

Article

Cryptocurrencies and Long-Range Trends

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Abstract: In this study we investigate possible long-range trends in the cryptocurrency market. We employed the Hurst exponent in a sample covering the period from 1 January 2016 to 26 March 2021. We calculated the Hurst exponent in three non-overlapping consecutive windows and in the whole sample. Using these windows, we assessed the dynamic evolution in the structure and long-range trend behavior of the cryptocurrency market and evaluated possible changes in their behavior towards an efficient market. The innovation of this research is that we employ the Hurst exponent to identify the long-range properties, a tool that is seldomly used in analysis of this market. Furthermore, the use of both the R/S and the DFA analysis and the use of non-overlapping windows enhance our research's novelty. Finally, we estimated the Hurst exponent for a wide sample of cryptocurrencies that covered more than 80% of the entire market for the last six years. The empirical results reveal that the returns follow a random walk making it difficult to accurately forecast them.

Keywords: cryptocurrencies; time series; autocorrelation; Hurst exponent; Rescaled Range Analysis; Detrended Fluctuation Analysis

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1. Introduction

Bitcoin has proven to be a successful experiment which has led to the gradual introduction of more than 22,000 digital currencies and 500 exchanges. The capitalization of the cryptocurrency market has reached 3 trillion USD according to the CoinMarketCap website. The market is unique and idiosyncratic, distinct from the other financial markets. The key feature of the cryptocurrency market is the absence of a centralized regulatory body. Central regulators, for example central banks (ECB, Fed), governmental agencies, financial institutions, etc. are intertwined with official economic policies and governed by detailed institutional and legal frameworks (Yang et al. 2020). The online marketplace of cryptocurrencies, based on the blockchain technology, dismantles structural axioms that are essential in traditional markets. The key axiom is that governments and financial institutions around the world are responsible for issuing currencies and ensuring the legitimacy and authenticity of transactions (Zhengyang et al. 2019). The unique characteristics of the cryptocurrency market make it an interesting subject of research by the academic and investment community.

Despite the fast growth in cryptocurrencies, the specific market is still relatively new and possibly shallow and inefficient in terms of the Fama's efficient market hypothesis (Papadimitriou et al. 2020). As a result, the cryptocurrency market is associated with high volatility and considerable risk, possibly due to the fact that it has limited interconnections to conventional financial assets (Qureshi et al. 2020). In addition, it provides easy and readily accessible information to any stakeholder as all market data is scattered across the global web and social networks, creating new dynamics in investment behavior and mindsets (Valencia et al. 2019).

The relevant research literature focuses on (a) forecasting cryptocurrency prices, returns, and risk; (b) qualitatively and quantitatively analyzing the cryptocurrency market; and (c) identifying short- and long-term patterns using statistical and machine learning tools (Zhengyang et al. 2019). A systematic survey on whether the pricing behavior of cryptocurrencies is predictable using the Hurst exponent was conducted by Kyriazis (2019). According to the results, most academic papers provide evidence for the inefficiency of Bitcoin and other major cryptocurrencies.

Long-range dependence and its presence in the financial time series has been discussed in several papers (Czarnecki et al. 2008; Grech and Mazur 2004; Carbone et al. 2004; Matos et al. 2008; Vandewalle et al. 1997; Alvarez-Ramirez et al. 2008; Peters 1994; Di Matteo et al. 2005; Di Matteo 2007; Krištoufek 2010). Long-short memory dependencies have been the subject of numerous studies, particularly with regards to stock markets, currency markets, commodities, and mutual fund performance in the past 30 years, especially after Peters (1991) and Hsieh (1991). For instance, Sirlantzis and Siriopoulos (1993) analyzed monthly returns for a small emerging market using the R/S method and concluded against the efficient market hypothesis. Similar results were found by Balci et al. (2022) in the Turkish stock market during the COVID-19 pandemic. Moreover, Siriopoulos (1996) used the same analysis to study long-range dependence in emerging capital markets and their connection with developed ones. Likewise, Siriopoulos and Skaperda (2020) analyzed the performance of American mutual funds from the perspective of long memory using R/S and using Surrogate Data Analysis (SDA), which resulted from the Shuffle Algorithm. The results strongly indicate that mutual fund investors should not base their choice on past performance alone when selecting which fund to invest in.

Arouxet et al. (2022) examined long-term memory for seven major cryptocurrencies. The research focused on the period of the pandemic using a wavelet-based method to estimate the Hurst exponent and noted the high frequency returns and volatility. Estimating the Hurst exponent using R/S and DFA is also used to analyze high-frequency-return data with varying lags. Zhang et al. (2019), in particular, used both methods on four leading cryptocurrencies for a period from 25 February 2017 to 17 August 2017. Researchers have also investigated the volatility clustering, leverage effect, autocorrelations, and heavy tails (Zhang et al. 2018) in similar schemes. Variations of the Hurst exponent and the DFA method are used, such as the generalized Hurst exponent and the multifractal version of DFA (MF-DFA) (Bariviera 2021). In this paper, daily prices of 84 cryptocurrencies were examined for a period spanning from 6 January 2018 to 5 March 2020. The author pinpoints the fact that among the cryptocurrencies both the long-range dependences and the stochastic processes that determine their dynamic behavior varies over time.

