

Article

The Impact of Geometric Continuities (C^1 , C^2 , and C^3) on the Trajectories of Industrial Robots

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Abstract

This article presents a mathematical analysis of the geometric continuity of industrial robot trajectories, highlighting the influence of continuity conditions on velocity, acceleration, and vibration profiles. This study proposes an original approach: isolating geometric continuity as an independent parameter and the comparative evaluation of the continuity levels C^1 , C^2 , and C^3 using a series of pre-computed kinematic metrics: vibration energy, acceleration variability, and trajectory curvature stability. The methodology is validated by numerical simulations performed with RoboDK for trajectory generation and by post-processing in Python for metric evaluation. The results indicate that C^1 trajectories exhibit discontinuities in the higher-order derivatives, which lead to undesirable kinematic behaviors, while C^2 continuity represents the minimum requirement for an acceptable, stable motion in industrial applications. Higher-order continuity, C^3 , brings greater regularity to the trajectories, but the practical advantages are limited and relatively insignificant for standard industrial applications.

Keywords: industrial robot; simulation; trajectory; continuity; geometry; RoboDK; jerk; impulsivity; discontinuity



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1. Introduction

Path planning is a fundamental component of industrial robot control, directly influencing the positioning accuracy, motion stability, and quality of technological processes. In modern industrial applications such as precision assembly, robotic welding, high-speed handling, or automated logistics processes, the trajectories generated for the robot's terminal point (Tool Center Point or TCP) must simultaneously satisfy requirements of geometric accuracy, kinematic smoothness, and operational robustness.

In the literature, trajectory planning for industrial robots has been approached from multiple perspectives, each focusing on a specific performance criterion. However, the comparative analysis of these directions highlights a common feature: geometric continuity is rarely treated as an independent design parameter.

The first category of works aims at optimizing the execution time or temporal parameterization in the presence of kinematic and dynamic constraints [1,2]. In these approaches, the trajectory geometry is considered fixed, and the problem is formulated as one of optimizing the velocity and acceleration profiles. Although these methods are effective in reducing the duration of the movement, they implicitly accept the level of continuity imposed by the motion law used (S-curve or polynomial type) without separately investigating the influence of the continuity order on the global kinematic behavior. Thus, continuity becomes a consequence of the chosen method, not an explicitly analyzed parameter.

A second major direction is represented by methods based on polynomial interpolation and splines [3,4]. Cubic, quintic, or B-spline interpolations provide different levels of continuity (C^1 , C^2 , or higher), but in most studies, the order of continuity is directly determined by the order of the interpolator and is not subject to systematic comparative evaluation [1,5]. For example, the use of quintic polynomials provides continuity of acceleration but is rarely justified by an explicit analysis of the differences with a cubic or higher-order solution. Comparison of the effects of these choices on the jerk distribution or on the local behavior of the trajectory is generally limited.

Another relevant category is constituted by works oriented towards the minimization of jerk or the limitation of acceleration variations [6]. These approaches introduce objective functions that penalize higher-order variations and can lead to trajectories with continuous jerk. However, they simultaneously modify both the temporal profile and the geometric structure of the motion, which makes it difficult to isolate the pure influence of the level of continuity. Furthermore, local jerk optimization is not always correlated with a global integrated energy analysis or with the assessment of the impulsive character of the kinematic variations.

In industrial practice, C^2 continuity is frequently adopted as the default standard due to its ability to ensure acceleration continuity and acceptable kinematic behavior [7]. However, this choice is more a result of technical convention than a rigorous comparison between C^1 , C^2 , and C^3 under identical geometry and duration conditions. The additional benefits of higher-order continuities are often assumed but rarely systematically quantified in relation to the associated energy cost or computational complexity [8,9].

Another critical aspect identified in the existing literature is the predominant use of integrated global metrics or local point-based assessments without explicit correlation between them. For example, acceleration or jerk energy are frequently used to assess smoothness, but these quantities may mask the existence of impulsive variations concentrated on short time intervals. The lack of an approach that combines global metrics with local discontinuity indicators makes it difficult to fully interpret the kinematic behavior [10].

Therefore, the comparative analysis of existing directions highlights a common limitation: although geometric continuity directly influences the behavior of higher-order derivatives, it is mostly treated implicitly, in correlation with other variables (geometry, time, dynamic constraints). There is a lack of a controlled framework in which the level of continuity is isolated as an independent parameter and its effects are simultaneously evaluated through global metrics and complementary local indicators.

This methodological limitation justifies the approach proposed in this paper, in which the continuity of C^1 , C^2 , and C^3 is comparatively analyzed under identical conditions of geometry, duration, and boundary conditions, allowing a clear separation of the effects due exclusively to the continuity order.

1.1. Research Gap

Although the literature offers numerous solutions for trajectory planning, a systematic analysis in which the level of geometric continuity is isolated as an independent parameter

under controlled conditions is lacking. In particular, there are not enough comparative studies that keep the trajectory geometry, execution time, and boundary conditions constant, modifying exclusively the continuity order to evaluate its real impact on the global and local kinematic behavior.

1.2. Problem Statement

If the trajectory geometry, execution duration, and boundary conditions are kept constant, what is the real impact of the continuity level (C^1 , C^2 , and C^3) on the kinematic behavior of TCP trajectories of industrial robots? Can global energy metrics accurately characterize the smoothness of the motion, or is it necessary to introduce complementary local indicators to highlight impulsive phenomena?

1.3. Objective

The objective of the paper is to perform a controlled comparative analysis between the C^1 , C^2 , and C^3 continuities by isolating continuity as the only variable parameter and evaluating its impact on kinematic behavior using both integrated global metrics (acceleration energy and jerk energy) and local indicators (discontinuities and impulsivity).

1.4. Main Contribution

The main contributions of the paper are:

- Explicit treatment of geometric continuity as an independent design parameter.
- Definition of a controlled comparative framework in which geometry, time, and boundary conditions are kept constant.
- Introduction and combined use of global and local metrics for evaluating kinematic behavior.
- Highlighting the difference between the uniform distribution of kinematic variations and their impulsive character.

1.5. Novelty Statement

The novelty of the paper does not lie in redefining the theory of geometric continuity but in systematically demonstrating, in a controlled setting, that global energy metrics can lead to incomplete interpretations if they are not correlated with local indicators of discontinuity and impulsivity. This dual approach provides a clearer mathematical and practical justification for choosing the optimal level of continuity in industrial applications.

2. Materials and Methods

2.1. Theoretical Set-Up

The analysis proposed in this paper is performed in the TCP space, considering the trajectory as a parametric curve defined over a finite time interval. The TCP trajectory is described by the position vector:

$$r(t) = [x(t), y(t), z(t)], \quad t \in [t_0, t_1] \quad (1)$$

where the component functions are continuous and differentiable depending on the level of continuity imposed. This formulation allows an analysis independent of the kinematic structure of the robot, the results being thus generalizable for different industrial platforms [11]. The higher-order derivatives of the trajectory (velocity, acceleration, and jerk) are defined as follows:

$$v(t) = r'(t) \quad (2)$$

$$a(t) = r''(t) \quad (3)$$

$$j(t) = r'''(t) \quad (4)$$

These quantities are used to evaluate the kinematic behavior associated with different levels of geometric continuity.

In this study, three levels of geometric continuity are analyzed, relevant for industrial robotics applications [12]:

- (a) Continuity C^1 , characterized by continuity of position and velocity but allowing discontinuities of acceleration.
- (b) Continuity C^2 , which ensures continuity of position, velocity, and acceleration, leading to a finite jerk.
- (c) Continuity C^3 , for which even the third-order derivatives are continuous, resulting in a smooth variation in the jerk.

Formally, at a connection point t_i (node between two segments of a trajectory), the continuity conditions are expressed as

$$\lim_{t \rightarrow t_i^-} r^{(k)}(t) = \lim_{t \rightarrow t_i^+} r^{(k)}(t), \quad k = 0, 1, \dots, n. \quad (5)$$

This mathematical condition can be translated as equality between the value of the limit of the derivative just before the point t_i (when t tends to t_i from the left side) and immediately after the point t_i (when t tends to t_i from the right side). The order of the level of continuity is denoted by k .

To allow an objective comparative assessment of different levels of continuity, we introduce the following kinematic metrics, which describe the behavior of the trajectory during execution.

Jerk energy: The energy that quantifies higher-order variations in acceleration and is directly correlated with the mechanical excitations induced in the system [13].

$$J = \int_{t_0}^{t_1} \|j(t)\|^2 dt \quad (6)$$

Acceleration variation: The way acceleration varies provides an overall measure of the kinematic demands associated with the trajectory.

$$A = \int_{t_0}^{t_1} \|a(t)\|^2 dt \quad (7)$$

Trajectory curvature is relevant for the analysis of variations in the direction of movement and the global geometric behavior. The stability of the curvature is evaluated by means of mean values and standard deviations over a considered time interval.

$$k(t) = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3} \quad (8)$$

To isolate the effect of geometric continuity, we must adopt a controlled comparative framework so that the analyzed trajectories respect the following conditions [14]:

- The geometry of the trajectory must be identical;
- The execution time must be kept constant;
- The boundary conditions for position, velocity, and acceleration at the ends must be equivalent.

The only variable parameter between trajectories is the level of geometric continuity. This approach allows the observed differences to be attributed exclusively to the continuity effect, eliminating the influence of other confounding factors.

Respecting these conditions, the trajectories were generated using the RoboDK v.5.7.4 offline programming and simulation software environment. This application allows both the precise definition of target points and the export of trajectories in a discretized format so that numerical analysis of the exported data and the calculation of previously defined kinematic metrics can be performed using Python 3.9.

2.2. Experimental Setup

To evaluate the impact of geometric continuity on kinematic behavior, trajectories with the same spatial geometry, execution programs with the same duration, and identical boundary conditions were generated. The only variable modified between the analyzed cases was the level of geometric continuity imposed on the trajectory, respectively, C^1 , C^2 , and C^3 .

In the RoboDK software application (v.5.7.4), a virtual station was created with an industrial robot model, ABB IRB 1300, which was selected due to its versatility and use in a wide range of industrial applications. RoboDK integrates the real robot post-processor, allowing the generation of target points, trajectories, and programs to execute movements identical to the real model.

Five sets of 20 randomly positioned points were generated in the robot workspace, and then trajectories were generated through these targets. Table 1 presents the coordinates of the first set of 20 target points, and Figure 1 shows the robot trajectory generated between them. Due to the large amount of data, the other four sets of points are not presented in tabular format, but only the trajectories generated between them can be seen in Figures 2 and 3.

Table 1. First set of points.

No.	X [mm]	Y [mm]	Z [mm]	RX [°]	RY [°]	RZ [°]
1	460.6447	137.8804	820.0924	0	90	0
2	605.7668	18.87753	605.294	0	90	0
3	846.1937	−67.3156	549.8528	0	90	0
4	528.5177	400.6719	174.0915	0	90	0
5	932.3085	−202.806	715.194	0	90	0
6	862.1414	312.9808	825.4004	0	90	0
7	477.6246	196.9541	648.7989	0	90	0
8	516.0405	−117.361	745.0748	0	90	0
9	607.0757	−300.478	780.6738	0	90	0
10	892.2601	−248.283	660.4557	0	90	0
11	513.8645	497.8536	584.5093	0	90	0
12	842.0969	134.5406	654.2602	0	90	0
13	865.7	68.71418	353.4616	0	90	0
14	1030.409	−492.53	490.7654	0	90	0
15	765.1458	118.809	239.8935	0	90	0
16	807.9702	−309.724	782.6877	0	90	0
17	647.3067	345.4127	640.1416	0	90	0
18	701.6887	240.297	845.3159	0	90	0
19	629.1637	−111.808	820.6739	0	90	0
20	822.3789	−388.914	329.5872	0	90	0

The orientation of the target points was maintained, keeping only the rotation about the Y axis so as to avoid reorientation and reconfiguration movements of the robot that could lead to substantial differences in the way the robot approaches from one target point to another.

