International Journal of
Financial Studies

MDPI

Article

Navigating Uncertainty in an Emerging Market: Data-Centric Portfolio Strategies and Systemic Risk Assessment in the Johannesburg Stock Exchange

John W. M. Mwamba [1,2], Jules C. Mba [1] and Anaclet K. Kitenge [1,*]

[1] School of Economics, University of Johannesburg, P.O. Box 524, Johannesburg 2006, South Africa; johnmu@uj.ac.za (J.W.M.M.); jmba@uj.ac.za (J.C.M.)
[2] The Haute École de Commerce de Kinshasa, Avenue de Libération, Gombe, Kinshasa B.P. 16596, Democratic Republic of the Congo
* Correspondence: anacletkite@gmail.com

Abstract: This study investigates systemic risk, return patterns, and diversification within the Johannesburg Stock Exchange (JSE) during the COVID-19 pandemic, utilizing data-centric approaches and the ARMA-GARCH vine copula-based conditional value-at-risk (CoVaR) model. By comparing three investment strategies—industry sector-based, asset risk–return plot-based, and clustering-based—this research reveals that the industrial and technology sectors show no ARCH effects and remain isolated from other sectors, indicating potential diversification opportunities. Furthermore, the analysis employs C-vine and R-vine copulas, which uncover weak tail dependence among JSE sectors. This finding suggests that significant fluctuations in one sector minimally impact others, thereby highlighting the resilience of the South African economy. Additionally, entropy measures, including Shannon and Tsallis entropy, provide insights into the dynamics and predictability of various portfolios, with results indicating higher volatility in the energy sector and certain clusters. These findings offer valuable guidance for investors and policymakers, emphasizing the need for adaptable risk management strategies, particularly during turbulent periods. Notably, the industrial sector's low CoVaR values signal stability, encouraging risk-tolerant investors to consider increasing their exposure. In contrast, others may explore diversification and hedging strategies to mitigate risk. Interestingly, the industry sector-based portfolio demonstrates better diversification during the COVID-19 crisis than the other two data-centric portfolios. This portfolio exhibits the highest Tsallis entropy, suggesting it offers the best diversity among the types analyzed, albeit said diversity is still relatively low overall. However, the portfolios based on groups and clusters of sectors show similar levels of diversity and concentration, as indicated by their identical entropy values.

Keywords: portfolio diversification; systemic risk; data-centric investment strategies; CoVaR; entropy measures; vine-copula; COVID-19 pandemic

check for updates

Academic Editor: Ronald Balvers

Received: 18 November 2024
Revised: 3 February 2025
Accepted: 5 February 2025
Published: 1 March 2025

Citation: Mwamba, J. W. M., Mba, J. C., & Kitenge, A. K. (2025). Navigating Uncertainty in an Emerging Market: Data-Centric Portfolio Strategies and Systemic Risk Assessment in the Johannesburg Stock Exchange. *International Journal of Financial Studies*, *13*(1), 32. https://doi.org/10.3390/ijfs13010032

Copyright: © 2025 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

1. Introduction

In recent years, the South African stock market has progressively emerged as the largest stock exchange in Africa; consequently, the South African financial system has also experienced rapid transformations. The South African government, in recent years, has kept promoting the reforms and opening up the process of its financial stock market not only to allow global economic exchange to be accelerated, but also to enhance investors' protection, and encourage foreign investors to participate in the local stock market.

Therefore, the South African financial stock market has not only attracted the interest of global investors, who are drawn in by its diverse opportunities and potential for high returns, but has also raised concerns regarding potential price fluctuations in some sectors and the market downturns that could have repercussions for other sectors in the Johannesburg Security Exchange (JSE) stock market (Mbatha & Alovokpinhou, 2022).

While the existing literature presents a substantial body of evidence on, on one hand, the correlation between the South African stock market and other stock markets (Ali et al., 2018; W. Zhang et al., 2020; Yousaf et al., 2022; Chowdhury et al., 2022; Insaidoo et al., 2023), and on the other hand, the international intra-sectorial dependency in numerous industries (Fasnacht & Louberge, 2007; Shahani et al., 2017; Braiek et al., 2022), there is a gap in research examining the connection among industry sectors within the JSE in the context of the COVID-19 pandemic, specifically focusing on data-centric investment approaches, portfolio diversification levels, and the precise measurement and assessment of the conditional effects of upward and downward movements in an industry sector on the entire portfolio of sectors.

To fill this gap, we use the entropy of returns to assess the risk profile of each type of data-centric strategy; the entropy of probability distributions (entropy of weights) to assess the portfolio diversification level, the GARCH-vine copulas to capture tail behavior and possible contagion effects between JSE industry sectors during the COVID-19 pandemic, and the conditional value-at-risk (CoVaR) as a systemic risk measure proposed by (Adrian & Brunnermeier, 2011) to assess the risk impact of each type of data-centric strategy on the portfolio of sectors or clusters.

Leveraging data-driven insights can lead to more accurate asset valuations, better-informed portfolio construction, and more effective trade execution. Hence, uncovering hidden patterns and subtle connections among JSE sectors can provide valuable insights for improving portfolio diversification strategies in times of uncertainty and randomness in a financial market (Odeyemi et al., 2024; Kumar et al., 2024). Therefore, it is crucial to complement the hidden patterns with a deep understanding of the underlying fundamentals and drivers of the sectors.

The transition towards data-centric strategies is driven by their proven effectiveness in enhancing decision-making processes, managing risks, and ultimately improving investment results. This transition has not only improved the accuracy of investment tactics but also brought forth new challenges and opportunities associated with the management of financial portfolios (Odeyemi et al., 2024). The consideration of data-centric investment strategies is crucial in financial risk management and portfolio optimization because data-centric approaches are more systematic, with portfolio construction and asset allocation decisions driven by quantitative models and optimization algorithms (Kumar et al., 2024). Moreover, integrating the clustering insights with sector-level financial market analysis can lead to more informed investment decisions and facilitate the formation of a portfolio.

A plethora of studies have performed clustering algorithms as data-centric models to leverage portfolio diversification. For instance, (Léon et al., 2017) use clustering techniques to sort out assets into categories from the Russell 1000 index and compare the perormance of different cluster portfolios. (Ji et al., 2018) combine cluster analysis and portfolio selection using components of DAX 20. (Khan & Mehlawat, 2022) use asset clustering as inputs for the portfolio optimization model to obtain the optimum asset allocations from the US stock market.

In line with the concept of diversification, previous studies focused on finding patterns in the dataset and testing the performance of portfolio optimization. However, our study examines the diversification level of portfolio clusters focusing on the tail dependence structures.

Unsupervised machine learning techniques such as Clustering algorithms aim to find and arrange sectors exhibiting close and similar features in separate groups named Clusters (Goudarzi et al., 2017). The underlying assumption of clusters is that sectors with proximity in this multidimensional feature are likely to have correlated returns, risk profiles, and investment dynamics.

In practice, an asset risk–return plot visually represents the relationship between expected return and the risk of different assets. As this visual tool allows investors to understand and compare the risk–return characteristics of various assets quickly, it can be seen as being closer to a data-centric approach. By analyzing patterns revealed by the asset risk–return plot, investors and portfolio managers can make more informed decisions about asset selection, portfolio construction, and risk management, ultimately enhancing the overall investment process.

To the best of our knowledge, no study has been conducted yet with consideration of insights from asset risk–return plots as an investment strategy. Moreover, less attention has been paid to in-depth comparisons of patterns revealed by clustering algorithms and the ones revealed by asset risk–return plots. This gap can give investors a better chance to diversify their portfolios and come up with a higher return on investments.

The ultimate goal for investors in the stock markets consists of protecting their portfolios from normal risks through the diversification of their assets. Nevertheless, in times of extreme circumstances, such as economic turmoil, all the financial markets are adversely affected. Consequently, almost all sectors are negatively affected by the crisis and investors, in such periods, seek to diversify their assets with the expectation of finding assets that have safe-haven properties.

Recent literature has acknowledged that the COVID-19 turmoil has caused distress to the whole global economy (Li & Yan, 2022). The coronavirus outbreak brought about significant consequences for many governments, such as the South African government, prompting them to enforce strict policy measures such as lockdowns and closing borders to contain the virus. These interventions had a substantial impact on the financial revenue of businesses and introduced higher levels of uncertainty into the trading of financial securities (Enow, 2022; Insaidoo et al., 2023).

Long before the outbreak of COVID-19, the South African stock market, specifically the JSE's All Share Index (ALSI) and ALSI Top 40, began to plummet as of 2 January 2020. The ALSI Top 40 experienced a loss of nearly 35%, but it has since recovered to approximately 85% of its pre-COVID-19 value as of July 2020. However, despite this recovery, the market experienced a cumulative loss of around ZAR 2.3 trillion at its worst (Mbatha & Alovokpinhou, 2022). Some researchers have documented the negative impact COVID-19 had on the financial system (Li & Yan, 2022; Hu et al., 2021; Ellul et al., 2020; Hassan et al., 2021).

The COVID-19 pandemic triggered severe distress with regard to financial assets. Contrary to the GFC, a financial crisis, the COVID-19 pandemic is viewed as a health crisis due to the fact that it did not originate in financial sectors. Nevertheless, both the GFC and COVID-19 pandemic have been perceived as systemic risks in the sense that the shocks of their crisis have been propagated over time and across institutions (Polanski et al., 2021).

The literature suggests that there is a close association between systemic risk and tail dependence (Kupiec & Guntay, 2016; Van Oordt & Zhou, 2019; Polanski et al., 2021; Aloui et al., 2022). Nonetheless, tail dependence refers to the fact that risky assets are likely to be intensely correlated during periods of stress, which can increase the risk of collapse (Trabelsi, 2017).

Over the past decade, empirical studies have explored the tail dependence structure using data from stock markets (Lahiani & Jlassi, 2021; Karmakar & Paul, 2023). Investigating

the tail dependence structure between industry sectors in a stock market is particularly important due to the high volatility and fat-tailed distributions associated with these sectors (Braiek et al., 2022). Therefore, the overriding question of whether the assets have significantly correlated downside risks remains the first-order focus when choosing assets to invest in.

To achieve efficient financial strategies, it becomes very useful to understand the tail behavior of asset returns (Hilal et al., 2011). The evidence suggests that an increase in dependence can amplify the likelihood of financial losses. Therefore, examining the dependence structure between industry sectors is worthwhile, as it will help people to manage investment risk better and allocate wealth efficiently for portfolio diversification (Supper et al., 2020).

Furthermore, the concept of contagion or spillover effects plays a critical role in understanding the interconnectedness of financial markets, particularly during periods of economic distress. When systemic risk is heightened, the transmission of shocks between asset classes or industry sectors can lead to widespread declines in asset values, exacerbating financial instability (Warshaw, 2019).

Although contagion may not always be evident, particularly in unique circumstances like the COVID-19 pandemic, the theoretical implications remain significant. This interconnectedness often manifests through increased correlations among assets, whereby negative returns in one sector can trigger adverse reactions in others, further amplifying overall market volatility (Yousaf et al., 2022). As such, identifying and analyzing these spillover effects is essential for investors aiming to mitigate risk and enhance portfolio resilience during turbulent market conditions.

The Pearson correlation coefficient is a commonly utilized measure of dependence structure between multiple financial assets within a portfolio. However, a plethora of research has demonstrated that the Pearson correlation coefficient is inadequate in describing dependence at the tail of distribution, particularly in extreme events. Therefore, in recent years, copulas, introduced by Sklar (1959), have been considered a tool for modeling dependence structures between multiple financial assets within a portfolio.

The challenge in practical applications lies in finding the appropriate copula. The complexity of using copulas increases significantly in higher dimensions, as traditional multivariate copulas offer limited flexibility. Nonetheless, vine copulas overcome the limitations of traditional multivariate copulas and can effectively capture complex tail dependency structures by using a diverse range of bivariate copulas (Hao & Chen, 2022).

There is a substantial body of literature on vine copula models. For instance, (Čeryová & Árendáš, 2024) explored the dependence structure within the US technology industry by employing vine copula modeling. Aslam et al. (2023) used vine copula to examine the sectoral dependence among Pakistani companies. Czado et al. (2022) focused on understanding the dependence structures of different environment, social, and governance (ESG) class indices from US companies using vine copulas.

The use of vine copulas in modeling tail dependence structure among sectors within a stock market is still rarely studied. This study uses vine copula models to compare tail dependence structures among sectors (or clusters) in different portfolio strategies from data-centric approaches. Additionally, numerous studies have used the CoVaR as a measure of systemic risk using different statistical techniques. For instance, estimated the CoVaR as the systemic risk in the banking system using the regime-switching technique in tails.

Tian and Ji (2022) modeled CoVaR using a GARCH copula quantile regression framework to access the risk spillovers from developed markets. Khiari and Ben Sassi (2019), Manguzvane and Muteba Mwamba (2019), and Bianchi and Sorrentino (2022) made use of the GARCH models to estimate CoVaR in the banking system. Luo et al. (2021) estimated

the CoVaR for the international stock markets using EMD-copula analysis. Ji et al. (2018) and Kumar et al. (2021) made use of a dependence-switching copula approach to estimate the CoVaR in commodity agriculture markets.