Other researchers tried to detect the efficiency of specific crypto markets using a time-varying generalized Hurst exponent based on rolling windows (Keshari Jena et al. 2022). Additional works investigated the day-of-the-week effect on the returns and volatility of Bitcoin using a linear stochastic process (Aharon and Qadan 2019), pricing efficiency, and response to calendar and seasonal effects (Qadan et al. 2022). Furthermore, the optimal number of indicators was investigated to achieve optimum trading results using Recurrent Neural Networks (RNNs), the Ichimoku Cloud (IC) indicator, Chaikin Money Flow (CMF), and Moving Average Convergence/Divergence (MACD), a trend momentum indicator (Cohen and Qadan 2022).

The aim of this study is to uncover possible long-range dependence in 37 of the most important—in terms of market capitalization—cryptocurrencies. This is achieved with the calculation of the Hurst exponent for the whole sample and for consecutive non-overlapping windows. The study of the Hurst exponents reveals potential changes in the long-term memory. It allows us to detect possible dynamic changes in the crypto-market during the last six years. Furthermore, since heavy tails in cryptocurrency returns can cause infinite auto-covariances, we test the Hurst exponent in sub-periods (Wendler and Betken 2018; Jach et al. 2011). Proper statistical tests of the Hurst exponent estimates will provide evidence about whether these cryptocurrencies are mean reverting, random walks, or

exhibit long-range persistence (memory). Additionally, the inclusion of 37 of the most important cryptocurrencies in the analysis allows for a more comprehensive understanding of the variation in long-range dependence across different cryptocurrencies. The study's goals have not been extensively studied in the past. The provided results are important for both academics and market participants.

2. Methodology and Data Set

The Hurst exponent (Hurst 1951), denoted H in honor of Harold Edwin Hurst, is widely used to study time series and the long-range correlations they may exhibit. The Hurst exponent, among other things, enables us to determine whether a time series exhibits positive or negative long-term autocorrelation or whether it is a random walk. The Hurst exponent takes values in the range $[0, 1]$. More specifically, when the Hurst exponent has a value of $0.5 < H < 1$, then the time series exhibits positive long-term memory and we say that it is characterized by persistence. This means that if a time series appears to be trending upward or downward it will most likely continue this trend in the next time period. When the Hurst exponent has a value of $0 < H < 0.5$, then the time series is characterized by anti-persistence or we say that it is mean reverting. This means that if the time series is increasing now, in the next period it will most likely start decreasing towards a long-term mean and vice versa. Finally, when the Hurst exponent is 0.5, then the time series exhibits persistent behavior and follows a random walk (Matos et al. 2008), i.e., it is random.

Several methods can be used to estimate the Hurst exponent. The most widely used are the Rescaled Range Analysis (R/S) and the Detrended Fluctuation Analysis (DFA), which we apply in this study.

The data used in this study were obtained from Yahoo Finance and CoinMarketCap. They include the daily closing prices of 37 cryptocurrencies that are reported in Table 1 for the period 1 January 2016 to 26 March 2021. The selected cryptocurrencies covered 80.6% of the total cryptocurrency market capitalization.

Table 1. List of cryptocurrencies.

	Cryptocurrency	Abbreviation
1	Bitcoin	BTC
2	Bitcoin Cash	BCH
3	Cardano	ADA
4	Celo	CELO
5	Chainlink	LINK
6	Dash	DASH
7	Decred	DCR
8	DigiByte	DGB
9	Dogecoin	DOGE
10	EOS	EOS
11	Etherium	ETH
12	Etherium Classic	ETC
13	ICON	ICX
14	IOST	IOST
15	IOTA	MIOTA
16	Lisk	LSK
17	Litecoin	LTC
18	Monero	XMR
19	NEM	XEM
20	NEO	NEO
21	OMG Network	OMG
22	Qtum	QTUM
23	Ravencoin	RVN
24	Stellar	XLM

25	Storj	STORJ
26	Tezos	XTZ
27	Theta Token	THETA
28	Tron	TRX
29	Voyager Token	VGX
30	WAVES	WAVES
31	White Coin	XWC
32	XRP	XRP
33	Zcash	ZEC
34	Ziliqa	ZIL
35	Bancor	BNT
36	Basic Attention Token	BAT
37	Binance Coin	BNB

We calculate the daily returns of these cryptocurrencies using the first logarithmic differences R_t of the closing prices:

$$R_t = \ln(P_t) - \ln(P_{t-1}) \quad (1)$$

where \ln is the natural logarithm and P is the closing price.

