

Article

A Simplified Tantalum Oxide Memristor Model, Parameters Estimation and Application in Memory Crossbars [†]

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Abstract: In this paper, an improved and simplified modification of a tantalum oxide memristor model is presented. The proposed model is applied and analyzed in hybrid and passive memory crossbars in LTSPICE environment and is based on the standard Ta₂O₅ memristor model proposed by Hewlett–Packard. The discussed modified model has several main enhancements—inclusion of a simplified window function, improvement of its effectiveness by the use of a simple expression for the *i*–*v* relationship, and replacement of the classical Heaviside step function with a differentiable and flat step-like function. The optimal values of coefficients of the tantalum oxide memristor model are derived by comparison of experimental current–voltage relationships and by using a procedure for parameter estimation. A simplified LTSPICE library model, correspondent to the analyzed tantalum oxide memristor, is created in accordance with the considered mathematical model. The improved and altered Ta₂O₅ memristor model is tested and simulated in hybrid and passive memory crossbars for a state near to a hard-switching operation. After a comparison of several of the best existing memristor models, the main pros of the proposed memristor model are highlighted—its improved implementation, better operating rate, and good switching properties.

Keywords: memristor model; tantalum oxide; memory crossbar; nonlinear dopant drift; window function; LTSPICE library memristor model



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1. Introduction

The memristor elements, based mainly on amorphous transition metal oxides, as TiO₂, HfO₂, Ta₂O₅, ZnO and other materials, have many potential applications, as in nonvolatile memory devices, artificial neural networks, and logical and reconfigurable electronic circuits [1–4]. The memristor was predicted in 1971 by Chua as the fourth fundamental, passive, and nonlinear one-port electronic element [5]. It has a memory effect and can retain its state after switching off the electric sources [5,6]. The first physical prototype of a memristor, based on TiO₂, was proposed by Hewlett–Packard (HP) Research Team, supervised by Stanley Williams [6]. In the scientific literature, information for generated polymeric, ferroelectric, and other types of memristors has been published [7–10]. Along with the oxide materials with resistance switching abilities already analyzed, the amorphous Ta₂O₅, doped by oxygen vacancies, has excellent switching properties, a sound dependability, great operating speed, small energy consumption, comparatively extensive memorizing time, and a good compatibility with the commonly used Complementary Metal Oxide Semiconductor (CMOS)-integrated circuits technology [11–14]. Due to this reason and to the increased interest of engineers and scientists to tantalum oxide memristors in the recent years, these elements and their promising applications are a central object of analysis in the paper. The tantalum oxide-based memory elements involve a high-conducting region and a

channel, based on non-stoichiometric Ta₂O₅ amorphous material [15–18]. The memristance (abbreviated from memory resistance) and the correspondent memristor status might be changed by applying outer voltage or current pulses [19–21].

For an accurate description of the tantalum oxide memristors' behavior in electronic digital and analog circuit and devices, a precise and simplified corresponding model is needed. Many attempts for adapting to the widely spread TiO₂ memristor models for approximate representation of the considered Ta₂O₅ memristors are available in the scientific literature [11]. Conversely, the structure and principal of operation of TiO₂ and Ta₂O₅ memristor elements are distinct [6,11]. This is the major cause for the generation of various special tantalum oxide memristor models [11,13,14]. The classical memristor model [11] suggested by Hewlett–Packard Research Labs has a high accuracy and sound switching representation. It applies the standard Heaviside step expression in the differential state equation and a non-differentiable modulus expression in the respective i – v relationship. Regrettably, these relationships are not flat and differentiable [11,13], and when this mathematical model is used for Simulation Program with Integrated Circuit Emphasis (SPICE) memristor model generation, many problems with the convergence exist [22,23]. This is a disadvantage of the traditional Ta₂O₅ memristor model. An improvement of the standard model is presented in [13]. The modified tantalum oxide memristor model [13] is related to a good precision and an enhanced behavior in SPICE environment. It uses continuous and differentiable relations in describing equations as an alternative of the applied non-smooth and non-differentiable expressions in [11]. While the enhanced tantalum oxide-based memristor model [13] is appropriate for SPICE incorporation, it is a computationally complex and time utilizing one, owing to the applied modulus-like complex expression. It demands many elementary calculations, corresponding to the high number of exponents in the state equation [13,14]. No window has been included in this model, and sometimes, the state variable x might go outside the interval between 0 and 1 when the memristor operates in a hard-switching mode. This is one more disadvantage of the discussed memristor models. A distinct memristor model, presented in [16], is appropriate for Ta₂O₅ memristor representation. It has good accurateness and correctly represents the switching processes in the memristor element. Disappointingly, it is founded on the classical non-differentiable step function, which is associated with convergence issues [20]. The memristor model represented in [15] is relatively simple and accurate, with a high operating rate.

The motivation for the present paper is the partial absence of simple and accurate Ta₂O₅ memristor models. The purpose of this research is to propose and consider an accurate, adjustable, fast operating, and simple model [12] for Ta₂O₅ memristors appropriate for generation of corresponding Linear Technology SPICE (LTSPICE) [23] library model. LTSPICE is a simple, user-friendly, and free software for analysis and design of electronic circuits and devices by preliminary computer simulations [23]. This software is offered by Analog Devices Corporation and could be freely downloaded and installed, using the next link: <https://www.analog.com/en/design-center/design-tools-and-calculators/ltspice-simulator.html> (last accessed on 5 November 2021). LTSPICE software is preferable for electronic device simulations by many design engineers and scientists. For adjustment of the offered memristor model, experimental current-voltage characteristics of tantalum oxide memristors, results derived from the use of several of the best existing models [11,13,14], and different methods for optimal parameters' estimation [22,24] are applied. The parameters' estimation procedure is realized in the MATLAB environment [22]. The least mean square error (MSE) between the experimental and simulated voltage–current relationships is used as an optimization criterion [22,24,25]. A corresponding LTSPICE library memristor model is successfully created. It is included in a unified and open LTSPICE memristor library, freely available for download and use at: <https://github.com/mladenovvaleri/Advanced-Memristor-Modeling-in-LTSpice> (last accessed on 9 November 2021) [26]. The proposed memristor model is analyzed and tested in passive and hybrid memristor memory crossbars for reading, writing, and erasing processes [27–30]. The derived current–voltage and state-flux relationships confirm the proper

operation of the considered tantalum oxide-based memristor model for soft-switching and hard-switching modes.

The rest of this paper is organized as follows. A description of the basic tantalum oxide memristor models is shown in the next section. The adjustment of the offered memristor model using experimental i – v characteristics, a procedure for extraction of the optimal model's parameters, and a procedure for coefficients' assessment in MATLAB—Simulink environment are discussed in Section 3. The related LTSPICE memristor model and its analysis are described in Section 4. The operation of the commented LTSPICE memristor library model in passive and hybrid memristor crossbars is shown in Section 5. The results are discussed in Section 6. The conclusion is given in Section 7.