Recently, some researchers utilized vine copula-based CoVaR frameworks to examine the magnitude of systemic risk and the marginal risk contribution triggered by one institution with regard to financial institutions as a whole. In particular, Kielmann et al. (2022) used vine copula-based CoVaR to investigate risk spillover between BRICS stock returns and different types of oil prices, while Braiek et al. (2022) utilized vine copula-based CoVaR in Islamic sector industries to capture their inter-linkages.

Studies modeling the intricate dependence relationships and tail risk across different sectors within a stock market context are rare. Our study wants to fill this gap by assessing the tail distributions of different data-centric approaches using the GARCH vine copula-based CoVaR model.

The questions to be dealt with in this study are as follows:

1. How do dynamic vine copula models enhance the assessment of tail dependence and systemic risk in the JSE market during periods of financial turmoil, and how does this impact portfolio optimization strategies?
2. To what extent do data-centric investment approaches (such as clustering and risk–return plot-based strategies) outperform traditional sector-based portfolios in terms of risk-adjusted returns and diversification benefits during crisis periods in emerging markets?
3. How does the integration of entropy measures with vine copula-based CoVaR models improve the identification of systemically important sectors and the construction of resilient portfolios in the JSE market?

This study makes a significant contribution to the field of portfolio management and systemic risk assessment in the JSE by integrating data-centric analysis with advanced statistical methodologies. The main contributions can be summarized as follows:

- Integration of Data-Centric Strategies

The study compares three distinct portfolio management strategies derived from data-centric investment approaches. The first is the sector-based portfolio, which is constructed based on industry sectors within the JSE that share conceptual similarities with supervised machine learning techniques. The second is the asset risk–return plot portfolio, which employs an asset risk–return plot. This strategy emphasizes data-driven investment decisions that highlight risk and return tradeoffs. The last one is the clustering technique portfolio, which uses an unsupervised machine learning clustering technique to group assets, enhancing diversification through data-driven insights.

- Advanced Risk Assessment Framework

A unique aspect of this research is the application of the ARMA-GARCH vine copula-based CoVaR approach combined with an entropy framework. This dual approach allows for tail dependence. The vine copula framework investigates the tail behavior of each portfolio, providing insights into the potential for extreme co-movements during periods of market stress; on the other hand, it allows for conditional value-at-risk (CoVaR). The CoVaR measure evaluates the portfolios' risk profile, focusing on their vulnerability when specific assets are under distress.

The study's main contribution is combining data-centric analysis with the ARMA-GARCH vine copula-based CoVaR approach and entropy framework. This contribution can be delved into via the following steps: the study compares three management portfolios obtained through data-centric investment strategies: the first portfolio is based on industry sectors within the JSE that share some conceptual similarities with supervised machine

learning. The second is based on an asset risk–return plot that can be considered as a part of a data-centric investment strategy. The last portfolio is based on a clustering technique that is considered to be an unsupervised machine learning tool. To compare these portfolios, we use both the entropy of returns and the entropy of weights and/or probability distributions to evaluate each asset's risk profile and the portfolio's diversification level. The vine copula framework is used to investigate the tail behavior of each portfolio. We also compute the CoVaR as a measure of risk for the three data-centric strategies to obtain information about a portfolio's vulnerability level conditional to an asset being in distress.

The consideration of the entropy framework is useful in portfolio diversification and risk management for numerous reasons. Firstly, entropy serves as a metric for portfolio diversification, where higher entropy denotes a more diversified portfolio. By calculating portfolio entropy, investors can evaluate the degree of concentration or dispersion of their asset allocations, which is a crucial consideration in risk management (Novais et al., 2022).

Secondly, entropy is closely related to the concept of uncertainty or unpredictability. A portfolio with higher entropy is perceived to have higher uncertainty or risk. Comparative analysis of the entropy across various portfolios can help investors understand the relative risk profiles and make more informed decisions regarding risk tolerance and investment strategies.

Finally, entropy can be leveraged to estimate the portfolio's performance against a benchmark or other investment options (Guilherme et al., 2022). By comparing portfolio entropy to that of a benchmark or market index, investors can assess whether their portfolio achieves the desired diversification and risk-adjusted performance. Markowitz's (1952) portfolio selection theory helps investors identify the optimal wealth allocation strategy for an efficient portfolio.

The rest of the paper is structured as follows. Section 2 reviews the literature on portfolio diversification, systemic risk, and CoVaR. Section 3 discusses the methodology of vine copulas, market risk modeling, entropy, and portfolio optimization. Section 4 presents and discusses the empirical results, and Section 5 concludes the paper with a summary of the key findings.

2. Literature Review

2.1. Theoretical Literature Review

The portfolio selection problem is about choosing assets to maximize risk-adjusted returns. In 1952, Markowitz presented the core idea of the mean-variance model (M-V), which seeks to optimize portfolios by boosting returns and reducing risk in wealth investment. This M-V model is widely accepted and used in both academic and real-world settings.

Markowitz (1952) solves two separate problems through the optimization problem. The first problem is a selection of market assets with the right M-V statistics to attain higher future returns. Similarly, the second problem is the optimization of market assets, which consists of seeking the right proportions of these market assets that could guarantee the achievement of a confident level of returns (Omane-Adjepong et al., 2019).

Diversification is a risk management strategy that involves spreading investments across different asset classes or sectors. Diversification's primary goal is to reduce a portfolio's exposure to any single investment or risk factor. Investors, therefore, aim to establish a more stable and well-diversified portfolio, potentially leading to improved risk-adjusted returns (Ljubownikow & Ang, 2020). In this way, data-centric models play a great role in portfolio diversification. The meaningfulness of a data-centric approach is threefold. First, the data-centric approach provides a more robust and data-driven approach to assessing and managing portfolio risk. It can help identify and quantify the

risk exposures of individual assets and the overall portfolio, enabling better diversification strategies. Second, the data-centric model can leverage historical data and other relevant information to optimize asset allocation decisions (Odeyemi et al., 2024). This can lead to more efficient portfolio construction and better risk-adjusted returns. Lastly, a data-centric approach can uncover hidden relationships and correlations between assets, revealing potential diversification opportunities. This allows investors to construct portfolios with lower overall risk through effective diversification (Kumar et al., 2024).

Among data-centric models, clustering algorithms are widely used to uncover patterns in datasets without relying on predefined classes, grouping similar objects together (Léon et al., 2017).

However, systemic risk can complicate diversification benefits. Systemic risk, which can lead to widespread financial disruptions (Haroon et al., 2021), increases correlations between assets, reducing diversification effectiveness (Warshaw, 2019; Abuzayed et al., 2021). Therefore, investigating the dependence structure of securities during market downturns is crucial.

Copula models are statistical tools mainly used for decoupling the marginal distribution associated with tail events from the dependence structure associated with systemic risk and modeling each separately with accuracy (Hao & Chen, 2022). Nevertheless, the challenge in practical implementations lies in the identification of the right copula. Different copula families have been extensively studied and are available for the bivariate scenario (Righi & Ceretta, 2013; Dewick & Liu, 2022). The use of copulas becomes notably complex in higher dimensions, as traditional multivariate copulas have limited flexibility.

Vine copulas overcome the limitations encountered by traditional multivariate copulas and can efficiently capture complex dependency structures by using a diverse range of bivariate copulas as basic components; they also enhance the precision of modeling of the interdependence among a greater number of variables (Hao & Chen, 2022; Aslam et al., 2023). Vine copulas, also known as R-vines or canonical vine (C-vine), were first introduced by Joe (1996) and subsequently refined in works by (Bedford & Cooke, 2002) for the first time. Vine copulas make use of a series of bivariate copula to construct multivariate copula, enabling the decomposition of a multivariate probability density into bivariate copula; specifically, the segregation of marginal and dependence modeling (Čeryová & Árendáš, 2024). Vine copulas have been widely used in many fields, such as finance, risk management, environmental modeling, and insurance, where modeling complex multivariate dependencies is crucial.

The assessment of systemic risk commonly involves the application of the CoVaR methodology of Adrian and Brunnermeier (2011). CoVaR is a widely used measure that captures the tail dependency of losses among financial institutions and systems, and it is often estimated through statistical methods like quantile regression or simulation approaches. CoVaR generally evaluates the value-at-risk (VaR) of the financial system contingent upon the distress of an individual institution (Righi & Ceretta, 2013). It offers a method for quantifying the impact of a financial entity on the overall systemic risk. By identifying the institutions that pose the greatest systemic risk, policymakers and regulators can better understand the sources of financial instability and implement appropriate regulatory measures (Kumar et al., 2021). In the case of this study, it is useful to obtain insights into the portfolio CoVaR conditional to a particular sector being in distress.

For several reasons, the consideration of CoVaR measures is crucial in financial risk management and portfolio optimization. Firstly, CoVaR provides a measure of downside risk that surpasses conventional risk measures such as standard deviation or VaR (Aloui et al., 2022). It quantifies the potential losses of a portfolio or investment under extreme market conditions by considering the tail dependence structure and capturing the joint

distribution of assets. As a result, CoVaR offers a more comprehensive assessment of systemic risk. Secondly, CoVaR proves particularly useful in assessing systemic risk, which pertains to the risk of widespread financial disruption or the collapse of an entire financial system (Manguzvane & Muteba Mwamba, 2019; Bianchi & Sorrentino, 2022).

Measuring the expected losses of other assets when the considered asset is in distress captures the potential impact of an individual asset or portfolio on the overall system. This aids in identifying systemically important assets and facilitates the monitoring and regulation of systemic risk. Lastly, CoVaR assists in portfolio optimization by incorporating the dependence structure among assets. By considering the tail of individual assets and their joint behavior, CoVaR enables investors to construct diversified portfolios resilient to extreme market events. This promotes risk reduction and enhances the stability of investment portfolios.

2.2. Empirical Literature Review

Since the pioneering work by Markowitz (1952), a plethora of studies has attempted to improve the modern portfolio theory (MPT). Regarding portfolio choice, various methodologies have been used, such as conditional mean-variance models (Wang, 2002), dynamic mean-variance frameworks (Gao et al., 2015), autoregressive integrated moving average (ARIMA) models (Rather et al., 2017), GARCH models (Aziz et al., 2019), and support vector regression (Mishra & Padhy, 2019).

In what may be seen as a dependence structure analysis, Righi and Ceretta (2013) estimate the dependence structure between economic sectors in the Brazilian financial market. The dataset that spans the 2007–2011 period was collected from the Bovespa stock market indices, represented by sectors such as the industrial, real estate, telecommunications, consumer, energy, basic materials, and financial sectors. They make use of pair copula models to construct efficient portfolios. In addition, they forecast daily VaR as a portfolio risk by considering distinct significance levels. The results of their study show that VaR predictions are correct and permit industry participants to construct portfolios with assets of these sectors under a proper diversification structure.

(Budiarti et al., 2023) implemented a study on extreme value copula and ARMA-GARCH models to quantify dependencies among financial assets under an extreme event. They collect data on daily returns from the IDX Composite Index (IHSG), Straits Times Index (STI), and Kuala Lumpur Stock Exchange (KLSE) spanning the period of the COVID-19 pandemic, from 1 March to 1 October 2020. The findings of their study show that the Gambel copula is the most appropriate extreme value copula for the selected indices.

Aslam et al. (2023) examined the sectoral dependence among 82 companies from the Pakistani stock market to capture the dynamic and complex relationship among equity sectors stemming from the nonlinear patterns of asymmetric behavior of returns that linear models cannot capture. Daily data were collected from 1 July 2014 to 17 December 2019, and the GARCH vine copula approach was used to analyze the dataset. The findings of their study show that the Pakistani stock market is driven not only by the oil and gas marketing sector but also by the oil and gas exploration sector.

(Čeryová & Árendáš, 2024) explored the dependence structure within the biggest U.S. technology sectors. They applied GARCH vine copulas models to daily data collected from 1 July 2009 to 1 August 2022, spanning the period over the decade after the global financial crisis. The results of their study indicate moderate to strong positive dependence within the technology sector, featuring substantial tail risk. The C-vine copula specification is selected as the best model and Microsoft is found to be the key player in describing the dependence structure.

Ji et al. (2018) examined the transmission of financial risk from China to 10 Asian stock markets by utilizing the CoVaR measure derived from copula models. He collected daily data from Morgan Stanley Capital International (MSCI) stock indices covering the period 8 May 2001–23 March 2016. The results of his study show evidence of a positive correlation between China and Asian stock markets, and the CoVaR values of Asian stock markets are significantly lower than their corresponding VaR values.

As a result, a downward price movement in China's stock market has a measurable impact on downward price movements in Asian stock markets. Overall, his findings suggest that investors should consider the presence of asymmetric spillover effects from China when managing the downside and upside risks of international portfolios in these Asian stock markets.

Aloui et al. (2022) utilizes a combination of bivariate copulas and CoVaR to examine the transmission of risk from China to G7 countries before and during the COVID-19 pandemic. By employing daily data on stock and equity sectors from 1 January 2013 to 9 June 2021, the primary empirical findings indicate that, before the COVID-19 pandemic, stock markets exhibited positive relationships and systemic risk was similar across all countries.