The analysis we perform in estimating the Hurst exponent is initially applied to the entire sample period from 1 January 2016 to 26 March 2021. Then, we divide the sample into three non-overlapping windows. Starting from the most recent observations, we create two windows spanning two years each, with the third window including all the remaining observations. Thus, the windows are as shown in Table 2.

Table 2. “Windows” of data.

Window	From	To
1	1 January 2016	24 March 2017
2	25 March 2017	25 March 2019
3	26 March 2019	26 March 2021

The last window includes data from 26 March 2019 to 26 March 2021, the second window covers the period 25 March 2017 to 25 March 2019, and the first window includes all the remaining observations from 1 January 2016 to 24 March 2017.

3. Empirical Results

We present the Hurst exponent for both the full sample and the three subperiods presented in Table 2. We estimate the Hurst exponent employing both the Rescaled Range (R/S) and the Detrended Fluctuation Analysis (DFA) methodologies. For all these Hurst exponent estimates, we also calculate the 90% and 95% confidence intervals (Weron 2002). If the respective confidence interval for a cryptocurrency includes the 0.5 value, then we cannot reject the hypothesis that the time series is a random walk. If the lower bound of the confidence interval is greater than 0.5, we find empirical evidence of a persistent time series, and when the upper bound of the confidence interval is less than 0.5, we find empirical evidence of an anti-persistent or mean reverting time series. As the results for both confidence levels are qualitatively similar, to keep the empirical results section concise and readable, in what follows, we only present the results for the 90% confidence level. The results for the 95% margin are available upon request. In the same manner and for the same reason, the results of the R/S methodology are available from the authors upon request.

3.1. Closing Prices

First, we estimate the Hurst exponent for the cryptocurrencies’ closing prices. The results for the whole sample and the three subperiods are presented in Tables A1–A4 in

Appendix A. We observe that the estimated Hurst exponents are greater than 1 in most cryptocurrencies. According to the relevant literature (Bryce and Sprague 2012), this implies that there is a residual short-term trend in the input series. This can happen if the input data series were not stationary, or the detrending did not work. Hurst can detect the long-range dependence of the time series and the calculation breaks down when strong short-term dependence is present. The cryptocurrencies' closing price data in our case are non-stationary and this is reflected in the estimated Hurst exponents that are, in most of the cases and for all periods examined, greater than 1. Table A9 in Appendix A summarizes the conclusions we reach from these results for the cryptocurrency prices. When the data set was too small to perform the DFA analysis, the results are missing¹.

In Table 3, we provide the results of the formal stationarity tests. We use both the ADF and the KPSS tests with no trend and with a trend. The null hypothesis in the ADF testing procedure is that the time series in question is non-stationary or I(1) in the terminology of Engle and Granger (1987). Thus, the null of non-stationarity can only be rejected when the test has enough power to reject a false null hypothesis. For this reason, it may be prone to Type II error, not rejecting a false null hypothesis due to low power. For this reason and for robustness, we also employ the KPSS test where the null hypothesis is that the time series is stationary or I(0) or integrated of order zero in the terminology of Engle and Granger (1987). An examination of the results from both tests sheds light on the true stationarity properties of our time series. When the two tests disagree, we conclude that the respective time series is non-stationary. According to Table 3, the levels of all of the cryptocurrencies are non-stationary in all three significance levels. Those that are stationary according to the ADF test are non-stationary according to the KPSS test.

Table 3. ADF and KPSS stationarity results for the closing prices and the returns of cryptocurrencies.

