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# Engaging Students in Mathematical Problem Solving with Technology during a Pandemic: The Case of the Tecn@Mat Club

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**Abstract:** The COVID-19 pandemic and the requirement for social distancing led to the closure of extracurricular activities that usually involve teamwork and collaboration, such as math clubs. Research on the design and effectiveness of extracurricular mathematical activities that aim to promote student interest and improve mathematical skills is limited, particularly in these challenging times. This exploratory case study examines an online after-school program, the Tecn@Mat Club, aimed at promoting middle grade students' ability in solving mathematical problems with digital technologies during the pandemic. Based on document analysis, observation of the club sessions, questionnaires, and interviews with the participants (students and a mathematics education expert), this research documents the design and implementation of the Tecn@Mat Club, as well as its participants' experiences and perspectives regarding the activity in which they engaged. The main results show the feasibility of adapting in-person, after-school math activities for an online setting, engaging middle grade students in mathematical problem-solving with technology by providing moderate mathematical challenges and promoting collaborative work. Results of a SWOT analysis (the acronym for strengths, weaknesses, opportunities, threats) allow for the identification of the key features to preserve and challenges to overcome in future replications of the club. Limitations of this study and future research directions are discussed.

**Keywords:** mathematical problem solving; digital technologies; techno-mathematical fluency; math club; after-school mathematics; 21st-century skills; COVID-19; online learning

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**Citation:** Jacinto, H. Engaging Students in Mathematical Problem Solving with Technology during a Pandemic: The Case of the Tecn@Mat Club. *Educ. Sci.* **2023**, *13*, 271. https://doi.org/10.3390/educsci13030271

Academic Editor: Christopher R. Rakes

Received: 24 January 2023
Revised: 25 February 2023
Accepted: 28 February 2023
Published: 3 March 2023

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## 1. Introduction

The COVID-19 pandemic has transformed mathematics teaching and learning worldwide [1–4]. The several lockdowns were extremely demanding for teachers, students, and their families. When schools reopened, the need to comply with social distancing led to the closing of extracurricular activities that typically involve interaction, collaboration, and teamwork, such as mathematics clubs and other similar initiatives.

Despite the diversity of math-related extracurricular activities that aim to promote students' interest and improve their achievement in mathematics, such as math clubs [5], math camps [6], math circles [7], math competitions [8], or online math communities [9,10], there is little empirical evidence on the design and ways of functioning of such initiatives, hence designers "are guided by anecdotal evidence and existing practices, but little underlying understanding of processes" [7] (p. 718).

Mathematics clubs, either developed within the school ground or beyond, are regarded as places where youngsters who enjoy mathematics and share similar ideas about mathematics get together [11]. In these clubs, that do not have the constraints of a typical math classroom and rely on voluntary participation [7], students find opportunities to develop their self-esteem, sense of belonging, and purpose [5].

Recent years have seen little attention from mathematics education researchers on the success of math clubs and its underlying factors. However, some studies have found

evidence supporting the idea that students' participation in math clubs can impact their learning and attitudes towards mathematics.

Based on data collected at an American school math club in which students tended to achieve excellent results on standardized mathematics tests, Papanastasiou and Bottiger concluded that participants had very positive attitudes towards both the subject of mathematics as well as the club itself [6]. The researchers also analysed what motivated students' participation, identifying reasons related to learning mathematics, reasons related to extrinsic factors such as being with friends, and others related to the characteristics of the club, e.g., because the use of calculators and group work were allowed. Another study aiming to understand the ways of reasoning of students involved in an informal after-school environment based on problem solving [12], concluded that when the tasks elicit exploration and collaboration, students seek to explain and justify their points of view and reasoning, both in small groups and in the large group discussion. Young participants not only build arguments collaboratively but also provide mathematical justifications for the solutions they develop. A different project, Pushing for Progression, provided in-service training to South African teachers to support the promotion of mathematics clubs in a particular region of the country [13]. The researchers studied the impact of this initiative on students' performance and concluded that it had a positive effect in the students' learning, both in terms of procedural fluency and conceptual understanding, in a relatively short period of time of involvement in one of the math clubs under analysis. What is more, the partnerships established between the academics, the district educational entities, and the teachers were decisive in the success of the project and in the students' learning. The impact of participating in extracurricular activities on student performance in mathematics was recently analysed by [11]. From a representative sample of over 23,000 grade 9 students, the researcher concluded that prior involvement in math competitions and math clubs lead to a significant increase in student proficiency.

Math clubs provide a space for students interested in mathematics to meet and share their interests, participate in games and competitions, and solve mathematical problems. According to the literature reviewed, it appears that students who have participated in math clubs and similar initiatives have improved their mathematical skills and demonstrated positive attitudes towards learning, the mathematics subject, and their own ability to succeed in mathematics.

The current study addresses a pioneering extracurricular math program in Portugal, the Tecn@Mat Club, that utilizes digital technologies to offer online enrichment activities for middle school students (ages 12–14). Its goal is to enhance students' mathematical problem-solving skills and, since the edition under study was developed during the pandemic, it also aimed to overcome social distancing restrictions by encouraging participation and interaction in a collaborative and technology-rich environment. This research study aims to examine the design and implementation of the Tecn@Mat Club, with the support of documentary data and the perspectives of the participants. To achieve this goal, the following research questions were developed:

(a) What are the main design features of the Tecn@Mat Club, in terms of its structure, content, and organization?
(b) What are the perspectives of the participants regarding problem solving and the use of digital technologies within the Tecn@Mat Club?
(c) Which are the strengths, weaknesses, opportunities, and threats to the Tecn@Mat Club?

Answering these questions will provide insight into the effectiveness of the club and inform the implementation of similar extracurricular programs. Furthermore, as it is unlikely that it will ever be possible to "formulate universal recommendations as to how to organize a problem-solving classroom so that it would fit the individual needs and traits of each student" [14] (p. 319), this study adds to the body of work on the development of mathematical problem solving with technology skills, by extending research to this particular context, where mathematical and technological resources are used to solve math challenges.

The following section delves into crucial concepts related to (i) the use of digital technologies in mathematics; (ii) the concept of non-routine mathematical problems; (iii) finding and explaining the solutions to problems; and (iv) the necessary skills to excel in this activity.

## 2. Theoretical Ideas on Solving Mathematical Problems with Digital Technologies

Studying the use and impact of digital tools on mathematical problem solving requires an examination of how interactions between individuals, technological tools, and the learning environment, either in the classroom or after-school settings, shape cognitive processes [8,15,16].

### 2.1. On the Role of the Digital Tools in Mathematical Activity

Many different perspectives have been presented on the role of technology in learning mathematics. The literature suggests that digital tools can serve as an extension of the user, for example when the student "incorporates technological knowledge as an integral part of their mathematical repertoire" [17] (p. 371). When technology is used to develop mathematical thinking, the boundary between the subject who thinks mathematically and the tool that supports that thinking becomes blurred, as the tool is incorporated "so naturally as an intellectual resource" [17] (p. 371). Along the same lines, Borba and Villarreal recognize the power of technology to transform and reorganize mathematical thinking, proposing that an indivisible unit—the *human being-with-media*—should be considered as the subject responsible for knowledge, thought, and action [18]. In fact, given the various possibilities for interaction between an individual and technology, it may not be possible to establish a clear boundary between the actions and the (mathematical) thinking that is triggered by the use of digital technologies [19].