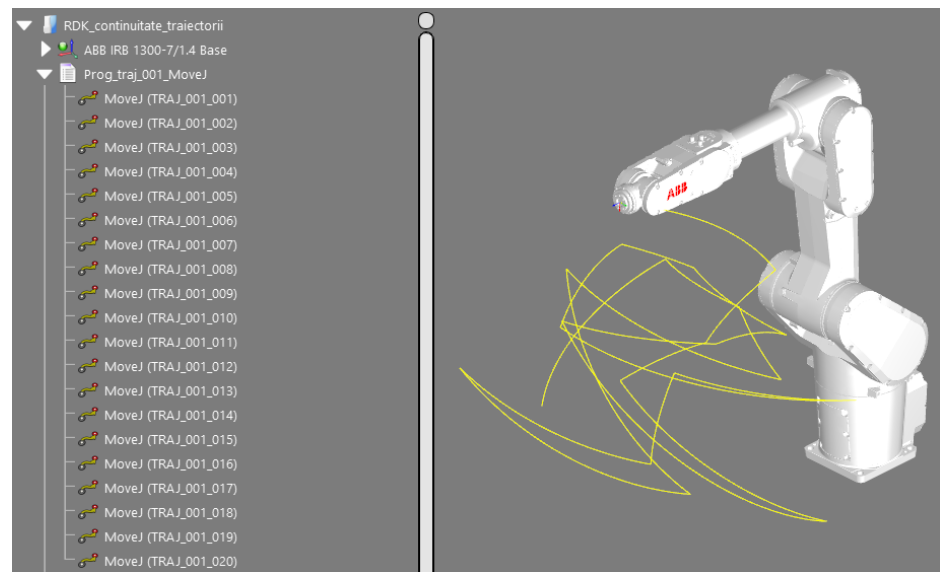


Figure 1. Robot trajectory generated for the first set of points.

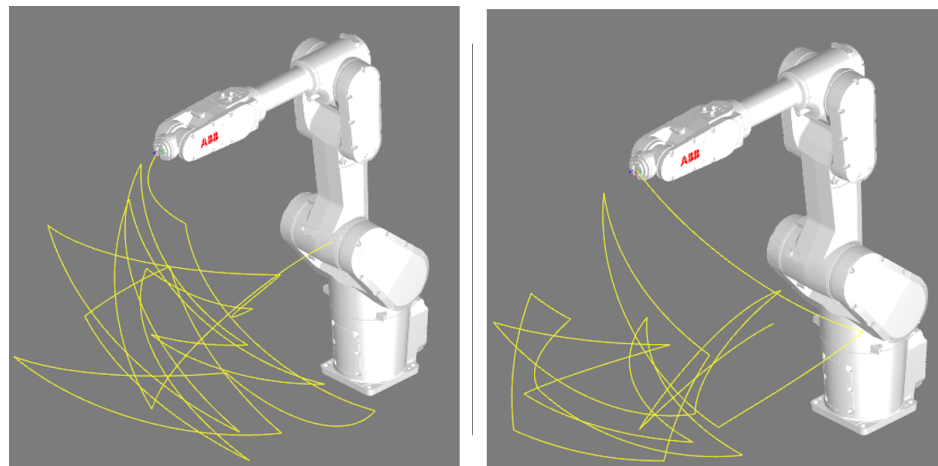


Figure 2. Robot trajectory generated for the second and third sets of points.

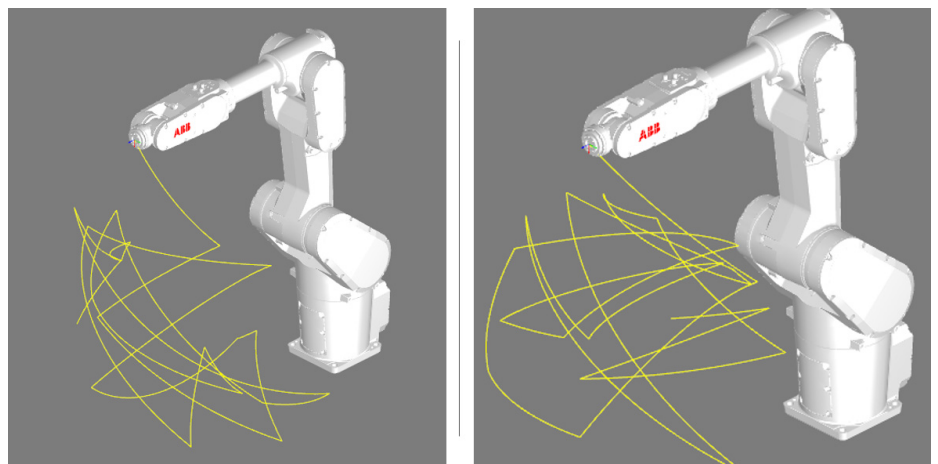


Figure 3. Robot trajectory generated for the fourth and fifth sets of points.

To isolate the influence of geometric continuity, each original trajectory was discretized and reconstructed using three different methods so as to ensure each level of continuity as follows:

- Cubic interpolation [15] was used to ensure continuity of position and velocity (C^1).
- Quintic interpolation [16] was used to ensure continuity of position, velocity, and acceleration (C^2).
- Seventh-order interpolation [17] was used to ensure continuity of position, velocity, acceleration, and jerk (C^3).

For each trajectory, an equal execution time ($t = 20$ s) was imposed for all continuity levels, and then a time parameterization was imposed so that the discretization points were distributed proportionally over the entire trajectory. This is a way to prevent speed variations caused by the non-uniform distribution of nodes on the trajectory. In order to analyze the sensitivity to different sampling frequencies, three study cases were conducted with time parametrizations of 0.01 s, 0.02 s, and 0.05 s, resulting in frequencies of 100 Hz, 50 Hz, and 20 Hz.

So, the original trajectories were re-discretized using a fixed sampling rate 3 times, once for each continuity type. After their reconstruction, each trajectory was exported in .CSV format for analysis, and metrics were calculated. In Figure 4, the first trajectory and the discretization points can be observed. All other trajectories were then discretized in a similar way.

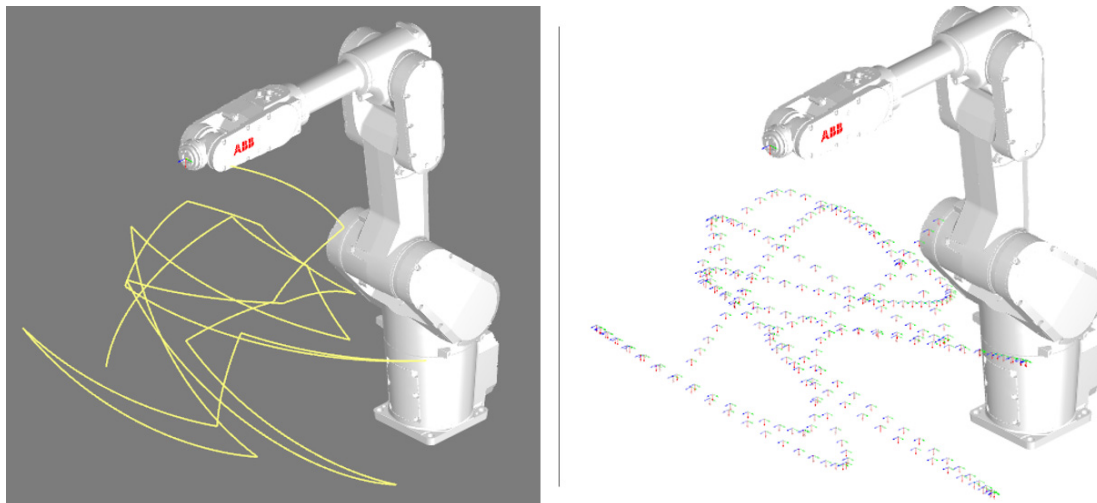


Figure 4. First trajectory and discretization points for C^1 .

3. Results

Each of the five original trajectories was discretized for each of the three types of continuity, resulting in a total of 15 distinct trajectories whose differences are due exclusively to the continuity conditions imposed by the interpolation method. The effects of continuity on the metrics are presented individually in the following subchapters.

3.1. Acceleration Energy

To quantify the global kinematic stress associated with each trajectory, the acceleration energy was evaluated for all five sets of trajectories, for each continuity level (C^1 , C^2 , C^3), and for the three sampling intervals analyzed ($\Delta t = 0.01$ s, 0.02 s, 0.05 s). Figures 5–7 present the acceleration energy values calculated individually for each of the five trajectories, for all continuity levels, and for the three sampling intervals. It is observed that the absolute values differ depending on the geometry of each trajectory, with some sets generating higher kinematic stresses than others. However, for each trajectory analyzed, the relative order between the methods remains constant: C^1 presents the lowest acceleration energy values, C^2 intermediate values, and C^3 the highest values.

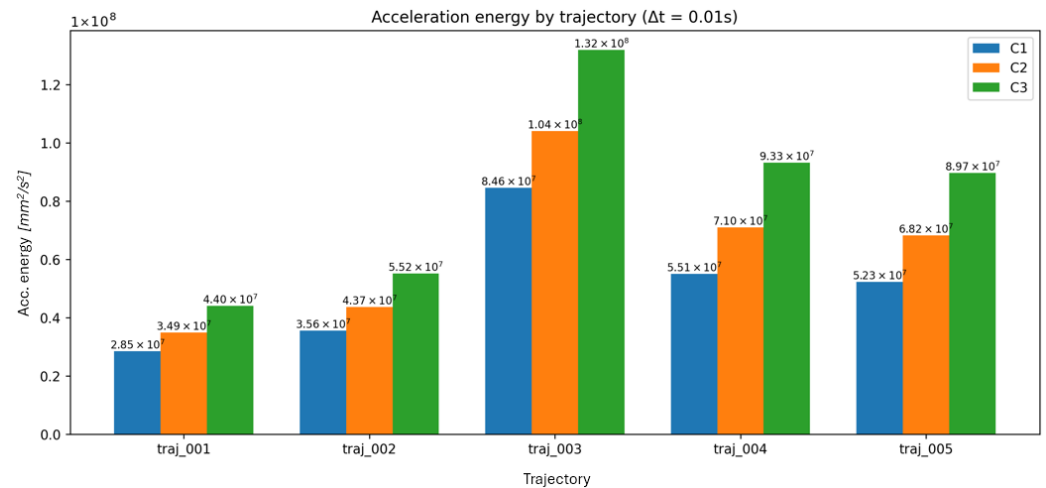


Figure 5. Acceleration energy at a sampling rate of 0.01 s.

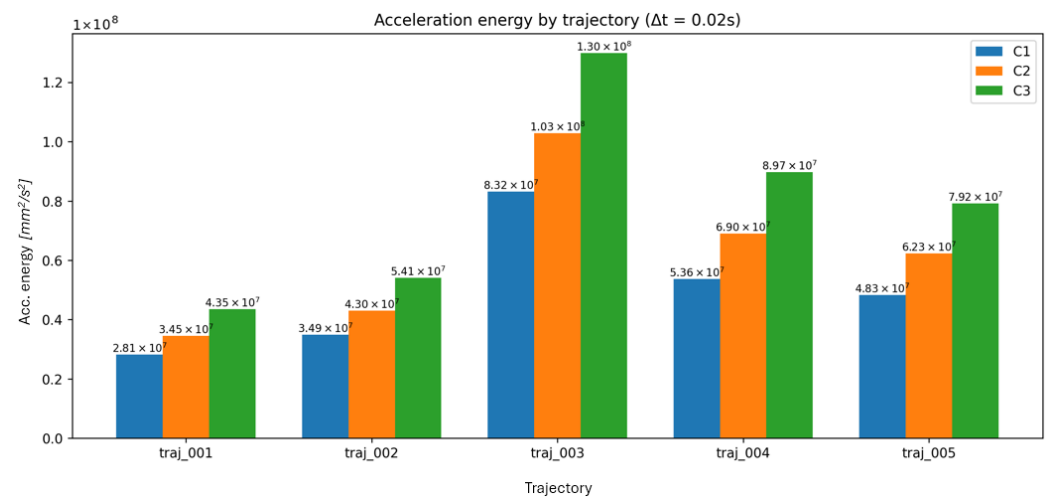


Figure 6. Acceleration energy at a sampling rate of 0.02 s.