Variable	Lags	ADF No-Trend		ADF with Trend		KPSS No Trend		KPSS with Trend		Decision
		Level	1st Diff.	Level	1st Diff.	Level	1st Diff.	Level	1st Diff.	
		Null Hypothesis I(1)				Null Hypothesis I(0)				
BTC	25	0.999	0.001 ***	0.999	0.001 ***	0.010 ***	0.100	0.010 ***	0.068 *	I(1)
ETH	25	0.973	0.001 ***	0.985	0.001 ***	0.010 ***	0.098 *	0.010 ***	0.021 **	I(1)
BNB	23	0.967	0.001 ***	0.976	0.001 ***	0.010 ***	0.100	0.010 ***	0.031 **	I(1)
ADA	23	0.640	0.001 ***	0.914	0.001 ***	0.032 **	0.100	0.010 ***	0.048 **	I(1)
XRP	25	0.004 ***	0.001 ***	0.012 ***	0.001 ***	0.010 ***	0.100	0.010 ***	0.100	I(1)
LTC	25	0.179	0.001 ***	0.259	0.001 ***	0.010 ***	0.100	0.010 ***	0.100	I(1)
THETA	22	0.999	0.001 ***	0.999	0.001 ***	0.010 ***	0.010 ***	0.010 ***	0.100	I(1)
LINK	23	0.999	0.001 ***	0.992	0.001 ***	0.010 ***	0.100	0.010 ***	0.100	I(1)
BCH	23	0.083 *	0.001 ***	0.060 *	0.001 ***	0.010 ***	0.100	0.010 ***	0.100	I(1)
XLM	25	0.096 *	0.001 ***	0.166	0.001 ***	0.010 ***	0.100	0.010 ***	0.100	I(1)
DOGE	25	0.996	0.001 ***	0.999	0.001 ***	0.010 ***	0.100	0.010 ***	0.100	I(1)
TRX	23	0.004 ***	0.001 ***	0.019 **	0.001 ***	0.042 **	0.100	0.010 ***	0.100	I(1)
XMR	25	0.196	0.001 ***	0.373	0.001 ***	0.010 ***	0.081 *	0.010 ***	0.096 *	I(1)
MIOTA	23	0.063 *	0.001 ***	0.115	0.001 ***	0.010 ***	0.100	0.010 ***	0.100	I(1)
EOS	23	0.102	0.001 ***	0.168	0.001 ***	0.010 ***	0.100	0.010 ***	0.100	I(1)
XTZ	23	0.214	0.001 ***	0.515	0.001 ***	0.010 ***	0.100	0.010 ***	0.100	I(1)
XEM	25	0.001 ***	0.001 ***	0.007 ***	0.001 ***	0.039 **	0.072 *	0.010 ***	0.024 **	I(1)
NEO	24	0.239	0.001 ***	0.524	0.001 ***	0.022 **	0.100	0.010 ***	0.042 **	I(1)
DCR	25	0.410	0.001 ***	0.595	0.001 ***	0.010 ***	0.100	0.010 ***	0.032 **	I(1)
DASH	25	0.019 **	0.001 ***	0.080 *	0.001 ***	0.010 ***	0.100	0.010 ***	0.066 *	I(1)
ZIL	22	0.945	0.001 ***	0.999	0.001 ***	0.010 ***	0.017 **	0.010 ***	0.100	I(1)
RVN	22	0.958	0.001 ***	0.996	0.001 ***	0.100	0.100	0.019 **	0.100	I(1)
BAT	23	0.545	0.001 ***	0.828	0.001 ***	0.100	0.100	0.010 ***	0.100	I(1)
ZEC	24	0.194	0.001 ***	0.243	0.001 ***	0.010 ***	0.100	0.010 ***	0.100	I(1)
ETC	24	0.224	0.001 ***	0.480	0.001 ***	0.010 ***	0.100	0.010 ***	0.100	I(1)
BNT	23	0.242	0.001 ***	0.621	0.001 ***	0.010 ***	0.035 **	0.010 ***	0.100	I(1)
ICX	23	0.010 **	0.001 ***	0.007 ***	0.001 ***	0.010 ***	0.100	0.010 ***	0.047 **	I(1)

WAVES	25	0.482	0.001 ***	0.710	0.001 ***	0.021 **	0.100	0.010 ***	0.100	I(1)
XWC	25	0.999	0.001 ***	0.999	0.001 ***	0.010 ***	0.100	0.010 ***	0.100	I(1)
VGX	13	0.902	0.047 **	0.601	0.161	0.010 ***	0.100	0.010 ***	0.080 *	I(1)
DGB	25	0.062 *	0.001 ***	0.073 *	0.001 ***	0.010 ***	0.100	0.010 ***	0.100	I(1)
STORJ	23	0.200	0.001 ***	0.564	0.001 ***	0.010 ***	0.100	0.010 ***	0.100	I(1)
OMG	23	0.215	0.001 ***	0.169	0.001 ***	0.010 ***	0.100	0.010 ***	0.014 **	I(1)
QTUM	23	0.016 **	0.001 ***	0.013 **	0.001 ***	0.010 ***	0.100	0.010 ***	0.100	I(1)
IOST	22	0.503	0.001 ***	0.975	0.001 ***	0.010 ***	0.100	0.010 ***	0.100	I(1)
CELO	13	0.740	0.007 ***	0.654	0.041 **	0.010 ***	0.100	0.024 **	0.100	I(1)
LSK	25	0.118	0.001 ***	0.336	0.001 ***	0.010 ***	0.100	0.010 ***	0.032 **	I(1)

Note: The optimal lag length was calculated with the Schwert criterion (Schwert 1989). *, **, and *** denote a rejection at 10%, 5%, and 1% significance level, respectively. The critical values for the ADF test without trend are −3.43, −2.87 and −2.57 for the 1%, 5%, and 10% significance level, respectively, and −3.97, −3.42, and −3.13 for the ADF with trend. For the KPSS without trend, values are 0.739, 0.463, and 0.347, and for the KPSS test with trend, values are 0.216, 0.146, and 0.119, for the 1%, 5%, and 10% significance level, respectively.

3.2. Returns

In a second step, we used the returns of the cryptocurrencies, calculated as the first differences in the log levels. The first differences remove the non-stationarity (Table 3), and the data are now better fitted for calculating the Hurst exponents. The detailed results are presented in Tables A5–A8 in Appendix A. In Table 4, we summarize the results of the calculated Hurst exponents and their respective confidence intervals on the returns of the 37 cryptocurrencies for the whole data sample and the three subperiods.