### 2.2. On Non-Routine Mathematical Problems

Similar to other curriculum enrichment experiences such as the problem-solving competitions documented in [8], Tecn@Mat aims to provide a space that fosters the development of mathematical problem-solving skills. It is therefore important to discuss what is meant by *problem*, *problem-solving* and *problem-solving with technologies*. A non-routine problem is a task that entails a challenging situation for the student who does not have a direct mathematical method that guarantees finding a solution [20,21]. This definition, which emphasizes a close connection to the mathematics classroom, was challenged by Lesh and Zawojewski who argue that problem-solving should also facilitate the understanding of the connection between mathematical ideas and real-world situations, beyond the classroom [22]. From their perspective, "a task, or goal-directed activity, becomes a problem (or a problematic situation) when the solver, which may be a group of collaborating experts, needs to develop a more productive way of thinking about the given situation" [22] (p. 782).

### 2.3. On Solving Non-Routine Problems and Expressing the Solutions with Digital Tools

Solving mathematical problems using technology, similar to using paper-and-pencil, involves engaging in a process of mathematization where the solver, a youngster-with-media, must develop a productive way of thinking about the problem, which leads to the development of a conceptual model of the situation [22]. As conceptual models convey mathematical understandings of situations, they are generally "expressed using a variety of representational means" [23] (p. 159) so they may incorporate paper-based diagrams or graphs, but also tables, written text, symbols, drawings, images, graphics, or dynamic figures produced with digital technologies. However, "it is the descriptive and explanatory quality of thought that makes it work as a model" [24] (p. 55), so the way youngsters-with-media externalize their interpretation of the situation and explain their approaches refers to considering that the phases of solving and expressing the solution are closely linked. Obtaining solutions to problems thus includes creating explanatory descriptions that "are not simply supplements that students include after the 'answer' has been produced.

They ARE the most important components of the responses that are needed" [25] (p. 3, original emphasis). In this vein, solving-and-expressing is a key concept that summarizes a conceptualization of problem-solving as a simultaneous process of mathematizing the situation and expressing the mathematical thinking developed [8,15,16].

Research has also shown that problem solving and expression through digital technologies results in the production of a narrative, a story that tells how the solution of a given problem was developed [15]. This expository digital mathematical discourse [9] is thus driven by the mathematical and technological tools used, and stands out for the use of colours, drawings, images or photos, the use of natural and symbolic language, and the use of files produced with other programs such as dynamic geometry or spreadsheets [8].

Another factor to consider is the level of challenge that non-routine mathematical problems should ideally contain, since "the predisposition to solve a task seems to decrease in two situations: when expectations about the probability of success are too high (the task is too easy) or when they are too low (the task is too difficult)." [24] (pp. 545–546). The concept of moderate mathematical challenge [26], adopted within the scope of the Problem@Web project [8], is particularly useful as it contributes to creating a certain balance, which is necessary and desirable, i.e., the challenge should be engaging enough to motivate students to want to solve it [27] and, although it requires effort, the solution of the problem must be achievable for all students.

### *2.4. On the Skills Needed to Succeed in Mathematical Problem Solving with Digital Tools*

To be successful in non-routine mathematical problem solving with digital technologies involves, among others, the use of adequate mathematical resources [21] but it also involves considering that digital tools are equally indispensable artifacts [15]. It is therefore necessary to discuss the proficiency of a youngster-with-media in solving moderate challenges using mathematical and technological tools. Hoyles and colleagues analysed the ability to use technological and mathematical knowledge to solve everyday problems or problems related to the professional world, developing the concept of *techno-mathematical literacy* [28]. This ability refers to a combination between mathematical and technological knowledge as well as communication skills, to solve problems in a particular context, which seems to be in line with the present discussion on the activity of solving mathematical problems with digital technologies in an extracurricular program. Previous studies [15,16] have highlighted the concept of *fluency* as a key characteristic in solving-and-expressing problems using technology. Fluency, as introduced by Papert and Resnick, refers to the ability to effectively communicate complex ideas and create relevant outcomes with digital tools [29].

These two concepts have inspired the notion of *techno-mathematical fluency* (TmF), which has been used to designate the ability to combine mathematical and technological knowledge to solve and express mathematical non-routine problems [15,30]. As with digital fluency [31], TmF involves the ability to select appropriate resources from a range of options, both mathematical and technological, to recognize their affordances and limitations, and to know how a given tool can be used to create a techno-mathematical solution to a problem [30]. It is essential to note that mathematical knowledge plays a crucial role in guiding the use of technology, as it enables the identification of affordances that shape the approach and the conceptual model developed in finding the solution [32].

## 3. Research Methods

The main purpose of this study is to document the key features in the design of the Tecn@Mat Club, particularly understanding the perspectives of the participants in terms of the club's strengths, weaknesses, threats, and opportunities that are fundamental in its implementation. Considering the lack of preliminary research on this topic [7], an exploratory case study [33] seems to be an appropriate research design as it allows a thorough examination of a particular phenomenon within a real-world context.

This study adopted a convergent mixed-methods design [34] in the form (QUAL + quant), where qualitative and quantitative data were collected concurrently but the

qualitative strand was emphasized. The data sets were analysed separately, and the findings were subsequently compared and integrated to provide a comprehensive understanding of the Tecn@Mat Club that can guide decision-making. The following provides a description of the participants, outlines the specific data collection methods and instruments used, and details the analysis processes employed.

### 3.1. The Participants

The first edition of the club consisted of 12 participants, 4 boys and 8 girls, aged between 12 and 14 years old. Participation was voluntarily and required the informed consent of the students' parents. Seven participants were in seventh grade, four were in eighth grade, and one student was in ninth grade. The participants came from schools in the Algarve, Greater Lisbon, and Minho regions. Participation in the club and the study was obtained through outreach to mathematics teachers and class directors of the participating students. A mathematics educator also served as an external observer and attended the club sessions, observing different breakout rooms without interfering in the organization or reasoning processes of the participants.

### 3.2. Data Collection

Data collection occurred at various points in time, resorting to several methods, namely, observation of the sessions, questionnaires applied to the participants, individual semi-structured interviews with the participants and the mathematics educator, and documental analysis.

During the club sessions, conducted via Zoom, both the collective work and the autonomous work in four breakout rooms were video recorded. The digital files created by the participants for solving problems and expressing solutions were also collected. At the end of each session, participants, anonymously and voluntarily, completed an online questionnaire which provided quantitative and descriptive data about the problem solved (e.g., reasons supporting the team's choice, personal's level of challenge and enjoyment felt), team member participation, strategies, and technological tools used (e.g., the different tools used and each tool's purpose throughout the activity).

After concluding the activities, the youngsters and the mathematics educator were interviewed. The students' interview took place through Zoom, which enabled videorecording, and was aimed at probing students on their participation and learning experiences in the Tecn@Mat Club. The interview was organized into three major themes: solving mathematical problems, using digital technologies, and engaging in collaborative work throughout the club sessions. The mathematics educator was probed regarding the main advantages that Tecn@Mat provides to the participants, what kind of problems Tecn@Mat may face, in which ways could Tecn@Mat be improved, and which external initiatives could benefit Tecn@Mat in future editions.

Furthermore, all documents supporting the development of the club were collected, namely, session plans, the outputs produced (e.g., the GeoGebra and spreadsheet files), and the presentations used.

### 3.3. Data Analysis

The qualitative data were organized and processed using NVivo (Release 1.5.1). In particular, the software was used to capture coding outcomes. Segments of the documental data were examined in terms of the structuring, content, and organization characteristics of the Tecn@Mat Club. The participants' statements were segmented, coded, and aggregated around two main themes: perspectives on mathematical problems and perspectives on the use of technologies to solve mathematical problems. In this paper, the descriptive reporting was further supported by presenting transcribed raw data segments.