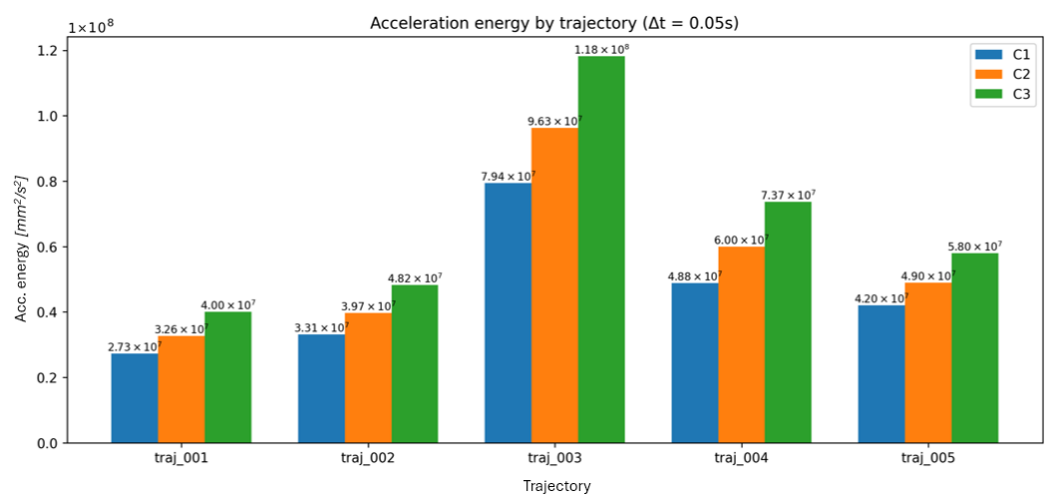


Figure 7. Acceleration energy at a sampling rate of 0.05 s.

Figure 8 shows the average values of the acceleration energy ($\pm$ standard deviation calculated over the five trajectories) for each continuity level and for each sampling interval.

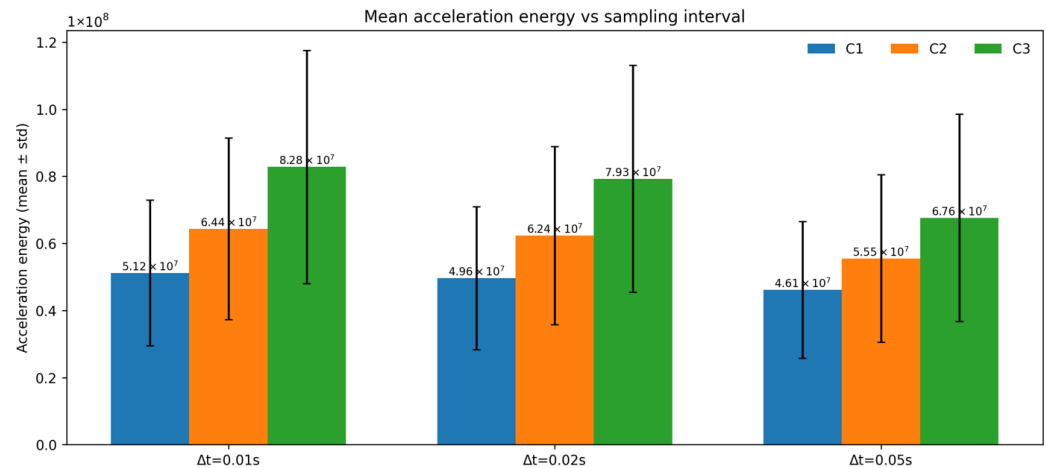


Figure 8. Average values of the acceleration energy.

The statistical analysis confirms the trend observed at the individual level: the relative order $C^1 < C^2 < C^3$ is maintained for all three sampling intervals analyzed. Changing the discretization step leads to moderate numerical variations in the absolute values but does not produce inversions of the hierarchy between methods. The results indicate that, under identical conditions of geometry, duration, and boundary conditions, the increase in the continuity order is systematically associated with an increase in the integrated acceleration energy both at the individual and average levels. It is observed that the values differ depending on the trajectory geometry, but for each analyzed trajectory, the same general trend is maintained: C^1 records the lowest jerk energy values, C^2 records intermediate values, and C^3 records the highest values.

3.2. Jerk Energy

To evaluate the acceleration variations over time and the kinematic stresses associated with them, the jerk energy was calculated for all five sets of trajectories, for each continuity level (C^1 , C^2 , and C^3), and for the three sampling intervals analyzed ($\Delta t = 0.01$ s, 0.02 s, 0.05 s).

Figures 9–11 present the jerk energy values calculated individually for each of the five trajectories, for all continuity levels, and for the three sampling intervals.

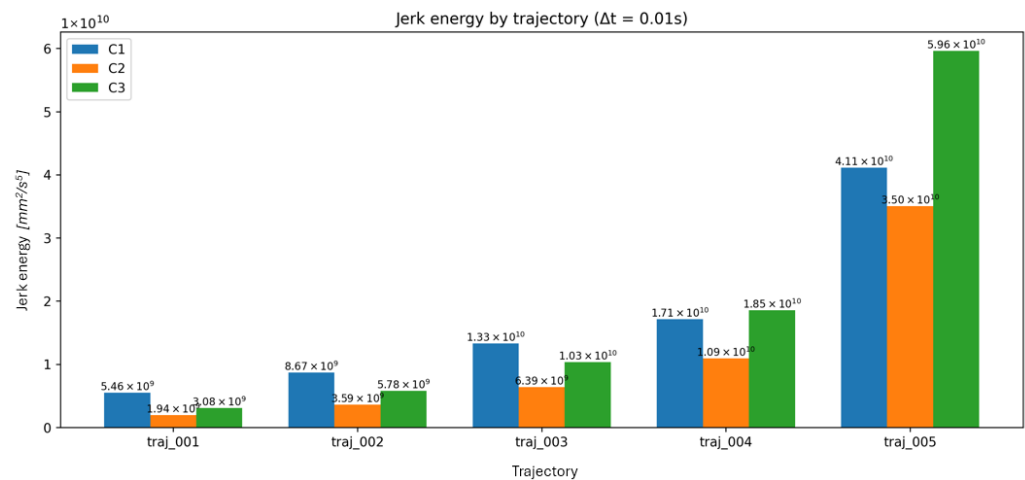


Figure 9. Jerk energy at a sampling rate of 0.01 s.

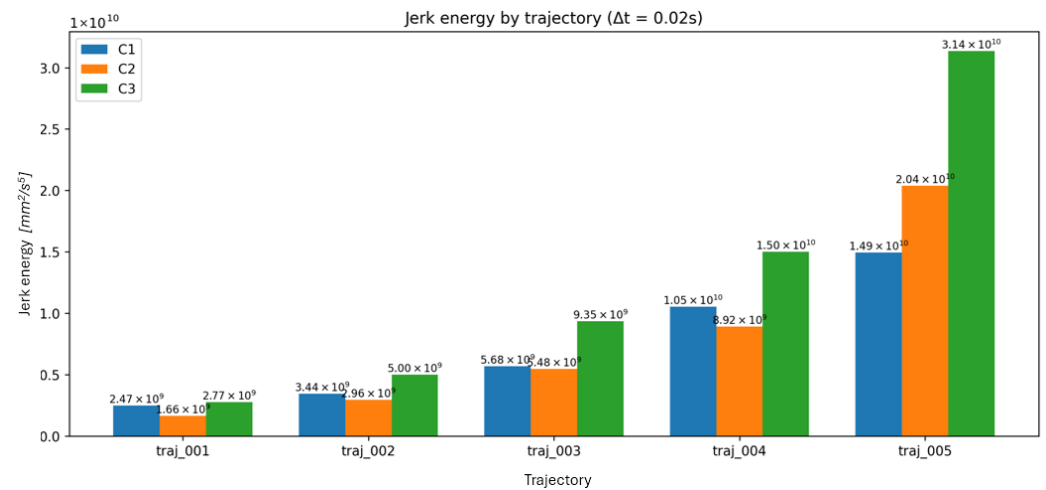


Figure 10. Jerk energy at a sampling rate of 0.02 s.

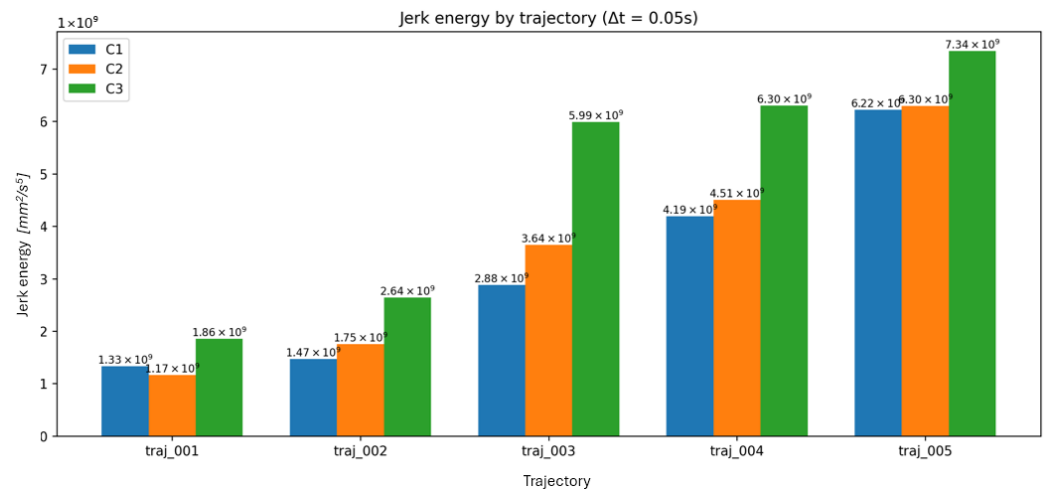


Figure 11. Jerk energy at a sampling rate of 0.05 s.

Figure 12 shows the average values of the jerk energy ($\pm$ standard deviation calculated over the five trajectories) for each continuity level and for each sampling interval.

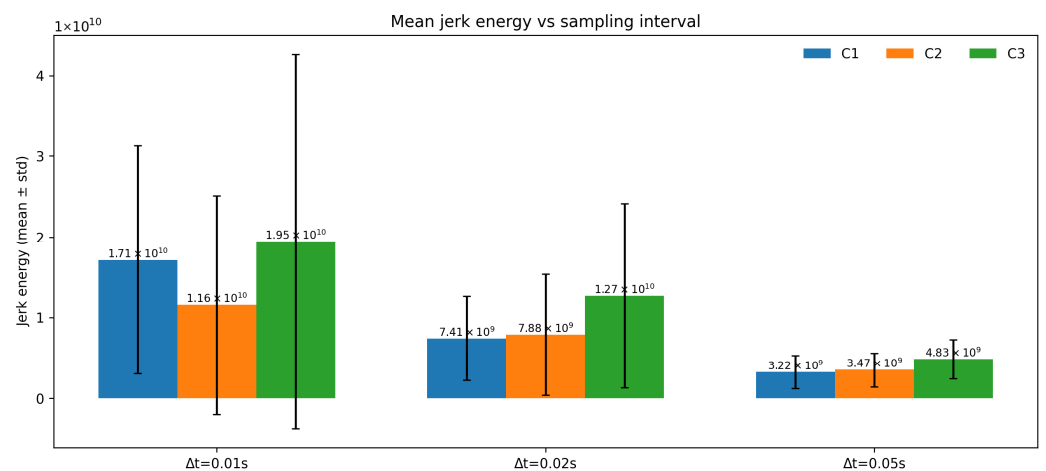


Figure 12. Average values of jerk energy.

