education sciences

Article

Contextual Mathematical Modelling: Problem-Solving Characterization and Feasibility

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Abstract: The current study investigates contextual mathematical modelling (MM) problems that were retrieved from authentic workplace situations and simplified for formal secondary school math lessons. First, the study aims to characterize contextual MM problems according to Schoenfeld's framework of problem-solving (PS). Second, it aims to investigate the perceptions of two stakeholder groups: (1) math experts and policymakers and (2) math teachers with respect to the characteristics of the contextual MM problems and their feasibility regarding implementation in secondary school education. Based on the Delphi methodology, we employed two phases for our analysis: an open-ended questionnaire to interview ten stakeholders and, subsequently, a Likert-type questionnaire to collect data from 122 stakeholders. The main results indicate that the contextual MM problems are characterized by PS. A similar view was expressed by different stakeholder groups, and no differences were caused by various background variables, such as educational level or STEM background. Additionally, the findings revealed that both stakeholder groups perceived that it is highly feasible for these problems to be integrated into secondary school education. This study contributes theoretically to the interrelationship between MM and PS frameworks, and provides practical recommendations for the implementation of contextual MM problems in secondary schools by applying PS skills.

Keywords: authentic workplace mathematics; mathematical modelling; problem solving

Citation: Kohen, Z.; Nitzan-Tamar, O. Contextual Mathematical Modelling: Problem-Solving Characterization and Feasibility. *Educ. Sci.* **2022**, *12*, 454. https://doi.org/10.3390/educsci12070454

Academic Editors: Ana Barbosa, Isabel Vale and Susana Carreira

Received: 1 June 2022
Accepted: 26 June 2022
Published: 29 June 2022

1. Introduction

Mathematics often reveals hidden patterns that help us to better understand the world around us. Mathematics involves more than just arithmetic measurements, as it cycles between data, deduction, and the applications in which it is applied, from everyday tasks to major problems in industry [1,2]. Although mathematics is based on rules, it is crucial that students move beyond the rules, and learn how to tackle higher-order mathematics, which involves logical, deductive, creative, and higher thinking, rather than just calculation or deduction. This entails developing problem-solving (PS) skills, requiring one to seek appropriate solutions and exploring patterns, as well as formulating and validating conjectures, rather than just memorizing patterns, problems, and procedures, or completing exercises [3].

Researchers suggest teaching PS skills in a context in which these skills can be applied, such as workplace mathematics, which is the mathematics underlying authentic problems taken from actual workplaces [4–6]. The utility of studying contextual problems in schools is well-established and is mainly referred to in the literature as mathematical modelling (MM). MM is a cyclic process involving the transition from an authentic situation to a mathematical problem by understanding and simplifying (idealizing) the authentic situation to obtain a real-world model, mathematizing the real-world model by translating it into a mathematical model, applying mathematical routines and processes, and interpreting and validating the mathematical solution to verify that it is in accordance with reality [7–10].

However, many researchers, e.g., [11–13], emphasize the difficulty of incorporating contextual MM problems in school mathematics, since authentic (particularly workplace) situations cannot simply be reproduced, as they are for school education. The application of PS in a mathematics classroom, particularly through contextual MM problems, remains rare. Changes in both the curricular content and instructional practices are usually required to make PS instruction feasible and effective in school settings [14–16].

Teachers play a significant role in preparing students to have strong mathematical skills. However, students should also know how to apply these skills in situations other than those in which they were learned [17]. However, engaging students in contextual MM problems is often incorporated into teaching in a didactical manner that sometimes reduces the heuristics of the problem to procedural algorithms [18,19].

Moreover, teachers' willingness to use contextual MM problems in their classes often does not coincide with international trends. Thus, it is crucial to learn how to convince teachers that it is worthwhile to conduct applicable mathematics classes, and demonstrate the didactic connection between these problems and the curriculum [20].

Contextual MM problems for secondary school students could be implemented differently by various stakeholders, in terms of the correspondence between the problems' characteristics and the standard math curriculum, as well as teachers' willingness to use such problems in class. For example, novice teachers tend to struggle most with the challenge of sticking to the curriculum [21,22]. Thus, the contextual MM problems might be seen as not corresponding with the secondary school curriculum. Alternately, veteran teachers are more traditional and might be more resistant to changes and new materials [23,24].

It is, therefore, critical that the mathematics curriculum is adjusted by embedding realistic applications of mathematics rather than teaching mathematics as a finished product. However, changes in the curriculum are not trivial; the basic and embedded mathematics content is generally validated due to the years spent establishing it in the educational system. A change in curriculum should not be a burden to either the students or the teachers, who generally will neither accept nor support radical changes in the curriculum with which they are familiar. Thus, the contents of contextual MM problems should be incorporated into the mathematics curriculum by careful selection, while focusing on the essential concepts, rather than meticulous details [1].

