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Sámi Body-Based Measuring in the Mathematics Classroom—The Development of a Model for Teaching

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Abstract: According to Norway's Core Curriculum all students are to learn about Sámi issues, and the subject of mathematics is not exempt from it. Central to the Sámi tradition is non-standardized individual body-based measuring. Two Sámi pre-service teachers taught Sámi traditional measuring in an urban school where most students are non-Sámi. Their idea was for students to experience the usefulness of using their own bodies as measuring tools. Afterwards, a teacher educator interviewed the pre-service teachers about their experiences. The teaching unit was planned as if it were to be taught in a Sámi school, except that the language of instruction was Norwegian. Analysis shows how the teaching unit aligns with the suggestions and recommendations of Norway's Truth and Reconciliation Commission. Analysis with respect to the Theory of Didactical Situations framework, TDS, which has not been developed for Indigenous education, show that TDS contributes to improving the teaching unit's plan. We recommend that Sámi traditional measuring should be taught by Sámi teachers to more non-Sámi students, and we present a revised teaching plan. However, because there is a significant lack of Sámi teachers, we recommend that non-Sámi teachers use our plan as a model for teaching Sámi traditional measuring.

Keywords: body-based measuring; non-standardized measuring units; Theory of Didactical Situations; Indigenous mathematics; Sámi; Norway's Truth and Reconciliation Commission



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1. Introduction

The Sámi are an Indigenous people in the Arctic who inhabit the northern part of Scandinavia and the Kola Peninsula of Russia. Sámi University of Applied Sciences (SUAS) offers Sámi teacher education programs based on Sámi culture, language, and social life [1]. Sámi traditional measuring is a cultural practice that is expressed in the Sámi language(s). This cultural practice is an example of Sámi intangible cultural heritage [2] that is integrated into mathematics teacher education at SUAS. Dina and Aile, who are pre-service teachers at SUAS, conducted an in-depth course on Sámi ethnomathematics. As part of the course, they investigated the use of Sámi traditional measuring, focusing on individual body-based measuring units. Based on their investigations, Dina and Aile conducted a teaching unit on Sámi traditional measuring for four Grade 8 classes at a school in Tromsø, the largest city in Northern Norway. Most of the students at this school are non-Sámi. The teacher educator Anne observed the last of these lessons, and, later the same day, Dina and Aile shared their experiences with her in a semi-structured interview.

The purpose of this article is to discuss the relevance of including Sámi body-based length measuring in mathematics education and to present a plan for a teaching unit. Norway's Truth and Reconciliation Commission [3] suggests a broad investment in knowledge about the Sámi, Kven/Norwegian Finns, and Forest Finns. This suggestion is supported by the Sámi Parliament's action plan to address incitements against the Sámi [4]. The inclusion of one or more Sámi issues in the national mathematics curriculum could represent a serious response to the Commission's [3] suggestion. To investigate whether the inclusion

of Sámi issues in mathematics teaching in Norwegian schools is sensible, our first research question is: How did the teaching unit relate to the suggestions and recommendations of Norway's Truth and Reconciliation Commission regarding the Sámi?

The teaching unit is planned as if it were to be taught to Sámi students, except that the language of instruction is Norwegian. One central aim in Sámi education and child upbringing, is to support the development of independent individuals [5,6]. The Theory of Didactical Situations, TDS [7,8], is a framework developed to support teacher development in mathematics education for non-Indigenous students. Central to TDS is that teachers plan for different phases in a lesson where they transfer control of the work to the students in the first phase and take back control in the final phase. The second research question is: How can TDS explain and improve the plan for the teaching unit? We present Aile's and Dina's plan for the teaching unit and analyze their experiences with respect to TDS.

The remainder of the introduction consists of four sections; the first presents Sámi culture, tradition, and education; the second outlines recommendations and suggestions by Norway's Truth and Reconciliation Commission. These are followed by a more thorough section about measuring. This includes (a) an overview of the international research literature on the role of measuring in mathematics education; (b) body-based units of measuring and their advantages with examples mainly from Sámi traditional measuring, including how it is used and has been used; and (c) a discussion of Sámi traditional measuring in relations to the described literature. We highlight contradictions in certain findings within existing research literature. We argue that Sámi individual body-based measuring, which is part of Sámi traditional knowledge, can provide a valuable approach to measuring in the education system. The fourth section presents the Theory of Didactical Situations.

1.1. Sámi Culture, Tradition, and Education

Traditional Sámi livelihoods are reindeer herding, and combinations of smallholdings and fisheries. *Meahcceealáhus* (fishing, hunting, and gathering) and *duodji* (Sámi handicraft) have strong traditions. Sara [9] explains Sámi traditional knowledge as Sámi common knowledge about making the best possible use of the nearby nature, and how to adapt and transform purchased material for use. Guttorm [10] refers to a time when people lived exclusively on resources from the nearby area. The people managed or survived, *birget* in Sámi. Javo et al. [11] explain this from a reindeer herding perspective where you often are alone in demanding situations. The point is that when you have faith in yourself, you seldom panic. According to Aikio [12], upbringing has an impact on how you *birget* (cope) in life. The ultimate goal of Sámi upbringing is for the child to develop into a versatile and self-reliant individual. From a Sámi perspective, specialization and narrow competence prevent a person from becoming overall harmonious.

By living in and off an area, people gradually enhanced their skills and knowledge and learned how to manage this knowledge as well as the surrounding area. This is not only a question of traditional knowledge, but also of the management of that knowledge. According to Sara [9], traditional knowledge is a new and widely used concept that refers to knowledges that have been maintained since ancient times. Sámi traditional knowledge is part of Sámi knowledge, culture, and identity. Sámi body-based measuring is one aspect of Sámi traditional knowledge. It is a way of measuring that has been developed over generations for practical use.

Sara [9] explains that Sámi traditional knowledge includes more than just practical knowledge and competence; it is also knowledge related to exchange of information, cooperation, participation, discussions about practical tasks, all kinds of stories, and communication issues. The term *knowledge* here means knowledge as a process, not just as the outcome of a process. Guttorm [10] distinguishes between two aspects of knowledge. The Sámi concept *diehtit* (to know about something) is primarily likened to theoretical knowledge, while *máhttit* (to know something or to master something) is likened to practical knowledge or skills. According to Nielsen [13], *máhttit* means to be able (speaking of ability, knowledge, skill). Guttorm [10] points out that knowing about something, *diehtit*,

does not guarantee that one has the skill to do or perform it. She provides an example of how Sámi traditional knowledge develops over time as a part of the culture; it is not static: Sámi traditional handcraft, *duodji*, is part of Sámi traditional knowledge. *Duodji* artisans and craftspeople carry with them their *duodji* experiences, in which creativity begins in the woods when they are searching for materials. They know what to do and how to do it, but they add their personal style and create the product, so it can be useful in contemporary societies.

Balto [5] describes the Sámi traditional upbringing of children, where the main idea is to develop independent individuals who can survive in a given environment, and to give the children self-confidence, a zest for life, and joy. Consequently, much effort is put into giving the learners the opportunity to gain their own experience. Children are “allowed to try and do something and to fail. This is seen as an important part of the learning process” (p. 107). Keskitalo and Määttä [6] explain what characterizes Sámi pedagogy and contrasts it with what they denote as ‘ordinary teaching’. They point out that the students’ role in Sámi pedagogy is active, autonomous and flexible, as opposed to a more passive and responsive role in ordinary teaching. The role of the teacher also differs; in Sámi pedagogy the teacher’s role is to advise, guide, and trust the students, while the teacher in ordinary teaching is more dominant and distributes information. Because Keskitalo and Määttä are educated in Finland, we interpret their term ‘ordinary teaching’ to mean traditional teaching in Finnish schools. Their point is in line with Balto’s [5] claim, that in Sámi education the focus is on the learning process and less on the teaching; experience oriented learning is favored. From her perspective, this contrasts with the traditional ‘school way’ of teaching.

Sara [9] points out that nowadays, Sámi traditional knowledge challenges us and invites us to consider what different knowledges mean today. Teaching Sámi body-based measuring to non-Sámi students is expected to challenge the idea of the SI system as the superior way of measuring in all contexts.

1.2. The Report by Norway’s Truth and Reconciliation Commission

The inheritance from the Norwegianization policies during the 19th and 20th centuries, is that minorities experience prejudice, mobbing, hate, harassment and discrimination in interactions with the majority population. Schools were, and still are, arenas for such experiences. Norway’s Truth and Reconciliation Commission was appointed by the Norwegian Parliament in June 2018. The purpose of the Commission was to investigate the Norwegianization policy and injustice committed against the Sámi people, the Kven people/Norwegian Finns, and the Forest Finns [3]. The Commission handed over its report to the Norwegian Parliament’s presidency in June 2023. The Commission’s recommendations and suggestions aim to foster a more reconciled society with equity in language, culture, and identity [3]. These suggestions are addressed to the Norwegian Parliament and society at large.