2. The Basic Existing Ta₂O₅ Memristor Models and the Proposed Modification

For better understanding and completeness of the models' description, the basic existing standard and modified Ta₂O₅ memristor models are briefly discussed in this section. The tantalum oxide memristor element has two terminals—top electrode (TE), also known as anode, and bottom electrode (BE), namely the cathode [11,18]. These electrodes are made of platinum or tantalum-layered materials [11]. Several modified physical realizations of tantalum oxide-based memristors with layered structure are available [19]. This two-terminal passive electronic element has a cross-section with a square shape. The injection of oxygen vacancies is conducted by electroforming-like process at comparatively low-level voltage pulses [11]. In the Ta₂O₅ memristor nanostructure, several parallel-oriented conducting channels are available [18,19]. The peripheral region is based on pure and stoichiometric Ta₂O₅ amorphous material [11]. The central conducting channel is founded on a consistent solution of O atoms in a crystalline Ta material. A transitional region, made of a non-stoichiometric Ta₂O₅ and partially doped with O₂ vacancies, is created among the central and the peripheral channels of the tantalum oxide memristor element [11,19]. The memristor status could be changed by the use of outer voltage pulses. According to several physical factors, as the effective ionic dopant mobility, the memristor length and the resistance in the operating state, the memristor could function at different frequencies and amplitudes of the applied signals, representing the change of the memristance and the correspondent state variable. The following tantalum oxide memristor models are able to properly represent the memristor behavior for low, middle and higher frequencies for hard- and soft-switching modes. The memristor state variable x [11,13] is stated as a ratio of the areas of the intersections of the low-resistance channel, denoted by a_1 , and the area of the whole nanostructure, expressed by a_2 [11]:

$$x = \frac{a_1}{a_2} \quad (1)$$

According to physical considerations ($a_1 \leq a_2$), the memristor state variable x is limited in the range (0, 1). The described physical limitation of the memory element's state variable for the memristor models could be mathematically realized, using an appropriate window function [7,8,28]. If a voltage signal with comparatively low level and high frequency is applied to the memristor, then its state variable x does not reach its limiting values. This operation is also known as a soft-switching mode. In such a case, the correspondent current–voltage relationship is a multi-valued pinched hysteresis loop, and the respective state-flux relation is a single-valued curve. For higher-level and lower-frequency signals, the state variable x could reach its boundary values—zero and unity. In this case, the memristor operates in the so-called hard-switching mode. This functioning is related to a rectifying effect [7,8,31]. The terminal-state problems are related to the impossibility of the state variable x to be changed when it reaches its limiting values. Some window functions, as these proposed by Joglekar and Williams [6,31], are related to terminal state problems, while others as Biolek's [7] are able to correctly resolve these problems. In the present case, terminal state issues are not established, owing to the use of a modified Biolek-based window function [12,26].

2.1. The Standard Model of Ta₂O₅ Memristor, Proposed by HP

The highest value of the *memductance* (abbreviated from *memristor conductance*) of the Ta₂O₅ memristor is denoted by G_{max} [11,13]. The conductivity of the doped layer of the memristor could be represented by the Frenkel–Poole equation [11,13]. According to the traditional HP model of tantalum oxide memristors [11], the state-dependent current–voltage relationship could be described as:

$$i(v, x) = v \cdot G_{eq} = v \cdot \left[x \cdot G_{max} + a \cdot (1 - x) \cdot \exp\left(b \cdot \sqrt{|v|}\right) \right] \quad (2)$$

where i is the memristor current, v is the applied voltage across the memristor, G_{eq} is the equivalent memductance, and a and b are tuning parameters. The state differential equation of the standard tantalum oxide memristor has the following expression [11]:

$$\begin{aligned} \frac{dx}{dt} = & A \cdot \sinh\left(\frac{v}{\sigma_{OFF}}\right) \cdot \exp\left(\frac{1}{\beta \cdot v \cdot i + 1} - \frac{x_{OFF}^2}{x^2}\right) \cdot stp(-v) + \\ & + B \cdot \sinh\left(\frac{v}{\sigma_{ON}}\right) \cdot \exp\left(\frac{v \cdot i}{\sigma_P} - \frac{x^2}{x_{ON}^2}\right) \cdot stp(v) \end{aligned} \quad (3)$$

where $A, B, \sigma_{OFF}, \sigma_{ON}, \sigma_P, x_{ON}, \beta$ and x_{OFF} are coefficients for adjustment of the model [11,13,31]. The memristor model is based on a nonlinear dopant drift representation [11]. The applied in the model Heaviside step function $stp(\cdot)$ is [7,8,25]:

$$\begin{cases} stp(v) = 1, & v \in [0, +\infty) \\ stp(v) = 0, & v \in (-\infty, 0) \end{cases} \quad (4)$$

The classical HP tantalum oxide memristor model [11] is completely characterized by (1) and (2). This model holds good accuracy, related to an RMS error of about 2.86% [11,13]. It might correctly describe the performance of memristors, based on amorphous tantalum oxide in electronic schemes and devices [11,13]. The main drawbacks of the model, related to SPICE realization, are the application of an interrupted and non-differentiable step function and a non-smooth and non-differentiable modulus function [11,13]. The analysis of the memristor model is realized by computer simulations in MATLAB environment [22]. The simulations are prepared on a desktop computer system with Intel i5, 2.4 GHz 4-core microprocessor, Windows 10 system, and 8 GB Random Access Memory (RAM) [12]. The time for simulation of this memristor model is $t_1 = 49.5$ ms.

2.2. Enhancements of the Classical HP Memristor Model

The major improvement included in [13], which enhances the SPICE realization of the memristor model, is the replacement of the classical modulus expression and the standard step function by differentiable and flat replacements [25]. A simple step-like differentiable relation, applied in [13,14], is:

$$stpp(v) = [\exp(kv) + 1]^{-1} \quad (5)$$

where k is a tuning coefficient [13,25]. It determines the steepness of the step-like function $stpp(\cdot)$ in the region of switching [25]. For SPICE realization, it usually has a value between -50 and -1000 [13,14]. A differentiable and flat analog of the classical modulus function, used in (2) is [13,14]:

$$f_m(v) = v \cdot \left[\frac{1}{\exp(-\rho v) + 1} - \frac{1}{\exp(\rho v) + 1} \right] \quad (6)$$

where ρ is a parameter for fitting the modulus function [13]. Usually, for SPICE realization, its value is between 100 and 1000 [13,14]. The main improvement in this memristor model is the realized prevention of convergence difficulties in SPICE environment [13]. This model is with good accuracy, and the related error is about 2.88%. A disadvantage of the described

memristor model is its comparatively high computational complicity. The necessary time for simulation of the considered tantalum oxide memristor model is $t_2 = 16.8$ ms.

