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Dynamic Modeling and Simulation of Urban Domestic Water Supply Inputs Based on VES Production Function

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Abstract: The Gompertz growth curve is used to describe the urban water population, the linear function is used to represent the per capita disposable income, and the domestic water demand is described combined with the factors of population, income, and the water-saving consciousness. The VES production function is used to describe the production function of the domestic water supply. Combined with system dynamics, the supply and demand management model of urban domestic water in Jiangsu province, China, is developed. The process of water supply investment and labor input in the urban domestic water system is studied with two depreciation methods: the straight-line depreciation method and the sum of years digits method. In the case that the water consumption population is expected to decline, four water demand scenarios composed of different per capita disposable income and the growth rate of water-saving consciousness are investigated. Investment and labor input are taken as control variables to conduct water supply and demand simulations for the four scenarios. The results show that the control schemes in all four scenarios reach a balance between water supply and demand. Moreover, the investment of the sum of years digits method is larger than that of the straight-line depreciation method in 2005–2019 but less than that of the straight-line depreciation method in 2020–2034. The sum of years digits method has the characteristics of more depreciation in the early stage and less depreciation in the later stage, which is conducive to timely compensation for the large loss of fixed assets in the early stage.

Keywords: domestic water; supply and demand management; VES production function; system dynamics; depreciation method

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1. Introduction

In recent years, rapid economic development, urbanization, and the continuous improvements of people's living standards have made the contradiction between domestic water supply and demand increasingly prominent in China. The issue of domestic water has become a critical limit factor to urban economic development. Therefore, taking Jiangsu Province of China as the subject investigated, considering the factors affecting domestic water demand, such as urban and rural residents' water consumption population, per capita disposable income and water-saving consciousness, this paper constructs a system dynamics (SD) model to study the balance of domestic water supply and demand in different scenarios.

At present, single or combined models are often used to predict water demand or supply in the short-term, medium-term and long-term, such as regression analysis model [1], artificial intelligence optimization algorithm [2–4], time series model [5–7], grey model [8,9] and system dynamics model [10,11]. Among these methods, the system dynamics model is more suitable for dynamic analysis, reflecting factors' interaction and restriction. The system dynamics method has the characteristics of multivariable, non-linear, high-order, and multi-feedback, which can describe complex relationships between social economy, ecological environment, and population systems [12]. Huang et al. developed a system dynamics model framed across the nexus of climate, water, energy and food sectors for a rooftop farm in Taipei City [13]. Dong et al. developed a system dynamics management model of a water resources–socioeconomic–environmental (WSE) system [14]. Jia et al. constructed a system dynamics model from the perspective of socio-hydrology to predict and understand the coevolution trajectories and dynamics of the WPE coupling system [15].

The production function model is widely used in calculating economic benefits, technological progress, input-output and economic forecasting. It is also used for the study of urban water supply and demand. The most widely used production function is the Cobb-Douglas (C-D) production function. Li used the C-D function to represent the domestic water supply and studied the dynamic optimization of urban domestic water input [16]. A Cobb–Douglas stochastic cost frontier model including technical effects is estimated to compare cost efficiencies of Brazilian public and private companies of water supply [17]. Li et al. used the urban water demand function and the C-D function to design an economic control model of the urban integrated water system with multiple input and multiple output non-linear systems [18]. Xie et al. developed the model for measuring the relationships among water demand, population, and GDP based on the C-D production function [19]. Li et al. combined the C-D function with the system dynamics model to manage the supply and demand of urban industrial water [20]. Wu et al. calculated the value of water resources of different water use sectors through C-D function and energy theory and accurately estimated the benefits of water resources [21]. As an essential function, however, the Variable Elasticity of Substitution (VES) production function has not been widely applied in water management. The VES production function improves the deficiency of the constant substitution elasticity coefficient between the input factors of the C-D production function. Therefore, it is more in line with the actual economic situation [22].

From the literature of recent years, firstly, it can be found that the total water supply is normally calculated by adding the amounts of surface water, groundwater, and reclaimed water; thus, it cannot clearly identify which part is used for domestic water. Secondly, there is little literature that uses the VES production function to measure domestic water supply. However, through the use of the VES function, we can adjust domestic water supply by changing investment and labor input. Thirdly, the water industry is a long-term asset industry, but there is little research on the application of the accelerated depreciation method in this industry in China. Compared with the existing research, this paper will discuss the applicability of the sum of years digits method to the water industry. Moreover, much of the existing literature considers the population, economic, and natural factors when developing the domestic water demand subsystem, but few studies consider the influence factor of water-saving consciousness. Therefore, this paper will investigate the impact of water-saving consciousness on the water industry.

The contributions of this paper are as follows. Firstly, it investigates the changes in urban water demand under the combined effect of population, income, and water-saving consciousness. Secondly, the VES production function is used to analyze water supply production, and the investment and labor input are dynamically adjusted to meet water demand. Thirdly, the straight-line depreciation method is widely used to depreciate fixed assets in Chinese water supply projects, but the accelerated depreciation method is rarely applied. Therefore, this paper also considers the application of the sum of years digits method, one of the accelerated depreciation methods of fixed assets, and compares the two methods in simulation.

2. Study Area and Data

2.1. Study Area

Jiangsu Province, located in the eastern coastal area of China, is a significant economic province in China. Pollution-induced water shortage in southern Jiangsu and water resource shortage in northern Jiangsu coexist. The rainfall displays an uneven characteristic in time and space, and it is a province with water shortages. In addition, water in Jiangsu

Province has problems such as excessive exploitation of groundwater, water pollution, aging projects, and insufficient investment in water resource projects.

2.2. Data and Sources

The data on the domestic water consumption population and water input is collected from Jiangsu Statistical Yearbook in 2005–2019 and the amount of money calculated according to the fixed price in 2005. Jiangsu Statistical Yearbook only gives fixed capital and labor force of the water production and supply industry. It does not provide the corresponding value of the domestic water. In this paper, the domestic water's fixed capital and labor force are set according to the ratio of domestic water to total water consumption. The data are shown in Table 1.

Table 1. Population, income, fixed capital, and labor force of Jiangsu Province.