However, during the COVID-19 outbreak, the level of interdependence increased for all G7 countries and the transmission of risk between upside and downside became, on average, greater for almost all stock markets. Their findings also provide evidence of higher exposure to market risk from China for the technology and energy sectors. Additionally, the study observes an asymmetrical transmission of risk from China to the G7 stock markets, with a higher intensity of downside risk spillovers before and during the spread of COVID-19.

Braiek et al. (2022) perform vine copula-based CoVaR on data from Islamic sector industries to capture their inter-linkages. The collected data concerned the Dow Jones Islamic Market index prices and the related sector indices covered the period from 3 January 2000 to 31 December 2015. They estimated CoVaR values for each sector conditional on the VaR of the Islamic market in order to evaluate the systemic impact of the sector distress and its effects across different periods.

The results of their work indicate that the technology, industrial, and financial sectors are the most systemically risky sectors in the pre-GFC. During the GFC and POST-GFC period, the energy, financial, utilities, and telecommunication sector weights decreased to a contribution of zero. The mean-CoVaR performs better than the mean-variance model, and systemic risk influences the minimum risk portfolio allocation, the interdependence structure between sectors, and the optimization model.

The other study is the work of Ji et al. (2018), where the authors apply tail dependence clustering to divide candidate assets into different groups during the crisis period from 3 January 2006 to 20 December 2014. The daily closed prices of the components of DAX 20 were collected and transformed into returns. The authors made use of the combination of cluster methodology with the ARMA-GARCH vine copula opinion pooling (AGVCOP) approach to select the minimum conditional value-at-risk portfolio. Then, they constructed diversified portfolios at the 1-day forecast horizon by adding the investor's information into the portfolio selection procedure based on minimizing the CVaR. The findings of their study indicate that many possible portfolios do not show stronger lower tail dependence than upper tail dependence.

It is revealed that Ji et al.'s (2018) work is limited when it comes to drawing an inclusive portfolio diversification level-based management recommendation and the magnitude of systemic risk that can be captured not by CVaR but by CoVaR measures. With this in mind, our study aims to investigate each cluster's risk impact on the portfolio through CoVaR and assess the portfolio diversification level using the entropy-based approach.

While our findings align with some previous studies, it is important to note that our research makes several novel contributions to the existing literature. Firstly, we extend the application of vine copulas and CoVaR to the specific context of the Johannesburg Stock Exchange during the COVID-19 pandemic, providing insights into an emerging market during a unique crisis period. Secondly, our comparative analysis of three data-centric investment strategies offers a new perspective on portfolio optimization in volatile markets.

Thirdly, our integration of entropy measures with copula-based risk assessments provides a more comprehensive framework for evaluating both diversification and systemic risk. Lastly, our focus on industry-level analysis in the JSE fills a gap in the literature, which has predominantly focused on inter-market dependencies or specific sectors. These contributions advance our understanding of risk management and portfolio diversification strategies in emerging markets during times of global crisis.

Despite extensive research exploring international stock market correlations, including studies on the South African market's relationship with global markets (Ali et al., 2018; W. Zhang et al., 2020; Yousaf et al., 2022; Chowdhury et al., 2022; Insaidoo et al., 2023), and intra-sectorial dependencies in various industries (Fasnacht & Louberge, 2007; Shahani et al., 2017; Braiek et al., 2022), there remains a gap in understanding regarding the specific dependence structure within the JSE, particularly during periods of high market stress like the COVID-19 pandemic.

This study addresses this gap by examining the tail dependence and systemic risk within the JSE using vine copulas and CoVaR, while also integrating entropy measures to provide a more comprehensive assessment of diversification benefits across different data-centric investment strategies.

3. Empirical Results

3.1. Data Description

This study analyzes the daily prices of 10 industry sectors in the Johannesburg stock market during the COVID-19 pandemic, collected from the IRESS I-NET DATABASE. We start with 10 sectors (the number of JSE industry sectors) but, to avoid the ubiquitous problem of a singular matrix, 1 industry sector is dropped during the data-cleaning process. Nine sectors are retained, including basic materials (basic mat), consumer discretionary (cons discret), energy, financial, healthcare, industrial, technology (techno), telecommunication (telecom), real estate, and consumer staples (cons staples).

All the data used are sourced from IRSS-INET-DATABASE. The classification of these sector indices is performed based on the Industry Classification Benchmark (ICB) created by the Financial Times Stock Exchange (FTSE). Stock returns are calculated as the logarithm difference between two consecutive daily prices, yielding 629 daily return data for each stock.

The decision to drop one sector was made to prevent the occurrence of a singular covariance matrix, which is a frequent challenge in portfolio optimization when the number of observations is limited compared to the number of assets. This singularity can arise as a result of factors such as high collinearity between sectors or low variance within a particular industry. Although omitting one sector may introduce slight bias, avoiding the unreliability associated with a singular matrix is preferable. Future studies with a larger dataset could investigate alternative methods, such as dimensionality reduction techniques, to further mitigate this issue.

Figure 1 below illustrates the volatility of each of the nine sectors under consideration. All the sectors are more volatile and show volatility clustering.

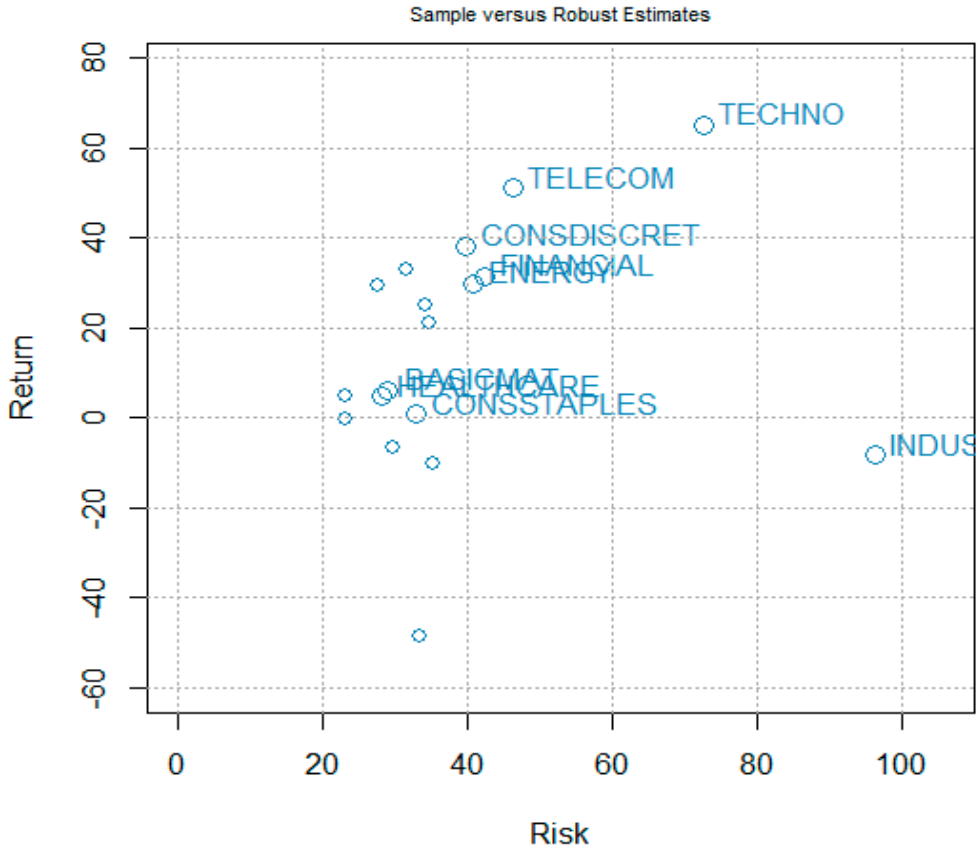


Figure 1. Asset risk–return plot.

The graphical assessment of the asset risk–return tradeoff (as shown in Figure 1) reveals four groups of sectors with very close risk characteristics. The first group includes basic materials, healthcare, and consumer staples, and the risk scale varies between 20 and 40. The second group includes consumer discretionary, energy, financial, and telecommunication, and the risk scale varies between 40 and 60. The third group and fourth group include the industrial and technology sectors, respectively. The technology sector is between 60 and 80 on the risk scale, and the industrial sector is between 80 and 100.

The grouping of sectors into different clusters based on the hierarchical clustering algorithm brings forth four clusters. The first cluster includes the basic materials, healthcare and consumer staples, consumer discretionary, energy, and financial sectors. The second cluster consists of the industrial sector. The third cluster and fourth cluster include the technology and telecommunication sectors, respectively. Therefore, this study investigates three portfolios of assets based on different aspects: the portfolio of JSE sectors, the portfolio of clusters of sectors, and the portfolio of groups of sectors.

Figure 2 presents a time series plot displaying the trends of various assets over time. Each asset shows significant variability, suggesting different levels of stability and trends over the observed period.

Tables 1 and 2 contain the descriptive statistics for the returns observations for each portfolio, in which the mean values of returns are positive for all of the sectors except for the industrial sector. Almost all the returns show high kurtosis, suggesting fat tails in the return distribution. The values of skewness are positive in many cases. The Jarque–Bera normality test rejects the null hypothesis of return normality in many cases.

Each cluster and group are represented by a time series obtained using the following procedure: First, we calculate the Markowitz mean-variance portfolio weights for the stocks in each cluster and group. Second, we calculate the weighted daily return by multiplying

the daily returns of each stock by its corresponding Markowitz MVP weight. Lastly, we compute the average of the weighted returns to obtain the daily average return of the cluster or group. This procedure provides a more nuanced representation of the cluster's performance, as it accounts for the risk-based optimal allocation of the stocks within the cluster or group.

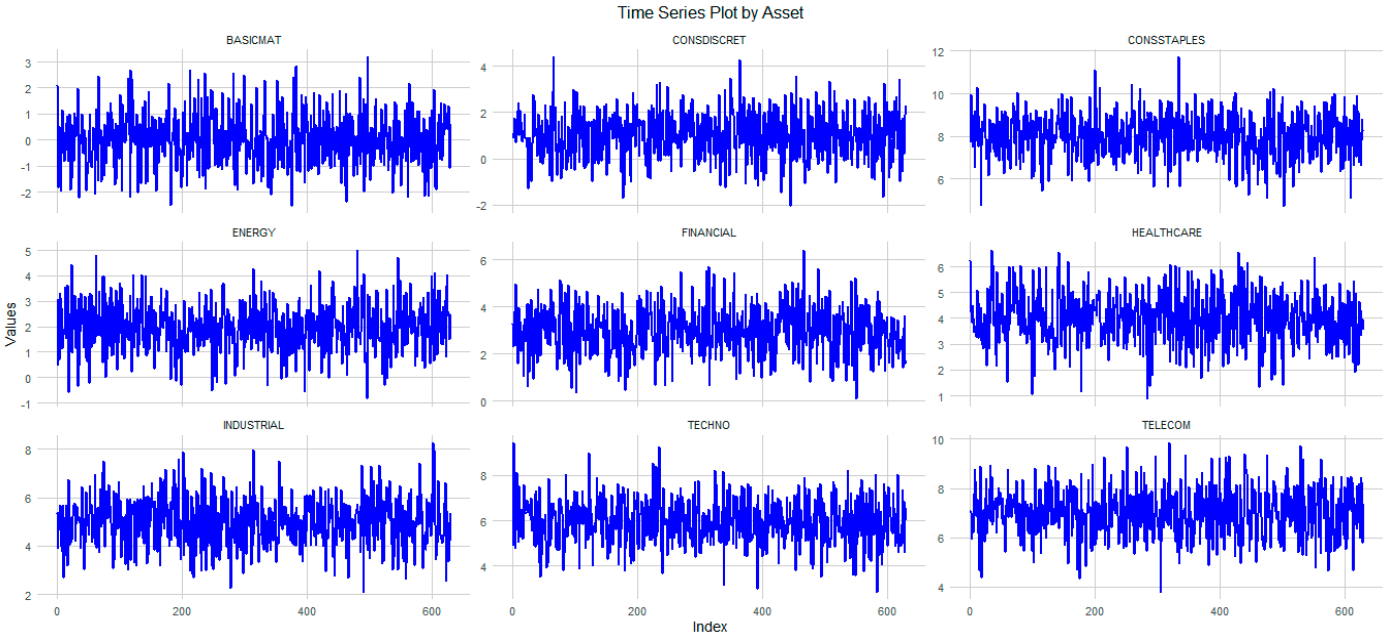


Figure 2. Time series of 9 industry sectors.

Table 1. Descriptive statistics of sectors.

