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Parental Expectation, Attitudes, and Home Numeracy Environment in Korea and in the U.S.: Potential Sources of Asian Math Advantages

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Abstract: This present study examined relations among parental math attitudes, expectations, and practice, and preschool children's math achievement (i.e., parental math practice as a mediator) that may differ between Korean and U.S. samples. We examined measurement invariance to minimize the bias and inaccurate estimates in scores in two samples, which is a common barrier in cross-cultural studies. The Korean sample comprised 232 children (mean age = 54.58 months) and their parents from large urban cities and two other provinces. The U.S. sample included 146 preschool children (mean age = 52.49 months) and their parents in an urban area. We utilized measurement invariance to investigate whether the parental math practice has the same meaning across the two samples, along with mediation group comparisons. U.S. parents tended to have more positive math attitudes and higher expectations, while Korean parents were likely to be more involved in math practice with children at home. Korean children scored significantly higher on math achievement than the U.S. children. Among the Korean sample, higher parental math expectations were associated with higher levels of parental math practice, which was, in turn, associated with higher levels of child math outcomes after controlling for child age and SES. On the contrary, parental math expectations were unrelated to their math practice or child math outcomes in the U.S. sample. Parent math attitudes were not associated with any variables in both samples. These findings explain Asian math advantages that emerge early and offer insights into cultural processes (i.e., the importance of parental math practice) that may play a different role in children's math outcomes.

Keywords: parental expectation; home numeracy environment; Asian math advantages; culture

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1. Introduction

Children demonstrate a rapid increase in math skills such as counting and number sense in early years [1,2]. The important predictive value of early mathematics skills in later mathematics and literacy skills, even stronger than that of early reading skills for later reading ability, has been well-documented [3–5]. However, children display substantial variation in their math knowledge and skills and growth rates as early as preschool age [1,6,7]. This individual variation is often explained by differences in the environment such as the quality of home learning environments (HLE) [8,9]. Further, some studies found that individual differences in math achievement vary by race, ethnicity, and culture. For example, in cross-cultural studies, researchers often include East Asian countries (e.g., Chinese, Korean, and Japanese) as non-Western counterparts for their distinctive developmental and academic outcomes. Results from international studies such as the Program for International Student Assessment (PISA) [10] and the Trends in International Mathematics and Science Study (TIMSS) [11] and cross-cultural studies [1,12–14] have shown that East Asian students consistently outperform students in other developed countries in mathematics. In the United States, Asian American children consistently outperform

other racial and ethnic groups in mathematics, although they have been educated in the same school and educational systems [15,16]. In general, East Asian children also show a higher growth rate in math skills than other racial, ethnic, or cultural groups in their early years [1,2]. This gap exists irrespective of age and begins to emerge as early as primary grades and even before formal schooling in various math skills [1,14,17–19]. Therefore, it is critical to explore the cultural assets and differences that explain these early variations between East Asian and Western children.

A handful of cross-cultural studies comparing East Asians and their Western counterparts have sought an answer to potential differences in what parents expect and do with their children [19,20] that may stem from their cultural values and emphasis on learning and education [21]. For example, East Asian parents of four-year-olds have tended to have higher academic expectations than North American parents [22]. Even within the same context of North America, those with East Asian heritage have higher academic expectations and involvement with their children (aged 5) than those with European backgrounds [23]. These varying parental expectations and involvement may reflect specific cultural values and norms [24] and are likely to play a critical role in children's math outcomes. However, cross-cultural studies with large groups of children from diverse socio-economic backgrounds and rigorous designs that compare the role of parents and home numeracy environments and how they are differently associated with children's math outcomes in both cultural contexts are scant. Thus, this current cross-cultural study examined similarities and differences in parental expectations, attitudes, and home numeracy environments (HNE) and preschoolers' mathematics outcomes in Korea and the U.S. Specifically, it aims to compare direct and indirect effects of parents' mathematics expectations and attitudes on preschoolers' mathematics outcomes.

1.1. Parental Attitudes and Expectations

Parenting, parent–child interactions, and HLE have consistently been among the strongest factors associated with children's early math learning and development [25,26]. Yet, research on parental attitudes and expectations' role in early math learning has been much less explored than reading and literacy skills [27]. While parents value literacy and believe reading in the home is generally important, many parents feel less positive about math [28]. Regardless, the limited available research indicates that these parental variables predict children's numeracy scores [29].

Parental attitudes towards math include their feelings about math (e.g., whether they find math enjoyable) [30] and their beliefs about their abilities to do math themselves [29,30]. Another study [31] found that the more positively the mother perceived math, the more actively the child learned math. Conversely, parents with higher math anxiety or more negative feelings towards math may engage in fewer activities in the home.

In addition, studies have consistently found that parental expectations about their children's educational futures predict student achievement [32] across families with diverse racial and ethnic backgrounds [33,34]. Parental math expectations can include what parents believe to be the necessary math skills for children at a particular age and whether it is important to meet benchmarks before a given age [35]. It includes a question of whether parents of preschool-aged children expect them to be able to count to 100, for example. Another study [29] found that parents believed daily exposure to reading was more important than daily exposure to math and had no clear expectations or goals for their children's math learning, indicating lower parental expectations for math than literacy. Parents' beliefs about the importance of their children's math skills significantly predicted their preschool-aged children's math performance even when controlling for all other predictors [36]. Nevertheless, less is known about how parental expectations influence their interactions with children, particularly in relation to the development of young children's mathematics knowledge [37,38].

Parental attitudes and expectations are related constructs [27]. For example, parents who value math also have higher expectations about children's ability to engage in math

content [39]. Research demonstrates a wide variety among parents' expectations, attitudes, and practices related to math. Much of the research regarding parent expectations and attitudes towards math is still emerging, and the relationships among the variables are often unclear [40,41].

1.2. The Home Numeracy Environment

The HNE, as part of the HLE, is an umbrella concept that includes the many ways parents and young children interact around math at home, including counting, number recognition, and logic games [41]. This consists of formal math activities, where parents intentionally teach children about numbers, quantity, or shapes [42], and informal, less structured activities where math instruction is not an explicit focus. For example, families play card games, measure cooking materials, count money, or even work with calculators. Although these informal activities do not intentionally and explicitly teach specific math skills, they allow children to interact with math more naturally because they may be part of what children were already engaged in [43].

The quality of the HLE has been strongly associated with numeracy skills in the first year of preschool [8] and kindergarten [44,45]. Several studies have demonstrated that family home engagement is broadly related to children's numeracy [46] and number knowledge [42]. More specifically related to math, parents' reports of engaging in numeracy and spatial activities were positively correlated with their preschool child's broad math knowledge [27] and elementary schoolers' math skills [47,48]. Finally, engaging in parent-child numeracy activities has been significantly related to later performance in counting and addition [19].

There are variations in the associations between HLE and children's math achievement [49–52]. These heterogeneous findings could be due to sample features, methodological variation in how the HNE is measured, or a relationship between the two that is not as strong as other influences [26,53]. However, the existing literature, including several recent meta-analyses, supports and weighs the positive associations between various aspects of the HNE and children's math achievement outcomes [26,36,53–56].

Some researchers suggest that the HNE plays a significant role as a mediator between parent beliefs and children's academic outcomes; parents who value math or have higher expectations regarding their children's math engagement report doing math at home more often, which in turn, is associated with improved math outcomes [57]. Another study [48] found that parents with higher academic expectations had more positive attitudes toward numeracy and reported engaging more frequently in numeracy activities. Parental attitudes may influence how parents approach math in the home. Parents who value math may directly teach their children math skills. They may also provide more opportunities for informal exposure to math by offering materials that encourage counting and sorting or by doing activities like cooking or shopping, which involve math. Parents may introduce mathematical language, i.e., words like more, less, or how many, during play or conversations. Parents who believe their child can engage in math tasks tend to provide math activities at home more often [51,57] and engage in more advanced math activities [30].

1.3. Cultural Differences in Parental Expectations, Attitudes, and Practices

Children's numeracy knowledge and skills greatly vary across cultures [2,14,17–19]. Many studies have focused specifically on the extraordinary academic achievement (i.e., in mathematics) among Asian students, especially East Asian students [1,17,58]. Although the potential role of genetic and biological factors in explaining racial and cultural differences in cognitive skills, in general, has not been fully ruled out [59], a large body of research has suggested that these differences are primarily attributed to environmental factors. These factors may be more contextual and culturally bound, such as language spoken, family demographics, cultural values, parenting practices, and educational systems [2,17,60].

Among these environmental factors, prior studies point out that parents' expectations and the amount and type of parental involvement in their children's learning associated

with their academic outcomes and meanings may differ across cultures [60,61]. These varying expectations and practices may explain large variations in children's math achievement in different countries or cultures. There may, however, be cultural differences in home numeracy environments [26]. For instance, studies have found that some Chinese American, Black, and Hispanic or Latino caregivers emphasized more direct math activities and formal education practices than European American and White caregivers [62]. Culture may contribute to families' participation in direct versus indirect instruction [63]. Another study [64] found that home numeracy activities mediated the relationships between parental attitudes toward learning and children's numeracy interests.