Further research is required to explore how contextual MM problems can be applied in the context of workplace mathematics, in terms of PS skills. The current study suggests characterizing contextual MM problems for a secondary school from the perspective of various stakeholders, based on Schoenfeld's theoretical framework [3]. Moreover, due to the challenges faced by teachers when applying MM problems in class, various stakeholders' attitudes should be explored in relation to the feasibility of incorporating contextual workplace MM problems into school mathematics.

Mathematical Problem-Solving

The investigation of mathematical PS was pioneered by Pólya [25], who defined it as a heuristic with a four-step process: understanding the problem, devising a plan for solving the problem, carrying out the plan, and looking back to examine the solution. Schoenfeld [3,26] refined this definition at a practical and empirical level that goes beyond a coherent and general heuristic to a more flexible view. Schoenfeld's framework provides the knowledge and skills that are necessary to adequately characterize a mathematical problem-solving performance. The performance of mathematical PS is based on the learner's dialogue between his prior knowledge, called resources, and his attempts and thoughts throughout the problem-solving process. However, in order to be resourceful, the learner needs to be familiar with a variety of heuristics, which are general problem-solving techniques. Three other dimensions of this framework are control, methods, and affect. Control refers to decisions made at the metacognitive level as well as global decisions that affect the solution path. Method refers to strategies used when working on a problem, and Affect refers to attitudes, beliefs, emotions, and values.

For this study, which aims to characterize MM problems regarding their fit to PS criteria from the objective perspective of various stakeholders, we focused on two criteria related to the knowledge required for PS, namely, "Resources" and "Heuristics", rather than the problem-solving processes that take place in class, namely, "Control", "Methods", and "Affect".

According to Schoenfeld [3], resources refer to the mathematical knowledge needed to solve a math problem. This includes the following: intuitions and informal knowledge regarding the domain, facts, algorithmic procedures, routine non-algorithmic procedures, and understandings (propositional knowledge). Heuristics are the strategies and techniques needed for effective PS, including the following: drawing figures and introducing suitable notations, exploiting related problems, reformulating problems, working backwards, and testing and verification procedures.

In an historical review, Stanic and Kilpatrick [19] identified three main themes regarding the use of PS: "problem solving as context", "problem solving as skill", and "problem solving as art". PS as a context can be defined as the use of mathematical problems to reach other curricular goals, such as motivating students to study math, and justifying its value, and why it is studied. PS as a skill is when it, by itself, is considered worthy of instruction. PS as an art promotes real PS as the essence of mathematics.

In this study, we applied PS by solving MM problems in the context of workplace mathematics. These are authentic problems taken from the work of tech engineers, which represent an educational interface between workplace mathematics and school mathematics [4]. As described in the methodology section, these problems express the cyclical nature of MM.

The focus on MM problems stems from the premise that these problems are considered non-routine. Students present a mathematical solution for a problem formulated in mathematical terms; however, it is embedded within a meaningful, real-world context [27]. Reeff [28] stated: "The problem solver has a more or less well-defined goal, but does not immediately know how to reach it. The incongruence of goals and admissible operators constitutes a problem. The understanding of the problem situation and its step-by-step transformation, based on planning and reasoning, constitute the process of problem solving" (p. 48). Lesh, Post, and Behr [29] mentioned that good problem-solvers are flexible regarding the use of different representational systems during the solution process. Researchers explain that a translation between mathematical representations provides students with more opportunities to explore concrete objects through imagery, modelling real-life situations, giving meaning to visualizations of abstract and conceptual ideas, and creating relationships between these mathematical ideas [29–32].

In this study, we aimed to address the following research questions:

RQ1) RQ1.1: To what extent do contextual MM problems, targeted for secondary school mathematics, fit Schoenfeld's framework of PS? RQ1.2: What are the differences (if any) between the perceptions of various stakeholders regarding the characteristics of contextual MM problems?

RQ2) RQ2.1: What is the feasibility of integrating contextual MM problems into secondary school classes? RQ2.2: What are the differences (if any) between the perceptions of various stakeholders regarding the feasibility of integrating contextual MM problems in class?

2. Materials and Methods

2.1. The Context of the Study

This study investigates authentic mathematics problems developed as part of the i-MAT project held at the Faculty of Education in Science and Technology at the Technion. Taken from authentic scenarios from the Hi-Tech industry, the i-MAT team simplifies problems that are solved by Hi-Tech employees (inventions as well as regular work problems), so that the problems can be used by teachers in formal secondary school math lessons.

