

Article

Analysis of Preschool Teachers' Dialogue with Children During Combinatorial Activities

Tatjana Hodnik *  and Adrijana Mastnak 

Faculty of Education, University of Ljubljana, Kardeljeva Ploščad 16, 1000 Ljubljana, Slovenia;
adrijana.mastnak@pef.uni-lj.si

* Correspondence: tatjana.hodnik@pef.uni-lj.si

Abstract

This article deals with the quality of dialogue between preschool teachers and children during mathematical activities. Thoughtful and attentive dialogue captures children's attention and allows them to reason, express thoughts and disagreements, and ask questions. The aim of the study is to determine how well preschool teachers are able to utilize the potential of a combinatorial problem to ask questions and promote children's mathematical thinking. Video recordings of eleven preschool teachers formed the basis of the study. Qualitative and quantitative analysis of the transcribed recordings provided an insight into the quality of the dialogues. The results of the study show that the preschool teachers mainly focused on combinatorics during the dialogue, although they also frequently asked the children questions about numbers, as well as questions from a non-mathematical context. In the analyzed dialogues, there was a lack of focusing questions, problem-solving questions, and questions requiring the children to justify their explanations. Unlike other studies that mainly investigate children's thinking when solving selected combinatorial problems, the present study highlights the role of the teacher as the main actor in the process of children's mathematics learning.

Keywords: children; combinatorics; dialogue; mathematical thinking; preschool teacher



Academic Editors: Penelope Kalogeropoulos and Ann Downton

Received: 7 September 2025

Revised: 22 October 2025

Accepted: 3 November 2025

Published: 5 November 2025

Citation: Hodnik, T., & Mastnak, A. (2025). Analysis of Preschool Teachers' Dialogue with Children During Combinatorial Activities. *Education Sciences*, 15(11), 1489. <https://doi.org/10.3390/educsci15111489>

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1. Introduction

On both the national and international level, there is a lack of consensus about how early mathematics instruction should be designed and what constitutes appropriate content (Palmér & Björklund, 2016). Most preschool programs foster number sense, enumeration, and arithmetic reasoning, including addition and subtraction (Grant & McLaughlin, 2001). In recent years, the development of preschoolers' combinatorial thinking has attracted the interest of researchers in mathematics education (Frantzeskaki et al., 2020). Although children's approaches to combinatorial problems are characterized by a high frequency of errors (Batanero et al., 1997; English, 1993, 1998, 1999a, 1999b), engagement with such problems can foster critical thinking, encourage flexibility, shift the focus to structure, and promote the development of thinking skills and mathematical systematic thinking (English, 2005; Sriraman & English, 2004).

In the revised curriculum for kindergartens in Slovenia (Antič & Cotič, 2025), the following objectives regarding very basic or intuitive combinatorial concepts are pursued in the area of mathematics, among others: children discuss their experiences and observations and develop mathematical language; they create and count simple combinatorial situations and find different and multiple solutions to a problem when solving mathematical problems.

Although problem solving in mathematics is emphasized in preschool programs in many countries (Lesh & Zawojewski, 2007), few programs provide mathematically challenging activities (Cross et al., 2009; Perry & Dockett, 2008).

A preschool that only encourages children to handle and experience objects for themselves would be considered inferior today (Sheridan et al., 2009). Regarding the role of the teacher, Biesta (2017) clearly states that “the teacher should be understood as someone who, in the most general sense, brings something new to the educational situation, something that has not been present before” (p. 74). The development of early mathematical knowledge is important, as research shows that early learning is of great importance for children who are later confronted with mathematics at school (Duncan et al., 2007; Dowker, 2005a, 2005b; Starkey et al., 2004; Sylva et al., 2004). One problem in the development of mathematical knowledge in preschool children is the inadequate mathematical knowledge of preschool teachers, which may be related to their own negative experiences with learning mathematics during their school years (Doverborg, 1987; Doverborg & Pramling, 1988; Doverborg & Samuelsson, 2011). Due to its abstract nature, combinatorics, which is the subject of the present study, is undoubtedly among the mathematical content that causes learners considerable difficulties. This is partly, or perhaps primarily, due to the fact that teachers have not had an opportunity to familiarize themselves with this content during their school years in a way that is appropriate for them: from the basics to in-depth or at least fundamental knowledge.

1.1. Literature Review

In the present paper, we explore teachers’ competences in guiding children’s learning of specific mathematical content through their involvement in activities. In particular, we focus on the ways in which the teacher engages in dialogue with the children when it comes to the basic content of combinatorics. The following two sections present an overview of the literature on early learning of combinatorics and on conducting quality dialogue in the context of preschool teachers’ questions in this regard.

1.2. Combinatorics and Systematic Thinking in Preschool Education

Combinatorics is the branch of mathematics that deals with counting and organizing the elements of a given finite set. By solving problems in combinatorics, students/children develop the concept of counting and make connections between concepts, as well as learning mathematical generalization, optimization and systematic thinking (Bräuning, 2019; English, 1991, 2005). Several authors (Bräuning, 2019; Lockwood et al., 2020; Zapata-Cardona, 2018) emphasize the importance of combinatorial content and its inclusion in the mathematics curriculum. Borba et al. (2021) argue that knowledge of combinatorics can develop in the early years of life, and that children should be offered activities that include all possible combinatorial situations: the combinatorial rule of the product, permutations, variations, and combinations. The authors reason that these four types of combinatorial situations contain the basic ideas of combinatorics, and that exposing children to basic situations that represent this content enables them to develop combinatorial thinking. Teaching combinatorics at this age requires children to recognize and solve various intuitive, exploratory, and creative problems, but not to memorize and apply the mathematical formulas required to solve them (Frantzeskaki et al., 2020). Although children are unable to understand the basic principles and concepts of combinatorics at a theoretical level, this should not prevent their intuitive understanding and the use of basic ideas and models of combinatorics at a young age (Krekić et al., 2015).

A number of studies of preschool education have addressed this issue. Research findings (English, 1991, 2005; Palmér & van Bommel, 2016; van Bommel & Palmér, 2018)

have shown that combinatorics content is suitable for introduction at the preschool level. Several arguments are presented in the literature to support this thesis: the content of combinatorics is independent of other mathematical content (e.g., arithmetic) and often only requires counting all of the possibilities (English, 2005; Lockwood et al., 2020); furthermore, it does not involve complex mathematical concepts. Problems in combinatorics can be solved in different ways using a variety of representations (Bräuning, 2019; English, 2005; Lockwood et al., 2020), including concrete representations, to adjust their complexity (English, 2005). Moreover, it is relatively easy to relate the content of combinatorics to problem-solving in children's everyday play and to everyday life situations (English, 2005; Krekić et al., 2015; Lockwood et al., 2020). According to English (2005), the main difficulty for young children working on combinatorial tasks is to systematize their representations, which involves both problem solution and the acquisition of domain-specific knowledge of the combinatorics structure (English, 1996). Like English (2005) and Zapata-Cardona (2018), Borba et al. (2021) found that preschool children are able to systematically solve combinatorial problems when they are presented in a realistic, meaningful context, supported by an appropriate explanation aimed at teaching the children to systematically search for possibilities. Learners need to be able to clearly define and distinguish between the possibilities in a given situation (which elements to choose), while recognizing the sameness of the elements and determining whether the order in which they are chosen is important. Bräuning (2019) found that by repeatedly solving the task with appropriate support from the preschool teacher, children developed a more systematic way of finding all of the possible arrangements of three elements of a set by asking questions over time.

