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Technologies for Increasing the Control Efficiency of Small Spacecraft with Solar Panels by Taking into Account Temperature Shock

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Abstract: The problem of the effective control of a small spacecraft is very relevant for solving a number of target tasks. Such tasks include, for example, remote sensing of the Earth or the implementation of gravity-sensitive processes. Therefore, it is necessary to develop new technologies for controlling small spacecraft. These technologies must take into account a number of disturbing factors that have not been taken into account previously. Temperature shock is one such factor for small spacecraft with solar panels. Therefore, the goal of the work is to create a new technology for controlling a small spacecraft based on a mathematical model of the stressed/deformed state of a solar panel during a temperature shock. The main methods for solving the problem are mathematical methods for solving initial/boundary value problems, in particular, the initial/boundary value problem of the third kind. As a result, an approximate solution for the deformation of a solar panel during a temperature shock was obtained. This solution is more general than those obtained previously. In particular, it satisfies the symmetrical condition of the solar panel. This could not be achieved by the previous solutions. We also observe an improvement (as compared to the previous solutions) in the fulfillment of the boundary conditions for the whole duration of the temperature shock. Based on this, a new technology for controlling a small spacecraft was created and its effectiveness was demonstrated. Application of the developed technology will improve the performance of the target tasks such as remote sensing of the Earth or the implementation of gravity-sensitive processes.

Keywords: motion control technology; temperature shock; small spacecraft; solar panel



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1. Introduction

Spacecraft control technologies are constantly being improved. This is mainly due to two important processes. The first is the process of expanding the areas of application of space technology. Spacecraft are faced with ever more complex tasks as a result of this process. These tasks necessitate the improvement of control technologies. For example, the first interplanetary spacecraft Luna-1 was launched on 2 January 1959 and did not reach the surface of the Moon. This was because the time required for the command signal to travel from the ground control station to the spacecraft was not accounted for [1]. At that time, the distance to the spacecraft was less than 400,000 km. The New Horizons spacecraft was launched to the planet Pluto 47 years later, on 19 January 2006. Its orbit correction was successfully completed on 2 December 2018. At that time, the distance to the spacecraft was more than 40 AU [2]. This fact clearly demonstrates the powerful development of spacecraft motion control technologies.

The second process, inextricably linked to the first, is the development and improvement of space technology itself. The trend in the miniaturization of space technology has led to the emergence and rapid development of small spacecraft. They are becoming increasingly popular due to their low cost, as well as the short time it takes them to implement a project compared to a larger spacecraft [3,4]. These are the two most important advantages of small spacecraft, thanks to which their development will continue in the future. Other specific advantages ensure the improved performance of individual target tasks in addition to general advantages. For example, the design of a small spacecraft and the technology for controlling its motion can be developed by taking into account the main features of a specific gravity-sensitive process carried out onboard [5]. In this case, the principle of individuality is only applicable to a small spacecraft and cannot be applied in the case of multifunctional spacecraft of other classes [6].

The combination of these two processes leads to the fact that some control technologies are unsuitable for small spacecraft. For example, a small spacecraft has a much denser layout compared to other spacecraft. This leads to the target and supporting equipment affecting magnetic measuring instruments [7,8]. Therefore, new control technologies are being developed based on magnetic measuring instruments that take this influence into account [9,10]. The vibrations of solar panels caused by various disturbing factors, for example, temperature shock [11,12], disrupt the favorable conditions for the implementation of gravity-sensitive processes [13,14]. This factor should be considered when developing new technologies for controlling the motion of a small spacecraft.

The current state of the problem is that images of the Earth's surface are of limited resolution [15,16]. This is due to a lack of appropriate technology for controlling the motion of small spacecraft. This can partially be solved by ground processing of telemetry information, for example, using artificial neural networks [17,18]. However, the possibilities of improving the quality of images by improving small spacecraft control technologies are far from exhausted.

Creating conditions for micro-accelerations for the realization of gravity-sensitive processes onboard a small spacecraft is a more difficult task than for other larger spacecraft in the field of space technology [19,20]. The fact is that the proportion of flexible elements for a small spacecraft is significantly higher. That is why the development of new control technologies is required to effectively solve the target tasks. These technologies should be incorporated into a prospective spacecraft during the design stage. Similar ideas are discussed in the works [14,21].

The aim of this work is to develop a new technology for controlling the motion of a small spacecraft by taking into account the influence of temperature shock. The work presents a model of the stressed/deformed state of a solar panel as the result of a temperature shock to achieve this goal. This paper derives approximate dependencies for the components of the displacement vector of points on the solar panel as the result of a temperature shock. These dependencies refine previously ones. This increases the accuracy of estimating disturbances in the motion of a small spacecraft due to temperature shock. An approximate solution is obtained for the components of the displacement vector of points on the solar panel from a temperature shock. This is more general compared to previously presented solutions. This makes it possible to develop a control law to reduce the impact of temperature shock. The increase in stabilization accuracy is shown when performing Earth remote sensing tasks using the example of the Aist-D small spacecraft. Previous estimates made it possible to implement a rotation speed of up to 0.00015 deg/s. The proposed method makes it possible to reduce this value to 0.0001 deg/s. This improves the performance of the spacecraft's target tasks. First of all, this refers to tasks of remote sensing of the Earth and the implementation of gravity-sensitive processes.