An example of this is an authentic problem exemplifying the usefulness of mathematics in the context of uploading and streaming a YouTube video by connecting the problem to motion problems. The task begins with a description of the authentic situation: YouTube makes use of technology that allows for us to stream content (e.g., videos or clips) on the Internet while the content is being loaded (i.e., without loading the entire content in advance). Then, a problem situated in reality is presented, e.g., how to proceed when a video gets stuck while being streamed. This occurs when the loading bar 'catches up' with the progress bar before the end of the video, i.e., the former is quicker than the latter (see Figure 1).

Figure 1. An example of a contextual MM problem: the YouTube task.

In accordance with the modelling cycle, the task developers idealize the problem; this includes making assumptions that simplify the authentic situation in order to match the problem to formal mathematics lessons. Regarding the YouTube task, several assumptions were made in order to make it accessible to the students, including the assumption that both the loading speed and the viewing speed were constant and uninterrupted by noise; the assumption that streaming is standard, i.e., streaming occurs at a standard speed where the video is like reality; and the assumption that the loading speed is shorter than or equal to the streaming speed; today, technology allows for a greater loading speed than a streaming speed.

When moving to the mathematical world, i.e., in the mathematization process, the connection to the motion problem is created by explaining the term 'streaming' as technology that allows for us to stream content while downloading it, with no need to download it all in advance. The mathematization is carried out through the following modelling question: How can continuous and direct streaming of a YouTube video be guaranteed? This question can be answered with the mathematical model used for solving motion problems, by relating to: (1) the video's volume, i.e., "the place where the information is taken up in the computer memory, as measured by megabytes", in parallel with the concept of 'distance'; (2) the time it takes to upload, i.e., "the time in seconds needed to move the information", in parallel with the concept of 'time'; (3) the speed, i.e., "the amount of information moved per second", in parallel with the concept of 'velocity'. Since responding to the modelling question necessitates dealing with both the amount of information being downloaded and the amount of information being streamed, this task requires two types of velocities to be calculated: the streaming speed and the downloading speed. Solving the modelling question is carried out by relating to the 'wait time', which is the time when early downloading takes place before streaming can begin. The mathematical model needed to ensure continuous, uninterrupted streaming is calculated as the shortest wait time from the beginning of the downloading process to the moment that streaming begins.

Then, the process of applying mathematical routines comes into play, and the students are asked to construct and solve level 1 equations; this allows for them to calculate the time, based on velocity and the video volume. With the mathematical answers they obtain, the students move on to the interpretation process, where they must determine whether their solution is suitable for the authentic situation. They must make sense of the mathematical solution in terms of the actual situation, and must decide whether it addresses the real problem satisfactorily. For example, will the video get stuck?

This study focuses on four contextual MM problems from different fields of mathematics (algebra, geometry, and trigonometry) that fit the secondary school math curriculum. The problems are presented as teaching units for secondary school mathematics classes, in a 15-min presentation format that includes a short and simple explanation about the key concepts of the workplace context, and mathematics exercises that lead the students throughout the full modelling cycle. One of the teaching units in Algebra was based on the YouTube problem that is illustrated above.

2.2. Participants and Procedures

The study is based on the Delphi methodology [33], which is based on responses that were iteratively retrieved by anonymous group interactions. This methodology was chosen for the current study, since it is well-suited to studies with incomplete knowledge about a problem or phenomenon [34], such as characterizing contextual MM problems according to the problem-solving framework proposed by Schoenfeld [3].

The iterative process for characterizing the contextual MM problems using the Delphi methodology was as follows. In the first phase, we used an open-ended questionnaire to interview ten stakeholders in the field of mathematics: experts and policymakers (n = 5), and teachers (n = 5). In the second phase, we converted the collected information into a Likert-type questionnaire, which served as a survey instrument for collecting data from 122 additional stakeholders: 35 experts and policymakers (28.7%), and 87 mathematics teachers (71.3%). The average age of the experts and policymaker group was 53 (SD = 6.5), with a minimum age of 36, and a maximum age of 68, whereas the average age of the teacher group was 41 (SD = 9.4), with a minimum age of 24, and a maximum age of 65. Table 1 presents descriptive statistics for this sample.

Table 1. Descriptive statistics regarding the study participants (the second phase) (n = 122).

Background Variables		Frequency	%
Gender	Male	24	19.7%
	Female	98	80.3%
STEM background	Engineering	28	24.1%
	STEM	41	35.3%
	None	47	40.5%
Level of education	B.A.	48	40.3%
	M.A. or Ph.D.	71	59.7%

Note. Several frequencies do not add up to 100% due to missing data. Data collection for the first phase was conducted at the Technion as a personal meeting with each of the 10 participants, whereas data collection for the second phase was conducted in different school settings as part of weekly mathematics team meetings, or in professional meetings such as workshops or conferences for experts in mathematics education. In both phases, before interviewing or administering the questionnaire, the authors presented the participants with a few examples of contextual MM problems, taken from the i-MAT project. The presentation of problems lasted for about 40–50 min, with about 20–40 min of a one-on-one interview (for the first phase) or individual responses to the questionnaire, totaling 1–1.5 h of data collection.

