problems-formulated in a real context-it was necessary to clearly interpret the data to identify the length of two sides of the right triangle and then use the Pythagorean Theorem to find the unknown side. Finally, this value had to be interpreted, in the first case, estimating the space occupied by a person in a human chain and, in the second, reasoning from the price. It should be noted that, in the second problem (in which the length of cable for the cable car had to be found), only one participant took into account that the line of a cable railway is double. Therefore, the resolutions that did not take into account this element of complexity in the resolution were considered correct. In the third problem, the procedure was similar; they could reason either from an indeterminate radius or from a fixed value for the radius (which they could estimate or obtain directly by measuring). However, in both cases, it was necessary to obtain the dimensions of the square inscribed to the circle by means of the Pythagorean Theorem.

Thus, two categories were established from the start: those resolutions that did not include any error in the approach or the resolution procedure were assessed as correct. On the contrary, those others that were either blank or had errors, for example, in the case of the second problem because they had not identified the vertical distance between the two stations of the cable car, were considered incorrect. Again, to guarantee the reliability of the resolution analysis, a total of 24 resolutions were analyzed independently by two researchers, obtaining an adequate reliability value, kappa with quadratic weighting equal to 0.913 (standard error 0.0704).

## 5. Results

### 5.1. Influence of Context in Problem Posing

### 5.1.1. Problem Posing Performance

Table 1 shows the absolute and relative frequencies obtained in the analysis of the problems posed by future teachers from each of the three proposed contexts.

Table 1. Problem posing performance.

|  | Near Real Context | Remote Real Context | Visual Poem |
| :---: | :---: | :---: | :---: |
| Non-mathematical problem | $9(16.4 \%)$ | $4(7.3 \%)$ | $15(27.3 \%)$ |
| Unsolvable mathematical problem | $3(5.5 \%)$ | $1(1.8 \%)$ | $9(16.4 \%)$ |
| Solvable mathematical problem | $43(78.1 \%)$ | $50(90.9 \%)$ | $31(56.4 \%)$ |

For each of the contexts presented to the students, the graph shown in Figure 8 shows: (1) the proportion of productions that do not propose a problem, (2) those that propose a problem that cannot be solved, and (3) those that do pose a problem. In this way, it can be seen that the multimodal representation included in the visual poem is the one that, based on these results, is more complex. However, surprisingly, the remote real context is the one that promotes to a greater extent that future teachers are able to pose a mathematical problem that can be solved.

The data presented in Table 1 and in Figure 8 suggest that the context influences the type of mathematical problem posed. A chi-square test of independence was performed to examine the relation between problem posing performance and contextual characteristics of multimodal representation of the situation. The relation between these variables was significant, $X^{2}(4, N=165)=18.9677, p=0.000797$.

Problem posing performance


Figure 8. Problem posing performance depending on the context.

### 5.1.2. Influence of Affective Factors in Problem Posing

After posing each of the three problems based on the contexts presented through a multimodal representation, the students had to rate, on a scale of $1-5$, the degree of agreement with four statements related to enjoyment, boredom, interest, and value. Table 2 below shows the means and standard deviations of each of these factors depending on the context presented.

Table 2. Affective factors in problem posing.

|  | Near Real Context (NRC) | Remote Real Context (RRC) | Visual Poem (VP) |
| :---: | :---: | :---: | :---: |
| Enjoyment | $3.22($ sd 1.100$)$ | $3.44(\operatorname{sd} 1.151)$ | $3.58($ sd 1.329$)$ |
| Boredom | $2.25($ sd 1.109$)$ | $2.15(\operatorname{sd} 1.079)$ | $2.04(\operatorname{sd} 1.232)$ |
| Interest | $3.85($ sd 0.951$)$ | $3.76(\operatorname{sd~} 0.942)$ | $3.91($ sd 1.143$)$ |
| Value | $4.38($ sd 0.828$)$ | $4.51(\operatorname{sd~} 0.717)$ | $4.35($ sd 0.844$)$ |

Based on the values of each of the affective factors, it can be inferred that, at least apparently, the context based on a visual poem is more fun, less boring, and more interesting than the other two real contexts. However, when we look at the value that students give to posing a problem, the highest average is obtained in the remote real context.