1.3. Preschool Teachers' Questions That Promote Children's Mathematical Thinking

Teachers who have pedagogical content knowledge related to mathematics (Gifford, 2005) know which concepts are the most fundamental and which analogies can best support conceptual understanding; they can engage with new ideas according to the children's interests and ensure that the children use mathematics and mathematical language by asking the right questions (McCray & Chen, 2012). Researchers have postulated that interactions rich in mathematical language are important for the development of children's early mathematical knowledge (Baroody et al., 2019). Under the guidance of an adult, children's mathematical ideas can be explored more comprehensively and explicitly (e.g., Björklund et al., 2018; Lee & Ginsburg, 2009; van Oers, 1996). Dovigo (2016) argues that the teacher's use of conversational moves is crucial if argumentation is to emerge in the conversation between the children and the teacher. Krummheuer (1995) suggests that the teacher should "try to bring communication as close as possible to the point of breakdown" (p. 263), in order to encourage justification, clarification, and evaluation of arguments.

Teachers' questioning is regarded as a core professional competency for extending children's mathematical thinking and deepening their understanding (Cheeseman, 2018). Effective questioning strategies require both the use of preplanned questions and the ability to create spontaneous questions in response to the children's answers (Zhang et al., 2025). Research on questioning primarily focuses on different types of questions and their significance for the development of children's mathematical thinking. Below we present some classifications of questions, as well as research on the importance of specific types of questions for children's progress in mathematical knowledge.

In early mathematics education, different types of questioning serve various purposes. One of the best-known classifications is to divide questions into open-ended and closed questions. Open-ended questions do not have a predetermined answer and therefore allow for multiple possible responses; they tend to promote critical thinking, creativity, and the exploration of mathematical ideas (Aziza, 2018). A number of studies (e.g., Björklund et al.,

2018; DeJarnette et al., 2020; Di Teodoro et al., 2011; Lee & Ginsburg, 2009) indicate that children can explain their mathematical reasoning more explicitly when teachers guide them with open-ended questions. The results show that when teachers ask open-ended questions, children are prompted to expand their explanations. Confirming the findings of other studies, Bay and Hartman (2015) found that preschool teachers mostly do not ask open-ended questions. In contrast to open-ended questions, closed questions—often referred to as factual questions—typically elicit a single correct answer and can be answered succinctly to assess children’s basic understanding of mathematical concepts and principles (Sahin & Kulm, 2008).

There are other types of question depending on the purpose. For instance, probing questions help teachers gain further clarity about children’s explanations, which in turn facilitates meaningful discussion (Franke et al., 2009; Sahin & Kulm, 2008; Sukmadewi, 2014). Tienken et al. (2009) categorized questions as productive (providing students with an opportunity to create, analyze, or evaluate; these questions are usually open-ended and divergent in nature) and reproductive (prompting students to imitate, recall, or apply knowledge and information taught by the teacher through a mimicking process). Di Teodoro et al. (2011) renamed these categories as “surface” (reproductive) and “deeper” (productive) questions. Tienken et al. (2009) suggested that 50 percent or more of the questions asked in a lesson should be productive (deeper) and emphasized the importance of preparing questions in advance, as productive questions are more difficult to generate spontaneously while teaching. Ball et al. (2008) identified asking productive mathematical questions as one of the recurrent mathematical tasks of teaching. From their analysis of preschool teachers’ questions, Saebbe and Mosvold (2015) suggested that the task of asking productive questions is highly complex. In their concluding discussion, they highlighted three issues: there are different types of questions that might be asked to facilitate children’s further reflection and exploration of mathematics; there are different possible purposes underlying the asking of questions; and the task of asking productive mathematical questions is often intertwined with other teaching tasks, which the preschool teacher needs to address instantly as they arise.

For the purposes of the present research, we have distinguished between funneling questions and focusing questions (Hattie et al., 2017; Herbel-Eisenmann & Breyfogle, 2005). We do not attribute particular value judgements to these question types or favor one type in particular, as the appropriate type of question is related to the goals, the context, and the child’s abilities. In adopting these question types, we avoid using the labels closed and open questions, which can themselves convey a value polarization. However, we should stress the need for a diverse range of questions, both in terms of content and regarding their effect on the development of children’s mathematical thinking. It is important that one type of question does not dominate the other, but that both are meaningfully intertwined so that the learning effect is maximized.

Funneling questions guide children along the teacher’s path to find the answer, while focusing questions allow children to do the cognitive work of learning by encouraging them to develop their thinking. Funneling questions combine the features of closed and probing questions. Teachers use these questions to check for understanding; the information they receive from the children is limited to whether they have answered correctly or incorrectly. These questions usually start with asking “How many...”, “Do you...”, “Can you...”, “Are you...?”, and are answered with a ‘yes’ or ‘no’, or with just a few words. In contrast, focusing questions are designed to advance children’s learning, not simply to assess it. This type of question combines the features of open-ended, probing, and productive questions, allowing children to consider different ways of solving a problem and to explain their solutions, rather than leading them to a specific solution. This type of question usually

starts with “How . . . , Why . . . , Which . . . , What . . . ?”. Table 1 provides examples of focusing and funneling questions using the context of combinatorial situations in preschool.

Table 1. Examples of funneling and focusing questions.

Funneling Questions	Focusing Questions
How many combinations did you get?	How do we know if we have all of the combinations?
How many combinations will we have if we add one more?	What happens if we add one more combination?
Are these two combinations different?	How do these two combinations differ?
Do we have a combination of fish and potato?	Describe the combinations you have found.
Can you make a combination that differs from his combination?	In what other ways can this solution be represented?
Who has a combination of fish and potato?	How did you get these combinations?
Can I give fish to potato?	What are you trying to find?
Did you make a new combination?	Why did you decide to do this combination?

1.4. Definition of the Research Problem

Children need challenges and communication to learn to reason, formulate their own explanations, and draw their own conclusions. In order to create learning opportunities centered on particular subject matter and to challenge children’s ideas, the teacher needs two kinds of skills and knowledge: (1) to know what early mathematics might be; and (2) to know how to communicate with children and challenge them with tasks that interest them and have meaning in their everyday lives (Doverborg & Samuelsson, 2011). This involves not only getting children to talk, think and share their ideas, but also knowing how to ask questions and communicate in order to strengthen their understanding. Although communication and interaction are two of the main features of developmental pedagogy, we must remember that children cannot necessarily articulate what they know (Doverborg & Samuelsson, 2011). Preschool teachers need to have knowledge of combinatorics in order to deliver high-quality and successful combinatorics activities. Several studies support the view that preschool children are capable of solving combinatorial problems, as long as the problems are properly planned and guided by the preschool teacher. In the present study, we investigated how preschool teachers engage in dialogue with children when covering the most basic combinatorics content. Preschool teachers undoubtedly know how important concrete representations are for preschool learning; they know the contexts that are attractive for children to learn mathematics, and they know how to create a desire to learn in children. However, they are less successful in implementing the mathematical aspect of the planned activity, especially with content that requires more thinking, e.g., counting, sorting, etc. We wanted to know what kind of questions teachers ask children during organized activities, how varied their questions are, and how they promote preschool children’s mathematical thinking. Stated more generally, our aim was to determine how well teachers know how to utilize the potential of a well-posed problem from the point of view of questioning and the development of mathematical thinking in children. Based on the definition of the research problem and the research objectives, the following research questions were established:

1. What types of questions (related to mathematical content and non-mathematical context) do preschool teachers use in dialogue with children when addressing content from combinatorics, and how diverse are these questions?
2. Which types of questions (funneling or focusing) predominate when preschool teachers engage in dialogue with children?