The recommendations are subdivided into five pillars: (i) knowledge and dissemination, (ii) language, (iii) culture, (iv) prevention of conflicts, and (v) implementation of rules. Several of the recommendations and suggestions regarding Sámi issues are relevant for teachers in the compulsory school, but we find the first pillar most relevant for our study because knowledge is a central aspect of education. To increase students’ knowledge about the Sámi people and their culture, students can be offered to carry out an activity that is part of Sámi traditional knowledge instead of reading about Sámi topics or listening to a lesson. The Commission provides suggestions and recommendations that are relevant for teachers in the Norwegian school system. However, a search for the Commission’s report on the Norwegian Directorate for Education and Training’s web pages [14] yielded no results. We interpret this as an indication that bureaucracy is progressing slowly.

1.3. Measuring or Measurement—A Domain in Mathematics

According to Goldenberg et al. [15], the role of measurement in geometry is to specify “how much” by assigning a number to attributes such as length, area, volume, and angle. Philpot et al. [16] explains measurement for children in Grade 4 and below as the process of quantifying attributes of objects and phenomena (e.g., length and time). Regarding lengths, they specify ‘measurement’ as to measure, estimate, add, and subtract lengths. The units referred to are millimeter, centimeter, meter, kilometer, which all belong to the SI system. Van den Heuvel-Panhuizen [17] describes measurement as the connecting link between arithmetic and geometry.

Bishop [18] explains ‘measuring’ as one of six basic activities that together shape the cultural product that we call mathematics. He found these six activities in all cultures studied so far (in 1988). The other activities are ‘counting’, ‘locating’, ‘playing’, ‘designing’, and ‘explaining’. Bishop prefers to use the -ing form of the verbs instead of the noun forms. When asked about the reason why he named the activity ‘playing’ and not ‘play’, his reply was “By using playing I wanted to focus attention on the processes involved in playing. For too long nouns were used to describe the curriculum” [19]. Kawagley [20] describes the Yupiaq’s use of mathematics. Like Bishop, he also uses the verb form ‘measuring’. We chose to use ‘measuring’ and not ‘measurement’ for the students’ work in this paper.

Bakker et al. [21] studied the use, nature, and purposes of measurement in occupations for which a school-level qualification is required. In these occupations, the need for measurement can be categorized as meeting quality standards, making something fit (furniture, textile, assembly), monitoring, safety, and problem solving. Because mathematics in general and measurement in particular are often implicit and invisible in work processes, there is a risk that teachers in vocational education do not consider measurement to be important and do not give it enough attention. Bakker et al. suggest that the teaching of measurement in school should have a purpose that students can engage with, and that occupations and situations that appeal to students can offer inspiration.

1.3.1. Body-Based Units of Measuring

The use of body-based measuring units is an example of embodied mathematics; when measuring something using a body-based unit you count the number of times you repeatedly use that part of your body. According to Freudenthal [22] (p. 76) “[t]he best palpable material you can give the child is its own body”. However, palpable teaching materials are of great value, provided that the children can structure these themselves. When the child is measuring something using parts of its own body, in Freudenthal’s words it is structuring this palpable material. Lakoff and Núñez [23] point out that human ideas are largely based on sensory motor experiences. When measuring, the child uses its sensory motor experiences. This is supported by Van den Heuvel Panhuizen [17], who recommends connecting mathematics to our physical world and the phenomena that occur within it.

Kaaronen et al. [24] have documented the use of body-based measuring units in 186 cultures and found that fathom (arm span), hand span, and cubit/ell are the most frequently used measuring units. Similar techniques of using fathoms for measuring nets and ropes are documented. Fathoms are practically useful for these purposes because nets and ropes can then be measured without being stretched out, even if they are relatively long. Fishermen in Norway use fathoms to measure sea depth [25], and so do fishermen around the world. The Alaskan study group Math in a Cultural Context, MCC, has developed mathematics teaching materials based on Yupiaq body-based measuring units [26] and the group’s research [27–29] provide information about the use of these materials.

Regarding availability, body-based units of measuring do not require any additional tools; they are always available.

1.3.2. Sámi Body-Based Units of Measuring

The body-based measuring of lengths is embedded in the work of *duojárat* (Sámi handicrafters) and Sámi reindeer herders [30–32]. Table 1 provides an overview of some

body-based units of measuring, in the North Sámi language [31,33]. There are nine Sámi languages in total. When we refer to Sámi language, we mean North Sámi unless otherwise is explicitly stated.

Table 1. Some Sámi body-based units of measuring.

English Term	North Sámi Term
hand span (index finger to- thumb)	<i>goartil, čuvdegoartil</i>
hand span (long finger to- thumb)	<i>gaskasuorbmagoartil</i>
fathom (arm span)	<i>salla</i>
foot	<i>juolgi</i>
cubit/ell	<i>gardnjil mihttu, állan</i>
step	<i>lávki</i>
finger width	<i>suorpma govddu</i>

The first published text about Sámi mathematics was Nystad's [34] description of Sámi measuring and geometry. Her work was inspired by the Yupiaq [20,26] people in Alaska who also inspired Jannok Nutti's [32] text about Sámi calculating and measuring. Jannok Nutti's licentiate thesis [31] about *duojárat* and reindeer herders' mathematical practice went into depth about these issues. *Duojárat* use body-based measuring units in for instance leather sewing, sewing the traditional Sámi garment *gákti*, and measuring the length of Sámi fur shoe bands, *vuoddagat* [33]. Figure 1 shows *goartil*, the hand span between thumb and index finger. Aile's and Dina's video of Sámi boat builders' work shows how they use body-based measuring units when working with more solid materials [35]. Dunfjeld et al. [36] document that a two-digit number of South Sámi body-based measuring units is used among *vytnesjæjjah* (South Sámi handicrafters).



Figure 1. *Goartil*, or *čuvdegoartil*. *Čuvdi* means index finger. Photo: Dina N. Somby.

Jannok Nutti [31] and Ryd [30] document how *goartil* (hand span between thumb and index finger) is widely used for measuring snow depth among reindeer herders on the Swedish side of Sápmi: six *goartil* is the maximum snow depth for a reindeer to find food. To measure snow depth, reindeer herders traditionally turned a ski pole upside down and pushed it through the snow, to the ground. Next, they marked or checked where on the pole the snow surface was, before pulling the pole back up again. Then they could *goartilastit* (measure by *goartil*) the length of the ski pole that was below the snow surface. Nowadays, the herders use snow scooters instead of skis, but they bring their hands and a pole with them for measuring snow depths. Figure 2 provides an overview of several Sámi traditional ways of measuring. The subtitle states that it is important to remember that everyone have their own measures. The underlined passage emphasizes the importance of knowing your own personal body measures.

SÁMI ÁRBEVIROLAŠ MIHTIDANVUOGIT

Muitte fal dán: Juohkehaččas leat iežas mihtut gorutguhkkodagaid ektui ja lea dehálaš diehtit iežas gorutmihtuid!



1 Bealgi – Mihttu lea bealggi govdodat dahje sáhtttá maiddái leat bealggi guhkkodat. Dán mihtu atnit dávjá duddjodettiin. (1)



2 Čuvdi – Mihttu lea čuvddi govdodat dahje guhkkodat. Dán mihtu atnit dávjá duddjodettiin. (2)



3A Goartil – Mihttu lea gaska bealggi geažis čuvddi dahje gaskasuorpma geažčái. Dán mihtu atnit dávjá duddjodettiin. (3)



3B Gardnjil – Mihttu lea gaska gardnjilis giehtalađđasii dahje suorbmageahčái. Dáinna mihtuin lea vuogas mihtidit juoidá man ii sáhte mihtidit salain dahje goartiliin. (4)



4 Juolgi – Mihttu lea guhkkodat šušmmis juolgesuorbmageahčái. (5)



5 Lávki – Mihttu lea gaska ovttá juolggi juolgesuorpmaid rájes nuppi juolggi juolgesuorpmaid rádjái, dahje šušmmis šušmái go leat lávken okte. (6)



6 Salla – Mihttu lea guhkkodat giedaid gaskkas go doalat giedaid njuolga olggos gorudis. Okta salla lea dávjá seamma guhkki go olmmoš guhte lea salasteame. Dávjá go salasta de doallá juoidá giedain ja danin mihttun ii leat guhkkodat suorbmageažis suorbmageahčái. Dán mihtu atnit dávjá duddjodettiin dahje suohpana oasttedettiin. (7)



7 Čorbma – Sáhtttá leat mihttun golmma sierra ládje. Mot don doalat du čorpma, dat váikkuha man olu čorbmi čáhká. Čorbma mihtu sáhtttá geavahit go galgá sáhtet biepmu dahje mihtidit gákteoaji galljodaga. (8)



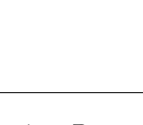
8 Suohpanbadji – Mihttu lea suohpana guhkkodat. Dán geavahat go háliidat mihtidit guhkes guhkkodagaid dahje gaskkaid, dahje man guhkin lea juoga man it veaje lávkut. (9)



9 Rievssatlávki – Mihttu lea guhkkodat rievssaha lávkkiiid gaskkas. Dán geavahat go háliidit govvidit hui oanehis gaskkaid.