To confirm that these relationships are significant, we performed an inferential analysis. First of all, we verified if the valuations of each one of the affective factors follow a normal distribution in any case in each context. To do this, a hypothesis test was carried out using the Shapiro-Wilk normality test, which in all cases yields a $p$-value less than 0.001 , which allows us to rule out the assumption of normality with a reliability of $99.9 \%$. Therefore, to analyze the existence of significant differences between the values of enjoyment, boredom, interest, and value depending on the context, it is necessary to carry out a non-parametric test. In this case, we did a Friedman test for each of the couples in relation to each of the four affective factors. In Table 3 below we show the mean differences (md) and the $p$-values obtained in each case.

Table 3. Mean differences and significance values of the Friedman test for significant differences between dependent samples.

|  | NRC vs. RRC | NRC vs. VP | RRC vs. VP |
| :---: | :---: | :---: | :---: |
| Enjoyment | $\begin{aligned} m d & =-0.22 ; \\ p & =0.465 \end{aligned}$ | $\begin{aligned} m d & =-0.36 ; \\ p & =0.049 \end{aligned}$ | $\begin{aligned} m d & =-0.14 ; \\ p & =0.096 \end{aligned}$ |
| Boredom | $\begin{aligned} m d & =+0.10 ; \\ p & =0.670 \end{aligned}$ | $\begin{aligned} m d & =+0.21 ; \\ p & =0.088 \end{aligned}$ | $\begin{aligned} m d & =+0.11 ; \\ p & =0.273 \end{aligned}$ |
| Interest | $\begin{aligned} m d & =+0.09 ; \\ p & =0.450 \end{aligned}$ | $\begin{aligned} m d & =-0.06 ; \\ p & =0.433 \end{aligned}$ | $\begin{aligned} m d & =-0.15 ; \\ p & =0.162 \end{aligned}$ |
| Value | $\begin{aligned} m d & =-0.13 ; \\ p & =0.083 \end{aligned}$ | $\begin{aligned} m d & =+0.03 ; \\ p & =0.782 \end{aligned}$ | $\begin{aligned} m d & =+0.16 ; \\ p & =0.109 \end{aligned}$ |

Taking a maximum value of $p=0.05$ as a reference, only significant differences were identified in the degrees of enjoyment in relation to posing problems from a nearby real context or from a visual poem. In other words, for the sample of future teachers who have participated in this study, it is significantly more fun to pose a problem from a multimodal representation that includes a visual poem than from another based on a real context close to the reality of the students. In the rest of the factors, the results of the test on the contrast hypothesis do not allow the null hypothesis of equality between distributions to be ruled out.

### 5.2. Influence of Context in Problem Solving

### 5.2.1. Problem Solving Performance

Table 4 shows the absolute and relative frequencies obtained in the analysis of the resolutions proposed by the future teachers to the three problems formulated in different contexts.

Table 4. Problem solving performance.

|  | Near Real Context | Remote Real Context | Visual Poem |
| :--- | :--- | :--- | :--- |
| Incorrect or blank | 13 | 33 | 39 |
| Correct | 42 | 22 | 16 |

The graph shown in Figure 9 offers, for each of the contextualized problems, the proportion of productions that manage to solve it. We can see that the problem formulated in a near real context, although mathematically equivalent, is more accessible than the problem formulated in a remote real context and even more so than the problem based on a visual problem.

The data presented in Table 4 and Figure 9 suggest that the context influences the problem solving performance. A chi-square test of independence was performed to examine the relation between problem solving performance and contextual characteristics of multimodal representation of the situation. The relation between these variables was significant, $X^{2}(2, N=165)=26.9824, p<0.00001$.

Problem solving performance


Figure 9. Problem solving performance depending on the context.

### 5.2.2. Influence of Affective Factors in Problem Solving

After solving each of the three problems based on the contexts presented through a multimodal representation, the students had to rate, on a scale of $1-5$, the level of agreement with four statements related to enjoyment, boredom, interest, and value. Table 5 shows the means and standard deviations of each of these factors depending on the context presented.