3. In what way and to what extent do the questions asked by preschool teachers when they engage in dialogue with children promote children's mathematical thinking?

2. Materials and Methods

The sample included all of the preschool teachers enrolled in the course Early Learning Mathematics who met two additional criteria: (1) they had experience working in a kindergarten, and (2) they had chosen combinatorial problems as the content of their activity. All of the teachers had acquired relevant theoretical knowledge within the Early Learning Mathematics course. As part of their coursework, they were assigned the task of conducting mathematics activities in a kindergarten, which they executed in the spring semester of the 2023/24 academic year. The teachers themselves determined the day on which the activities took place. They were required to record an activity (at least 10 min long) with a group of children (aged 4–6 years) that they considered to represent high-quality dialogue with children in kindergarten. Insights into each teacher's implementation of mathematical dialogue were gained through coding and qualitative analysis of the transcribed recordings of their dialogue with the children during the implementation of the mathematical activities. A total of 11 videos (of 11 teachers) of the planned activities were analyzed. Each video was first transcribed. The transcripts were then analyzed with regard to the nature and appropriateness of the (mathematical) questions and the preschool teacher's encouragement of the children's mathematical thinking. Every question posed by the teacher during the activity was analyzed. The questions were categorized according to an inductive process consisting of three stages:

1. In the first stage, the transcripts of the videos of four teachers were evaluated, and those indicating certain types of questions were grouped together. The codes for the questions were created by the researchers and authors of the present paper based on the research questions.
2. In the second stage, the transcripts of the questions of the remaining seven videos were classified and additional categories/codes were created where necessary.
3. In the final stage, the researchers switched roles and classified the teachers' questions according to the categories/codes created. This stage was repeated until there was complete consensus on the categorization of the questions.

The categories and subcategories were analyzed using quantitative methods by calculating frequencies and percentages. Tables 2 and 3 provide basic information on the analyzed activity recordings.

Table 2. The content and organization of the activities of the analyzed dialogues.

Name of Activity	Mathematical Context	Age of Children	Number of Children
Dialogue 1: Balls	Combinations	4–5 years	8
Dialogue 2: Balls	Permutations	5–6 years	9
Dialogue 3: Dress	The combinatorial rule of product	5–6 years	6
Dialogue 4: Dress	The combinatorial rule of product	5–6 years	16
Dialogue 5: Ice cream	Combinations	5–6 years	4

Table 2. *Cont.*

Name of Activity	Mathematical Context	Age of Children	Number of Children
Dialogue 6: Ice cream	Combinations	4–6 years	20
Dialogue 7: Dress	The combinatorial rule of product	5–6 years	12
Dialogue 8: Lunch Bouquets	The combinatorial rule of product	3–5 years	5
Dialogue 9: Dress	The combinatorial rule of product	4–5 years	10
Dialogue 10: Fruit	Combinations	3–5 years	8
Dialogue 11: Dress, Tower	The combinatorial rule of product	5–6 years	7

Table 3. Brief description of the activities.

Dialogue	Brief Description of Activities
Dialogue 1	The children look for combinations of two colors of balls out of four.
Dialogue 2	The children look for possible different arrangements of balls in three colors in a 3×3 grid.
Dialogue 3	The children look for combinations of a headdress with a top and a bottom, choosing between two tops and three bottoms.
Dialogue 4	The children look for combinations of a hat and scarf, choosing between two colors of hat and two colors of scarf. The activity is extended by choosing two out of four gifts for each snowman.
Dialogue 5	The children look for combinations of two out of four ice cream flavors.
Dialogue 6	The children look for combinations of two ice creams out of three, then all possible arrangements of the three ice creams.
Dialogue 7	The children look for combinations of a hat with a scarf, choosing between three colors of hats and three colors of scarves.
Dialogue 8	The children look for combinations of a main dish with a side dish, choosing between two main dishes and three side dishes. The activity is extended by decorating the table and making bouquets of flowers. They can choose two flowers out of three.
Dialogue 9	The children look for hat and scarf combinations and choose between three hat colors and three scarf colors.
Dialogue 10	The children look for combinations of two out of four fruits.
Dialogue 11	The children first look for clothing combinations, choosing between two shirt colors and two trouser colors. Extension activity: the children look for different arrangements of the three colors of cubes in a tower.

3. Results

The results of the research are presented in the following sections: content and diversity of questions in leading dialogue; types of questions from the perspective of leading dialogue (funneling and focusing questions); and ways to promote mathematical thinking in children.

3.1. Content and Diversity of Questions in Leading Dialogue

An inductive approach of open coding was used to determine the types of questions from the point of view of content and diversity. Based on the codes obtained, three main

categories of questions were formed from a content perspective: combinatorics, numbers, and non-mathematical content. Within each category, the questions were categorized into subcategories of questions according to their diversity (Tables 4–6). Each subcategory contained several variants of a question with identical content.

Table 4. Categorization of questions for the content of combinatorics.

Combinatorics	Content Subcategory of Questions	Example of Question	Number and Proportion of Questions		Number of Dialogues ¹
			<i>f</i>	<i>f</i> %	<i>f</i>
1.	Search for any combination ²	<i>“How would you dress the snowmen so that no two are dressed alike?”</i>	54	12.4	9
2.	Search for missing combinations	<i>“Which fruit plate could you make from the leftover fruit?”</i>	29	6.7	7
3.	Systematic search for combinations	<i>“What would you change to change the order of the balls?”</i>	12	2.8	4
4.	Describing and recognizing combinations	<i>“Who chose the combination of green and blue?”</i>	24	5.5	5
5.	Comparison of combinations	<i>“How do the ice creams differ from each other?”</i>	50	11.5	8
6.	Synthesis of combinations	<i>“Do we have all of the combinations? Which fruits have we combined with blueberry? And with the strawberry?”</i>	38	8.8	10
7.	Extension by adding, repeating elements	<i>“I found another hat color in the wardrobe. What other snowmen can we make?”</i>	24	5.5	7
8.	Extension with a new context	<i>“What lunch could we put together if we choose two out of three dishes? What bouquets could we make from two (of three) flowers?”</i>	4	0.9	2
9.	Extension with a new combinatorial situation	<i>“Which ice cream can we make if we choose two out of three flavors? What different ice creams can we make from three scoops of three flavors of ice cream?”</i>	8	1.8	4
10.	Summarizing knowledge about combinations	<i>“What is important when we make combinations? Do we have to pay attention to the order of the balls?”</i>	3	0.7	1
11.	Naming the term combination	<i>“What do you call it when the squirrel has a different colored hat and a scarf?”</i>	4	0.9	1
12.	The number of combinations	<i>“How many different color combinations of balls do we have?”</i>	43	9.9	9
13.	Comparison of the number of combinations	<i>“Has the number of combinations increased or decreased?”</i>	3	0.7	2
Total in combinatorics content			296	68.2	11

Table 5. Categorization of questions for the numbers content.