2. Materials and Methods

A small spacecraft for remote sensing of the Earth [15,16] or a small spacecraft for technological purposes [5,6] is considered as the object of this research. A feature of this

spacecraft is the presence of solar panels [11,12]. The influence of temperature shock is negligible for spacecraft without solar panels [8,19]. In this case, the construction of a new control technology is not required.

A temperature shock occurs as the result of the small spacecraft being plunged into the Earth's shadow or when it exits the Earth's shadow [11]. In this case, the heat flux of solar radiation appears or disappears in a short period of time. It is approximately equal to 1400 W/m^2 in Earth orbit [11]. A temperature shock causes a sudden change in the stressed/deformed state of the solar panel and excites thermal vibrations [20,22]. It significantly affects the dynamics of the rotational motion of the small spacecraft around the center of mass [23,24] and requires improvement in the technology for controlling the small spacecraft. The need to improve control technologies was especially clearly demonstrated during experiments with the promising ROSA solar panel [25,26]. Thermal vibrations were so intense that they did not make it possible for ROSA to be rolled up at the end of the experiment. At that time, ROSA was successfully applied to the DART small spacecraft [27]. ROSA has good prospects for its further use due to its significantly lower mass compared to classical solar panels [28].

The technology for controlling the motion of a small spacecraft can be attributed to mechatronic technologies [29]. These are information technologies for motion control. They imply the implementation using information technologies of complex laws of motion which, for one reason or other, could not be implemented using traditional technologies earlier [30].

The control technology includes the following positions:

1. Situation analysis (analysis of disturbing factors, analysis of motion parameters and their evolution, analysis of the capabilities of the motion control system, and analysis of the capabilities of the information and measurement control system).
2. Information preparation (quantitative assessment of factors affecting control, assessment of the need and feasibility of control, and formation of the target values of motion parameters).
3. Formation of control laws.
4. Implementation of control by actuators of the motion control system.
5. Supervision of control efficiency using an information and measurement control system.

These positions define control technology in a broad sense. They imply the formation of control laws (control technology in the narrow sense) and mechanisms of its implementation (they include a wider range of issues). Implementation mechanisms take into account the influence of disturbing factors on related processes (data transfer [31], functioning of devices [32], etc.).

At that time, the capabilities of the motion control system and the information and measurement control system are often quite high. Therefore, the quality of control depends on the accuracy of assessing the disturbing factors and the motion parameters of the spacecraft. Therefore, the accuracy of three-axis stabilization of the Aist-2D small spacecraft in the orbital coordinate system was 0.0002 deg/s (in angular velocity) and 0.004 deg (in angle) for successful solution of remote sensing of the Earth [16]. It is necessary to adequately assess its inertial and mass parameters for successful removal of space debris [33,34].

Many authors (for example, [35–40]) note that it is a temperature shock that significantly reduces the performance of the target tasks of remote sensing of the Earth. Therefore, this work analyzes the stressed/deformed state of the solar panel during the temperature shock in order to improve the technology for controlling the motion of the spacecraft around its center of mass.

The one-dimensional heat conduction problem was solved in the work [41]. This case is the most dangerous. It is characterized by the maximum influence of a temperature shock on the rotational motion of the spacecraft around the center of mass. However, it is not enough for building the effective control technology. Rather, it serves as an assessment of the need and feasibility by taking into account a temperature shock.

The two-dimensional heat conduction problem was solved with some limitations in the work [42]. It was assumed that half of the plate ($l/2 < x \leq l$) moved freely during a temperature shock. The other half of the plate ($0 \leq x \leq l/2$) was in equilibrium. It resulted in additional internal forces in the x -axis direction (Figure 1). This representation of the plate deformation was discussed in [23] and made it easier to obtain an approximate solution for the components of the point displacement vector. However, the boundary conditions were satisfied only at small values of time after a temperature shock. It will not always lead to a quantitative assessment of the factors affecting control with the required accuracy using the approximate dependencies obtained in [42]. Therefore, it is necessary to remove these limitations when improving the technology for controlling a small spacecraft. This paper will consider a more general case. It will increase the accuracy of quantitative assessment for the components of the displacement vector during a temperature shock.

