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Article

Bringing Together Mathematics and Philosophy with Logic and Poly-Universe

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Abstract: In this paper we report on an activity developed in the context of the Erasmus+ PUNTE Project, using the Poly-Universe material, which led to the learning of logic and mathematics among the 10th grade students of a school in the central region of Portugal. Three teachers, two of mathematics and one of philosophy, and 21 students of secondary education participated in the study. The data obtained, following the observation and application of questionnaires and pre- and post-tests, reveal the usefulness of Poly-Universe for the acquisition of notions about logical operations, for analysing arguments and their validity and solving mathematical problems, motivating the students for carrying out the activity.

Keywords: interdisciplinarity; mathematics; logical operations philosophy; Poly-Universe; Erasmus+ project PUNTE

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1. Introduction

During the period of compulsory education, it is possible to find students who like mathematics but do not like philosophy, and vice versa, as well as students who like neither philosophy nor mathematics, and others still who may like both subjects, which influences their performance and school path, e.g., [1,2]. Despite all these possibilities, it is known that mathematics is a particularly shunned in course choices and, in this sense, it is infrequent to find mathematics and philosophy teachers from different cultures, academic and scientific backgrounds, and cognitive styles carrying out joint planning, given also the relative absence of a culture of collaboration in schools [3].

In fact, the evolution of modern science has led to a vertiginous specialization of knowledge, which is at the origin of a progressive compartmentalization or even fragmentation of knowledge, which has been reflected in the way of conceiving teaching and teacher training in different subject areas [2]. However, that same evolution and the complexity of issues and problems it encompasses has also evidenced the need to establish bridges and an articulation that enhance the integration of several perspectives, as [4] or [5], among others, propose. In this sense, and particularly in the case of Mathematics, which often appears isolated despite its usefulness being defended, an interdisciplinary mathematics education has been gaining relevance and is intended to be promoted and expanded [1].

In Portugal, the most recent curricular reorganization expressed in documents such as the Profile of the Student Exiting Compulsory Schooling (PASEO) [6], the Decree-Law No. 55/2018 [7] or the Order No. 6944-A/2018 of July 19 [8], refer to the importance of the interdisciplinary approach as a way to achieve the consolidation of learning and an integrated management of knowledge. This interdisciplinary approach aims to articulate the knowledge of the various disciplines, with a view to a greater and better appropriation of knowledge. Among the proposals to implement the interdisciplinary approach we can mention the principle of autonomy and curricular flexibility or the domains of curricular autonomy. In this regard, we can refer, for example, to the studies of [9] that associate the interdisciplinary approach with cooperation and the promotion of academic success, and we can also refer to [10,11]. However, despite the guidelines towards the implementation of interdisciplinary practices, which become more likely following the recent curriculum guidelines, systematic studies on these are still rare, both internationally and nationally.

It should be noted that interdisciplinarity, despite its complexity., [12] refers in its simplest sense to the interaction between various fields of knowledge, the proposal of new knowledge in the production of scientific knowledge. In an educational context, it can be characterised by involving several disciplines for the analysis of complex issues, not cancelling disciplinary knowledge, but seeking to mobilise it for problem-solving [13,14], which situates interdisciplinarity at a midpoint between multidisciplinarity and transdisciplinarity, emphasising, on the one hand, the integration of knowledge underlying interdisciplinarity, but also the autonomy simultaneously maintained by each area.

In relation to mathematics and philosophy, this connection has already been highlighted [15,16], and is even the object of theoretical modelling [2] in the construction of a didactic model of interdisciplinarity, around argumentation and reasoning. Logic may constitute the touchstone in this interdisciplinary adventure [17]. Specifically, mathematics and philosophy are closely related when it comes to logical operations as both disciplines rely heavily on logical operations to analyse and evaluate ideas. Logical operations can be considered an essential part of both mathematics and philosophy, allowing us to reason, analyse, and make deductions [18,19]. In mathematics, logical operations are used to construct mathematical proofs and to derive new conclusions from existing information [20,21]. In philosophy, logical operations are used to analyse arguments and evaluate their validity, to construct logical proofs, and identify fallacies in arguments, also involving language and semantics. In this scope, logical operations can be used to explore and promote interdisciplinarity as a tool that can be used in both disciplines. While it is essential to teach each discipline individually, an integrated approach to teaching philosophy, logic, and mathematics can offer several advantages. As argued in [19], such an approach can enhance students' ability to recognize the connections between these subjects, appreciate the broader implications of their studies, and foster interdisciplinary thinking—a crucial skill in our rapidly changing world.

In turn, the Poly-Universe material, due to its characteristics, thoroughly described in [22,23], has led to several studies within the PUNTE Project, which show its potential in the teaching–learning of mathematics, but also of several other disciplines, namely within a STEAM education [24]. Preliminary research has shown that the tool can be widely used in education, especially in information visualization in the fields of geometry, combinatory, logic, graphs, and algorithms [25]. Other studies have highlighted the possibility of using Poly-Universe in learning computational thinking or biology [24]. It is in this context that the activity proposed here arises, involving logical operations, mathematics, and philosophy, using the Poly-Universe material, of which we try to give an account in this article, describing its planning, implementation, and the results obtained.