Table 4. DFA Hurst exponent inference on the returns of cryptocurrencies.

Currency	From	1 January 2016	1 January 2016	25 March 2017	26 March 2019
	To	26 March 2021	24 March 2017	25 March 2019	26 March 2021
BTC		persistent	random walk	persistent	random walk
ETH		persistent	persistent	persistent	random walk
BNB		random walk	-	random walk	persistent
ADA		persistent	-	persistent	persistent
XRP		random walk	random walk	random walk	random walk
LTC		persistent	anti-persistent	random walk	random walk
THETA		random walk	-	random walk	random walk
LINK		random walk	-	random walk	random walk
BCH		random walk	-	random walk	random walk
XLM		random walk	anti-persistent	random walk	random walk
DOGE		random walk	anti-persistent	random walk	random walk
TRX		random walk	-	persistent	random walk
XMR		random walk	random walk	random walk	random walk
MIOTA		random walk	-	random walk	random walk
EOS		random walk	-	persistent	random walk
XTZ		random walk	-	random walk	random walk
XEM		persistent	random walk	persistent	random walk
NEO		persistent	random walk	persistent	random walk
DCR		persistent	random walk	random walk	persistent
DASH		persistent	persistent	random walk	random walk
ZIL		random walk	-	random walk	persistent
RVN		random walk	-	random walk	persistent
BAT		random walk	-	random walk	random walk
ZEC		random walk	random walk	random walk	random walk
ETC		random walk	random walk	random walk	random walk
BNT		random walk	-	random walk	persistent
ICX		random walk	-	random walk	random walk
WAVES		persistent	random walk	random walk	random walk

XWC	random walk	anti-persistent	persistent	anti-persistent
VGX	random walk	-	-	random walk
DGB	random walk	random walk	random walk	random walk
STORJ	random walk	-	random walk	random walk
OMG	random walk	-	random walk	random walk
QTUM	random walk	-	random walk	random walk
IOST	random walk	-	random walk	random walk
CELO	random walk	-	-	random walk
LSK	random walk	anti-persistent	persistent	random walk

Full sample

According to these results, we find empirical evidence that nine cryptocurrencies are persistent in the full sample: BTC, ETH, ADA, LTC, XEM, NEO, DCR, DASH, and WAVES. The lower bound of the 90% confidence interval of the estimated Hurst exponent for these nine series is greater than the value of 0.5 that implies a random walk series. Thus, we find evidence of persistence for these 9 cryptocurrencies and a random walk behavior for the other 28.

Period 1 January 2016–24 March 2017

In this period, we find evidence for only two cryptocurrencies that are persistent: ETH and DASH. Moreover, five cryptocurrencies appear to be anti-persistent or mean reverting: LTC, XLM, DOGE, XWC, and LSK. For the rest of the cryptocurrencies, we cannot reject the hypothesis that the returns follow a random walk.

Period 25 March 2017–25 March 2019

For this period, nine cryptocurrencies appear to be persistent: BTC, ETH, ADA, TRX, EOS, XEM, NEO, XWC, and LSK. For the rest of the cryptocurrencies, we cannot reject the hypothesis that the returns follow a random walk.

Period 26 March 2019–26 March 2021

In this subsample, there is evidence that six cryptocurrencies are persistent: BNB, ADA, DCR, ZIL, RVN, and BNT. Moreover, one appears to be anti-persistent or mean reverting: XWC. For the rest of the cryptocurrencies, we cannot reject the hypothesis that the returns follow a random walk.

4. Discussion

In this paper we examine the long-range trends of the closing prices and returns for 37 cryptocurrencies. We implement both the R/S and DFA method to calculate the Hurst exponent. The period under investigation is 1 January 2016 to 26 March 2021. We calculated the Hurst exponent for the whole period, and for three consecutive time-windows.

In a similar study, Arouxet et al. (2022) focused on the returns and volatility for seven cryptocurrencies. Although the authors investigated a smaller sample, both in time length and in number of cryptocurrencies, their results are similar to our findings. Zhang et al. (2019) investigated four cryptocurrencies both with R/S and a DFA-based Hurst exponent. Yet again, our results coincide. For example, BTC and ETH are reported to have similar behavior. Zhang et al. (2018) reported Hurst values close to 0.5, using the Hurst exponent and the rolling-window DFA method. Similarly, our estimations show that the BTC time series present a random walk behavior. Bariviera (2021) investigated daily price data using the generalized Hurst exponent and a multifractal version of DFA analysis. The conclusion of the author is that larger cryptocurrencies, according to traded volume, present random walk behavior, which coincides especially with our results for the third rolling window. The generalized Hurst exponent with a rolling-window framework was also employed also in Keshari Jena et al. (2022). Daily prices for the top six cryptocurrencies, based on the market capitalization, were used, and the Hurst exponent values in most of the cases varied from the efficient 0.5 value and were either persistent or anti-persistent. Other studies investigated the day-of-the-week effect and concluded that BTC seems independent of speculative factors (Aharon and Qadan 2019). These findings, in combination with

the efficient market hypothesis, are aligned with the results in our paper, as we find in most cases that the BTC has a random walk behavior. Furthermore, Cohen and Qadan (2022) designed machine learning (ML) systems that can trade for Bitcoin, Ethereum, BNB, and Solana. They conclude that more indicators do not necessarily mean better trading performance, meaning that cryptocurrencies are efficient enough. These results agree with our DFA results for the returns of cryptocurrencies.