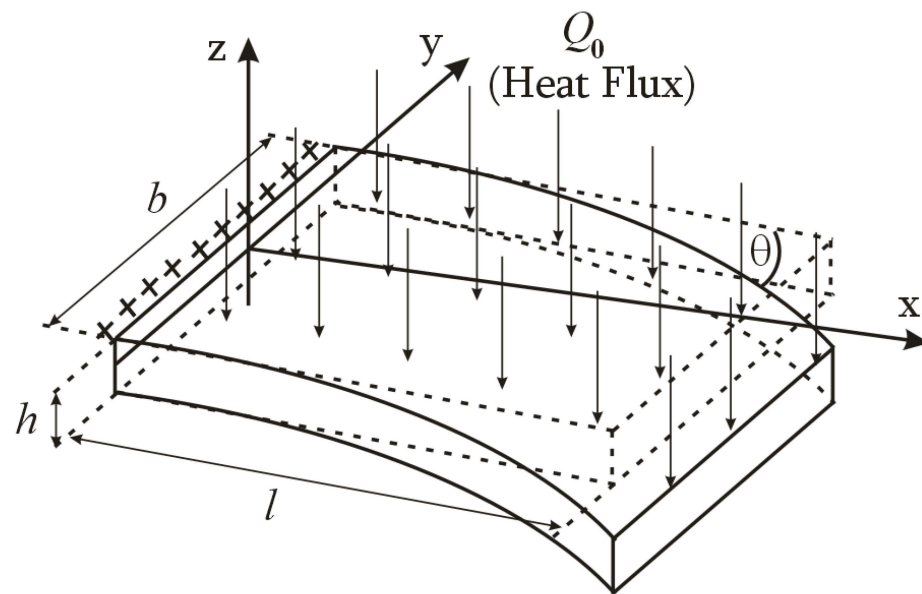


Figure 1. Temperature shock calculation scheme.

The approximate methods for solving initial/boundary value problems are used to solve the problem. They are generally accepted and well known (for example, [43–45]). Numerical calculations are carried out in the high-level programming language Wolfram Mathematica.

3. Mathematical Formulation of the Problem and Its Approximate Solution

3.1. Analysis of the Problem and Simplifications

In real conditions, a spacecraft while moving in its orbit regularly finds itself in the Earth's shadow and exits it [46,47]. If the spacecraft has large elastic elements then they deform under the influence of a temperature shock. Large elastic elements are most often solar panels. Their temperature deformations can significantly affect the dynamics of the rotational motion of the spacecraft and the technology for controlling this motion.

In real conditions, solar panels oscillate under the influence of various disturbing factors [48,49]. The spacecraft exits the Earth's shadow with an arbitrary value of the angle between the normal to the surface of a solar panel and the direction to the Sun. However, the following facts must be taken into account when mathematically setting up the problem.

1. A large deviation of the normal from the direction to the Sun is not allowed when orienting a solar panel relative to the Sun. Otherwise, the efficiency of generating electrical energy using solar panels will be low [50,51].
2. The characteristic time of a temperature shock is significantly lower than the period of oscillations of solar panels. This allows the use of a static formulation of the

initial/boundary value problem for heat conduction [42,52]. The solar panel over a short period of time (the characteristic time of a temperature shock) moves only under the influence of a temperature shock within the framework of such a formulation. In this case, the oscillations are taken into account through the initial deflection. The shape of the solar panel at the moment of a temperature shock is curved and not flat as in the one-dimensional heat conduction problem [41].

3. The solar panel can be represented as a rectangular homogeneous plate as a first approximation. It is rigidly fixed at one edge. Its three other edges are free. This simplification is often used in the literature [11,12,22,24,25]. It does not introduce significant errors in the assessment of disturbances from solar panel vibrations or a temperature shock.

Thus, let us adopt the calculation scheme (Figure 1). The assumptions used in this paper to derive the equations of the mathematical model are described in detail in [42].

The appearance of the initial deflection (angle θ in Figure 1) makes it possible to apply the one-dimensional heat conduction problem. The fixed and free edges of the plate will be heated unequally because of the deflection.

3.2. Analysis of Two-Dimensional Heat Conduction Problem

We will obtain a more general solution of approximate dependencies for the components of the displacement vector of points for the solar panel as the result of temperature deformations.

The third initial/boundary value problem of heat conduction can be represented as follows [42]:

$$\left\{ \begin{array}{l} \frac{\partial T(x, z, t)}{\partial t} = a^2 \left(\frac{\partial^2 T(x, z, t)}{\partial x^2} + \frac{\partial^2 T(x, z, t)}{\partial z^2} \right); \\ \left(\lambda \frac{\partial T(x, h, t)}{\partial n} \right) = Q_0 \cdot \cos(\theta(x, t)) - e\Theta(T^4(x, h, t) - T_C^4), \quad z = h; \\ \left(\lambda \frac{\partial T(x, 0, t)}{\partial n} \right) = -e\Theta(T^4(x, 0, t) - T_C^4), \quad z = 0; \\ T(x, z, 0) = T_0 = \text{const.} \end{array} \right. \quad (1)$$

The range of variables in the system of Equation (1) was assumed to be as follows: $0 \leq x \leq l$, $0 \leq y \leq \frac{b}{2}$, $0 \leq z \leq h$, $t > 0$. If there are no additional constraints in the equation then, further, these ranges are assumed to be the same.

The first equation is a two-dimensional heat conduction equation in the system of Equation (1). Here $T = T(x, z, t)$ is a three-dimensional function for temperatures of points of the solar panel depending on two spatial coordinates x and z (Figure 1) and time coordinate t ; a is the coefficient for thermal diffusivity of the material of the solar panel; l and h are the length and thickness of the solar panel, respectively.

The second Equation (1) represents the boundary condition on the upper surface ($z = h$) of the solar panel where the solar radiation flux falls (heat flux in Figure 1) (λ is the thermal conductivity; n is the outward normal to the element of the surface of the solar panel; Q_0 is the maximum value of the incident solar flux (1400 W/m^2); $\theta(x, t)$ is the function for the angles of rotation of the tangent to the curved surface of the solar panel, depending on the longitudinal coordinate x and time t ; e is the blackness of the surface of the solar panel; Θ is the Stefan–Boltzmann constant; T_C is the temperature of near-Earth space).