Figure 2 illustrates the two phases of the study.

2.3. Research Tools

The *open-ended questionnaire* consisted of two parts, aimed at responding to the two research questions: characterizing the contextual MM problems according to a PS view, and the feasibility of integrating these problems into secondary school classes. The first part includes questions based on the two criteria of Schoenfeld's framework: Resources, e.g., relating to 'intuitions and informal knowledge regarding the domain'; we asked: "What information is intuitive and what requires an explanation and clarification?" Regarding heuristics, relating to 'Analogies and exploiting related problems'; we asked: "Is the analogy given to the problem sufficient?" The second part of the questionnaire included questions based on the Strength, Weakness, Opportunity, and Threat (SWOT) methodology, since it serves as an objective tool to obtain a critical perspective of a pedagogical approach by specifying internal and external factors that might affect its success [35,36]. For example: To what extent can this problem be of interest to students? [S]; What difficulties may teachers face when implementing the contextual MM problems? [W]; How can teachers be encouraged to use such problems during class? [O]; What external factors can impair the implementation of these problems? (T).

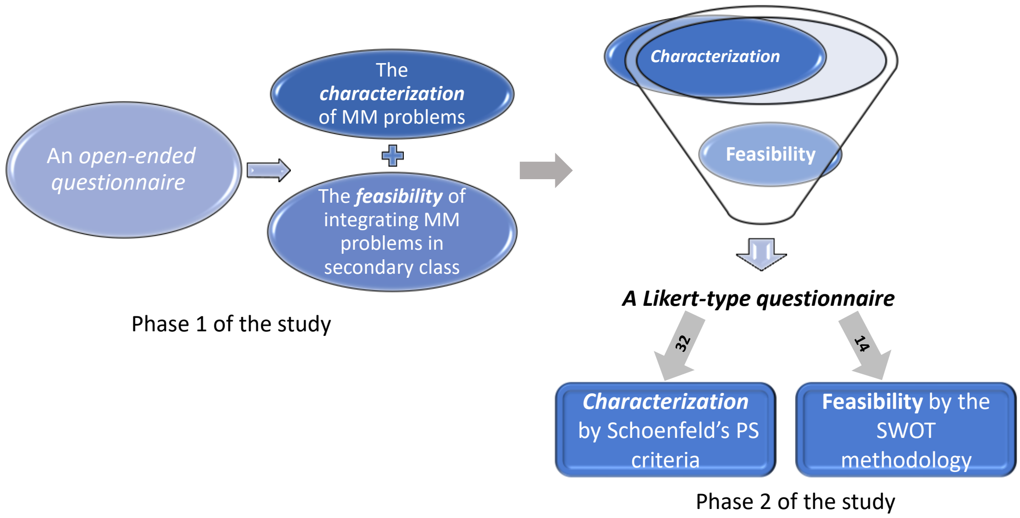

Figure 2. The two-phase study procedure.

The *Likert-type questionnaire* comprised 32 questions to assess the characterization of the problems, and 14 questions to assess their feasibility, on a scale from 1 (to a low extent) to 6 (to a very high extent). The questionnaire was constructed according to the responses retrieved from the open-ended questionnaire; it was generally phrased to adjust all four contextual MM problems, as detailed in the data analysis section. For example, with respect to the question "Is the analogy given in the problem sufficient?", we received responses such as 'In the YouTube problem, the analogy to a motion problem is excellent, but I would have preferred a more explicit analogy to a traffic problem (speed, time, and distance calculation), which is more intuitive'. Therefore, a quantitatively related question was phrased: "To what extent do you find the analogy that was presented in the problem intuitive to students?" (the resource criteria). Another example relates to the following response: "The problem contributes to the understanding of Geometry in general and to the context of Geometry in everyday life... It presents the students with the idea that sometimes an unexpected solution can be found by thinking out of the box . . . ". Following this response, a quantitative question related to the strengths of the contextual MM problems was phrased as follows: "To what extent are such problems linked to reality and relevant to students' experience in everyday life?"

2.4. Data Analysis

To analyze the *open-ended questionnaire* responses, we used a thematic analysis [37] to encode each statement to various categories that generally characterize the contextual MM problems, i.e., by disconnecting the response from the specific problem's context. A recursive process for this analysis was repeated by two experts in mathematics education until 90% agreement was reached for all statements, along with an inter-rater reliability of $\kappa = 0.83$. After the categories were determined and validated, we calculated the frequencies of the statements associated with each of the categories. We then compared the frequency of statements within each category to examine the differences (if any) between the various stakeholders, regarding both the characterization of the contextual MM problems, and the feasibility of integrating these problems into secondary school classes.