The statistical analysis indicates that changing the discretization step produces moderate numerical results, and the results indicate that, under identical conditions of geometry,

duration, and boundary conditions, the increase in the continuity order is systematically associated with an increase in the integrated jerk energy.

3.3. Sensitivity to Sampling Interval

For a more rigorous quantification of the influence of the discretization step, the percentage variations in the average values were calculated relative to the reference case $\Delta t = 0.01$ s. The results are illustrated in Figures 13 and 14 for the acceleration energy and, respectively, for the jerk energy.

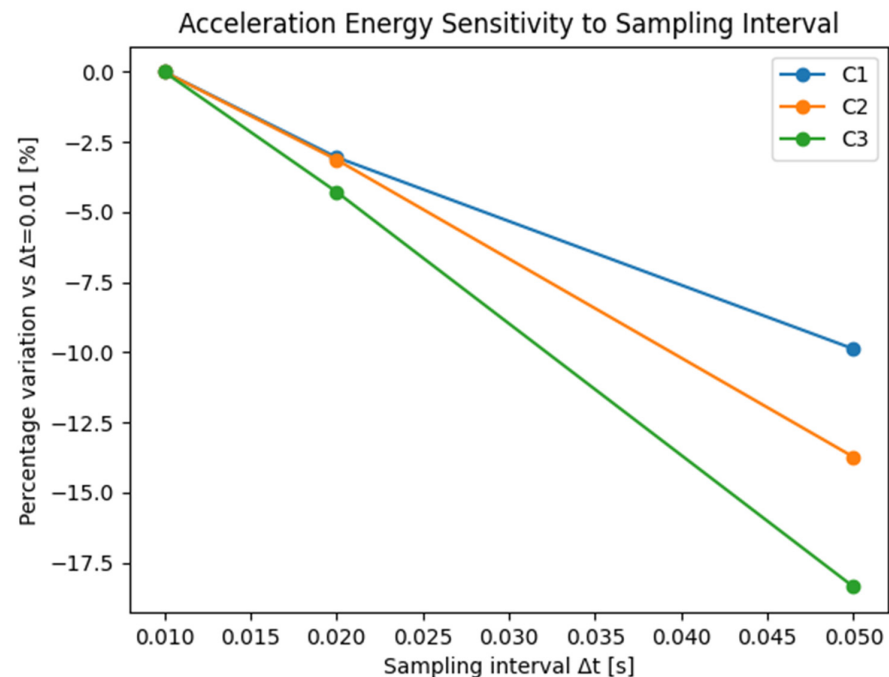


Figure 13. Acceleration energy percentage variation.

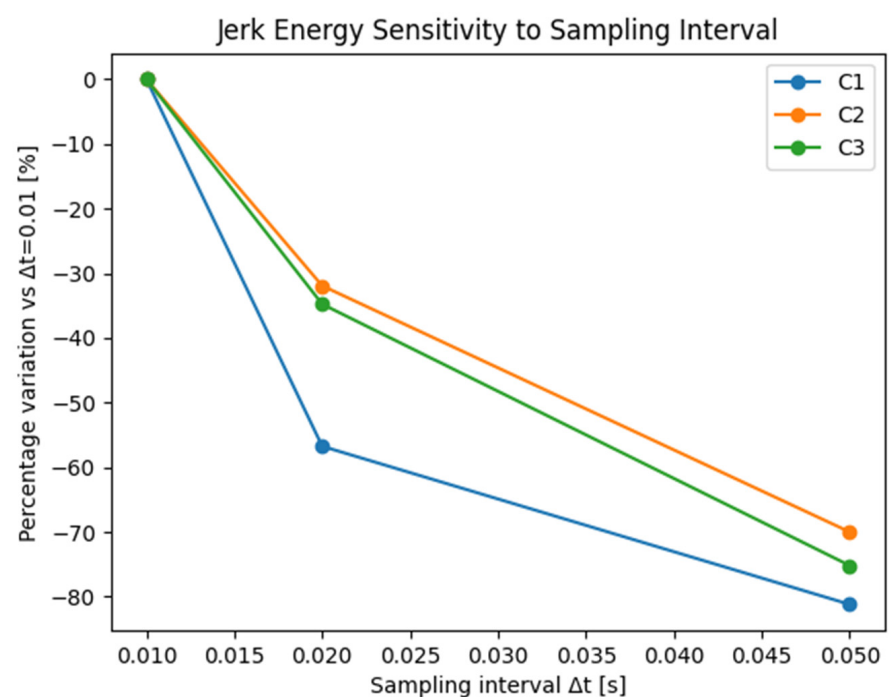


Figure 14. Jerk energy percentage variation.

Figure 13 presents the percentage variation in the acceleration energy as a function of the sampling interval. It is observed that for $\Delta t = 0.02$ s, the deviations are reduced, indicating a good numerical convergence. In the case of coarser discretization ($\Delta t = 0.05$ s), more pronounced variations in the absolute values appear, but without changing the hierarchy between the continuity levels.

Figure 14 presents a similar behavior for the jerk energy. In this case, too, although the absolute values are influenced by the sampling step, the comparative trend between the methods remains unchanged for all three analyzed values.

The results demonstrate that the comparative analysis of continuity levels is robust to the temporal discretization in the studied interval. Thus, the conclusions regarding the relative behavior of the C^1 , C^2 , and C^3 trajectories are not dependent on the specific choice of the sampling step used in post-processing.

3.4. Discontinuity Metrics

The previously analyzed energy indicators provide a global perspective on the kinematic load but do not allow the identification of the local behavior of the trajectories in the transition areas between segments. To highlight these local effects, discontinuity indicators for acceleration and jerk were calculated [18]. Discontinuities were quantified by determining the maximum difference between two consecutive samples:

- max. Δa = maximum acceleration difference between two successive points.
- max. Δj = maximum jerk difference between two successive points.

These quantities reflect the sudden nature of kinematic variations and are sensitive to the occurrence of impulsive phenomena.

Figures 15–17 present the maximum values of acceleration discontinuity for each trajectory and for each continuity level.

It is observed that trajectories of type C^2 and C^3 present reduced values of acceleration discontinuity compared to C^1 . In the case of C^1 , allowing acceleration discontinuity at the connection points leads to higher values of this indicator for most of the analyzed trajectories.

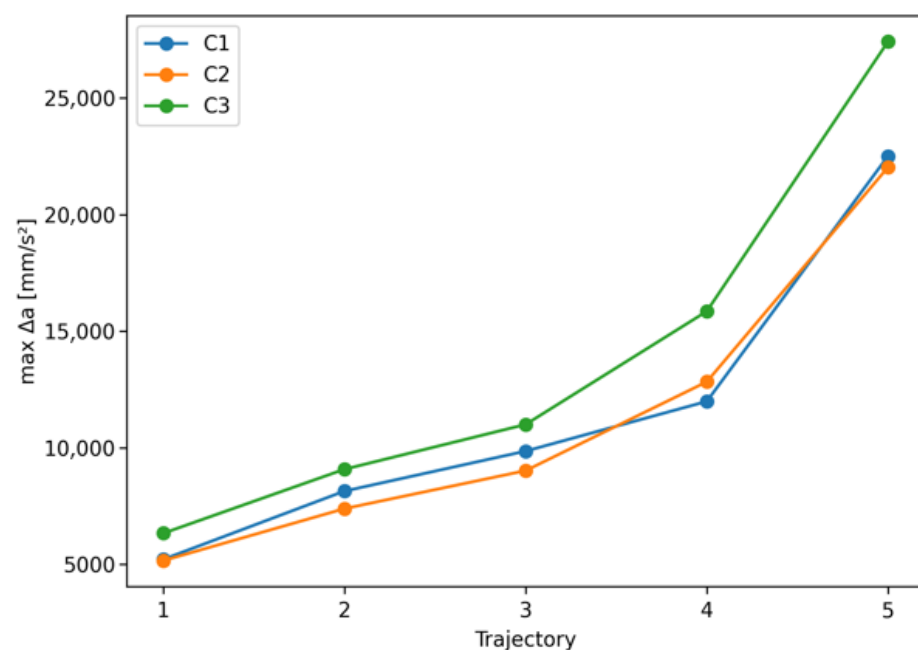


Figure 15. Maximum acceleration difference for $\Delta t = 0.01$ s.

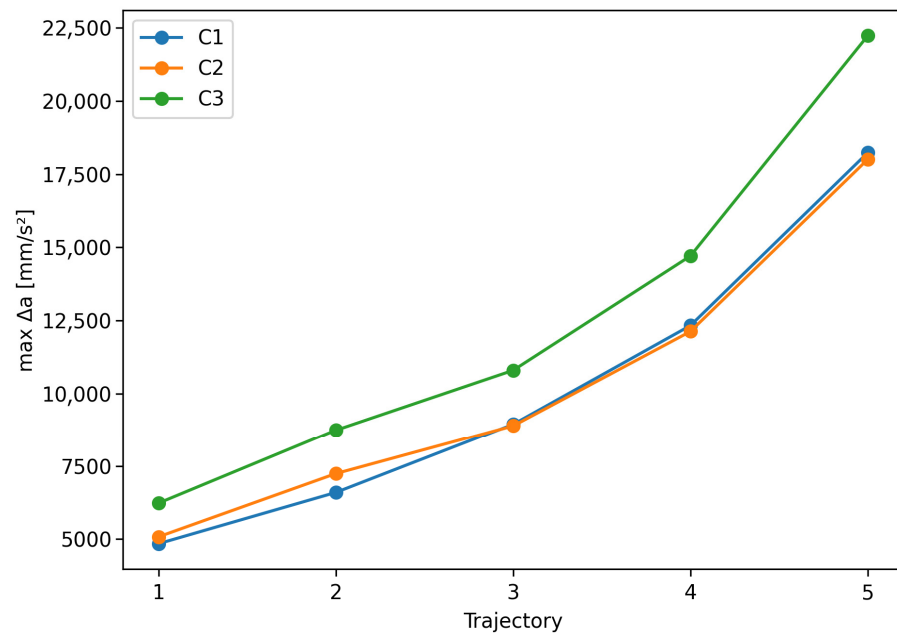


Figure 16. Maximum acceleration difference for $\Delta t = 0.02$ s.

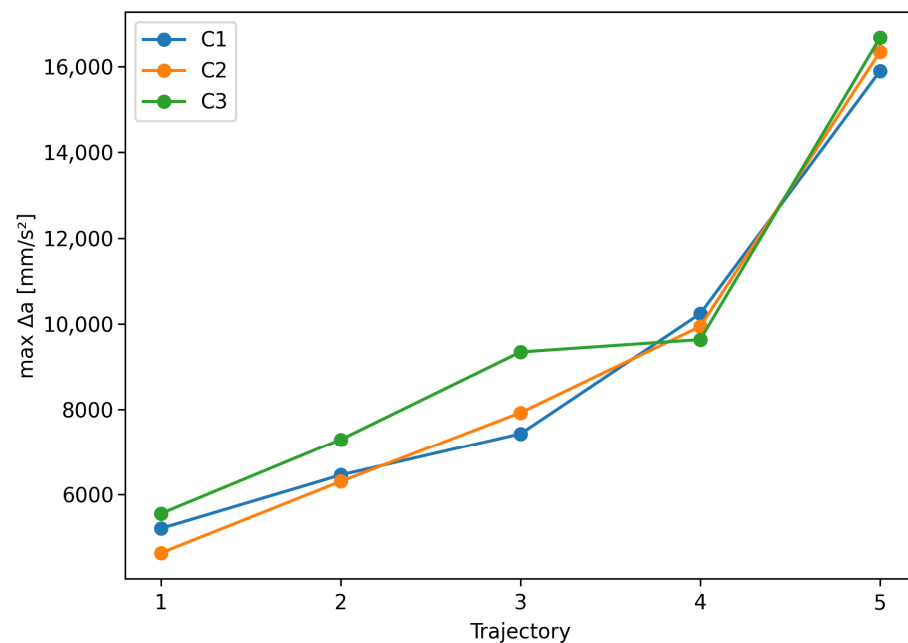


Figure 17. Maximum acceleration difference for $\Delta t = 0.05$ s.