10 Fanasbadji – Mihttu lea sullii fatnasa guhkkodat. Dan mihtu sáhtttá geavahit jus áiggut mihtidit man guhkkálaga muhtun dinggat leat.



11 Čalbmeravkaleapmi – Áigemihttu. Gowida man guhkes áiggi dus sáhttet čalmmit leahkit rahpasat ravkalkeahtá. Okta čalbmeravkaleapmi manná oalle jodánit, sullii sekunda.

Beanagullan – Mearkkaša man guhkás gullo go beana ciellá. Árbevirolaččat lohket ahte beanagullan lea sullii 10 km. Dálut, muorat, dinggat ja dálki sáhttet hehttet jienaid, dalle ii soaitte beanagullan leahkit nu guhkki. Geasset ja arvin/dálkin beanagullan lea oaniit, dálvet lea guhkit.

Iditbodda – Áigemihttu. Sáhtttá mihtidit man guhká juoidá lea ádjánan bargat. Ovdamearkka dihte, “mun ádjánin iditbotta hoigat muohttaga”, mihttui váikkuha ahte goas olmmoš jurddaša ahte lea idit ja man guhká idit bistá.

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Figure 2. Sámi árbevirolaš mihtidanvuogit–Sámi traditional ways of measuring. Poster by I. E. Hætta, J. Käsälä and P. M. Labba. Sámi Lohkanguovddaš, UNESCO and Sámi University of Applied Sciences, 2024. Reprinted with permission.

Kaaronen et al. [24] refer to Schefferus' 16th century description of Sámi skis; the skiers used two different skis where the kicking ski's length was the height of the skier, while the sliding ski's length was the skier's height plus foot. They point out that the Karelian skis, however, are longer; their length is the skier's fathom plus six finger spans. According to Birkely [37] and Vorren [38], in earlier days (as well as now) the Sámi used different types of skis depending on snow conditions and purposes. On some occasions they used skis of equal length, while for other purposes they used one sliding ski together with one shorter kicking ski, like Schefferus described.

Fyhn and Nystad [39] refer to the shopkeeper at Maritex Gávpi in Guovdageaidnu/Kautokeino, describing a typical situation when a customer buys a *suohpan* (lasso). The customer state how many *salla* (fathom) the lasso should be. The saleswoman measures by her individual fathom, and if it is a tall person she adds a bit (an individual's fathom

is approximately the same as body height). Then she must convert the measurement to meters, because the price is given per meter. In other words: the customer asks for a lasso of a certain number of fathoms and pays for how many meters that is.

Traditional Sámi ways of measuring have survived the introduction of the standardized SI-system which Norway adopted in 1960 [40]. This is remarkable, given Denmark's [41] documentation of a widespread use of Norwegian number words in day-to-day Sámi language in Guovdageaidnu/Kautokeino, where Sámi is the official municipality language. The survival of Sámi body-based ways of measuring as opposed to Sámi number words, indicates the usefulness, functionality, and robustness of Sámi body-based measuring. Other countries may have succeeded less than Norway in adopting the SI system. Readers from these countries may have more experiences with body-based measuring than Norwegian readers.

1.3.3. Body-Based Versus Standardized Units of Measuring

Kaaronen et al. [24] suggest that pressure to standardize grows primarily in large-scale societies. The early adoption of standardized measuring units is related to governance and administration, while body-based measuring units are more commonly used by practical workers and artisans. They argue that today's use of body-based measuring units has had, and may still have, advantages over standardized systems. Their study supports four explanations as to why body-based measuring is still preferred over standardized measuring units. The explanations are: (i) custom-made ergonomic design; (ii) the affordance of convenient motor routines; (iii) availability; and (iv) integration with local culture.

The researchers found references to ergonomic design by body-based units of measure in 25 cultures and "take indigenous ergonomics to be an especially favorable domain for the use of body-based units" (p. 949). The Yupiaq use proportionality properties of the body when making clothes as well as kayaks; things that need to fit a particular individual [27]. The length of a *suohpan* (lasso) is related to body size and the Sámis use *suohpan* of different lengths for different purposes. Your main *suohpan* is claimed to be 13 *salla* (fathom), which means the length of your *suohpan* is proportional to your body height. The Sámi boat builder Ove Stødle explains how Sámi traditional boats are made to fit one particular individual's length of arms and legs [35]. In modern-day Norway, it is common knowledge that the length of your ski poles (for cross-country skiing) should reach up to your armpit when you have shoes on. Most people can decide whether a pair of ski poles will fit them or not, but they most likely have no idea of the ski poles' length in centimeters. These examples show the functionality of individual body-based units of measuring.

1.3.4. The Role of Estimation in Indigenous People's Measuring

Juuso [42] explains the North Sámi term *meroštallat*, estimation, in a Sámi context. Estimation has a more largescale and wide-ranging use in Sámi culture than what is found in Norwegian mathematics textbooks. Eye measuring is a central part of *meroštallan*, in combination with contextual knowledge and experiences. Kawagley [20] presents a discussion among a group of Yupiaq elders on how to define mathematics. They agreed that the best Yupiaq definition would be "the process of measuring and estimating in time and space" (p. 51). He provides more information about the role of estimation: In ancient times it was not important to measure things precisely, but it was more convenient to use common-sense estimations and measurements. The land and water were collectively owned, so it was not necessary to subdivide and measure land parts. Consequently, the importance of estimating is a common feature of Sámi and Yupiaq cultures.

1.3.5. Introduction to Measuring in School

Students' understanding of measurement is often weak; for example, many are taught to simply read numerals from a ruler [43]. Szilágyi et al. [44] explain length as a comparative property of objects that embodies the amount of one-dimensional space between endpoints of the objects, which can be compared or quantified (measured). Sarama et al. [43] studied

the instruction when measuring was taught. Their study is based on three levels of understanding:

1. Length direct comparer. Children at this level can physically align two objects and decide which is the longest or whether they have equal length. The child uses the terms long, longer, longest;
2. End-to-end length measurer. The child can lay units end to end and indicate how many physical units will fill a linear space;
3. Length unit relater and repeater. The child understands that if you measure with cm and not inches, you will need more because each one is smaller.

One issue in Sarama et al.'s [43] work was the comparison of learning outcomes for children who use standard versus non-standard units of measuring. They designed two similar sets of instructional activities to support growth between the levels. They found that a greater percentage of those who used standard units made the level transition from length direct comparer to end-to-end length measurer. Sarama et al. point out that their study is consistent with those suggesting that initial length instruction should use familiar standard units. However, the non-standard units in their study included paper clips, connecting cubes, and toy trucks. These are examples of things that were not necessarily related to real world situations or other purposes that the students could engage with.

Sarama et al.'s [43] study did not include the premise that the measuring situations or the material should be relevant or meaningful for the students. This might be a weakness with their study, since Bakker et al. [21] suggest that the teaching of measurement should include a purpose that students can engage with. This, together with Freudenthal's [22] point that the students' own bodies are the best palpable material that they can be provided with, points to reasons for using or designing real-world situations where students use their individual body-based units of measuring. According to Kaaronen et al. [24], researchers have rarely studied how or why body-based measuring has been used. Because Sarama et al.'s [43] study was not designed to investigate body-based teaching units, their findings do not state how these body-based measuring units would perform.

The availability of body-based measuring units is important, because children always have their bodies with them, while the ruler is not always present. When the students use their body parts as tools for mathematics, the schools do not have to buy any teaching material for those lessons. The use of body-based measuring units is often related to particular cultural practices [24]. This might provide opportunities for teaching, because the students are then provided with examples that give meaning in different cultural practices. This might meet Bakker et al.'s [21] point, that measuring should be taught in relation to contexts that could make sense for the students.

1.4. Theory of Didactical Situations

The Theory of Didactical Situations (TDS) as described by Mangiante-Orsola et al. [7] and Strømskag [8], is a five-phase categorization of the teaching-learning process. Phase 1 is the teacher's introduction at the start of a lesson; here the 'didactical contract' between teacher(s) and students is clarified: the behavior that the teacher expects from the students and the behavior that the students expect from the teacher [45]. In an old-fashioned traditional mathematics lesson, a contract could be that the teacher will present one or more new rules for the students and how to use them. Then, the students will solve some similar textbook tasks (or computer tasks) where they apply these rules, and check whether they found the correct answers. Then some homework with more of these tasks might follow.