To analyze the *Likert-type questionnaire* data, we used various ANOVA tests to examine the differences between the various criteria of the questionnaire, as well as to compare the various stakeholders' perceptions.

3. Results

Responding to RQ1, we first characterized the contextual MM problems according to Schoenfeld's framework, as revealed from the two-phase procedure of the study: (1) the classification of categories retrieved from the open-ended questionnaire and (2) the intensity of the different criteria according to the data retrieved from the Likert-type questionnaires. We then compared the views of experts and policy makers, vs. teachers, regarding various criteria, and then presented the findings relating to the difference between all stakeholders according to various background variables. Responding to RQ2, we followed the same procedure for displaying the findings, except that we focused on the feasibility of integrating contextual MM problems into secondary school classes.

3.1. The Characteristics of the Contextual MM Problems

Table 2 shows the categories that arose when characterizing the contextual MM problems. To focus our discussion of the findings, we presented the most frequent category for each criterion as a representative example.

Table 2. Characterization of contextual MM problems, based on Schoenfeld's framework.

Criteria (by Schoenfeld [3])	Sub-Criteria (by Schoenfeld [3])	Examples of Categories Retrieved in This Study	
		Experts and Policy-Makers	Teachers
Resources	Intuitions and informal knowledge regarding the domain	Suitable for high-level students	Suitable for high-level students
	Facts, rules, and algorithmic procedures	Being precise in sketches	Being precise in sketches
	"Routine" non-algorithmic procedures	Suitable as a summary, enrichment, or research question	Suitable as a summary or as enrichment
Heuristics	Appropriate representations	The use of dynamic illustrations	The use of dynamic illustrations
	Analogies and exploiting-related problems	Using a relevant analogy and a story that will motivate students	Using a relevant analogy and a story that will motivate students
	Testing and verification procedures	Using a real scale in building the mathematical problem	Attach a student help page

According to Table 2, all stakeholders consider the contextual MM problems to be compatible with the sub-criteria that arose from Schoenfeld's theoretical framework. Additionally, it was found that the perceptions of the two stakeholder groups were consistent for most sub-criteria. Differences were found with regard to the 'testing and verification procedures' sub-criterion, in which teachers' perceptions were targeted to the aspect of learning in the classroom, as opposed to the experts' perceptions, which were targeted to the characteristics of the mathematical problem itself. Another difference was revealed in the 'routine non-algorithmic procedures' sub-criterion, when experts and policy-makers considered the contextual MM problems as encouraging inquiry, whereas the teachers did not raise this point.

Furthermore, we applied descriptive analysis, and presented the average score for each PS criterion by Schoenfeld [3], by using the data obtained from the Likert-type questionnaire (see Figure 3).

Generally, all stakeholder groups perceived the contextual MM problems as being more characterized by their heuristics than their resources, $t(118) = -4.6,\ p < 0.0001$. Further ANOVA with repeated-measures analyses were performed to investigate the differences between the various sub-criteria, and were conducted separately for the resources and heuristic criteria. The results revealed significant differences between the different sub-criteria, for both the resource criteria, $F(2,\ 118) = 103.19,\ p\ <\ 0.001, \eta^2 = 0.64$, and the heuristic criteria, $F(2,\ 118) = 16.40,\ p\ <\ 0.001, \eta^2 = 0.22$. Bonferroni analysis revealed that the most positively perceived sub-criteria of resources were facts, rules, and algorithmic procedures, followed by routine non-algorithmic procedures, and then intuition and informal knowledge regarding the domain. Furthermore, the most positively perceived sub-criteria of heuristics were the appropriate representations and analogies and exploiting related problems, followed by testing and verification procedures.

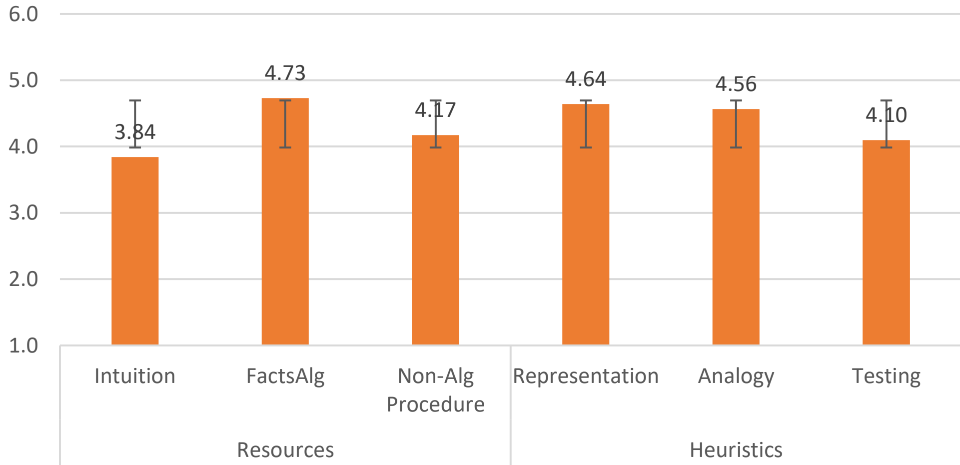

Figure 3. Distribution of PS criteria, according to Schoenfeld's framework [3].