The third Equation (1) represents the boundary condition on the lower surface ($z = 0$) of the solar panel where the solar radiation flux does not enter.

The fourth Equation (1) determines the initial temperature distribution of the solar panel. This distribution is taken to be uniform (T_0 is the initial temperature of the panel, the same at all its points) within the framework of this work.

If we neglect the magnitude of the temperature deformations compared to the initial deflection of the solar panel, then the second Equation (1) can be rewritten as follows:

$$\left(\lambda \frac{\partial T(x, h, t)}{\partial n} \right) = Q_0 \cdot \cos \left(\frac{du_{z0}(x, 0)}{dx} \right) - e\Theta \left(T^4(x, h, t) - T_C^4 \right), z = h \quad (2)$$

where $u_{z0} = u_{z0}(x, 0)$ is the function for the initial deflection of the solar panel, which depends on the longitudinal coordinate x ($\theta(x, t) = \frac{\partial(u_{z0}(x, 0) + u_z(x, t))}{\partial x} \approx \frac{du_{z0}(x, 0)}{dx}$).

3.3. Analysis of the Thermoelasticity Problem

The dynamics for deflections of points of the solar panel $u_z = u_z(x, t)$ is determined by the Sophie Germain equation:

$$D \frac{\partial^4 u_z(x, t)}{\partial x^4} + \rho h \frac{\partial^2 u_z(x, t)}{\partial t^2} = -2\mu\alpha \int_0^h \left[2 \frac{\partial T(x, z, t)}{\partial z} + z \frac{\partial^2 T(x, z, t)}{\partial z^2} \right] dz + \frac{\partial^2 u_{z0}(x, 0)}{\partial x^2} \sigma_{xz} \quad (3)$$

where D is the cylindrical stiffness of the solar panel in bending; ρ is the density of the material of the solar panel; μ is the Lamé coefficient; α is the coefficient of linear expansion; σ_{xz} is the shear stress from the initial deflection.

We write the equilibrium equation after a temperature shock in vector form to determine other component of the displacement vector $u_y = u_y(x, y, t)$:

$$\frac{3(1-\nu)}{1+\nu} \text{grad div } \overset{\circ}{u} - \frac{3(1-2\nu)}{2(1+\nu)} \text{rot rot } \overset{\circ}{u} = \alpha \text{ grad } T, t = \infty \quad (4)$$

where $t = \infty$ means a large time interval after a temperature shock during which the temperature deformations will be negligible; $\overset{\circ}{u} = (0, u_y, u_z)$ is the displacement vector of the points of the solar panel; ν is Poisson's ratio.

It can be rewritten as follows in scalar Formula (4):

$$\begin{cases} \frac{\partial^2 u_y(x, y, t)}{\partial x \partial y} = \alpha \frac{\partial T(x, z, t)}{\partial x}, z = \frac{h}{2}, t = \infty; \\ \frac{3(1-2\nu)}{2(1+\nu)} \frac{\partial^2 u_y(x, y, t)}{\partial x^2} = 0, t = \infty; \\ \frac{3(1-2\nu)}{2(1+\nu)} \frac{\partial^2 u_z(x, t)}{\partial x^2} = \alpha \frac{\partial T(x, z, t)}{\partial z}, t = \infty. \end{cases} \quad (5)$$

where b is the width of the solar panel.

This is due to the symmetry of the stressed/deformed state of the solar panel relative to the x -axis; only one half of it is considered $0 \leq y \leq \frac{b}{2}$.

In this case, the first equation of system (5) is satisfied only in the middle surface of the solar panel ($z = h/2$) since the left side of the first equation of system (5) does not depend on z . However, such a formulation turns out to be sufficient due to the small thickness of the solar panel.

The boundary conditions are the geometric conditions of rigid fixing:

$$\begin{cases} u_z(0, t) = 0, x = 0; \\ \frac{\partial u_z(x, t)}{\partial x} = 0, x = 0. \end{cases} \quad (6)$$

$$\begin{cases} u_y(0, y, t) = 0, x = 0; \\ \frac{\partial u_y(x, y, t)}{\partial x} = 0, x = 0. \end{cases} \quad (7)$$

It is also static conditions of the free edge of the solar panel:

$$\begin{cases} \frac{\partial^2 u_z(x, t)}{\partial x^2} = 0, x = l; \\ \frac{\partial^3 u_z(x, t)}{\partial x^3} = 0, x = l. \end{cases} \quad (8)$$

$$\begin{cases} \frac{\partial^2 u_y(x, y, t)}{\partial x^2} = 0, & x = l; \\ \frac{\partial^3 u_y(x, y, t)}{\partial x^3} = 0, & x = l. \end{cases} \quad (9)$$

These results are described in detail in [42]. The dynamic equation was obtained for the change in the component of the displacement vector $u_y = u_y(x, y, t)$ using the D'Alembert principle:

$$\frac{3(1-2\nu)}{2(1+\nu)} \frac{\partial^3 u_y(x, y, t)}{\partial x^3} + \frac{6\rho(1-\nu^2)}{Eh^2} \frac{\partial^2 u_y(x, y, t)}{\partial t^2} b = \frac{12(1-\nu^2)}{h^3} \alpha \int_0^h [T(x, z, t) - T_0] dz \quad (10)$$

where E is the Young's modulus.