Table 5. Affective factors in problem solving.

|  | Near Real Context (NRC) | Remote Real Context (RRC) | Visual Poem (VP) |
| :---: | :---: | :---: | :---: |
| Enjoyment | 3.06 (sd 1.089) | 3.48 (sd 1.128) | 2.96 (sd 1.243) |
| Boredom | 2.35 (sd 1.102) | 2.13 (sd 1.100) | 2.46 (sd 1.059) |
| Interest | 3.59 (sd 1.091) | 3.61 (sd 1.123) | 3.44 (sd 1.127) |
| Value | 4.26 (sd 0.758) | 4.35 (sd 0.705) | 4.06 (sd 0.998) |

Based on the mean values of each of the affective factors, it can be inferred, at least apparently, that when solving the problem formulated in a multimodal representation that includes elements from a remote real context, it is more fun, less boring, more interesting, and more important than the resolution of the other two problems raised. It is curious because the data is, except in the case of value, inverse to those obtained in the first phase of the experience.

To confirm that these relationships are significant, we performed an inferential analysis. First, we verified if the valuations of each one of the affective factors follow a normal distribution in any case in each context. To do this, a hypothesis test was carried out using the Shapiro-Wilk normality test, which in all cases yields a $p$-value less than 0.001 , which allows us to rule out the assumption of normality with a reliability of $99.9 \%$. Therefore, to analyze the existence of significant differences between the values of enjoyment, boredom, interest, and value depending on the context, it is necessary to carry out a non-parametric test. In this case, we carried out a Friedman test for each of the pairs in relation to each of the four affective factors. In Table 6 we show the mean differences and the $p$-values obtained in each case.

Taking a maximum value of $p=0.05$ as a reference, significant differences are identified in the degrees of enjoyment and boredom. In particular, it is inferred that for the sample of future teachers who participated in this study it is significantly more fun to solve a problem formulated through a multimodal representation that includes elements from a distant real context than to solve either of the other two versions (near real context or based on a visual
poem). It is also inferred that it is significantly more boring to solve the problem whose statement is formulated from a visual problem than another whose statement is formulated from a real context far from the reality of the participants.

Table 6. Mean differences and significance values of the Friedman test for significant differences between dependent samples.

|  | NRC vs. RRC | NRC vs. VP | RRC vs. VP |
| :---: | :---: | :---: | :---: |
| Enjoyment | $\begin{gathered} m d=-0.42 \\ p=0.001 \end{gathered}$ | $\begin{aligned} m d & =+0.10 \\ p & =0.505 \end{aligned}$ | $\begin{aligned} m d & =+0.52 \\ p & =0.008 \end{aligned}$ |
| Boredom | $\begin{aligned} m d & =+0.22 ; \\ p & =0.223 \end{aligned}$ | $\begin{aligned} m d & =-0.11 ; \\ p & =0.398 \end{aligned}$ | $\begin{aligned} m d & =-0.33 \\ p & =0.023 \end{aligned}$ |
| Interest | $\begin{aligned} m d & =-0.02 ; \\ p & =1.000 \end{aligned}$ | $\begin{aligned} m d & =+0.15 ; \\ p & =0.450 \end{aligned}$ | $\begin{aligned} m d & =+0.17 \\ p & =0.336 \end{aligned}$ |
| Value | $\begin{aligned} m d & =-0.09 \\ p & =0.317 \end{aligned}$ | $\begin{aligned} m d & =+0.20 \\ p & =0.433 \end{aligned}$ | $\begin{aligned} m d & =+0.29 ; \\ p & =0.194 \end{aligned}$ |

## 6. Discussion

In the two experiences developed in this study, students were presented with a sequence of mathematical activities stated in a multimodal representation. The first experience is presented as a problem posing activity, while the second is a problem-solving activity. Certainly, students are more familiar with solving than with posing problems since, as pointed out in [14], problem posing activities are still rare in classrooms, even in teacher training colleges.

In both experiences, the independent variable that made it possible to design the sequence is the context in which the problem posing and problem-solving activities are formulated. The variable we refer to as "context" takes three values: near real context, far real context, and visual poetry. The design is meant to have a growing degree of abstraction: from a real context, close to the reality of the students, to an abstract context that is presented through visual poetry. Next, the results obtained are discussed, first in relation to performance and, later, in relation to the affective aspects analyzed.