The parental role at home is valued in all racial/ethnic groups. Still, parental investment and dedication to their children's academic achievement are more highly emphasized in Asian parent groups than in other racial/ethnic groups [2]. For example, in general, East Asian parents tend to have higher academic expectations than North American parents [22]. Although they are less involved in school activities (e.g., volunteering), Asian parents report more time devoted to their children's education, provide more explicit teaching and effective support, and make a more significant educational investment [34,60]. These reflect pro-education cultural values and beliefs rooted in the Confucian philosophy, which highly emphasizes success and self-improvement through continuous hard work and effort, the parental obligation to monitor and support learning, and educational achievement for the sake of the family [2].

Similarly, mathematics achievement is highly valued in Asian culture in general. Therefore, parents and schools in East Asian countries focus on teaching math skills and providing children with formal math instruction in the early years [3,19,65]. These strong values in mathematics skills and the importance of supporting these skills early lead to frequent math instruction at home among East Asian parents [66]. For example, cross-cultural research found that East Asian parents not only involved their children in math activities (e.g., counting, identifying shapes, using ordinal numbers and spatial numbers, comparing sizes) but also used more effective instructional strategies and more encouragement for their involvement in math activities, compared to other racial/ethnic groups [19,22,67]. These studies suggest that it is critical to consider how cultural values and norms may impact parents' expectations, attitudes, and practices as they shape children's experiences at home and academic outcomes [37].

Varying parental expectations, attitudes, and practices across cultures may affect children's outcomes differently. A few studies have shown that parental expectations, attitudes, and practices are differently associated with one another and with children's math achievement [19]. For example, researchers [19] found that while there is a significant association between parental math instruction and children's learning of numbers among Chinese participants, the association was not crucial for American participants. Although they did not focus on East Asian parents, there are different patterns of associations among parental attitudes, math practices, and children's math outcomes between Greek and Canadian families [41]. While parental attitudes toward math were not significantly associated with children's math outcomes among Greek families, these associations were significant for Canadian families.

On the other hand, while parent math practices at home mediated the associations between their attitudes and children's math outcomes among Greek families, there was only a partial mediation effect among Canadian families. The associations among high parental values and expectations, positive attitudes toward math, the quality of the HLE, the amount of numeracy talk, and children's numerical knowledge have been well-established in many different cultural backgrounds [17,23,25,37,42]. Still, much of this research has been conducted primarily in Westernized contexts (e.g., Canada, U.S., Finland, and Greece). More comparative work is needed to uncover how the role of the HNE in the associations of parental attitudes and expectations with child math outcomes differs across cultures.

1.4. Present Study

Most studies on parental expectations, attitudes, and math practices at home have been conducted in Western countries with limited cross-cultural studies available, and thus, the generalization of the results may be limited to different cultural and educational contexts. The extant cross-cultural studies that examine the differences between East Asian countries and Western countries primarily focus on the Chinese or Chinese American population. Other East Asian countries, such as South Korea, have rarely received attention in research. Although South Korea shares similar cultural traditions and values with China (e.g., value in and high expectations for children's math skills), it is distinct in many ways, including (a) extremely high parental educational focus and investment, known as educational fever; (b) availability of and easy access to various tutoring and educational resources in the private sector; (c) low birth rate, low employment, and highly competitive job markets for recent college graduates [13]. Thus, this cross-cultural study examined relations among parental practice, expectation, and attitudes toward children's academic skills, home environment, and early mathematics skills after controlling child (age, ethnicity) and family demographics (e.g., parents' education) in Korea and the United States. We asked the following three questions.

Research question #1: Are there differences between Korea and the U.S. in preschoolers' math outcomes and parental math attitudes, expectations, and practices at home?

Research question #2: Are parental math attitudes, expectations, and practices associated with preschoolers' math outcomes in Korean and U.S. samples?

Research question #3: Do parental math practices at home mediate the association between parental math attitudes and expectations and preschooler's math outcomes in Korean and U.S. samples?

2. Method

2.1. Participants

We recruited participants from South Korea and the United States. The Korean sample comprised 232 children (mean age = 54.58 months; 53.4% male). The participants were sampled from large urban cities in Seoul and two other provinces in South Korea. The U.S. sample included 146 preschool children (mean age = 52.49 months; 50.7% male) and their parents (93.15% mothers) in childcare settings in a large urban setting. Families came from diverse racial (43.8% Black, 33.6% White, 6.8% Latino) and socio-economic backgrounds. Although there are more children in the Korean sample, the characteristics of the Korean and U.S. samples are comparable regarding gender and age, parent income, and parent education (see Table 1). Korea is a homogenous group, while the U.S. sample represents a racially diverse urban population in the Southeastern region. Both samples came from diverse family SES backgrounds.

Table 1. Participant demographics for Korean and U.S. samples.

	Korea (N = 232)		**U.S. (N = 146)**	
Child Age (Months)	**Mean (SD)** **54.58**	**Range** **42–63**	**Mean (SD)** **52.49**	**Range** **42–63**
Demographic Variable	*n*	%	*n*	%
Child gender				
Male	124	53.4%	74	50.7%
Female	108	46.6%	72	49.3%
Child races/ethnicity				
White	0	0%	49	33.6%
Korean/Asian	225	97.8%	7	4.8%
Black	0	0%	64	43.8%
Hispanic	0	0%	10	6.8%
Other race/ethnicity	5	2.2%	16	11.0%

Table 1. *Cont.*

	Korea (N = 232)		U.S. (N = 146)	
Child Age (Months)	**Mean (SD) 54.58**	**Range 42–63**	**Mean (SD) 52.49**	**Range 42–63**
Parent income				
$0 to $20,000	5	2.2%	22	16.8%
$20,001 to $40,000	28	12.2%	20	15.3%
$40,001 to $60,000	47	20.5%	8	6.1%
$60,001 to $80,000	61	26.6%	7	5.3%
$80,001 to $100,000	29	12.7%	10	7.6%
over $100,001	59	25.8%	64	48.9%
Parent education				
No high school	0	0%	1	0.7%
High school graduate or GED	30	13%	27	19.9%
Some college	56	24.3%	16	11.8%
Bachelor's degree	115	50.0%	34	25.0%
Master's degree or above	29	12.6%	31	22.8%
Doctorate	0	0%	27	19.9%

2.2. Procedures

After obtaining approval from the University Institutional Research Board (IRB) that allows a human-subject study, the researchers in Korea and the U.S. invited center-based early childhood settings to participate. The data collection process in Korea and the U.S. was identical. After we received permission from a center director, we visited the centers and explained the purpose of the study and procedure to the director and teachers. We also distributed a packet containing a consent form and a questionnaire for parents. Once parents agreed to participate by returning the consent form, they were asked to complete a paper and pencil questionnaire on family demographic information (e.g., child age, parental education, family income) and their math expectations, attitudes, and home numeracy environment. They returned the questionnaire to an envelope placed in the classroom.

Once we obtained parental consent and a questionnaire, we scheduled a visit to assess children's math skills at the center. Trained undergraduate and graduate research assistants administered two math assessments in a quiet room at the early childhood center. Before beginning the assessment, children were instructed about what they would do. We also informed them that they could stop and return to the classroom anytime if they did not want to participate. Considering their attention span, each child's assessment did not exceed 15–20 min.

2.3. Measures

2.3.1. Early Math Skills

Two measures, including the Applied Problems subscale of the Woodcock–Johnson III (WJ III) [68] and the Preschool Early Numeracy Skills Test—Brief Version (PENS-B) [69,70] were used to assess children's mathematical understanding and knowledge. We also translated and back-translated both measures initially developed in English to Korean for the Korean sample. The WJ III is a nationally normed math assessment, and we only utilized the portion of the applied problem of the evaluation. This section measures children's abilities to solve oral problems, including counting pictured objects and simple story problems (e.g., 'Show me two fingers' 'How many ducks are in the water?'). Numerous studies have used this measure to assess math achievement and have an average Cronbach's α for preschool-age children of 0.91 [71]. In this current study, the internal consistency reliability is 0.85, and the total standardized score, which accounts for the child's age, was used. We used a total raw score, not a standardized score, for the U.S. and Korean samples because it was not standardized in Korea.

Preschool Early Numeracy Skills Test—Brief Edition is a 24-item measure of children's early numeracy skills (PENS-B) [69,70]. Items included 'How many dogs are there?' and 'When you count, what number comes before 5?', which addresses multiple mathematical

domains, including one-to-one counting, cardinality, counting subsets, subitizing, number comparison, set comparison, number order, numeral identification, set-to-numerals, story problems, number combinations, and verbal counting. This covers a broader range of early numeracy skills than other standardized math tests such as Woodcock–Johnson III. The Cronbach's alpha in this current study is 0.85. While we used a standardized score for the U.S. sample, we used the total raw score for the Korean sample because the standardized score was normalized based on the U.S. population.