3.4. Obtaining an Approximate Solution of the Thermoelasticity Problem

The right side of this equation in the general case does not correspond to the free edge of the solar panel. However, it was possible to achieve obtaining an approximate dependence for the component of the displacement vector $u_y = u_y(x, y, t)$ [42] in the form:

$$u_y(x, y, t) = \alpha \left[-Mxy + \frac{C_1 h E}{b} \frac{1}{\rho} \left\{ \frac{t^2}{2} - a(t + \beta)[\ln(t + \beta) - 1] \right\} \right] + C_2 t, \quad x \approx 0. \quad (11)$$

where M , C_1 , C_2 , and β are some positive constants which are parameters and depend on the properties of the material of the solar panel.

Let us remove this limitation on the right side and obtain a more general solution. It will make it possible to modernize the technology for controlling a spacecraft during a temperature shock. We present it in the following form since Equation (10) must correspond to the free edge:

$$\frac{3(1-2\nu)}{2(1+\nu)} \frac{\partial^3 u_y(x, y, t)}{\partial x^3} + \frac{6\rho(1-\nu^2)}{Eh^2} \frac{\partial^2 u_y(x, y, t)}{\partial t^2} b = 0 \quad (12)$$

It should be taken into account that the first equation of system (5) requires a linear dependence for the function $u_y = u_y(x, y, t)$ on the coordinate y when forming an approximate solution. Otherwise, the temperature on the right side of the first equation of system (5) must also depend on y . It is possible only within the framework of a three-dimensional heat conduction problem. The two-dimensional formulation used in this work enables us to limit ourselves only to a linear dependence of $u_y = u_y(x, y, t)$ on y . We should replace the expansion used in [42] $u_y = u_y(x, y, t)$ by taking into account (12):

$$u_y(x, y, t) = u_{1y}(x) y + u_{2y}(t) \quad (13)$$

We propose to use the following expression:

$$u_y(x, y, t) = u_{1y}(x, t) y \quad (14)$$

This structure is more complex than (13). It contains a function of two variables. Let us determine the partial derivatives of $u_y = u_y(x, y, t)$ by taking into account the expansion (14):

$$\frac{\partial^2 u_y(x, y, t)}{\partial x \partial y} = \frac{\partial u_{1y}(x, t)}{\partial x} \quad (15)$$

$$\frac{\partial^3 u_y(x, y, t)}{\partial x^3} = y \frac{\partial^3 u_{1y}(x, t)}{\partial x^3} \quad (16)$$

$$\frac{\partial^2 u_y(x, y, t)}{\partial t^2} = y \frac{\partial^2 u_{1y}(x, t)}{\partial t^2} \quad (17)$$

This is according to the first equation of system (5):

$$\frac{\partial u_{1y}(x, t)}{\partial x} = \alpha \frac{\partial T(x, z, t)}{\partial x} \quad (18)$$

Let us differentiate (18) twice with respect to x :

$$\frac{\partial^3 u_{1y}(x, t)}{\partial x^3} = \alpha \frac{\partial^3 T(x, z, t)}{\partial x^3} \quad (19)$$

The approximation of the temperature proposed in [42] assumes its linear dependence on the coordinate x :

$$T(x, z, t) = C_1 z \frac{t}{t + \beta} - Mx + T_0 \quad (20)$$

However, now, its substitution into (19) will give a zero value for the third derivative $u_y(x, y, t)$. Then, it will be equal to zero and the acceleration $\frac{\partial^2 u_y(x, y, t)}{\partial t^2}$ according to (12). This means the rest of the solar panel is in contradiction. This problem was solved due to the right side of Equation (10) in the formulation [40]. However, the right side is missing in a more general setting. Therefore, it will be necessary to adjust the approximation (20). It should be understood that this approximation converges well with the data of the computational experiment [41,42,52]. Therefore, the following approximation is proposed instead of (20) in this work:

$$T(x, z, t) = C_1 z \frac{t}{t + \beta} - \frac{M}{\eta} \sin \eta x + T_0 \quad (21)$$

There is for a sufficiently small parameter η : $\sin \eta x \approx \eta x$. Then, dependence (21) goes into (20). On the other hand, we have $\frac{\partial^3 T(x, z, t)}{\partial x^3} \neq 0$ now. Thus, it was possible to eliminate the problem not by artificially introducing the right side [42] but by improving the structure of the dependence approximating the temperature field of the solar panel.