### 6.1. Performance of Pre-Service Teachers in Posing and Solving Problems

There are some previous studies that compare the influence of the context on the performance of students when posing a problem. For example, in the work of English [58], the author designs an experience with third grade children; she presents three situations and proposes to the students that they pose a problem based on them: one based on a "formal context" (which consists of giving a mathematics operation), and two based on an "informal context": one based on an image and the other on a verbally described situation. In our present study, we tried to advance in this line by proposing three situations that combine different multimodal representations and include text, numerical data, and images. In addition, we incorporated the particularity that one of the images shown is a visual poem, since we were interested in identifying the reaction of the students to this type of aesthetic stimulus which is unfamiliar to them.

Our initial hypothesis when designing the first part of the study was that students would have an easier time posing problems from the multimodal representation of a familiar situation for them, such as the one presented in the first activity of problem posing in phase 1 of the experience. Consequently, we also assumed that based on the image of the visual poem they would have more difficulties in posing a mathematical problem. If we pay attention to the results shown in Table 1 and Figure 8, as well as the results of the inferential analysis, it follows that there is indeed a relationship between the characteristics of the situation and the problem posing performance. However, the hypothesis that the close context facilitates the creation of mathematical problems is not confirmed. This may be due to the particular characteristics of the information provided in the near real context. Indeed, although the information was presented in a very similar way to that
of the remote real context (numerical data, explanatory text and image), the first case included information referring to numerical proportionality. Most of the problems that have been categorized as non-mathematical or unsolvable include questions that have to do with scale information. A participant proposes, in relation to the nearby real context, the following question: "If 272 slices are added to the existing ones, what scale would the model be?" This question posed from the initial situation highlights the difficulty that students have in reasoning with proportions, since apparently, they have not understood that the slices to which the statement refers are made on a 1:35 scale, therefore, increasing their number will not change the value of this ratio. Another student, faced with this same situation, proposes the following problem: "Knowing that in Gulliver Park we worked with a 1:35 scale model divided into 68 slices, what do you think its real dimension will be?" Again, this problem highlights a difficulty with the meaning of the scale, which he interprets as a relationship between number of slices rather than as a relationship between measurements. It is also curious that this student poses this problem in this way, taking into account that the statement included an image with a scale of the park, that is, it was to be expected that they would ask for the real dimensions of the Gulliver not based on the scale of the model, but on the scale of the image. Regarding the context based on the visual poetry of Toni Prat, the results were adjusted to the hypothesis, but we did not expect to find such a high proportion ( $16.4 \%$ ) of unsolvable problems. As explained in the section describing the qualitative analysis methodology, we had to incorporate this category when we observed that a significant number of students proposed, particularly in this third context, problems that although they seemed to make sense from a mathematical point of view, were unsolvable by the incorporated data. In most cases, this error is due to the fact that students ignore the relationships between the dimensions of elementary geometric figures. This reveals serious limitations regarding the understanding of the measurement of the area in relation to other magnitudes (such as the radius or the perimeter).

Regarding the design of the second phase of the experience, our starting point was based on the works of Rellesman and Schukajlow [6] and Elfringhoff and Schukajlow [59]. Indeed, the results of these studies infer the importance of introducing a real context in the statement of a problem to promote motivation (particularly in the two studies cited, a single affective factor was analyzed, interest). Thus, following the same design strategy as in the first phase of the study, we proposed a sequence of three problems varying the context: first, a problem based on a context close to the students' reality; second, a problem based on a real context but not close to the reality of the students; and, finally, a problem based on a visual poetry. The first two problems can be considered modeling activities, since they require interpreting data (which are real in the first two cases and figurative, in the case of visual poetry), identifying the variables of the problem, establishing a model to mathematize the initial situation (in the three cases can be based on the Pythagorean Theorem), working mathematically and interpreting the solution. Regarding the third problem, although it is formulated from a visual poetry whose title "Truncated Square" is directly related to the statement of the problem, it is certainly not necessary to understand (or even read) the title to be able to address its resolution. We can, therefore, consider that this problem is an intra-mathematical problem.

To assess the performance of the students, we took into account the mathematical correction of the resolution, considering as correct only those in which all the phases of the modeling process were carried out without errors. Our starting hypothesis was, again, that the performance would be better in the first problem and that it would decrease in the second and, to a greater extent, in the third problem. Based on the results shown in Table 4 and Figure 9 and according to the results of the inferential analysis, we can confirm our hypothesis.