2.3. The Proposed and Considered Improved Tantalum Oxide Memristor Model

The proposed modified memristor model [12] contains several main replacements. First, the classical step expression $stp(v)$ in (3) is substituted by a smooth and differentiable analogue $s(v)$ [12,25]:

$$s(v) = \frac{1}{2} \left(1 + \frac{v}{\sqrt{v^2 + m}} \right) \quad (7)$$

where m is a coefficient with a typical value between 0.01 and 0.0001 [12,25]. This coefficient determines the steepness of the step-like function in the region of switching [25]. The function expressed by (7) is an alternative of (5). An advantage of such a step-like continuous function is the partial avoidance of convergence problems in SPICE environment. Conversely, the correspondent region of switching is not as sharp and rigorously defined, as in the case of the classical Heaviside function [25]. This could lead to decreasing the accuracy of the respective memristor model, especially if it operates at low-voltage signals and soft-switching mode. A compromise between the model's accuracy and the prevention of convergence issues must be introduced, and the coefficient m could be used as a parameter for adjustment of the memristor model. Due to the use of simple mathematical expressions and avoiding the exponential function applied in (5), the memristor model based on (7) has a slightly higher operating speed [12]. The operating rate is related to the number of the elementary mathematical operations in the memristor model, when it is functioning in a software environment, as MATLAB [22] or SPICE [23]. Second, the fragment $a \cdot \exp(b \cdot \sqrt{|v|})$ in (2) is approximated with a low-order polynomial:

$$F(v) = a \cdot \exp(\sqrt{|v|} \cdot b) \approx h_1 v^4 + h_2 v^2 + h_3 \quad (8)$$

which holds almost equal values in the interval $(-1 \text{ V}, 0.5 \text{ V})$. The RMS error among the original term and its approximation in (8) is about 2.8% [12]. The coefficients h_1 , h_2 and h_3 in the right-hand position of (8) are parameters for adjustment of the polynomial [12,22]. The applied approximation in (8) ensures lower complexity of the considered tantalum oxide memristor model, compared to the model described in [11]. Third, a modified and comparatively simple window representation $f_{Bmod}(x, i)$, founded on both the classical window expression proposed by Biolek [7] and the described step-like function $s(i)$, is used in the considered memristor model [12]:

$$f_{Bmod}(x, i) = 1 - [s(-i) - x]^2 \quad (9)$$

The applied window function is able not only to restrict the state variable x in the interval $(0, 1)$ but also to correctly represent the boundary effects and to solve the terminal state problems [7,12]. The terminal state problems are related to several memristor models, as Strukov–Williams, Joglekar and others [8]. Sometimes, when the state variable reaches the physical limits of zero and unity, it cannot be changed, although the applied voltage and the correspondent flux linkage are with sufficient values and polarities. Other models, such as Biolek and Boundary Condition Memristor (BCM) models, resolve successfully this issue [8]. The term $s(-i)$ is a smooth and differentiable step-like function, used for prevention of convergence issues in SPICE environment [13,14]. The memristor state differential Equation (3) [12] of the described model might be expressed in the next form:

$$\frac{dx}{dt} = [M(v)E(i, x, v)s(-v) + C(v)J(i, v)s(v)]f_{Bmod}(i, x) \quad (10)$$

where the terms $M(v)$, $E(x,i,v)$, $C(v)$, $J(x,i,v)$, are expressed according to (3) as follows:

$$M(v) = A \cdot \sinh\left(\frac{v}{\sigma_{OFF}}\right) \quad (11)$$

Here, A and σ_{OFF} are fitting parameters [11,13]. Unfortunately, the steepness of this term as a function of v is high in some voltage ranges and cannot be correctly approximated, unlike the term $C(v)$. The next term in (10) $E(x,i,v)$ is expressed by (12):

$$E(x,i,v) = \exp\left(\frac{1}{\beta v i + 1} - \frac{x_{OFF}^2}{x^2}\right) \quad (12)$$

where β and x_{OFF} are parameters for adjustment of the memristor model [13]. The term (12) is also not appropriate for approximation and simplification [13]. The term $C(v)$ is successfully approximated by a low-order polynomial [12]:

$$C(v) = B \cdot \sinh\left(\frac{v}{\sigma_{ON}}\right) \approx k_1 v^3 + k_2 v \quad (13)$$

where B , σ_{ON} , k_1 , k_2 are fitting coefficients [12]. The coefficients k_1 and k_2 are derived using the least squares approximation method, realized in MATLAB environment [22]. The RMS error between the original and the approximated values of $C(v)$ in the interval $(-1.45 \text{ V}, 1.45 \text{ V})$ is about 2.4% [12]. The next term $J(x,i,v)$ is [11–13]:

$$J(x,i,v) = \exp\left(\frac{v i}{\sigma_P} - \frac{x^2}{x_{ON}^2}\right) \quad (14)$$

Here, σ_P , x_{ON} are fitting parameters [11,13]. The final term in the state equation, presented by (14) could not be appropriately simplified. The original and the approximated expressions of the memristor current i as a function of the applied voltage v and the equivalent memductance of the tantalum oxide memristor $G_{eq}(x,v)$ are [12]:

$$\begin{aligned} i(x,v) &= v \cdot G_{eq} = v \cdot \left[x \cdot G_{\max} + a \cdot (1-x) \cdot \exp\left(b \cdot \sqrt{|v|}\right) \right] = \\ &= v \cdot \left[x \cdot G_{\max} + (h_1 v^4 + h_2 v^2 + h_3) \cdot (1-x) \right] \end{aligned} \quad (15)$$

The proposed simplified and improved tantalum oxide memristor model is completely described by (10) and (15) [12]. The model has good precision, and the obtained error is about 2.93%. It is able to properly operate in high-frequency mode, representing the respective alteration of the state variable x for both soft-switching and hard-switching operation [12].

3. The Fine Tuning and Parameters Estimation of the Suggested Memristor Model

The enhanced tantalum oxide-based memristor model [12], considered in this paper and fully described by (10) and (15), includes several parameters for fine adjustment. It is tuned in accordance with voltage–current characteristics, obtained by experimental data [11,19], the original HP model [11] and several of the best and accurate models [11,13,14,19], and it employs a procedure for changing the model's factors until reaching the total minimum of the RMS error [12,22]. Several researchers [31,32] have applied simulation annealing and gradient descent techniques for deriving the best parameters of the optimized memristor models. Additional comparison to several of the best tantalum oxide memristor models [11,13,14] is conducted, applying current–voltage and the correspondent state-flux relationships. Considering the terms (8), (13), and (10), (15), it could be easily concluded that the evolution of the memristor state variable x depends mainly on its initial value x_0 and the history of the applied voltage signal. The retention of the memristor state and the related information stored in the element are dependent on the state variable and the corresponding memductance. The processes of internal diffusion of dopant ions could affect the memristor state, but over a long interval, practically about ten years [11,19]. Owing to this, additional terms for considering

the reliability and retention [33] related to the tantalum oxide memristor are not included; thus, the model must be simple and applicable for analysis of memristor-based schemes and devices. A technique for extraction of the memristor model's parameters in the MATLAB environment [22] is also applied. The considered method for optimization of the tantalum oxide memristor model's tuning is based on altering the coefficients and searching for the global minimum of the RMS error among the experimental and the obtained-by-simulations $i-v$ relationships. At each iteration, one of the parameters of the memristor model is changing by a little constant increment [12]. The corresponding RMS error between the experimental and the simulated current-voltage characteristics is calculated. The other coefficients for tuning the memristor model are also altering. After finishing the course of this tuning process, a graphical observation of the simulated $i-v$ relation and its closeness to the experimental current-voltage characteristic is also established [12], paying attention to the shape of the obtained pinch hysteresis loop and especially its regions of switching the memristance [12]. The corresponding time diagrams of the experimental and the simulated memristor currents are compared as well, applying the squared differences between them in the sampling points.