In the first TDS phase, the teacher transfers the problem or task to the students together with information about rules for action, with the aim of transferring the responsibility for solving the task to the students. This responsibility includes choosing strategies for solving the task. This phase of transferring responsibility, Phase 1, is called the 'devolution phase'. In Phases 2 to 4, the students work without considerable support from teacher(s), hence these phases are denoted as adidactical. Phase 5 is the teacher-led closing session of the lesson, which is also a didactical phase. The idea of TDS is that in a given situation, students

can take over ownership of a given problem if some target knowledge appears to them as an optimal solution to it [7]. This means that the students must find a purpose to engage in the situation. In addition, the target knowledge must appear as a useful tool for solving the problem.

A central issue in TDS is the ‘milieu’; all kinds of elements that may influence the situation when the teacher(s) transfer(s) control of the lesson to the students. The ‘milieu’ includes informative texts, teaching material and tools, other students, the students’ backgrounds and previous knowledge, rules for co-operation, and the physical environments [8]. In preparing for Phase 1, the devolution phase, the teacher needs to consider all parts of the ‘milieu’ [7,8]. The ‘milieu’ is important; it is crucial for the teacher to plan the lesson so that the ‘milieu’ can provide feedback to the students independently of the teacher. This means the teacher needs to consider the clarity of written information, combining students into groups, students’ previous knowledge, noise in the corridor, and supporting tools, among other things, when preparing the lesson. The reason for these considerations is that the aim is to hand over the problem to the students in a way that causes them to take over ownership. Once students have taken over ownership, three adidactic phases follow. The adidactic phases can be understood as a game with rules described by the teacher in the devolution phase. There are rules for this game, and to win the game you need to apply the knowledge being learned, in combination with your prior knowledge.

In the three adidactic phases, the students’ knowledge appears in three different ways, respectively; implicitly as an action, explicitly as a formulation and mathematically as a reason for validity [8]. These are denoted as ‘methods’ that the students choose for solving the problem or the task. In Phase 2 the ‘method’ is implicit or intuitive; the students’ use trial and error or something similar. In Phase 3, the ‘method’ is explicit, which means the students explain their ‘method’, while they use mathematics to argue why the ‘method’ is correct in Phase 4.

In the ‘action’ phase, students try out implicit solutions on their own with no assistance from the teacher. It is not necessary to communicate to solve a given task in this phase. In the ‘formulation’ phase, the students, if they work in pairs or triplets, will formulate their suggested solution to their co-operation partner(s). If the given problem is a task where co-operation is useful, there are stronger reasons for the students to formulate their suggested solution. They can then proceed to the ‘validation’ phase, where they calculate their solutions and discuss and check if their solutions sound reasonable.

If the students get stuck in one of the adidactic phases, there might be a need for support from the teacher. The teacher can then refer to the rules given in the first didactical phase, or the teacher can provide additional information. The fifth and final phase, ‘institutionalization’, is didactic. The teacher’s aim in this phase is to help the students recognize the knowledge gained in the game and to transform it into knowledge that can be used to solve other problems. For the teacher to do this, there is a need for discussion of what the *target knowledge* is when the lesson is designed.

2. Materials and Methods

Dina and Aile are native speakers of North Sámi. When they enrolled in teacher training, they had experiences from other professions and had children of their own. The lesson in our study took place in the second half of their fourth year of teacher training, so Dina and Aile had experiences with teaching practice in different Sámi schools before the actual period of practice. They wanted to try out a period of practice in a city school where Sámi students constitute a minority, to experience how that would contrast with Sámi schools, which are primarily in more rural areas. The term ‘Sámi school’ is described in Norway’s Core Curriculum [46]:

The Sami school shall ensure that the pupils receive education and training based on Sami values and the Sami languages, culture and societal life. The values in the objectives clause are also Sami values and apply in the Sami school. It is important to have a holistic-Sami perspective and an indigenous-people’s perspective in the Sami school, and

to focus on material and immaterial cultural heritage, such as traditional knowledge, duodji/duodje/duedtie [Sámi handicraft] and the importance of familial relations.

(p. 4)

The word Sámi is written with the letter á, which is not included in the Norwegian alphabet and neither in English. However, some texts, like this one, use the spelling Sami and other texts use the spelling Saami.

The pre-service teachers carried out a three-week period of practice at an *ungdomsskole* (a school for grades 8 to 10 in Norway's ten-year compulsory school) in the Northern Norwegian city of Tromsø, that has around 80,000 inhabitants [47]. As part of this period of practice, Dina and Aile developed and tested a teaching unit on Sámi traditional measuring in four of this school's Grade 8 classes.

Immediately after observing the pre-service teachers' teaching in the last of the four classes, Anne had a supervision meeting with them. During this meeting, Aile and Dina got feedback on their teaching, which they discussed with Anne. The main aim of such supervision is to provide pre-service teachers with an arena for self-reflection. The supervisor and the pre-service teachers review the lesson together, to identify what worked out well and to discuss possibilities for improvement. This meeting took the form of a semi-structured group interview, an interview with a set agenda and open-ended questions [48]. Because the interview questions are part of a discussion about the lesson, some questions were indirect, while others were more direct. Anne recorded the supervision meeting, which lasted for one hour, and transcribed it. Because the interview focuses on one actual lesson, we present the pre-service teachers' plan for this teaching unit before presenting the teacher educator's interview guide.

Our study's focus is the teaching unit and its plan, while the data is the audio recording of the interview. The context is a mathematics classroom and the discussions between the pre-service teachers and the teacher educator about what happened in the classroom. Our study of the audio recording of the interview is presented as a descriptive case study, because we provide a description of a phenomenon in its context [49–51]. Studies about Sámi teachers' teaching of Sámi issues in non-Sámi schools is an unexplored field of educational research. This justifies the choice of a descriptive case study as a design in order to provide insight into this innovative practice.

2.1. Indigenous Issues in Education and Education Research

Statistics reveal that around half of the Sámi population has experienced discrimination, and a significant proportion of the incidents take place in Sámi everyday life and in their local communities such as in classrooms and schoolyards [4]. This means that a worst-case scenario would be that the introduction of Sámi cultural issues in the mathematics classroom would evoke negative reactions from one or more students and/or parents. Due to the relatively high frequency of Sámi intimidation in Norway, which actually is one of the reasons for establishing the Truth and Reconciliation Commission [3], the pre-service teachers ran a risk when they decided to teach about a Sámi issue in this city school. This indicates that they are confident in their knowledge about Sámi traditional measuring, as well as with their own teaching. It also indicates that they support each other and are brave.

Research on Indigenous issues must be carried out in a way that is respectful and ethically sound from an Indigenous perspective [52]. Because Aile and Dina are Sámi pre-service teachers who grew up in Sámi culture, they are conscious of this issue in a different way than Anne, who had limited knowledge about Sámi issues before she was an adult. This is one important reason why the pre-service teachers were invited to be co-authors of this paper. They know how to express and explain Sámi cultural practices in a respectful way.

Dina and Aile will be aware of every possibility of tiny misinterpretations on the part of the researcher, and through their role as co-authors they are invited to correct these. Anne has experience from other studies where non-researchers are co-authors [19,53,54] of articles where their cultural practices are in focus. The quality of these studies benefits

from the co-authors' engagement when working with the analysis; they dare to disagree with the researcher's interpretations and explain why they disagree. Sámi school principal Elle, who has co-written several papers with Anne, described the difference between being an informant and a co-author in a personal communication: "We are tired of being research objects, we want to participate in the research that is about us". Dina and Aile read the transcribed interview data more carefully than they would have had they not been co-authors, which helps to improve the quality of the transcription. Their contribution to the analysis is considerable because many details are discussed from different perspectives. Because they are native speakers of North Sámi, Aile and Dina also add a valuable language perspective to the paper.

2.2. The Pre-Service Teachers' Plan for the Teaching Unit

Regarding mathematics, the aim of the teaching unit was for students to learn about Sámi traditional ways of measuring and to investigate the use of their own body-based units of measuring. This two-fold aim includes both the *diehtit* (know about) and *máhttit* (know by one's own experience) aspects of knowledge, as described by Guttorm [10]. Dina and Aile had carried out work with Sámi traditional ways of measuring from different perspectives: A book chapter in a Norwegian textbook for mathematics teacher education, a video [35] about skilled Sámi artisans' production of traditional boats, a conference paper [55] for an international audience, as well as presentations for Sámi mathematics teachers and Norwegian teacher educators. Through these experiences, the pre-service teachers acquired in-depth knowledge about how to present and explain these ways of measuring to different audiences.