2.3.2. Parents' Math Expectation, Attitudes, and Home Numeracy Environment

Parents completed questionnaires modified from those used in studies regarding mathematics practices at home and their attitudes toward academic skills [41,72]. We also translated and back-translated the questionnaires developed in English for the U.S. sample to Korean for the Korean sample, including questions regarding HNE, parental attitudes toward mathematics, and parental math expectations. The measure includes three subscales: positive parent attitudes toward math (4 items), parent math expectation about their child's numeracy skills (7 items), and home numeracy environment (9 items) that are relevant in both cultural contexts. For example, we compared items from the Korean version of the Home Environment Measure developed and validated in South Korea [73] with the Home Numeracy Environment Measure from Western culture [41]. We identified items common to both measures, such as a child's ability to count to 20 and perform addition up to 10, which are likely less influenced by cultural factors. Example items for the parent attitudes subscale include 'When I was in school, I was good at mathematics', 'When I was in school, I enjoyed mathematics', and 'I find mathematics activities enjoyable'. This scale is a 5-point Likert scale with ratings ranging from 1 = strongly disagree to 5 = strongly agree. For parent math expectations, parents were asked to what extent they expect their child to master specific numeracy skills at the end of kindergarten (e.g., child's ability to count till 20, both forward and backward, child's ability on addition to 10). The list of home numeracy activities included nine items that asked parents to indicate the frequency with which their child participated in the various math-related activities using a 5-point Likert scale with ratings ranging from 1 = never to 5 = more than once a day. These math-related activities include counting objects (e.g., how many spoons?), guessing the names of numbers in books and flyers, learning simple addition (e.g., 2 + 2), collecting objects (e.g., cards, stamps, spinning tops), reading storybooks that involve numbers (e.g., *The Colorful Zoo*, *Twelve Baby Ducks Are Too Many*), singing number songs (e.g., Carrot Song), and discussing time using calendars and clocks. The psychometric properties in our study are 0.80 (Cronbach's α).

2.3.3. Control Variables

We included a child's age in months and parental income in our analysis model as control variables. To ensure comparability between the Korean and American samples, parental monthly income was standardized. Income was scaled from 1 to 5 for each sample: in Korea, categories ranged from 1,000,000 won to over 5,000,000 won, and in the U.S., from $20,000 to over $100,000. This standardization facilitated meaningful comparisons across both cultures. These covariates were included because their associations with children's math achievement have been demonstrated in previous studies [47,74,75].

2.4. Data Analysis

The data analyses involved five steps. First, we conducted tests of homoscedasticity and normality of the Korean and U.S. samples. The descriptive statistics and the result from the independent sample *t*-test are presented in Table 2. Second, we used parcels of items as manifest variables to increase reliability and to get a more stable parameter estimate [76,77]. Item parceling is a structural equation modeling technique aggregating the average of two or more items. The advantages of item parceling included more significant commonalities, a higher ratio of common-to-unique factor variance, lower

likelihood of distribution violations, less violation of normality assumptions, and reduction in idiosyncratic characteristics of items [76–78]. The empirical study indicated that item parcels best represented the multifaceted scale [79].

Table 2. Means, standard deviations of child outcomes, and explanatory variables by country.

	Korea (N = 232)		The U.S. (N = 146)		
	Mean	**SD**	**Mean**	**SD**	***t*-Test**
Parent Math Attitude					
Good at Mathematics in school	2.84	1.14	3.69	1.17	−6.83 ***
Enjoy math in school	2.62	1.16	3.43	1.27	−6.26 ***
Math-related job	2.22	1.28	3.12	1.39	−6.22 ***
Importance of exposing math	2.65	1.14	3.54	1.20	−7.04 ***
Total	10.34	4.11	13.79	4.26	−7.61 ***
Parent Math Expectation					
Count till 20	3.48	0.74	3.80	0.54	−4.46 ***
Count without hands	3.42	0.70	3.61	0.71	−2.45 *
Count in groups of 2, 5, or 10	2.67	0.93	3.16	0.83	−5.13 ***
Addition till 10	2.87	0.90	3.26	0.88	−3.98 ***
Addition till 20	2.48	0.93	2.93	0.90	−4.49 ***
Subtraction till 10	2.76	0.90	3.01	0.89	−2.54 *
Subtraction till 20	2.40	0.91	2.68	0.98	−2.80 **
Total	20.07	4.90	22.48	4.72	−4.56 ***
Home Numeracy Environment					
Counting objects	4.10	0.75	4.06	0.79	0.49
Measuring when cooking	3.25	1.02	2.46	1.02	0.71 ***
Identifying written numbers	3.52	1.12	3.20	1.16	2.56 *
Learning simple sums	3.05	1.23	2.82	1.27	1.72
Making collections of objects	2.45	1.35	2.99	1.10	−3.91 ***
Reading number storybooks	2.92	0.94	3.00	1.12	−0.79
Rehearsing counting rhymes	3.26	0.77	2.92	1.21	3.25 **
Using calendars and dates	3.20	0.18	2.66	1.19	4.17 ***
Sorting by size, color, shape	3.42	0.94	3.53	1.07	−1.05
Total	24.72	24.08	15.49	37.96	2.89 **
Children's Math Achievement					
PENS-B	15.18	5.50	14.19	5.43	1.71
WJ: Applied Problems	16.01	4.64	14.24	4.70	3.60 ***

Note: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. PENS-B: Preschool Early Numeracy Skills—Brief, WJ: Woodcock–Johnson.

In this study, items were parceled with a 'uniqueness distributed strategy', which distributed similar items equally across parcels, as all the latent constructs satisfy the unidimensionality assumption. The uniqueness of the distributed strategy for parcel items is that it distributes equality to the items that most contribute to the variance of the factor loadings in each parcel. Because all four latent constructs satisfy the unidimensionality assumption, each latent construct was divided into three parcels, using all items from each scale.

Third, we utilized measurement invariance to investigate whether latent constructs have the same meaning across the Korean and U.S. samples. The measurement invariance was tested by imposing a series of parameter constraints into CFA models using Mplus [80]. Although the specific steps vary by scholars, we followed a three-step model used in cross-cultural research. We tested the three models [81]: configural invariance, metric invariance, and scalar invariance. The first step is to test the configural invariance model to examine whether the basic model is invariant across Korea and the U.S. The second step is to run metric invariance to examine if Korean and U.S. participants respond to the indicators similarly. Specifically, metric invariance tests whether the strengths of the

relations between measured indicators and their respective underlying constructs are the same across the Korean and U.S. samples. Third, we tested partial invariance to examine whether some model parameters are invariant while others vary across groups. Partial invariance allows using a scale in which there may be some difference in measurement between the groups. Finally, scalar invariance indicates cross-national comparisons of means. Meeting the requirement of scalar invariance indicates that "the same score on the latent construct would obtain the same score on the observed variable regardless of their group membership" [81].

Fourth, we calculated the correlations among latent constructs and compared latent mean differences in the two countries. Researchers often use mean comparison in a cross-cultural study without testing whether the instrument measures the same psychological construct across cultures. Without testing measurement invariance, mean differences across cultures can be attributed to differences in the construct. However, composite variable scores differ from latent variable scores [82]. We employed a multi-group structural equation modeling (SEM) approach, which allowed us to compare latent means while accounting for measurement invariance.

Finally, we conducted a mediation group comparison across the Korean and U.S. samples with Mplus 7. We handled missing data using Full Information Maximum Likelihood (FIML), which is suitable given the different patterns of missingness observed in our samples. A robust mean and variance-adjusted weighted least square (WLSMV) estimator was employed to account for the non-normality of the data structure in the U.S. sample. We also tested the significance of the indirect effects by using a bootstrapping strategy. Bootstrapping examines the standard errors of the indirect effects 1000 times and assesses the confidence interval of the mediated coefficients [83]. The uses of bootstrap prevent the problems from the assumption of normality inherent in the *z-test* for the indirect effect [84].

3. Results

3.1. Differences in Korea and the U.S. at the Item Level

The preliminary analyses of univariate and multivariate normality and outlier screening were conducted using SPSS. The homogeneity of variance (homoscedasticity) was tested to see whether the Korean and U.S. samples had equal variance. Leven's test indicated no significant equality of error variance differences in the U.S. but significant differences in the Korean sample. Table 2 presents means, standard deviations, and independent *t*-tests between the Korean and U.S. samples at the item level. Despite the similarities between the two countries, the U.S. parents had higher scores in two items. Specifically, the U.S. parents had slightly higher math expectations for their children to count to 20 (M = 3.80, SD = 0.54) compared to Korean parents (M = 3.84, SD = 0.74), which was a statistically significant difference (t = 4.46, $p < 0.001$). The U.S. parents also tended to have higher math expectations for their children to count in groups of 2, 5, and 10 (M = 3.16, SD = 0.84) than Korean parents (M = 2.67, SD = 0.93).

3.2. Measurement Invariance of the Structural Equation Models

Measurement invariance tests were performed to test the equivalence of factorial validity across the Korean and U.S. samples. Measurement invariance tests examine if differences in observed variables (e.g., scale scores) reflect the differences between groups [85]. Measurement invariance follows sequential analysis, where the subsequent analyses add more invariance constraints on the sets of parameters across groups. As a result, a series of nested model tests with a chi-square difference test is utilized. The model selection requires considerations of model indices and improvement in the fit by comparing a target model with a more constrained nested model (see Table 3) [86].

Table 3. Measurement invariance test for home numeracy environment model across Korea and the U.S.

	χ^2_M	$\Delta\chi^2$	df_M	Δdf	RMSEA (90% CI)	CFI	CFI	SRMR
Configural invariance	127.222		77		0.040 0.077	0.979		0.046
Metric invariance	146.674	19.45	84	7	0.046 0.079	0.974	−0.005	0.064
Partial invariance	188.270	41.596	87	3	0.063 0.094	0.957	−0.016	0.071
Scalar invariance	283.454	95.18	95	8	0.089 0.116	0.921	−0.036	0.161

First, the measurement invariances tests began with testing configural invariance as the baseline. The test for configural invariance indicated that χ^2 (77) = 127.22, $p < 0.001$, comparative fit index (CFI) = 0.98, and Standardized Root Mean Square Residual (SRMR) = 0.05, suggesting that the configural invariance model fits the data well [87]. Specifically, it means that both groups had the same factor structural model.