Then, we obtain, according to (19) by taking into account the expansion (21):

$$\frac{\partial^3 u_{1y}(x, t)}{\partial x^3} = \alpha M \eta^2 \cos \eta x \quad (22)$$

Further, we transform Equation (12) by taking into account (17) and (22).

$$\frac{3(1-2\nu)}{2(1+\nu)} \alpha M \eta^2 y \cos \eta x + \frac{6\rho(1-\nu^2)}{Eh^2} \frac{\partial^2 u_{1y}(x, t)}{\partial t^2} by = 0 \quad (23)$$

Then, the acceleration for the points of the solar panel in the direction of the y -axis is

$$\frac{\partial^2 u_{1y}(x, t)}{\partial t^2} = -\frac{Eh^2(1-2\nu)\alpha M \eta^2}{4\rho(1-\nu^2)(1+\nu)b} \cos \eta x \quad (24)$$

We obtain the component of the displacement vector after integration over time:

$$u_{1y}(x, t) = -\frac{Eh^2(1-2\nu)\alpha M \eta^2}{8\rho(1-\nu^2)(1+\nu)b} \cos \eta x t^2 + C_3(x)t + C_4(x) \quad (25)$$

In the general case, $C_3(x)$ and $C_4(x)$ are functions that depend on x which are determined using the boundary conditions (7) and (9), and the initial condition of the absence of deformation along the y -axis at the initial moment of time. Then, we have according to the expansion (14):

$$u_y(x, y, t) = \left[-\frac{Eh^2(1-2\nu)\alpha M \eta^2}{8\rho(1-\nu^2)(1+\nu)b} \cos \eta x t^2 + C_3(x)t + C_4(x) \right] y \quad (26)$$

We suppose that $C_3 = \text{const}$ and determine $C_4(x)$. We find the mixed derivative to perform this:

$$\frac{\partial^2 u_y(x, y, t)}{\partial x \partial y} = \frac{Eh^2(1-2\nu)\alpha M\eta^3}{8\rho(1-\nu^2)(1+\nu)b} \sin \eta x t^2 + \frac{\partial C_4(x)}{\partial x} \quad (27)$$

We neglect the first term on the right side of (27) due to the smallness of η^3 and substitute the right side of (18) by taking into account the approximation of the temperature field (21):

$$\frac{\partial C_4(x)}{\partial x} = -\alpha M \cos \eta x \quad (28)$$

We obtain the function $C_2(x)$ integrating (28):

$$C_4(x) = -\alpha \frac{M}{\eta} \sin \eta x + C_5 \quad (29)$$

Then expression (29) takes the form:

$$u_y(x, y, t) = \left[-\frac{Eh^2(1-2\nu)\alpha M\eta^2}{8\rho(1-\nu^2)(1+\nu)b} \cos \eta x t^2 + C_3 t - \alpha M \sin \eta x + C_5 \right] y \quad (30)$$

It is necessary to set $C_5 = 0$ to achieve exact satisfaction of the initial condition at $t = 0$. Then it (30) will take the following form:

$$u_y(x, y, t) = \left[-\frac{Eh^2(1-2\nu)\alpha M\eta^2}{8\rho(1-\nu^2)(1+\nu)b} \cos \eta x t^2 + C_3 t - \alpha M \sin \eta x \right] y \quad (31)$$

The approximate dependence (31) differs from the dependence (11). Moreover, it was emphasized that (11) well describes the displacements of points only near the clamped edge of the solar panel ($x \approx 0$). However, it may not be enough to develop the effective technology for controlling the rotational motion of a small spacecraft. Expression (31) is free from this limitation. We managed to obtain an approximation for $u_y(x, y, t)$ for the entire range of variation of the coordinate x ($0 \leq x \leq l$) in this work. This important result will be investigated in a numerical example in the following sections of this work.

4. Results of Numerical Simulation

The parameters presented in Table 1 were chosen for numerical simulation corresponding to the parameters of the ROSA solar panel [25,53]. The calculations were carried out in the Wolfram Mathematica programming language.

Figure 2 shows the dependences for $u_y(x, y, t)$ using approximations (31) and (11). They correspond to the values of the longitudinal coordinate $x = l/5$.

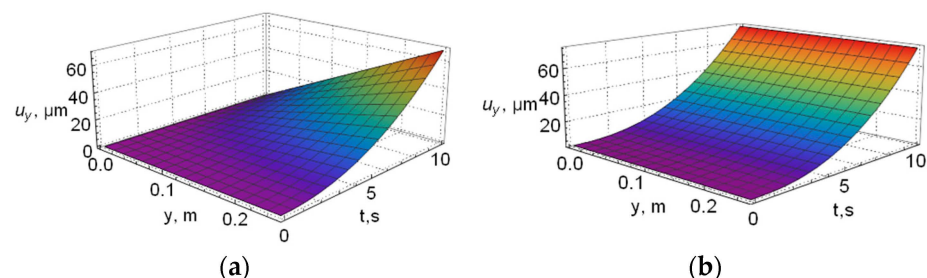
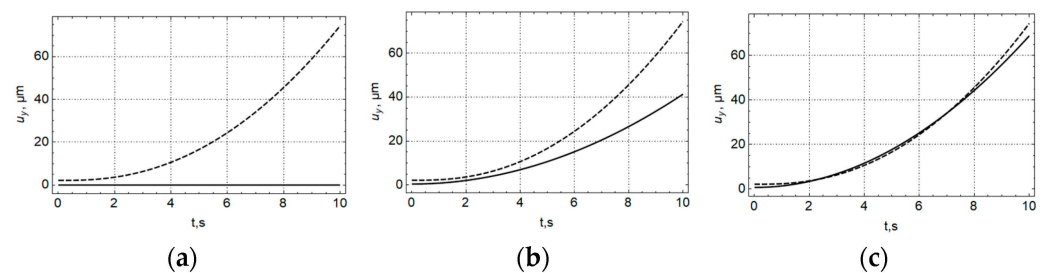


Figure 2. There is dependency $u_y(x = \frac{l}{5}, y, t)$ on $\eta = 0.1$: (a) according to Formula (31), (b) according to Formula (11).