Figures 18–20 show the maximum jerk discontinuity values. The differences between the methods are more pronounced than in the case of acceleration. C^1 trajectories generally record the highest jerk discontinuity values, while C^2 and C^3 show lower and more stable values.

It is also observed that certain geometries (especially trajectory 5) generate significantly higher discontinuity values for all methods, indicating a direct influence of geometric complexity on the local behavior of the motion.

The results show that although global energy indicators may suggest lower values for C^1 , the discontinuity analysis highlights a less stable local behavior of this type of trajectory [19].

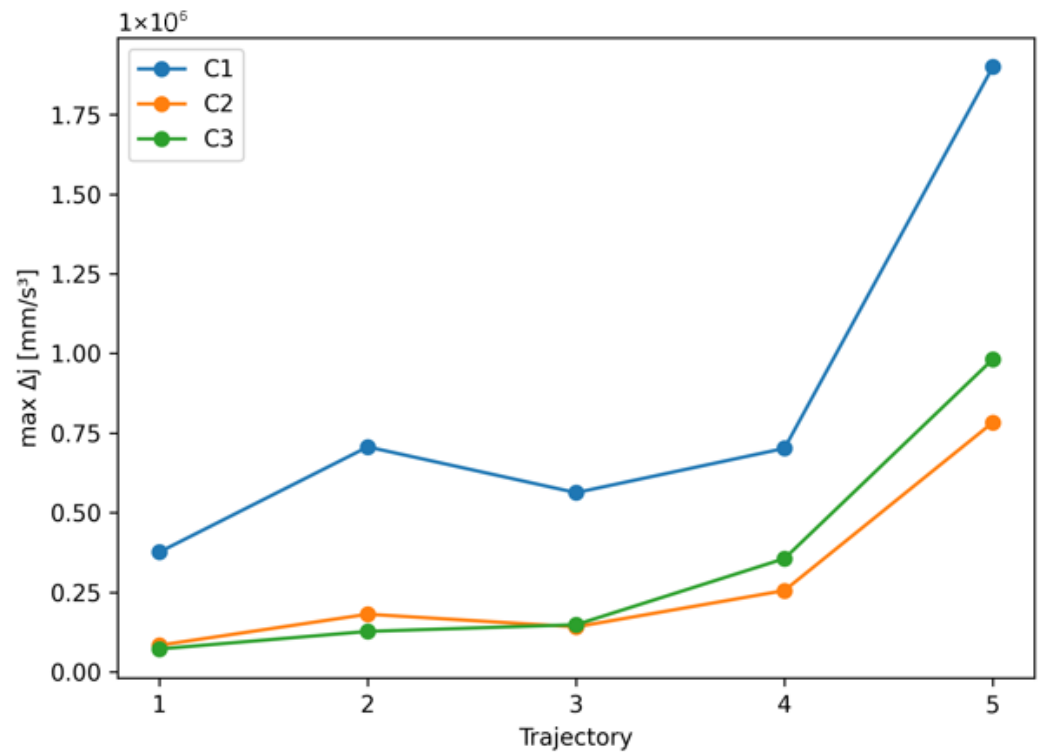


Figure 18. Maximum jerk difference for $\Delta t = 0.01$ s.

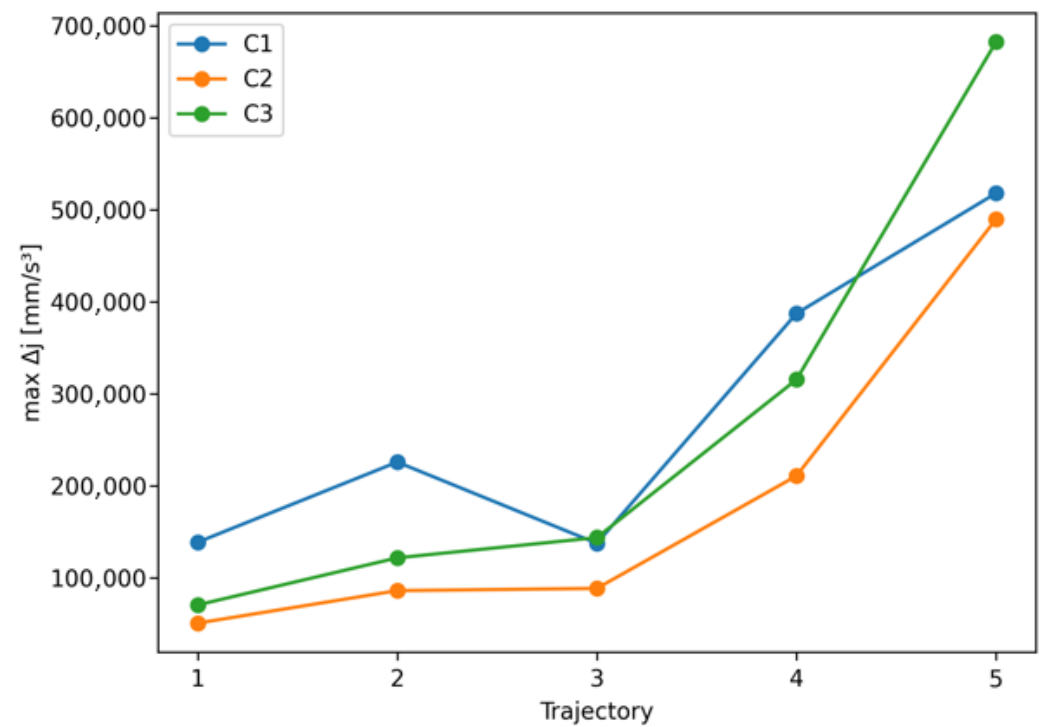


Figure 19. Maximum jerk difference for $\Delta t = 0.02$ s.

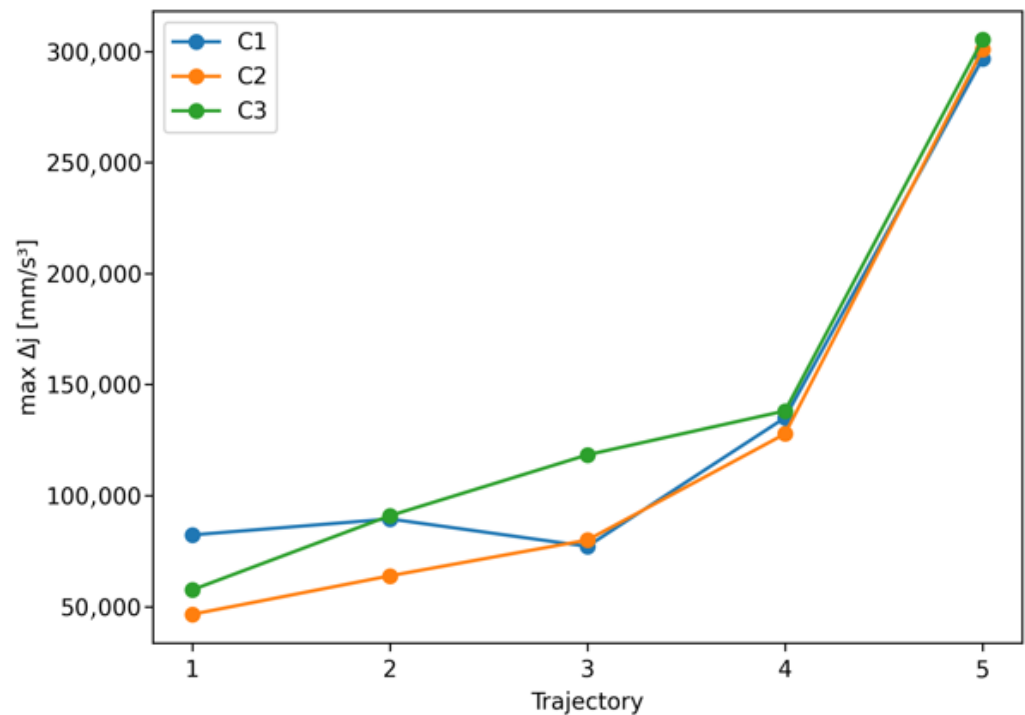


Figure 20. Maximum jerk difference for $\Delta t = 0.05$ s.

3.5. Impulsiveness of the Jerk

To capture the local and impulsive character of the jerk, we introduced an impulsivity indicator I_j to capture the “spike-like” character. This dimensionless indicator is defined as the ratio between the maximum peak and the average level of the jerk.

$$I_j = \frac{j_{max}}{j_{rms}} \quad (9)$$

where the average jerk level is calculated with the formula:

$$j_{rms} = \sqrt{\frac{1}{N} \sum_{k=1}^N \|j_k\|^2} \quad (10)$$

where k = time step index, j_k = jerk value at discrete time t_k , and N = number of samples. The indicator was calculated for all three sampling intervals analyzed: $\Delta t = 0.01$ s, 0.02 s, and 0.05 s.

For each of the 5 trajectories, both impulsivity values and jerk profiles were calculated and plotted over the entire duration of the trajectories. At this point, we have all the necessary information so that we can highlight both the temporal distribution of the jerk and the mechanical disturbances (possible shocks) induced by abrupt acceleration variations. Tables 2–4 present the impulsivity values calculated for each trajectory, with minimum, maximum, and average values highlighted in the last column by the corresponding colors green, red, and yellow.

Table 2. Impulsivity of each trajectory at $\Delta t = 0.01$ s.

Trajectory	Method	$j_{\max}$ [mm/s ³]	j_{rms} [mm/s ³]	I_j
Traj. 1	C1	376,334.0857	19,936.55915	18.8765816
Traj. 1	C2	83,836.94772	10,187.40394	8.229471237
Traj. 1	C3	72,211.87575	12,581.10041	5.73971063
Traj. 2	C1	706,495.2134	25,725.48969	27.46284802
Traj. 2	C2	181,269.423	13,996.10865	12.95141582
Traj. 2	C3	127,305.568	17,171.33772	7.413841024
Traj. 3	C1	562,892.3383	29,890.33773	18.8319163
Traj. 3	C2	141,556.7783	18,273.14856	7.74670976
Traj. 3	C3	148,383.9898	22,852.87615	6.493011594
Traj. 4	C1	701,873.0284	33,312.66599	21.06925421
Traj. 4	C2	255,624.418	23,777.32482	10.75076443
Traj. 4	C3	355,894.4441	30,476.86423	11.67752829
Traj. 5	C1	1,899,335.531	54,250.61326	35.01039744
Traj. 5	C2	783,243.4748	44,626.50331	17.55108325
Traj. 5	C3	981,546.6265	55,263.18661	17.76131068

Table 3. Impulsivity of each trajectory at $\Delta t = 0.02$ s.

Trajectory	Method	$j_{\max}$ [mm/s ³]	j_{rms} [mm/s ³]	I_j
Traj. 1	C1	138,414.1163	12,815.5885	10.8004495
Traj. 1	C2	50,449.43798	9313.614317	5.416741155
Traj. 1	C3	70,212.03333	11,806.60899	5.946841585
Traj. 2	C1	225,545.6819	14,941.37175	15.09537984
Traj. 2	C2	85,776.28951	12,529.22466	6.84609717
Traj. 2	C3	121,420.3198	15,860.77564	7.655383476
Traj. 3	C1	137,212.6137	18,024.77709	7.612444416
Traj. 3	C2	88,187.69331	16,699.39134	5.28089267
Traj. 3	C3	143,434.8854	21,614.718	6.635982271
Traj. 4	C1	387,099.9602	26,678.96348	14.50955771
Traj. 4	C2	210,495.8166	22,232.64364	9.467871661
Traj. 4	C3	315,179.532	27,672.14596	11.38977557
Traj. 5	C1	517,828.9147	29,391.6779	17.61821549
Traj. 5	C2	489,556.6807	33,066.65114	14.80514851
Traj. 5	C3	682,260.0015	40,500.02427	16.84591587

Table 4. Impulsivity of each trajectory at $\Delta t = 0.05$ s.