Numbers	Content Subcategory of Questions	Example of Question	Number and Proportion of Questions		Number of Dialogues
			<i>f</i>	<i>f</i> %	<i>f</i>
1.	Counting objects	<i>“How many snowmen are on the table? How many different colors do we have?”</i>	42	9.7	9
2.	Addition, adding objects	<i>“How many snowmen would we have if we had two more snowmen?”</i>	3	0.7	1
Total in number content			45	10.4	9

Table 6. Categorization of questions for non-mathematical content.

Non-Mathematical Content	Content Subcategory of Questions	Example of Question	Number and Proportion of Questions		Number of Dialogues
			<i>f</i>	<i>f</i> %	<i>f</i>
1.	Attitudes	<i>“Mija, which color do you like the most? And which one do you like the least?”</i>	26	6.0	5
2.	Organizational questions	<i>“Can you all see what I have on the table? Are you ready yet? Who will help me?”</i>	21	4.8	6
3.	Getting to know the context	<i>“What color is the ball/hat/shirt? What have I got here?”</i>	46	10.6	8
Total in non-mathematical content			93	21.4	9

A total of 18 different content subcategories of the 434 questions in the dialogues were identified. As shown in Table 4, the content category combinatorics has 13 subcategories and 296 questions. The subcategories include questions that relate to the following: search for any combination, search for missing combinations, systematic search for combinations, describing and recognizing combinations, comparison of combinations, synthesis of combinations, extension (by adding elements, repeating elements, extension with a new context, extension with a new combinatorial situation), summarizing knowledge about combinations, naming the term combination, number of combinations, and comparison of the number of combinations. The content category of questions about numbers has two subcategories (Table 5): questions about counting objects, and addition, adding objects. The non-mathematical content category (Table 6) has three subcategories of questions: attitudes, organizational questions, and getting to know the context.

As can be seen in Tables 4–6, analysis of all of the dialogues revealed that the preschool teachers mostly (68.2%) asked questions related to combinatorics, while one fifth (21.4%) of the questions were related to non-mathematical content, and the remaining tenth (10.4%) were related to numbers, mainly counting objects. As many as 9 of the preschool teachers (out of 11) included number content and non-mathematical content. In terms of non-mathematical content, the largest proportion of questions were related to familiarization with the context (8 out of 11 preschool teachers). Questions on attitudes and organization accounted for a smaller proportion and were only used by half of the preschool teachers (5—attitudes, 6—organizational questions) (Table 6).

With the subcategories in the combinatorics category, we defined the key knowledge that the preschool teachers developed in the children when they led the activities. Table 4 shows that in the activities analyzed, the largest proportion of questions asked by the

preschool teachers concerned searching for combinations (subcategory 1, 2, and 3, representing 21.9% of the questions), comparing combinations (11.5%), counting combinations (10.6%), synthesizing combinations (subcategory 7, 8, and 9, representing 8.8% of the questions) and extension (8.3%). In terms of searching for combinations, the least frequently presented subcategories were systematic search for combinations and search for missing combinations. All of the preschool teachers asked at least some questions about searching for combinations, while 6 also asked about missing combinations. Only 4 of the preschool teachers encouraged the children to undertake a systematic search for combinations. The majority of the preschool teachers (10 out of 11) asked the children at least some questions about synthesizing all of the combinations formed, counting combinations (9 out of 11), and comparing combinations (8 out of 11). Questions with extensions were asked by 7 of the 11 preschool teachers, mainly extension by adding elements. Other subcategories that the preschool teachers included less frequently in their questions were: describing and recognizing combinations (5 out of 11), extensions with a new context (2 out of 11) or a new combinatorial situation (2 out of 11), comparing the number of combinations (2 out of 11), naming the term combination (1 out of 11), and summarizing knowledge about combinations (1 out of 11). The diversity of the questions used in the dialogue was analyzed from the point of view of the content coverage of combinatorics, as well as from the perspective of the diversity of the questions in relation to the quantity of all of the questions asked in the dialogue.

The content coverage of the questions provides information about the proportion of the subcategories of the combinatorics content used in the dialogue, e.g., if a preschool teacher included 7 out of 13 subcategories, the content coverage was around 54%.

The diversity of questions in relation to the set of all questions was calculated as the ratio between the number of all questions and the number of subcategories. The ratio (r) indicates how often the variant of a question from the same content subcategory was repeated on average. Of course, the fact that the teachers repeat certain questions more often than others must be taken into account, e.g., if a preschool teacher takes questions from 10 content subcategories of questions and asks 25 questions in the dialogue, the diversity of questions is 2.5, which is rounded to the integer value 3; this means that, on average, every third question asked by the preschool teacher comes from a different subcategory.

As shown in Table 7, the teachers' dialogues differ both in the number of questions and the number of subcategories included, and consequently also in the content coverage and diversity of questions. The number of questions that the preschool teachers asked the children during the 10–15 min activities differs greatly, ranging from 19 to 71 questions. Significant differences are also observed between the total number of subcategories of questions (from 5 to 15), as well as between the subcategories of combinatorics content questions (from 3 to 10). The diversity of the questions in the dialogues ranges from 2 to 7, which means that in some dialogues an average of every second question comes from a different subcategory, while in others only every seventh question comes from a different subcategory. The dialogues therefore showed a different number of repetitions of the variant of a question from the same subcategory. There are several reasons for this. One of the content-related reasons is the number of different subcategories included by the preschool teachers. This can be illustrated by comparing Dialogues 2 and 3. Both dialogues had approximately the same number of questions, but quite a different number of subcategories. The second reason for the diversity of questions is related to the organization of the activity: the preschool teachers involved a different number of children in the dialogue, and also conducted the dialogue in different ways. This can be illustrated by comparing Dialogues 6 and 9. In both dialogues there were 10 subcategories, but in Dialogue 9, the number of questions asked was much larger than in Dialogue 6. In Dialogue 9, the preschool

teacher led the dialogue in such a way that the children in the group searched for (missing) combinations one after the other and compared them, whereas in Dialogue 6, the children first searched for combinations on their own (during which time there was no guided dialogue) and then compared them together. In Dialogue 9, there was also a greater tendency for the preschool teacher to involve all of the children in the dialogue, which was reflected in the fact that each child in the group had to answer a particular variant of a question from the same subcategory, and therefore the same subcategory of question was repeated several times. The content coverage of the questions reveals the breadth and depth of the discussion of the content from combinatorics, which the preschool teacher incorporates into the dialogue with questions. The higher the content coverage, the more opportunities the child has to develop knowledge about the combinatorial content in the activity. Two preschool teachers had a high content coverage of questions in the dialogue; five preschool teachers covered about half of the combinatorics content with questions; and the remaining four preschool teachers had a significantly lower content coverage.