Table 1. The main parameters of the solar panel.

Parameter	Designation	Value	Dimension
Solar panel length	l	1	m
Solar panel width	b	0.5	m
Solar panel frame thickness	h	1	mm
Young's modulus	E	4×10^{10}	Pa
Lamé coefficient	μ	7.5×10^{11}	Pa
Cylindrical bending stiffness	D	3.66	N·m
Density	ρ	1780	kg/m ³
Coefficient of linear expansion	α	1.35×10^{-5}	K ⁻¹
Poisson's ratio	ν	0.3	–
Coefficient of thermal diffusivity	a	4.786×10^{-5}	m ² /s
Thermal conductivity coefficient	λ	16.3	W/(m·K)
Degree of blackness	e	0.2	–
Model parameters:	$C1$	200	K/m
	β	1	s
	M	3	K/m

Let us carry out a comparative analysis of the results of dependences (31) and (11). Figure 3 shows various two-dimensional projections of the three-dimensional dependence of Figure 2 at fixed values of longitudinal and transverse coordinates.

**Figure 3.** Projections of the dependencies presented in Figure 2 for different values of the y coordinate. The solid line is approximation by Formula (31). The dotted line is approximation by Formula (11).

$$(a) u_y(x = \frac{l}{5}, y = 0, t); (b) u_y(x = \frac{l}{5}, y = \frac{b}{4}, t); (c) u_y(x = \frac{l}{5}, y = \frac{b}{2}, t)$$

Joint analysis of Figures 2 and 3 shows that dependence (31) more adequately describes the deformation model of the plate. It consists of the stationarity of the center line (at $y = 0$). In other words, the symmetrical condition of the plate is fulfilled. It was not observed when using it (11). The displacements of the plate points in the areas $0 < y \leq \frac{b}{2}$ and $-\frac{b}{2} \leq y < 0$ have different signs. They must be equal to zero by virtue of continuity of deformations at $y = 0$. This can be clearly seen in Figure 3a.

Modeling of $u_y(x, y, t)$ was carried out using approximations (31) for different values of the parameter η . They are presented in Figure 4.

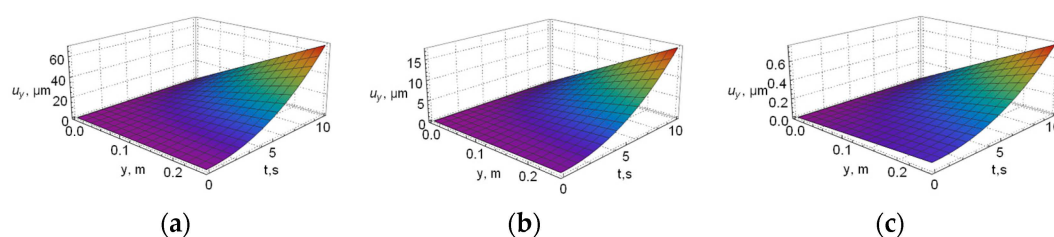
**Figure 4.** The dependence $u_y(x, y, t)$ obtained by Formula (31) for different values of the parameter η ($x = \frac{l}{5}$): (a) $\eta = 0.1$, (b) $\eta = 0.05$, and (c) $\eta = 0.01$.

Figure 5 shows two-dimensional projections of the dependencies presented in Figure 4 for the initial moment of time $t = 0$ for clarity of comparative analysis.

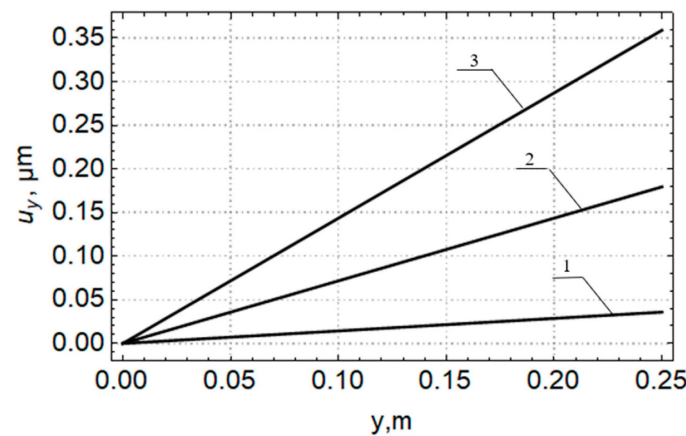


Figure 5. The dependence $u_y(x, y, t)$ obtained by Formula (31) at the initial moment of time ($t = 0$) for different values of the parameter η $x = \frac{l}{5}$: 1— $\eta = 0.1$, 2— $\eta = 0.05$, and 3— $\eta = 0.01$.

Figure 5 demonstrates the fulfillment of the initial condition of the modeled problem. It consists of the absence of displacements at the initial moment of time. The analysis of Figures 4 and 5 shows that the reduction of the parameter η leads to deterioration of the fulfillment of this condition.