The decisive factor for finishing the tuning procedure is the minimization of the RMS error [12,22,24]. Supplementary tests and simulations are made in the proximity of the derived optimal levels of the tantalum oxide memristor model's coefficients, using decreased increments for their changing [12]. The precise and optimal levels of the memristor element model's coefficients are also established, applying the least squares method in Simulink environment, using the Optimization Toolbox [22]. The technique for memristor model's parameters estimation could be approximately summarized in several basic steps:

1. Initialization of the values of the model's parameters, using the coefficients in the original sources [11,13,19] and the coefficients in the polynomials (8), (13) after their approximation in MATLAB environment [22];
2. Defining the satisfactory root mean square error for stopping the procedure, the respective tolerances and the maximal iteration steps;
3. Starting simulation and calculating the values of the memristor current, according to the applied model;
4. Estimation of the root mean square error (the cost function) between the experimental and the calculated memristor currents;
5. Changing the model's parameters, according to the gradient descent of the cost function, then proceed to step 4;
6. If the satisfactory root mean square error is reached, or the maximal iteration steps are finished, stopping the simulation and estimation of the obtained model's parameters; if not, proceed to step 5.

The applied voltage signals in the Simulink model of the memristor are hitherto sampled, and the time step is 10 μ s. The voltage drop across the element v_1 is applied as an input signal, applied to the Simulink model of the considered memristor [12,22]. The empirically recorded memristor's current is denoted by i_{mes} . The simulated output of the Simulink memristor model is equivalent to the simulated current of the tantalum oxide memory element i_{calc} . The cost function S_{cost} is represented by a sum of the squares of the differences of the calculated and experimental memristor current's values [12,22]:

$$S_{cost} = \sum_{k=1}^N [i_{calc}(k) - i_{mes}(k)]^2 \quad (16)$$

where $N = 200,000$ is the total amount of samples of the considered signals, and k is the actual sample. The benchmark for finishing the parameters' estimation procedure minimizes the cost function S_{cost} [22,32]. The best possible values of the extracted memristor model's coefficients are presented in Table 1 for the generation of the LTSPICE tantalum oxide memristor library model, as discussed in the next section. The derived optimal values of the tantalum oxide memristor model's coefficients are similar to those obtained in other scientific works on tantalum oxide memristors [11,13,14]. The obtained values of the

coefficients k_1, k_2, h_1, h_2 and h_3 after approximations in MATLAB environment [22] ensure that the RMS error between the respective terms is lower than 0.8%.

Table 1. Memristor model parameters derived after the optimization procedures.

Memristor Parameters	A	σ_{OFF}	β	x_{OFF}	k_1	k_2	σ_P
Units	s^{-1}	V	W^{-1}	-	$s^{-1}V^{-3}$	$s^{-1}V^{-1}$	W
Optimal values	1.47×10^{-7}	0.042	476.5	0.27	-1.3×10^{-7}	0.0001	7.05×10^{-5}
Memristor parameters	x_{ON}	m	x_0	G_{max}	h_1	h_2	h_3
Units	-	V^2	-	S	SV^{-4}	SV^{-2}	S
Optimal values	0.04	6.2×10^{-8}	0.1	0.0227	-1.98×10^{-6}	1.35×10^{-4}	3.31×10^{-4}

The scaled values of the trajectories of the model's parameters during the optimization are shown in Figure 1a to present their change with time. The initial values of the model's parameters are chosen to be close to those obtained in [11,13], and the memristor state variable x is in the space $(0, 1)$.

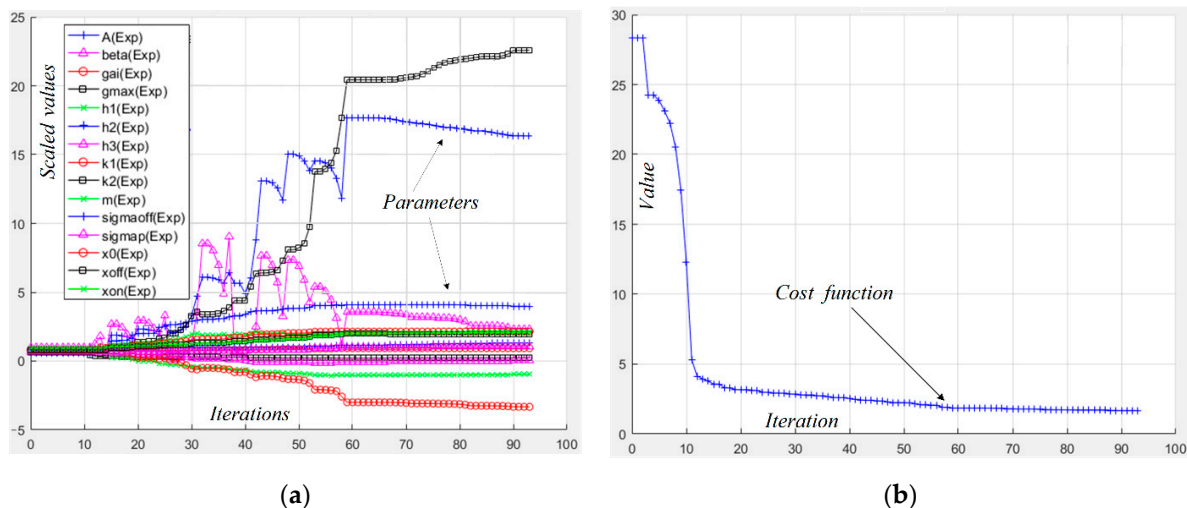


Figure 1. (a) Trajectories of the memristor model parameters during the optimization procedure, expressed in scaled values. (b) Minimization of the cost function.

For a brief comparison of the presented model to some of the existing and frequently used tantalum oxide memristor models [11,13–15], several basic criteria, as simulation time, operation speed, accuracy and convergence are presented in Table 2.

Table 2. A brief comparison of the presented model to several of the best Ta₂O₅ memristor models.

Models	Accuracy	Simulation Time, ms	Operating Speed	Convergence Issues
[11]	very high	49.52	average	rarely observed
[13]	high	16.84	high	not observed
[14]	high	16.42	high	not observed
[12]	high	16.31	high	not observed

The minimization of the cost function S_{cost} is given in Figure 1b to show the correctness of the optimization process [12]. The time diagrams of the memristor current (experimental and simulated) and voltage after the optimization are presented in Figure 2a for confirmation of the sufficient closeness between them. The experimental current–voltage characteristic [11] and the simulated i – v relationship obtained in the MATLAB environment [22] are presented in Figure 2b to present the proximity between the characteristics.