In any case, in view of the errors made by the participants, a more detailed analysis is key to interpreting and understanding these global results. Indeed, in the second problem, the errors derive from an incorrect interpretation of the information presented in the statement. It is possible to observe that many students have not realized that the height
of the right triangle is the difference between the heights with respect to sea level of the top of the mountain and the base station of the cable car. These students have reasoned as if the base of the cable car were at sea level. This incorrect resolution is key to identify two difficulties of the students when they are faced with solving modeling activities. The first is the identification of the variables of the problem (which derives from an incorrect interpretation of the data) and, in this case, has to do with understanding the statement. The second has to do with validation. Students who reason incorrectly assuming that the difference in height between the base of the cable car and the top is greater than 3500 m , correctly applying the Pythagorean Theorem, arrive at the cable car having a length of more than 4 km . However, the statement presents another piece of information that allows the solution to be validated and, therefore, to verify that it is not possible for the route to have a length of more than 4 km (since it is stated that the trip lasts between 5 and 8 min at a speed of $8 \mathrm{~m} / \mathrm{s}$ ). These types of errors in the interpretation of the data and in the validation of the solution have already been identified in studies carried out with future teachers who were faced with solving modeling tasks [60,61].

The third problem, whose statement is contextualized by a visual poem, had a significantly lower performance with less than $30 \%$ of the resolutions correct. In this case, the difficulties are different from those of the other two problems. In many cases, students have ignored any mathematical reasoning and have limited themselves to deducing the answer directly from imprecise and incorrect measurements on the image that constitutes the visual problem. In this problem, taking into account the profile of the participants (future teachers with symbolic calculation skills), we expected that the correct solutions would be based on reasoning from an undetermined radius R. However, the formulation of the statement certainly invited reasoning from the measurements of the image that could be estimated because the center of the circle was not identified (no student obtains this measurement by geometrically finding the center). However, as our aim in this study is to compare performance in solving mathematically equivalent problems with different contexts, we considered both approaches (from a radius R or from an estimated measure) to be correct. It is worth pointing out that this statement should be modified in the case of using this problem for the same purpose in the future; it could read: "From an indeterminate value of a dimension of the image that appears in the poem, find the dimensions of: ... ". The proportion of students who are not able to tackle the problem and left it blank is also considerable ( 12 in total, almost $22 \%$ of the resolutions of this problem), while in the other two problems, all the participants are able to at least identify (albeit incorrectly) the variables and establish (albeit incorrectly) a mathematical model. At this point, it should be noted that the differences between the three problems are not limited to the context in which they are formulated. Although in all three cases the resolution involves reasoning from the Pythagorean Theorem-this would correspond to thinking of the modeling process and the mathematization phase-the previous process, the transition between the real situation and the mathematical situation, is different in each case. Indeed, the representation accompanying problem 1 (see Figure 4) includes a drawing of the right triangle whose dimensions must be estimated from the statement, and in problem 2 it is almost drawn (only the hypotenuse is missing, see Figure 5). However, in problem 3, it is necessary to identify and represent the right triangle that makes it possible, by means of the Pythagorean Theorem, to deduce the value of the side of the inscribed square as a function of the radius of the circle (see, for instance, Figure 6). This might be an alternative reason that could explain, to some extent, the lower performance in the third problem. It follows that, in this case, the absence of numerical data and graphical representation of the triangle in the statement influences the performance of students who are not able to find a strategy to deduce them (measure), nor to work with a generic variable.

### 6.2. Influence of the Context in Affective Aspects: Study of Motivation

As previously mentioned, despite numerous studies emphasizing its pedagogical interest, problem posing activities are still rare in classrooms. It is important to take this aspect into account in order to understand the results of the study focused on affective aspects, particularly the differences between the two phases of this study.

When designing the study on the impact of modifying the context variable in the sequence of problem posing activities introduced in the first phase of the experience, our initial hypothesis was that the positive affective impact (corresponding to the items of enjoyment, interest, and value) would be higher in the case of the situation based on a visual poem. Indeed, the results of studies carried out in $[47,48]$ show that the incorporation of visual poetry in the development of classroom activities had an impact on the interest of the students. Based on the results of our study with future teachers, we deduce that although the highest values of enjoyment and interest appear in the activity that asks to pose a problem from a visual poem (see Table 3), these differences are only significant in the case of the enjoyment factor when comparing them with the nearby real context (see Table 4). These results surprised us because we expected that the differences would be more evident in relation to remote context. However, the results show that future teachers give more value to posing problems from a context that is far from their reality (although the differences with the data from the other two contexts are not significant). In any case, it is important to highlight that, despite the previously mentioned difficulties when posing problems from a visual poem, this activity is significantly more fun for the future teachers who participated in the study than posing a problem from a real and familiar context to all of them. It is inferred, therefore, that the aesthetic characteristics of the visual poem have a positive influence on this factor linked to motivation.