Table 7. Content coverage and diversity of questions in leading dialogue.

Dialogue	Number of Questions	Number of Content Subcategories of Questions	Number of Subcategories of the Combinatorics Content	Content Coverage	Diversity of Questions
	<i>f</i>	<i>f</i>	<i>f</i>	<i>f</i> %	<i>r</i>
1	58	10	7	53.8	6
2	28	5	3	23.1	6
3	25	9	6	46.2	3
4	29	7	3	23.1	4
5	44	9	5	38.5	5
6	34	10	10	76.9	3
7	47	8	4	30.8	6
8	19	8	7	53.8	2
9	71	10	6	46.2	7
10	41	9	7	53.8	5
11	38	15	10	76.9	3
Total	434	18	13	100.0	

Two dialogues with high content coverage should be highlighted, namely Dialogues 6 and 11.

In Dialogue 6, all of the questions from the content of combinatorics, and as many as 10 out of 13 categories were covered. In the dialogue, the preschool teacher asked the children to find different combinations of two out of three types of ice cream. All of the children searched for combinations simultaneously. They then compared their findings with each other and searched for all of the different combinations. This was followed by an extension of the situation with the possibility of repeating a flavor. At the end, the children also explored how the three different flavors of ice cream could be arranged (Figure 1). The focus of the dialogue was therefore not only on the search for different combinations, but also on the extension of combinatorial situations. In Dialogue 6, the search for missing combinations, the requirement that the child should name the term combination, and the extension with a new context were not covered. As two extension situations had already

been used in the dialogue, this was not necessary. Moreover, we do not consider it necessary for the children to name the mathematical term combination based on a given description (e.g., “What do you call it when the squirrel has a different colored hat and a scarf?”). As the children had already found all of the combinations when they began to form combinations, they only systematically checked how they had combined the individual elements. The preschool teacher therefore sees an opportunity to improve the content of Dialogue 6 by asking the children whether they are still missing a combination, and how they know that they have all of the combinations.

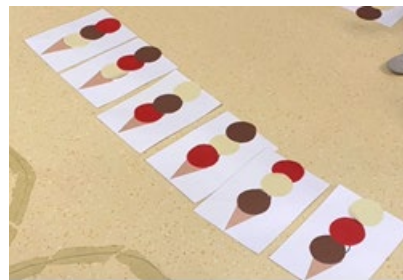


Figure 1. Permutations/arrangements in Dialogue 6.

In the analysis, we would also like to highlight Dialogue 11, which covers the largest number of different subcategories and has a large diversity of questions, as well as many subcategories from the field of combinatorics (10 out of 13). The dialogue took place as part of an activity in which the children first searched for clothing combinations for a teddy bear, choosing between two colors of shirts and two colors of trousers. The children were involved in this activity as they wished. They then added a new shirt color to the task. This was followed by a new task in which the children had to find all of the different towers they had built from three different colored cubes. All of the combinations were created by the children together with the support and guidance of the preschool teacher. In the dialogue, the preschool teacher omitted the following subcategories: summarizing knowledge about combinations, naming the term combination, and comparing the number of combinations. All of the important subcategories of combinatorics content were therefore included. We would, however, suggest including a question to compare the number of combinations, especially since an extension situation was used by adding elements and thus increasing the number of combinations. For each guided activity, it is important that the teacher concludes with a summary of knowledge about the content addressed. In the dialogues analyzed, this was mostly missing or was only present as a summary in the form of an explanation, not in the form of a question that would involve the children in the summary of the activity. In this respect, we also see an opportunity to improve Dialogue 11.

It is important to highlight dialogues with low content coverage. Dialogues 2 and 4 had the lowest content coverage, as the teachers only included 3 out of 13 subcategories of questions.

In Dialogue 2, the children had to arrange colored balls in a given 3×3 grid (Figure 2), so that there were three different colored balls in each row and column. The activity was carried out through movement, i.e., the children carried the balls to the grid. During the activity, the preschool teacher asked the children questions about the correct arrangement of the colors of the balls and about the search for a missing arrangement. At the end, they produced a synthesis of all of the arrangements by checking the correctness of the completed grid. The dialogue could be improved by encouraging the children to compare the arrangements with each other, in order to determine the number of all of the different arrangements, and generally to think about the order of the colors of the balls and the search for a new arrangement. With the method the preschool teacher used in this activity,

the children were running from the grid to the set of balls, each time moving quite a long way away from the grid on the floor. The activity focused only on completing the grid and following the rule about the different colors of the balls in the row or column. Despite the fact that the content of combinatorics (89.3%) predominated in the dialogues, mainly with questions on searching for arrangements, the children did not develop knowledge of searching for different arrangements or the number of all possible arrangements.



Figure 2. Arranging balls in a 3×3 grid in Dialogue 2.

In Dialogue 4, the children were given the task of dressing up snowmen in such a way that no two were dressed the same. They combined two different colored hats and two different colored scarves. The children sat down in groups of four so that they could compare their solutions and make all four different combinations at the same time. Once everyone had made their own combination, the preschool teacher continued to lead the dialogue. The subcategories of questions she included were: search for combinations, extension by adding elements, and the number of combinations. In the dialogue, the preschool teacher did not focus further on combinatorial content, but on the content of numbers (37.5% of all questions), as she asked for the number of snowmen that have a certain property, e.g., “How many snowmen have a yellow hat?” There were no questions that included a description of a combination. The number of questions about snowmen with a particular characteristic was greater than the number of questions about the number of combinations, as this question was only asked after the activity was completed. The teacher then continued with questions about the number of snowmen with a particular characteristic, also adding questions about addition, e.g., “How many snowmen would we have if we had two more?” A large proportion of questions about non-mathematical content (31.3%) were also present in the dialogue, mainly to familiarize the children with the context (25.0%). With such questions, the preschool teacher moved away from the combinatorics content and missed the opportunity to develop the knowledge that was the main objective of the activity.

3.2. Types of Questions from the Perspective of Leading Dialogue

A deductive approach of closed coding was used to determine the types of questions. The teachers’ questions were divided into the fixed, predetermined categories of “funneling questions” and “focusing questions”.

Funneling questions were defined as all questions asked by the teacher that directed the child’s thinking in a predetermined direction, regardless of the children’s answers, i.e., questions with a predetermined, unambiguous answer. These are closed questions that often begin with the question words “Are you/is that...?”, “How many...?”, or “Who...?”. Focusing questions were defined as all questions with which the teacher stimulates the children’s thinking and continues the dialogue on the basis of the answers, i.e., the teacher adjusts the direction of the dialogue according to the children’s answers. As a rule, these

are open questions to which several answers are possible. The questions often start with “How would...?”, “What kind of combination...?”, “Why...?”, or “What...?”. Examples of both categories of questions can be found in Table 8.

Table 8. Examples of funneling and focusing questions in the analyzed dialogues.