Time (Year)	Resident Population (10^4People)	Per Capita Disposable Income (Yuan)	Fixed Capital (10^8Yuan)	Labor Force (10^4People)
2005	7588.24	8712.20	10.8363	0.1350
2006	7655.66	9790.51	13.5876	0.1373
2007	7723.13	10,922.30	14.7201	0.1450
2008	7762.48	11,851.52	16.8281	0.1507
2009	7810.27	13,171.82	20.5288	0.1688
2010	7869.34	14,780.98	23.1173	0.1816
2011	7898.80	16,383.77	24.8845	0.1602
2012	7919.98	18,147.30	27.0982	0.1741
2013	7939.49	19,668.84	35.3195	0.2071
2014	7960.06	21,161.48	39.9850	0.2197
2015	7976.30	22,744.75	44.6090	0.2368
2016	7998.60	24,225.92	43.0127	0.2482
2017	8029.30	26,071.11	47.6872	0.2633
2018	8050.70	27,759.55	49.2170	0.2683
2019	8070.00	29,346.03	62.3315	0.2839

3. Model

3.1. Water Demand Subsystem

Urban water demand is affected by many factors, such as economic development level, policy adjustments, water prices, water-saving technologies and consciousness. The level of economic growth can be expressed as the per capita disposable income of residents. Cui et al. believed that domestic water consumption is positively correlated with income but has a complicated relationship with water prices [23]. There are some differences in water prices among cities in Jiangsu Province, but water prices have not been adjusted frequently over the years. Therefore, the influence of water prices is not considered in this paper. The urban and rural residents' water consumption population, the per capita disposable income of residents, and the water-saving consciousness are considered factors that affect the domestic water demand in Jiangsu Province.

3.1.1. Gompertz Population Function

The growth process of the population often conforms to the growth curve model, such as the Gompertz growth curve [24]. This paper also uses the Gompertz curve to predict the population of Jiangsu Province:

$$M(t) = \omega \exp(-be^{-kt}) \tag{1}$$

where t is time; $M(t)$ is the water population; ω is the upper limit of water consumption population; b and k are undetermined coefficients. The Public Comment Announcement on Land and Space Planning of Jiangsu Province (2021–2035) sets the goal that the permanent resident population of Jiangsu Province will reach 90 million in 2035. Therefore, set $\omega = 9000 \times 10^4$ people.

According to Table 1, the Gompertz growth curve model of the water population in Jiangsu Province is:

$$M(t) = 9000 \exp(-0.1624e^{-0.03t}) \tag{2}$$

The coefficient of determination of Equation (2) is $R^2 = 0.967$.

3.1.2. Per Capita Disposable Income Function

The per capita disposable income of residents in Table 1, $R(t)$, can be expressed as the linear function of t:

$$R(t) = 7759.433 + 1508.063t \tag{3}$$

The coefficient of determination of Equation (3) is $R^2 = 0.996$.

3.1.3. Annual Growth Rate Function of Domestic Water Demand

The annual growth rate function of domestic water demand, $\varphi_D(t)$, can be expressed as:

$$\varphi_D(t) = (d \cdot \varphi_{POP}(t) + f \cdot \varphi_{PCDI}(t))^{\phi(t)} \tag{4}$$

where $\varphi_{POP}(t)$ is the growth rate of the water population; $\varphi_{PCDI}(t)$ is the growth rate of per capita disposable income; $\phi(t)$ is the water-saving consciousness; d and f are undetermined coefficients.

3.1.4. Water-Saving Consciousness Function

Water-saving consciousness can be expressed as [25]:

$$\frac{d\phi}{dt} = \theta\left(\frac{n-\phi}{n}\right) \tag{5}$$

where θ reflects the changing rate of water-saving consciousness; n is the upper limit of water-saving consciousness. $\left(\frac{n-\phi}{n}\right)$ is a value between 0 and 1. Equation (5) means that the change rate is lower when the water-saving consciousness is closer to its upper limit.

Solving Equation (5), we obtain:

$$\phi(t) = n - ze^{-\frac{\theta}{n}t} \tag{6}$$

where z is the undetermined coefficient. The determination steps of each coefficient are as follows: (1) Determine n. Set the range of n to $[0, 1]$, first let $n = 1$. (2) Determine z. $\phi(0) = n - z$ when $t = 0$, let the range of z be $[0, n]$, and different values of z will give different values of ϕ. (3) Substitute different values of ϕ, n, z and θ into the water demand prediction model, and select the z value that has the minimum fitting error with the historical water demand. If a satisfactory solution cannot be obtained when $n = 1$, the value of n should be adjusted and recalculated according to the above steps until the fitting result is satisfactory [25].

3.1.5. Water Demand Function

The water demand in Jiangsu Province, $D(t)$, can be expressed as:

$$D(t) = D(t - \Delta t) + \Delta t \cdot \varphi_D(t - \Delta t) \tag{7}$$

After adjusting, we obtain $d = 0.335$, $f = 0.184$, $n = 1$, $z = 0.52$ and $\theta = 0.03$.

3.2. Depreciation Method

3.2.1. Straight-Line Depreciation Method

The straight-line depreciation method is a simple and commonly used fixed asset depreciation method. According to this method, there is:

$$d = \delta_1 K_0 = ((1 - C/K_0)/T)K_0 \tag{8}$$

where d is the annual depreciation; K_0 is the initial value of fixed assets; C is the residual value; T is the depreciation period; $\delta_1 = (1 - C/K_0)/T$ is the annual depreciation rate. Set $T = 30$ and $C/K_0 = 0.04$, then $\delta_1 = 0.032$.

3.2.2. Sum of Years Digits Method

The sum of years digits method is a method of accelerated depreciation of fixed assets [20]. According to this method, we have

$$d_t = \delta_2(t)K_0 = (1 - \frac{C}{K_0})\frac{2(T-(t-1))}{T(T+1)}K_0 \tag{9}$$

where d_t is the depreciation at t, and $\delta_2(t) = (1 - \frac{C}{K_0})\frac{2(T-(t-1))}{T(T+1)}$ is the corresponding annual depreciation rate. The base year 2005 is set to $t = 0$.

The following simulation will compare the impact of the above two different fixed asset depreciation methods on Jiangsu's water supply and demand.