Vars	Basic Mat	Cons Discret	Energy	Financial	Healthcare	Industrial	Techno	Telecom	Cons Staples
Mean	0.02	0.15	0.12	0.12	0.02	−0.03	0.26	0.20	0.00
Sd	1.83	2.51	2.56	2.66	1.77	6.06	4.58	2.92	2.07
Median	0.01	0.00	0.18	0.12	−0.02	0.00	0.00	0.00	−0.08
Min	−6.99	−9.55	−11.82	−12.86	−6.16	−32.23	−14.08	−11.99	−7.06
Max	7.86	11.33	11.10	16.96	6.75	84.73	46.79	13.80	12.71
Skew	0.12	0.32	−0.08	0.30	0.12	3.99	3.97	0.42	0.51
Kurtosis	1.63	2.88	2.41	5.32	1.15	63.78	34.30	2.59	3.12
ADF Test	−20.40 *	−17.50 *	−19.50 *	−19.80 *	−21.60 *	−19.10 *	−17.40 *	−17.50 *	−18.00 *
KPSS Test	0.271	0.091	0.098	0.037	0.090	0.088	0.148	0.358	0.091
ARCH Test	27.30 **	69.60 **	17.10 **	127.50 **	24.40 **	1.20	7.50	17.00 **	20.50 **
White test	16.41 *	25.88 *	7.52 **	70.29 *	12.99 *	5.80	3.94	19.67 *	20.52 *
Sample	629	629	629	629	629	629	629	629	629

Notes: ADF refers to the Augmented Dickey–Fuller test statistic. The asterisk * indicates statistical significance at the 1% level. The KPSS test stands for the Kwiatkowski–Phillips–Schmidt–Shin test. It is a statistical test used to assess the stationarity of a time series. Both ADF and KPSS tests indicate that all the series are stationary at a specific level. Moreover, the null hypothesis of the KPSS test posits that the time series is stationary around a constant mean; this is referred to as level stationarity. In this study, the *p*-values obtained from the KPSS test consistently remained around 10%. This indicates a failure to reject the null hypothesis, suggesting that all series can be regarded as stationary. Consequently, the results reinforce the conclusion that the time series does not display non-stationarity. ARCH refers to the Autoregressive Conditional Heteroskedasticity test statistic. The double asterisks (**) indicate the 95% confidence level significance. The results show that the industrial and technology sectors do not exhibit ARCH effects. In contrast, the other seven sectors do display ARCH effects.

In the literature on finance, the Brock–Dechert–Scheinkman (BDS) test is commonly used to detect nonlinearity in time series data. As shown in Table 3, the BDS test assesses whether a time series is independent and identically distributed (i.i.d.) by examining the correlation of data in a multidimensional space. The BDS test effectively identifies nonlinearity in time series data, making it a valuable tool in this context.

Table 2. Descriptive statistics of clusters and groups of sectors.

	Clustering Sectors				Grouping Sectors			
Vars	**Cluster1**	**Cluster2**	**Cluster3**	**Cluster4**	**Group1**	**Group2**	**Group3**	**Group4**
Mean	0.08	−0.03	0.26	0.20	0.01	0.15	−0.03	0.26
Sd	1.33	6.06	4.58	2.92	1.28	1.65	6.06	4.58
Median	0.06	0.00	0.00	0.00	−0.03	0.13	0.00	0.00
Min	−6.44	−32.23	−14.08	−11.99	−4.24	−7.61	−32.23	−14.08
Max	6.33	84.73	46.79	13.30	5.29	7.14	84.73	46.79
Skew	0.29	3.99	3.97	0.42	0.27	0.12	3.99	3.97
Kurtosis	3.27	63.78	34.30	2.59	1.36	2.59	63.78	34.30
ADF Test	−18 *	−19.1 *	−17.4 *	−17.5 *	−18.9 *	−17.8 *	−19.1 *	−17.4 *
KPSS Test	0.179	0.09	0.15	0.36	0.22	0.26	0.09	0.15
ARCH Test	46.40 **	1.20	7.50	17.00 **	36.00 **	44.90 **	1.20	7.50
White test	53.45 *	5.80	3.94	19.67 *	33.63 *	33.54 *	5.80	3.94
Sample	629	629	629	629	629	629	629	629

Notes: ADF refers to the Augmented Dickey–Fuller test statistic. The asterisks * and ** indicate statistical significance at the 1% and 5% levels, respectively. Both ADF and KPSS tests indicate that all the series are stationary at a specific level. The ARCH results show that cluster2, cluster3, group3, and group4 do not exhibit both ARCH effects. In contrast, cluster1, cluster4, group1, and group2 do exhibit ARCH effects and forms of heteroskedasticity.

Table 3. BDS test results.

Sector	Embedding Dimension	Epsilon	Test Statistic	*p*-Value
Basic mat	2	0.91	4.82	0.00
Basic mat	2	1.83	4.88	0.00
Basic mat	2	2.74	4.07	0.00
Basic mat	2	3.66	2.95	0.00
Basic mat	3	0.91	5.51	0.00
Basic mat	3	1.83	5.97	0.00
Basic mat	3	2.74	5.16	0.00
Basic mat	3	3.66	4.19	0.00
Cons discret	2	1.25	6.30	0.00
Cons discret	2	2.50	6.36	0.00
Cons discret	2	3.76	6.65	0.00
Cons discret	2	5.01	6.49	0.00
Cons discret	3	1.25	8.05	0.00
Cons discret	3	2.51	7.14	0.00
Cons discret	3	3.76	7.01	0.00
Cons discret	3	5.01	6.74	0.00
Energy	2	1.28	3.63	0.00
Energy	2	2.56	4.81	0.00
Energy	2	3.85	2.70	0.00
Energy	2	5.13	2.43	0.01
Energy	3	1.28	2.39	0.01
Energy	3	2.56	2.64	0.00
Energy	3	3.85	2.92	0.00
Energy	3	5.13	4.93	0.00
Financial	2	1.33	6.40	0.00
Financial	2	2.66	6.77	0.00
Financial	2	3.99	7.24	0.00
Financial	2	5.33	8.55	0.00
Financial	3	1.33	6.74	0.00
Financial	3	2.66	7.33	0.00
Financial	3	3.99	7.95	0.00
Financial	3	5.33	9.27	0.00
Healthcare	2	0.89	4.69	0.00
Healthcare	2	1.77	4.62	0.00
Healthcare	2	2.66	4.65	0.00
Healthcare	2	3.54	4.12	0.00
Healthcare	3	0.89	4.50	0.00
Healthcare	3	1.77	4.67	0.00
Healthcare	3	2.66	4.94	0.00
Healthcare	3	3.54	4.84	0.00

Table 3. *Cont.*

Sector	Embedding Dimension	Epsilon	Test Statistic	*p*-Value
Industrial	2	3.03	5.86	0.00
Industrial	2	6.06	5.59	0.00
Industrial	2	9.10	6.24	0.00
Industrial	2	12.13	6.98	0.00
Industrial	3	3.03	6.93	0.00
Industrial	3	6.06	6.63	0.00
Industrial	3	9.10	7.18	0.00
Industrial	3	12.13	7.56	0.00
Techno	2	2.30	5.77	0.00
Techno	2	4.58	6.03	0.00
Techno	2	6.88	5.20	0.00
Techno	2	9.17	4.44	0.00
Techno	3	2.30	7.12	0.00
Techno	3	4.58	6.86	0.00
Techno	3	6.88	5.39	0.00
Techno	3	9.17	4.38	0.00
Telecom	2	1.46	4.08	0.00
Telecom	2	2.92	3.34	0.00
Telecom	2	4.38	3.12	0.00
Telecom	2	5.84	2.36	0.02
Telecom	3	1.46	4.24	0.00
Telecom	3	2.92	3.71	0.00
Telecom	3	4.38	3.62	0.00
Telecom	3	5.84	3.04	0.00
Cons staples	2	1.04	4.40	0.00
Cons staples	2	2.07	4.99	0.00
Cons staples	2	3.11	5.62	0.00
Cons staples	2	4.15	6.36	0.00
Cons staples	3	1.04	6.05	0.00
Cons staples	3	2.07	6.68	0.00
Cons staples	3	3.11	7.10	0.00
Cons staples	3	4.15	7.46	0.00

The test is conducted for embedding dimensions of 2 and 3, allowing the BDS test to evaluate data structure using pairs or triples of points. Epsilon values represent the distance threshold for identifying close points in the time series. The test statistics for each combination of embedding dimension and epsilon are presented in the table under the standard normal heading, indicating the deviation of observed statistics from the expected values under the null hypothesis of i.i.d.

The BDS test results show that all the *p*-values are less than 5%, this indicates nonlinearity in the sector's returns, prompting the consideration of techniques such as GARCH modeling to capture the series dynamics.

3.2. ARMA-GARCH Model Selection

We employ an ARMA(1,1)-GJR-GARCH(1,1) to model the series that display conditional heteroskedasticity to capture the stylized facts often observed in financial time series data, including volatility clustering, leptokurtosis, and leverage effects. While other GARCH specifications were considered, the ARMA(1,1)-GJR-GARCH(1,1) model was selected based on the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), which balance model fit with complexity. Nevertheless, we will use a simpler model such as ARMA(1,1) for series that do not exhibit ARCH effects. The estimated coefficients for ARMA (1,1)-GJR-GARCH (1,1) are shown in Table 4.

In summary, the GARCH models demonstrate significant volatility persistence (high β), but provide relatively limited evidence of immediate relationships in returns (low

significance in AR and MA components). The findings indicate that $\alpha + \beta < 1$, which implies the model is stationary and that volatility shocks diminish over time. The parameters for skewness and shape suggest the model can effectively capture asymmetry and heavy tails, characteristics which are commonly found in financial data. Additionally, certain parameters, such as mu, AR(1), and omega, lack statistical significance, indicating they may not have a meaningful impact on the model.

Table 4. (**a**) Estimated coefficients for ARMA (1,1)-GJR-GARCH (1,1). (**b**) Estimated coefficients for ARMA (1,1)-GJR-GARCH (1,1).

(a)

	B. Mat	Discr	Energy	Financial	HC	Telecom	Staple
mu (μ)	0.02	0.22	0.10	0.08	0.00	0.13	−0.04
	(0.34)	(3.02)	(1.03)	(0.90)	(−0.11)	(1.16)	(−0.59)
AR(1)	−0.59	0.09	−0.69	−0.18	0.30	0.01	−0.87
	(−1.22)	(0.33)	(−1.11)	(−0.25)	(1.93)	(0.01)	(−5.03)
MA(1)	0.62	−0.20	0.69	0.20	−0.54	0.03	0.89
	(1.33)	(−0.76)	(1.10)	(0.28)	(−3.98)	(0.06)	(5.56)
Omega	1.01	0.19	0.15	0.31	0.78	0.20	0.43
(ω)	(2.59)	(1.59)	(1.04)	(1.42)	(2.70)	(1.38)	(0.67)
Alpha1	0.25	0.10	0.03	0.07	0.19	0.03	1.02
(α)	(2.30)	(2.28)	(1.63)	(1.26)	(2.63)	(1.79)	(0.31)
Beta1	0.51	0.85	0.94	0.85	0.54	0.94	0.97
(β)	(3.70)	(20.54)	(30.30)	(10.98)	(4.30)	(31.85)	(44.75)
Gamma1	−0.09	0.08	0.01	0.07	0.04	0.02	0.03
(γ)	(−0.74)	(1.30)	(0.38)	(1.32)	(0.38)	(0.59)	(1.15)

(b)

Parameters	Cluster1	Cluster4	Group1	Group2
mu (μ)	0.06	0.13	−0.02	0.15
	(1.25)	(1.16)	(−0.34)	(2.52)
AR(1)	−0.33	0.01	−0.49	−0.88
	(−1.05)	(0.01)	(−1.10)	(−12.28)
MA(1)	0.41	0.03	0.53	0.93
	(1.35)	(0.06)	(1.21)	(17.01)
Omega	0.07	0.20	0.11	0.24
(ω)	(1.77)	(1.38)	(1.30)	(1.48)
Alpha1	0.06	0.03	0.08	0.12
(α)	(1.73)	(1.79)	(1.38)	(1.93)
Beta1	0.87	0.94	0.83	0.81
(β)	(17.81)	(31.85)	(9.09)	(7.52)
Gamma1	0.06	0.02	0.05	−0.04
(γ)	(1.15)	(0.59)	(0.72)	(−0.78)

Note: Table 4a provides the efficient estimates for ARMA(1,1)-GJR-GARCH(1,1) model applied to a portfolio of seven return series. The values of t-statistics are reported in parentheses and can be compared to the critical values of 2.33, 1.645, and 1.28, corresponding to the confidence levels of 99%, 95%, and 90%, respectively. Table 4b provides the efficient estimates for arma(1,1)-GJR-GARCH(1,1) applied to portfolios comprising two clusters and two groups of return series, respectively. The t-statistics are reported in the parenthesis and can be compared to the critical values of 2.33, 1.645, and 1.28, corresponding to confidence levels of 99%, 95%, and 90%, respectively. To clarify the distinction between the two data-centric approaches, clusters from the asset risk–return plot approach referred to as GROUPS. In contrast, those obtained from the clustering method are referred to as CLUSTERS.

The gamma parameter is utilized to capture the leverage effect, which reveals that negative shocks, such as price declines, have a different, often greater, impact on volatility compared to positive shocks like price increases. A positive gamma indicates that negative shocks result in a larger increase in future volatility than positive shocks of the same magnitude. Within the GJR-GARCH framework, a non-zero gamma demonstrates the model's ability to account for the observation that volatility tends to rise more significantly after negative news than after positive news.

The results demonstrate that the gamma parameters are recorded as zero across all sectors, suggesting that the model does not provide evidence of asymmetry in the volatility responses for these specific datasets. This implies that, according to the model, negative and positive shocks have similar effects on future volatility for the variables represented. If gamma consistently remains at zero, it may indicate that the return distributions do not exhibit the typical leverage effect often seen in many financial time series.