2.2.1. Issues to Consider

Even though work with standardized and non-standardized units of measuring was part of the students' primary school curriculum [56], Dina and Aile knew from experience that Scandinavian teachers and teacher educators see non-standardized units of measuring as something 'primitive'; it is seen as something childish that precedes 'the real measuring' with centimeters and meters. They had to plan the teaching unit in a way that students would experience their individual body-based units of measuring as useful. This was important, because they wanted the students to experience that Sámi traditional measuring has a value. This is an example of Sara's [9] point, which states that Sámi traditional knowledge challenges us and invites us to consider what different knowledges mean.

Due to the school's organization, the lesson would last for 60 min. Through their background in Sámi teacher education, Aile and Dina were aware that one aim of education is to contribute to students' *iešbirgejeaddji*, independence, as described by Balto [5]. In order for students to become independent, they need to try things out in their own ways. This is not in line with the traditional mathematics task paradigm [57], where the teacher controls what happens; lessons that start with the teacher presenting an example and the students then work on solving similar tasks under the supervision of the teacher. Consequently, a considerable part of Aile's and Dina's lesson would consist of independent work by the students.

The pre-service teachers observed from the other classes that groups of four students often caused situations where two students worked while the other two focused on irrelevant issues or just sat and talked. Therefore, they considered a group size of three individuals to be the most favorable here. Because a considerable part of the lesson would be independent work on the part of the students, Aile and Dina sought to create situations where the groups themselves would find the solution to the given task, without any significant assistance from the pre-service teachers. The pre-service teachers would organize the groups so that students with special needs were in groups together with students who could provide necessary information to them.

Because most of the students were non-Sámi, it was expected that most students had never heard about Sámi traditional measuring. Consequently, centimeters and meters were

included because these were central to the students' previous knowledge about length measuring. When the actual students were in Grades 1 to 4, they followed the previous mathematics curriculum [56], which claimed that students should work with measuring lengths using standardized and non-standardized units of measuring throughout the first four years of primary school. When they started Grade 5, they met the present curriculum [58], which has no explicit focus on measuring lengths during Grades 5 to 7. Sámi traditional measuring is not mentioned at all in these mathematics curricula. This means the students most likely had not worked with length measuring in their mathematics lessons since Grade 4.

2.2.2. The Teaching Unit's Content and Structure

The teaching unit consists of four parts and is carried out during one lesson. Part 1 is an introduction by the pre-service teachers where they prepare the ground for the students' independent work, and Part 2 is group work where the students get familiar with their own body-based measuring units without any considerable assistance from the teachers. The main part of the lesson is a 30-min competition between the groups, Part 3. Part 4 concerns the 'optimal' solutions and this is where the winners are awarded.

In Part 1, the pre-service teachers present a selection of Sámi traditional ways of measuring, *goartil*, *salla*, *lávki*, which are found in Table 1, and also *čibberádjai* (up to your knees), *bahtarádjai* (up to your bum), *čalbmemihttu* (eye measure), *beanagullan* (the distance from which you can hear a dog bark), and *kaffekok* (how far you could travel during the time you needed to brew coffee). Aile and Dina show these in a PowerPoint presentation with four slides. They also provide examples of how and for what purpose these measures are in use. One reason for this is that, in a Sámi context, the use of measuring is part of a cultural activity. Thus, it is necessary to introduce a concept such as body-based measuring by including a cultural activity to which it belongs. A further reason is to provide information about how individual units of measuring are more useful than SI units for some purposes. The penultimate slide in Part 1 shows an overview of the members in each group. Part 1 ends with a slide containing necessary information about Part 2, as presented in Figure 3.

Group work
Gruppearbeid

Hint:
goartil, fathom, step, foot,
cubit/ell, thumb width
What else could be smart to
measure?

Tips:
goartil, favn, skritt, fot, alen,
bredde på tommel.
Hva mer kan være lurt å måle?

Measure and make an overview of your body measures. (e.g.: cm, m)
Mål og lag en oversikt over deres kroppsmål. (f.eks.: cm, m)

- - This overview will be used when measuring without measuring tape and ruler .
- - Oversikten skal brukes når dere skal måle uten målebånd og linjal.

Dere har 10 minutter på dere.
You have 10 minutes in total.

Figure 3. Information about the group work. English translations have been added by the authors.

Part 2 lasts for ten minutes. The PowerPoint slide in Figure 3 is on the screen during this part. Each group is given a measuring tape and a ruler. The students measure their own body-based units of measure and write these down, without any considerable assistance from the teachers. After ten minutes have passed, the preservice teachers collect the measuring tapes and rulers.

The following 30 min are dedicated to the competition itself; Part 3 in the teaching unit's plan. In order to win the competition, the group members need to cooperate. The PowerPoint slide for this task is titled 'competition', as presented in Figure 4.

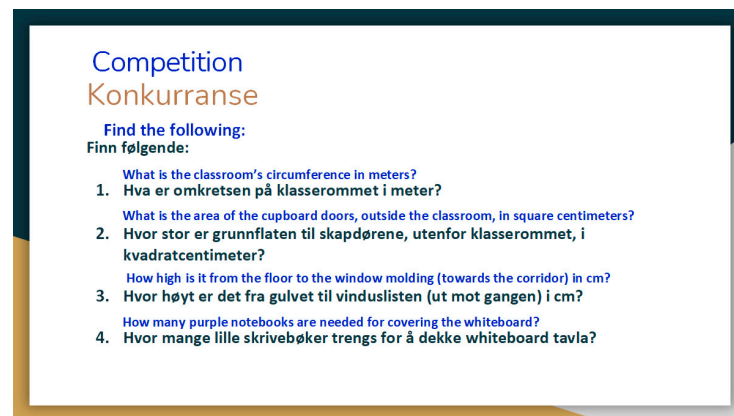


Figure 4. Information about Part 3, the competition. English translations are added by the authors.

The students are told that each group must measure some lengths without using any measuring tape or ruler. The only tool available will be their personal overviews of their own body-based measuring units. The aim of the competition is to find the four measures listed in Figure 4. Here, the students must formulate their suggested solutions with the other group members and the groups have to decide whose suggestions sound the most reasonable. They then need to formulate the group's solution in writing on a Post-it note and give it to the pre-service teachers.

After the competition, Part 4 begins. Here the pre-service teachers decide who the winners are and award them a prize. To award a winner, Dina and Aile have to measure using SI tools and calculate 'optimal' answers to the tasks. These calculations are carried out before the lesson. The pre-service teachers expect that the usefulness of individual body-based measuring units will emerge when answers that are almost correct are compared with the 'optimal' answers, as these answers are more or less the same.

During Part 4, the pre-service teachers explain which group was closest to each answer and reveal the winners' answers. Here, Aile and Dina aim to support the students by recognizing the knowledge gained and used through the group work. Thanks to their experiences from previous teaching, the pre-service teachers underline the importance of only telling the solution from the group closest to the correct answer. This is to protect those groups who had tried but "failed" from receiving comments and negative utterances from their peers. This is in line with Balto's [5] point that one aim of education is to develop independent individuals who experience a zest for life and joy. After the results from each question are read aloud, the pre-service teachers summarize the task and decide which group is the winner. They close the lesson by giving a prize to each member of the winning group. Because the teaching unit has a Sámi focus, the prize for each winner is a key ring attached to a Sámi braided colored cord called *ruvden*. Dina and Aile braid these gifts themselves, making the prizes personal.

2.3. Guide for the Group Interview

Because Dina and Aile had worked thoroughly with individual body-based units of measuring in different contexts, Anne did not need to ask about the teaching unit's plan before she arrived. The interview guide consisted of ten questions and Anne was prepared for the possibility that some questions would result in a variety of useful data, while other questions might not result in any useful data at all. The interview guide questions are as follows:

1. What was the focus of the teaching unit?
2. Did you experience differences in the four classes?
3. Did your teaching concern the SI system or just individual body-based measuring units?
4. What was the aim of the teaching unit?

5. Was the teaching unit “childish” for ungdomsskole students (Grades 8 to 10)?
6. Did you consider the students’ application of the knowledge in new contexts?
7. How did you experience oral activity among the students?
8. How did you experience the lesson’s relevance for the students?
9. How did you address the point that this teaching unit was about a Sámi issue?
10. How did you experience the inclusion of all students in the class?

Three of the questions started with the word ‘did’. These questions had an implicit ‘how’. By asking just ‘did’ and not ‘how did’, the questions enabled the pre-service teachers to elaborate on ‘how’. In addition to these three questions, four questions started with ‘how’. The use of the interrogative pronoun ‘how’ supports the idea of choosing a descriptive case study as research design [49].

The questions can be categorized primarily as mathematics education (questions 1, 3, 4, 5 and 6), and mainly education in general (questions 2, 7, 8, 9 and 10). The supervision meeting also included questions about the use of time and suggestions for relevant literature for the students’ further schoolwork. These questions are excluded because they are not considered relevant to the interview guide.