3.3. Water Supply Subsystem

The VES production function is used to describe the domestic water supply in Jiangsu. The general form of the function is:

$$S(t) = AK(t)^{\frac{m}{1+c}}(L(t) + (\frac{b}{1+c})K(t))^{\frac{cm}{1+c}} \tag{10}$$

where $S(t)$ is the domestic water supply; A is the comprehensive technical level; $K(t)$ is the fixed assets of the domestic water supply industry, that is, the capital stock; $L(t)$ is the annual average number of employees of the domestic water supply industry, that is, the labor stock; b, c and m are undetermined coefficients.

The logarithmic form of Equation (10) is:

$$\ln S(t) = \ln A + \frac{m}{1+c}\ln K(t) + \frac{cm}{1+c}\ln L(t) + \frac{cmb}{(1+c)^2}\frac{K(t)}{L(t)} \tag{11}$$

$\dot{K}(t)$ can be expressed as:

$$\dot{K}(t) = \mu I(t) - \delta K(t), K(0) = K_0 \tag{12}$$

where $I(t)$ is the investment of the urban domestic water supply industry; μ is the capital formation rate, δ is the capital depreciation rate, $0 < \mu, \delta < 1$; K_0 is the initial value of the investment.

$\dot{L}(t)$ can be expressed as:

$$\dot{L}(t) = \sigma P(t) - \gamma L(t), L(0) = L_0 \tag{13}$$

where $P(t)$ is the labor input of the urban domestic water supply industry; σ is the labor force formation rate; γ is the labor force abolition rate; $0 < \sigma, \gamma < 1$; L_0 is the initial value of labor force.

According to Table 1, ordinary least square regression (OLS) was performed on Equation (11) first, but multicollinearity was found among the independent variables. Therefore, by using ridge regression to solve the multicollinearity problem, we obtain:

$$\ln S(t) = 3.5108 + 0.0446 \ln K(t) + 0.1078 \ln L(t) + 0.0005 \frac{K(t)}{L(t)} \tag{14}$$

The coefficient of determination of Equation (14) is $R^2 = 0.95$.

Therefore, according to Equation (14), the production function of domestic water in Jiangsu is:

$$S(t) = 33.4750 K(t)^{0.0446} (L(t) + 0.0046 K(t))^{0.1078} \tag{15}$$

The other parameters are set as follows: $\mu = 0.85$, $\sigma = 0.98$, $\gamma = 0.02$ [18,20].

From Equation (12), we obtain:

$$I(t) = \frac{\dot{K}(t) + \delta K(t)}{\mu} \tag{16}$$

Due to $\dot{K}(t) \approx \frac{K(t+1)-K(t)}{(t+1)-t}$, Equation (16) can be approximately described as:

$$I(t) = \frac{K(t+1) + (\delta - 1)K(t)}{\mu} \tag{17}$$

$K(t)$ in Table 2 can be expressed as a quadratic function of t:

$$K(t) = 10.660 + 2.007t + 0.102t^2 \tag{18}$$

The coefficient of determination of Equation (18) is $R^2 = 0.977$.

Substituting Equation (18) into Equation (17), the investment $I_1(t)$ at $\delta = \delta_1(t)$ with the straight-line depreciation method can be obtained as:

$$I_1(t) = 2.882 + 0.316t + 0.004t^2 \tag{19}$$

The coefficient of determination of Equation (19) is $R^2 = 1$.

Similarly, the investment $I_2(t)$ at $\delta = \delta_2(t)$ with the sum of years digits method can be calculated as:

$$I_2(t) = 3.329 + 0.36t \tag{20}$$

The coefficient of determination of Equation (20) is $R^2 = 0.999$.

Similar to the process of solving $I(t)$, $P(t)$ can be obtained as:

$$P(t) = \frac{L(t+1) + (\gamma - 1)L(t)}{\sigma} \tag{21}$$

For $L(t)$ in Table 2, it can be expressed as a linear function of t:

$$L(t) = 0.120 + 0.011t \tag{22}$$

The coefficient of determination of Equation (22) is $R^2 = 0.95$.

Substituting Equation (22) into Equation (21), $P(t)$ can be calculated as:

$$P(t) = 0.014e^{0.015t} \tag{23}$$

The coefficient of determination of Equation (23) is $R^2 = 0.999$.

Table 2. The results of the historical test.

Parametric Variable		Year				
		2015	**2016**	**2017**	**2018**	**2019**
Population	Actual value/10^4 people	7976.30	7998.60	8029.30	8050.70	8070.00
	Simulation value/10^4 people	7979.57	8008	8035.68	8062.63	8088.88
	Relative error/%	0.04	0.12	0.08	0.15	0.23
Per Capita Disposable Income	Actual value/Yuan	22,744.75	24,225.92	26,071.11	27,759.55	29,346.03
	Simulation value/Yuan	22,840.10	24,348.10	25,856.20	27,364.30	28,872.30
	Relative error/%	0.42	0.50	−0.82	−1.42	−1.61
Water demand	Actual value/$10^8 m^3$	36.60	37.50	38.70	39.20	40.60
	Simulation value/$10^8 m^3$	38.92	39.59	40.24	40.86	41.47
	Relative error/%	6.35	5.58	3.97	4.24	2.13
Water supply by straight-line method	Actual value/$10^8 m^3$	36.60	37.50	38.70	39.20	40.60
	Simulation value/$10^8 m^3$	36.11	36.53	36.94	37.34	37.72
	Relative error/%	−1.34	−2.59	−4.55	−4.76	−7.09
Water supply by the sum of years digits method	Actual value/$10^8 m^3$	36.60	37.50	38.70	39.20	40.60
	Simulation value/$10^8 m^3$	36.10	36.51	36.92	37.32	37.71
	Relative error/%	−1.37	−2.63	−4.60	−4.80	−7.12

4. Dynamic Supply and Demand Model of Domestic Water System

Integrated domestic water demand subsystem and water supply subsystem, the domestic water system with different depreciation methods in Jiangsu province are obtained. The models are shown in Figure 1.

Figure 1 shows two flow charts of the population-economy-water resources system based on the causal relationship of each subsystem. Figure 1a is the domestic water supply and demand model in Jiangsu Province with the straight-line depreciation method. Figure 1b is the domestic water supply and demand model with the sum of years digits method.