Type of Question	Examples
Funneling questions	<i>“How many different types of fruit will you put on one plate?”</i>
	<i>“How many clothing combinations does this squirrel have? Count them.”</i>
	<i>“How many differently dressed snowmen did we get?”</i>
	<i>“Are the plates the same?”</i>
	<i>“Can I give the penguin a red hat?”</i>
	<i>“Do we already have this combination?”</i>
	<i>“Is one scoop smaller?”</i>
	<i>“Shall we make another combination by putting two chocolate scoops?”</i>
	<i>“Do we have to pay attention to the order of the ice cream scoops?”</i>
Focusing questions	<i>“Who has an ice cream in the combination chocolate and vanilla?”</i>
	<i>“How would we find out how many different fruit plates we can prepare?”</i>
	<i>“How can we change the order of the ice cream scoops so that they look different?”</i>
	<i>“How can we dress the snowmen so that no two are dressed the same?”</i>
	<i>“How do the ice cream creations differ from each other?”</i>
	<i>“What fruit do we have left, and can we put it next to the strawberry?”</i>
	<i>“Which combinations have we not used yet?”</i>
	<i>“Why did you decide this way?”</i>
	<i>“Why did you put the strawberry scoop in this place?”</i>
	<i>“What could this penguin have worn differently?”</i>
	<i>“What else can I change to get a new combination?”</i>
	<i>“What is important when we make combinations?”</i>
	<i>“What will you change to make it different?”</i>
	<i>“What hat can it have if it has an orange scarf?”</i>
	<i>“What kind of ice cream do you put next to the strawberry?”</i>

As Table 9 shows, funneling questions predominate in the majority of the dialogues analyzed, beginning with the interrogatives “Do/is/is...?”, “Who...?”, and “How many...?”. In all of the dialogues together, an average of 33% of the questions were focusing questions, but the proportion of focusing questions in the dialogues differs significantly (from 14.3% to 72.0%). In addition to Dialogue 3, Dialogues 6, 8, and 10 also have a relatively high proportion of focusing questions, while the proportion of focusing questions in the other dialogues is low. Funneling questions therefore predominate in almost all of the dialogues examined, indicating room for improvement in this respect. Examples of focusing questions from dialogues with a slightly higher proportion of such questions are therefore provided below.

Table 9. Proportion of focusing questions in the dialogues analyzed.

Dialogue		1	2	3	4	5	6	7	8	9	10	11	Total
Focusing questions	<i>f</i>	14	4	18	5	14	14	12	9	22	20	13	145
	<i>f %</i>	24.1	14.3	72.0	17.2	31.8	41.2	25.5	47.4	31.0	48.8	34.2	33.4
Funneling questions	<i>f</i>	44	24	7	24	30	20	35	10	49	21	25	289
	<i>f %</i>	75.9	85.7	28.0	82.8	68.2	58.8	74.5	52.6	69.0	51.2	65.8	66.6

In all of the dialogues, the majority of focusing questions were related to the search for combinations (also in extension situations) and to the comparison of combinations.

In Dialogue 3, the teacher asked the children searching for a new combination questions such as “What else can we swap?”, “What combination will you make to make it different again?” Several different answers were possible in these cases, as well as different ways for each child to find the answer. Dialogue 6 also included slightly different focusing questions: “How would you change the order of the scoops to get a different combination?”, “Which combinations are still missing?”, “What is important when we make combinations?” In Dialogue 10, similar focusing questions emerged as in Dialogue 3, but with some variants, e.g., “Why did you decide on this combination?”, “How would you represent this combination on paper?”, “What is different on your plate?”, “How are the combinations different from each other?”

3.3. Ways to Promote Mathematical Thinking in Children

In the study, we were also interested in the ways and the extent to which the questions asked by the preschool teachers promote the children’s mathematical thinking. We therefore categorized all of the questions that promote the children’s mathematical thinking.

When determining the types of questions from the point of view of promoting children’s mathematical thinking, an inductive approach of open coding was used, i.e., the codes were formed spontaneously and grouped into broader categories using thematic coding.

Based on the codes obtained, the questions used to promote the children’s mathematical thinking were divided into five categories: number, comparison, mathematical terms, reasoning, and problem-solving questions. The number category included all of the questions related to recognizing the number of elements of a set, counting, and adding elements of a set in simple situations (e.g., “How many combinations did we get? How many different types of fruit do you put on the plate? Did we get four different combinations? How many different hats do we have? How many different colors of balls do we have? How many snowmen do we have if we add two?”). Questions categorized in the comparison category were mainly related to comparing the properties of elements and combinations, and comparing the size of numbers (e.g., “Which combination is different from this one? Is it OK if the colors of the balls have to be different? How are the ice cream creations different? Are these two the same combination? What is different on your plate? Has the number of combinations increased or decreased?”). The mathematical terms category included questions that required the child to use the term combination in their answer (e.g., “What can you call it when the squirrel can wear hats and scarves in different colors? What do we have 16 of?”). The reasoning category included questions that require the children to provide an explanation (e.g., “Why did you put the red ice cream scoop on top? Why do you think we already have all the combinations? Why isn’t that right? Why do you think I have six friends?”). The problem-solving question category covered questions that had several possible correct answers and no predetermined problem-solving strategy (e.g., “How many different plates of fruit can you make from two types of fruit? How would you find out if I can make different gifts for all 12 of my friends? Can we make a different combination? What combination can we make that is not the same as this combination? How could we dress this penguin differently? How would you change the order to get a different ice cream? What other combination can you make if you can use the same type of ice cream twice? Which combination with kiwi is still missing? Do you think there will be more combinations if we add a new color?”). All of the questions in this category were derived from combinatorics.

Based on the categorization of the questions on encouraging mathematical thinking and the subsequent quantitative analysis (Table 10), it was found that the teachers encouraged the children’s mathematical thinking in just over two thirds of the questions

on average. They did so primarily with problem-solving questions (34.3%), and to a slightly lesser extent, using questions related to numbers (18%) and comparisons (11%), while the smallest proportion of questions focused on reasoning (2.5%) and mathematical terms (0.9%).

Table 10. Categorization of questions for encouraging children’s mathematical thinking.

Dialogue	Problem-Solving Question		Number		Comparison		Reasoning		Mathematical Terms		Total	
	<i>f</i>	<i>f</i> %	<i>f</i>	<i>f</i> %	<i>f</i>	<i>f</i> %	<i>f</i>	<i>f</i> %	<i>f</i>	<i>f</i> %	<i>f</i>	<i>f</i> %
1	13	22.4	4	6.9	4	6.9	0	0.0	0	0.0	21	36.2
2	11	39.3	2	7.1	13	46.4	0	0.0	0	0.0	26	92.9
3	11	44.0	5	20.0	4	16.0	0	0.0	0	0.0	20	80.0
4	4	12.5	15	46.9	0	0.0	0	0.0	0	0.0	19	65.5
5	19	43.2	11	25.0	5	11.4	1	2.3	0	0.0	36	81.8
6	14	41.2	4	11.8	9	26.5	1	2.9	0	0.0	28	82.4
7	14	29.8	10	21.3	0	0.0	0	0.0	4	8.5	28	59.6
8	9	47.4	5	26.3	3	15.8	0	0.0	0	0.0	17	89.5
9	23	32.4	16	22.5	11	15.5	5	7.0	0	0.0	55	77.5
10	14	34.1	4	9.8	11	26.8	4	9.8	0	0.0	33	80.5
11	17	44.7	2	5.3	10	26.3	0	0.0	0	0.0	29	76.3
Total	149	34.3	78	18.0	70	11.0	11	2.5	4	0.9	312	71.9

Comparing the individual dialogues, the proportion of questions that stimulate mathematical thinking ranges from 36.2% (Dialogue 1) to 92.9% (Dialogue 2). A few examples will illustrate how problem-solving questions were included in the dialogues. These questions are particularly interesting, as searching for combinations during the preschool years is quite a challenge for children because it requires them to develop a strategy for searching, comparing, and summarizing the number of combinations. It should be emphasized that the final result of all combinations is not the most important thing here.