2. Materials and Methods

2.1. Corpus, Data Gathering, and Analysis

The implemented activity aims to promote learning the logical operations of conjunction, disjunction and implication using Poly-Universe materials. The activity was planned by mathematics and philosophy teachers according to the curricular guidelines defined in the Essential Learning Outcomes, that is a term used in the Portuguese education system to refer what students are expected to achieve in each subject and grade level [26]. In the Philosophy Essential Learning Outcomes (10th grade), logic appears in the domain "Argumentative rationality of Philosophy and the discursive dimension of philosophical work". In relation to the Essential Learning Outcomes in Mathematics A, logic is a cross-cutting theme, and is therefore, introduced as and when it is needed. In the subject of Mathematics Applied to Social Sciences (MACS), logic appears as a tool for developing reasoning.

The activity was implemented in two classes of the 10th grade; one was MACS and the other was Mathematics A. In the former, twelve secondary school students participated, eight females and four males. In the Mathematics A class, nine students participated, six males and three females. All the students answered a questionnaire before and after the activity, and pre- and post-tests were also carried out, in addition to the recording of field notes, photos, and videos of the ongoing process, which allowed for the collection of indicators of the motivation and involvement of the participants and the pedagogical dialogue.

The questionnaire that was completed prior to the activity included a set of questions about participation in interdisciplinary activities throughout one's school career, the disciplines involved, how they took place, etc. It also included questions about the perception of the degree of relation between philosophy and mathematics and the importance given to the use of manipulative materials in both subjects, also seeking to find out more about the use of these types of materials in these disciplines.

In turn, the questionnaire that was completed after the activity was concerned with global perceptions about the process. It included statements adapted from the Centre for Self-Determination Theory (2022) on intrinsic motivation [27], namely on interest, 'I enjoyed doing this activity very much', perceived competence, 'I think I did pretty well at this activity', effort, 'I put a lot of effort into this', pressure? 'I was very relaxed doing theses activities', perceived choice, 'This activity was doing following my ideas', and usefulness 'This activity is useful for learning mathematics'. Additional items were also included about the experience of using Poly-Universe, namely 'It was easy to start using Poly-Universe' and the assessment of 13 skills that the activity potentially developed, namely group work, creativity, critical thinking, understanding, problem-solving, sharing/cooperation, concentration/attention, innovation, decision-making, autonomy, responsibility, logical reasoning, visual perception and others that were indicated. To the previous statements, participants were asked to give their degree of agreement on a five-point Likert scale (1—total disagreement; 5—total agreement). The questionnaire also included four open questions about what they liked best about the activity, difficulties, learning, and suggestions for improvement. Finally, Likert-type scales were again used to assess the perceived degree of relation between philosophy and mathematics in the activity carried out, the importance of using Poly-Universe materials for acquiring concepts, learning logic, and for solving mathematical exercises.

The pre-test and post-test included a set of three exercises on logical operations, one on conjunction, one on disjunction, and one on implication. Each exercise consisted of several paragraphs, the answers to which allowed the completion of a truth table. The exercises included a philosophy dimension as they involved evaluating the logical value (truth or falsity) of these propositions using logical reasoning. The use of logical operations and propositions to analyse and evaluate claims is a central aspect of philosophical inquiry. In addition, the exercises also included a math problem, as in the case of the exercise about implication, focusing on the statical analysis of the results of the elections for the School Students Association, or in Analytic Geometry in the plane and in space, more specifically, in the notion of the slope of a line.

The conjunction exercise involved propositions related to general knowledge, particularly about the Mondego River, such as 'The Mondego River begins in Serra da Estrela', 'The Mongelo River flows into Figueira da Foz', 'The Mondego River begins in Coimbra', 'The Mondego River flows into Lisbon'. After indicating the logical value of the propositions, the students were asked to formalise and indicate the logical value of the propositions involving the conjunction of previous propositions, for example, 'The Mondego River is born in Serra da Estrela and flows into Figueira da Foz'. Based on the conclusions resulting from this classification, in the last paragraph of the exercise, students were asked to fill in the truth table concerning the logical operation of the conjunction. The exercise on implication involved notions of statistics based on the lists from the School Students' Association elections. The structure of the exercise was similar to that referred to for conjunction and disjunction. Additionally, in the case of the last two exercises the aim was to arrive at the truth table of the logical operation referred to in each of these exercises (disjunction and implication).

During the implementation of the activity, field notes were also prepared and photos and videos were taken of the ongoing process, which allowed us to collect the indicators of the motivation and involvement of the participants and of the pedagogical dialogue.

2.2. Pedagogical Approach

2.2.1. Poly-Universe Materials

Poly-Universe is a geometric ability development game invented by János Saxon Szász. It includes three base forms, circle, triangle, square, four colours, red, yellow, blue, green, a basic element considered a basic object designed from the base forms with shapes of various sizes and colours (Figure 1). A set is a package of 24 elements of the same base form (Figure 2). Considering these elements, endless figures can be created, with participants of any age and in different contexts. In fact, 'although the tool consists of simple elements (basic shapes, basic colors, proportions), it is extremely complex because there are virtually endless possibilities for changing proportions, combining colours, and connecting all of them' [28].