4.1. Historical Test

The year of 2005 is taken as the base year, the time step is one year, and the simulation period is 2005–2034. The key variables of the model from 2015 to 2019 are selected for historical testing. The verification process compares historical data with modeled data. The variable of the resident population is the key index of the population subsystem. Per capita disposable income is the key index of the economic subsystem. Water demand and water supply are the key indexes used to evaluate the domestic water balance in Jiangsu Province. As the key indexes of the system, these variables are selected for verification. Table 2 shows the results of the historical test.

In the historical test, if the relative error of the key variables is not more than 10%, the accuracy of the model is usually considered acceptable and the model can be used for predictive analysis [26]. The absolute value of relative errors of all the key variable are less than 10% in Table 2, so the model is supposed to pass the historical test and can be used to forecast domestic water supply and demand in Jiangsu Province.

Table 2 shows that the relative errors between the predicted value and the actual value of water supply with the straight-line depreciation method and the sum of years digits method both are small. Therefore, these two methods are applicable to the depreciation of fixed assets in the water industry.

In addition to the historical test, sensitivity analysis can also be used to test the effectiveness of the system dynamics model. The sensitivity analysis of the model will be carried out below.

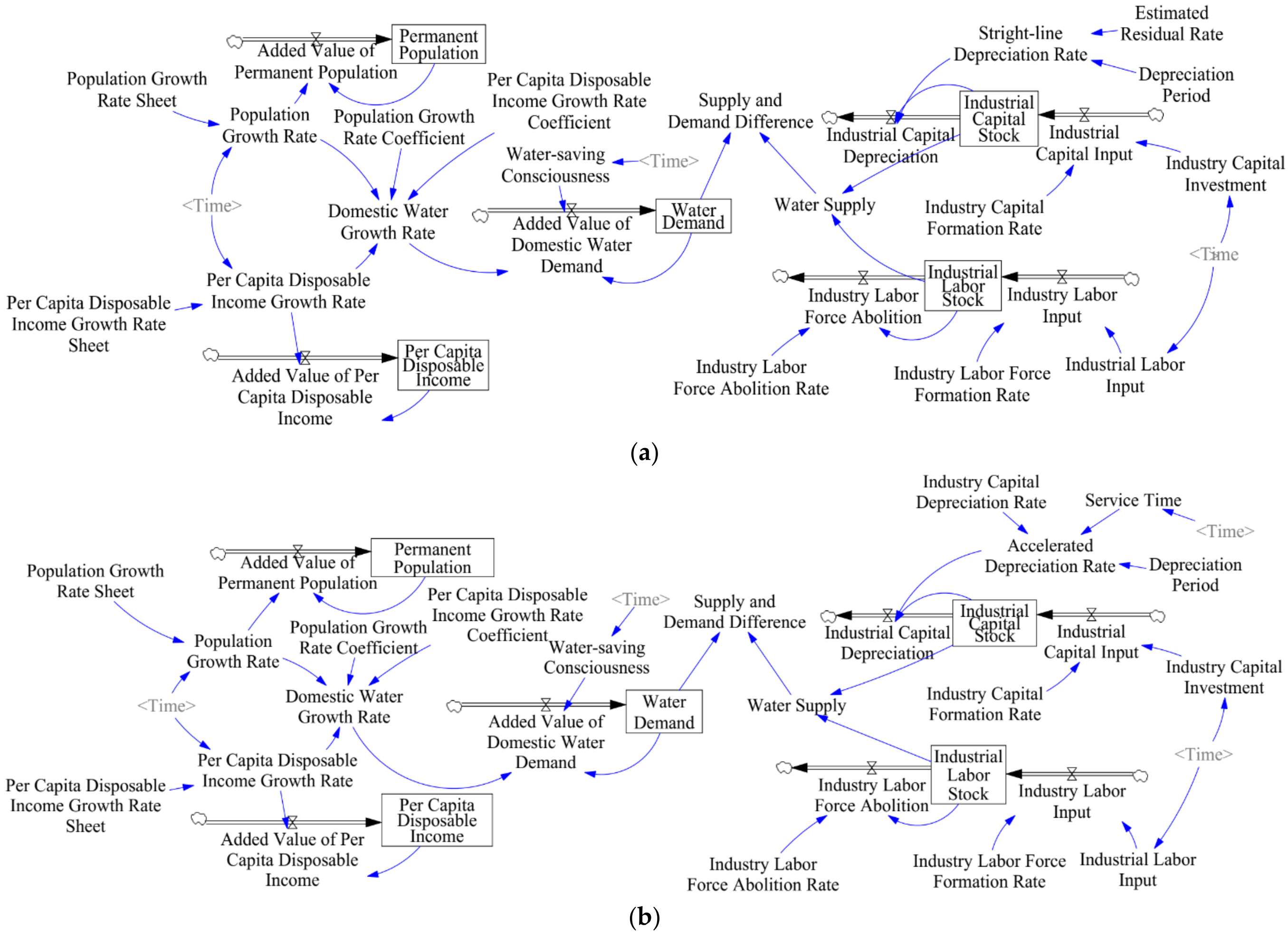

Figure 1. Domestic water supply and demand model. (**a**) Straight-line depreciation method. (**b**) Sum of years digits method.

4.2. Sensitivity Analysis

Sensitivity analysis is used to study the impact of parameter changes on the behavior of the water supply and demand system. Changing one parameter and the other parameters remaining unchanged at a time, if there is a significant difference in model behavior, the parameters need to be accurately estimated or the model structure needs to be readjusted. The function of sensitivity degree S_Q is [27]:

$$S_Q = \left| \frac{\Delta Q(t)}{Q(t)} / \frac{\Delta X(t)}{X(t)} \right| \tag{24}$$

where $\frac{\Delta X(t)}{X(t)}$ is the relative change rate of parameter X at time t; $\frac{\Delta Q(t)}{Q(t)}$ is the relative change rate of system state Q at time t.

For n state variables $(Q_1, Q_2, \ldots, Q_n)$, the general sensitivity degree $|S|$ of a parameter at time t can be defined as follows [27]:

$$|S| = \frac{1}{n} \sum_{i=1}^{n} S_{Q_i} \tag{25}$$

where S_{Q_i} is sensitivity degree of state Q_i.

Four parameters including industry capital formation rate, labor force formation rate, labor force abolition rate, and capital residual value rate are selected for sensitivity analysis.