2.4. Analysis

The first part of the analysis intends to reveal the relevance of the teaching unit; we search through the interview data to identify utterances that relate to the following list of proposals from Norway’s Truth and Reconciliation Commission. The Commission proposes

- Increasing knowledge about the Sámi people and the history of the Norwegianization and injustice, because the population’s general level of knowledge is very insufficient
- A broad dissemination of solid knowledge about the Sámi people to reduce stereotypes, prejudices, and negative attitudes
- A dissemination of knowledge about the Norwegianization policies and injustice, and the consequences of the Norwegianization policies
- Providing the Sámi societies with better opportunities to participate in knowledge development and dissemination of their own history and culture
- That national institutions establish a tighter cooperation with Sámi institutions, to contribute to visibility, safeguarding, and dissemination of Sámi culture
- An aim directed focus on making Sámi languages visible in the public area
- A language lift and an emphasis on multilingualism in the society to provide constructive frameworks for revitalization of language for both schoolchildren and grown-ups
- Increasing visibility of the Sámi people and increasing knowledge about them in school

During the observation, Anne experienced that the pre-service teachers followed an inquiry-based approach where the students started by trying to perform examples of body-based units of measuring and started discussing this with each other as they were working in groups of two or three. Dina’s and Aile’s care and attention to detail when preparing for the teaching unit, in combination with the separation between students’ individual work with their personal measuring units and the following competition with group communication, prompted Anne to raise TDS [7,8] and its five phases during the supervision meeting. Three TDS phases concern the students’ independent work, while the first and the last phases concern how the teachers’ work frames these three phases. This enabled the pre-service teachers to add a new analytical perspective to their work, hence why the Theory of Didactical Situations, TDS [7,8], has been chosen as the framework for the second part of the analysis in our study. Two elements in TDS, ‘milieu’ and ‘method’, will be identified in the teaching unit. In addition, the analysis also concerns how the four parts of the teaching unit’s plan relate to the five phases in TDS. TDS will be used to explain what happens in the teaching unit and to provide suggestions for improving the plan. TDS provides an opportunity to explore whether a framework developed for non-Indigenous mathematics education may contribute to Sámi mathematics education.

Because the relevance of TDS appeared during the interview, the interview guide was not planned with respect to TDS.

3. Results

The focus of this research is to investigate the relevance of a teaching unit about Sámi traditional measuring for students in a Northern-Norwegian city, and to study how the plan for a teaching unit on this topic can be improved. The research questions are: (a) how did the teaching unit relate to the suggestions and recommendations of Norway's Truth and Reconciliation Commission regarding the Sámi? and (b) how can TDS explain and improve the plan for the teaching unit? The analysis shows how the teaching unit fulfils the listed suggestions and recommendations by Norway's Truth and Reconciliation Commission [3]. The analysis with respect to Theory of Didactical Situations, TDS [7,8] shows how the 'milieu' influences the students' co-operation, their independent work, and how the students' different ways of working correspond with the 'methods' in TDS. TDS is used to explain how the lesson proceeds and in addition, TDS also provides opportunities for improving the teaching unit's plan.

3.1. Suggestions and Recommendations from Norway's Truth and Reconciliation Commission

The pre-service teachers' completed teaching unit about Sámi traditional measuring is an example of what happens when the teachers and the principal at a non-Sámi school facilitate two representatives from SUAS, a Sámi institution, to disseminate knowledge about their culture. Dina and Aile made a point of presenting themselves as Sámi to the students, and to say that they wanted to show Sámi traditional ways of measuring, which are part of their culture. This is an example of what Norway's Truth and Reconciliation Commission recommends, when stating that Sámi societies must have better opportunities to disseminate knowledge about their culture [3]. In order to contribute to the visibility, safeguarding, and dissemination of Sámi culture, the Commission's report [3] recommends that national institutions establish tighter co-operation with Sámi institutions. However, the number of pre-service teacher students at SUAS is low, so it is not possible to establish this way of teaching as a large-scale project.

During the introduction part of the lesson, Dina and Aile explained how the different ways of measuring are still in use in current cultural practices. This kind of teaching prompts part of the majority population to learn and acquire knowledge about Sámi issues which, according to the Commission [3] might remedy some of the experiences of being hidden and neglected.

Anne asked the pre-service teachers how they would have responded if someone asked why they did not wear the traditional Sámi dress, *gákti*. The reply was that they wanted to show that Sámi culture is not limited to the traditional dress, *gákti*, and the reindeer, which together with the traditional housing, *lávvu*, constitutes the mainstream media's usual presentation of Sámi culture. Aile and Dina pointed out that Sámi culture also includes ways of measuring and units of measuring. Their claims are interpreted to mean that by focusing on Sámi ways of measuring, their lesson sought to reduce stereotypes, which is a recommendation of Norway's Truth and Reconciliation Commission [3].

Aile said that one reason why they chose to teach Sámi traditional measuring was to make this Sámi topic visible. Dina added that this was important because they had met teacher educators who said that when teaching non-standardized measuring units, the students ask when they will return to 'real' mathematics. Dina and Aile pointed out that many teachers and teacher educators consider body-based measuring more as a curiosity or historic approach than as something useful. Consequently, they wanted the students to experience the usefulness of non-standardized units of measuring and wanted the students to recognize and have experiences with this value of Sámi traditional measuring. This is an example of professional solid knowledge about the Sámi, which Norway's Truth and Reconciliation Commission [3] suggests as a tool for countering negative attitudes and prejudices.

The Truth and Reconciliation Commission's [3] recommendation of a broad investment in knowledge about the Sámi in education comes to surface regarding implicit work with Sámi issues. The lesson implicitly contributed to students' *máhttít* and *diehtit*, the two Sámi aspects of knowing body-based measuring as outlined by Guttorm [10]. Dina (transcript from interview, 23 April 2024, author's translation) expressed it this way, "And they [the students] could indeed do something that is Sámi, they joined in, in Sámi ways of measuring . . . and they learned *goartil*". Here, the pre-service teachers challenged the stereotype even more, by providing the students with experiences of body-based measuring in practical situations; the students were on their way to *máhttít* Sámi body-based measuring, and they were able to achieve some *diehtu* (knowledge) about Sámi body-based measuring. This was done by creating situations where students work independently on the activities, as Balto [5] as well as Keskitalo and Määttä [6] explain as being central in Sámi pedagogy. The teaching unit was designed so that all students experienced a Sámi cultural practice by performing Sámi traditional measuring themselves.

Aile said that the students now know they can measure things without using a measuring tape or a ruler; they know (*diehtit*) how to measure with their own body in situations where no standardized measuring tool is available. Knowing that you can measure without standardized tools is an expected learning outcome from the lesson, which encourage the students' creativity and autonomy in practical situations. Autonomy in practical situations is a valued skill in the Sámi upbringing of children [8] as well as in Sámi pedagogy [6]. A teaching unit which is focused on students' autonomy shows one aspect of Sámi child upbringing and pedagogy. This focus on students' autonomy is an implicit example of Norway's Truth and Reconciliation Commission's [3] recommendation of a broad investment in knowledge about the Sámi in education. The example is implicit because the pre-service teachers did not point out that this was a Sámi way of teaching.

The main part of the teaching unit was a competition where each group wrote down their solutions to the tasks. Based on the groups' answers, Dina and Aile chose the winners for each question in the competition. Each member of the winning groups received a prize, which was a key ring with a Sámi braided woolen cord attached, braided by Aile and Dina. They wanted to give the winners a Sámi prize. All the winners smiled when they got their prize. This Sámi prize is an example of how Sámi topics become more visible in society, as recommended by the Truth and Reconciliation Commission [3].

The lesson ended with everyone present counting aloud from one to ten in Sámi. Because all students participated in the counting, the pre-service teachers experienced this activity as a success. This is an example of making Sámi language visible in society, as the Commission [3] recommends. When Anne asked if there were any Sámi students in the class, Aile replied that it was possible, because she observed one student who knew how to count in Sámi. Dina added that the school organized the classes so that almost all Sámi students were together in one class. This was one of the other classes and not where the final teaching took place.

Aile and Dina experienced that the students appreciated this lesson, and some students even asked for more. This indicates a positive attitude towards Sámi issues, which is the opposite of prejudices and negative attitudes. This strongly indicates that the teaching unit is in line with more than one of the Truth and Reconciliation Commission's [3] recommendations.

3.2. Analysis with Respect to the Theory of Didactical Situations, TDS

The teaching unit was conducted in one lesson. The main component consisted of students' independent work where the teachers' role was to support and to stay in the background. The teaching unit consists of four parts, in which the students' independent work takes place in the second and third parts. TDS consists of five phases, in which the students' independent work takes place in the second, third and fourth phases.

3.2.1. Different Aspects of the 'Milieu'

Part 1 in Dina and Aile's work concerns transferring the group work task to the students, and preparing the ground for the competition. From a TDS [7,8] perspective, this corresponds to Phase 1. One intention of Phase 1 is to make sure that the students are familiar with the 'milieu'. This prepares the ground for students to work without any considerable support from the teachers through the following three phases. A variety of aspects of the 'milieu' were identified, and these were organized into four categories.