One-Way MANOVA analyses, used to compare the two stakeholder groups' attitudes, revealed a similarity between the experts and policy-makers vs. the teachers, for both the resource criteria, $F(3,\ 116) = 0.43,\ p\ >\ 0.05, \eta^2 = 0.01$, and the heuristic criteria, $F(3,\ 116) = 0.84,\ p\ >\ 0.05, \eta^2 = 0.02$.

Furthermore, One-Way MANOVA analyses were performed to determine the differences in attitudes toward the contextual MM problems, for both the resource and heuristic criteria, by various background variables that represent the various stakeholder groups, in particular: (a) the level of education: B.A. vs. M.A. or Ph.D.; (b) STEM background: engineering (such as program engineering), STEM (such as a physicist), or none; and (c) gender: male or female. In addition, Pearson correlations were conducted between the participants' age and each of the PS criteria. The findings revealed no significant differences in all the explored background variables, $0.64 < F(6,\ 158) > 1.90$, as well as no correlation between age and each of the PS criteria, $0.03 < r(120) < 0.17$. Appendix A presents descriptive statistics for the background variables (besides age), in relation to the PS criteria.

3.2. The Feasibility of Incorporating the Contextual MM Problems in Classes

Table 3 shows the categories that arose regarding the feasibility of integrating the contextual MM problems into secondary school classes.

According to Table 3, in general, the various stakeholders' views regarding the feasibility of integrating the contextual MM problems differed. Regarding strengths, the teachers emphasized the learner's aspect, whereas the experts and policy makers emphasized the characteristics of the contextual MM problem itself. Another difference was found in the sub-criteria of both the weaknesses and threats; the teachers' attitude was negative (the teachers' lack of confidence and the lack of flexibility in the educational system), whereas the experts and policy-makers exhibited a positive attitude (a belief in the teachers' ability to assimilate the problems). However, the two stakeholder groups agreed on the need to train teachers so that these problems would provide an opportunity for success.

Table 3. The feasibility of integrating the contextual MM problems into secondary school classes, based on the SWOT methodology.

Criteria	Examples of Categories Retrieved in This Study	
	Experts and Policymakers	Teachers
Strengths	Authenticity and relevancy	Increasing students' motivation
Weaknesses	Convincing teachers that they can use the contextual MM problems	Teachers' lack of confidence in explaining the related scientific knowledge of the contextual MM problems
Opportunities	Teacher training sessions	Teacher training sessions
Threats	Availability of teachers to apply the contextual MM problems	Conservative view of the education system and the teachers

Next, we applied a descriptive analysis, and then presented the average score for the advantages, i.e., the strengths and opportunities, and the disadvantages, i.e., the weaknesses and threats, using the data obtained from the Likert-type questionnaire (see Figure 4).

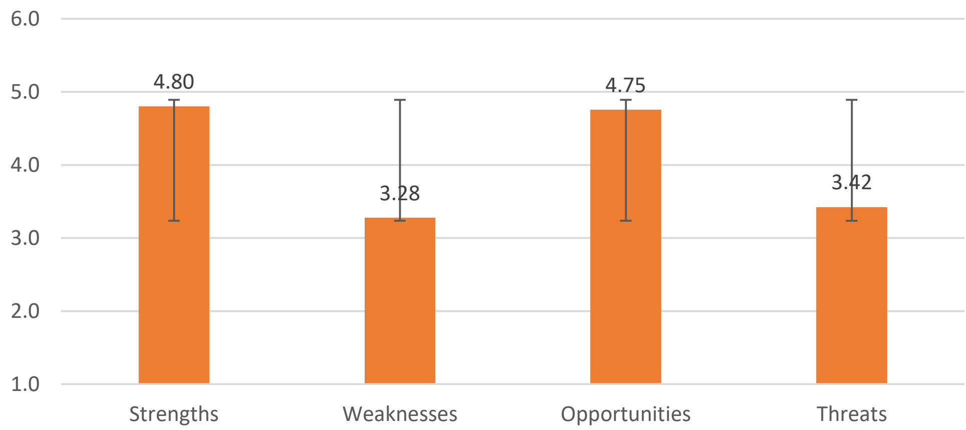

Figure 4. Distribution by the average score for the strengths, opportunities, weaknesses, and threats.