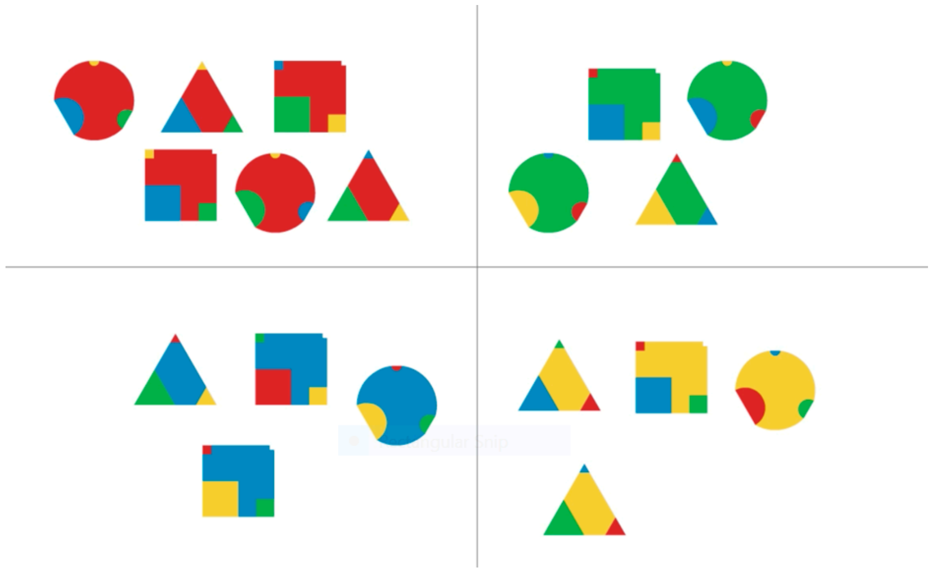

Figure 1. Examples of Poly-Universe elements.

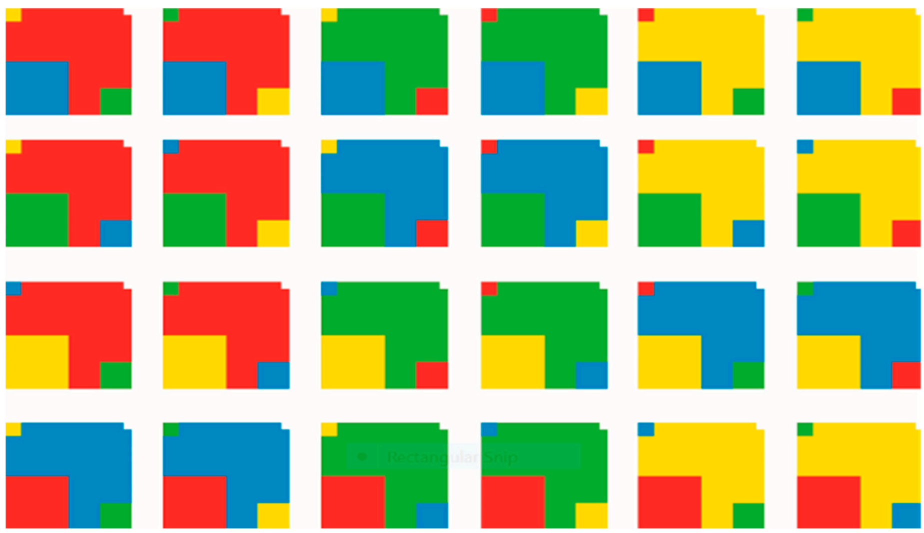

Figure 2. Set of square shapes with cut-out corners from Poly-Universe.

The glossary integrating the definitions of the fields was also presented [29].

2.2.2. Exercises Implemented

In Tables 1–3, we present the tasks proposed with the Poly-Universe material. It is a set of three exercises, with a structure similar to the pre-test, involving logic operations and the calculation of areas based on the fields of the Poly-Universe pieces, based on propositions to determine their logical value.

Table 1. Exercise on conjunction.

Questions			
1. Select the set of triangular pieces for which the proposition	P: The base field is green.		
1.1. For the set of fields obtained previously, indicate, justifying, the logical value of the following propositions.	Q: The smaller field is an equilateral triangle. R: The largest field is yellow. S: The middle field is red.		
1.2. Given the meanings of P, Q, R, and S, formalise the following propositions and state their logical value.	1.2.1. The base field is green and the smaller field is an equilateral triangle. 1.2.2. The base field is green and the larger field is yellow. 1.2.3. The middle field is red and the base field is green. 1.2.4. The largest field is yellow and the middle one is red.		
1.3. Calling A and B any two propositions, and taking into account the conclusions obtained in Question 1.2, complete the following table.	A V V F F	B V F V F	$A \wedge B$

Table 2. Exercise on disjunction.