The obtained RMS error is about 3.24%. The time for simulation of the memristor model t_3 is about 16.3 ms [12].

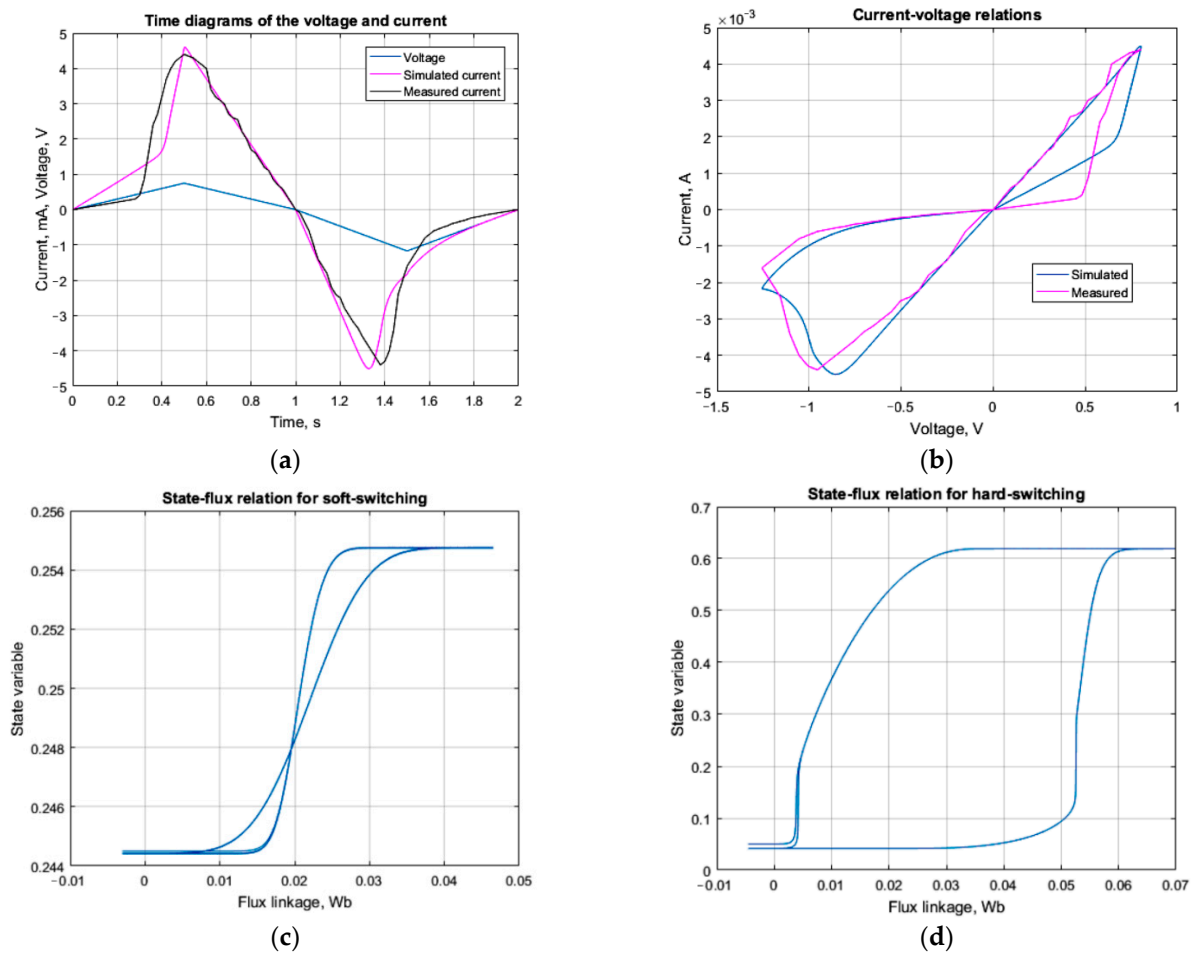


Figure 2. (a) Time charts of the memristor voltage and current (experimental and simulated) after the optimization process. (b) Current–voltage relationships obtained after the parameters estimation procedure. (c) State–flux relation for soft-switching; (d) State–flux relation for hard-switching.

Owing to the modified and simple equations, the suggested model of Ta_2O_5 memristor operates more rapidly than the formerly expressed memristor models [11,13,14]. The derived state–flux relationships for soft-switching and hard-switching operations are presented in Figure 2c,d, respectively, for confirmation of the proper functioning of the suggested modified memristor model [12]. The state–flux characteristic of the tantalum oxide memristor for soft-switching mode is a pinched hysteresis loop, while for the hard-switching operation, it is a hysteresis curve, correspondent to the memristor boundary effects [31].

4. The Corresponding LTSPICE Memristor Library Model

Based on the proposed improved mathematical memristor model, described by (10) and (15), an LTSPICE [23] library model of the described tantalum oxide-based memristor element was created [12,26]. The basic functional units in the LTSPICE environment are employed for realization of the relevant math calculus, in accordance with the presented memristor model. The substituting schematic of the generated LTSPICE model is shown in Figure 3a for further discussion. The memristor variable x is realized as the voltage $V(Y)$ of the capacitive element C_1 [12,26]. Its current corresponds to the time derivative of x . The two-port voltage controlled current source G_1 represents the memductance G of the tantalum oxide memory element. The internal own resistance of the applied voltage source V_1 is represented by the resistance R_2 . The resistor R_1 prevents the occurrence of

convergence issues [12,13]. The generated LTSPICE memristor library model in a simple electric circuit is presented in Figure 3b. The main terminals are the top electrode (TE, anode) and the bottom electrode (BE, cathode). The additional electrode Y is applied for measuring the memristor state variable x [12,13].

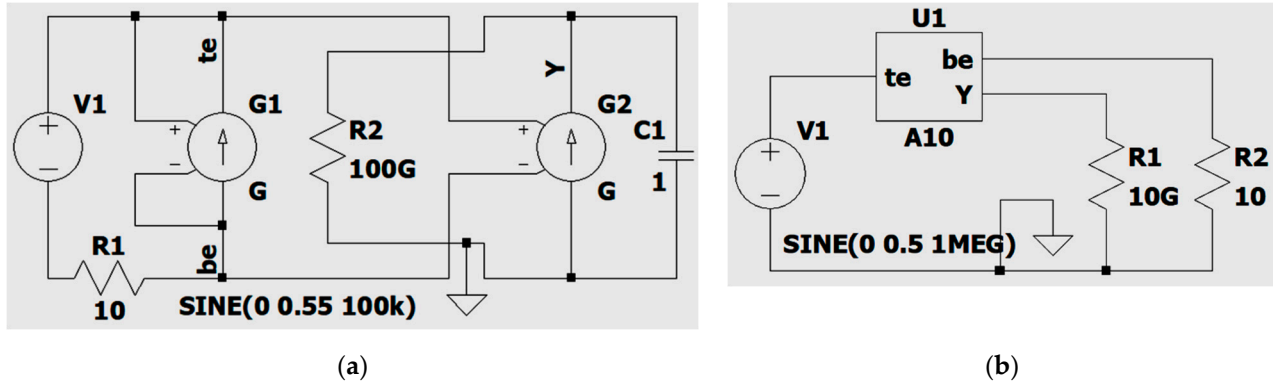


Figure 3. (a) An equivalent LTSPICE schematic of the considered tantalum oxide-based memristor model. (b) The generated LTSPICE memristor model, included in a simple test electric circuit.