In Dialogue 1 (Figure 3), the children had to choose two out of four colored balls: the one that they liked the most, and the one that they liked the least. With this question, the teacher missed an opportunity for a good problem-solving question at the beginning, i.e., for the development of mathematical thinking in general, as she did not ask the children to think about finding possible color combinations of the balls; only later were the resulting combinations compared by the children. Although problem-solving questions did appear later, the opportunity for a good question was again not utilized: the children were asked to think about which color combinations would result if they chose three out of four balls; the question was hypothetical and therefore the children did not answer it. In this case, the teacher should have provided the children with material that they could use to select three out of four ball colors and find out the number of all possible combinations. A similar situation arose at the end of the activity, when the teacher asked the children: “What if I add purple color balls for you. Will you have more or fewer combinations?” The answer was given by the teacher herself, as the children did not know how to answer without any concrete material.



Figure 3. Creating combinations of two balls out of four in Dialogue 1.

It is clear from the above that it is not enough for the teacher to ask a good problem-solving question; she must create all of the necessary conditions for the children to explore the given problem situation.

In Dialogue 4, the teacher started the activity with a good initial question (“How would you dress the snowmen so that no two are dressed alike?”), but later drew attention mainly to questions about counting snowmen with a particular characteristic. Despite two further problem-solving questions (“How could you dress the snowman differently if I gave you a new scarf color? How could you give the snowmen two presents?”), questions about counting and getting to know the context predominated in the dialogue. The reason for the low proportion of problem-solving questions is also due to the fact that various problem-solving questions were only asked once to all of the children at the same time. Dialogue 2, on the other hand, has a high proportion of problem-solving questions, as almost all of the questions asked were aimed at finding arrangements of three-colored balls, which represents a problem-solving situation for children. However, it must be pointed out that the children did not have many opportunities to think together about the appropriateness of the arrangement. The question was posed to each child individually as they brought their ball into the 3×3 grid, where they had to arrange it. Therefore, even in this dialogue, which otherwise has a high proportion of problem-solving questions, there are opportunities for improvement. We suggest that the teacher ask more varied problem-solving questions and that more children participate in finding the answer at the same time. The children should listen to each other and complement each other in finding the solution for all of the arrangements of the three ball colors in the 3×3 grid.

Dialogue 8 had a high proportion of problem-solving questions, but they were more varied than in Dialogue 2. Using concrete material, the children first had to explore the different combinations of lunch they could make when choosing between meat and fish, to which they could add rice, potatoes, or pasta as a side dish. They also investigated which combinations were missing and whether they had covered all of the possibilities. The teacher used questions to suggest how they could systematically exhaust all of the possible combinations (e.g., “What can we add to fish?”). In addition, she posed two problem-solving questions with extension situations (first extension question: “What would you do if two more guests came and wanted a different lunch than all of the other guests?”; second extension question: “What bouquets can we make from two different flowers if we choose from three?”). The children explored these questions with concrete material.

4. Discussion

In the present study, the main aim was to explore how preschool teachers use a given mathematical situation to ask high-quality questions, in the sense of stimulating thinking about combinatorics in a child-friendly way. Most importantly, we can see that the contexts chosen to address the content of combinatorics are meaningful, drawn from children’s lives, and presented with concrete objects, which undoubtedly enables children’s exploration in a context that is meaningful to them.

With the first research question, we wanted to determine what types of questions (in terms of content) preschool teachers use when engaging in dialogue with children when it comes to combinatorics content, and how diverse these questions are. We found that most of the questions were related to the combinatorics content, while some were concerned with number content and non-mathematical content (questions about children's attitudes, organizational questions, questions about getting to know the context). Although the combinatorics content is considered to be quite independent from other mathematical content, the analysis of the dialogues confirmed the findings of [Bräuning \(2019\)](#) and [English \(1991, 2005\)](#) that children also develop the concept of counting in combinatorics activities. The teachers developed the children's concept of counting with questions about counting combinations and by counting the elements of the set from which the children made combinations. [Zapata-Cardona \(2018\)](#) notes that it is important for children to distinguish between the number of elements they combine and the number of combinations. Some teachers in our sample did in fact ask the children about the number of elements in the set and the number of combinations, but they failed to synthesize the results by highlighting the difference between the two numbers. It would be useful to include this content question as a new subcategory in combinatorics content in the future. Content analysis of the questions also showed that only a small proportion of the dialogues contained questions related to the development of the children's systematic thinking. [English \(2005\)](#) states that this is what children have the most difficulty with when solving combinatorial problems. We therefore suggest that preschool teachers guide children through the activity in a way that teaches them systematic thinking through questions. In order to encourage the systematic searching of new combinations, teachers should select a specific element and identify all of the elements that can potentially be combined with it. Some examples of questions are: "If you have a blue scarf, what colors of hat could you wear with it?" "You already have meat and potatoes. What could we use instead of potatoes to create a new combination?" By asking these questions, we are applying the principle of constancy, which, according to [English \(1996\)](#), states that different combinations result when at least one element is kept constant while at least one other is systematically varied.

From the point of view of content coverage, we can indeed say that with the formed content subcategories of questions within the content category of combinatorics, we have defined all of the necessary frames of knowledge that the teacher develops in children with this content. The subcategories precisely outline the required knowledge under the combinatorics content, complementing the knowledge that some other authors suggest is necessary for dealing with this content (e.g., [Borba et al., 2021](#); [English, 2005](#); [Zapata-Cardona, 2018](#)). The most common subcategories included in the majority of the teachers' dialogues were the search for combinations, synthesis of combinations, comparison of combinations, and the total number of combinations. In addition, we emphasize the importance of certain other subcategories that were included in a smaller number of dialogues: the inclusion of extension situations (by adding, repeating elements, extension with a new context, extension with a new combinatorial situation), the description of combinations, and the comparison of the number of combinations in relation to the addition of elements when searching for new combinations. Very important, and even less present in the dialogues, is the subcategory of questions that summarize the knowledge at the end of an activity based on combinatorics. In the future, it will be important to sensitize preschool teachers to include more differentiated questions in their dialogues with children. The research findings encompass all subcategories, accompanied by sample questions designed to assist teachers in implementing preschool activities related to combinatorics. Teachers who train future teachers may also find them useful, helping them to guide students towards asking meaningful questions related to combinatorics.