Overall, the GARCH models show strong volatility persistence (high β) but relatively weak evidence for immediate relationships in returns (low significance in AR and MA terms). The results show that $\alpha + \beta < 1$, suggesting that the model is stationary, and shocks to volatility decay over time. The skewness and shape parameters indicate that the model can accommodate asymmetry and heavy tails, which are typical in financial data. Some parameters, like mu, ar1, and omega, are not statistically significant, suggesting that they may not contribute meaningfully to the model.

Entropy of Returns

The entropy of returns helps us understand the dynamics and predictability of the investment portfolio. A lower entropy value may be preferred for more stable and predictable investments, whereas a higher level of entropy is linked to investments that exhibit volatility and unpredictability. Additionally, the entropy of returns can provide valuable insights into one's risk management approaches and facilitate the process of making well-informed investment choices.

Table 5 gives the level of entropy for each asset within a portfolio. The results show that the Energy sector has the highest entropy value (6.4441) in the portfolio of sectors, followed by the financial and consumer staples sectors, whereas the industrial sector exhibits the lowest entropy value among sectors (5.4534). In the case of portfolios of clusters and groups of sectors, it appears that cluster1, Group1, and Group2 exhibit higher entropy values while cluster2 and Group2 show lower entropy values.

Table 5. Entropy of returns.

	Basic Mat	Cons Discret	Energy	Financial	Healthcare	Industrial	Techno	Telecom	Cons Staples
Entropy	6.40	6.17	6.44	6.44	6.34	5.45	6.04	6.17	6.44
	Group1	Group2	Group3	Group4		Cluster1	Cluster2	Cluster3	Cluster4
Entropy	6.44	6.44	5.45	6.04		6.44	5.45	6.04	6.17

3.3. Vine Copula Selection

This study focuses on C-vine and R-vine copulas due to their flexibility in modeling complex dependence structures. C-vine copulas are well suited for capturing star-shaped dependencies, where a central variable influences others, while R-vine copulas offer a more general approach. We compare these two vine copula types to determine the most appropriate representation of the dependence structure within the JSE sectors.

Table 6 presents details of the trees from the C-vine copula. The detailed tables of the trees exhibit the C-vine tree structure, selected copula family, and the fitted Kendall's tau value. In Tree 1, the most common pair copula families are normal, I, Student-t, Frank, and SBB8. These copula families are often detected in financial datasets. Tree 1 identifies the financial sector (4) as the root node in the dependence structure network. This implies that the financial sector influences all the sectors (except the consumer staples sector).

Table 7 presents the estimated parameters of the R-vine copula description for all pair copula families instigated, revealing that the most frequently selected copula families are G, t, N, and SBB8.

Table 6. Optimal C-vine copula specifications of each return pair within the portfolio of 9 sectors.

Tree	Edge	Family	Cop	Par		Par2	Tau	Utd	Ltd
1	4.1	1	N 0.33	(0.03)		-	0.21	-	-
	4.8	0	I	-			0.00	-	-
	4.7	2	t 0.19	(0.04)	7.45	(2.54)	0.12	0.04	0.04
	4.2	2	t 0.14	(0.04)	7.71	(2.92)	0.09	0.03	0.03
	4.6	2	t 0.21	(0.04)	14.67	(9.08)	0.14	0.01	0.01
	4.5	5	F 1.15	(0.24)			0.13	-	-
	4.3	20	SBB8 1.97	(0.38)	0.80	(0.11)	0.19	-	-
	9.4	0	-	-		-	0.00	-	-

Type: C-vine; logLik: 0; AIC: 36; BIC: 128.47. Note: "Tau" represents Kendall's tau, a measure of rank correlation. "Utd" and "Ltd" denote the upper and lower tail dependence coefficients, respectively, which quantify the strength of dependence in the tails of the joint distribution. Upper Tail Dependence (Utd): This coefficient measures the probability of one asset experiencing an extreme upward movement given that another asset has also experienced an extreme upward movement. A higher Utd indicates a stronger tendency for assets to move together during extreme positive market events (e.g., booms or rallies). Lower Tail Dependence (Ltd): This coefficient measures the probability of one asset experiencing an extreme downward movement given that another asset has also experienced an extreme downward movement. A higher Ltd. indicates a stronger tendency for assets to move together during extreme negative market events (e.g., crashes or crises). 1 <-> BASICMAT, 2 <-> CONSDISCRET, 3 <-> ENERGY, 4 <-> FINANCIAL, 5 <-> HEALTHCARE, 6 <-> INDUSTRIAL, 7 <-> TECHNO, 8 <-> TELECOM, 9 <-> CONSSTAPLES.

Table 7. Optimal R-vine copula specifications of each return pair within the portfolio of 9 sectors.

Tree	Edge	Family	Cop	Par		Par2	Tau	Utd	Ltd
1	7.4	2	t 0.19	(0.04)	7.45	(2.54)	0.12	0.04	0.04
	8.9	0		-	-	-	0.00	-	-
	1.2	4	G 1.09	(0.03)		-	0.09	0.11	-
	4.6	2	t 0.21	(0.04)	14.67	(9.08)	0.14	0.01	0.01
	4.1	1	N 0.33	(0.03)			0.21	-	-
	3.5	1	N 0.23	(0.04)			0.15	-	-
	4.3	20	SBB8 1.97	(0.38)	0.80	(0.11)	0.19	-	-
	1.8	0		-	-	-	0.00	-	-

Note: "Tau" represents Kendall's tau, a measure of rank correlation. "Utd" and "Ltd" denote the upper and lower tail dependence coefficients, respectively, which quantify the strength of dependence in the tails of the joint distribution. Type: R-vine; logLik: 0; AIC: 40; BIC: 142.75. 1 <-> BASICMAT, 2 <-> CONSDISCRET, 3 <-> ENERGY, 4 <-> FINANCIAL, 5 <-> HEALTHCARE, 6 <-> INDUSTRIAL, 7 <-> TECHNO, 8 <-> TELECOM, 9 <-> CONSSTAPLES.

Both the C-vine (Table 6) and R-vine (Table 7) models reveal generally weak tail dependence (utd and ltd) among the JSE sectors. This suggests that during the COVID-19 period, extreme market movements in one sector did not strongly translate to similar extreme movements in other sectors. This could indicate a degree of resilience within the JSE, potentially due to the diverse nature of the South African economy.

Comparison Between C-Vine Copula and R-Vine Copula

When working with a multivariate dataset, it can be advantageous to choose a reliable vine copula. This type of copula model can effectively capture different types of dependence structures. The comparison between the C-vine and D-vine copula is based on the values of Log-likelihood (Loglik), BIC, and AIC.

Regarding the fit of a statistical model to the observed data, as well as its complexity, C-vine copula models seem to be the most suitable for examining the dependence structure among industry sectors in the JSE.

The results from Table 8 confirm that C-vine copula models have the highest number of parameters and the lowest AIC value and are preferred over R-vine copula.

The C-vine model designates the financial sector as the root node, indicating it has the most significant overall dependence on other sectors (except consumer staples). This is consistent with the financial sector's central role in the economy. Conversely, the R-vine

model does not identify a single root node but still emphasizes the strong links of the financial sectors.

Table 8. Estimated Loglikelihood, number of parameters (par), AIC, and BIC for both fitted models.

Models	Log-Likelihood	Number of Parameters	AIC	BIC
C-vine copula	0	8	36	128.47
R-vine copula	0	6	40	142.75

The most frequently selected bivariate copula family is Student-t. The Student-t copula captures both upper and lower tail dependence, while the SG copula is particularly effective at modeling lower tail dependence (co-movements in extreme downward market events). The prevalence of these copulas suggests that while tail dependence is generally weak, it is not entirely absent, and the JSE sectors may exhibit some co-movements during extreme market downturns.

The Tau coefficients, representing rank correlation, are generally low to moderate, further supporting the finding of weak overall dependence. The highest Tau values are often associated with the financial sector, reinforcing its central role in the dependence structure.

It can be seen from Table 9 that the most common pair copula families are I (integrated copula) and SBB1. However, SBB1 is the unique bivariate copula family that provides parameters and upper and lower tail dependence values. The results show that the first group of sectors, as the root node, influences the second group, and their dependence structure is higher in the lower tail (24%). Based on the results, we can see that there is no dependence structure among other groups of sectors. The central node order for the C-vine specification is four for group4, one for group1, and three for group3.

Table 9. Optimal C-vine copula specifications of each return pair within the portfolio of 4 groups of sectors.

Tree	Edge	Family	Cop		Par		Par2	Tau	Utd	ltd
1	1.2	17	SBB1	0.13	(0.07)	1.23	(0.05)	0.24	0.01	0.24
	1.3	0	I		-		-	0.07	-	-
	4.1	0	I		-		-	0.00	-	-

Type: C-vine; log-Lik: 0; AIC: 4; BIC: 14.27. 1: GROUP1, 2: GROUP2, 3: GROUP3, 4: GROUP4.

It can be seen from Table 10 that SBB1 and I are the bivariate copula families that provide a good fit of clusters of the JSE sectors. Therefore, SBB1 is the one that estimates the dependence structure among clusters and the tail dependence structure. Based on the results, we can see that cluster1 appears as the root node, leading to cluster2 and Cluster4. The central node ordering for the C-vine specification is 1 for cluster1, 3, and 2 for cluster2.

Table 10. Optimal C-vine copula specifications of each return pair within the portfolio of 4 clusters of sectors.

Tree	Edge	Family	Cop			Par	Par2	Tau	Utd	Ltd
1	1.2	0	I		-	-	-	0.00	-	-
	1.4	7	SBB1	0.08	(0.06)	1.07	(0.03)	0.10	0.09	0.00
	3.1	0	I		-		-	0.00	-	-

Type: C-vine; logLik: 26.5; AIC: −49; BIC: −40.1. 1: CLUSTER1, 2: CLUSTER2, 3: CLUSTER3, 4: CLUSTER4.

The SBB1 copula in Tables 9 and 10 is prominent in these models. The SBB1 copula allows for asymmetric tail dependence, which could be relevant for capturing the different responses of groups to positive and negative shocks. The root nodes in these models (Tree 1

in Tables 9 and 10) indicate the groups/clusters that have the strongest influence on others within respective portfolios.

It has been observed, across Tables 6, 7, 9 and 10, that the Utd and Ltd. values are generally low, often close to zero. This reinforces the observation of weak tail dependence among JSE sectors, groups, and clusters. This suggests that even during extreme market movements (both positive and negative), the JSE sectors tend to behave relatively independently.

3.4. Estimates of CoVaR

In this study, we provide the estimates of CoVaR at 1%, 5%, and 10% confidence levels that measure the portfolio systemic risk, as shown in Tables 11 and 12.

Table 11. Portfolio CoVaR conditional on a specific sector being in distress and on sector weights.

Portfolio	Sector in Distress								
	Basic Mat.	Cons Discr.	Energy	Financial	Healthcare	Industrial	Techno	Telecom	Cons Staple
CoVaR (1%)	0.90	1.10 **	0.96	0.95	0.97	−4.63 *	−4.62 *	0.91	0.90
CoVaR (5%)	0.93	1.02 **	0.96	0.89	0.85	−2.65 *	−2.66 *	0.91	0.90
CoVaR (10%)	0.91	1.00 **	0.89	0.84	0.79	−1.83 *	−1.82 *	0.88	0.81

Notes: The asterisks * and ** indicate the negative and the highest values of the portfolio CoVaR, respectively, for each confidence level.

Table 12. Portfolio CoVaR conditional on a specific group/cluster being in distress.

Portfolio	Group/Cluster in Distress							
	Group1	Group2	Group3	Group4	Cluster1	Cluster2	Cluster3	Cluster4
CoVaR (1%)	0.81	0.83 **	−4.32 *	−4.30 *	0.81 **	−4.28 *	−4.27 *	0.79
CoVaR (5%)	0.72	0.74 **	−2.24 *	−2.24 *	0.78 **	−2.15 *	−2.15 *	0.74
CoVaR (10%)	0.65 **	0.64	−1.41 *	−1.42 *	0.61	−1.34 *	−1.35 *	0.71 **

Notes: The asterisks * and ** indicate the negative and the highest values of the portfolio CoVaR, respectively, for each confidence level.

Table 11 shows that the industrial and technology sectors have greater contributions to the negative portfolio CoVaR values at 1%, 5%, and 10% confidence levels. The consumer discretionary sector contributes more to the highest portfolio CoVaR value at 1%,5%, and 10% confidence levels. The negative value suggests that the overall portfolio may fare better or experience less severe losses when the industrial and technology sectors are stressed.

The industrial and technology sectors exhibit negative CoVaR across different confidence levels, emphasizing the need to understand their countercyclical nature. These sectors often perform well during wider economic downturns, driven by consistent demand, a shift in investment toward safer options, and long-term trends that promote innovation and sustainability. Grasping these dynamics is vital for investors and policymakers who are aiming to address economic challenges effectively.