Trajectory	Method	$j_{\max}$ [mm/s ³]	j_{rms} [mm/s ³]	I_j
Traj. 1	C1	82,254.27971	9689.375095	8.489121218
Traj. 1	C2	46,491.30138	8114.167101	5.729645544
Traj. 1	C3	57,498.52038	9744.90224	5.900369133
Traj. 2	C1	89,454.61879	9506.526732	9.409810892
Traj. 2	C2	63,833.20708	9928.786729	6.429104464
Traj. 2	C3	90,907.53173	11,856.75473	7.667151241
Traj. 3	C1	77,098.0485	12,875.90026	5.987779258
Traj. 3	C2	79,955.86961	13,776.4667	5.803800884
Traj. 3	C3	118,337.6251	17,353.71978	6.819150396
Traj. 4	C1	135,066.282	16,212.40852	8.331043586
Traj. 4	C2	127,779.8212	15,996.75985	7.987856443
Traj. 4	C3	138,130.242	18,437.59768	7.491770045
Traj. 5	C1	296,866.2328	21,250.66643	13.96973755
Traj. 5	C2	300,946.117	20,768.32105	14.49063293
Traj. 5	C3	305,307.8794	21,911.43906	13.93372104

Jerk profiles for each trajectory and for each sampling rate are presented in the following Figures 21–35.

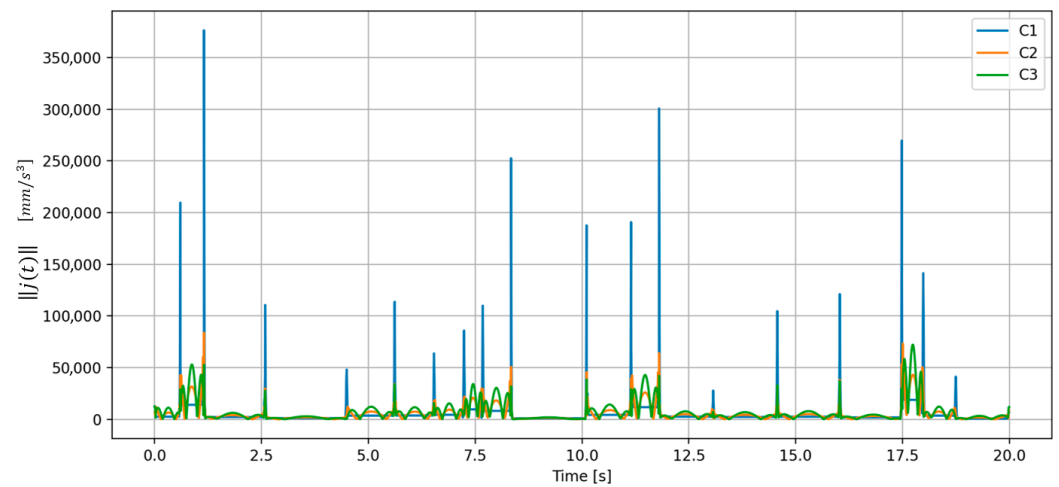


Figure 21. Jerk profile, trajectory 1, $\Delta t = 0.01$ s.

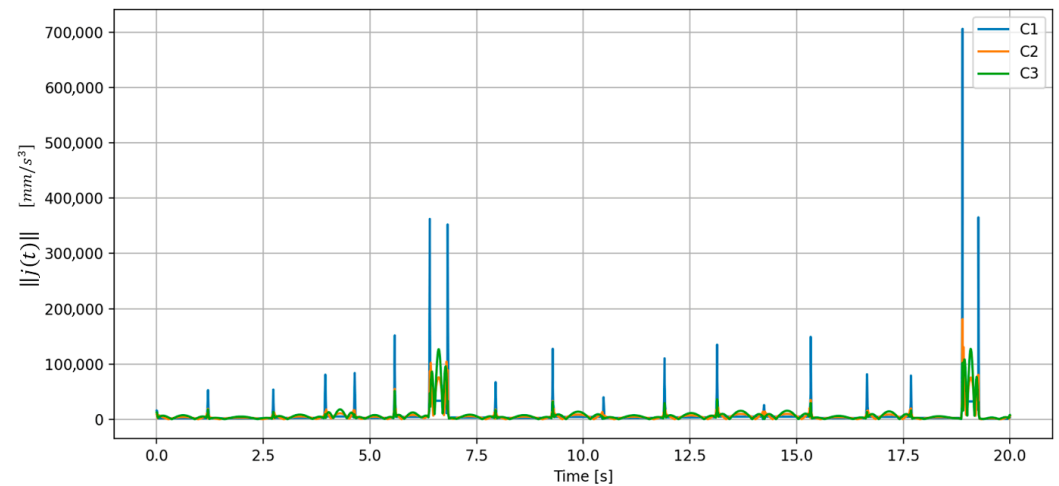


Figure 22. Jerk profile, trajectory 2, $\Delta t = 0.01$ s.

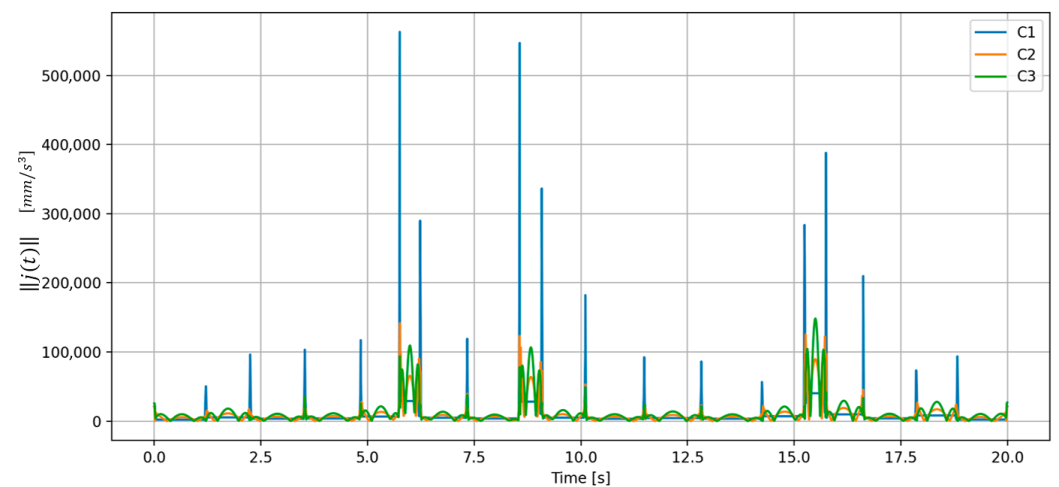


Figure 23. Jerk profile, trajectory 3, $\Delta t = 0.01$ s.

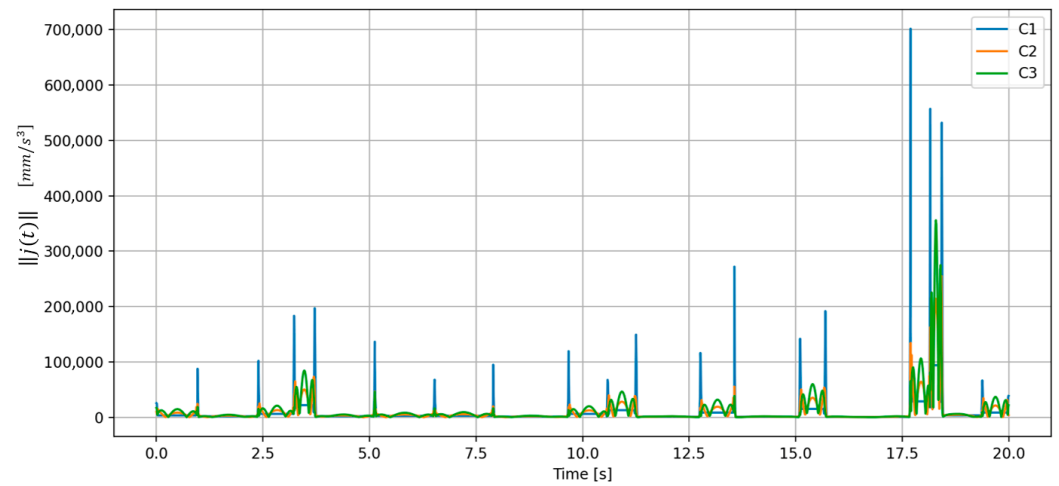


Figure 24. Jerk profile, trajectory 4, $\Delta t = 0.01$ s.

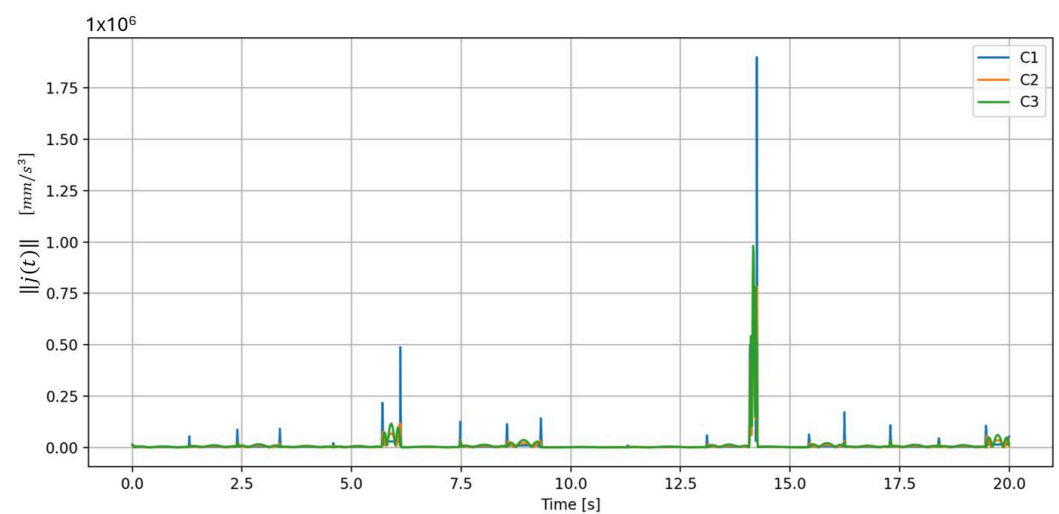


Figure 25. Jerk profile, trajectory 5, $\Delta t = 0.01$ s.

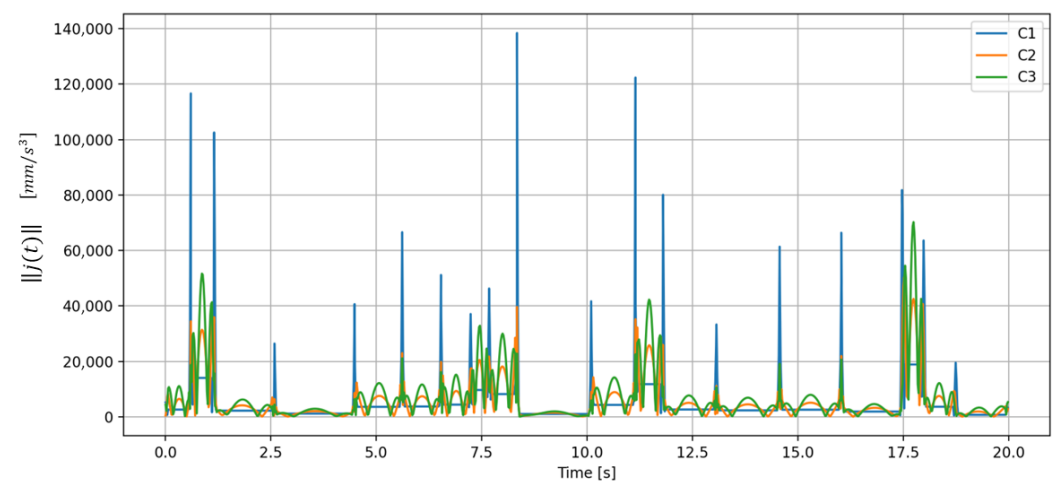


Figure 26. Jerk profile, trajectory 1, $\Delta t = 0.02$ s.