To identify the internal and external factors that may challenge or contribute to the success of Tecn@Mat in achieving its goals, a SWOT analysis (acronym for strengths, weaknesses, opportunities, threats) was conducted on the data from the interviews to the

students and the mathematics education expert. SWOT analysis is considered a strategic planning tool and has been used in the assessment of educational organizations [35], to evaluate and inform the redesign of micro-projects concerning prospective mathematics teacher education [36], and also in STEM education settings [37,38]. The analysis of the internal features of the Tecn@Mat Club, namely its strengths and weaknesses, as well as external features, such as opportunities and threats, is expected to be "an illuminating instrument in the exploration of the context of change" [35] (p. 3). NVivo assisted in conducting a deductive content analysis of the transcribed interviews, in order to identify and describe the strengths, weaknesses, opportunities, and challenges of the Tecn@Mat Club.

The quantitative data were analysed using descriptive statistics to identify patterns or changes in students' attitudes towards solving mathematics problems with digital technologies. Given the small number of participants and the exploratory nature of the study, the establishment of inferences or statistical significance was not pursued. Rather, the analysis aims to provide a description of the case that allows an in-depth understanding of the way the club works and the experiences of the participants, particularly focusing the development of the ability to solve mathematics problems with digital technologies.

Overall, the analysis process involved a thorough reading of the qualitative data and the subsequent search and identification of key utterances based on the central concepts discussed theoretically, guided by the elements of the SWOT analysis, combining and merging quantitative results whenever appropriate.

## 4. Results

### *4.1. The Design and Implementation of the Tecn@Mat Club*

This section accounts for the main characteristics of the Tecn@Mat Club, both from the point of view of the design and the implementation of its curriculum, as well as from the perspectives and experiences of the young participants and the external observer.

#### 4.1.1. How Tecn@Mat Works

The virtual club aimed at fostering collaborative mathematical problem solving with digital technologies, among middle grade students (12–14 years old). Its first edition took place during the COVID-19 pandemic. This after-school and extracurricular initiative had, at its core, an inclusive nature, as it was aimed at students who typically enjoy mathematical challenges but may or may not be high achievers in the school subject. This idea was patent in several documents (such as the advertisement leaflet and the informed consent form) and throughout the sessions.

> *The Tecn@Mat Club aims to be a special forum for students who are enthusiastic about mathematical challenges and enjoy using digital technologies, regardless of their achievement level in Mathematics. The students will have the opportunity to engage in an enjoyable, challenging, and exciting mathematical activity, in a relaxed yet learning atmosphere.* [translation of excerpt from the advertisement leaflet]

The club's goal is to promote math problem solving for all students, regardless of their performance in the subject. It aims to be inclusive, setting itself apart from initiatives that only focus on identifying highly talented or gifted students. The students who participated in the club were motivated and interested in mathematics, despite having varying levels of proficiency, as measured by their school classifications ranging from 3 to 5 on a scale where 1 is the lowest and 5 is the highest.

The club's activities were primarily conducted using Zoom for videoconferencing, and various Google tools such as Drive, Google Docs, and Sheets. The club met once a week for five synchronous sessions, each lasting 90 min. In addition, an asynchronous session was held between the second and third meetings, during which participants were given two challenges to solve independently. Each session was supported by a website that provided general information about the research project in which the club was anchored. A dedicated page for Tecn@Mat Club was provided, which had all the content for the sessions including the challenges, instructions, and a form for submitting solutions as well as access

to questionnaires. Additionally, the website had a page that covered basic concepts related to solving mathematical problems, such as Pólya-like problem-solving steps, heuristics, and examples of commonly used strategies.

#### 4.1.2. The Tecn@Mat Curriculum: Problems, Mathematical and Technological Resources

The proposed problems were selected or created so that, in a short period of time, they provided experiences with different technological tools and did not require the use of advanced mathematical knowledge, since the purpose was to keep the participants involved in the problem-solving activity with technologies and not the instruction of specific curricular contents (Table 1). Although some of the problems could be solved using more advanced mathematical knowledge or procedures (as is the case of the "Cages and parakeets" problem, whose solution can be obtained by solving a system of two equations with two unknowns), the fact that they can be approached in multiple ways and appropriate technological tools are allowed makes them more accessible.

**Table 1.** Summary of the content in each session of the Tecn@Mat Club (source: session plans).

| Session | Challenge | Mathematical Concepts | Technological Tools |
|---|---|---|---|
| 1 | United and cropped | Areas of polygons | GeoGebra (constructing robust figures; computing areas; spreadsheet) |
| | How many rectangles? | Quadrilaterals. Sum of the first n positive integers. | GeoGebra (constructing figures; spreadsheet). |
| 2 | Snap rings | Circle area. Ratio. | GeoGebra (constructing figures; computing areas; spreadsheet). |
| | The three houses | Logical reasoning. | Text and presentation editors (tabular representation). |
| 3 | Cages and parakeets | Numerical and algebraic relations, involving the concept of variable (system of two equations with two unknowns). | Spreadsheet (writing, cell formatting, simple formulas, autocomplete cells). |
| | The opening of the "Sombrero Style" | Numerical and algebraic relations, involving the concept of fraction, multiple, and variable. | Spreadsheet (writing, cell formatting, formulas, autocomplete). |
| 4 | A costumed pizza | Combinatorics. | Text, presentation, and video editors. |
| | Friends' reunion | Co-variational reasoning. | Spreadsheet (writing, cell formatting, formulas, autocomplete). |
| 5 | Cubes on the floor | Numerical and algebraic relations, involving the concept of perfect square. | Spreadsheet (writing, cell formatting, formulas, autocomplete). |
| | Keys and lockers | Combinatorics. | Text, presentation, and video editors. |

Although the use of digital tools chosen by the participants was allowed and encouraged, the mathematical challenges were also chosen to create opportunities to explore specific software that could support the development of mathematical approaches, such as dynamic geometry environments or spreadsheets. Additionally, the potential for using everyday software, such as text or presentation editors, to support mathematical work was also considered (e.g., organizing information using tables, creating diagrams, or creating videos and animations).

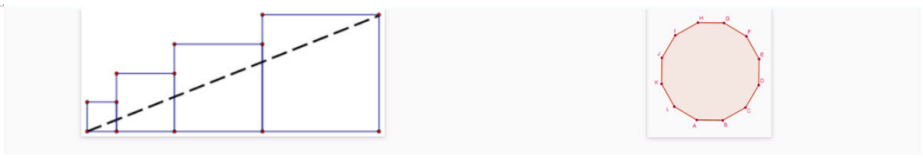
4.1.3. A Typical Session at the Club

Each session of the Tecn@Mat Club included three major moments. A first one involved all participants and aimed to establish the organization of the session and remember rules for participation in the club (and in the study). In the initial sessions, it was also discussed what it means to solve mathematical problems with technologies, namely, the importance of "presenting and explaining the solving process clearly" (presentation, Session 1), that it is allowed to use any technological tool as well as using a diversity of mathematical representations, such as diagrams, figures, calculations, or incorporating explanations that the participants deemed necessary and adequate. Thus, this initial moment legitimized the use of any tool that the participants considered appropriate and the use of a diversity of representations, encouraging the development of an expository digital mathematical discourse [9].

The second moment lasted approximately 60 min and involved the autonomous work of the participants, organized in small teams of three elements, in simultaneous rooms. Each team was asked to choose one of the two problems published in each session, solve it using the tools of their choice, create their solution, and submit the file(s) by email or on the website. The use of Google Drive, with exclusive access to each team, allowed participants to collaborate in real time in the co-construction of Google Docs and Google Sheets to solve problems or elaborate the final solutions for each problem. During the autonomous work of the students, the facilitator circulated through the breakout rooms, observing the collaborative work, and clarifying general questions.