Questions			
2. Select the set of circular pieces for which the proposition is true	P: The larger area is blue.		
2.1. Suppose that the areas (larger, intermediate, and smaller) in each of the pieces have diameters equal to 4, 2, and 1, respectively. For the set of pieces obtained previously, indicate, justifying, the logical value of the following propositions.	Q: The sum of the areas of the fields, major, intermediate, and minor is equal to $\frac{21}{8}\pi$. R: The base area is circular. S: The smaller area is yellow.		
2.2. Given the meaning of P, Q, R, and S, formalise the following propositions and state their logical value.	2.2.1. The larger field is blue or the sum of the areas of the regions, larger, intermediate, and smaller, is equal to $\frac{21}{8}\pi$. 2.2.2. The larger field is blue or the base region is circular. 2.2.3. The smaller field is yellow or the base field is circular.		
2.3. Calling A and B any two propositions, and taking into account the conclusions obtained in Question 2.2, complete the following table.	A V V F F	B V F V F	$A \vee B$

The exercises proposed included math, namely the concept of triangles and fields, which are geometric objects and can be studied using mathematical methods. The given exercise involves selecting and analysing sets of triangular pieces based on logical operations. The exercises included a philosophical component as the data included propositions that make claims about the properties of the triangular pieces and their fields, and the exercises involved evaluating the logical value (truth or falsity) of these propositions using logical reasoning.

Table 3. Exercise on implication.

Questions			
3. Select the set of square tiles for which the proposition is true P: The base region is blue. Suppose that the fields (largest, middle, and smallest) on each of the pieces have sides equal to 4, 2, and 1, respectively.			
3.1. For the set of pieces obtained previously, indicate, justifying, the logical value of the following propositions.	Q: The piece has three similar fields. R: The area of the larger region is twice the area of the intermediate region.		
3.2. Taking into account the meanings of P, Q, and R, formalise the following propositions and state their logical value.	3.2.1. If the base region is blue, then the piece has three similar fields. 3.2.2. If the base region is blue, then the area of the larger region is twice the area of the intermediate region.		
4. Now consider all the pieces (triangles, squares. and circles).			
4.1. State the logical value of the following propositions.	P: The larger field is blue. Q: The smaller field is not blue. R: The piece has 4 fields.		
4.2. Taking into account the meanings of P, Q, and R, formalise the following propositions and state their logical value.	4.2.1. If the larger field is blue, then the smaller field is not blue. 4.2.2. If the largest field is blue, the piece has 4 fields.		
5. Calling A and B any two propositions, and taking into account the conclusions obtained in Questions 3.2 and 4.2, complete the following table.	A	B	$A \Rightarrow B$
	V	V	
	V	F	
	F	V	
	F	F	

2.2.3. Implementation

In both classes, the activity was implemented during lesson time. The students were sitting in pairs.

The activity began with a review of logic concepts previously taught in two mathematics lessons, involving the symbols and notions of conjunction, disjunction, negation, double negation, and implication. The teacher asked questions such as 'What were the logical operations studied?', 'What is the logical value of a proposition?', writing on the board a truth table for each of the operations studied.

This was followed by answering the questionnaire and the pre-test sheet. One student wished to consult the dictionary. The teacher asked some questions about the mathematical component, for example, 'How do you calculate abstention?', about the logical value of the propositions, and about the steps needed to complete the task. The students were involved in the task, sometimes exchanging ideas with each other. This part of the lesson took approximately one hour.

This was followed by the presentation of the Poly-Universe material and the glossary. The activity was performed in groups; four groups of three were formed. The teacher introduced the materials and the terms used and distributed the tasks

After the presentation of the activity, the students immediately started reading the exercises and exploring the Poly-Universe materials. The teacher gave instructions to each group. In the end, all the groups completed the proposed tasks. Figure 3 shows photographs of the course of the activity.

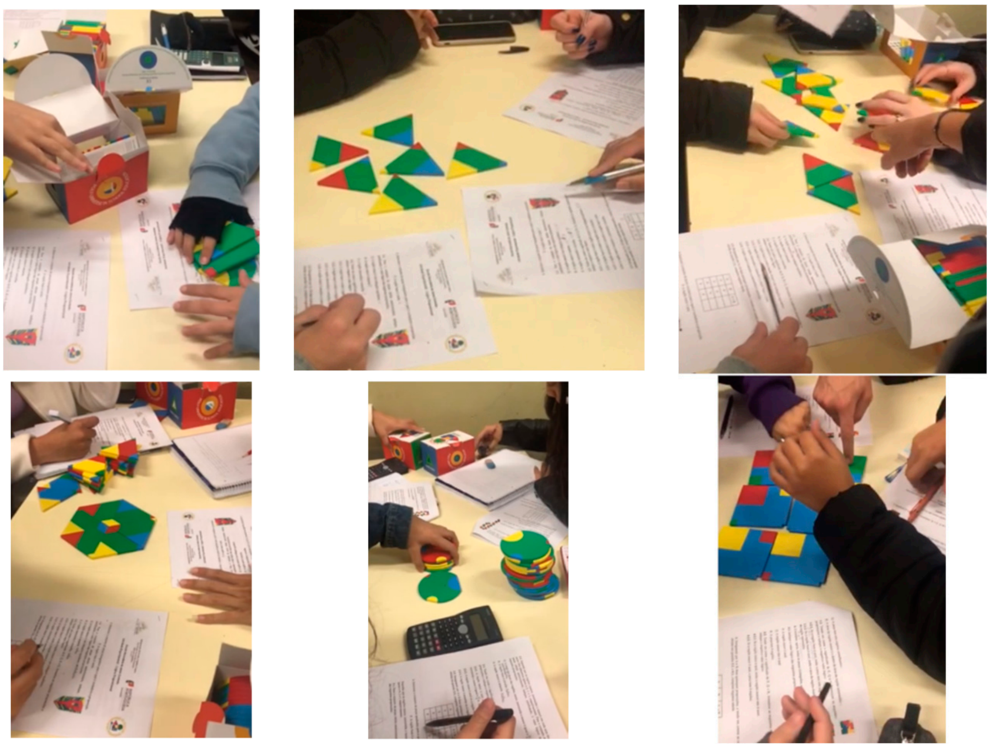

Figure 3. Participants developing the exercises.