Second, metric invariance was tested to see whether the strengths of the relations between measured indicators and their respective underlying constructs were the same across the Korean and U.S. samples. The metric invariance test indicated that χ^2 (84) = 146.67, $p < 0.001$, CFI = 0.97, and SRMR = 0.06; thus, the metric invariance test fits the data well. We conducted a chi-squared test comparing the configural and metric invariance test models. The resulting p-value was 0.00688, below the significance level of 0.01. As a result, we would reject the null hypothesis. Thus, the configural invariance model is not met.

Third, we tested partial invariance to examine whether some model parameters were invariant while others could vary across groups. We found that the partial invariance model showed a relatively large χ^2 = 41.60, although changes in the degree of freedom are only 3. Furthermore, the partial invariance model did not meet the requirement of a SRMR less than 0.06 [87]. Partial invariance allows using a scale in which there may be some difference in measurement between the groups while still considering the overall comparison. The partial invariance test indicated that χ^2 (87) = 188.27, $p < 0.001$, CFI = 0.96, and SRMR = 0.07, indicating the partial invariance test did not fit the data well.

Finally, the scalar invariance test indicated that χ^2 (95) = 283.45, $p < 0.001$, CFI = 0.92, and SRMR = 0.16, indicating the scalar invariance did not fit the data well. When the scalar invariance test fits the data well, differences in path coefficients across groups can be attributed to actual differences in the construct. However, the scalar invariance test indicated that the mean differences across the Korean and U.S. samples could not be attributed to differences in the construct.

This study compared latent means for fundamental constructs in two countries. Korean children had significantly higher math achievement scores ($Z = 0.38$, $p < 0.001$) than the U.S. children. On the contrary, the U.S. parents had a significantly more positive attitude towards math ($Z = -0.87$, $p < 0.001$) than the Korean parents. U.S. parents also had significantly higher math expectations ($Z = -0.55$, $p < 0.001$) than Korean parents. There was no difference between Korea and the U.S. in the HNE ($Z = 0.09$, $p > 0.5$).

Table 4 displays estimated correlations across the four latent constructs in this study. As indicated in Table 4, the parental expectations had some correlations with children's math achievement (for the Korean sample: $r = 0.17$, $p < 0.05$, for the U.S. sample: $r = 0.19$, $p < 0.05$). However, parent math attitudes did not have a relationship with children's math achievement (for Korean sample: $r = 0.01$, $p > 0.05$, for the U.S. sample: $r = 0.15$, $p > 0.05$). The pattern of correlations obtained from the Korean sample was quite different from the U.S. sample. For the Korean sample, correlations between parent math expectation and HNE are high ($r = 0.33$, $p < 0.05$). On the contrary, the correlation between these two latent variables was relatively low for the U.S. sample ($r = 0.09$, $p > 0.05$). Similarly, different patterns of correlations were found between HNE and children's math achievement from the Korean and U.S. samples. While the Korean sample had correlations between HNE and children's math achievement ($r = 0.33$, $p < 0.05$), the U.S. sample did not have correlations among those two latent variables ($r = 0.02$, $p > 0.05$).

Table 4. Correlations among latent constructs.

		1	2	3	4	5	6
		Korean sample (n = 232)					
1	Children's age	1					
2	Parent income	0.08	1				
3	Parent math attitude	−0.06	0.13	1			
4	Parent math expectation	−0.09	−0.06	0.17 *	1		
5	Home numeracy environment	0.10	0.17 *	0.12	0.33 *	1	
6	Children's math achievement	0.52 **	0.16 *	0.01	0.35 *	0.33 *	1
		U.S. sample (n = 146)					
1	Children's age	1					
2	Parent income	−0.01	1				
3	Parent math attitude	0.09	−0.03	1			
4	Parent math expectation	0.21 *	0.06	0.19 *	1		
5	Home numeracy environment	0.11	0.24 **	0.06	0.09	1	
6	Children's math achievement	0.35 **	0.34 **	0.15	0.29 *	0.02	1

Note: * $p < 0.05$, ** $p < 0.01$, The significance p value = standardized correlation coefficients/the standard error of correlations. If the ratio is greater than 1.96, it is significant. Using standardized score works!

3.3. Direct and Indirect Associations Before Controlling for Age and Parent Income

We ran two mediation models for the Korean and U.S. samples with and without control variables (e.g., child age and parental income). Figure 1 presents the mediation model after controlling for child age and parental income. For the Korean sample, we found no changes in the relationship between variables even after controlling for child age and parental income. The fit indices of the model predicting children's math achievement from parental math expectation, parental math attitude, and HNE after controlling for covariates were χ^2 (115) =237.59, RMSEA 0.07 (CI = 0.06–0.08), CFI = 0.95, TLI = 0.94, SRMR = 0.06, suggesting that the model fits the data well.

HNE predicted children's math achievement after controlling for the covariates (β = 0.32, SE = 0.12, p = 0.05). Furthermore, the HNE mediated the relationship between parent math expectations and children's math achievement (β = 0.12, SE = 0.05, p = 0.002). We examined the confidence interval of the indirect effect of bootstrapping. The result from the bootstrap indicated that the indirect path coefficients range from 0.01 to 0.25, indicating that the indirect effect is significant. However, for the Korean sample, parents' math attitude did not predict HNE (β = 0.11, SE = 0.22, $p > 0.05$) and children's math achievement (β = −0.30, SE = 0.22, $p > 0.05$). Also, HNE did not mediate the relationship between parents' math attitude and children's math achievement (β = 0.03, SE = 0.04, $p > 0.05$).

For the U.S. sample, parents' math expectation did not predict HNE after controlling for child age and parent income (β = 0.16, SE = 0.19, $p > 0.05$). Still, parents' math expectation predicted children's math achievement (β = 0.26, SE = 0.20, $p < 0.01$). Home numeracy environment did not predict children's math achievement after controlling for covariates (β = −0.04, SE = 0.10, $p > 0.05$). Also, HNE did not mediate the relationship between parent math attitude and children's math achievement after controlling for the covariates (β = 0.01, SE = 0.02, $p > 0.05$). Similarly, parent math attitude did not predict HNE (β = 0.11, SE = 0.22, $p > 0.05$) and children's math achievement (β = 27, SE = 0.22, $p > 0.05$). Also, HNE did not mediate the relationship between parent math attitude and children's math achievement (β = −0.01, SE = 0.01, $p > 0.05$).

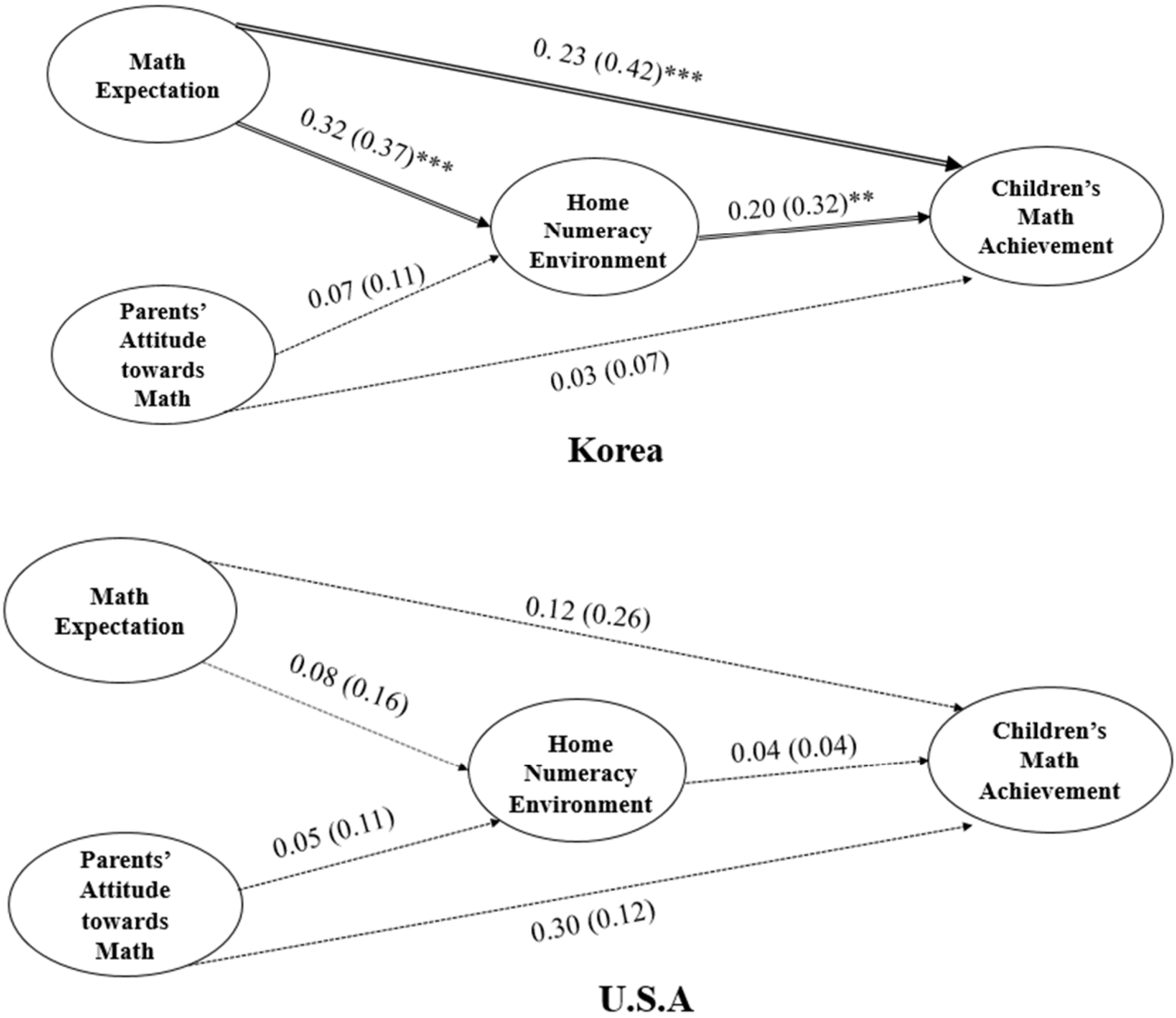

Figure 1. Mediational model after controlling for child age and parental income comparing Korea and the U.S. Note: The coefficients are standardized, while the unstandardized coefficients are presented in parentheses. ** $p < 0.01$, *** $p < 0.001$. Dotted line: non-significant relationship; solid line: significant relationship; double-dashed line: significant indirect effect; χ^2 (115) = 237.59, RMSEA 0.07 (CI = 0.06–0.08), CFI = 0.95, TLI = 0.94, SRMR = 0.06.