A third and final moment, again in a large group, involved the discussion of the solutions produced by the various teams. In addition, productions by other students who solved identical problems were also presented, mainly based on the results of the Problem@Web project [8], illustrating the diversity of approaches and technological tools that can be useful and effective in problem solving-and-expressing.

For example, in session 1, the problems "United and cropped" and "How many rectangles?" were proposed (Figure 1) to test the organization of the planned work and the management of the different virtual and collaborative spaces. At the end of that session, GeoGebra was presented to the participants, namely, going through some of its tools and affordances, and how to use them for tackling the problems. Finally, participants were invited to re-explore these problems by themselves using this tool.



**Challenge 1: "United and cropped"**
Consider a sequence of squares with sides 1, 2, 3, 4,…cm, arranged so that they are connected to each other, as illustrated in the image. Once together, cut up all the squares by a line drawn from the bottom left corner of the smaller square to the upper right corner of the larger square. What is the area above the cut line, if the sequence has 8 squares?
**Don't forget to explain your reasoning process!**

**Challenge 2: "How many rectangles"**

How many rectangles can you build inside the polygon, by connecting its vertices?

What if the regular polygon has 100 sides? What if it has $n$ sides?

**Don't forget to explain your reasoning process!**

**Figure 1.** Challenges proposed in Session #1 (translation).

In each session, two mathematical challenges were proposed (e.g., Figure 1), which can be considered as non-routine problems in the sense that they are not aligned with the school curriculum but aim to intellectually stimulate students and require the development of a strategy or approach that involves mathematical concepts or procedures [39]. Allowing choice seems to have two implications: on the one hand, it promotes a careful reading of each challenge that would feed an initial discussion on the problems, and, on the other hand, it contributes to higher expectations of being successful and thus persuades participants in trying to solve their preferred problem [8,24,26]. In fact, even though both problems

can be considered moderate mathematical challenges [8,24], their level of difficulty can be perceived differently by the participants as it depends on their knowledge and skills.

### *4.2. The Voices and Experiences of Participants in the Tecn@Mat Club*

This section presents results on the perspectives of the participant students regarding the mathematical problems posed along the club and the use of digital tools to solve the problems and to express the solutions developed. Data stemmed essentially from the questionnaires, where participants were asked for an appreciation of the work carried out in each session, and the final interviews conducted with the youngsters after the conclusion of the activities of the Tecn@Mat Club.

#### 4.2.1. On the Mathematics Problems

The participants appeared to have a moderate relationship with mathematics, particularly with problem-solving, despite the previously mentioned variations in their academic performance. The questionnaires examined the level of enjoyment they experienced while solving the problems presented in each session and their perceived level of difficulty. Figure 2 illustrates that participants reported high levels of enjoyment throughout the four sessions evaluated, with values ranging between 4.50 and 4.82. In the final interview, participants cited mathematical content and the challenge felt as two of the main reasons for enjoying their favourite challenges (e.g., "*I think logic problems are cool*" [CC]; "*I liked it because it was challenging*" [FN]).

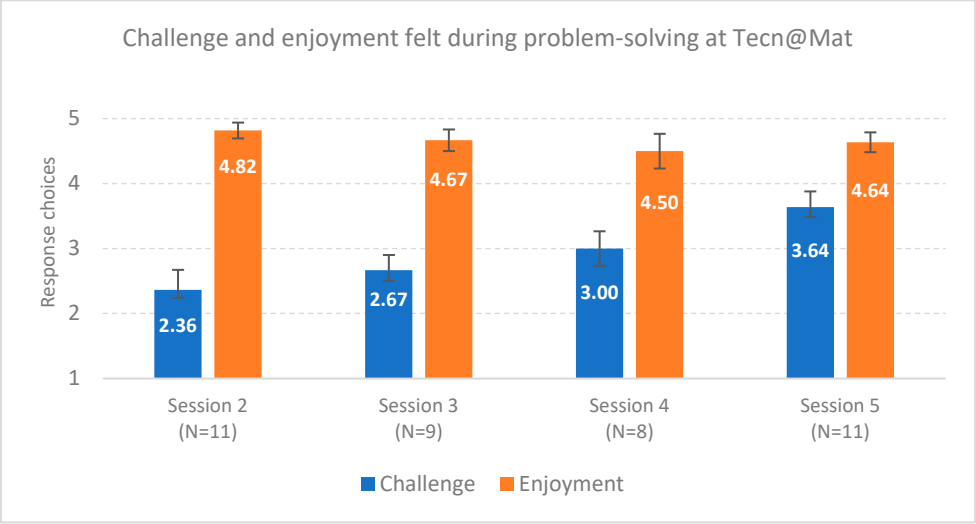



**Figure 2.** Mean values of the level of the challenge* and enjoyment** felt by participants while solving problems in each session of Tecn@Mat (* 1—Very easy; 5—Very difficult. ** 1—Did not enjoy; 5—Enjoyed a lot). N is the number of respondents in each session. Standard error bars are shown.

The level of challenge experienced by the participants increased over the course of the sessions, with a mean value of 2.36 in the second session and 3.64 in the last one (Figure 2). In the second session, many participants solved the problem "The three houses" which required analytical and deductive reasoning [40], and several participants stated during the interview that they enjoyed this type of problem. The mean value that characterizes the challenge felt in solving problems in the last session also reveals the difficulty experienced by the participants, but it is worth noting that their enjoyment did not decrease in comparison to the previous session or significantly change over the various sessions.

During the interviews, most participants believed that the level of difficulty of the proposed problems was appropriate. However, some problems were considered more manageable than others, as can be seen in the following quotes:

> *They weren't very difficult, but they weren't easy either, it's right there in the middle.* [FN, interview]

> *For example, the one with the tables, as I was saying, in the restaurant, it wasn't very easy but it wasn't very difficult either . . . the pizza one, I even thought it was easy.* [MP, interview]

Thus, the evidence suggests that the mathematical challenges selected for the Tecn@Mat Club curriculum are both enjoyable and moderately challenging [8,24,26].

#### 4.2.2. On the Use of Technologies to Solve-and-Express Mathematical Problems

Despite being proficient in the use of various digital tools (such as social media or gaming), having used some common tools in school mathematics (such as calculators), or being familiar with others through their teachers (such as GeoGebra), in general, these students were unaware of how to effectively use them to solve mathematical problems and express their reasoning. Some students mentioned that they had used spreadsheets in an ICT class, while others reported seeing presentations by their math teachers that included images created using GeoGebra.

In the questionnaire, participants were also asked to indicate their level of agreement with several statements related to the use of technology and/or paper-and-pencil in each session of the club. As Figure 3 shows, the use of paper-and-pencil decreased, particularly between the second and subsequent sessions, as the participants gradually realized the benefits of using technology in the development of their approaches, in finding solutions, and in expressing their reasoning and procedures. On the other hand, the use of technology increased as the participants became aware of its potential and effectiveness, and felt more confident in their ability to use it to solve-and-express problems.

Although participants did not completely abandon paper-and-pencil, they recognized the usefulness of technologies both to obtain the solution of the problems and to create explanations for those solutions, especially after the whole group discussions in which several affordances of the tools were discussed and validated for solving-and-expressing mathematics problems, as the following excerpts demonstrate.