Table 12 provides results of systemic risk measures for the portfolio of sectors clustering and the portfolio of sectors grouped according to risk–return characteristics. The results show that group3 (which includes the industrial sector) contributes more to the negative portfolio CoVaR value, whereas group2 (which includes the consumer discretionary, energy, financial, and telecommunication sectors) contributes more to the highest portfolio CoVaR value. In terms of portfolio clusters, the results indicate that cluster2 (which includes the

industrial sector) contributes more to the negative portfolio systemic measure value at 1% and 5% confidence levels, whereas cluster1 (which includes the basic materials, healthcare and consumer staples, consumer discretionary, energy, and financial sectors) contributes more to the highest portfolio systemic risk value at 1% and 5% confidence levels.

These findings have important implications for portfolio management. The industrial sector's lower CoVaR indicates that it could provide a degree of portfolio stability during market downturns.

Overall, across different confidence levels and portfolio constructions, the industrial sector (or groups/clusters containing it) remains the most significant contributor to the portfolio CoVaR. This highlights the industrial sector's vulnerability to systemic shocks and its potential to amplify risk within a portfolio.

Conversely, the healthcare sector often contributes the least to portfolio CoVaR, suggesting it might offer a degree of stability during market downturns. This is likely due to the sector's essential nature and relative insulation from economic fluctuations.

In analyzing portfolio weights for sectors on the Johannesburg Stock Exchange (JSE), we reference Table 13, which outlines these weights. It shows allocations in both a mean-variance efficient portfolio and an equal-weighted portfolio. The efficient portfolio emphasizes the Industrial and Technology sectors, while the equal-weighted portfolio distributes weights evenly across all sectors.

Table 13. Portfolio weights for JSE sectors.

Portfolio	Weights								
	Basic Mat	Cons Discr	Energy	Financial	Healthcare	Industrial	Techno	Telecom	Cons Staple
M-V Efficient portfolio	0.002	0.001	0.007	0.004	0.001	0.491	0.491	0.003	0.0003
Equal weighted portfolio	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9

Note: We use GARCH-Copula and mean-variance optimization to compute asset weights efficiently. An equal-weighted portfolio is another strategy that can equally distribute weights among all the assets.

Table 14 subsequently details the portfolio weights for sector groups. It indicates that the efficient portfolio gives notable weights to Group3 and Group4, maintaining an equal distribution in the equal-weighted portfolio.

Table 14. Portfolio weights for groups/clusters of sectors.

Portfolio	Weights							
	Group1	Group2	Group3	Group4	Cluster1	Cluster2	Cluster3	Cluster4
Efficient portfolio	0.003	0.003	0.505	0.489	0.003	0.504	0.489	0.003
Equal weighted portfolio	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25

Note: We use GARCH-Copula and mean-variance optimization to compute asset weights efficiently. An equal-weighted portfolio is another strategy that can equally distribute weights among all the assets.

Portfolio Diversification Levels

We use Shannon entropy and Tsallis entropy to measure portfolio diversification, quantify uncertainty or information content, and assess investment portfolio diversity. The dual approach, using probability distributions and asset weights, offers a nuanced understanding of diversification, identifying portfolios with diverse asset types and balanced investment allocations.

Table 15 compares the diversification levels among different types of portfolios using both the Tsallis entropy and Shannon entropy values for each portfolio type. While Tsallis entropy measures the diversity and uncertainty in the distribution of asset weights in a portfolio, Shannon entropy, on the other hand, also measures diversity and uncertainty, but it does so under the assumption of additivity and independence of the asset distributions.

The findings show that the portfolio of sectors exhibits the highest Tsallis entropy, suggesting it has the best diversity among the types analyzed, albeit said diversity is still relatively low overall. However, the portfolios of groups and clusters of sectors show similar levels of diversity and concentration, with identical entropy values.

Table 15. Comparison of diversification levels among portfolios.

Type of Portfolio	Tsallis Entropy Based on Asset Weights with q = 2	Shannon Entropy Based on Asset Weights
Portfolio of sectors	0.52	2.97
Portfolio of groups of sectors	0.51	1.05
Portfolio of clusters of sectors	0.51	1.05

Entropy findings highlight greater volatility and unpredictability in the energy sector (Group1, Group2, and cluster1), appealing to high-return investors but indicating a higher level of risk. Conversely, the industrial sector presents stable, predictable returns with lower growth potential.

3.5. Discussions and Implications of the Study

This study employs data-centric approaches and the ARMA-GARCH vine copula-based CoVaR model to robustly assess systemic risk, return, and diversification in the JSE market in the context of the COVID-19 pandemic. The integrated approach compares three investment strategies: industry sector-based, asset risk–return plot-based, and clustering-based strategies. The findings suggest that the industrial and technology sectors do not exhibit ARCH effects and remain isolated from other sectors as clusters, indicating potential diversification opportunities.

The ARMA-GJR-GARCH model captures the returns series dynamics, while the Vine-copula-based CoVaR method addresses extreme risk spillovers. This sophisticated and robust approach to risk modeling demonstrates the researchers' attention to model robustness. The selection of the significant Vine-copula, considering the number of estimated parameters, log-likelihood, and BIC criteria, further enhances the model's reliability. The use of C-vine and R-vine copulas reveals weak tail dependence among JSE sectors, suggesting that substantial fluctuations in one sector have minimal impact on others, highlighting the resilience of the South African economy. This finding is supported by Bonga-Bonga and Hendriks (2024), Csizmás et al. (2021) and Dewick and Liu (2022).

Entropy measures, such as the entropy of returns, Tsallis entropy, and Shannon entropy, provide insights into the analyzed portfolios' dynamics, predictability, and diversification levels. The implication that the energy sector, group1, group2, and cluster1 are more volatile and unpredictable due to the highest values of the entropy of returns is particularly relevant for risk management and investment decision-making. Interestingly, the portfolio of sectors exhibits the highest Tsallis entropy, suggesting it has the best diversity among the types analyzed, albeit said diversity is still relatively low overall. However, the portfolios of groups and clusters of sectors show similar levels of diversity and concentration to their identical entropy values.

The study's findings offer valuable insights for investors and policymakers, guiding investment strategies and portfolio optimization choices in the JSE market, especially during turbulent periods like the COVID-19 pandemic. The insights into the diversification potential and risk dynamics of different industry sectors can assist policymakers and market regulators in monitoring and managing risk within the JSE market.

Overall, the study's combination of data-centric approaches, advanced risk modeling techniques, and insights into industry sector dynamics and diversification provides

a valuable contribution to understanding risk and portfolio management in the JSE market downturns.

The limited tail dependence among JSE sectors indicates potential diversification benefits even during the COVID-19 crisis, suggesting that the JSE, as an emerging market, might be less susceptible to extreme co-movements compared to developed markets, although further research is needed. The industrial sector's consistently low CoVaR suggests potential stability that may encourage risk-tolerant investors to increase their exposure, while others might consider diversifying their portfolios or exploring hedging strategies during periods of heightened systemic risk. The entropy results reveal higher volatility and unpredictability in the energy sector, Group1, Group2, and cluster1, which are attractive for high-return seekers but imply higher risk. In contrast, the industrial sector's lower entropy suggests stable, predictable returns with lower growth potential.

4. Methodology

Modeling the interdependence within the JSE sectors involves a focus on South Africa, which is widely acknowledged as the most prominent public trade industry. The research scope covers the COVID-19 period, from 28 March 2020 to 30 September 2022, and concerns sectors with available daily closing price data throughout this timeframe, resulting in nine sectors being selected for analysis.

This timeframe is justified, as 28 March 2020 represents a crucial moment in the COVID-19 timeline, marking when many countries, including South Africa, began to enforce strict lockdown measures. This date provides a clear starting point for analyzing the initial impact of the pandemic on financial markets. By extending the research period to 30 September 2022, the study also captures the recovery phase, during which economies began to reopen and adapt to new norms. This broader timeframe allows for a comprehensive assessment of how markets stabilized or evolved in response to ongoing challenges, such as emerging variants and shifting regulations.

Daily data are transformed into return series through the mathematical expression given as follows:

$$r_t = ln\left(\frac{P_t}{P_{t-1}}\right) \times 100 \quad (1)$$

where r_t is the logarithmic return of the asset at time t; P_t and P_{t-1} are the daily closing prices of the stock index at time t and t $-$ 1, respectively.

However, the presence of features such as volatility, non-stationary, and serial dependence among data returns necessitates using an ARMA-GARCH model to derive residuals suitable for subsequent analyses (Zheng et al., 2022).

Given that copula functions are based on uniform margins, standardized residuals from the ARMA-GARCH model are transformed into pseudo-observations and subjected to fitting a range of vine copula to ascertain the most suitable model for representing the interdependence structure within the JSE industry sector.

4.1. ARMA-GARCH Model

The use of an ARMA(p,q) model combined with GARCH(1,1) enables the extraction of standardized residuals from the data, offering accurate predictions despite its simplicity. Within the GARCH framework, the Glosten–Jagannathan–Runkle (GJR) model and Skewed Student-t innovations are incorporated to capture the leverage effect, asymmetric, and leptokurtic features observed in many financial time series (Joo & Ghulam, 2023).

The mathematical expression of an ARMA(p,q) − GARCH(p,q) is given as follows:

$$\begin{aligned} Y_t &= \mu + v_t \\ v_t &= \sigma_t.z_t \qquad Z_t \sim i.i.d \\ \sigma_t^2 &= \omega + \sum_{i=1}^{p} \alpha_i.v_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \end{aligned} \tag{2}$$

where ω, α_i , and β_j are the parameters to be estimated. $\omega \geq 0; \alpha \geq 0; \beta \geq 0$; μ is a constant term, v_t symbolizes standardized residuals at time t, and σ_t is the conditional variance at time t.

4.2. Vine Copula Models

Using the copula methodology to develop multivariate models proves beneficial when it is necessary to construct a collective model encompassing a diverse range of elements, such as assets originating from various sectors, and market categories. In this scenario, it is feasible to employ arbitrary models for individual components interconnected through a copula that characterizes the interdependence among these elements. Defined as a multivariate distribution function with values within $[0,1]^d$ and uniformly distributed margins, a d-dimensional copula C plays a pivotal role. Assuming a continuous scenario, the copula density can be derived through partial differentiation. A crucial finding establishes a fundamental relationship between any distribution function, a copula, and its margins. Specifically, we consider a d-dimensional random vector X with a joint distribution function denoted as F and marginal distribution functions denoted as Fj, along with the copula C.

$$F(x_1, x_2, \ldots, x_d) = C\{F_1(x_1), F_2(x_2), \ldots, F_d(x_d)\} \tag{3}$$

This suggests that the multivariate density function f(x_1,. . ., x_d) can be represented as

$$f(x_1, \ldots, x_d) = C_{1,\ldots d}\{F_1(x_1), \ldots, F_d(x_d)\}.f_1(x_1), \ldots, f_d(x_d) \tag{4}$$

in which each fj represents marginal density and c denotes the copula density. Through the inversion of Equation (1), it is possible to utilize any d-dimensional distribution function for deriving the corresponding copula. Instances of such distributions include the multivariate Gaussian and the Student-t copula. The employment of generator functions is common for the class of parametric Archimedean copulas, encompassing the Gumbel, Clayton, and Frank copula families. The two-parameter BB copula class enables the modeling of distinct upper and lower tail behaviors, even when non-zero values are involved. The flexibility of bivariate copulas can be enhanced by utilizing their survival and reflection versions. For instance, the survival version of bivariate copula density c can be expressed as

$$\mathrm{c}(u_1, u_2) = \mathrm{c}(1 - u_1, 1 - u_2) \tag{5}$$

In this scenario, d = 2 holds significant importance, as it serves as the foundational element for vine copulas. Additionally, following Czado et al. (2022), the conditional density $f_{1|2}$, and distribution function $F_{1|2}$ are essential components that must be considered:

$$f_{1|2}(x_1|x_2) = C_{12}(F_1(x_1), F_2(x_2))f_1(x_1) \tag{6}$$

$$F_{1|2}(x_1|x_2) = \frac{\partial}{\partial V} C_{12}(F_1(x_1), v)|_{F_2(x_2)} \tag{7}$$

Vine structures are created by pairing marginal densities, d, resulting in $d(d-1)/2$ pair copulas organized in $d-1$ trees. The initial root node establishes the dependency by employing a single variable with bivariate copulas for each pair. Subsequently, we

model the pairwise dependency of the second variable, which is represented by the second root node. A root node is designated for each tree, with all pairwise relationships of this node being determined in relation to all preceding root nodes, culminating in a star configuration within C-vine trees. In accordance with the findings of Aslam et al. (2023), the decomposition utilized for C-vine density, for root nodes 1, …, d, is as follows:

$$f(x) = \prod_{k=1}^{d} f_k(x_k) * \prod_{i=1}^{d-1} \prod_{j=1}^{d-i} C_{i,i+j|1:(i-1)}\left(F(x_i|x_1, \cdots, x_{i-1}), F\left(x_{i+j}\middle|x_1, \cdots, x_{i-1}\right)\middle|\theta_{i,i+j|1:(i-1)}\right) \quad (8)$$

where f_k, (k = 1, $\cdots$, d), represent the marginal densities, and $C_{i,i+j|1:(i-1)}$ denote the bivariate copula densities with parameter(s) $\theta_{i,i+j|1:(i-1)}$. The outer product pertains to the d − 1 trees and root nodes i, while the inner product pertains to the d − 1 pair copulas in each tree i = 1, $\cdots$, $d-1$.