Table 5 also shows the results of the difference test from each path coefficient from the Korean and U.S. sample. The difference test indicated significant differences in the path coefficient from home numeracy to children's math achievement in the two groups. HNE in the Korean sample predicted children's math achievement significantly more than in the U.S. sample (*Diff* = 0.28, $p < 0.01$). Also, for the Korean sample, children's age was a more significant predictor of their math achievement compared to that of the U.S. sample, (*Diff* = 0.16, $p < 0.05$). Note that there is a significant difference in how much parent income predicted children's math achievement in the two groups. For the U.S. sample, parent income predicted children's math achievement significantly more than that of the Korean sample (*Diff* = −0.81, $p < 0.01$). However, there are no significant differences in the association between math expectation and HNE in the two groups (*Diff* = 0.22, $p > 0.05$). Similarly, we found no significant differences between the two groups in parent math expectation predicting children's math achievement (*Diff* = 0.16, $p > 0.05$).

Table 5. (Un)standardized path coefficients (SE) for structural model comparing Korea and the U.S. after controlling for child age and parent income.

	Korea			**The U.S.**			**Difference Test**
	Standardized Coefficients	***Unstandardized Coefficients***	***SE***	***Standardized Coefficients***	***Unstandardized Coefficients***	***SE***	
Direct Effect							
Math expectation → HNE	0.32 ***	0.37 ***	0.07	0.08	0.16	0.10	0.22
Math attitude → HNE	0.07	0.11	0.07	0.05	0.11	0.10	−0.02
Math expectation → Math	0.23 **	0.42	0.07	0.12	0.26	0.09	0.16
Math attitude → Math	0.03	0.07	0.06	0.12	0.30	0.09	−0.23
HNE → Math	0.20 **	0.32 **	0.07	0.04	0.04	0.09	0.28 $^{+}$
Child age → Math	0.55 ***	0.44 ***	0.05	0.37	0.28 ***	0.08	0.16 *
Parent income → Math	0.03	0.10	0.06	0.38	0.91 ***	0.08	−0.81 **
Indirect Effect							
Math expectation → HNE → Math	0.01	0.12 *	0.02	0.01	0.01	0.01	-
Math attitude → HNE → Math	0.07	0.03	0.03	0.01	0.01	0.01	-

Note: $^{+}$ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$, HNE: home numeracy environment, χ^2 (115) = 237.59, RMSEA 0.07 (CI = 0.06–0.08), CFI = 0.95, TLI = 0.94, SRMR = 0.06.

4. Discussion

This study sought to examine whether there were differences between a Korean and U.S. sample regarding parental attitudes, the HNE, and child math outcomes and their associations with one another. This is one of the few cross-cultural studies from diverse socio-economic statuses in South Korea and the United States on parental expectations, attitudes, and practices as a potential source of cultural differences in children's math outcomes in early years. This present study's findings provide additional support for the importance of parental math practices at home and offer insights into cultural processes that may play a different role in children's math outcomes.

4.1. Comparison Between South Korea and the United States

First, we compared parents' math expectations, attitudes, and practices and preschoolers' math outcomes between South Korea and the United States. The descriptive data and *t*-test results indicated that U.S. parents had significantly more positive math attitudes (e.g., enjoying math in school, the importance of exposing math) and higher expectations for all areas of math skills on the scale (e.g., from counting to 20 to subtraction to 20) for their preschoolers to master by the end of kindergarten than Korean parents in the sample. On the contrary, Korean parents had higher HNE than U.S. parents. These findings were counter-intuitive given the evidence in previous studies that East Asian parents place a high emphasis on math and high expectations for children's academic outcomes, especially math achievement, in general [22]. However, it is also feasible as it may reflect culturally different early childhood practices and expectations between two cultures [24]. It may be that Korean parents have a significant amount (maybe too much) of exposure and pressure to master math in school, which may contribute to developing negative attitudes toward math themselves. The parents' perception of being good at math is also based on the assessment of their own ability relative to others. As most Korean students spend a considerable amount of time studying math, their overall math achievement at a group level is high. Thus, even if they are doing well with math, they may feel that their performance is not high enough compared to other students in the highly competitive classroom or school context in Korea. This is consistent with previous studies that have found that, despite high scores in math, Korean students tend to have lower math self-concept, less self-efficacy, and more math anxiety [10,88]. In East Asian countries, academic achievement, especially math, is highly valued and emphasized, and competitions to excel in math are likely to be tense [89]. Thus, Asian students tend to compare their abilities to peers and feel pressure to do better, which contributes to negative attitudes and low expectations for their children.

The findings that Korean parents had lower expectations for math than U.S. parents are somewhat unexpected and different from the majority of findings in previous studies that have shown higher parental expectations toward their child's academic achievement and children's academic outcomes, including mathematics outcomes [2,14,17–19,22]. Typically, U.S. children enter formal schooling (i.e., Pre-K, kindergarten) one or two years earlier than Korean students who start their formal education in the first grade. Curriculum instruction and daily routines in kindergarten in the U.S. are structured similarly to upper grades, focusing on academic subjects. At the same time, Korean kindergarten is characterized by more open-ended and play-based activities with less focus on academics. As parents are exposed to the different school curricula, instruction, and expectations in the two countries, they may develop somewhat different views of what their children are expected to master by the end of kindergarten. This may explain why U.S. parents may have higher expectations for their child's math mastery early on than Korean parents. Still, this pattern may change once Korean children enter formal schooling, with increasing demand and parental expectations for academics.

It is also interesting that Korean parents had less positive math attitudes and lower expectations for their preschoolers. However, they tended to provide more math-related activities at home than U.S. parents. It is possible that even though Korean parents do not have positive attitudes toward math or do not have high expectations for their child to

master specific math skills, it may be natural for them to provide math-related activities or invest their time in engaging their children in math activities at home given their exposure to pressure on doing well with math in school, which is consistent with the findings of previous cross-cultural studies that have found East Asian parents involve their children in math activities (e.g., counting, identifying shapes, using ordinal numbers and spatial numbers, comparing sizes) more than other racial/ethnic groups [19,22].

Regarding child math outcomes, the Korean sample scored significantly higher in Woodcock–Johnson than the U.S. children. Although the score for PENS-B was higher for Korean children than the U.S. children, the differences for PENS-B did not reach a significant statistical level. Overall, this is in an expected direction and consistent with previous studies that have found Asian students' academic performance higher, especially in math, compared to U.S. students. The differences in math achievement in PENS-B and Woodcock–Johnson may stem from how children are assessed and their scope of assessing math skills (e.g., a broader range of math skills in Woodcock-Jones vs. focus on numeric skills only in PENS-B).

4.2. Associations Among Paternal Math Expectations, Attitudes, Practices, and Math Outcomes

Similar to the correlation results, our mediation analysis showed differences between the direct paths from math expectations to children's math achievement in the Korean and U.S. samples, where math expectations were significantly associated with children's math outcomes only for the Korean sample after controlling for child age and parent income. With the U.S. sample, parental expectations were directly associated with child math outcomes, but after controlling for child age and parent income, this link became no longer significant. Our study supports the literature [57,90] highlighting the importance of parental expectations on children's achievement, at least for the Korean sample, where higher expectations were associated with higher home numeracy scores and engaging in more math-related activities at home.

Unlike the results with parent expectations, there was no significant direct association for either group between parents' attitudes toward math and children's math achievement. Previous research has found links between parents' attitudes towards math, the HNE, and children's achievement [31,42]. It may be that the association between parents' attitudes towards math is not strong enough to link to children's achievement directly. As suggested by researchers [91], it is also possible that specific aspects of the HNE may be associated with particular math skills. It may be that the link between positive math attitudes and children's math achievement is more evident when children are engaged in more complex problem-solving or spatial reasoning tasks, which are not assessed through the PENS-B [36].

The HNE is only significantly associated with child math achievement in the Korean sample. There are consistent findings about the associations between parental math practices or the HNE and children's math outcomes, suggesting that parents can support their children's math development through direct teaching or math learning activities or indirectly through materials or activities [40,41]. This is also similar to the findings of previous cross-cultural studies that have found East Asian parents not only involve their children in math activities but also tend to use more effective math instructional strategies than other racial/ethnic groups [19,22], which is likely to contribute to Asian children's higher math achievement.

