

Different Approaches in Mathematics Teacher Education: Participants, Tools, Contexts and Professional Learning

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1. Introduction

This Special Issue focuses on different approaches to teacher education in mathematics. Specifically, it focuses on participants and key actors (preservice and in-service teachers), different institutional contexts, and learning and development processes. Mathematical knowledge for teaching and the specific professional practices for teaching mathematics are prominent issues in this field of research and are relevant to our understanding of how to educate mathematics teacher in different contexts. Furthermore, issues regarding the assessment of learning and the development of teaching competence in relation to different institutional contexts across the world are of great importance. Other issues include the development of different theoretical approaches with specific tasks across mathematics teacher education programs, as well as different methodological approaches.

The ten papers in this Special Issue are from seven different countries (Portugal, Brazil, UK, Mexico, Spain, Colombia and Serbia), with three of them having been written by international teams (Mexico/Spain, Brazil/Colombia and Brazil/Portugal). Five of the papers focus on preservice mathematics teachers (primary or secondary education), three papers focus on in-service teachers, and two papers address the literature regarding the facets knowledge needed to teach mathematics. The papers in this Special Issue examine various dimensions of how preservice teachers learn core practices—such as noticing skills and their ability to analyze tasks and reflect on their teaching. Furthermore, the contributors explore the facets of mathematical knowledge relevant for teaching, with emphasis on theoretical development, diverse methodological approaches (including lesson studies in initial teacher education and Fading Programs for professional development), and the factors influencing how preservice teachers develop a professional identity within the mathematics teacher community. Overall, these papers represent a practice-based approach to teacher education, focusing on the specific competencies required to manage the unique demands of teaching mathematics. The papers provide an insight into to how a teacher should act, not just what a teacher should know (Blomeke & Kaiser, 2017). Furthermore, this topic is explored through particular mathematical topics addressed through the study of professional practices (core practices), including noticing students' mathematical thinking, analyzing tasks as components of the curricular noticing, planning or analyzing mathematics lessons, and reflecting on the teaching of mathematics itself. The professional practices that comprise mathematics teaching are a specific set of teaching actions essential for supporting student learning (how a teacher acts). This is also developed using the mathematical topics and processes considered in the majority of the papers in this Special Issue: classification in geometry; mathematical problem solving and the role of



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the metacognitive strategies within this; relational thinking in arithmetics; teaching natural numbers in primary education; functions, area, and plane surfaces; 3D-geometrical solids; and STEM-oriented approaches to mathematics. These papers recognize the intricate nature of developing specialized mathematical knowledge and the complexities of managing the professional practices specific to mathematics teaching (Ponte & Chapman, 2008). That is to say, they address the specific mathematical skills required to teach effectively. This challenge highlights the need for research into what mediates the transformation of theoretical knowledge into classroom practice.

Across the papers in this Special Issue, two overarching themes emerge: the different ways of addressing the intertwined relationships among knowing, doing, and being, and the features of pedagogical reasoning as a knowledge-based process. Collectively, these papers offer critical insights into these themes, contributing to a broader understanding of the complex processes involved in becoming a mathematics teacher.

2. Relationships Between Knowing, Doing, and Being (a Focus on How Teachers Act)

The intertwined relationships between knowing, doing, and being are illustrated by examining the specific mathematical knowledge needed for teaching and the role of context and methodology in bridging the gap between theory and practice. Furthermore, identifying the aspects that lead preservice teachers to teaching help to explain how their identities are formed. In this Special Issue, three papers provide insights into 'knowing', two explore the relationship between 'knowing and doing', and one addresses how teaching preferences influence the process of 'becoming' a teacher.