D-vines follow a similar procedure to C-vines by establishing a specific sequence of variables. Initially, pairwise copulas are utilized in the first tree to model the dependence between the first and second variables, followed by the second and third variables, and so forth, resulting in the identification of a pathway structure. In the subsequent tree pairs, copulas are employed to depict the dependence between two variables conditioned on a third variable. This iterative process continues until (n − 1) trees have been identified.

This paper utilizes the R-Vine and C-Vine (regular and canonical, respectively) copula models to examine the interdependence characteristics within the JSE sectors. In contrast to previous research that focuses solely on R-Vine (see, for instance, X. Zhang et al., 2022), exclusively on C-Vine (Ma et al., 2021), or only on D-Vine (Q. Wang et al., 2022), we enhance the reliability of the analysis while providing a methodological contribution by systematically investigating, contrasting, and assessing all primary vine copula classes that are involved in modeling the interdependence structure within the JSE sectors. The approximate estimation of R-vine copulas is presented by Čeryová and Árendáš (2024) as illustrated below:

$$f(x_1, \ldots, x_d) = \left[\prod_{k=1}^{d} f_k x_k\right] * \left[\prod_{i=1}^{d-1} \prod_{e \epsilon E_i} C_{j(e)k(e)|D(e)}\left(F\left(x_{j(e)}\middle|x_{D(e)}\right), F\left(x_{k(e)}\middle|x_{D(e)}\right)\right)\right] \quad (9)$$

the multivariate density function $f(x_1, \ldots, x_n)$ is used to represent a part of the equation, while *Fi* denotes the marginal distributions, and $cj(e)$, $k(e) \mid D(e)$ describes a bivariate conditional density copula with $j(e)$ and $k(e)$ as conditioned nodes and $D(e)$ as the conditioning set. C-Vines, a subset of R-Vines, are characterized by their star-like shape, with each tree within them designating a root node variable chosen based on having the highest absolute correlation with the remaining variables. This property allows them to effectively capture the multivariate dependence, where one component influences the others significantly (Čeryová & Árendáš, 2024). On the contrary, D-Vines exhibit linear tree shapes, indicating that the nodes in each tree are connected to no more than two edges, making them suitable for modeling dependence structures where a subset of variables with crucial influence is identified (Czado et al., 2022).

4.3. Risk Metrics

One of the most used risk metrics is the value-at-risk (VaR). Mathematically, VaR can be written as

$$\mathrm{P}(R_i > -VaR_\alpha) = 1 - \alpha \quad (10)$$

where R_i is the asset return, and α is the significance level of the quantile, which is often 10%, 5%, or 1%. Conventional VaR solely addresses the risk associated with an individual sector in seclusion, overlooking the transmission of risk due to complex interrelations among diverse sectors in reality. To mitigate this constraint, (Tobias & Brunnermeier, 2016) introduced CoVaR and delta CoVaR as tools for gauging conditional risk. Consequently,

contemporary research on risk management is placing greater emphasis on integrating these metrics to evaluate systemic risk.

The variable $CoVaR_q^{j/i}$ represents the VaR of financial institution j conditioned on a specific event affecting financial institution (Tobias & Brunnermeier, 2016). CoVaR is mainly used to investigate the interconnection effects among an entire network of financial institutions. According to Fijorek et al. (2021), $CoVaR_q^{j/i}$ can be defined as a q-quantile of the conditional probability distribution given by

$$\mathrm{P}\left(X_j \leq CoVaR_q^{j/i} \middle| X_i = VaR_q^i\right) = \mathrm{q}. \tag{11}$$

This definition implies a probability of q% that the returns X_i will fall below $CoVaR_q^{j/i}$ within a given time frame, assuming that the returns X_i correspond to the q% VaR level.

4.4. Portfolio Optimization

This section delineates how the portfolio was constructed and enhanced with regard to the objective function. Exploring the depths of probability theory and quadratic programming in 1952, Harry Markowitz sought to create sophisticated investment models that could produce outcomes defined by both low-risk and high-risk returns. This forms the basis of present-day portfolio management. This section aims to find the optimal allocation of assets that minimizes portfolio risk while maximizing the portfolio's expected return. The quadratic mean-variance portfolio optimization problem is expressed as follows:

$$\text{maximize } F = W'.R - \frac{\alpha}{2} W'\Sigma\, W, \tag{12}$$

$$s.t. \begin{cases} W'I = 1 \\ W \geq O \end{cases}$$

where R is a vector of returns; W is a vector of asset weights, and Σ is the variance–covariance matrix that expresses the portfolio variance. α denotes the constant absolute risk aversion (CARA), which assumes that the investor's risk aversion factor can influence the optimal solution (Kabundi & Mwamba, 2012).

4.5. Shannon Entropy

The notion of entropy was introduced in engineering and mathematics by Shannon Claude in 1948, and it was given a probabilistic interpretation in the theory of communication and transmission of information (Shannon, 1948). As stated by the Shannon entropy, the idea of measuring information is similar to the context of a long-only portfolio, as long as stock weights exhibit the structure of an appropriate probability distribution (they sum to 100% and are all positive). Thus, asset weights are treated as a proxy of the probability mass function of a random variable, and the Shannon entropy of a given portfolio can be calculated as follows:

$$SE(w) = -\sum_{i=1}^{N} w_i log_2(w_i) \tag{13}$$

where $0 \leq SE(w) \leq log_2(N)$, and w_i represents the weight of a given asset i. The logic is that when the entropy is equal to zero, there is no uncertainty about the outcome for the random variable, as long as the portfolio is only made of one asset. In the case where the entropy reaches its maximum, the uncertainty about the value of the random variable is the highest, since each asset is equally weighted. The logic is that increased portfolio diversification reduces uncertainty and lowers entropy.

4.6. Tsallis Entropy

Tsallis Entropy is a generalization of the classical Boltzmann–Gibbs entropy, which was introduced by Constantino Tsallis in 1988. Tsallis entropy can provide valuable insights in the context of portfolio diversification, particularly in financial markets characterized by complex behaviors and nonlinear interactions among assets.

This method helps to quantify the diversity of assets in a portfolio. A higher Tsallis entropy indicates a more diversified portfolio, which can reduce risk. This is particularly useful when it comes to assessing portfolios with assets that may not follow normal distribution patterns.

The Tsallis entropy of a given portfolio can be calculated as follows:

$$S_q = \frac{1}{q-1}\left(1 - \sum_i p_i^q\right) \tag{14}$$

where q $\geq$ 1 is the parameter which adds a layer of complexity. For q = 1, Tsallis entropy converges to Shannon entropy.

While both Tsallis entropy and Shannon entropy can be used to analyze portfolios, they serve slightly different purposes and are based on different theoretical foundations.

4.7. Hierarchical Clustering Algorithm

Agglomerative hierarchical algorithms are types of clustering mainly used without predefining the desired number of clusters. They are unsupervised machine learning techniques and refer to algorithms that discover patterns and relationships in data without the use of labeled examples or a predefined target variable. The agglomerative hierarchical algorithms specially perform as follows: all the data points to be clustered are first considered as a unique cluster. Then, the algorithm compares data points among themselves using a distance measure; for instance, the Euclidian distance. The two clusters with smaller distances are joined. The algorithm only allows us to join two clusters in each step, and they cannot be disjointed after they are joined. Next, it keeps repeating over and over again until attaining the appropriate number of clusters. Finally, each step's clustering results are shown through a dendrogram.

In cases in which hierarchical clustering is going to be used for portfolio optimization issues, the convenient distance metric D_{ij} to be used is given as follows (Léon et al., 2017):

$$D_{ij} = \sqrt{2(1-\rho_{ij})} \tag{15}$$

where ρ_{ij} is the Pearson correlation coefficient between asset i and asset j.

5. Conclusions

This study investigates systemic risk, return patterns, and diversification within the JSE during the COVID-19 pandemic by employing data-centric approaches alongside the ARMA-GARCH vine copula-based CoVaR model, supported by an entropy framework. It compares three types of portfolios: one based on industry sectors, another on an asset risk–return plot, and the last using a clustering technique. The ARMA-GJR-GARCH model and vine copula-based CoVaR method effectively model returns and addresses extreme risk spillovers, while entropy measures assess dynamics, predictability, and diversification levels.

The application of C-vine and R-vine copulas reveals weak tail dependence among JSE sectors, suggesting that substantial fluctuations in one sector have minimal impact on others, highlighting the resilience of the South African economy. The findings challenge traditional views on market interdependence during downturns. CoVaR estimates further

enhance the understanding of systemic risk, revealing that sectors like industrial and technology can stabilize portfolios during economic stress. Different entropy measures also indicate increased volatility in specific sectors, underscoring the unpredictability of financial returns.

Insights from this analysis provide practical guidance for effective portfolio construction, encouraging investors to leverage the resilience of certain sectors to enhance diversification and mitigate risk. The study underscores the necessity of adaptable risk management strategies in volatile markets and suggests that policymakers should utilize these findings to develop frameworks that promote diversification and reduce systemic risks.

Furthermore, the analysis shows that the industrial and technology sectors exhibit no ARCH effects and remain largely isolated from other sectors and clusters, indicating potential diversification opportunities. In contrast, the energy sector, Group1, Group2, and cluster1 demonstrate higher volatility and unpredictability, while the industrial sector is the least volatile. The optimal vine copula approach identified is the C-vine, with the financial sector, Group1, and cluster1 emerging as root nodes, displaying limited tail dependence.

From a systemic risk perspective, the industrial and technology sectors significantly contribute to negative CoVaR values, suggesting they could provide a degree of stability during market downturns. The industry sector-based portfolio shows heightened unpredictability during the COVID-19 crisis. Additionally, this portfolio exhibits the highest Tsallis and Shannon entropy, indicating it has the greatest diversity among the analyzed types. The portfolio constructed using an asset risk–return plot approach shows similar levels of diversity and concentration to the cluster-based portfolio.

To validate these findings, future research should explore the sensitivity of results to various estimation methods for GARCH and copula models and analyze alternative periods or data frequencies to assess the stability of dependence structures and systemic risk (Budiarti et al., 2023) contributions.

Author Contributions: Conceptualization, J.W.M.M., J.C.M. and A.K.K.; methodology, J.C.M. and A.K.K.; software, J.C.M. and A.K.K.; validation, J.W.M.M.; formal analysis, J.W.M.M., J.C.M. and A.K.K.; investigation, J.W.M.M., J.C.M. and A.K.K.; resources, A.K.K.; data curation, J.C.M.; writing—original draft preparation, A.K.K.; writing—review and editing, J.W.M.M. and J.C.M.; visualization, A.K.K.; supervision, J.W.M.M. and J.C.M.; project administration, J.C.M. and A.K.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Informed Consent Statement: This study did not involve humans.

Data Availability Statement: The dataset consists of daily closing prices of the South African JSE industry sectors, including basic materials, consumer discretionary, energy, financial, healthcare, industrial, technology, telecommunication, and consumer staples. All the data used were sourced from IRSS-INET-DATABASE. The classification of these sector indices is based on the Industry Classification Benchmark (ICB) created by the Financial Times Stock Exchange (FTSE).

Conflicts of Interest: The authors declare no conflicts of interest.

References

Abuzayed, B., Bouri, E., AI-Fayoumi, N., & Jalkh, N. (2021). Systemic risk spillover across global and country stock markets during the COVID-19 pandemic. *Economic Analysis and Policy*, *71*, 180–197. [CrossRef]

Adrian, T., & Brunnermeier, M. K. (2011). *CoVaR*. Working Paper 17454. National Bureau of Economic Research.

Ali, S., Shahzad, S. J. H., Raza, N., & AI-Yhyaee, K. H. (2018). Stock market efficiency: A comparative analysis of Islamic and Conventional Stock markets. *Physica A: Statistical Mechanics and Its Applications*, *503*, 139–153. [CrossRef]

Aloui, R., Jabeur, S. B., & Mefteh-Wali, S. (2022). Tail-risk spillovers from China to G7 stock market returns during the COVID-19 outbreak: A market and sectoral analysis. *Research in International Business and Finance*, *62*, 101709. [CrossRef] [PubMed]

Aslam, F., Hunjra, A. I., Bouri, E., Mughal, K. S., & Khan, M. (2023). Dependence structure across equity sectors: Evidence from vine copulas. *Borsa Istanbul Review*, *23*(1), 184–202. [CrossRef]

Aziz, N. S. A., Vrontos, S., & Hasim, H. M. (2019). Evaluation of multivariate GARCH models in an optimal asset allocation framework. *The North American Journal of Economics and Finance*, *47*, 568–596. [CrossRef]

Bedford, T., & Cooke, R. M. (2002). Vines-a new graphical model for dependent random variables. *The Anals of Statistics*, *30*(4), 1031–1068. [CrossRef]

Bianchi, M. L., & Sorrentino, A. M. (2022). Exploring the systemic risk of domestic banks with δcovar and elastic-net. *Journal of Finance Services Research*, *62*(1), 127–141. [CrossRef]

Bonga-Bonga, L., & Hendriks, J. J. (2024). Sectoral dependence and contagion in the BRICS: An application of the R-vine copulas. *Studies in Nonlinear Dynamics and Econometrics*. [CrossRef]

Braiek, S., Rihab, B., & Belkacem, L. (2022). Islamic portfolio optimization under systemic risk: Vine copula-CoVaR based model. *International Journal of Finance, & Economics*, *27*(1), 1321–1339.