We also examined the indirect associations whether the HNE mediated the association between math expectations and/or attitudes and children's math achievement. In this model, there are two possible paths in which parental characteristics can contribute indirectly to children's math achievement. Our study found that the HNE mediated the association between parent math expectations and children's math achievement for the Korean sample only. The HNE did not mediate the association between parents' math attitudes and children's achievement in either sample. Previous research has also examined how the HNE is influenced by parents' attitudes and expectations, with parents with higher math anxiety typically not engaging in many math-related activities in the home [30]. These

findings highlight the influence of parents' attitudes on children's home experiences [31,48], which is consistent with our findings with the Korean sample.

A significant mediational link was not found in the U.S. sample. There are several reasons why parental expectations or attitudes did not indirectly influence math achievement in the U.S. sample. It may be because, while the U.S. sample maintained high expectations of their children's math abilities, these expectations did not translate to high levels of math practice at home, given their lower scores in HNE. Although parents do not have higher expectations for their child's math skills, they engage more in directly teaching math concepts and skills (e.g., identifying written numbers, rehearsing counting rhymes, using calendars and dates) at home. This direct teaching and math-related engagement would be more proximal and effective and contribute more to their child's math skills.

It is also possible that SES is driving the relationship between HNE and achievement [54]. Previous research has found that children from low-income backgrounds who have lower levels of math achievement also tend to have lower engagement in math activities at home. Although we included children from diverse socio-economic family backgrounds in both countries, parental income was a significant predictor for children's math outcomes only in the U.S. sample. Our U.S. sample was diverse regarding SES; initially, the path between math expectations and children's math achievement was significant; this significance disappeared after controlling for parental income. This finding underscores the potential impact of the family socio-economic status on parent HNE and child math achievement in the U.S. sample relative to the Korean sample and helps to explain the differential associations in the two contexts.

One final aspect is whether informal and formal activities are linked with children's math achievement. Our survey included both measures, as we hypothesized that informal and formal activities would contribute to children's math achievement. Previous research has found that direct activities have been more consistently linked to math achievement, while informal activities may tap different math skills than those assessed in standardized math assessments [47,92,93]. Nevertheless, most research linking formal activities to achievement has been conducted with higher-SES, White families [93,94]. To fully explore the relationship between parent characteristics, the HNE, and children's achievement, research may need to distinguish formal and informal activities in the models and examine differences across various math skills [91].

4.3. Limitations and Directions for Future Research

This current study has several limitations. First, there are some methodological pitfalls [37], including the correlational and cross-sectional nature of the study, that prevent us from drawing a causal influence on the associations between key variables. Parental self-report measures and frequency may not accurately capture what they believe or do at home (e.g., social desirability). Other limitations are related to an examination of the associations of parental math attitudes, expectations, and practices with child math outcomes in two different cultural contexts without considering the nested structure within each country (i.e., certain parents may be nested in the same province), potentially leading to biased or inaccurate coefficient estimates. The measures were validated and widely used with the U.S. sample, not the Korean one. Some constructs and items may not be readily understandable or may have slightly different meanings to Korean parents (e.g., good at math in school, enjoy math in school). Even though the authors have a solid understanding of and extensive experiences with both contexts, it is impossible to consider and control for numerous other culture-related factors and disentangle how culture plays a role in the differences in children's math learning and parental attitudes, expectations, and HNE. For example, East Asian children's high math performance can be explained by parental factors and other cultural systems, such as their numbering system, which is more straightforward and accessible to grasp [18].

More longitudinal investigations with larger sample sizes and more variables that enable us to account for the cultural differences are warranted to confirm this current

study's findings. Such research could, for example, explore differences between fathers and mothers in their attitudes towards math. Another direction to consider is whether math achievement should be the only indication of the value of the HNE or the product of parents' expectations about their child. Future research could continue to expand our understanding of how children develop into lifelong mathematicians by exploring their interests, feelings about math, and their understanding of what math is beyond a set of numeracy skills or names of shapes. The period before children enter formal schooling is a ripe time to develop children's numeracy skills alongside their mathematical identity.

5. Conclusions

This study examined the relationships among parental attitudes and expectations, the home numeracy environment, and children's math outcomes in a preschool-aged sample in Korea and the United States. We found that our model of the relationships was invariant across the two samples, indicating that the items and constructs are similar cross-culturally. Yet, we found differences between the samples across measures. The Korean sample had higher math achievement, while the U.S. had higher math attitudes and expectations. No differences were found in the HNE. These similarities and differences highlight the need for continued examination of families and the home environment's role in children's math achievement.

Much of the previous research on the role of the family in children's math outcomes paints a murky picture. Some research highlights how parents' feelings about math, their expectations of children's abilities, and their math competence contribute directly and indirectly to children's math achievement. Parents who value math because they see its importance or because they may have struggled with it previously encourage their children through informal and formal activities. On the other hand, the frequency of these activities, their content, and who initiates these activities have not been associated with children's math outcomes. Our study similarly presents mixed findings—in the U.S. sample, there was no indirect pathway from parents' attitudes to children's math achievement through the HNE. In contrast, in the Korean sample, parents' expectations led to increased activities at home, while it was associated with children's math outcomes. This present study provided additional evidence on the importance of considering cultural values and contexts to explain the differences in parental math attitudes, expectations, practices, and their children's math outcomes and how they are associated.

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References

1. Aunio, P.; Hautamäki, J.; Heiskari, P.; Van Luit, J.E.H. The early numeracy test in Finnish: Children's norms. *Scand. J. Psychol.* **2006**, *47*, 369–378. [CrossRef] [PubMed]
2. Sun, Y. Cognitive advantages of East Asian American children: When do such advantages emerge and what explains them? *Sociol. Perspect.* **2011**, *54*, 377–402. [CrossRef]

3. Aunola, K.; Leskinen, E.; Lerkkanen, M.-K.; Nurmi, J.-E. Developmental dynamics of math performance from preschool to grade 2. *J. Educ. Psychol.* **2004**, *96*, 699–713. [CrossRef]
4. Cross, C.T.; Woods, T.A.; Schweingruber, H.; National Research Council. *Mathematics Learning in Early Childhood: Paths toward Excellence and Equity*; The National Academies Press: Washington, DC, USA, 2009. [CrossRef]
5. Duncan, G.J.; Magnuson, K. The nature and impact of early achievement skills, attention skills, and behavior problems. In *Whither Opportunity: Rising Inequality, Schools, and Children's Life Chances*; Duncan, G.J., Murnane, R.J., Eds.; Russell Sage: Troy, NY, USA, 2011; pp. 47–69.
6. Dowker, A. Individual differences in numerical abilities in preschoolers. *Dev. Sci.* **2008**, *11*, 650–654. [CrossRef]
7. Klibanoff, R.S.; Levine, S.C.; Huttenlocher, J.; Vasilyeva, M.; Hedges, L.V. Preschool children's mathematical knowledge: The effect of teacher "math talk". *Dev. Psychol.* **2006**, *42*, 59–69. [CrossRef]
8. Anders, Y.; Rossbach, H.-G.; Weinert, S.; Ebert, S.; Kuger, S.; Lehrl, S.; von Maurice, J. Home and preschool learning environments and their relations to the development of early numeracy skills. *Early Child. Res. Q.* **2012**, *27*, 231–244. [CrossRef]
9. Gunderson, E.A.; Levine, S.C. Some types of parent number talk count more than others: Relations between parents' input and children's cardinal-number knowledge. *Dev. Sci.* **2011**, *14*, 1021–1032. [CrossRef]
10. OECD. *PISA 2018 Results: What Students Know and Can Do, PISA*; OECD Publishing: Paris, France, 2019. [CrossRef]
11. Mullis, I.V.S.; Martin, M.O.; Foy, P.; Arora, A. *TIMSS 2011 International Results in Mathematics*; TIMSS PIRLS International Study Center, Boston College: Boston, MA, USA, 2012.
12. Kim, S.W. Meta-analysis of parental involvement and achievement in East Asian countries. *Educ. Urban Soc.* **2020**, *52*, 312–337. [CrossRef]
13. Park, H.; Byun So, Y.; Kim, K. Parent involvement and students' cognitive outcomes in Korea: Focusing on private tutoring. *Sociol. Educ.* **2011**, *84*, 3. [CrossRef]
14. Ryoo, J.H.; Molfese, V.J.; Heaton, R.; Zhou, X.; Brown, E.T.; Prokasky, A.; Davis, E. Early mathematics skills from prekindergarten to first grade: Score changes and ability group differences in Kentucky, Nebraska, and Shanghai samples. *J. Adv. Acad.* **2014**, *25*, 162–188. [CrossRef]
15. Caplan, N.; Choy, M.H.; Whitmore, J.K. Indochinese refugee families and academic achievement. *Sci. Am.* **1992**, *266*, 36–45. [CrossRef]
16. Sue, S.; Okazaki, S. Asian-American educational achievements: A phenomenon in search of an explanation. In *The New Immigrants and American Schools*; Routledge: Brighton, UK, 2022; pp. 297–304.
17. Aunio, P.; Aubrey, C.; Godfrey, R.; Pan, Y.; Liu, Y. Children's early numeracy in England, Finland, and People's Republic of China. *Int. J. Early Years Educ.* **2008**, *16*, 203–221. [CrossRef]
18. Kung, M.; Schmitt, S.A.; Zhang, C.; Whiteman, S.D.; Yang, F.; Purpura, D.J. The role of mathematical language in mathematics development in China and the US. *Int. J. Educ. Res.* **2019**, *95*, 131–142. [CrossRef]
19. Pan, Y.; Gauvain, M.; Liu, Z.; Cheng, L. American and Chinese parental involvement in young children's mathematics learning. *Cogn. Dev.* **2006**, *21*, 17–35. [CrossRef]
20. Hu, X.; Leung, F.K.; Teng, Y. The influence of culture on students' mathematics achievement across 51 countries. *Int. J. Sci. Math. Educ.* **2018**, *16*, 7–24. [CrossRef]
21. Balestrini, D.P.; Stoeger, H. Substantiating a special cultural emphasis on learning and education in East Asia. *High Abil. Stud.* **2018**, *29*, 79–106. [CrossRef]
22. Zhou, X.; Huang, J.; Wang, Z.; Wang, B.; Zhao, Z.; Yang, L.; Yang, Z. Parent–child interaction and children's number learning. *Early Child. Dev. Care* **2006**, *176*, 763–775. [CrossRef]
23. Huntsinger, C.S.; Jose, P.E.; Larson, S.L.; Krieg, D.B.; Shaligram, C. Mathematics, vocabulary, and reading development in Chinese American and European American children over the primary school years. *J. Educ. Psychol.* **2000**, *92*, 745–760. [CrossRef]
24. Phillipson, S.; Phillipson, S.N. Academic expectations, belief of ability, and involvement by parents as predictors of child achievement: A cross-cultural comparison. *Educ. Psychol.* **2007**, *27*, 329–348. [CrossRef]
25. LeFevre, J.-A.; Fast, L.; Skwarchuk, S.L.; Smith-Chant, B.L.; Bisanz, J.; Kamawar, D.; Penner-Wilger, M. Pathways to mathematics: Longitudinal predictors of performance. *Child Dev.* **2010**, *81*, 1753–1767. [CrossRef]
26. Mutaf-Yıldız, B.; Sasanguie, D.; De Smedt, B.; Reynvoet, B. Probing the relationship between home numeracy and children's mathematical skills: A systematic review. *Front. Psychol.* **2020**, *11*, 557630. [CrossRef] [PubMed]
27. DeFlorio, L.; Beliakoff, A. Socioeconomic status and preschoolers' mathematical knowledge: The contribution of home activities and parent beliefs. *Early Educ. Dev.* **2015**, *26*, 319–341. [CrossRef]
28. Missall, K.; Hojnoski, R.L.; Caskie, G.I.L.; Repasky, P. Home numeracy environments of preschoolers: Examining relations among mathematical activities, parent mathematical beliefs, and early mathematical skills. *Early Educ. Dev.* **2015**, *26*, 356–376. [CrossRef]
29. Skwarchuk, S.-L. How do parents support preschoolers' numeracy learning experiences at home? *Early Child. Educ. J.* **2009**, *37*, 189–197. [CrossRef]
30. Del Rio, M.F.; Susperreguy, M.I.; Strasser, K.; Salinas, V. Distinct influences of mothers and fathers on kindergartners' numeracy performance: The role of math anxiety, home numeracy practices, and numeracy expectations. *Early Educ. Dev.* **2017**, *28*, 939–955. [CrossRef]
31. Lee, H.J.; Kim, J. A structural analysis on Korean young children's mathematical ability and its related children's and mothers' variables. *Early Child Dev. Care* **2016**, *186*, 1675–1692. [CrossRef]