The correspondent LTSPICE code of the memristor model is [12,26]:

```
.subckt A10 a c Y
*terminals—top electrode (a), bottom electrode (c) and additional
*electrode Y for measuring the memristor state variable
.params yon = 0.04 A = 1.37 × 10−7 sigmap = 7.05 × 10−5 sigmaoff = 0.042
.params Gm = 0.027 yoff = 0.27 m = 6 × 10−10 beta = 822.6 k1 = 0.0062
.params k2 = 0.0001 h1 = 1.98 × 10−4 h2 = 0.000135 h3 = 3.31 × 10−4
*memristor model parameters for tuning
G1 a c value = {(V(Y) × Gm + (1 − V(Y)) × (h1 × (pow(V(a,c),4)) + h2 × (pow(V(a,c),2))
+ h3)) × V(a,c)}
*voltage-controlled current source G1 for deriving the memristor current
G2 Y 0 value = {(A × sinh(V(a,c)/sigmaoff) × exp(1/(1 + beta × I(G1) × exp(−pow(yoff/V(Y),2))
× V(a,c))) × stpp(−V(a,c),m) + (k1 × (pow(V(a,c),3)) + k2 × (V(a,c))) × exp(I(G1) × exp(−pow(V(Y)/
yon,2)) × V(a,c)/sigmap) × stpp(V(a,c),m))) × (1 − (pow((V(Y) − stpp(−V(a,c))),10)))}
*deriving the state variable as a voltage across the capacitor C1 by G2
C1 Y 1 0 IC = 0.23
*a capacitor C1 for obtaining the state variable
R2 Y 0 10G
*additional resistor Rad for avoiding convergence issues
.func stpp(x,p) = {(1/2) × (1 + (x/sqrt(pow(x,2) + p)))}
*step-like differentiable function
ends A10
```

The described tantalum oxide memristor model is effectively analyzed by LTSPICE, version XVII, at various sinusoidal signals with various frequencies and amplitudes of the employed signal [12]. The obtained time diagrams and the i – v relationships are presented in Figure 4a–d to illustrate the proper operation of the described memristor model [12]. The state variable changes between 0 and 0.5. It starts from an initial value of 0.35 and, owing to the applied voltage and the generated flux linkage, their average value is decreased to a stable value of 0.25. This appropriate operation of the LTSPICE memristor model is supported by the detected decreasing of the area of the pinched current–voltage loop, while raising the frequency of the applied signal. Additional simulations at different amplitudes confirm the decrease of the area of the i – v loop. These are two of the main fingerprints of the memristor elements [7,12]. The SPICE programs simulate electronic schemes and devices using initialization and iterative procedures, numerically solving the corresponding differential equations by suitable solvers [23]. Convergence problems

occur in the SPICE environment, mainly due to several reasons. Frequently, convergence issues occur in electronic circuits with several stable operating points. The limited precision of the numbers' representation is a reason for obtaining errors and convergence issues. During the numerical integration of the respective differential equations, some models are sensitive to the errors. For example, the capacitive element C_1 , which is used as an integrator, is sensitive to the derived numerical errors and it accumulates them during the simulations. The truncation errors are also related to the simulations in SPICE. Such problems are also related to the rapid rise or fall of impulse currents and voltages and steep and discontinuous characteristics of electronic components. The convergence issues are related to erroneous results and sometimes to stopping the simulation before the previously defined end time, or to the impossibility of the simulation to start normally [23,33]. In some cases, the decrease of the time step or the change of the tolerances of some parameters could resolve these problems. In this sense, the use of a sigmoid step-like continuous function in modeling of memristors is another way to prevent the convergence problems in LTSPICE.

