

Table S1. Table of developed indicators and epistemic activities from the mathematical modeling cycle

Activities	Description	Indicators (Example of evidence from participants' work)
Understanding	Forming an idea about what the problem is asking	<ul style="list-style-type: none"> *Reading the task *Playing with simulation to know the environment *Making a sketch *Returning to the elements of the task to get more precise information
Simplifying/ Structuring	Simplifying the model and adding structure to it by introducing conditions and variables	<ul style="list-style-type: none"> *Identifying variables in the problem *Making assumptions to simplify the problem *Listing assumptions *Using maximum and minimum amount of each value to make an assumption *Making structured table of data *Fixing some variables to work with one variable at a time
Mathematizing	Presenting the model in mathematical language	<ul style="list-style-type: none"> *Speaking in terms of relationships between variables *Using appropriate procedures to answer the questions *Mathematically representing ideas (e.g., graphs, equations, tables, symbols) *Describing how a specific variable affects the behavior of the system
Running Simulation	Running the simulation with different settings each time	<ul style="list-style-type: none"> *Setting the value of each variable and running the simulation when working within the computer environment
Experience reconciliation	Arriving at an interpretation that is consistent with all the computer results students have seen	<ul style="list-style-type: none"> *Using computer results to design a new setting *Prioritizing variables based on the results of the simulation in each run *Using conditions to design a new setting *Using if then statements to change the settings of simulations
Interpreting	Recontextualizing the mathematical result	<ul style="list-style-type: none"> *Relating the results to real-world experiences *Speaking about the result in the context of the problem *Explaining the pattern observed and reason why
Validating	Verifying results against constraints	<ul style="list-style-type: none"> *Comparing the answer to previous answers *Running the simulation to verify the final results/predictions

Table S2. Example of an event map (Cannon Ball simulation)

Line#	Time	Speaker	Transcription	Indicator	Coding based on MMC
34	0:10:36	N	I found that the speed makes it go farther actually	Identifying the relationship between two variables; speed and distance	understanding, simplifying/structuring
35	0:10:41	A	Ok ... so how speed makes it [going] farther? Like how do you change the speed so it goes farther?		
36	0:10:48	N	For example if you are at zero (speed of zero) it'll go straight down but if you are at 1 (speed of 1) you go a little bit more and for 2 (speed of 2) the same thing ... you go a little bit more and it keeps going like that ...	Increasing the speed gradually to see the effect of the speed on distance, Running the simulation in each case	Running, Mathematizing, simplifying
37	0:11:06	A	So if you increase the speed, it goes farther you mean ...		
38	0:11:08	N	Yes!	Confirming that speed and distance has direct relationship	Mathematizing, Coordinating variables
39	0:11:09	A	Ok		
40	0:11:11	N	And then I realized that unlike the cat game, doubling ... like doubling all of these (showing the setting of the simulation) doesn't work.	Identifying the relationship between the variables	Simplifying/structuring, Coordinating variables
41	0:11:21	A	Ohhh ... OK		
42	0:11:22	N	So		
43	0:11:27	A	Uhum So how ... so you mean for example if you double some number here, it doesn't double others ... other numbers. What do you mean? Can you explain that more for me?		

44		N	So I start at 11 ... speed like ... the speed is 11 meters and 2 meters height and 10 degrees ... the cannon was at 10 degrees ... and the target was at 9 exactly. So ... umm ... then I doubled it ... 22 speed, 4 meters, 20 degrees and 19 meters ... or 18 meters and I realized when I did that it just went	Trying a specific setting and use the results of running the simulation to explain the relationship between variables	Simplifying/structuring, Coordinating variables
	0:11:36		...		
45	0:12:12	A	Oh it didn't hit the target?		
46		N	It didn't	Running the simulation to show her prediction is correct.	Running, Validating
	0:12:14				