32. Davis-Kean, P.E.; Sexton, H.R. Race differences in parental influences on child achievement: Multiple pathways to success. *Merrill-Palmer Q.* **2009**, *55*, 285–318. [CrossRef]
33. Hong, S.; Ho, H.-Z. Direct and indirect longitudinal effects of parental involvement on student achievement: Second-order latent growth modeling across ethnic groups. *J. Educ. Psychol.* **2005**, *97*, 32–42. [CrossRef]
34. Liu, C.; Georgiou, G.K.; Manolitsis, G. Modeling the relationships of parents' expectations, family's SES, and home literacy environment with emergent literacy skills and word reading in Chinese. *Early Child. Res. Q.* **2018**, *43*, 1–10. [CrossRef]
35. Kleemans, T.; Peeters, J.; Segers, E.; Verhoeven, L. Child and home predictors of early numeracy skills in kindergarten. *Early Child. Res. Q.* **2012**, *27*, 471–477. [CrossRef]
36. Silver, A.M.; Elliott, L.; Libertus, M.E. When beliefs matter most: Examining children's math achievement in the context of parental math anxiety. *J. Exp. Child Psychol.* **2021**, *201*, 104992. [CrossRef] [PubMed]
37. Cankaya, O.; LeFevre, J.-A. The home numeracy environment: What do cross-cultural comparisons tell us about how to scaffold young children's mathematical skills? In *Early Childhood Mathematics Skill Development in the Home Environment*; Blevins-Knabe, B., Austin, A., Eds.; Springer: Berlin/Heidelberg, Germany, 2016; pp. 87–104.
38. Ribeiro, L.A.; Donolato, E.; Aguiar, C.; Correia, N.; Zachrisson, H.D. Concurrent and Longitudinal Associations Between Parent Math Support in Early Childhood and Math Skills: A Meta-Analytic Study. *J. Cogn. Dev.* **2024**, *25*, 66–99. [CrossRef]
39. Seegers, E.; Kleemans, T.; Verhoeven, L. Role of parent literacy and numeracy expectations and activities in predicting early numeracy skills. *Math. Think. Learn.* **2015**, *17*, 219–236. [CrossRef]
40. Blevins-Knabe, B.; Austin, A.B.; Musun, L.; Eddy, A.; Jones, R.M. Family home care providers' and parents' beliefs and practices concerning mathematics with young children. *Early Child. Dev. Care* **2000**, *165*, 41–58. [CrossRef]
41. LeFevre, J.-A.; Skwarchuk, S.-L.; Smith-Chant, B.L.; Fast, L.; Kamawar, D.; Bisanz, J. Home numeracy experiences and children's math performance in the early school years. *Can. J. Behav. Sci. Rev. Can. Des Sci. Du Comport.* **2009**, *41*, 55–66. [CrossRef]
42. Skwarchuk, S.L.; Sowinski, C.; LeFevre, J.A. Formal and informal home learning activities in relation to children's early numeracy and literacy skills: The development of a home numeracy model. *J. Exp. Child Psychol.* **2014**, *121*, 63–84. [CrossRef]
43. Niklas, F.; Schneider, W. Casting the die before the die is cast: The importance of the home numeracy environment for preschool children. *Eur. J. Psychol. Educ.* **2014**, *29*, 327–345. [CrossRef]
44. Baker, C.E. Fathers' and mothers' home literacy involvement and children's cognitive and social emotional development: Implications for family literacy programs. *Appl. Dev. Sci.* **2013**, *17*, 184–197. [CrossRef]
45. Baker, C.E. African American fathers' contributions to children's early academic achievement: Evidence from two-parent families from the Early Childhood Longitudinal Study—Birth Cohort. *Early Educ. Dev.* **2014**, *25*, 19–35. [CrossRef]
46. Huang, Q.; Zhang, X.; Liu, Y.; Yang, W.; Song, Z. The contribution of parent–child numeracy activities to young Chinese children's mathematical ability. *Br. J. Educ. Psychol.* **2017**, *87*, 328–344. [CrossRef]
47. Huntsinger, C.S.; Jose, P.B.; Luo, Z. Parental facilitation of early mathematics and reading skills and knowledge through encouragement of home-based activities. *Early Child. Res. Q.* **2016**, *37*, 1–15. [CrossRef]
48. Susperreguy, M.I.; Di Lonardo Burr, S.M.; Xu, C.; Douglas, H.; LeFevre, J.-A. Children's home numeracy environment predicts growth of their early mathematical skills in kindergarten. *Child Dev.* **2020**, *91*, 1663–1680. [CrossRef] [PubMed]
49. Ciping, D.; Silinskas, G.; Wei, W.; Georgiou, G.K. Cross-lagged relationships between home learning environment and academic achievement in Chinese. *Early Child. Res. Q.* **2015**, *33*, 12–20. [CrossRef]
50. Elliott, L.; Bachman, H.J. SES disparities in early math abilities: The contributions of parents' math cognitions, practices to support math, and math talk. *Dev. Rev.* **2018**, *49*, 1–15. [CrossRef]
51. Hart, S.A.; Ganley, C.M.; Purpura, D.J. Understanding the home math environment and its role in predicting parent report of children's math skills. *PLoS ONE* **2016**, *11*, e0168227. [CrossRef]
52. De Keyser, L.; Bakker, M.; Rathé, S.; Wijns, N.; Torbeyns, J.; Verschaffel, L.; De Smedt, B. No association between the home math environment and numerical and patterning skills in a large and diverse sample of 5-to 6-year-olds. *Front. Psychol.* **2020**, *11*, 547626. [CrossRef]
53. Trickett, J.; Batchelor, S.; Brittle, B.; Foulkes, M.; Pickering, J.; Slocombe, F.; Gilmore, C. The role of parent-led and child-led home numeracy activities in early mathematical skills. *Cogn. Dev.* **2022**, *63*, 101189. [CrossRef]
54. Daucourt, M.C.; Napoli, A.R.; Quinn, J.M.; Wood, S.G.; Hart, S.A. The home math environment and math achievement: A meta-analysis. *Psychol. Bull.* **2021**, *147*, 565. [CrossRef]
55. Dunst, C.J.; Hamby, D.W.; Wilkie, H.; Dunst, K.S. Meta-Analysis of the relationship between home and family experiences and young children's early numeracy learning. In *Engaging Families as Children's First Mathematics Educators. Early Mathematics Learning and Development*; Phillipson, S., Gervasoni, A., Sullivan, P., Eds.; Springer: Singapore, 2017. [CrossRef]
56. Eason, S.H.; Scalise, N.R.; Berkowitz, T.; Ramani, G.B.; Levine, S.C. Widening the lens of family math engagement: A conceptual framework and systematic review. *Dev. Rev.* **2022**, *66*, 101046. [CrossRef]
57. Zippert, E.L.; Rittle-Johnson, B. The home math environment: More than numeracy. *Early Child. Res. Q.* **2020**, *50 Pt 3*, 4–15. [CrossRef]
58. Kao, G.; Thompson, J.S. Racial and ethnic stratification in educational achievement and attainment. *Annu. Rev. Sociol.* **2003**, *29*, 417–442. [CrossRef]
59. Neisser, U.; Boodoo, G.; Bouchard, T.J.; Boykin, A.W.; Brody, N.; Ceci, S.J.; Halpern, D.F.; Loehlin, J.C.; Perloff, R.; Sternberg, R.J.; et al. Intelligence: Knowns and unknowns. *Am. Psychol.* **1996**, *51*, 77–101. [CrossRef]