> *I think it was only on the first time we used paper and pencil. Yup. I think it was to visualize the problem more easily because I still I wasn't sure how to solve it with technology, but later realized it would be simpler with technology.* [CC, interview]

> *In my case, after solving it . . . and after the explanation of how it worked, I found it more convenient to do it on a computer, but before I preferred solving the problem on paper.* [LC, interview]

The demonstration of new tools' affordances was provided at the end of each session or at the beginning of the next one, which gave the participants ample opportunity to explore and use the software. However, their usage was not always the most suitable. For instance, GeoGebra was introduced at the end of the first working session to show its usefulness in solving the proposed challenges (Figure 1), but in the following sessions, team 2 chose to use GeoGebra to create representations of the problems under examination, even though the nature of the challenges did not align with the software's affordances, i.e., challenge 4 required analytical and deductive reasoning, while challenge 5 required numerical and algebraic relationships (Figure 4).

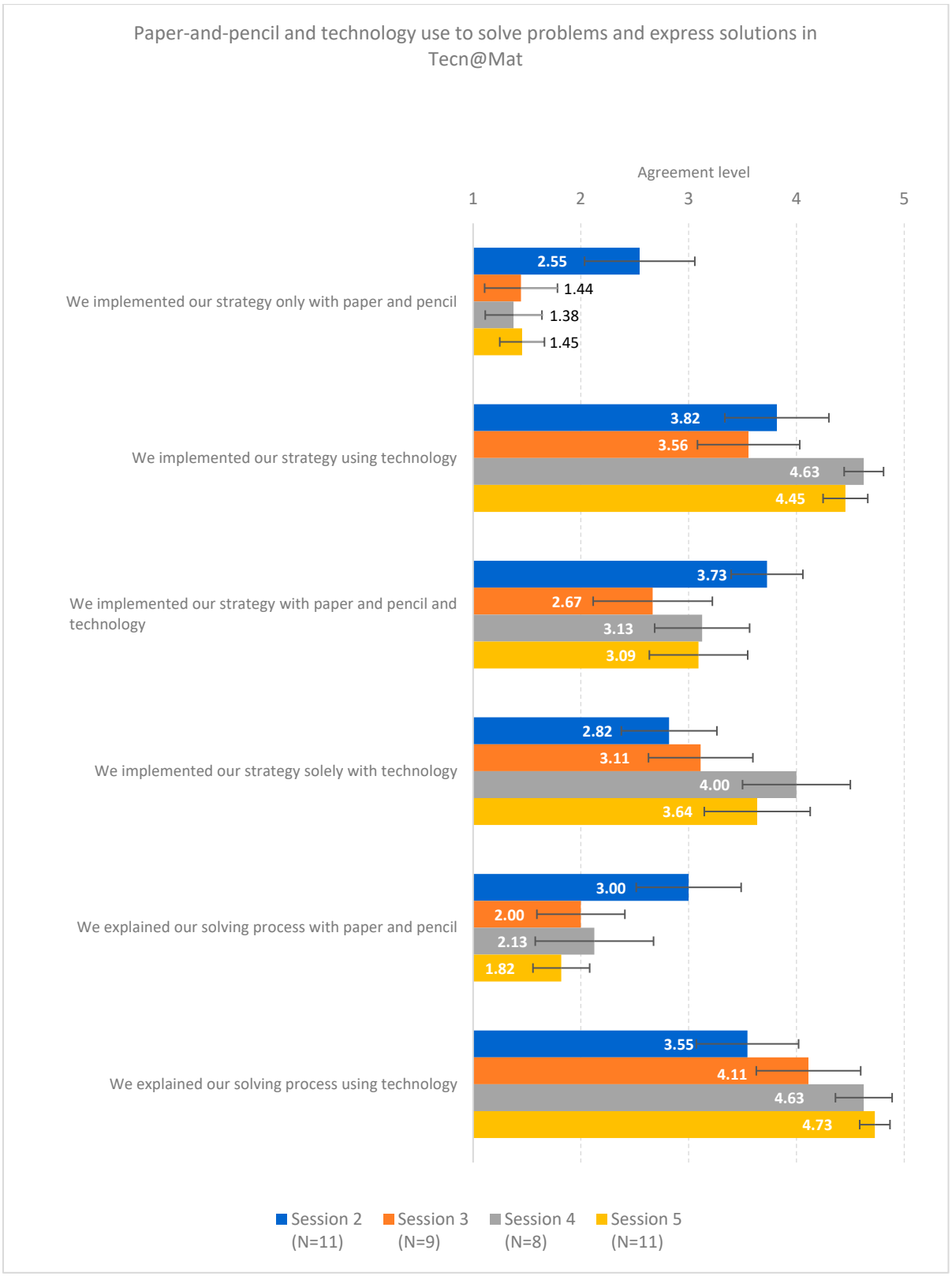



**Figure 3.** Mean value of the level of agreement with each statement about the use of paper and pencil and technology to solve and express the problems, per session (1—Totally disagree; 5—Totally agree). N is the number of respondents per session. Standard error bars are shown.




*First, we tried to find out the colors of the houses, we knew that the lawyer lived in the yellow house, which is next to the red and not the green, so we know that the red house was in the middle. Next, we found out that the bachelor lived in house 21, which was not yellow, so it had to be green, then we discovered that the owner of the workshop was neither married nor widowed, so he could only be single, we also knew that the widower lived next to the green house, By exclusion we found out that the teacher was widowed and lived in the red house* (translation).




*To have two parakeets per cage, we chose the number 4 and divided that number by cages so that there were two parakeets per cage. Then, in order to have an empty cage, we added one cage to the already two occupied cages* (translation).

**Figure 4.** Solutions developed by Team #2 to challenge 4 (on the **left**) and challenge 5 (on the **right**).

A key requirement emphasized throughout the sessions, in each challenge, was the instruction "Don't forget to explain your reasoning process!" (Figure 1), referring to the creation of a descriptive account of the procedures and reasoning used. Along with exploring tools such as GeoGebra and a spreadsheet, the participants also noted that *explaining* was one of the most valuable aspects of their learning, as constructing a clear and complete explanation of the solution process, including a justification for the reasoning, was an activity that was largely absent from their previous mathematics classroom experiences. It's worth noting that not all participants responded to this request in the same way, and some pointed out the need to use paper-and-pencil to express their reasoning.

> *[to explain,] paper and pencil may be easier because we are not so used to express our ideas easily . . . with technology I think it is easier to find the solution to the problem, but yes . . . I didn't use paper and pencil because I realized that I could find my way with technology but sometimes it is useful to make diagrams faster.* [CC, interview]

When asked about the characteristics of a good explanation, participants emphasized the importance of clarity and completeness in their answers. These aspects seem to align with their previous classroom experiences. Even though the use of different representations and digital outputs was encouraged and validated, portraying inspirations from various theoretical perspectives [8,23,25], the participants did not regard them as essential components of a good explanation of their solutions:

> *That includes every step . . . it has to be well explained. As my older sister says: "Explain it as if you were explaining it to a 5-year-old kid". State the [final] answer. I think it has to be complete. Also, showing a sequence, we can't present loose calculations.* [LA, interview]

> *First [put] the topics that the problem deals with . . . then explain our thinking process, then see if everything is ok and I think that's it.* [LC, interview]

When asked about the Tecn@Mat features they appreciated the most, participants identified two main themes. On one hand, they highlighted the social aspect of the club, expressing that they enjoyed interacting and collaborating with students from other regions of the country. One participant mentioned in the interview that the sense of belonging to such a community that enjoys doing mathematics was one of the most positive aspects of the experience.