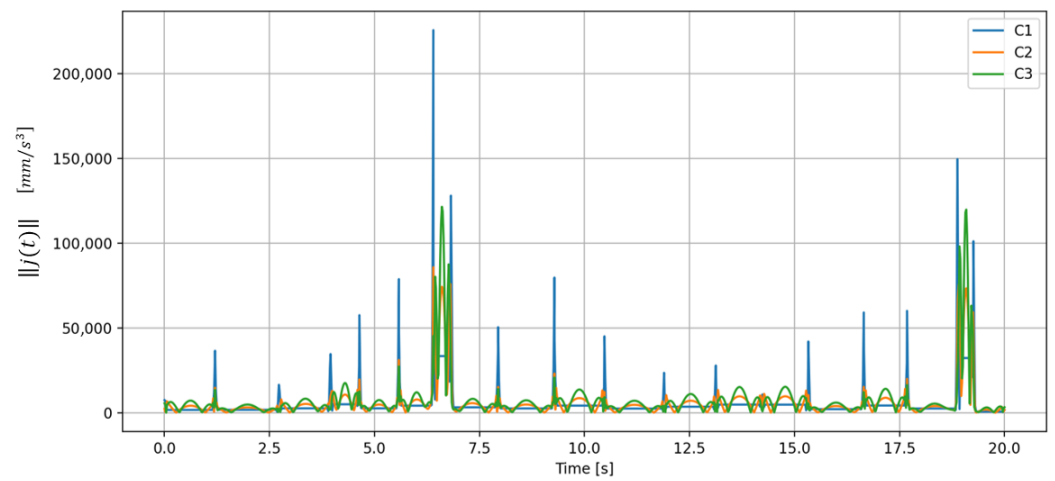


Figure 27. Jerk profile, trajectory 2, $\Delta t = 0.02$ s.

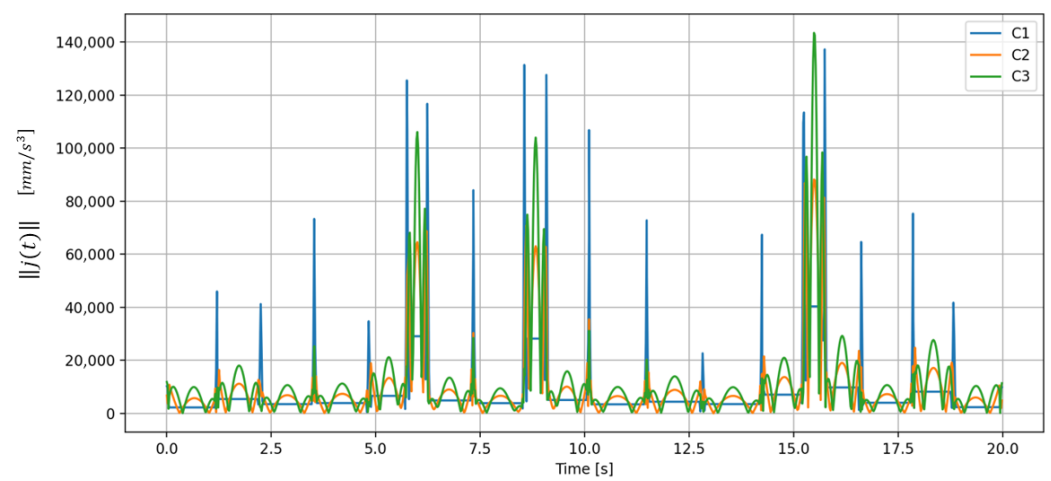


Figure 28. Jerk profile, trajectory 3, $\Delta t = 0.02$ s.

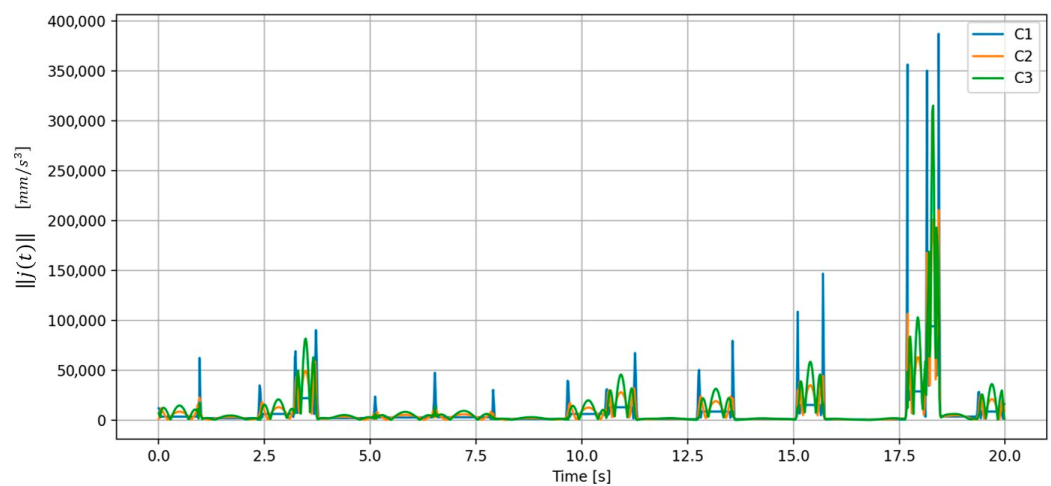


Figure 29. Jerk profile, trajectory 4, $\Delta t = 0.02$ s.

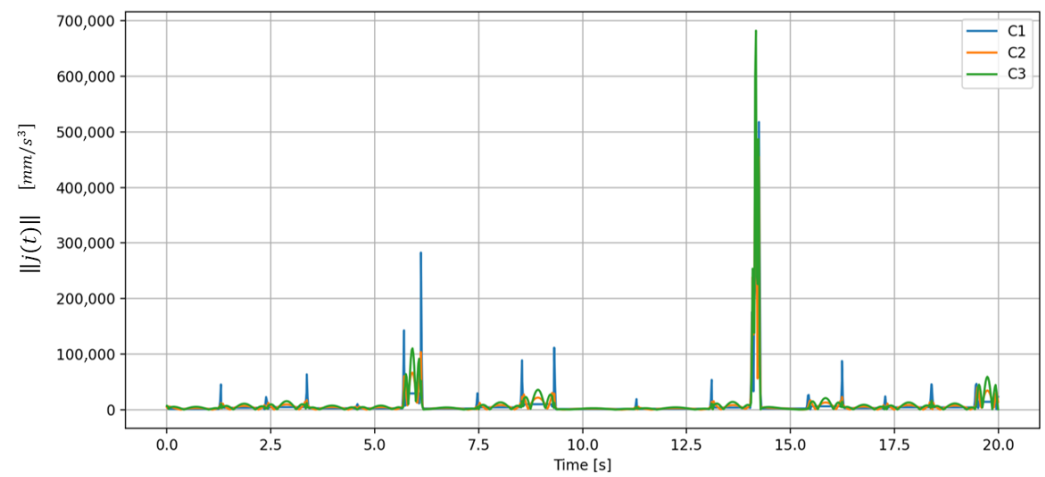


Figure 30. Jerk profile, trajectory 5, $\Delta t = 0.02$ s.

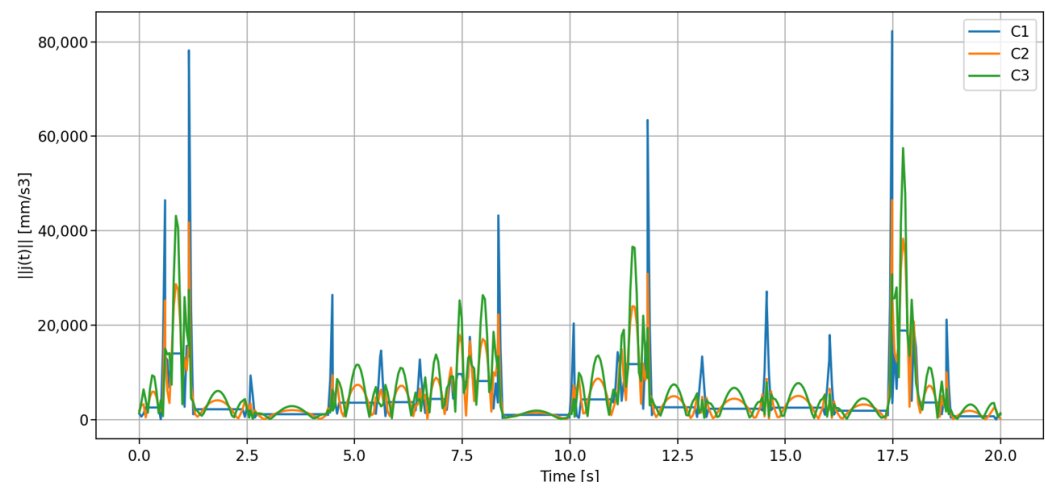


Figure 31. Jerk profile, trajectory 1, $\Delta t = 0.05$ s.

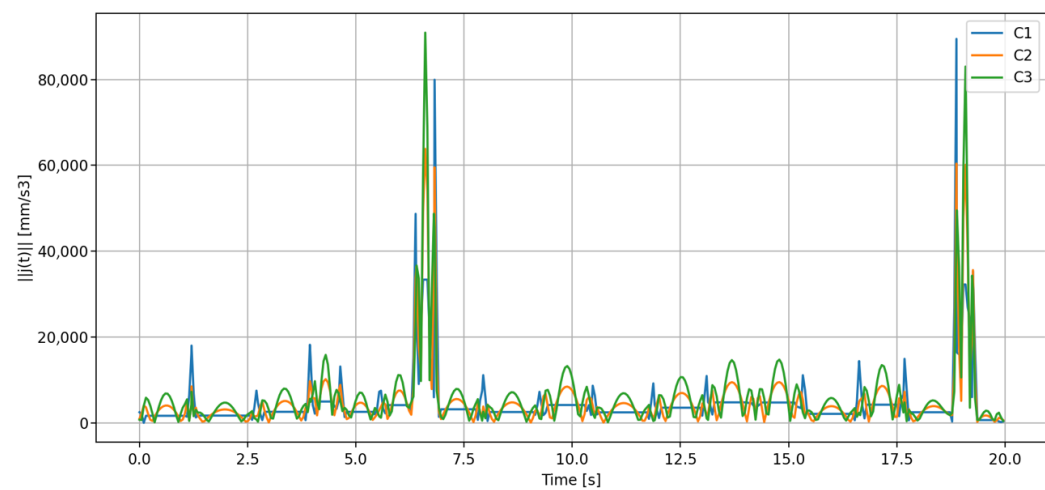


Figure 32. Jerk profile, trajectory 2, $\Delta t = 0.05$ s.