Physical items: Rulers and measuring sticks, PowerPoint slides on the board, students' body parts, other measuring items.

Human relations: Previous lessons in this class, the organization of the groups, the needs for co-operation.

Students' previous knowledge: Knowledge about measuring from the first four years of primary school, knowledge that differs from what can be expected in Sámi schools.

The school: The building and the classroom, rules and regulations, the time schedule.

The pre-service teachers' varied experiences with working with Sámi individual body-based measuring offered them knowledge that reduced the risk of possible situations where the students could end up depending on teacher support. These experiences meant that they could prepare for a 'milieu' that could support the students' independent work in Part 2 and Part 3 of the lesson.

Teaching a similar teaching unit in three other classes caused the pre-service teachers to consider more details in 'the milieu'. When teaching the first of the classes, Aile and Dina experienced that the students had no idea what else to measure after having measured their heights. Consequently, the pre-service teachers added the 'hint' at the top of Figure 3. Aile said this 'hint' provided the students with a concrete starting point. The 'hint' provides some examples, followed by the question "What else can be smart to measure?" In the classroom, this question worked as an invitation for the students to investigate what other body parts they could measure. The question's target group was everyone; anyone could answer it without knowing any mathematics beforehand. The question was open because a variety of answers could be considered 'correct'.

The need for the extra information, or 'hints', was explained as a difference that Dina and Aile realized regarding basic mathematical knowledge among students in Sámi schools and students in non-Sámi schools. The students at this non-Sámi school did not grow up with body-based measuring as an integral part of their day-to-day life. "... our children in Guovdageaidnu grow up with performing *goartilastit* [measuring by *goartil*] in kindergarten. So, they have grown up with this" (Dina, transcript from interview, 23 April 2024, author's translation). The information given in Figure 3 is an important part of the TDS 'milieu' [7,8]. The 'milieu' is not the same in a rural Sámi school as in a more urban non-Sámi school.

Measuring your *salla* is complicated to do alone because you cannot use a measuring stick properly while your arms are stretched out. This task is easier to do when you stand up, so the task invites the students to leave their chairs. This task contributes to co-operation among the group members; it is an important issue in the 'milieu'. The group members experience the usefulness of co-operating with each other before they meet the competition tasks.

There were enough physical items such as rulers and measuring sticks for all students, and the little pink notebook was on the board. Aile and Dina had planned the time schedule for the lesson carefully, and they presented this to the students to make the frameworks of the lesson as predictable as possible. They were also aware of expected questions. Centimeters and meters were important to include because that is the students' previous knowledge, and part of 'the milieu'. The target knowledge was the use of Sámi body-based units of measuring. Due to their previous experiences, the pre-service teachers were aware of how to present the values embedded in the target knowledge; its usefulness and the fact that you always bring your body with you.

Dina said that when explaining *goartil* and *salla* (fathom) to the students, they presented examples of how these measuring units are used. She interpreted the lack of discussion

and questions related to this presentation as an indication that the students grasped the concept of how these units of measuring are used; the ‘milieu’ supported the students in their work during Part 2 and Part 3.

The analysis with respect to the TDS ‘milieu’, explains how the pre-service teachers’ previous knowledge and experiences regarding the teaching unit resulted in a lesson plan that prepared the ground for students’ independent work.

3.2.2. The ‘Methods’

The students took responsibility for solving the tasks in Part 2 and Part 3 without considerable assistance from the pre-service teachers; from a TDS perspective, they progressed to the didactic phases. The group work in Part 2 of the plan corresponds to Phase 2 in TDS, the phase of action. Because of the extra information, the ‘hint’, the students measured a variety of body parts, not just the suggestions on the slide. This ‘hint’ was a crucial part of the ‘milieu’. The ‘method’ here was implicit or intuitive; the students used trial and error or something similar to what was presented in Part 1:

When we had provided them a list of things to measure, and they had done that, many of them continued to then measure, they measured their arm length from the shoulder, and measured from the knee to the foot, and many more measures like that, which they added by themselves.

(Aile, transcript from interview, 23 April 2024, author’s translation)

While working with the group task in Figure 3, Anne observed TDS activities where all students participated. They tried measuring the length of their own body parts in cm and m, but they did not come up with any problem-solving strategies, which according to Strømskag [8] is a characteristic of Phase 3’s explicit ‘method’, to explain. They had to work together because when measuring *salla* (fathom) it was complicated to work alone. So, the students got up and talked with each other at normal volume while they were measuring. The students measured their own body-based measuring units in cm and m. Measuring units such as the fathom is complicated to investigate alone, so co-operation was useful in that task.

Aile and Dina told that in the first three classes, all students measured their height. In this class, however, one boy said he would check whether his fathom equaled his height. Dina and Aile observed that more of the students investigated this. “They [the students] went over to the wall and held their *salla* (fathom) [vertically], then they checked if this equaled their height” (Aile, transcript from interview, 23 April 2024, author’s translation). This is an example of how a task opens up opportunities for students to formulate and check their own hypothesis. Based on a student initiative, the ‘method’ changes from ‘implicit’ to ‘explicit’; the students explain what they find. They do not need any ‘proof’ or formal validation; they explain the hypothesis by referring to their experience. None of them measured their height using standardized measuring units; they did not need any measuring units from the SI system to check whether their height equaled their *salla*.

In Part 2 of the teaching unit, the students alternate between implicit and explicit ‘methods’. This can be explained by the ‘milieu’: (a) the ‘hint’ text in Figure 3 invites all students to try out their own suggestions independently of their previous mathematical knowledge, (b) the use of arms in measuring *salla* invites the students to co-operate with peers, (c) the use of homogenous groups ensured that all students participated.

Regarding Part 3 in the teaching unit, the competition, the pre-service teachers observed groups in all classes that solved task 4 in Figure 4 without using any unit from the SI system. The task was “How many purple notebooks are needed to cover the whiteboard?” In one group, the students used one *goartil* and three fingers for measuring the book’s length and some other measure for its width. “They did not use centimeters or anything, they just used, ... they transferred from book to board. How many books is there room for this way [direction] and how many books do you have room for the other way” (Dina, transcript from interview, 23 April 2024, author’s translation). Dina and Aile observed that

the groups solved this task in a variety of ways. The groups had to agree on a solution to each task and write their solution down on a Post-it note to participate in the competition. The ‘method’ was explicit, which means the students formulated an explanation for their ‘method’, as in TDS Phase 3. In addition, they discussed and argued why their ‘method’ was correct; their ‘method’ was validation, as in TDS Phase 4.

The group members worked together on solving the tasks in Part 3. All group members participated in the measuring process. Some students worked together by checking each other’s measuring. Other students checked their own measuring by repeating it using a different way of measuring, to check if the results varied. These ‘methods’ are explicit; the problem to be solved is to find out whether different ways of measuring yield the same result. As in TDS, the target knowledge, which is the students’ knowledge about the usefulness of individual body-based measuring units, appears as useful for solving the problem, because the students had no standardized measuring tools available. After the measuring part, they discussed which calculations would be appropriate. Then, one or two group members performed the calculations using pen and paper. In TDS, this is the mathematical part of the problem-solving process, this ‘method’ is validation. The other group members commented on the calculations and solutions, and after an oral discussion they agreed on a solution. This measuring and calculation work was an interplay between the explicit and verification ‘methods’ in TDS, Phases 3 and 4. The students oscillated between formulation and validation before they agreed on a solution. Then each group wrote down their solutions on a Post-it note and gave it to the pre-service teachers.

The final part of the lesson consisted of (a) the pre-service teachers’ presentation of the solutions that were most appropriate for each task and (b) awarding the winners. The pre-service teachers read through the answers. They found that all solutions were close to the ‘optimal’ answers, so the usefulness of individual body-based units of measuring emerged through the groups’ solutions. Anne observed that the students paid close attention when Dina and Aile presented the optimal answers; the students compared these with their own results and compared their own validation with the winners’. This ‘method’ is validation. For each task, Dina presented the winner’s solution; the group that was closest to the ‘optimal’. In the previous lessons, the groups had their calculations in front of them while the ‘optimal’ solutions were presented. The group members discussed how their solutions looked compared to the presented solution. In this class, the group members returned to their seats once the tasks were solved. The students had to catch a bus immediately after the lesson, so the pre-service teachers ran out of time because the lesson ended a couple of minutes earlier. This meant that an important discussion was left out. The information about the bus belongs to the ‘milieu’, the school’s time schedule. The pre-service teachers did not consider the consequences of the bus for the time schedule when planning the lesson.