ANOVA with repeated-measures analyses, followed by Bonferroni analysis, performed to investigate the differences between the various SWOT criteria. These revealed that the study participants similarly valued the advantages of the contextual MM problems, namely, the strengths and opportunities, and valued the disadvantages significantly less, namely, the weaknesses and threats, which they perceived as equally important, $F(3,\ 114) = 120.84$, $p < 0.001$, $\eta^2 = 0.76$. Further One-Way MANOVA analyses, performed to compare the two stakeholder groups' attitudes, revealed a similarity between the experts and the policy-makers, vs. the teachers for both the advantages, $F(2,\ 114) = 0.69$, $p > 0.05$, $\eta^2 = 0.01$, and the disadvantages, $F(2,\ 115) = 1.36$, $p > 0.05$, $\eta^2 = 0.02$. These findings indicate that both stakeholder groups perceived that there was high feasibility of integrating the contextual MM problems into classes, since they provided more weight to the strengths and the opportunity to use these problems, and significantly less weight to the weaknesses and threats.

Lastly, One-Way MANOVA analyses were performed to investigate the differences in attitudes regarding the feasibility of integrating the contextual MM problems into classes by using the various background variables, i.e., the level of education, the STEM background, and gender. In addition, Pearson correlations were conducted between the participants' age and each of the SWOT criteria. The findings revealed no significant differences in all the investigated background variables, $0.08 < F(4,\ 218)$, $F(2,\ 218) < 2.30$, as well as no correlation between age and each of the SWOT criteria, $0.04 < r(120) < 0.07$. Appendix B presents descriptive statistics for the background variables (besides age), in relation to the SWOT criteria.

4. Discussion

This study aimed to characterize contextual MM problems within the framework of the PS framework [3], as well as to examine the feasibility of integrating these problems into classes. Overall, according to two stakeholder groups from the mathematics field, namely, (1) experts and policymakers, and (2) teachers, the contextual MM problems were found to correspond to Schoenfeld's framework for PS. Both stakeholder groups agreed that the problems demonstrate the problem resources; however, they mainly perceived the problems as being appropriate for more advanced students. It also seems that although the algorithmic procedures precisely reflect the problem (particularly the sketches), these problems are perceived as more appropriate for non-formal learning, such as enrichment or inquiry learning. This finding strengthens the challenges of incorporating MM problems into formal school education [11,12,38].

In addition, there was mutual agreement concerning the heuristics of the problems, since all stakeholder groups considered the strategies and techniques in the problems as necessary for effective PS. More specifically, they were positively influenced by the use of appropriate representations such as dynamic illustrations and the relevant analogy presented in the problems, which they considered as factors that motivate students. This is in accordance with the definition of Stanic and Kilpatrick [19] for the use of PS as a context; it describes its value in motivating students to study, which also justifies why one should study mathematics, due to its relevancy to students' lives [39].

Furthermore, all stakeholder groups perceived the problems as least reflecting the testing and verification procedures; however, this was for different reasons. Apparently, the teachers perceived the significance of maintaining the heuristics of the problems and not reducing them in a procedural manner [18,19], and they perceived their role in facilitating students in using the PS process [17]. Thus, they specifically suggested adding a student help page. Furthermore, the experts and policy-makers especially valued the use of a real scale to construct the mathematical problem, which indicates the student's value of being a "good" problem-solver, and who has the opportunity to inquire about other representational systems outside (but related to) the world of mathematics [29–32].

Finally, considering that the contextual MM problems are retrieved from authentic workplace scenarios, it is probable that they could differently affect the attitudes of stakeholders with different backgrounds. Since various demographic variables did not affect the participants' attitudes, this indicates that the contextual MM problems were well-characterized from the viewpoint of PS, which might also affect the application of MM problems in practice [11,38,40].

Regarding the feasibility of integrating the contextual MM problems into classes, our findings indicate that all stakeholders view the strengths and opportunities positively, which means that they agree on the high feasibility of applying these problems in formal school education. However, the two stakeholder groups understand the weaknesses and threats of the contextual MM problems differently: the teachers considered the aforementioned criteria as inhibiting factors, whereas the experts and policy-makers considered these factors as more of an opportunity. This highlights the extensive challenges that teachers face when applying PS in mathematics classrooms. More specifically, in practice, they need systemic support in the form of appropriate curricular content, and professional support to make the required changes in their instructional practices [14–16,22].

Clearly, engaging students in PS is influenced by the teacher's instructional decisions e.g., [41]. Future research should acknowledge the significant role of teachers in supporting students' engagement in PS; therefore, future policies should involve ways of supporting teachers in implementing PS in classes, particularly through contextual MM problems. This suggestion refers to one limitation of this study, since our goal was to characterize the problems, rather than to investigate the processes that are actually taking place in class. Thus, this study focused on the PS processes of *resources* that describe our current understanding of cognitive structures, i.e., the constructive nature of cognition, cognitive architecture, memory, and access to this, and *heuristics*, which refer to mathematical problem-solving strategies. We also suggest focusing on students as the main participants in further research, which will allow for one to characterize the contextual MM problems through their actual implementation in class, using all the PS framework components.