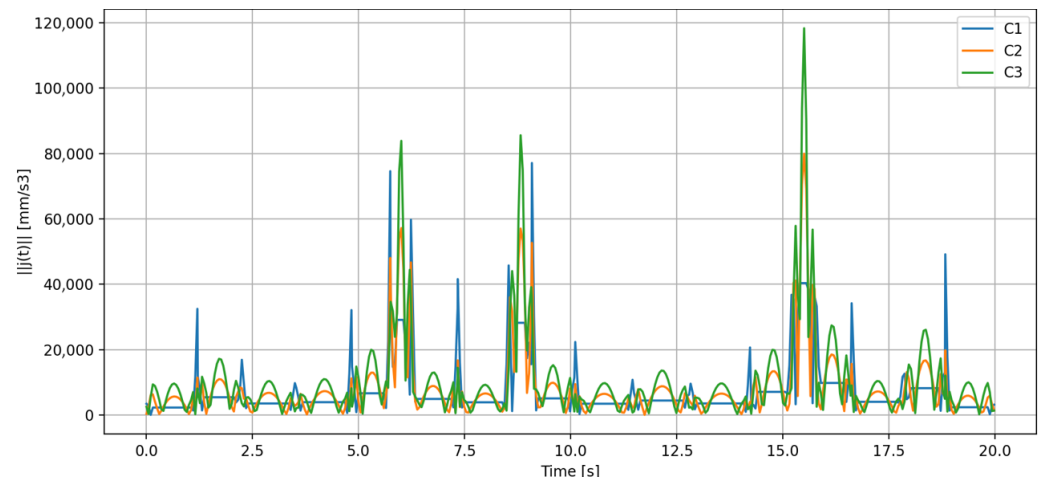


Figure 33. Jerk profile, trajectory 3, $\Delta t = 0.05$ s.

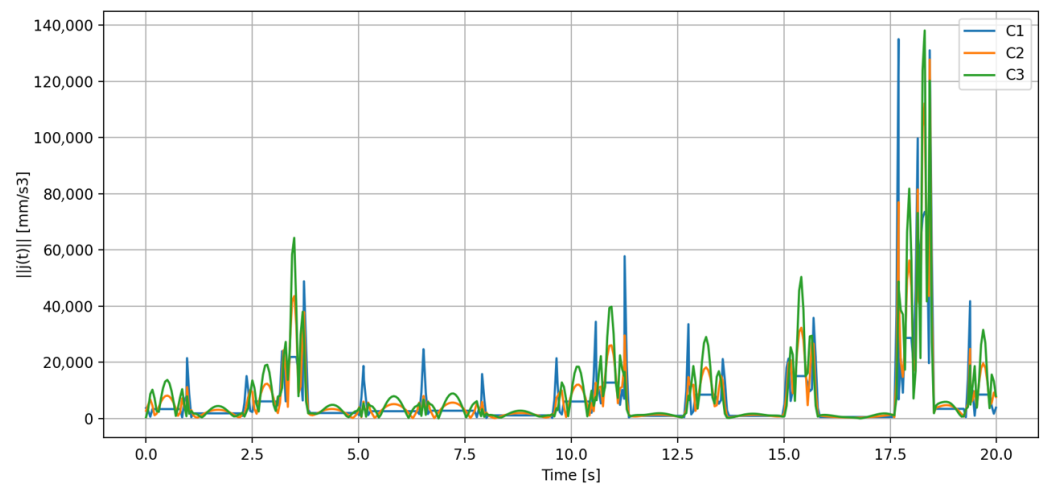


Figure 34. Jerk profile, trajectory 4, $\Delta t = 0.05$ s.

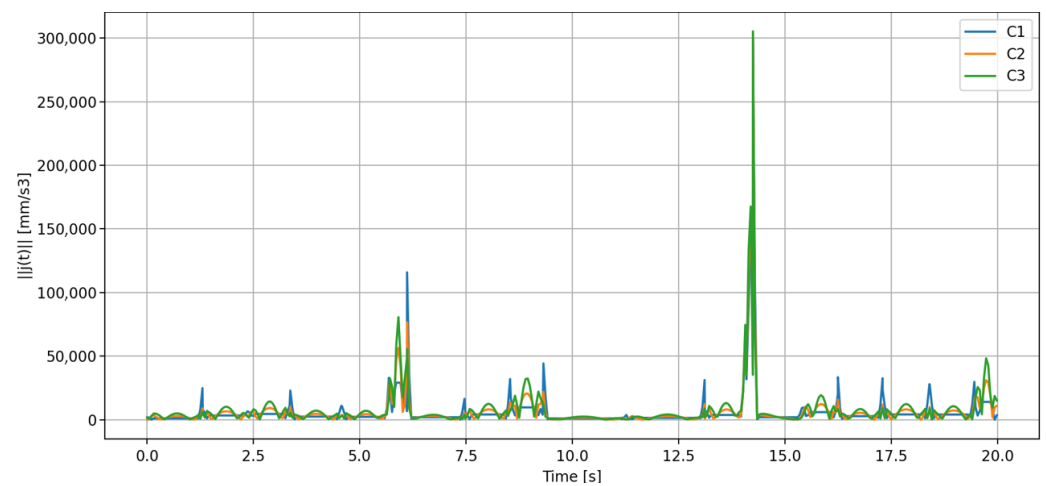


Figure 35. Jerk profile, trajectory 5, $\Delta t = 0.05$ s.

Figure 36 shows the statistical analysis graph (mean $\pm$ standard deviation).

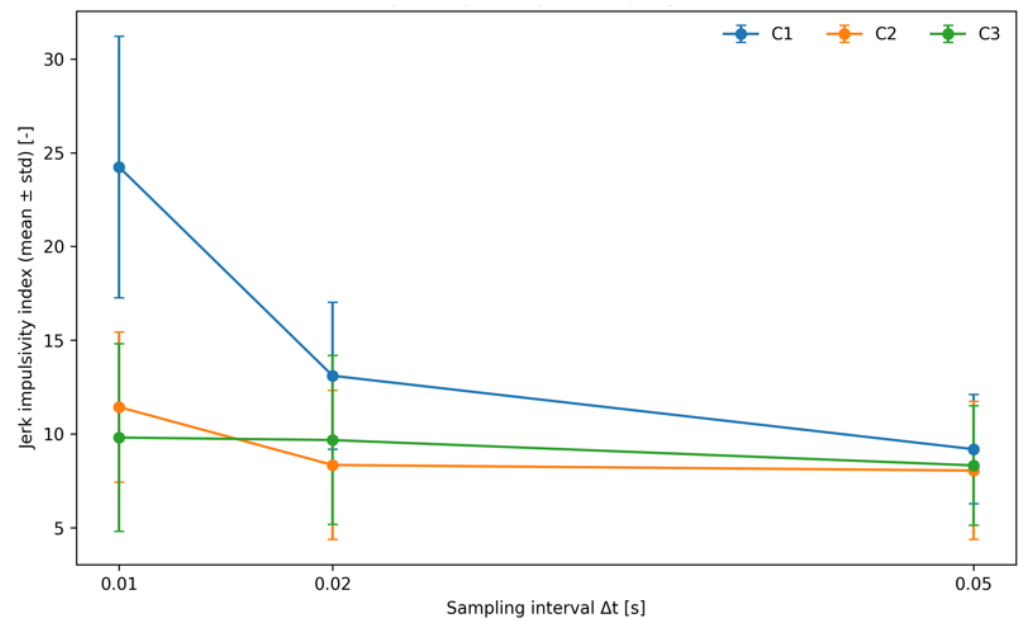


Figure 36. Mean jerk impulsivity vs. sampling interval.

4. Discussion

The results obtained in this study provide a perspective on the influence of geometric continuity on the kinematic behavior of industrial robot trajectories. By isolating the continuity level as the only variable parameter, keeping the trajectory geometry, execution duration, and boundary conditions identical, the observed differences can be attributed exclusively to the continuity order.

The analysis of the global metrics (acceleration energy and jerk energy) indicates that increasing the continuity order systematically leads to increasing integrated energy values. At first glance, this result could suggest that C^1 trajectories are more energy efficient. However, the analysis of the local discontinuity metrics (max. Δa and max. Δj) and the jerk impulsivity indicator (I_j) highlights an essential complementary aspect: C^1 trajectories show pronounced local variations and an impulsive behavior that is masked by the global integration.

The combined interpretation of the global and local metrics demonstrates that the C^2 level represents an optimal structural compromise. It eliminates acceleration discontinuities, significantly reduces jerk impulsivity, and ensures a more uniform distribution of kinematic variations. Although the continuity of C^3 guarantees the mathematical continuity of the jerk, the additional benefits over C^2 are limited in relation to the increase in integrated energy and the complexity of interpolation.

Sensitivity analysis with respect to the sampling interval ($\Delta t = 0.01$ s, 0.02 s, 0.05 s) shows that even if numerical values are influenced by the discretization, the comparative hierarchy between C^1 , C^2 , and C^3 remains unchanged. This consistency strengthens the validity of the conclusions and demonstrates that the results are not an artifact of a particular numerical resolution.

Overall, the results highlight that global energy metrics are not sufficient to fully characterize the smoothness of the motion. Only by correlating the integrated measures with local indicators of discontinuity and impulsivity can a correct interpretation of the kinematic behavior of the trajectories be made.

Limitations

First, the analysis was performed exclusively in TCP space. This approach allows the generalization of the results for different robotic platforms but does not directly capture the effects in the joint space, such as motor torques, inertias, structural flexibility, or the influence of control loops.

Second, the study is based on offline simulation (RoboDK) and numerical post-processing. Although the methodological framework is controlled and reproducible, experimental validation on a real industrial robot was not included in the current version. Under real conditions, factors such as filtering in the controller, servo dynamics, mechanical compliance, or vibration couplings can influence the practical manifestation of jerk discontinuities.

Third, the imposed execution duration (20 s) and the identical temporal parameterization were chosen to ensure a rigorous comparative framework. Different optimal timing strategies or dynamic constraints may lead to quantitative variations in the results, although the qualitative comparative trends are expected to remain similar.

Last but not least, the analysis was limited to three continuity levels (C^1 , C^2 , and C^3). Extension to higher-order continuities or to other smoothing methods could be relevant directions for future studies.

Future work will include experimental validation on a real industrial robot, correlation with dynamic quantities (torques and vibrations), and extension of the analysis to dynamic performance indicators.

5. Conclusions

In this paper, an analysis of the influence of geometric continuity on the kinematic behavior of industrial robots' trajectories was performed. Unlike most existing studies in the literature, where continuity is analyzed implicitly or in correlation with other parameters (geometry, execution time, and dynamic constraints), in the present work, continuity was explicitly isolated as an independent design parameter. The trajectory geometry, execution time, and boundary conditions were kept constant, allowing a correct and reproducible comparison between trajectories with C^1 , C^2 , and C^3 continuity.

The results obtained show that global energy metrics, such as acceleration energy and jerk energy, are not sufficient to assess the true degree of kinematic smoothness. Although C^1 trajectories can lead to lower energy values, this apparent advantage is associated with pronounced impulsive variations and local discontinuities, which are masked by the global energy integration. In contrast, C^2 and C^3 trajectories redistribute kinematic variations over time, eliminating local shocks and leading to a more stable structural behavior, even if this implies higher global energy values.

An important original contribution of the work is the introduction and use of discontinuity metrics ($\max \Delta a$ and $\max \Delta j$) and the impulsivity indicator of the jerk. These quantities allow a clear separation between the global and local behavior, providing a much more faithful interpretation of the effects of continuity on the trajectories. The analysis demonstrated that C^1 trajectories are characterized by discontinuities and impulsive jerk, while C^2 minimizes both discontinuities and impulsiveness, representing an optimal compromise between smoothness and efficiency. The continuity of C^3 ensures a continuous jerk from a mathematical point of view, but the additional benefits over C^2 are limited in relation to the energy cost and increased complexity.

The study also highlighted the sensitivity of all metrics to trajectory geometry, demonstrating that trajectories with more aggressive geometric variations can generate high kinematic loads regardless of the chosen level of continuity. This result highlights the need

for a combined assessment, which takes into account both geometry and continuity in trajectory planning processes.

Through the proposed approach, the paper provides a mathematical and practical framework that can be directly used in offline programming and simulation applications for trajectory analysis. The introduced indicators can be integrated into algorithms for automatic evaluation or optimization of trajectories, contributing to the reduction in vibrations and mechanical wear and the improvement of precision in high-speed or high-precision industrial applications. Future studies will aim at extending the analysis to the joint space, the correlation with dynamic quantities (torques and vibrations), and the experimental validation on real robots.

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