In relation to the second part of the experience, our initial hypothesis was that the positive affective impact (related to enjoyment, interest, and value) would be greater in the case of the problem based on a close real context and, moreover, familiar to the students. In effect, the problem statement refers to a location well known to the study participants. Furthermore, in this case-unlike in the other two problems-they were the protagonists of the statement. Modeling problems whose context is familiar to the students presumably generate greater interest on the part of the students, since it helps them to connect the mathematical procedures involved in the resolution with a known situation [62], although it is true that there are some studies in which the effect of introducing a real context is not so evident [6]. At this point, the results are again surprising, since the results show that in this experience the problem formulated in a context far removed from the reality of the students is significantly more fun than the other two. Indeed, when designing the problem formulated in a close context, we not only considered the context of the problem from a geographical point of view, but also the fact that the solvers are, in some way, the key players in the problem statement. Obviously, making a human chain can be as far from their reality as using a cable car, but we consider the first problem to be closer because it refers directly to the students' own reality. In fact, as can be deduced from the study by Yu and Sullivan [63], personalized statements (based on the interests of the solvers and including them as protagonists) promote higher performance and more positive attitudes. We understand that, in our case, it would be necessary to complete the sequence with more problems of each of the three types and expand the sample of participants in order to confirm or refute the results of this study. In the other positive affective aspects (interest and value), it is also observed that the outcome is better for the same problem, but the differences are not significant (see Table 4). In relation to boredom, it is observed that the problem that arouses it is, to a greater extent, the problem based on a visual poem, in fact, the difference is significant when compared to the problem formulated in a remote real context. It is likely that, contrary to what happened in the first phase of the experience, during the resolution process (less creative than problem posing), the visual poem, which in this case was made up of an image and a suggestive title that was related to the statement, has not been engaging enough to stimulate the participants in the study. It is quite likely,
therefore, that they did not even stop to reflect on the relationship between the title of the visual poem ("Cuadrado truncado", which means truncated square) and the statement of the problem posed, since it was not necessary to answer the problem nor was it referred directly to them in the sentence.

In the field of educational research, an extensive body of literature has been generated focusing on motivation in relation to learning and teaching mathematics. In the review carried out by Schukajlow, Rakoczy, and Pekrum [64] different theoretical approaches to this topic are explained in detail, which is still evolving to this day. Motivation is a complex concept that involves different constructs, among which enjoyment, boredom, interest, or value can be identified (although others such as self-efficacy also have an influence). In the present work, we focused on four motivational factors that, due to their characteristics, have different valence. Thus, in order to globally analyze the results of the two experiences developed in this study, we defined a new variable that we call "motivation" and that depends on enjoyment, interest, and value (the three with positive valence), and boredom (with negative valence). This variable, once normalized so that it takes values between 1 and 5 , allows us to globally compare the affective variables associated with the experiences of approaching and solving problems depending on the context. Table 7 shows the mean values and standard deviations in each case and Figure 10 shows the data relative to the mean values of motivation in each of the two phases of the experience.

Table 7. Mean values of the motivation variable in the problem posing and solving phases for each of the three situations.

|  | NRC | RRC | VP |
| :--- | :--- | :--- | :--- |
| Problem posing <br> (Cronbach alpha 0.863) <br> Problem solving <br> (Cronbach alpha 0.822) | 3.80 (sd 0.103) | 3.89 (sd 0.102) | 3.95 (sd 0.121) |

As we can see, in each phase there are differences both depending on the context and whether you are asked to pose or solve a problem. In the graphs shown in Figure 10, it is possible to visually compare between contexts and between experiences.