The values of the four parameters are reduced by 10% to investigate their impact on the total water supply.

According to Equations (24) and (25), the general sensitivity of the four parameters in the two depreciation methods is calculated, and the result is shown in Table 3.

Table 3. Sensitivity analysis of parameters.

Parameters	$\vert S \vert$	
	Straight-Line Depreciation Method	**Sum of Years Digits Method**
Capital formation rate	0.0719	0.0745
Labor force formation rate	0.0282	0.0279
Labor force abolition rate	0.0080	0.0079
Capital residual value rate	0.0007	0.0103

The classification of sensitivity is shown in Table 4 [28].

Table 4. Classification of sensitivity.

$\vert S \vert$	**Sensitivity Classification**
$[0, 0.05)$	Insensitive parameter
$[0.05, 0.2)$	Medium parameter
$[0.2, 1)$	Sensitive parameter
$[1, +\infty)$	High sensitivity parameter

Table 3 shows that the general sensitivity of capital formation rate is less than 0.2, and the labor force formation rate, labor force abolition rate and capital residual value rate are all less than 0.05, indicating the system responds to the parameter changes with a quite low sensitivity degree. This means that the model can effectively predict the system's behaviors.

4.3. Simulation

After the model has passed the historical test, it will be further used to predict the supply and demand of domestic water in the next 15 years. Let $S_1(t)$ and $S_2(t)$ represent the water supply with the straight-line depreciation method and the sum of years digits method, respectively; $I_1(t)$ and $I_2(t)$ represent the investment of the two methods, respectively. The water supply and demand and changes in input factors under the current situation in 2005–2034 are shown in Figure 2.

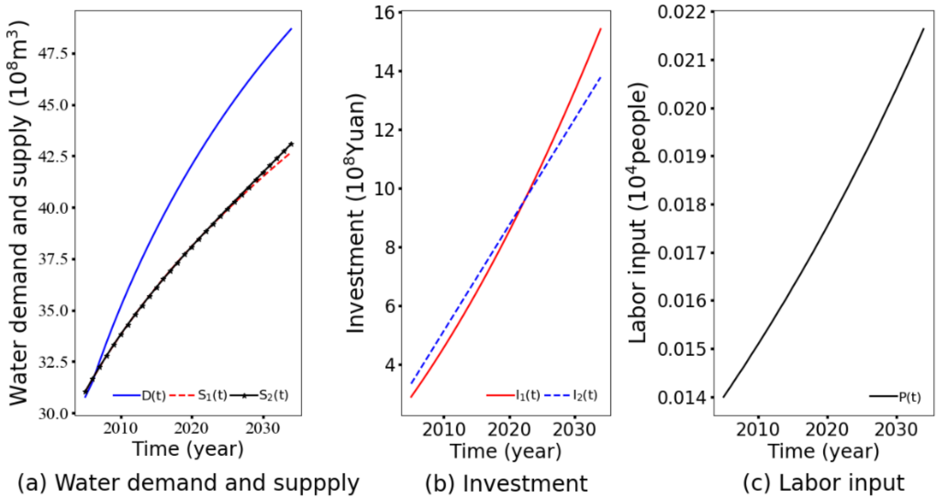

Figure 2. Current water supply and demand and changes in input factors. (**a**) Water demand and supply; (**b**) Investment; (**c**) Labor input.

Figure 2 shows that the growth rates of domestic water supply with the two depreciation methods are less than that of water demand. Supply and demand for both depreciation methods were approximately in balance in 2005–2006, but supply has become increasingly inadequate to meet demand since 2007. By 2034, the water supply with the straight-line method and the sum of years digits method will be 5.999×10^8 m^3 and 5.5885×10^8 m^3 less than the water demand, respectively. The investment of the sum of years digits method is more than that of the straight-line depreciation method in 2005–2022, and it is less than straight-line depreciation in 2023–2034. Labor input is increasing in 2005–2034. Regarding the current input level of production factors, increasing investment or labor force is necessary to balance water supply and demand.

Therefore, the investment and the labor input are adjusted as below. The labor input of the straight-line depreciation method is consistent with that of the sum of years digits method. $P^*(t) = P(t)$ in 2005 and $P^*(t) = 1.2P(t)$ in 2006–2034. Table 5 shows the adjusted investment of the two depreciation methods in 2005–2034.

Table 5. Adjusted investment of the two depreciation methods.

Time	$I_1^*(t)$	$I_2^*(t)$
2005	$I_1(t)$	$I_2(t)$
2006–2009	$2I_1(t)$	$1.9I_2(t)$
2010–2015	$2.55I_1(t)$	$2.5I_2(t)$
2016–2019	$3I_1(t)$	$3I_2(t)$
2020–2027	$3I_1(t)$	$2.8I_2(t)$
2028–2034	$3.1I_1(t)$	$2.7I_2(t)$

The changes in water supply and improved input factors with the two depreciation methods are shown in Figure 3.

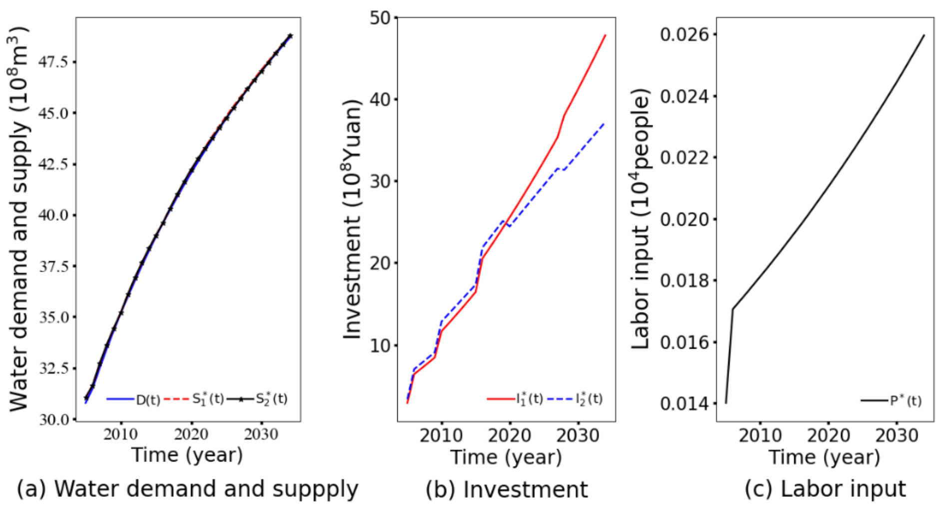

Figure 3. Current improved water supply, water demand, and two input factors.