Our research indicates that the efficient market hypothesis may apply in the crypto markets (random walk), in contrast to several papers. Aggarwal (2019) analyzed the market efficiency of the daily BTC returns for the time frame of July 2010 to March 2018, using multiple robust tests (multiple unit root tests and volatility persistence measures). She found evidence of market inefficiency. The findings of the study reveal that the bitcoin returns do not follow a random walk model and hence are characterized by market inefficiency. Palamalai et al. (2021) investigated the weak-form efficiency of the top-ten most highly capitalized cryptocurrencies (Bitcoin, Ethereum, Ripple, Litecoin, Stellar, Monero, Dash, Ethereum Classic, NEM, and Maker) using non-parametric and parametric random walk testing methods that are robust to structural breaks and asymmetric effects. The findings do not support the random walk hypothesis, hence validating the weak-form inefficiency for daily cryptocurrency returns. Amirat (2021), using daily closing prices from 1 January 2015 to 31 January 2019 of the eight large cryptocurrencies (Bitcoin, XRP, Ethereum, Litecoin, Stellar, Monero, Dash, and NEM) and MVIS Crypto Compare Digital Assets for the large cap index, applied a battery of 13 robust tests to check randomness and correlation in returns. The results show that all cryptocurrencies are inefficient except the Bitcoin, which showed weak efficiency in more than 50% of the tests. Verma et al. (2022) empirically tested the behavior of the cryptocurrency returns, inferring its market efficiency. For this purpose, daily data of five cryptocurrencies (Bitcoin, Ethereum, Litecoin, Tether, and Ripple) were collected from 1 January 2016 to 31 March 2021 to investigate the random walk hypothesis. To provide statistical evidence and ensure the robustness of results, analysis was performed using the variance ratio test, augmented Dickey–Fuller test, Philip–Perron test, Breusch–Godfrey serial correlation LM test, and ARIMA model. The statistical results illustrated strong evidence refuting the presence of the random walk hypothesis in this emerging market, thus implying inefficiency in the cryptocurrency market. Magner and Hardy (2022) tested the random walk hypothesis and evaluated whether cryptocurrency returns are predictable using the Meese–Rogoff puzzle. They conducted in-sample and out-of-sample analyses to examine the forecasting power of their model, which was built with autoregressive components and lagged returns of BTC, compared with the random walk benchmark. To this end, they considered the 13 cryptocurrencies with the highest market capitalization between 2018 and 2022. Their results indicate that the models significantly outperform the random walk benchmark; in particular, cryptocurrencies tend to be far more persistent than regular exchange rates.

Our findings indicate that the efficient market hypothesis applies in the crypto market, with most cryptocurrencies showing a random walk behavior.

5. Conclusions

In this study, we attempted to uncover evidence about the long-range behavior of the prices and daily returns of the cryptocurrency market. To do so, we estimated the Hurst exponent for 37 of the most important cryptocurrencies, in terms of market capitalization, as they account for more than 80% of the total market capitalization. The estimates of the Hurst exponent were made using both the R/S and DFA methodologies. Moreover, instead of performing the analysis only once for the whole data set, we created three consecutive non-overlapping time windows. This was undertaken in order to provide evidence about the dynamic changes in the cryptocurrency market. The windows that were used are from 1 January 2016 to 24 March 2017, 25 March 2017 to 25 March 2019, and 26 March 2019 to 26 March 2021. For the time series of closing prices and daily returns, we

calculated the Hurst exponent and estimated the corresponding 90% and 95% confidence intervals.

The goal of this study was to determine whether the time series of the 37 cryptocurrencies' closing prices and daily returns exhibit persistence or mean reversion, or follow a random walk. Such evidence is important to both academics and market participants who need to optimize their investment portfolios.

The empirical results show that the cryptocurrencies closing price data are non-stationary and this is reflected in the estimated Hurst exponents that are greater than 1.

For the cases of the daily returns and the 90% confidence interval, we have evidence from both methodologies and all of the time windows that the time series generally follow a random walk. This means that they move in a random manner and there is no possibility of predicting them. It is worth mentioning that for the period 1 January 2016 to 24 March 2017 we found five time series of returns and for the period 26 March 2019 to 26 March 2021 we found one time series of returns that showed negative autocorrelation, meaning that a negative or positive trend was followed by the exact opposite trend in the future period. Similarly, for the time series of returns, our findings for the 95% confidence interval with both methods, for all periods, were consistent with the original ones.