3. Results

3.1. Students' Perceptions of the Activity That Was Carried Out

Based on the responses to the initial questionnaire, it was found that in MACS class, nine of the participants had already been involved in interdisciplinary activities, four of them in the 10th year of schooling, three in the 9th year and two in the 8th year. The subjects involved in the interdisciplinary activities were Mathematics Applied to Social Sciences and Philosophy, in the case of three participants, but also Mathematics and Portuguese, Physical Chemistry and Mathematics, Geography and Citizenship, Physical Education and Biology, and Physical Education and Citizenship. In the last case, the contents involved were sustainability and development, and the contents were logic in the case of mathematics and philosophy.

In the Mathematics A class, two participants referred to their previous involvement in interdisciplinary activities, giving a short description of the activity as a 'soup of letters with synonyms'.

Figure 4 shows the results concerning the perception of the degree of relation between mathematics and philosophy before and after the activity, and also the importance of the use of manipulative materials in learning these subjects. In both classes, MACS and Mathematics A, participants perceived a higher degree of relation between mathematics and philosophy in the post-test, and also attributed a greater importance to the use of manipulative materials in the learning of mathematical and philosophical concepts.

Figure 5 presents the results concerning the perception of motivation and other dimensions related with participation in the activity in MACS and Mathematics A classes. The score of interest corresponds to the average score on Item 1. 'I enjoyed doing this activity very much' and Item 6. 'This activity was fun to do'. On average, students rated the activity as interesting; however, MACS students gave a slightly higher rating (4.25) than Mathematics A (4.05) students.

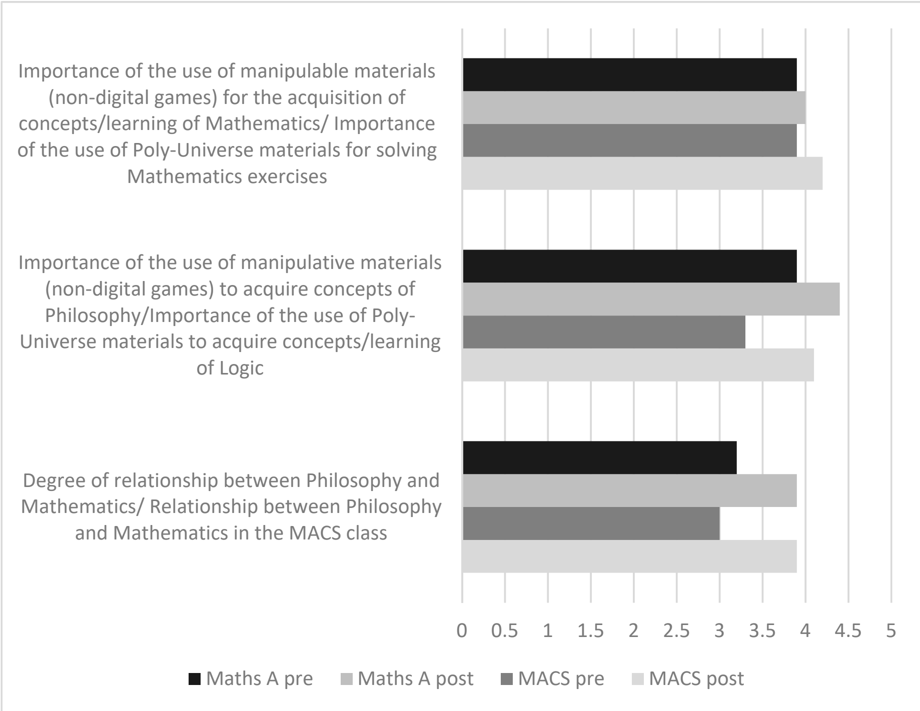

Figure 4. Perception of the degree of relation between mathematics and philosophy and the importance of the use of manipulative materials in the learning of the subjects before and after the activity.