Figure 3 shows that after the change of input variables, except that the water supply-demand balance with the two methods slightly exceeded 0.25×10^8 m^3 in 2005, the balance in other years was controlled within this value. Moreover, the balance was less than 0.1×10^8 m^3 in 2015–2034. In 2034, the supply is greater than the demand by 0.0647×10^8 m^3 with the straight-line depreciation method and 0.0517×10^8 m^3 with the sum of years digits method. Compared with the current situation, the improved inputs have dramatically closed the difference between supply and demand. The adjusted investment with the two depreciation methods has increased significantly in the current situation. The investment of the straight-line depreciation method has been shown in a stepwise way. The investment of the sum of years digits method shows a stepwise increase before 2028. Compared with the investment in 2027, it has experienced a certain degree of decline in 2028 and then continues to increase in 2029–2034. The investment of the sum of years digits method is more than that of the straight-line depreciation method from 2005 to 2019, and less than that from 2020 to 2034. The investment difference has increased from 2020 and reaches its maximum in 2034, which is 11.9716×10^8 Yuan. The labor force has also increased significantly compared with the current situation.

4.4. Scenario Simulation

Xu predicted the population change trend of Jiangsu Province after the implementation of the universal two-child policy [29]. The prediction shows that the population will reach its peak around 2022. The total population will decline year by year from 2023. By 2030, the total population will be roughly equivalent to that when the universal two-child policy was implemented in 2016 [29]. However, the country introduced the three-child policy in 2021, so the forecast may be affected to some extent.

According to experience, in the year after the birth policy adjustment, the number of births is relatively small and relatively large in the second and third years, then declines in the fourth and fifth years. It is expected that the three-child policy will have a negligible impact on the population change in Jiangsu Province but will not make drastic changes to the long-term trend of population. Therefore, we consider the following scenarios. The total population of Jiangsu Province will decline year by year starting from 2026. It is estimated that the total population in 2034 will be roughly the same as that in 2016, when the universal two-child policy was implemented.

Moreover, we also examine two scenarios of per capita disposable income growth rate $\varphi_{PCDI}(t)$ after 2020, which are 0.7 times and 1.3 times of the current growth rate, representing low-speed and high-speed income growth, respectively.

Furthermore, two scenarios of the growth rate of water-saving consciousness, θ, are considered, which are 0.01 and 0.05, representing that the growth rate of water-saving consciousness of residents slowed down and accelerated, respectively. The water demand of residents is positively correlated with the income level. With the increase in income, the sensitivity to water expenditure will generally be reduced, resulting in the rise in water consumption and the weakening of the growth of water-saving consciousness, $\theta = 0.01$. However, there may also be another situation. Due to the Chinese Government's comprehensive promotion of constructing a water-saving society, the residents' water-saving consciousness grows faster and $\theta = 0.05$ occurs.

Therefore, four scenarios are obtained by combining the above values of $\varphi_{PCDI}(t)$ and θ, as shown in Table 6.

Table 6. The setting of different water demand scenarios.

Scenario	Parameter Setting in 2020–2034
Scenario (a)	$1.3\varphi_{PCDI}(t), \theta = 0.01$
Scenario (b)	$1.3\varphi_{PCDI}(t), \theta = 0.05$
Scenario (c)	$0.7\varphi_{PCDI}(t), \theta = 0.01$
Scenario (d)	$0.7\varphi_{PCDI}(t), \theta = 0.05$

The water demand in Scenario (i) is represented by $D_i(t), (i = a, b, c, d)$, and the domestic water supply and demand in the four scenarios are shown in Figure 4.

Figure 4 shows that the water demand under four scenarios and the current water demand are ranked as follows: $D_a(t) > D_b(t) > D(t) > D_c(t) > D_d(t)$. If the production is carried out according to $I_1^*(t)$ and $I_2^*(t)$, $S_1^*(t)$ and $S_2^*(t)$ cannot meet water demand well except that Scenario (b) is relatively in balance, but there are still some years in short supply. Supply exceeds demand in Scenario (a) but supply is less than demand in Scenario (c) and Scenario (d). Therefore, it is necessary to adjust investment and labor input further to make the water supply better meet the water demand.

For Scenario (i), let $P_i^*(t)$ represent labor input, $I_{i1}^*(t)$ and $I_{i2}^*(t)$ represent the investment of the straight-line depreciation method and the sum of years digits method, respectively. Set the value of labor input after debugging as follows. $P_i^*(t) = 1.2P(t)$ in

2020–2025, $P_i^*(t) = P(t)$ in 2026–2034. Since the water consumption population of Jiangsu province is expected to show a negative growth from 2026, it may not be easy to increase labor input, $P_i^*(t)$ will not be adjusted after 2026, and it is more appropriate to adjust the investment.