Cryptocurrencies are considered attractive to unconventional investors due to the absence of a formal and central regulating authority. They are considered high- and fast-earning investments. However, in our tests, we found that the returns follow a random walk, making it difficult to accurately forecast them. Therefore, the investment community, whether in the form of individual investors or organizations and governments entering this market, should bear in mind that while the market has shown signs of stabilization, its high volatility makes it risky. The time series of returns are overwhelmingly characterized as random walks, which means that we cannot make safe and reliable future predictions.

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Appendix A

Table A1. Hurst exponent, closing prices from 1 January 2016 to 26 March 2021, DFA method and 90% Confidence Intervals.

N	Currency	Lower	Upper	Hurst	Behaviour
1908	BTC	1.2982	1.4262	1.3660	persistent
1908	ETH	1.2772	1.4051	1.3450	persistent
1337	BNB	1.1178	1.2659	1.1961	persistent

1269	ADA	1.1871	1.3385	1.2670	persistent
1908	XRP	1.1524	1.2804	1.2202	persistent
1908	LTC	1.2095	1.3374	1.2772	persistent
1161	THETA	1.0530	1.2102	1.1360	persistent
1280	LINK	1.3613	1.5122	1.4410	persistent
1339	BCH	1.2501	1.3982	1.3284	persistent
1908	XLM	1.2535	1.3814	1.3212	persistent
1908	DOGE	1.0049	1.1328	1.0726	persistent
1287	TRX	1.0201	1.1706	1.0996	persistent
1908	XMR	1.3052	1.4331	1.3730	persistent
1379	MIOTA	1.2287	1.3749	1.3060	persistent
1361	EOS	1.2220	1.3690	1.2997	persistent
1268	XTZ	1.1757	1.3272	1.2558	persistent
1908	XEM	1.1869	1.3148	1.2547	persistent
1656	NEO	1.2732	1.4087	1.3449	persistent
1868	DCR	1.2652	1.3942	1.3335	persistent
1908	DASH	1.2953	1.4233	1.3631	persistent
1153	ZIL	1.2548	1.4126	1.3381	persistent
1109	RVN	1.0676	1.2281	1.1523	persistent
1391	BAT	1.0069	1.1525	1.0838	persistent
1606	ZEC	1.1649	1.3022	1.2375	persistent
1703	ETC	1.2725	1.4064	1.3434	persistent
1374	BNT	1.1228	1.2692	1.2001	persistent
1243	ICX	1.2318	1.3845	1.3124	persistent
1755	WAVES	1.2311	1.3634	1.3012	persistent
1908	XWC	1.0104	1.1383	1.0781	persistent
158	VGX	1.4725	1.9038	1.6967	persistent
1908	DGB	1.1370	1.2649	1.2048	persistent
1360	STORJ	1.2103	1.3573	1.2880	persistent
1348	OMG	1.2963	1.4439	1.3743	persistent
1399	QTUM	1.2292	1.3745	1.3061	persistent
1162	IOST	1.1560	1.3132	1.2390	persistent
158	CELO	1.3010	1.7324	1.5252	persistent
1812	LSK	1.2650	1.3956	1.3342	persistent

Table A2. Hurst exponent, closing prices from 26 March 2019 to 26 March 2021, DFA method and 90% Confidence Intervals.

N	Currency	Lower	Upper	Hurst	Behaviour
728	BTC	1.4480	1.6416	1.5499	persistent
728	ETH	1.4210	1.6146	1.5229	persistent
728	BNB	1.1531	1.3467	1.2550	persistent
728	ADA	1.3055	1.4991	1.4074	persistent
728	XRP	1.0957	1.2893	1.1976	persistent
728	LTC	1.3212	1.5148	1.4231	persistent
728	THETA	1.0672	1.2608	1.1691	persistent
728	LINK	1.4024	1.5960	1.5043	persistent
728	BCH	1.1234	1.3170	1.2253	persistent
728	XLM	1.3104	1.5040	1.4123	persistent
728	DOGE	1.1323	1.3259	1.2342	persistent
728	TRX	1.2804	1.4740	1.3823	persistent
728	XMR	1.3680	1.5616	1.4699	persistent
728	MIOTA	1.2925	1.4861	1.3944	persistent
728	EOS	1.2281	1.4217	1.3300	persistent
728	XTZ	1.2153	1.4089	1.3172	persistent
728	XEM	1.1860	1.3796	1.2879	persistent
728	NEO	1.3049	1.4985	1.4068	persistent