Budiarti, R., Intansari, K., Purnaba, I. G. P., & Septyanto, F. (2023). Modelling depencies of stock indices during covid pandemic by extreme * value copula. *Journal Teori dan Aplikasi Matematika*, *7*(3), 805–819. [CrossRef]

Chowdhury, E. K., Dhar, B. K., & Stasi, A. (2022). Volatility of the US stock market and business strategy during COVID-19. *Business Strategy, & Development*, *5*(4), 350–360.

Csizmás, G. D., Margit, E., & Kovács, E. A. (2021). The effect of dependence structure on risk measures. *Gradus*, *8*(3), 157–171. [CrossRef]

Czado, C., Bax, K., Sahin, Ö., Nagler, T., Min, A., & Paterlini, S. (2022). Vine copula based dependence modeling in sustainable finance. *The Journal of Finance and Data Science*, *8*, 309–330. [CrossRef]

Čeryová, B., & Árendáš, P. (2024). Vine copula approach to the intra-sectoral dependence analysis in the technology industry. *Finance Research Letters*, *60*, 104889. [CrossRef]

Dewick, P. R., & Liu, S. (2022). Copula modelling to analyse financial data. *Journal of Risk and Financial Management*, *15*(3), 104. [CrossRef]

Ellul, A., Erel, I., & Rajan, U. (2020). The COVID-19 pandemic crisis and corporate finance. *The Review of Corporate Finance Studies*, *9*(3), 421–429. [CrossRef] [PubMed]

Enow, S. T. (2022). Overreaction and underreaction during the COVID-19 pandemic in the south African stock market and its implications. *Eurasian Journal of Business and Management*, *10*(1), 19–26. [CrossRef]

Fasnacht, P., & Louberge, H. (2007, December 20). *International stock market correlations: A sectoral approach*. Finance Inernational Meeting AFFI-EUROFIDAI, Paris, France.

Fijorek, K., Jurkowska, A., & Jonek-Kowalska, I. (2021). Financial contagion between the financial and the mining industries—Empirical evidence based on the symmetric and asymmetric CoVaR approach. *Resources Policy*, *70*, 101965. [CrossRef]

Gao, J., Li, D., Cui, X., & Wang, S. (2015). Time cardinality constrained mean-variance dynamic portfolio selection and market timing: A stochastic control approach. *Automatica*, *54*, 91–99. [CrossRef]

Goudarzi, S., Jafari, M. J., & Afsar, A. (2017). A hybrid model for portfolio optimization based on stock clustering and different investment strategies. *International Journal of Economics and Financial*, *2*(7), 602–608.

Guilherme, G., Wanke, P., & Tan, Y. (2022). A higher order portfolio optimixation model incorporating information entropy. *Intelligent Systems with Applications*, *15*, 200101.

Hao, X., & Chen, Z. (2022). Systemic risk in Chinesse financial industries: A vine copula groupe CoVaR approach. *Economic Research-Ekonomka Istrazivania*, *35*(1), 2747–2763.

Haroon, O., Ali, M., Khan, A., Khattak, M. A., & Rizvi, S. A. R. (2021). Financial market risks during the COVID-19 pandemic. *Emerging Markets Finance and Trade*, *57*(8), 2407–2414. [CrossRef]

Hassan, M. B., Mahi, M., Sarker, T., & Amin, M. R. (2021). Spillovers of the COVID-19 pandemic: Impact on global economic activity, the stock market, and the energy sector. *Journal of Risk and Financial Management*, *14*(5), 200. [CrossRef]

Hilal, S., Poon, S.-H., & Tawn, J. (2011). Hedging the black swan: Conditional heteroskedasticity and tail dependence in S&P500 and VIX. *Journal of Banking, & Finance*, *35*(9), 2374–2387.

Hu, C.-y., Tang, Y.-w., Su, Q.-m., Lei, Y., Cui, W.-s., Zhang, Y.-y., Zhou, Y., Li, X.-y., Wang, Z.-f., & Zhao, Z.-x. (2021). Public health measures during the COVID-19 pandemic reduce the spread of other respiratory infectious diseases. *Frontiers in Public Health*, *9*, 771638. [CrossRef] [PubMed]

Insaidoo, M., Ullah, A., Dziwornu, R. K., Amoako, S., & Abdul-Mumuni, A. (2023). COVID-19 pandemic and stock market performance: A comparative study of emerging economies. *Heliyon*, *9*(5), e16054. [CrossRef]

Ji, H., Wang, H., & Lseo, B. (2018). Portfolio diversification strategy via tail-dependence clustering and ARMA-GARCH Vine Copula approach. *Australian Economic Papers*, *57*(3), 265–283. [CrossRef]

Joe, H. (1996). Families of m-variate distributions with given margins and m (m-1)/2 bivariate dependence parameters. *Distributions with Fixed Marginals and Related Topics*, *28*, 120–141.

Joo, B. A., & Ghulam, Y. A. (2023). The predictability, volatility persistence and leverage effects in stock market returns: A study of BRICS stock market indices. *American Journal of Finance and Accounting*, *7*(3–4), 188–213. [CrossRef]

Kabundi, A., & Mwamba, J. M. (2012). Applying a genetic algorithm to international DIVERSIFICATION of equity portfolios: A south AFRICAN investor perspective. *South African Journal of Economics*, *80*(1), 91–105. [CrossRef]

Karmakar, M., & Paul, S. (2023). Downside risk and Portfolio optimization of energy stocks: A study on the extreme value theory and the vine copula approach. *The Energy Journal*, *44*(2), 139–180. [CrossRef]

Khan, A. Z., & Mehlawat, M. K. (2022). Dynamic portfolio optimization using technical analysis-based clustering. *International Journal of Intelligent Systems*, *37*(10), 6978–7057. [CrossRef]

Khiari, W., & Ben Sassi, S. (2019). On identifying the systemically important Tunisian banks: An empirical approach based on the delta CoVaR measures. *Risks*, *7*(4), 122. [CrossRef]

Kielmann, J., Manner, H., & Min, A. (2022). Stock market returns and oil price shocks: A CoVaR analysis based on dynamic vine copula models. *Empirical Economics*, *62*(4), 1543–1574. [CrossRef] [PubMed]

Kumar, S., Datta, S., Singh, V., Singh, S. K., & Sharma, R. (2024). Opportunity and challenges in data-centric AI. *IEEE Access*, *12*, 33173–33189. [CrossRef]

Kumar, S., Tiwari, A. K., Raheem, I. D., & Hille, E. (2021). Time-varying dependence structure between oil and agricultural commodity markets: A dependence-switching CoVaR copula approach. *Resources Policy*, *72*, 102049. [CrossRef]

Kupiec, P., & Guntay, L. (2016). Testing for systemic risk using stock returns. *Journal of Financial Services Research*, *49*, 203–227. [CrossRef]

Lahiani, A., & Jlassi, N. B. (2021). Nonlinear tail dependence in cryptocurrency-stock market returns: The role of Bitcoin futures. *Research in International Business and Finance*, *56*, 101351. [CrossRef]

Léon, D., Aragón, A., Sandoval, J., Hernández, G., Arévalo, A., & Niño, J. (2017). Clustering algorithms for risk-adjusted portfolio construction. *Procedia Computer Science*, *108*, 1334–1343. [CrossRef]

Li, S., & Yan, Y. (2022). DATA-driven shock impact of COVID-19 on the market financial system. *Information Processing, & Management*, *59*(1), 102768.

Ljubownikow, G., & Ang, S. H. (2020). Competition, diversification and performance. *Journal of Business Research*, *112*, 81–94. [CrossRef]

Luo, C., Liu, L., & Wang, D. (2021). Multiscale financial risk contagion between international stock markets: Evidence from EMD-Copula-CoVaR analysis. *The North American Journal of Economics and Finance*, *58*, 101512. [CrossRef]

Ma, H., Lin, L., Sun, H., & Qu, Y. (2021). Research on the dependence structure and risk spillover of internet money funds based on C-vine copula and time-varying t-copula. *Complexity*, *2021*(1), 3941648. [CrossRef]

Manguzvane, M., & Muteba Mwamba, J. W. (2019). Modelling systemic risk in the South African banking sector using CoVaR. *International Review of Applied Economics*, *33*(5), 624–641. [CrossRef]

Markowitz, H. (1952). The utility of wealth. *Journal of Political Economy*, *60*(2), 151–158. [CrossRef]

Mbatha, V. M., & Alovokpinhou, S. A. (2022). The structure of the South African stock market network during COVID-19 hard lockdown. *Physica A: Statistical Mechanics and Its Applications*, *590*, 126770. [CrossRef]

Mishra, S., & Padhy, S. (2019). An efficient portfolio construction model using stock price predicted by support vector regression. *The North American Journal of Economics and Finance*, *50*, 101027. [CrossRef]

Novais, R. G., Wanke, P., Antumes, J., & Tan, Y. (2022). Portfolio optimization with a mean-entropy-mutual information model. *Entropy*, *24*(3), 369. [CrossRef]

Odeyemi, O., Mhlongo, N. Z., Nwankwo, E. E., Scholatica, U. C., & Okoye, C. C. (2024). Big data applications in portfolio management: Areview of techniques and strategies. *International Journal of Science and Research Archive*, *11*(1), 1984–1999. [CrossRef]

Omane-Adjepong, M., Ababio, K. A., & Alagidede, I. P. (2019). Time-frequency analysis of behaviourally classified financial asset markets. *Research in International Business and Finance*, *50*, 54–69. [CrossRef]

Polanski, A., Stoja, E., & Chiu, C.-W. (2021). Tail risk interdependence. *International Journal of Finance, & Economics*, *26*(4), 5499–5511.

Rather, A. M., Sastry, V. N., & Agarwal, A. (2017). Stock market prediction and Portfolio selection models: A survey. *Opsearch*, *54*(3), 558–579. [CrossRef]

Righi, B. M., & Ceretta, P. S. (2013). Analyzing the dependence structure of various sectors in the Brazilian market: A pair Copula Construction approach. *Economic Modelling*, *35*, 199–206. [CrossRef]

Shahani, R., Talwar, A., & Gupta, S. (2017). Dynamics of the co-movement of the returns of the sectorial stock indices of India, US and European Economies. *IUP Journal of Financial Risk Management*, *14*(3), 7–21.

Shannon, C. E. (1948). Amathematical theory of communication. *The Bell System Technical Journal*, *27*(3), 379–423. [CrossRef]

Sklar, A. (1959). Fonctions de répartition à n dimensions et leurs marges. *Publications de l'Institut de Statistique de l'Université de Paris*, *8*, 229–231.

Supper, H., Irresberger, F., & Weib, G. (2020). A comparison of tail dependence estimators. *European Journal of Operational Research*, *282*(2), 728–742. [CrossRef]

Tian, M., & Ji, H. (2022). GARCH copula quantile regression model for risk spillover analysis. *Finance Research Letters*, *44*, 102104. [CrossRef]

Tobias, A., & Brunnermeier, M. K. (2016). CoVaR. *The American Economic Review*, *106*(7), 1705–1741.
Trabelsi, N. (2017). Asymmetric tail dependence between oil price shocks and sectors of Saudi Arabia System. *The Journal of Economic Asymmetries*, *16*, 26–41. [CrossRef]
Van Oordt, M., & Zhou, C. (2019). Systemic risk and bank business models. *Journal of Applied Econometrics*, *34*(3), 365–384. [CrossRef]
Wang, K. Q. (2002). Nonparametric tests of conditional mean-variance efficiency of a benchmark portfolio. *Journal of Empirical Finance*, *9*(2), 133–169. [CrossRef]
Wang, Q., Liu, M., Xiao, L., Dai, X., Li, M. C., & Wu, F. (2022). Conditionalsovereign CDS in market basket risk scenario: A dynamic vine-copula analysis. *International Review of Financial Analysis*, *80*, 102025. [CrossRef]
Warshaw, E. (2019). Extreme dependence and risk spillovers across north american equity markets. *North American Journal of Economics and Finance*, *47*, 237–251. [CrossRef]
Yousaf, I., Patel, R., & Yarovaya, L. (2022). The reaction of G20+ stock markets to the Russia-Ukraine conflict "black-swan"event: Evidence from event study approach. *Journal of Behavioral and Experimental Finance*, *35*, 100723. [CrossRef]
Zhang, W., Zhuang, X., Lu, Y., & Wang, J. (2020). Spatial linkage of volatility spillovers and its explanation across G20 stock markets: A network framework. *International Review of Financial Analysis*, *71*, 101454. [CrossRef]
Zhang, X., Zhang, T., & Lee, C.-C. (2022). The path of financial risk spillover in the stock market based on the R-vine-Copula model. *Physica A:Statistical Mechanics and Its Applications*, *600*, 127470. [CrossRef]
Zheng, T., Xiao, H., & Chen, R. (2022). Generalized autoregressive moving average models witn GARCH errors. *Journal of Time Series Analysis*, *43*(1), 125–146. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.