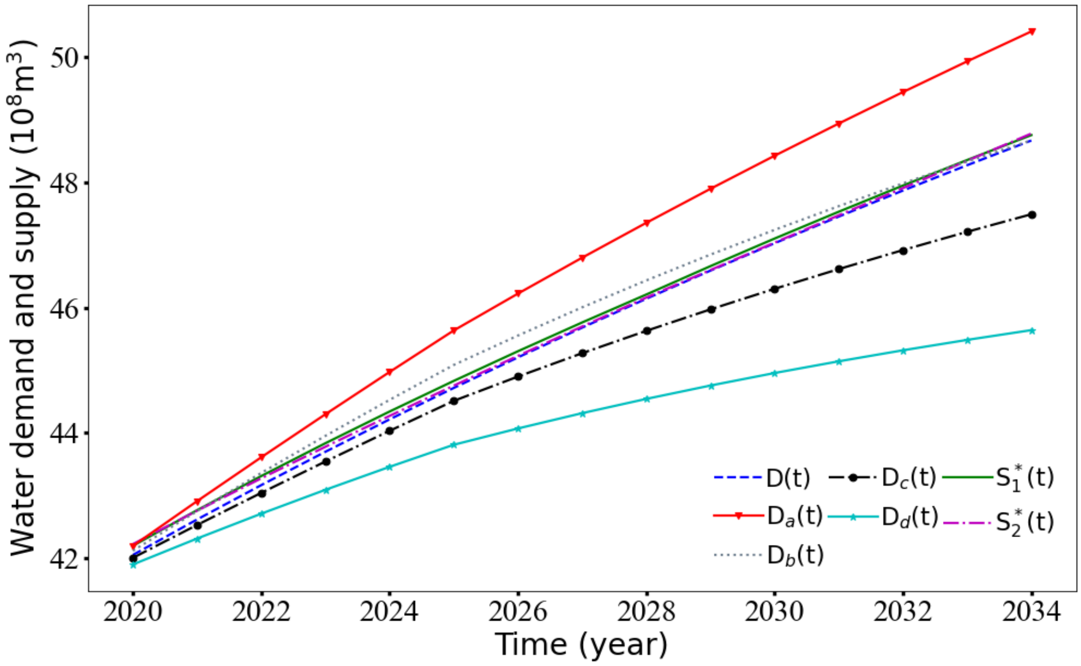

Figure 4. Water supply and demand under four scenarios.

The investment is consistent with the current improvement model in 2005–2019, and the adjusted investment in 2020–2034 is set as in Table 7.

Table 7. Adjusted investment in different scenarios.

Time	Scenario (*a*)		Scenario (*b*)		Scenario (*c*)		Scenario (*d*)	
	$I_{a1}^*(t)$	$I_{a2}^*(t)$	$I_{b1}^*(t)$	$I_{b2}^*(t)$	$I_{c1}^*(t)$	$I_{c2}^*(t)$	$I_{d1}^*(t)$	$I_{d2}^*(t)$
2020–2027	$4.1I_1(t)$	$3.9I_2(t)$	$3.32I_1(t)$	$3.2I_2(t)$	$2.65I_1(t)$	$2.55I_2(t)$	$1.9I_1(t)$	$1.8I_2(t)$
2028–2034	$4.4I_1(t)$	$3.9I_2(t)$	$2.83I_1(t)$	$2.4I_2(t)$	$2.2I_1(t)$	$1.7I_2(t)$	$1.2I_1(t)$	$0.8I_2(t)$

Let $S_{i1}^*(t)$ and $S_{i2}^*(t)$ represent the water supply of Scenario (i) with the straight-line depreciation method and the sum of years digits method, respectively. The changes in water demand, supply and adjusted investment under the four scenarios are shown in Figure 5.

Figure 5a shows that since the water supply is far less than the water demand after 2020 in Scenario (a), $I_{a1}^*(t)$ and $I_{a2}^*(t)$ have increased compared with $I_1^*(t)$ and $I_2^*(t)$, respectively, since 2020. Furthermore, $I_{a1}^*(t)$ has been greater than $I_{a2}^*(t)$ since 2020, and the difference will be 14.1049×10^8 Yuan in 2034. The water supply with the straight-line depreciation method and the sum of years digits method is 0.0657×10^8 m^3 and 0.0890×10^8 m^3 greater than the water demand, respectively, in 2034.

Figure 5b shows that as the water supply is slightly less than the water demand in 2022–2032 in Scenario (b), $I_{b1}^*(t)$ and $I_{b2}^*(t)$ will be greater than $I_1^*(t)$ and $I_2^*(t)$ in 2020–2027, but less than $I_1^*(t)$ and $I_2^*(t)$ in 2028–2034, respectively. $I_{b1}^*(t)$ is 10.5647×10^8 Yuan greater than $I_{b2}^*(t)$ in 2034. The water supply with the straight-line depreciation method is 0.0103×10^8 m^3 greater than the demand in 2034, while the value of the sum of years digits method is 0.1161×10^8 m^3.

Figure 5c shows that since the water supply is greater than the water demand after 2020 in Scenario (c), $I_{c1}^*(t)$ and $I_{c2}^*(t)$ have both experienced a decrease in 2020 and 2028 and then increased. Both $I_{c1}^*(t)$ and $I_{c2}^*(t)$ are less than the corresponding $I_1^*(t)$ and $I_2^*(t)$.

$I_{c1}^*(t)$ is 10.4947×10^8 Yuan greater than $I_{c2}^*(t)$ in 2034. The water supply with the straight-line method and the sum of years digits method is, respectively, 0.0619×10^8 m^3 and 0.063×10^8 m^3 greater than the demand in 2034.

Figure 5d shows that as $D_d(t)$ is less than $D_c(t)$ in 2020–2034 in Scenario (d), $I_{d1}^*(t)$ and $I_{d2}^*(t)$ have experienced similar but more downward adjustments than $I_{c1}^*(t)$ and $I_{c2}^*(t)$ in Scenario (c) in 2020 and 2028, respectively. $I_{d1}^*(t)$ is 7.4768×10^8 Yuan greater than $I_{d2}^*(t)$ in 2034. The water supply with the straight-line method is 0.0625×10^8 m^3 higher than the demand in 2034, and the value of the sum of years digits method is 0.1038×10^8 m^3.