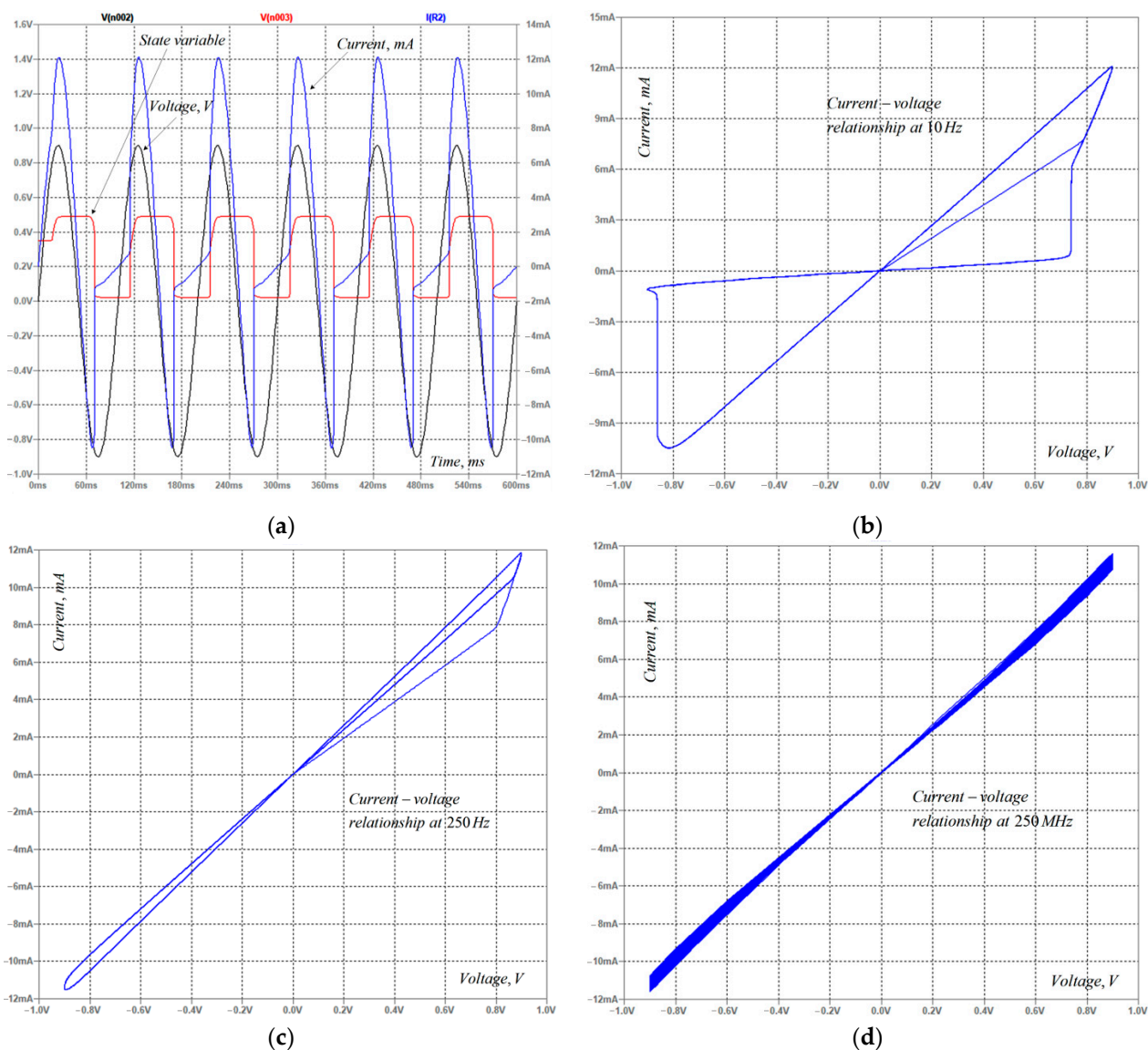


Figure 4. (a) Time graphs of memristor current and voltage with a frequency of 10 Hz. (b) Respective $i-v$ relation of the modified model of memristor. (c) Voltage–current relation at a frequency of 250 Hz. (d) Current–voltage characteristic obtained for a frequency of 250 MHz.

The time step, used in the simulation setup, must be as low as possible, but it is related to the obtained output file and the simulation time of the respective scheme. According to the other SPICE products, LTSPICE is not sensitive to convergence issues. The computational efficiency of the considered memristor models is related to the amount of time and memory for a given iterative step in the simulation process. Conversely, the needed time for simulations is proportional to the number of elementary mathematical operations for obtaining the respective solution at a given iteration step [23,33].

The considered memristor model is also analyzed by rectangular pulse voltage signals with different duty cycles and amplitudes in the LTSPICE environment, and no convergence issues were observed [12].

5. Hybrid and Passive Memristor Memory Crossbars

In this section, the application of the considered modified tantalum oxide memristor model in hybrid and passive memory crossbars [12,29,30] is described, paying attention to the memristor operation in a hard-switching mode [7,8]. The main procedure for writing, reading, and erasing information is related to applying short and rectangular voltage pulses to the respective memristors and changing their resistance [12,29]. For reading the stored information without changing the corresponding content, voltage pulses with a level lower than the activation threshold are used [12,29,30].

5.1. A Simple Hybrid Memristor Crossbar

The considered model of the Ta_2O_5 memory element is tested and considered in a hybrid matrix [12,29,30]. A section of the memristor crossbar is presented in Figure 5a for better explanation of the basic writing, reading, and erasing procedures.

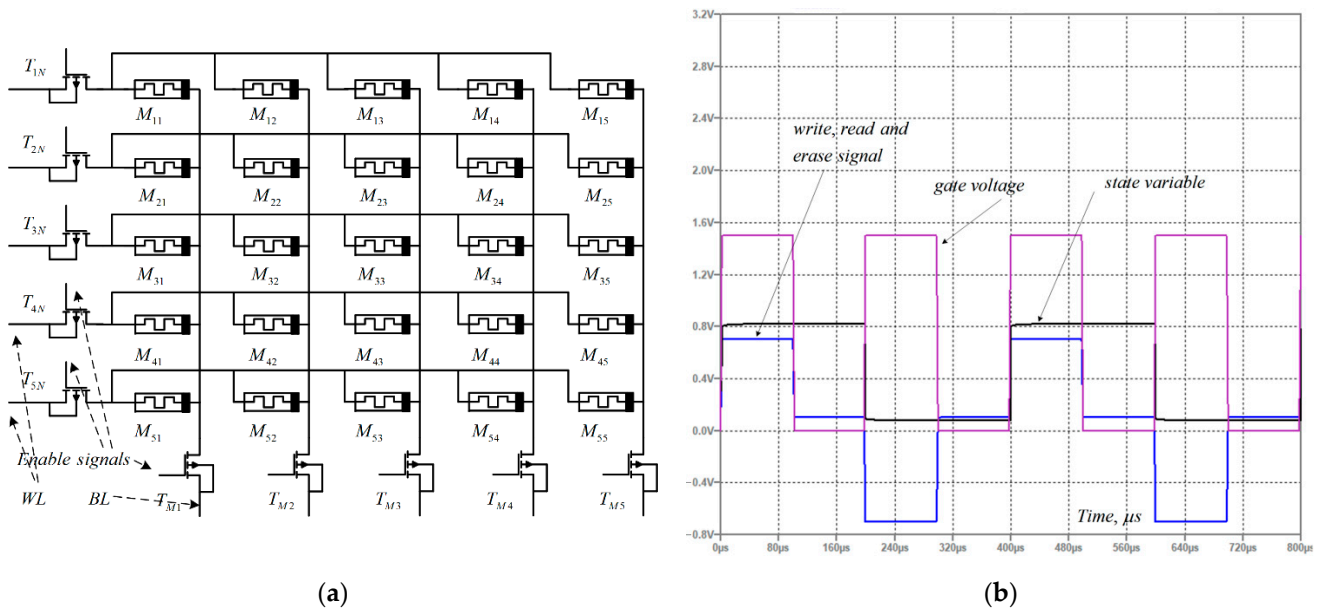


Figure 5. (a) A fragment of a simple hybrid memristor memory crossbar. (b) Time illustrations of the voltage across the memristor, the state variable x and the gate voltage.

Applying a positive rectangular voltage impulse with a duration of 100 μs and level of 0.7 V ensures the storing of a logical unity in the tantalum oxide memory component. The memristance of the element is changed to a minimal level, correspondent to ON-resistance status. When a voltage impulse with a negative polarity and identical duration and level is applied to the described memory element, it runs to its OFF-resistance state. Then, the correspondent memristance has a high-level value, and the logical data collected in the memristor is zero [12,30]. The technique for extracting the information accumulated in the respective memory element is accomplished by applying a voltage pulse with a duration of 100 μs and a level of 0.1 V. Due to the minimal level of the applied reading pulse, the

memristor's status is not affected, and the correspondent data are not altered. The used MOS transistors are with N-channel, type Si4866DY. They are applied for realizing the read-allow, write-enable, and deleting of the written information in the memristor elements [12]. The applied gate voltage is a sequence of pulses with an amplitude of 1.5 V and a duration of 100 μ s. It ensures the reading, writing, and erasing processes. When the gate voltage is zero, the respective memristor is isolated, and its state cannot be affected. The time graphs of the signal v and the state variable x of the element M_{11} are shown in Figure 5b. Their behavior validates the correct functioning of the memristor elements in a state close to a hard-switching mode.