3.2.3. Other Remarks Regarding TDS

When TDS was presented to the pre-service teachers, Anne said that in Part 4 of the teaching unit they could ask the students if there was anything they could use this newly acquired knowledge for. Dina replied that they had explained this to the students in Part 1 of the teaching unit, “... when we explain *goartil* and *salla*, what it can be used for” (Dina, transcript from interview, 23 April 2024, author’s translation). Anne replied that this was about the students’ actions and not what the teachers did, and that the introduction part concerned how they as teachers would explain things. The pre-service teachers nodded and asked who had written about this. After Anne explained a bit about TDS, Dina said, “We have followed a theory that we did not know existed, that is ... hehe” (transcript from interview, 23 April 2024, author’s translation), and the pre-service teachers smiled. This utterance shows that TDS is relevant for explaining how the teaching unit was carried out and for improving the plan for the teaching unit. After the lesson, in light of TDS, the pre-service teachers suggested that they could have closed the lesson by giving a homework question like: ‘How do you expect to use this newly gained knowledge in your life?’. Then

the next day could start with a discussion where the students present their ideas about how they can use this new knowledge and experience. This suggestion is an example of how TDS contributed to improving the plan for the teaching unit. The pre-service teachers had not considered the importance of students' institutionalization.

Four issues from TDS clearly contributed to the analysis of the teaching unit: (i) How to get a structured overview of the 'milieu'; all factors that influence the students' independent work, (ii) the differences between students' individual work and co-operation in the didactic phases, (iii) the usefulness of the 'methods' for distinguishing between different ways the students work, and (iv) how the students' newly gained knowledge in the lesson can be transformed into knowledge they can use for solving other problems in their future life. In total, TDS served as a tool for improving the lesson plan.

4. Discussion

The analysis in this paper seeks to show the relevance of teaching Sámi traditional measuring for students in a Northern-Norwegian city and to contribute to improving the plan for a teaching unit about this topic. Two research questions are analyzed: (a) how did the teaching unit relate to the suggestions and recommendations of Norway's Truth and Reconciliation Commission regarding the Sámi? and (b) how can TDS explain and improve the plan for the teaching unit?

The analysis shows how the teaching unit fulfils several of the listed suggestions and recommendations by the Truth and Reconciliation Commission [3] regarding the Commission Report's pillars (i) knowledge and dissemination, (ii) language, (iii) culture, (iv) prevention of conflicts, and (v) implementation of rules. The pillar 'knowledge and dissemination' is most relevant for our study because dissemination of knowledge is a central issue in education. The pre-service teachers are two representatives of Sámi societies, who were given the opportunity to disseminate knowledge about their own culture. The teaching unit contributed to a group of young Norwegian individuals' knowledge about the Sámi people. The data shows several examples of a broad dissemination of solid knowledge about the Sámi people, which in turn may contribute to reducing stereotypes, prejudices, and negative attitudes. The students' counting activity at the end of the lesson was an example of how to make Sámi language more visible in society. The teaching unit was an example of co-operation between a national institution and a Sámi institution. Some proposals from the Commission were not met in this teaching unit: There was no focus on the Norwegianization policies because that was outside the scope for the lesson. There was neither any focus on a language lift nor on multilingualism for the same reason.

Regarding TDS, the analysis identifies issues from the 'milieu' and the 'method' that surfaced. These issues are relevant for explaining the students' independency and the development of the teaching plan. Analysis with respect to the 'milieu' shows that the 'hint' in Figure 3 invites all students to measure their body parts, the task invites everyone to participate independently of their previous mathematical knowledge. The inclusion of *salla* as one of the body-based measures contributes to co-operation among the group members; the group members experience the need for co-operation before they meet the competition task. The use of homogenous groups ensured that all students participated because there was no need for everyone in the groups to be able to read or write any advanced texts. With respect to the TDS 'methods', we see that the students alternate between implicit and explicit 'methods' in Part 2 of the teaching unit. In Part 3, the students' 'method' involved an interplay between the explicit and verification 'methods' in TDS.

The pre-service teachers designed and carried out the teaching unit based on their Sámi background and their knowledge about (a) Sámi traditional knowledge [9–12,59], (b) Sámi traditional measuring [31,32,39] (c) Sámi traditional child upbringing [5], (d) Sámi pedagogy [6], and (e) their experiences with creating a video about this topic [35]. Their presentations of Sámi traditional measuring at conferences provided them valuable feedback regarding the usefulness of individual body-based measuring. TDS shows how the teaching unit can be further developed by providing the students with a homework task,

the teaching unit lacked a focus on students' internalization of their newly achieved knowledge about Sámi traditional body-based measuring. When solving the competition tasks, the students used measuring units that they found appropriate; we interpret this as an indication to mean that they chose units that made sense to them. This is what Bakker et al.'s [21] point out, that the teaching of measurement has a purpose the students can engage with.

Adams and Harrell [60] point out that estimation provides a foundation for measurement. The role of estimation in Norwegian school mathematics is limited to deciding whether a solution to a given task is appropriate or not and is not explicitly related to measuring. Adamses and Harrell's study of the use of estimation among selected professionals in their work revealed clear links between estimating and measuring. This largely corresponds with the activity of estimation as an important part of Indigenous cultures [20,42]. The links between estimation and measuring in Sámi cultural practices constitute an unexplored field of study in Indigenous mathematics education. Investigating the links between estimation and measuring in Sámi and non-Sámi schools could provide a basis for future research. Such research is expected to contribute to the growing field of study about the relationship between language and culture in Sámi mathematics education.

In contrast to Sarama et al.'s [43] work, our study is based on individual body-based units of measuring; units that can make sense to the students. Our study challenges their study due to the choice of measuring units. Measuring using one's own body parts is expected to resonate with students' lives, due to Freudenthal's [22] point, that students own bodies are the best palpable material you can give them. Students' own body parts are relevant for measuring in their future lives; they can use these everywhere because they always bring their bodies with them. Body-based units of measuring are developed over time in practical situations, as opposed to the non-standard measuring units that are used in Sarama et al.'s study. A future study based on Sarama et al.'s teaching plan could replace their choice of non-standard measuring units with body-based units of measuring.

Aile and Dina were satisfied to observe that the four competition tasks were won by three different groups. Because there were three different winners, the competition tasks supported the idea that everyone could solve the tasks, regardless of their mathematics grades. Although the idea of demonstrating the usefulness of individual body-based units of measuring formed part of the basis for the teaching unit, unfortunately the teacher educator did not ask any questions about this in the interview. A follow-up study would benefit from including an explicit question about this issue. However, because the pre-service teachers are co-authors of this paper, they were able to confirm that most of the answers were close to the 'optimal' solution.

5. Conclusions

The Theory of Didactical Situations, TDS, is a framework designed for teachers' development of student-active education in non-Indigenous schools. The analysis in our study shows how TDS contributes to an improved plan for the Sámi pre-service teachers' teaching unit about Sámi traditional measuring. This new plan has five phases.

Phase 1: Planning, and literature studies about the central issues Sámi traditional knowledge, Sámi body-based ways of measuring, and Sámi traditional child upbringing. Karlsen et al.'s video [35] about Sea Sámi traditional boat builders can serve as an alternative to a meeting with Sámi elders. It is important in this phase that teachers pay particular attention to the need for co-operation when measuring your *salla*, and other factors that may contribute to the 'milieu'.

Phase 2: A didactic phase where the teacher presents examples of Sámi traditional measuring and relates these to the cultural contexts to which they belong. The teacher presents examples that show the functionality of individual body-based units of measuring. This phase lasts a maximum of 10 min. The control is being transferred to the students in this phase.

Phase 3: An adidactic phase where the students work in groups. They investigate their own body-based units of measuring and check how many meters and centimeters their measures are. The teachers' plan must consider how to support the students' alternations between implicit and explicit 'methods' in this phase.

Phase 4: An adidactic phase where the students work in groups to solve some competition tasks without access to any standardized measuring tool. The plan must consider how to support students' alternation between explanation and validation 'methods' in this phase; for instance, a solution that looks appropriate at first sight might be outperformed by a solution from a different point of view. Each group must write down their final solution to each task on a piece of paper and hand this to the teacher.

Phase 5: A didactic phase where the teacher, for each task, presents the solution that is closest to the 'optimal', while the group members can discuss their own solutions related to it. The winners are revealed and awarded. The teacher then asks what the acquired knowledge about body-based measuring can be used for in the future. This question is homework to be answered the next day.

Based on the findings, we suggest that our revised plan for this teaching unit can serve as a framework for teaching about Sámi traditional body-based measuring in non-Sámi schools. This plan will be in line with several of Norway's Truth and Reconciliation Commission's recommendations [3]. According to the Commission's report, teaching about Sámi issues should be carried out by Sámi teachers. Because there is a huge lack of Sámi mathematics teachers, we recommend that non-Sámi teachers use our plan with support from this article as a model for teaching about Sámi traditional measuring.

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