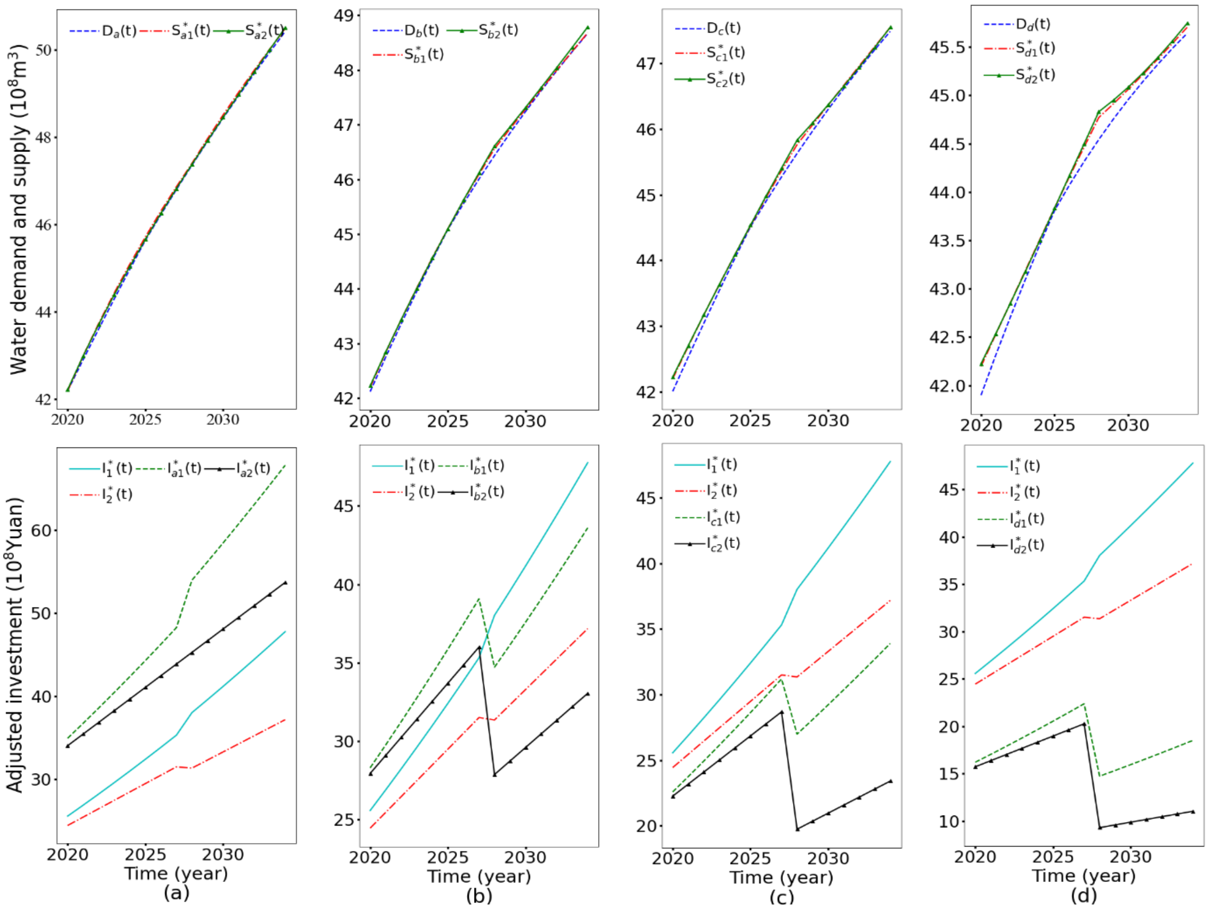

Figure 5. Water demand, supply and adjusted investment in the four scenarios. (**a**–**d**) show the water demand, supply and adjusted investment of the two methods in Scenarios (**a**–**d**), respectively.

The investment of the two methods has been increasing in a stepwise way in Scenario (a). With the increase in demand, more investment is needed to make the water supply meet the water demand. However, the investment is not simply increased in Scenarios (b), (c) and (d). A timely decision to reduce investment should be made at a time when the accumulation of capital will lead to an oversupply of water by maintaining previous investment levels.

The investment of the sum of years digits method is more than that of the straight-line method in 2005–2019 and less than the latter in 2020–2034 in the four scenarios. The reason is that the depreciation of the sum of years digits method is more than that of the straight-line method in the early period. Therefore, more investment is required to make up for the depreciation than the straight-line method to meet production needs in this period. On the other hand, the depreciation of the sum of years digits method is less than that of the straight-line method in the later period, so the investment is less at this time. The sum of years digits method has the characteristics of larger depreciation in the early period and less depreciation in the later period, which is conducive to timely compensation for the larger loss of fixed assets in the early period.

The differences between the domestic water supply and demand of the two depreciation methods in all the four scenarios have been controlled within 0.35×10^8 m^3. This means that the supply meets the demand fairly well in all the scenarios.

It can provide some references for the regulation of urban domestic water consumption from the model simulation. For example, with the trend of decreasing population, the comparison between Scenario (a) and Scenario (b) shows that the increasing speed of water-saving consciousness can effectively reduce the domestic water demand. By 2034, the gap between supply and demand in Scenario (b) would be 1.7397×10^8 m^3 less than that in Scenario (a). Compared with Scenario (a) and Scenario (c), when the growth rate of water-saving consciousness remains unchanged, the greater the per capita disposable income, the greater the water demand will be. By 2034, $D_c(t)$ is 2.9206×10^8 m^3 less than $D_a(t)$. Scenario (d) requires the least water demand in the four scenarios, but this is at the expense of economic development. From the perspective of promoting social development, it is hoped that people's water-saving consciousness can be strengthened, and meanwhile, the economy can develop rapidly. Therefore, it can be considered that Scenario (b) is the best in the four scenarios. The change trend shows that while ensuring the development of economy, measures such as improving water-saving consciousness can promote the balance of water demand and supply in the long term.

5. Conclusions

The urban domestic water demand function is examined in this paper with the combined effect of the water consumption population, the per capita disposable income of the residents, and the water-saving consciousness. Then, the VES production function is used to represent the domestic water supply. Combined with system dynamics, the domestic water supply and demand management models of the straight-line depreciation method and the sum of years digits method for Jiangsu Province of China are developed, respectively. With the current investment and labor input, the simulated domestic water supply with the two depreciation methods has not met the water demand well since 2007. Therefore, investment and labor input are used as control variables to make the water supply meet the water demand better.

Assuming that the water consumption pattern includes low-speed and high-speed income growths, combined with the change of growth rate parameter of water-saving consciousness, four water demand scenarios are investigated in this paper. Through the change of investment and labor input, the corresponding water supply production plan is obtained. The simulation results show that the schemes in all scenarios achieve the goal of supply-demand balance quite well.

Compared with the straight-line depreciation method, the investment of the sum of years digits method is larger in 2005–2019 and less in 2020–2034 in the four scenarios. The results show that the sum of years digits method, as an accelerated depreciation method, can make more timely compensation for the loss in the early use stage of fixed assets than the straight-line depreciation method.

Scenario (b) is the best of the four scenarios. When ensuring the economic development, measures such as improving water-saving consciousness should be taken to advance the balance of water demand and supply.

Although the historical test and sensitivity analysis are used to test the model in this paper, the verification method of system dynamics models of water resources management has not always been discussed in detail. This may be a problem worthy of further study.

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