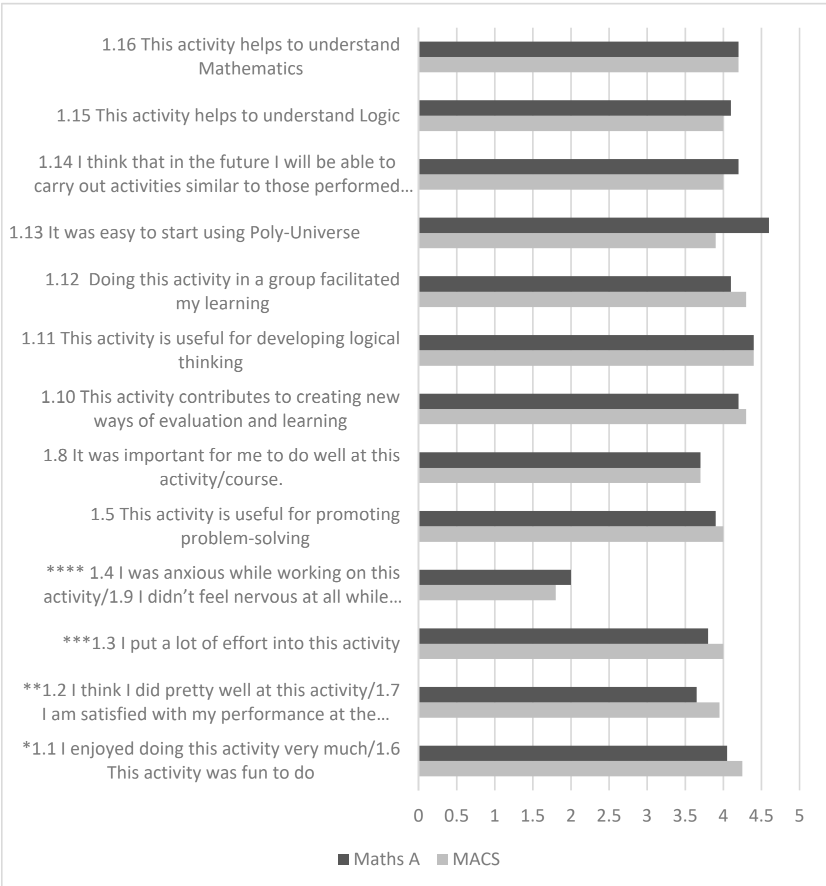

Figure 5. Perception of motivation and other dimensions related with the participation in the activity. Indicators are mentioned with *, namely *—interest; **—self-efficacy; ***—effort; and ****—pressure.

The perception of the self-efficacy score corresponds to the average score on Item 2. 'I think I did pretty well at this activity' and Item 7. 'I am satisfied with my performance at the activity'. Students rated themselves as doing quite well and being satisfied with their performance in both courses. Again, MACS students gave a slightly higher rating (3.95) than Mathematics A (3.65) students.

The perception of pressure corresponds to the average score obtained from Item 4. 'I was anxious while working on this activity' and Item 9. 'I didn't feel nervous at all while doing this course'. On average, students reported feeling less anxious while working on Mathematics A (2.0) compared to MACS (1.8). Students also rated themselves as putting a lot of effort into both courses, with MACS participants giving a slightly higher rating (4.0) than Mathematics A (3.8).

Items 1.5, 1.10, 1.11, 1.15, and 1.16 focus on the perception of the usefulness of the activity for different aspects, such as problem-solving, development of logical reasoning, or new ways of teaching and learning. Participants considered the activity useful for promoting problem-solving, with MACS students giving a slightly higher rating (4.0) than Mathematics A (3.9) students, and believed that it may contribute to new ways of evaluation and learning; again, MACS students giving a slightly higher rating (4.3) than Mathematics A (4.2) students. Developing logical thinking skills was rated high by students from both classes. The potential of the activity for the development of logical reasoning and the understanding of mathematical concepts was also highlighted.

The remaining items focused on the general conditions of use of the materials and the way in which the activity was performed, in this case highlighting the importance of the activity that was performed in a group. Mathematics A students also found Poly-Universe slightly easier to use (4.6) than their colleagues in the MACS class (3.9) and expressed confidence in their ability to carry out similar activities in the future.

Overall, the data suggest that both MACS and Mathematics A students appreciated the activity, considered it useful in developing logical thinking skills, in promoting problem-solving, and in facilitating group learning. However, MACS appeared to have a slight edge in terms of student interest, self-efficacy, and contribution to new ways of evaluating and learning, while Mathematics A was perceived as being less anxiety-inducing and easier to use with Poly-Universe.

Figure 6 shows the result of the assessment of the activity's potential for the development of skills. In general, students from the MACS class perceived the activity to have higher potential for skill development, with the exception of innovation. The competences perceived to have a higher potential development by student from the MACS class were group work, creativity, and logical thinking. In addition to these, competencies with scores higher than 4 were sharing/cooperation, understanding, visual perception, and concentration/attention.

Students from the Mathematics A class perceived the activity to have a high potential to develop skills such as group work, sharing, understanding, and innovation.

Based on the answers to the open-ended questions on what they liked best in the activity, what difficulties they experienced, what they learned, and what improvements they would suggest, we can mention several aspects. The content analysis of the answers to the question about what they liked most in the activity allows us to identify three themes in the case of the MACS class: manipulation of materials, group work, and positive experience. The most frequent was the first one. Participants appreciated "understanding logic through manipulative materials", "being able to touch the shapes", and "solving problems through manipulative materials", as well as collaboration and teamwork, "working with the Poly-universe materials", "in a group, I was able to get help from others to better understand the subject and finish the work quickly". The fact that it was a positive experience was also mentioned as "overall, I liked everything. I think it's a unique, interesting activity that requires attention, especially visual attention", "I liked it because it helped with developing reasoning and it was easier to learn with the pieces".