Three papers in this Special Issue provide insights into the different facets of knowledge needed to teach mathematics (knowledge that supports a teacher's actions). Juarez-Ruiz and colleagues identify connections among different facets of teacher knowledge from a Piagetian perspective. By utilizing the concept of schema structure, they examine the professional practice of analyzing lesson plans for teaching multiplication and division of whole numbers in primary education. Their research reveals significant variations in how mathematical knowledge and didactic knowledge are integrated by analyzing different lesson plans. The remaining two papers also address teacher-specific knowledge but through different methodological lenses. Inglada and colleagues explore how epistemic meta-didactic mathematical knowledge leads to epistemically suitable teaching, while Ferreira and colleagues differentiate between school and academic mathematics. Both papers conduct literature reviews across different databases (Web of Science and Scopus, and the CAPES Theses and Dissertations Catalog in Brasil) and consider different mathematical topics: the concept of functions in secondary education and the concept of the area of plane surfaces in primary education, respectively. Through different analytical approaches, both studies identify the characteristics of the specialized knowledge required to teach these topics, offering generalizable conceptual reflections. Inglada and colleagues highlight the knowledge needed to teach functions by drawing on the history of mathematics and mathematical curricula, considering specific mathematical practices (multiple approaches that use functions in different contexts) and professional practices (task analysis and reflections on teaching practice). On the other hand, Ferreira and colleagues argue that theoretical approaches to teaching and learning processes are essential for framing reflections on the nature of teacher knowledge. Specifically, they employ Douday's "interplay of setting" (Jeux of cadres) and the tool-object (outi-objet) perspective, as discussed by Perrine-Glorian, to analyze the professional practice of designing and analyzing specific tasks.

Regarding the contexts and methodological approaches used to support prospective and in-service teacher learning, two papers present two different approaches that highlight the nuanced relationship between knowing and doing. Mulholland and colleagues analyze the design of a Fading Program within the context of professional development for primary school teachers, while Duarte and colleagues examine the use of lesson studies in initial primary teacher education to strengthen the link between knowing and doing. Mulholland et al. explicate the relationship between teachers being aware of different metacognitive strategies for problem solving and enacting these strategies in their teaching and lesson planning. They argue that the 'Fading' design and the use of successful examples, supported by peer collaboration, allow teachers to adhere to the core program principles while adapting to contextual factors. On the other hand, Duarte and colleagues highlight how lesson studies articulate the relationship between theory and practice (between knowing and doing). Specifically, based on cohorts of students learning the topics of length and area, they focus on the skills of designing effective tasks and anticipating student's difficulties and strategies as essential components of planning, implementation, and reflection.

Finally, Colic and colleagues examine the differences between primary and secondary preservice mathematics teachers regarding the factors that shape their teacher identity and their process of becoming a teacher. Their findings highlight the significant role that prior experiences with mathematics play in framing how preservice teachers understand the subject matter they eventually teach. Furthermore, the results show that the educational level (primary versus secondary) influences the focus of their teaching—whether centered on children's learning or on mathematics as a formal science-. These findings suggest that variables linked to identity formation may affect how the relationship between knowing and doing is ultimately established.

3. Specific Pedagogical Reasoning as a Knowledge-Based Process

A mathematics teacher's pedagogical reasoning can be defined as the mental processes used to make-sense and navigate various teaching situations. As Loughran et al. (2016) argue, understanding pedagogical reasoning, how it develops, and the manner in which it influences practice is important. Making this clear for others, especially students of teaching, is a challenge that should not be eschewed in teacher education programs. Pedagogical reasoning serves as a mediatory process between knowledge and action, revealing the relevance of a teacher's skills to perceive, interpret, and make decisions based on the conceptual tools and professional materials provided by teacher education programs (Llinares & Chapman, 2020). Collectively, these skills and the influence of knowledge on the development of pedagogical reasoning define one aspect of teaching competence (Gegenfurtner & Stahnke, 2025).