> *I think it's always better this way because we end up meeting students from schools from north to south of the country and not just from the school where I go to. Even though I'm not much friendly and not very good at working in a group, I think it was good. I liked the feeling that I'm not alone.* [LA, interview]

*I really enjoyed working on the explanations and including images there and so on.* [LC, interview]

*I thought it was cool to be with colleagues from other parts of the country and . . . and to get to know new tools that I didn't know about . . .* [CC, interview]

On the other hand, the participants also expressed appreciation for the club's technological aspect and the opportunity to learn specific tools, as well as the nature of the problems posed during the sessions, as highlighted in the previous examples. These factors can be considered extrinsic and intrinsic, respectively, and are aligned with the findings of [5], supporting the idea that mathematics clubs are also venues that foster mathematical experiences where the affective and cognitive dimensions are closely intertwined.

### *4.3. Strenghts, Weaknesses, Opportunities, and Threats: Inspiring the Future of the Tecn@Mat Club*

The following subsections summarize the main results of a SWOT analysis conducted on the data collected through interviews with the participating students and the mathematics educator, who acted as an external observer of the club and of the participants' activity, both during collective work and in breakout rooms.

#### 4.3.1. On the Strengths and Weaknesses of Tecn@Mat

One of the most highly regarded aspects of the club, acknowledged by both the students and the mathematics educator, is that the activities are conducted online, which enables participation from multiple schools across the country, allowing them to connect with other students who share an interest in mathematics, problem-solving, and the use of digital tools.

*It gives the participants the opportunity to meet and make friends with other fellow mates that share the same interest and enjoyment for mathematical problem solving and technology use.* [Maths Ed, interview]

*I thought it was fun to be with colleagues from other parts of the country and get to know new tools that I didn't know* [CC, interview]

The development of skills was also emphasized as Tecn@Mat provided opportunities for students to enhance their skills and knowledge on the use of specific technological tools to solve the challenges and effectively communicate the solutions. Additionally, the discussion and sharing of the solutions fostered the emergence of new ideas and promoted the development of new approaches that are beneficial for the students' learning and achievement but may be absent in their school experiences.

*I would like to explore GeoGebra more at school because I found it useful. I also understand Excel better now, but I didn't use to understand the mathematical formulas well, I used to use it to draw tables.* [CC, interview]

*I enjoyed using [Google] Drive.* [LC, interview]

*Problems are also nicer than the ones at school.* [CC, interview]

*The possibility of comparing solutions, ideas and strategies is quite important to many students . . . [they] may profit from new ideas, strategies, methods, and approaches that are not prevalent in the regular mathematics classes, unfortunately.* [Maths Ed, interview]

While there are other initiatives that involve similar content or aim to promote a somewhat similar set of skills, such as mathematical problem-solving competitions (e.g., Olympiads) or technology-related clubs (e.g., robotics clubs), Tecn@Mat appears to have unique advantages and benefits. In addition to being "*inclusive and not selective*", it is viewed as a workshop that operates in a "*friendly and enthusiastic*" atmosphere, brings together a more diverse range of skills, as "*it may include programming but is not limited to that*", and only requires a computer with an internet connection and some easily accessible and free software. Furthermore, according to the external observer, one of the main advantages of

Tecn@Mat is that it "*targets techno-mathematical fluency*", which aligns with the specialized research-based design of the curriculum and learning experiences.

However, some areas for improvement were also identified. One of the challenges highlighted by the external expert was creating a "*good blend between coach, independent work, and collaborative work*", as some participants lack fundamental problem-solving skills and are not familiar with the technological tools from a mathematical perspective. The main and unanimous recommendation from the participants is to extend the duration of the sessions to allow more time to work on both the challenges presented.

[My recommendation would be] *A little more time to solve the problems.* [SS, interview]

*I can handle two hours! No problem for me.* [LC, interview]

Regarding the factors that could lead participants to leave the club, the external expert identified the workload, that is, "*the feeling of too much work needed to be able to solve some of the problems*", as well as the lack of perceived usefulness in this experience as students could think "*that it is not useful to their school mathematics and eventually becoming time consuming for them*". These two issues have been considered in the design of the Tecn@Mat Club, namely by selecting and creating problems of moderate mathematical challenge that, while not aligned with the school curriculum, are related to mathematical concepts and procedures learnt in school, and make use of digital tools that are also recommended for classroom learning, such as the spreadsheet and GeoGebra.

#### 4.3.2. On the Opportunities of and Threats to Tecn@Mat

The external expert suggested several new features and exciting trends that the club could try, such as opening "*a small international window*" by inviting international experts to participate in sessions to share ideas, tools, and solutions with the participants; or supplementing the club's activities with a webinar or a "*tecn@mat summer studio*", which would provide students with the "*the opportunity to show what they have learnt and were able to do*" to mathematics teachers, parents, and other students.

As computational thinking is becoming a trend in mathematics curricula in many countries, incorporating the use of Scratch to solve computable problems would expand the range of challenges presented at the Tecn@Mat Club, foster the development of computational and algorithmic thinking, and encourage the use of other digital tools, such as block-based programming tools. Additionally, the use of technology to solve mathematical problems is now recommended, for the first time, in a national syllabus, so these are two of the recent changes to national educational policy that can be beneficial to Tecn@Mat. However, as the mathematics educator pointed out, "*projects, as this one, have to be the driving force for changes in teachers and in schools*".

The club may face several challenges, one of which is a possible invisibility, as the perceived benefits of participating may not be "*clear to many people, regarding the gains that such an experience may mean to the students, both from the intellectual and affective perspectives*", according to the maths educator. Another challenge is the potential "*heterogeneity of participants*" as the club is not specifically targeting highly talented students or high-achievers in school mathematics. Additionally, a "*widespread culture of learning to the test or the examination*" and a "*strong culture of private tutoring where students repeat series of routine exercises from the textbook or from national examinations*" was mentioned, which may make developing a 21st-century mathematical way of thinking, through participation in the club, less valued by teachers or parents. Another significant concern is the sustainability of the Tecn@Mat Club, which may be hindered by the lack of funding to hire developers or human resources, resulting in a scalability issue.

## 5. Discussion and Conclusions

This study documented the design and implementation of a web-based, after-school math club for middle grade students during the COVID-19 pandemic, with the goal of

improving their skills on solving mathematical problems with technology. The following sections address the research questions that guided this exploratory study.

### *5.1. Design Features of Tecn@Mat: Structure, Content, and Organization*

The Tecn@Mat Club consisted of a set of five synchronous sessions, taking place through a videoconferencing tool and supported by free, accessible technological tools—either to afford communication and collaboration among participants (e.g., Zoom, a website, Google Drive, Google Docs, Google Sheets) or to support the development of techno-mathematical solutions of the challenges posed in each session (e.g., GeoGebra, spreadsheet, text, presentation or video editors) (Figure 5).