60. Huntsinger, C.S.; Jose, P.B. Parental involvement in children's schooling: Different meanings in different cultures. *Early Child. Res. Q.* **2009**, *24*, 398–410. [CrossRef]
61. White, M.J.; Glick, J.E. Generation status, social capital, and the routes out of high school. *Sociol. Forum* **2000**, *15*, 671–691. [CrossRef]
62. Dearing, E.; Casey, B.M.; Ganley, C.M.; Tillinger, M.; Laski, E.; Montecillo, C. Young girls' arithmetic and spatial skills: The distal and proximal roles of family socioeconomics and home learning experiences. *Early Child. Res. Q.* **2012**, *27*, 458–470. [CrossRef]
63. Harkness, S.; Super, C.M. Themes and Variations: Parental Ethnotheories in Western Cultures. In *Parenting Beliefs, Behaviors, and Parent-Child Relations: A Cross-Cultural Perspective*; Rubin, K.H., Chung, O.B., Eds.; Psychology Press: London, UK, 2006; pp. 61–79.
64. Hao, Y.; Chen, X.; Qi, Y.; Huang, T.; He, W.; Yang, X. How do Chinese parental attitudes influence children's numeracy interests? What matters is home numeracy activities, not extracurricular participation. *J. Appl. Dev. Psychol.* **2022**, *83*, 101475. [CrossRef]
65. Cheung, S.K.; McBride, C. Effectiveness of parent–child number board game playing in promoting Chinese kindergarteners' numeracy skills and mathematics interest. *Early Educ. Dev.* **2017**, *28*, 572–589. [CrossRef]
66. Ng, S.S.N.; Rao, N. Chinese number words, culture, and mathematics learning. *Rev. Educ. Res.* **2010**, *80*, 180–206. [CrossRef]
67. Huntsinger, C.S.; Jose, P.B.; Liaw, F.-R.; Ching, W.D. Cultural differences in early mathematics learning: A comparison of Euro-American, Chinese-American, and Taiwan-Chinese families. *Int. J. Behav. Dev.* **1997**, *21*, 371–388. [CrossRef]
68. Woodcock, R.W.; McGrew, K.S.; Mather, N. *Woodcock Johnson III Tests of Achievement*; Riverside Publishing: Rolling Meadows, IL, USA, 2001.
69. Purpura, D.J.; Lonigan, C.J. Informal numeracy skills: The structure and relations among numbering, relations, and arithmetic operations in preschool. *Am. Educ. Res. J.* **2013**, *50*, 178–209. [CrossRef]
70. Purpura, D.J.; Lonigan, C.J. Early numeracy assessment: The development of the preschool early numeracy scales. *Early Educ. Dev.* **2015**, *26*, 286–313. [CrossRef] [PubMed]
71. Rhoades, B.L.; Warren, H.K.; Domitrovich, C.E.; Greenberg, M.T. Examining the link between preschool social–emotional competence and first grade academic achievement: The role of attention skills. *Early Child. Res. Q.* **2011**, *26*, 182–191. [CrossRef]
72. Kleemans, T.; Segers, E.; Verhoeven, L. Relations between home numeracy experiences and basic calculation skills in children with and without specific language impairment. *Early Child. Res. Q.* **2013**, *28*, 415–423. [CrossRef]
73. Kim, J. The development and validation of a scale to measure the mathematical interaction of young children's parents. *Korean J. Child Stud.* **2015**, *36*, 95–113. [CrossRef]
74. Bornstein, M.H.; Bradley, R.H. *Socio-Economic Status, Parenting, and Child Development*; Erlbaum: Mahwah, NJ, USA, 2008.
75. Kang, H.D. Primary school English education in Korea. In *Primary School English-Language Education in Asia*; Routledge: Brighton, UK, 2012; pp. 59–82.
76. Hau, K.T.; Marsh, H.W. The use of item parcels in structural equation modelling: Non-normal data and small sample sizes. *Br. J. Math. Stat. Psychol.* **2004**, *57*, 327–351. [CrossRef]
77. Little, T.D.; Rhemtulla, M.; Gibson, K.; Schoemann, A.M. Why the items versus parcels controversy needn't be one. *Psychol. Methods* **2013**, *18*, 285. [CrossRef]
78. Nasser, F.; Wisenbaker, J. A Monte Carlo study investigating the impact of item parceling on measures of fit in confirmatory factor analysis. *Educ. Psychol. Meas.* **2003**, *63*, 729–757. [CrossRef]
79. Bagozzi, R.P.; Edwards, J.R. A general approach for representing constructs in organizational research. *Organ. Res. Methods* **1998**, *1*, 45–87. [CrossRef]
80. Muthén, B.; Muthén, L. *Mplus, Version 7*; General Latent Variable Modeling Program; Muthén & Muthén: Los Angeles, CA, USA, 2012.
81. Milfont, T.L.; Fischer, R. Testing measurement invariance across groups: Applications in cross-cultural research. *Int. J. Psychol. Res.* **2010**, *3*, 111–130. [CrossRef]
82. Bollen, K.A.; Curran, P.J. *Latent Curve Models: A Structural Equation Perspective*; John Wiley Sons: Hoboken, NJ, USA, 2006.
83. Preacher, K.J.; Hayes, A.F. SPSS and SAS procedures for estimating indirect effects in simple mediation models. *Behav. Res. Methods Instrum. Comput.* **2004**, *36*, 717–731. [CrossRef]
84. Hayes, A.F.; Scharkow, M. The relative trustworthiness of inferential tests of the indirect effect in statistical mediation analysis: Does method really matter? *Psychol. Sci.* **2013**, *24*, 1918–1927. [CrossRef] [PubMed]
85. Klee, W.; Teresi, J.A. An essay on measurement and factorial invariance. *Med. Care* **2006**, *44*, S69–S77.
86. Hu, L.T.; Bentler, P.M. Evaluating model fit. In *Structural Equation Modelling. Concepts, Issues, and Applications/Sage*; Sage Publications, Inc.: Troy, NY, USA, 1995.
87. Hu, L.T.; Bentler, P.M. Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Struct. Equ. Model. A Multidiscip. J.* **1999**, *6*, 1–55. [CrossRef]
88. Lee, J. Universals and specifics of math self-concept, math self-efficacy, and math anxiety across 41 PISA 2003 participating countries. *Learn. Individ. Differ.* **2009**, *19*, 355–365. [CrossRef]
89. Ho, H.; Senturk, D.; Lam, A.G.; Zimmer, J.M.; Hong, S.; Okamoto, Y.; Chiu, S.; Nakazawa, Y.; Wang, C. The Affective and Cognitive Dimensions of Math Anxiety: A Cross-National Study. *J. Res. Math. Educ.* **2000**, *31*, 362–379. [CrossRef]
90. Sonnenschein, S.; Sun, S. Racial/ethnic differences in kindergartners' reading and math skills: Parents' knowledge of children's development and home-based activities as mediators. *Infant Child Dev.* **2017**, *26*, e2010. [CrossRef]

91. Leyva, D.; Libertus, M.E.; McGregor, R. Relations between subdomains of home math activities and corresponding math skills in 4-year-old children. *Educ. Sci.* **2021**, *11*, 594. [CrossRef]
92. Ramani, G.B.; Rowe, M.L.; Eason, S.H.; Leech, K.A. Math talk during informal learning activities in Head Start families. *Cogn. Dev.* **2015**, *35*, 15–33. [CrossRef]
93. Thompson, R.J.; Napoli, A.R.; Purpura, D.J. Age-related differences in the relation between the home numeracy environment and numeracy skills. *Infant Child Dev.* **2017**, *26*, e2019. [CrossRef]
94. Sonnenschein, S.; Galindo, C. Race/ethnicity and early mathematics skills: Relations between home, classroom, and mathematics achievement. *J. Educ. Res.* **2015**, *108*, 261–277. [CrossRef]

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