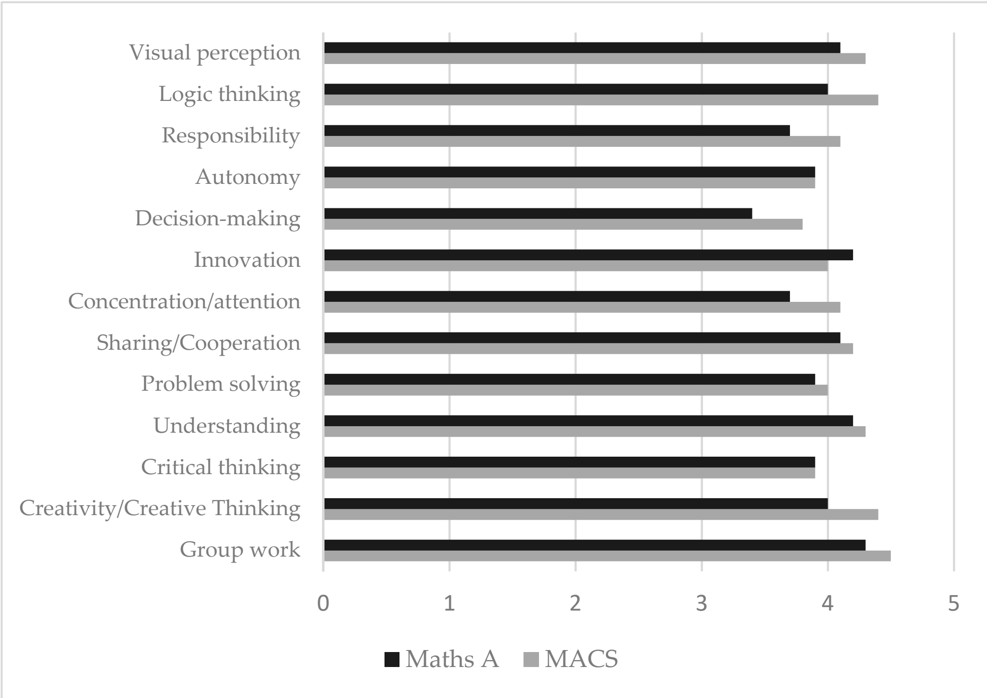

Figure 6. Perception of satisfaction and motivation when participating in the activity, namely perception of interest, self-efficacy, effort, stress, and usefulness.

In the Mathematics A class, the three categories mentioned for MACS were also identified, namely group work, manipulation of materials, and positive experience and a fourth category named creativity was also identified. The most frequent reason for appreciating the activity was collaboration and teamwork, "working with my colleague next to me", "being able to discuss opinions until reaching a consensus", "working in a group", "being able to work in a group".

Participants also mentioned the experience of manipulative materials and hands-on learning, "playing with the pieces helps to understand better and give better answers", "I really liked the Poly-universe material because it's very intuitive and helped with learning". Participants also mentioned that it was a positive experience, "everything. making the participant feel good". Another reason for the positive appreciation was creativity, "creativity was very advantageous in understanding",

Most of them reported having had no difficulties, although three of them identified difficulties in aspects related to Mathematics, 'I had difficulties with the value of pi and the fraction operation with pi', 'knowing the areas', 'when it came to doing the calculations, I couldn't understand'. In the Mathematics A class participants referred to an initial difficulty in understating what was requested, saying that "initially, I had difficulty understanding the purpose of the activity, but over time I understood", "at first, I didn't understand the usefulness of the pieces", "At first, I had more difficulty understanding how the pieces worked". Other difficulties that students referred to were related to logical operations, namely "I didn't understand the exclusive and inclusive disjunction at first, but now I do", "I had the most difficulty with truth tables". Reading and understating meanings were also reported as challenges, "understanding and reading the exercise sentences was a slight challenge".

When asked 'What did I learn?', in MACS class, students referred to a better understanding of logical reasoning as a learning outcome, "I learned logical reasoning", as well as and an increased ability to use truth tables, "The truth tables". They also referred to an improvement in understanding math concepts, "I learned that there are various ways to learn math' and an improvement in problem-solving skills, "Everything becomes easier to solve problems when being able to observe the materials", "I learned that if we do the work and cooperate, it goes much better." They also expressed positive perception of the subject matter, "The colors are beautiful", "The areas (with a smiley face drawing)", "I was able to acquire a better understanding of the subject matter". In the Mathematics A class, students focused on the learning outcomes related with logical operations, referring to an improvement in their understanding of logic, "I learned better about logic as a discipline and its reasoning", "I learned about logic and new and easier ways to understand the subject", "I learned to improve the use of my logic", as well an enhanced ability to create truth tables, namely "Knowing how to make a truth table, the subject of logic" or "Truth tables and logic". Participants also referred to an awareness of the limitations of their own logic, "I learned that our logic may not correspond to the truth" and an increase in problem-solving skills, "With mathematics, I learned to solve problems better".

In general, participants enjoyed the activity and had no suggestions for improvement, although one participant said she felt a little lost.

3.2. Evaluation of Learning

The assessment of learning was based on the answers to the pre-test and post-test, as well as results on the activity proposed.