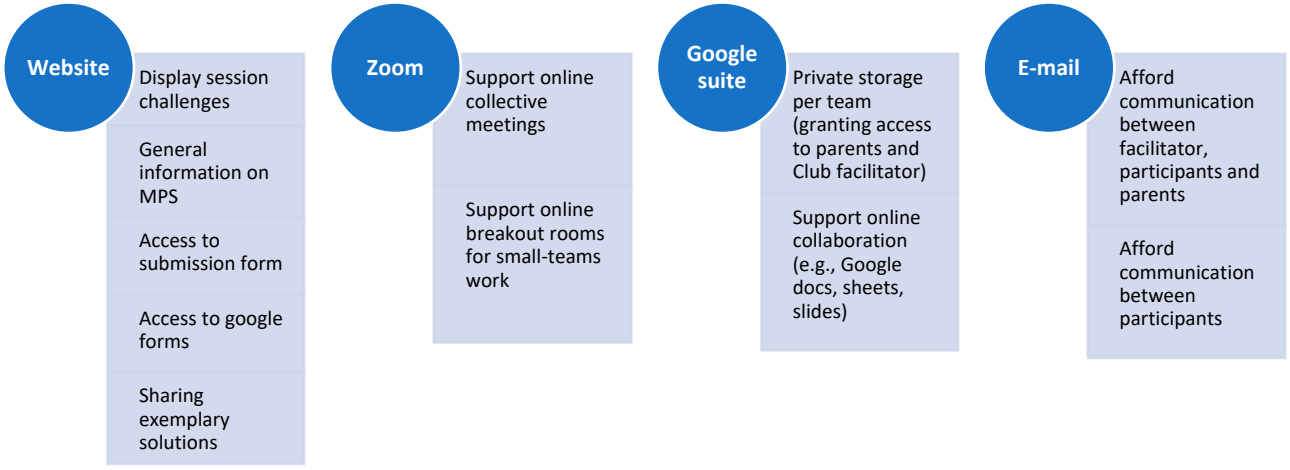



**Figure 5.** The structuring features of the Tecn@Mat Club.

The club's content was designed based on principles derived from research on the use of digital technologies to solve mathematical problems (Figure 6). The primary learning goal at Tecn@Mat was to engage participants in solving mathematical problems with digital tools, with the aim of enhancing their techno-mathematical fluency [15,30]. The sessions were supported by specific tasks, referred to as challenges, that had the following characteristics: they were non-routine mathematical problems not necessarily aligned with the school curriculum, they could be solved using various approaches and mathematical and technological tools, and they presented a moderate level of difficulty [24,26]. Another unique aspect of the activity was the requirement to create a narrative that documented the process of solving a problem [9]. This narrative included several types of representations, as the requirement for a problem to be considered "solved" was that the solution had to be effectively "expressed" meaning it had to be communicated and explained to others. As expected, participants used a variety of representations in their techno-mathematical solutions, thereby developing and utilizing an expository digital mathematical discourse [9].

A typical session was organized into three parts: (1) a whole-group introduction and organization of the session, (2) small-group work on the session's problem(s), and (3) a collective discussion of the solutions and coaching on new tools as needed. Each session offered two challenges for participants to choose from, and they were free to select the technological tools they preferred for solving the problems and creating their solutions. This was intended to boost participants' confidence in finding solutions to challenging situations. Thus, by allowing participants to choose the problems, tools, approaches, and their role in collaborative work, the Tecn@Mat Club can be seen as a choice-affluent environment [14] that fosters the development of 21st-century skills.

Overall, the Tecn@Mat Club that was designed and implemented embodies the features of a community of mathematical practice [41]. The young problem solvers who participated in the club were exposed to a set of prevailing values that defined the process of solving mathematical problems by means of digital technologies.

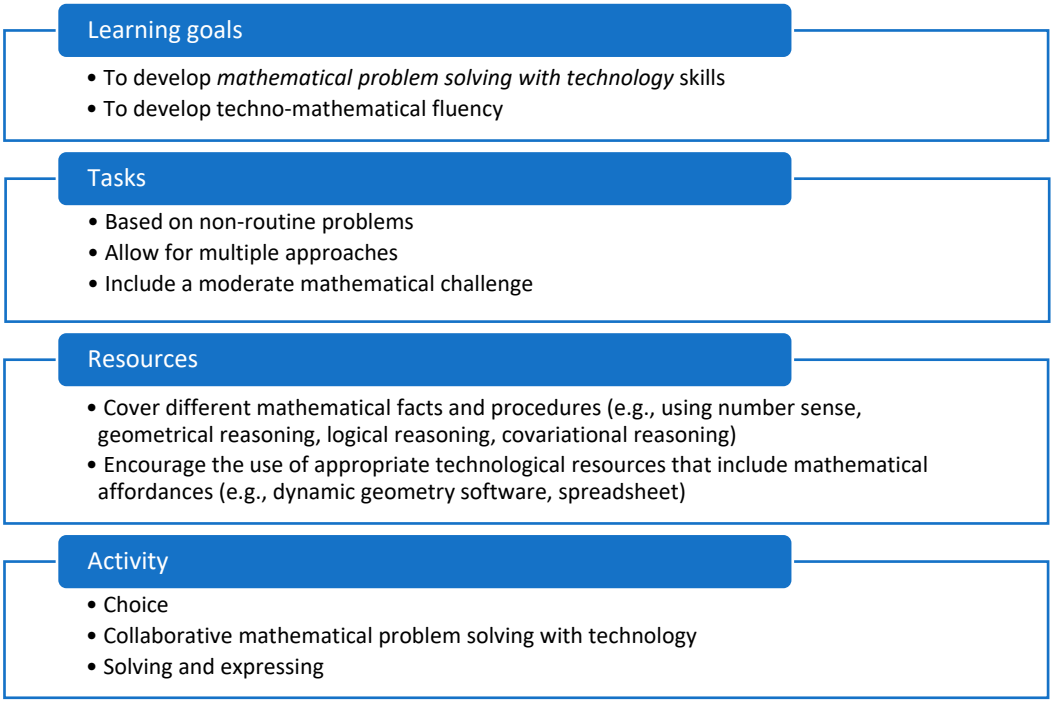



**Figure 6.** The content features of the Tecn@Mat Club.

### *5.2. Participants' Perspectives on Problem Solving with Digital Tools at Tecn@Mat*

The problems presented in the club were perceived as more challenging than those at school, as they required the development of an approach and the selection of appropriate resources (mathematical and technological) for solving and expressing the solution. On the contrary, at school, students are required to solve problems more closely aligned with the content and to use procedures they are studying at the time.

While the level of challenge felt increased over the course of the sessions, the participants still found the problem-solving experiences enjoyable. Since they considered the level of difficulty to be appropriate, the non-routine problems posed at Tecn@Mat presented a moderate level of mathematical challenge [8,26] for these youngsters.

Regarding the use of technologies to solve-and-express the problems, the participants gradually sought to understand the mathematical affordances in each tool and make good use of them either to obtain solutions or to explain their procedures and reasoning. During the sessions, they were able to explore different affordances of tools to support mathematical thinking or to communicate effectively, which was valued by several participants. In general, participants think their experience in the club allowed them to increase their repertoire of techno-mathematical tools and expand their knowledge about the mathematical affordances and constrains of several digital technologies.

The participants' techno-mathematical fluency, despite incipient, is consistent with a genesis of youngsters-with-media [15,18], as they were able to use mathematical resources (concepts and procedures) to solve the problems using digital tools but were still exploring the full range of affordances in the media. Even though they knew superficially some of the tools explored, they had never used them to engage in mathematical activity nor to solve problems. Although in a frail way, collectives of youngsters-with-media emerged throughout the club which was evident in the way they gradually abandoned the use of paper-and-pencil, began to recognize mathematical affordances in the tools, and to explore them more independently.

Despite its short implementation period, the Tecn@Mat program helped the participants develop their techno-mathematical fluency. This fluency includes knowledge of mathematical facts and procedures, proficiency in using technology to apply mathemat-

ical knowledge, and the ability to effectively combine these skills to solve and create techno-mathematical solutions to the problems [15,16,30].

### *5.3. Tecn@Mat's Strengths, Weaknesses, Opportunities, and Threats*

The SWOT analysis conducted (Figure 7) allowed to identify the strengths of the Tecn@Mat Club, the features that need to be improved, the ones that may have a positive impact in achieving its goals, and the obstacles it faces, which, taken together, may be useful in improving the design and implementation of future editions of the club.