Thus, it can be seen that there is a noticeable improvement in the adequacy for the description of the approximation $u_y(x, y, t)$ in this work compared to [42]. Dependence (31) has a wider application than (11). The analysis of the simulation results will be presented in the next section of the work.

The simulation conducted shows that, because of a temperature shock, points on the solar panels move. This movement causes a shift in the center of mass of both the solar panel and the small spacecraft as a whole. Such a shift deteriorates the accuracy of the small spacecraft orientation. It negatively affects the quality of its execution of the target tasks, for example, Earth remote sensing.

5. Discussion

The function $u_y(x, y, t)$ using approximation (11) is practically independent of y , as can be seen from Figure 2b. This coordinate is included in the first term on the right side of (11) enclosed in square brackets. At the same time, it is believed that (11) adequately describes the deformation when the coordinate x is small. It significantly reduces the influence of y on the structure of $u_y(x, y, t)$ and simplifies it. The influence of y on the function $u_y(x, y, t)$ is more obvious in Figure 2a. The structure of this function is more complex. Moreover, such a picture should be observed due to the symmetrical deformations in the direction of the y -axis. Points having a positive y coordinate (Figure 1) will move in the direction of the positive y -axis. Conversely, points having a negative y coordinate will move in the direction of the negative y -axis. Points lying on the x -axis (Figure 1) should not move in the direction of the y -axis. It is exactly what is observed in Figure 2a. It is also embedded in the structure of the expansion of $u_y(x, y, t)$ (14). Therefore, it can be argued that the obtained approximation (31), unlike the approximation (11), satisfies the conditions of the symmetrical deformation in the direction of the y -axis. It is clearly demonstrated in Figure 3a. Further, the difference in the description of $u_y(x, y, t)$ using approximations (31) and (11) decreases significantly (Figure 3b) with an increase in the coordinate y and approaching from the middle of the solar panel ($y = 0$) to its edge ($y = b/2$), and the results practically coincide (Figure 3c) for the edge of the solar panel ($y = b/2$). Consequently, the dependences (31) and (11) describe the function $u_y(x, y, t)$ with approximately the same accuracy near the edge of the solar panel. The accuracy of the approximation (11) is

significantly inferior to the approximation (31) obtained in this work (by more than 30% of the deformation value) closer to the middle of the solar panel.

Varying the parameter η showed that its decrease worsens the fulfillment of the initial conditions of the absence of deformations (Figures 3 and 4). At that time, the temperature approximation (21) approaches (20). In this work, the best consistency with the initial conditions was achieved at $\eta = 0.1$. Since $l = 1$ m (Table 1), then the maximum value of the sine: $\max_{0 \leq x \leq l} \{\sin \eta x\} = \sin \eta = \sin 0.1 \approx 0.0998$. Reducing η leads to a divergence with the initial conditions increasing. This makes the difference between the sine and its argument greater.

Finally, let us go back to spacecraft motion control technologies. The results obtained in this work relate to the analysis of the results and the preparation of information for the formation of control laws. This refers to the first two points of the spacecraft control technology described in the Materials and Methods section. More accurate models of disturbing factors affecting the motion of the spacecraft enable the creation of more effective laws for controlling the motion of the spacecraft. Therefore, the error in modeling $u_y(x, y, t)$ using the approximate dependence (11) enables the implementation of a control law that provides angular velocity values no higher than 0.00015 deg/s for the Aist-2D spacecraft [16,54,55]. This value has been validated during flight tests. The results are presented in [56]. It is enough to meet the orientation requirements. They were incorporated into the design requirements for the motion control system of the Aist-2D small spacecraft [23]. The error in modeling $u_y(x, y, t)$ using the approximate dependence (31) enables the implementation of a control law that provides angular velocity values no higher than 0.0001 deg/s. This is possible due to a more accurate mathematical model for the displacement of points of the solar panel during a temperature shock. First of all, it enables the satisfaction of the symmetrical conditions of the solar panel and the removal of restrictions on the magnitude of the coordinate x when satisfying the boundary conditions. This is precisely the advantage of the proposed method compared to existing methods. Since in the future the orientation requirements will only increase, such an increase in accuracy is quite relevant.

It is possible to complicate the form of the decomposition (14) in further studies. It will lead to a more cumbersome solution of the problem. However, it will increase the accuracy of the realized spacecraft motion control technology.

6. Conclusions

In conclusion, this paper discussed the technology for controlling a small spacecraft by taking into account a temperature shock of the solar panels. The need to improve the technology for controlling the motion of a small spacecraft was justified, for example, to solve the problems of Earth remote sensing. The method for increasing the efficiency of control technology was proposed. It more accurately describes the deformation of the points of the solar panels during a temperature shock compared to known methods. The proposed approximation of the components of the displacement vector (31) for the Aist-2D small spacecraft surpasses the known approximation (11) in terms of the accuracy of fulfilling the boundary conditions of the thermoelasticity problem (more than 30%). Due to this, it is possible to provide an angular velocity of up to 0.0001 deg/s (it was 0.00015 deg/s). Therefore, the application of the proposed method in the technology for controlling a small spacecraft will improve the quality of the images obtained due to a decrease in its angular velocity of rotation.

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