5.2. Analysis of a Passive Memristor Crossbar

A simplified representation of a fragment of a passive memristor-based crossbar [12,29] is shown in Figure 6a for further explanation and discussion. This equivalent electric circuit is presented for further explanation of the memristor matrix structure and functioning of pulse and hard-switching mode. The memristor memory matrix is analyzed in the LTSPICE environment, applying the classical tantalum oxide memristor model [11], and its improved modification [12] is discussed in this work. The rectangular pulse voltage signals, used for writing, reading, and erasing logical information in the corresponding memristor cells, are applied between the respective word line and bit line of the selected memory element [12]. Due to the high resistance of the memristor elements in a reverse-biased state, the flowing sneak path currents do not significantly disturb the normal functioning of the considered memory crossbar. The respective time graphs of the applied voltage v , the state variable of the memristor x , and the corresponding current–voltage characteristic are illustrated in Figure 6b. These figures are presented to prove the proper memristor operation in a state near to a hard-switching mode [12]. The applied classical tantalum oxide memristor model [11], denoted by K_8 and its modified analogue and presented as A_{10} [26], have comparable behavior. For the writing, reading, and erasing processes in the memory matrix, different levels of voltage pulses are used. The amplitude of the voltage pulses for writing information is about 0.63 V. The duration of the applied pulses is about 200 μ s. The respective alteration of the memristor state variable x is about 0.83. The correspondent change of the memristance is about 80% of the full resistance, ranging between R_{ON} and R_{OFF} . For reading the information accumulated in the respective memristors, positive pulses with lower amplitude (about 0.08 V) with the same duration are used. Due to the lower level of the reading voltage pulses, the state variable of the memristor element and the respective memristance does not alter; thus, the accumulated information is not altered.

For the process of erasing the information in the memristor cells, voltage pulses with negative polarity, a duration of 200 μ s, and an amplitude of 0.63 V are applied. Due to the negative polarity impulses, the memristor state variable returns to its previous value. According to the alteration of the state variable in a broad range (approximately between 0.08 and 0.82), the memristor elements operate in a hard-switching mode.

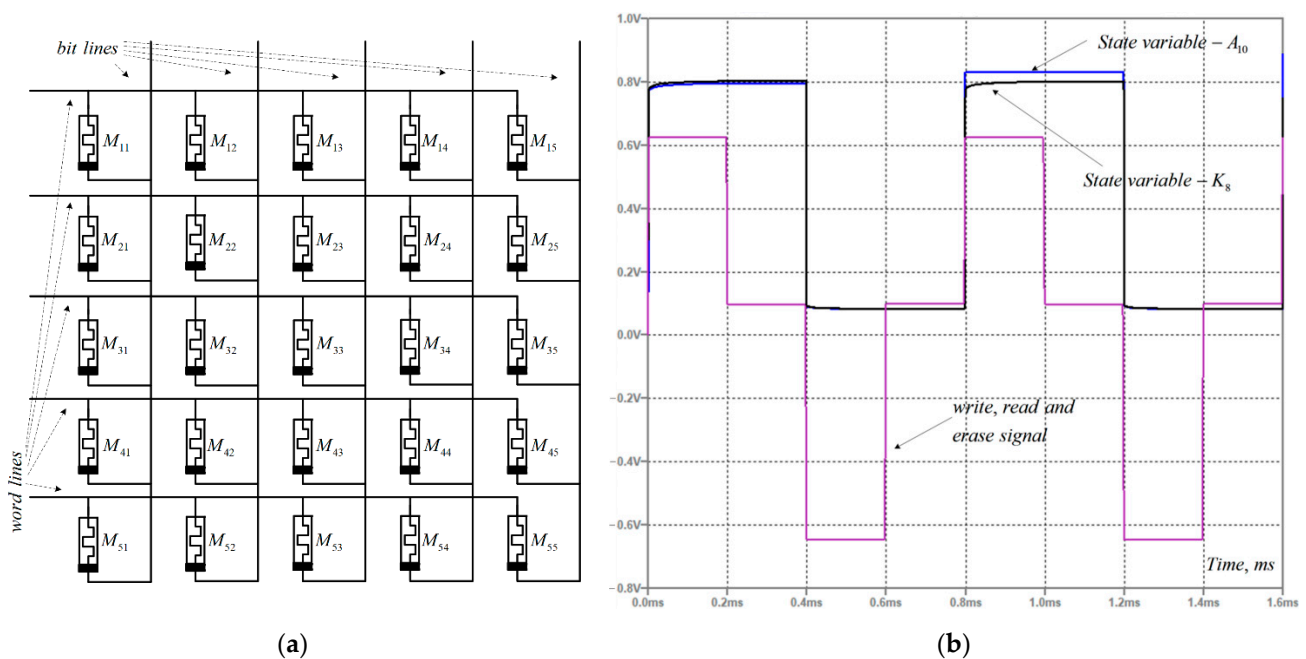


Figure 6. (a) A simple passive memristor memory crossbar. (b) Time diagrams of the memristor state variable according to models K_8 and A_{10} and the applied voltage.

6. Discussion

The modified and enhanced tantalum oxide memristor model considered in this paper is initially adjusted according to experimental current–voltage characteristics to derive the optimum values of the memory element model’s factors. The optimization procedure is built on changing the model’s coefficients and searching for the lowest RMS error among the current–voltage characteristics. The derived coefficients are used during the construction of the corresponding LTSPICE model. The generated LTSPICE memristor library model is analyzed and simulated for sinewave and impulse modes and for soft-switching and hard-switching representation. During the tests in an LTSPICE environment, no convergence issues are observed. The considered memristor model is investigated together with the classical and the existing tantalum oxide memristor models, and their identical behavior was confirmed. The detected reduction of the pinched current–voltage hysteresis loop’s area with rising the operating frequency confirms the correctness of the considered memristor model. The state-flux relationships are also analyzed. For soft-switching mode, they are pinched hysteresis curves with a small area, and for hard-switching, they are hysteresis multi-valued curves with a broad area. For future work on oxide memristors’ modeling and applications, additional attempts for simplification, generalization and optimizing the models will be conducted.

7. Conclusions

In the suggested modified and improved tantalum oxide memristor model, a simple altered window function built on the traditional Biolek’s window is included. The offered model of the tantalum oxide memristor has a higher operation rate, regarding the classical and existing scientific literature modified models. It is tuned according to experimental current–voltage characteristics of Ta_2O_5 memristor nanostructures. It has a sufficient amount of coefficients for tuning and good correctness of the representation of the basic memristor’s characteristics—current–voltage and state-flux relations. The described tantalum oxide memristor model has a relatively simple mathematical model and higher switching rate, according to the classical Hewlett–Packard model of the Ta_2O_5 memristor and its available modified versions. The used simplified window function successfully limits the memristor state variable x in the interval $(0, 1)$, and it properly characterizes

the boundary effects for a hard-switching operating state. The state terminal problems are resolved as well. An LTSPICE library memristor model is created in accordance with the considered model's mathematical equalities. It was suitably analyzed for sinusoidal and impulse modes, and convergence problems were not observed. The offered LTSPICE library memristor model was productively analyzed in hybrid and passive memory cross-bars. The capability of the discussed tantalum oxide memory element model for suitable functioning in complex electrical circuits for both hard-switching and soft-switching and modes was proven.

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