The Study's Contribution

This study contributes theoretically to the relationship between mathematical modelling [7–10] and PS as a context [3,19], particularly regarding the context of workplace mathematics [4–6]. This investigation of the theoretical relationship, based on the perspectives of two key stakeholder groups from the mathematics field, supports claims made by scholars, e.g., [11,15,40,42] that MM and PS should be further theorized to better address students' practical needs.

Methodologically, our use of the Delphi methodology facilitated the elaboration of our knowledge on characterizing contextual MM problems for secondary school, based on Schoenfeld's [3] PS framework. Moreover, the two-phase methodology that was applied in this study, starting with the application of the Delphi methodology, yielded a quantitatively valid Likert-type questionnaire that can be further used to characterize math problems in other contexts.

Finally, from a practical viewpoint, we highlighted the importance of incorporating contextual MM problems into classes, especially because they correspond to PS skills, which require students to seek solutions, explore patterns, and formulate conjectures [3]. There is evidence that the study level and success in school mathematics predicts future success in the workplace [43,44]. Hence, contextual MM problems can better prepare students to enter the workforce, particularly in STEM-oriented contexts [13].

Furthermore, the use of contextual MM problems is valuable for students, since it can help them to understand the role that mathematics plays in the world, and how to apply mathematics in situations that are likely to arise in their current and future lives and professions, particularly in the contemporary science and technology-driven workforce [45]. Thus, it has the potential to attract more students to engage in high-level mathematics, as well as in STEM-related fields.

Author Contributions: Conceptualization, Z.K. and O.N.-T.; methodology, O.N.-T.; software, Z.K. and O.N.-T.; validation, Z.K. and O.N.-T.; formal analysis, Z.K. and O.N.-T.; investigation, Z.K. and O.N.-T.; resources, Z.K. and O.N.-T.; data curation, O.N.-T.; writing—original draft preparation, Z.K. and O.N.-T.; writing—review and editing, Z.K.; visualization, O.N.-T.; supervision, Z.K.; project administration, Z.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Approval for the study was received from the university's Institutional Review Board (#IRB- 2019-034) before beginning of the study (28 May 2019).

Informed Consent Statement: Informed consent was obtained from all subjects involved in the study.

Data Availability Statement: Due to ethical reasons data cannot be provided.

Acknowledgments: The authors highly acknowledge the past and current MtED lab research group, and the R&D group from the i-MAT project for their great support and contribution to this research.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. Means and SD for background variables, in relation to PS criteria [3].

PS Criteria	PS Sub-Criteria	Gender				Level of Education				STEM Background					
		M		F		B.A.		M.A. or Ph.D.		Engineering		STEM		None	
		M	SD	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
Resources	Intuitions and informal knowledge regarding the domain	3.95	0.51	3.82	0.57	3.79	0.53	3.86	0.58	3.97	0.53	3.87	0.54	3.79	0.54
	Facts, rules and algorithmic procedures	4.67	0.63	4.76	0.69	4.78	0.70	4.69	0.66	4.75	0.62	4.83	0.66	4.68	0.72
	"Routine" nonalgorithmic procedures	3.99	0.59	4.22	0.69	4.11	0.73	4.24	0.61	4.22	0.69	4.17	0.58	4.20	0.69
Heuristics	Appropriate representations	4.43	0.80	4.69	0.76	4.63	0.87	4.67	0.70	4.67	0.73	4.73	0.67	4.56	0.86
	Analogies and exploiting related problems	4.55	0.84	4.57	0.82	4.56	0.85	4.58	0.79	4.56	0.87	4.68	0.62	4.52	0.93
	Testing and verification procedures	4.13	1.13	4.09	1.03	4.11	0.98	4.10	1.06	4.00	0.88	4.04	0.97	4.19	1.16

Appendix B

Table A2. Means and SD for background variables, in relation to SWOT criteria [35,36].

SWOT Criteria	Gender				Level of Education				Engineering		STEM Background			
	M		F		B.A.		M.A. or Ph.D.				STEM		None	
	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
Strength	4.82	0.85	4.80	0.79	4.74	0.91	4.85	0.74	5.07	0.80	4.91	0.74	4.59	0.81
Weakness	3.33	0.78	3.27	0.86	3.31	0.80	3.27	0.90	3.37	0.82	3.38	0.71	3.10	0.96
Opportunity	4.66	0.64	4.78	0.74	4.59	0.80	4.88	0.64	4.86	0.61	4.78	0.65	4.69	0.82
Threat	3.39	0.80	3.42	0.75	3.40	0.79	3.41	0.75	3.52	0.65	3.36	0.75	3.39	0.82

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