The mean number of correct answers to the exercise involving the logical operation conjunction is presented in Table 4. Although in the identification of the logical value of propositions in the pre-test of the MACS class most students answered correctly, in the post-test, all of them answered correctly. In the case of filling in the truth table, in the pre-test, the average value of correct answers was 0.6, still showing some difficulties in finding the correct answer, but his was overcome in the post-test. In the exercise on conjunction, all groups answered correctly in identifying the logical value of the propositions and completing the truth table.

Table 4. Mean of correct answers to questions about the logical operation conjunction in the pre- and post-test and in the PUNTE activity.

	Tasks	**Determining the Logical Value of Propositions—Conjunction**				**True Table Construction**
		1.2.1 V∧V	1.2.2 V∧F	1.2.3 F∧V	1.2.4 F∧F	1.3
MACS	Pre-test	0.9	0.9	0.8	0.8	0.6
	Activity PUNTE	1.0	1.0	1.0	1.0	1
	Post-test	1.0	1.0	1.0	1.0	1.0
Mathematics A	Pre-test	1.0	1.0	1.0	0.9	0.9
	Activity PUNTE	1.0	1.0	1.0	0.9	0.9
	Post-test	1.0	1.0	1.0	1.0	1.0

The results concerning the exercise on the logical operation disjunction are presented in Table 5. These results show that in the pre-test of the MACS class, most participants did not correctly identify the proposition involving F∨V, and about half also did not correctly identify the propositions involving the operations V∨F and F∨F. The average value of correct answers is also very low in the case of the truth table construction. However, in the post-test, the mean values of correct answers vary between 0.6 and 0.8 in the case of the truth table, which shows a learning of the concepts in question. All groups answered correctly in identifying the logical value of the propositions and completing the truth table, except for the identification of the logical value of the proposition V∨F, in which one of the groups did not answer correctly.

Table 5. Mean of correct answers to questions about the logical operation disjunction in the pre- and post-test and in the PUNTE activity.

	Tasks	Determining the Logical Value of Propositions—Disjunction				True Table Construction
		1.2.1 V∨V	1.2.2 V∨F	1.2.3 F∨V	1.2.4 F∨F	1.3
MACS	Pre-test	0.8	0.5	0.3	0.6	0.2
	Activity PUNTE	1.0	1.0	0.67	1.0	1.0
	Post-test	0.8	0.7	0.7	0.7	0.6
Mathematics A	Pre-test	0.8	0.3	0.2	0.8	0.6
	Activity PUNTE	0.9	0.9	0.9	0.9	0.9
	Post-test	0.9	0.8	1.0	1.0	0.9

The results of the case of the exercise involving the logical operation implication are presented in Table 6. In the MACS class, there was also progress in the mean values of correct answers between the pre- and post-tests, with the exception of the logical operation F→V. In the case of implication, one group did not correctly identify the logical value of V→F, two groups in the case of F→F, and only one group correctly completed the truth table. In the Mathematics A class, the results of the pre- and post-tests do not indicate the aimed change so clearly. However, in both cases, MACS and Mathematics A, the results of the activity using Poly-Universe materials are much better than in the pre-test, pointing towards the reasons for the interest of using the material for this learning process.

Table 6. Mean of correct answers to questions about the logical operation implication in the pre- and post-test and in the PUNTE activity.

	Tasks	Determining the Logical Value of Propositions—Implication				True Table Construction
		1.2.1 V∧ → V	1.2.2 V∧ → F	1.2.3 F → V	1.2.4 F → F	1.3
MACS	Pre-test	0.4	0.8	0.8	0.3	0.3
	Activity PUNTE	1.0	1.0	0.67	0.0	0.33
	Post-test	0.6	0.8	0.5	0.3	0.7
Mathematics A	Pre-test	1.0	0.9	0.3	0.3	0.6
	Activity PUNTE	1.0	1.0	0.7	0.7	0.9
	Post-test	0.6	0.6	0.6	0.6	0.6

4. Conclusions

The data suggest that the use of the Poly-Universe material contributed to the engagement and motivation of performing the proposed tasks, which was revealed in the perception of interest and self-efficacy, and also led to the learning of logical notions. It should also be noted that the activity proposed using the Poly-Universe material increased the participants' perception of the relationship between philosophy and mathematics and the importance assigned to the manipulation of materials in the learning of logical and mathematical concepts. These data highlight the potential of the Poly-Universe material in interdisciplinary learning [18,19], especially for disjunction in the Mathematics A class, and for implication in the case of the MACS class. The data also suggest that the group that reported familiarity with previous interdisciplinary experiences, the MACS class students, had better pre- and post-test results and better results in the activity itself, both in terms of motivation and the skills they consider to have acquired. These results reinforce the importance of these types of interdisciplinary activities, which are more advocated than practised in the classroom.

It should also be noted that group work, logical reasoning, and creativity, as well as comprehension, visual perception, concentration, sharing, and responsibility emerged as the competencies most often reported by the participants in this study as being developed in the implemented activity, which corroborates other studies involving the use of the Poly-Universe material in the development of transversal competencies [17].

It remains, however, to test, in the future, the hypothesis of the effect of the Poly-Universe material on the acquisition of logical and mathematical notions in an experimental or quasi-experimental plan, using a wider variety of tasks in which the logical propositions closely involve the contents of philosophy and mathematics.

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Data Availability Statement: The data presented in this study are available on request from the corresponding author. The data are not publicly available due to privacy issues.

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