Figure 10. Problem solving and posing motivation depending on the context.
We observe that although there are no significant differences depending on the context in the approach phase, motivation is higher when students are asked to pose a problem based on a visual poem. In the problem-solving experience, since normality cannot be assumed (the Shapiro-Wilk test only allows us to assume normality for problem-solving motivation from the context based on a visual poem), non-parametric tests have to be completed. A hypothesis test using the Friedman test confirms that there are significant differences between at least two variables $(p=0.031)$. To confirm that the far real context
significantly influences motivation, we performed a Wilcoxon signed rank test. As a result, it is found that there are significant differences in motivation when solving a problem in a near real context or in a far real context ( $p=0.025$ ); likewise, there are also significant differences in motivation when solving a problem formulated in a remote real context or from a visual poem ( $p=0.005$ ).

## 7. Conclusions

This study analyzed the result of an experience with pre-service teachers of Primary Education at the University of Valencia. It is important that, during their initial training, future teachers are acquainted with different ways of representing information. In particular, it is key that they are aware of the potential that multimodal representation offers not only in literary contexts, but also in relation to other disciplines such as the resolution of mathematical problems. In this sense, the focus of our experience is novel, since in the experiences that we developed and analyzed in this study a visual poem was introduced within the framework of a multimodal representation. This type of mathematical problem is of great interest due to its aesthetic content and the potentiality of the message that can be deduced through its interpretation.

As can be deduced from the results of the first phase of the experience, the introduction of a visual poem in a mathematical problem posing activity promotes both interest and enjoyment. It is also particularly striking that the affective impact is not at all correlated with performance, that is, the interest and enjoyment in posing a mathematical problem based on visual poetry does not depend in any case on the ease with which students have when approaching this activity. The problem posing activity also allows us to detect some gaps in the mathematical knowledge of teachers in relation to the difficulty in asking questions in situations of proportionality, which is associated with conceptual and procedural errors that had already been identified in studies with other types of activities with students with the same academic profile [65].

Regarding the problem-solving activity, the results confirm that in relation to both performance and affective aspects there is a certain relationship between performance and motivation. Although it was conceivable that the participants were more interested in and better solved the formulated problem in a context close to their reality, we found that even though the performance is significantly better, the motivation is not. In addition, we observe that no positive effect is identified in the affective aspects in relation to the problem that is formulated through a visual poem. It is possible that this is due to the poem selected and the problem posed: indeed, when posing the problem, we try to relate it both to the graphic representation (a circle) and to the phrase that accompanies it (a truncated square). However, it is very likely that the participants have ignored this relationship, since it was not necessary to solve the problem. Thus, what was initially presented as a contextualized problem in a visual poem has remained an intra-mathematical problem that based on the results is not very motivating for students and is significantly more difficult than the two contextualized problems in real environments. This can be seen as a limitation of the work. Therefore, it would be convenient to propose an alternative sequence in the future to confirm or complete the results obtained with it.

Finally, it is important to consider the limitations of this study. Our work is the result of a first collaboration between researchers from different areas (mathematics education and literary education); it is an initial study and, as such, it is possible to identify some aspects that can be improved in future work. Regarding the design of the sequence of activities, in both phases of the study we adapted problems or situations that had already been used in previous investigations and which, therefore, ensured a robust design. However, in direct relation to visual poetry, since there are no studies that analyze the impact of these resources in relation to performance and affective aspects in the resolution of problems, we identified limitations that can be overcome in the future. Particularly in the problem-solving part, the problem based on the visual poem by Antonio Ledesma ("Truncated square") is significantly more difficult than the other two. This is so because when analyzing the
solving procedure as a modeling process, it requires students to identify (without any visual support) the geometric elements to be considered in order to find the dimensions of the inscribed square: the hypotenuse, which corresponds to the diameter, and the two equal legs, which correspond to the two sides of the square. To overcome this limitation, it would be useful, in the future, to complete these sequences with at least one more problem (and one more situation) of each type, in line with Schukajlow's work [31]. On the other hand, in relation to the qualitative analysis of the students' productions both in the problem posing phase and in the resolution phase, the categorizations used in the analysis of the problems posed could be further refined, considering categories based on the degree of relationship between the problem posed and the context provided, as done in [53]. In the analysis of resolutions, it could also be further refined by, for example, identifying the elements of complexity considered in the resolution or categorizing the errors made. It would also be interesting, in order to complete the information on the influence of the context on affective or performance aspects, to complete the study with the analysis of interviews with some of the study participants. It is, therefore, a study that opens up several lines of future work that may contribute to a better understanding of the impact of context on affective aspects or performance.

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