When analyzing the dialogues, it also emerged that knowledge of the context in which children solve a combinatorial problem is important. Solving a combinatorial problem often takes place in everyday situations (English, 2005; Krekić et al., 2015; Lockwood et al., 2020), which became clear in the dialogues analyzed. Among the dialogues analyzed, only two had a mathematical context (Dialogue 1, Dialogue 2). Understanding the context is a prerequisite for successfully solving a combinatorial problem, but questions about the context must not override the combinatorics content in the dialogue. It proves to be problematic if questions with numerical or non-mathematical content predominate in a dialogue, as illustrated in the presentation of the results using selected dialogue examples. Major differences were found between the dialogues analyzed in terms of diversity of questions and coverage of content. Despite the teachers leading the activity for approximately equal periods of time, there were significant differences in the number of questions asked and in the content coverage of the questions. Differences in the pedagogical knowledge of the teachers were evident in the way they conducted the dialogue, and in the content coverage of the questions. Only two of the teachers demonstrated good coverage of the content of the questions, while four showed very low coverage. In the dialogues analyzed, differences in the diversity of questions arose due to the different number of questions asked, the different number of content subcategories of the questions, the organization of the delivery of the activity, and the expected level of the children's involvement in the dialogue. The number of questions may also depend on which questions the teacher asks in relation to the openness of the answer. With alternative questions—questions that require a short, usually predetermined answer—the exchange of involvement in the dialogue between the child and the teacher is usually faster, enabling the teacher to ask more questions in less time.

At this point, we are already touching upon the second research question, regarding which questions predominate in dialogues (funneling questions or focusing questions). On average, funneling questions predominate in the dialogues studied, as they require brief reflection or answers. Such questions also give the teacher greater control over the dialogue, as the answer is usually clear and known in advance. In only 4 dialogues (out of 11) did we find that the proportion of focusing questions was approximately equal to or greater than the proportion of funneling questions. We believe that teachers should ask more focusing questions in dialogues, but we are aware that they need both the appropriate pedagogical content knowledge and self-confidence to do so. Focusing questions do not offer a predetermined answer, and therefore require more flexibility and professional confidence from teachers. Examples of focusing questions are provided in the research results. It was found that these questions usually begin with 'how', 'what', 'which', 'why', or 'what', while funneling questions typically begin with 'how many', 'do/is/are', or 'who'. Both categories of questions (funneling questions and focusing questions) enable the development of children's mathematical thinking.

With the third research question, we wanted to determine to what extent and how preschool teachers promote mathematical thinking in dialogue with children. It was found that the teachers promoted the children's mathematical thinking in more than two thirds of the questions. The largest proportion of questions promoted mathematical thinking with problem-solving questions, while a slightly smaller proportion used questions related to number knowledge and comparisons (properties of elements of a set or numbers). The smallest proportion of questions promoted mathematical thinking with questions on reasoning and on naming mathematical terms. The content of combinatorics is not sophisticated from a language use perspective, nor is it necessary for children to use sophisticated terms such as combinations, arrangements, etc., so we do not suggest any improvements in this regard. From the point of view of language use and mathematical

expression, the only improvement necessary is the teachers' way of expressing themselves when formulating the questions. The analysis of the results regarding this aspect of asking questions in dialogue with children is presented in [Mastnak and Hodnik \(2024\)](#).

In all of the dialogues analyzed, we identify room for improvement from the point of view of including a greater proportion of reasoning and problem-solving questions. The nature of combinatorics content is such that the teacher can present it to children as a mathematical problem. Other important factors are what questions the teacher asks in the dialogue, and whether they give the children an opportunity to explore the mathematical problem. We suggest avoiding the form of questions that start with "Is...?" unless followed by the question "How would you find out...? Why do you think that?" When faced with an "Is...?" question alone, the child will simply answer 'yes' or 'no'. Consequently, the teacher does not gain any insight into the child's thinking and cannot support them in continuing the dialogue. In the presentation of the research results, we have therefore provided some examples of good questions that promote children's mathematical thinking in different ways, especially through problem solving and reasoning.

5. Conclusions

The results presented in the present article relate to the teaching of combinatorics at preschool age. As we have argued, pedagogical competences related to mathematics are important in addition to pedagogical competences when engaging in a dialogue with a child. Due to the nature of the mathematical content, i.e., combinatorics, which is one of the more challenging topics, conducting a dialogue is specific and, one could say, more demanding. We assume that we would obtain different results if the teachers were to conduct a dialogue about other mathematical content. Apart from being restricted to specific content in combinatorics, one of the study's limitations is that the results cannot be generalized due to the small sample size and the specific national context, which is characterized by particular features of preschool education and the training of preschool teachers. Furthermore, it was not possible to identify patterns in dialogue management among individual teachers, as dialogue management was analyzed in only one activity. The results do, however, offer an insight into the content structure of the questions, which, in our view, are fairly predictable, since they relate to mathematical content and would probably not differ significantly in terms of content if other teachers conducted the dialogue. Nevertheless, they could certainly vary in the complexity of the questions and the challenges they present to children. Another limitation of the study is that the observed pedagogical work was analyzed without including the teacher's perspective on his/her intentions. Introducing the teacher's voice from a follow-up interview could have been interesting; however, this is beyond the scope of the present paper, which focused strictly on unpacking the observed teaching. Interviews with kindergarten teachers could, however, provide further information about teachers' beliefs and knowledge, which would also be a relevant field of investigation. The findings offer a theoretical framework for more extensive analysis in this field, as well as guidelines for developing teachers' didactic competence, particularly with regard to facilitating quality dialogue with children.

The contribution of our work to understanding the management of mathematical content at preschool age lies in the analysis of dialogues between teachers and children, in which we have formed content categories and subcategories of questions that, on the one hand, provide an in-depth insight into the development of mathematical concepts from combinatorics at preschool age and, on the other hand, can guide teachers in formulating questions that primarily stimulate children's thinking and develop mathematical discourse in dialogue with children.

Most of the research in the area of combinatorial content in preschool education that we have cited in the literature review is concerned with how children develop an understanding of this content. However, we have focused on the role of teachers, who significantly influence the child's experience and understanding of concepts through the way they lead dialogue. Thus, our research also contributes to understanding the role of teachers and their pedagogical content knowledge.

Author Contributions: Conceptualization, T.H. and A.M.; methodology, T.H. and A.M.; validation, T.H. and A.M.; formal analysis, T.H. and A.M.; investigation, T.H. and A.M.; resources, T.H. and A.M.; writing—original draft preparation, T.H. and A.M.; writing—review and editing, T.H. and A.M. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: The study was reviewed and approved by the Ethics Commission of the Faculty of Education of the University of Ljubljana (Protocol 27/2025, 5 September 2025).

Informed Consent Statement: Informed consent was obtained from all subjects involved in the study.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

Notes

- ¹ The number of dialogues in which this subcategory was present. The total number of dialogues is 11.
- ² Although only the term “combination” is used, there are various combinatorial situations, including arrangements, permutations, and the product rule.

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