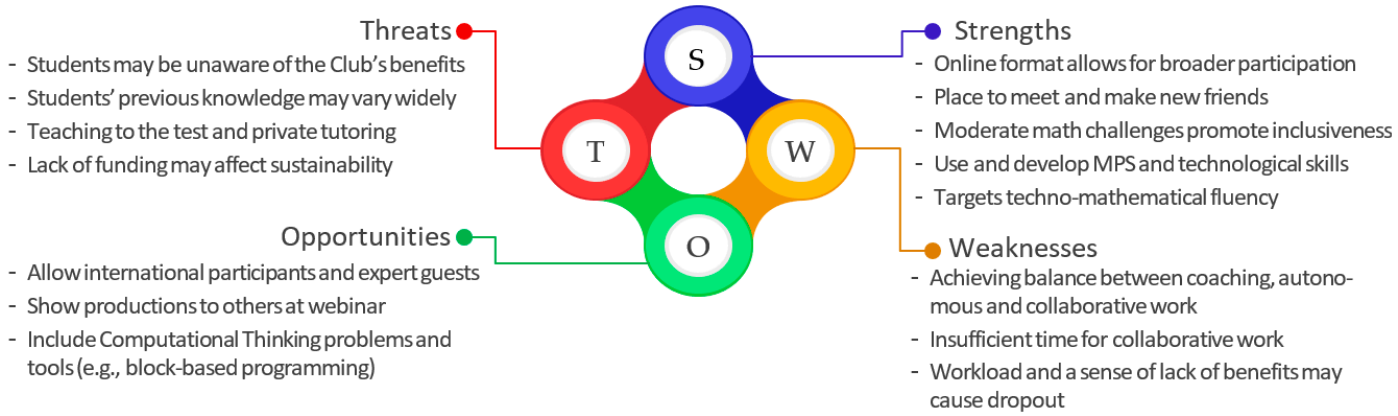



**Figure 7.** Synthesis of the SWOT analysis conducted.

The Tecn@Mat Club has a variety of assets, including a combination of key features identified in the literature as crucial for successful after-school initiatives, that should be preserved in future replications. Its online format allows for nationwide, voluntary participation for young students [7] who are interested in solving mathematical challenges and learning about digital technologies, as well as the opportunity for them to connect and make new friends [11]. The tasks in Tecn@Mat Club present a moderate level of mathematical challenge [26], thus are suitable for students of varying mathematical abilities, also contributing to the development of students' techno-mathematical fluency [30].

There are some constraints to address in future editions of the club in order to prevent participant dropout, such as achieving a balance between guided instruction, independent work, and collaboration. The sessions should be extended to provide participants with more time to work together on the problems. The workload should be appropriate for participants' backgrounds and align with the learning opportunities offered, and the value and benefits of the club should be made clear to participants.

Possible new additions to the Tecn@Mat Club that could be considered for future editions include opening the club to international participants or inviting guest experts in specific areas. To culminate the experience, the club could host a final event such as a webinar or an in-person summer camp where participants present their solutions to an audience, either colleagues, teachers, or family members. The club could also incorporate more resources such as computational problems and block-based programming activities to foster computational thinking skills among participants.

Among the challenges that could potentially threaten the continuity of the Tecn@Mat Club is the diversity in participants' previous knowledge (either regarding problem solving, mathematics, or technology), the lack of knowledge about the benefits of participating in the club, particularly the contribution of these activities to school learning, or the school-based experiences where "teaching to the test" dominates as a classroom norm, boosted by private tutoring. Additionally, future editions of Tecn@Mat may be at risk due to a lack of funding—needed either to hire teachers or trainee teachers to assist in managing or accompanying the groups of students; to hire developers or improve the website—which may affect the club's sustainability and scalability.

### *5.4. Final Considerations*

This exploratory case study portrays the feasibility of adapting in-person, extracurricular mathematical initiatives for an online setting, in tune with recent research [42].

Several innovative features of the Tecn@Mat Club were identified. Firstly, the web-based nature of Tecn@Mat, combined with the use of free, open access digital tools, enables students nationwide to access an inclusive learning space where they can meet and engage in enjoyable collaborative mathematical activities. Moreover, the club is structured around several key facets that have been recognized as important for the success of home-schooling during the pandemic, including creating familiarity with resources and tools, promoting individual or collaborative autonomous work on problem-solving tasks, and analysing the affordances and constraints of technological tools to develop solutions to problems [43].

Furthermore, the planning and delivering of the Tecn@Mat Club were based on several theoretical perspectives and empirical results regarding the specificities of using digital technologies in mathematical problem-solving. Thus, its literature-informed design allowed the development of a curriculum concerning the development of skills in the use of several digital tools to solve problems and to communicate the solutions, with a particular focus on the development of students' techno-mathematical fluency. The findings of this exploratory case study also show that it is possible to engage middle-grade students in mathematical problem solving in an after-school online setting, by providing moderate mathematical challenges, blending in, and acknowledging independent and collaborative work. This paper also contributes with the identification of several features that may be accounted for in future replications of the Tecn@Mat Club, among which are the possibility of hosting an in-person closure event; engaging participants with different mathematical and technological abilities in a meaningful techno-mathematical activity; or broadening the scope of the curriculum to include computational problems and the development of computational thinking skills.

Future research is needed to validate the set of characteristics pertaining the design and implementation of the club, as identified in this exploratory case study. Prolonging the activities, involving a higher number of participants, or including elementary or secondary school students, would yield a rich database to examine the students' engagement in the activity, to characterize their processes of collaborative problem solving with technologies, as well as to describe the development of their techno-mathematical fluency, for instance, with a particular digital tool.

The COVID-19 pandemic has forced many educational institutions to adopt emergency remote teaching methods, and it remains uncertain whether the previous in-person approach to teaching and learning will fully return [44]. This study contributes to our understanding of how to keep students connected and engaged in a technology-driven mathematical environment during a global crisis, where over 1.5 billion students have been impacted by disruptions to their education [45]. The study highlights the necessity of reconsidering the role of technology in post-pandemic mathematics education, whether in face-to-face or online classrooms. While transmission models of teaching may be more easily adapted to remote learning, student-centred learning models that require them to engage in problem-solving and justification of their reasoning may be more challenging to implement in online environments [46].

Thus, the results of this study raise important questions about how mathematical problem-solving and technology skills can be effectively developed in instructional settings. This may be an opportunity to re-evaluate the way mathematical problem solving with technology is taught in the mathematics classroom, aiming to equip students with the skills they need to succeed academically and in their future careers in a post-pandemic world.

**Funding:** This work was partially supported by Fundação para a Ciência e a Tecnologia, under the Stimulus of Scientific Employment Program (CEECIND/01584/2017) and within the scope of Unidade de Investigação e Desenvolvimento em Educação e Formação [Research and Development Center on Education and Training, UIDB/04107/2020].

**Institutional Review Board Statement:** The study was approved by the Ethical Commission of the Institute of Education of the University of Lisbon, on the 16 January 2020, with the approval code nr. 186.

**Informed Consent Statement:** Informed consent was obtained from all participants involved in the study.

**Data Availability Statement:** Anonymized data are available upon request to the author.

**Acknowledgments:** The author expresses gratitude to: (i) the participants and the mathematics education expert, for their collaboration, (ii) the math teachers, J.T., L.C. and P.D., for encouraging their students to participate in the club; and to (iii) J.R. and J.B., for providing assistance with transcribing the interviews and conducting literature search.

**Conflicts of Interest:** The author declares no conflict of interest.

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