Four studies in this Special Issue examine the development of pedagogical reasoning in both preservice and in-service teachers as a means of establishing a relationship between knowing and doing. In this case, cognitive skills act as mediators between knowledge and actions (or in the decisions made). These studies focus on key cognitive skills—specifically the ability to perceive relevant mathematical situations, interpret their meaning, and decide on appropriate actions—and explore how teacher education can effectively develop them. Particular attention is given to how preservice teachers use their knowledge to analyse representations of practice to notice and interpret relevant events in specific teaching situations. The studies characterize how prospective teacher's noticing of student's mathematical thinking across specific topics—such as classifications in geometry and relational thinking in arithmetic—and analyze tasks. One aspect of developing pedagogical reasoning is the coherence between skills when preservice teachers are analyzing students' thinking and different mathematical tasks. Coherence in reasoning implies a logical

consistency in how prospective teachers articulate their observations and ensure that each statement follows logically from the previous one.

Oliveira and colleagues examine the coherence between the noticing skills, illustrating a certain dependence on preservice teachers' knowledge. Their findings seem to support the idea that the knowledge needed to teach should be developed alongside the development of skills in analyzing students' thinking processes and choosing tasks related to their learning objectives. Furthermore, they propose that thematic coherence—maintaining a common mathematical/pedagogical thread across skills—can be considered a higher level of noticing. Similarly, when focusing on mathematical task sequences, Fernandez and colleagues argue that the coherence between task analysis and subsequent teaching decisions depends heavily on a teacher's knowledge—in the sense of how the teacher can enhance the learning opportunities offered by the initial tasks. Collectively, these studies support the hypothesis that knowledge and skills can be developed together and the level of coherence between skills identifies specific pedagogical reasoning as a knowledge-based process.

On the other hand, Moreno and colleagues examine how the theoretical knowledge provided in teacher education programs serves as a conceptual tool to support the development of preservice teachers' pedagogical reasoning. This paper analyzes features that bridge the gap between merely noticing an event and understanding its meaning. Specifically, they investigate how preservice teachers selectively attend to students' responses regarding relational thinking in arithmetic and provide meaningful explanations for those characteristics. The findings highlight different ways in which preservice teachers interpret the information provided—such as the aspects of relational thinking—and how these interpretations influence their pedagogical reasoning and the justification of their instructional decisions. In this context, the interplay between noticing students' mathematical thinking and curricular noticing (noticing of mathematical tasks) emerges as a key driver of pedagogical reasoning. Shifting the focus to in-service teachers, Falcó-Solsona and colleagues explore how teachers of non-mathematical subjects assess interdisciplinary learning situations. By examining the relationships between initial reflections and conceptual tools, the authors revisit existing theories to better facilitate the bridge between knowing and doing. This analysis of instructional tasks provides fresh insights into potential avenues for enhancing the pedagogical reasoning of in-service teachers across disciplines.

The studies reported here—through their design and their findings—seem to support the hypothesis that an integrated approach (combining knowledge, skills, and situational aspects) could facilitate the synergy between teacher knowledge and cognitive skills of noticing. These four studies use various representations of practice in the design of professional learning tasks, alongside the introduction of theoretical knowledge as a professional resource or conceptual tool. By effectively using these resources along with the cognitive skills of noticing, teacher education programs can create learning opportunities that utilize integrated approaches. Ultimately, this alignment of knowledge and skills within specific contexts serves as a powerful framework for developing professional teaching competence.

4. Final Remarks

The papers within this Special Issue generate new insights into how prospective and in-service teachers learn to reason and to make teaching decisions using the conceptual tools provided by teacher education programs (Gegenfurtner & Stahnke, 2025). The key professional practices examined specifically in these papers—which are central to a teaching competence—include the ability to identify and interpret significant events in mathematics

teaching situations, analyze tasks and lesson plans, and evaluate students' mathematical thinking. These findings shed new light on how prospective and in-service mathematics teachers learn to use the knowledge to notice and interpret relevant teaching and learning events, which is a key component of professional teaching and supports new developments and insights into teacher education (Fernández et al., 2026) and professional development.

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