728	DCR	1.3271	1.5208	1.4290	persistent
728	DASH	1.1786	1.3722	1.2805	persistent
728	ZIL	1.3274	1.5210	1.4293	persistent
728	RVN	1.0901	1.2838	1.1920	persistent
728	BAT	1.0946	1.2882	1.1965	persistent
728	ZEC	1.2141	1.4077	1.3160	persistent
728	ETC	1.0407	1.2343	1.1426	persistent
728	BNT	1.2489	1.4425	1.3508	persistent
728	ICX	1.3106	1.5042	1.4125	persistent
728	WAVES	1.4296	1.6232	1.5315	persistent
728	XWC	1.2669	1.4605	1.3688	persistent
158	VGX	1.4725	1.9038	1.6967	persistent
728	DGB	1.2880	1.4816	1.3899	persistent
728	STORJ	1.1667	1.3603	1.2686	persistent
728	OMG	1.2732	1.4669	1.3751	persistent
728	QTUM	1.1947	1.3884	1.2966	persistent
728	IOST	1.1716	1.3653	1.2735	persistent
158	CELO	1.3010	1.7324	1.5252	persistent
728	LSK	1.1935	1.3871	1.2954	persistent

Table A3. Hurst exponent, closing prices from 25 March 2017 to 25 March 2019, DFA method and 90% confidence intervals.

N	Currency	Lower	Upper	Hurst	Behaviour
731	BTC	1.2990	1.4923	1.4007	persistent
731	ETH	1.3582	1.5514	1.4599	persistent
609	BNB	1.2458	1.4562	1.3564	persistent
541	ADA	1.2238	1.4465	1.3408	persistent
731	XRP	1.1323	1.3255	1.2340	persistent
731	LTC	1.2572	1.4505	1.3589	persistent
433	THETA	1.1143	1.3628	1.2446	persistent
552	LINK	1.0681	1.2887	1.1840	persistent
611	BCH	1.2238	1.4340	1.3343	persistent
731	XLM	1.2615	1.4548	1.3632	persistent
731	DOGE	1.1162	1.3094	1.2179	persistent
559	TRX	1.0929	1.3121	1.2080	persistent
731	XMR	1.3446	1.5378	1.4463	persistent
651	MIOTA	1.2280	1.4320	1.3353	persistent
633	EOS	1.2506	1.4572	1.3592	persistent
540	XTZ	1.1915	1.4144	1.3086	persistent
731	XEM	1.2359	1.4292	1.3376	persistent
731	NEO	1.3521	1.5453	1.4538	persistent
731	DCR	1.3455	1.5388	1.4472	persistent
731	DASH	1.3603	1.5536	1.4620	persistent
425	ZIL	1.2571	1.5079	1.3886	persistent
381	RVN	1.0553	1.3204	1.1941	persistent
663	BAT	1.1468	1.3490	1.2531	persistent
731	ZEC	1.2590	1.4522	1.3607	persistent
731	ETC	1.2475	1.4407	1.3492	persistent
646	BNT	1.0608	1.2655	1.1684	persistent
515	ICX	1.2829	1.5110	1.4026	persistent
731	WAVES	1.2362	1.4294	1.3379	persistent
731	XWC	1.3268	1.5201	1.4285	persistent
731	DGB	1.1072	1.3004	1.2089	persistent
632	STORJ	1.2201	1.4269	1.3288	persistent
620	OMG	1.2937	1.5024	1.4034	persistent
671	QTUM	1.1999	1.4010	1.3056	persistent

434	IOST	1.0798	1.3279	1.2099	persistent
731	LSK	1.3327	1.5260	1.4344	persistent
0	VGX	-	-	Sample is too small	-
0	CELO	-	-	Sample is too small	-

Table A4. Hurst exponent, closing prices from 1 January 2016 to 24 March 2017, DFA method and 90% confidence intervals.

N	Currency	Lower	Upper	Hurst	Behaviour
449	BTC	1.4105	1.6545	1.5385	persistent
449	ETH	0.8999	1.1439	1.0278	persistent
449	XRP	0.9940	1.2380	1.1219	persistent
449	LTC	1.0766	1.3207	1.2046	persistent
449	XLM	0.9762	1.2203	1.1042	persistent
449	DOGE	0.8119	1.0559	0.9398	persistent
449	XMR	1.2502	1.4943	1.3782	persistent
449	XEM	1.1759	1.4199	1.3038	persistent
197	NEO	0.8971	1.2761	1.0945	persistent
409	DCR	1.0622	1.3179	1.1962	persistent
449	DASH	1.0176	1.2616	1.1455	persistent
147	ZEC	0.5961	1.0467	0.8301	persistent
244	ETC	1.0036	1.3395	1.1788	persistent
296	WAVES	0.8022	1.1046	0.9602	persistent
449	XWC	0.9036	1.1476	1.0316	persistent
449	DGB	1.0437	1.2877	1.1716	persistent
353	LSK	0.8970	1.1727	1.0413	persistent
0	BNB	-	-	Sample is too small	-
0	ADA	-	-	Sample is too small	-
0	THETA	-	-	Sample is too small	-
0	LINK	-	-	Sample is too small	-
0	BCH	-	-	Sample is too small	-
0	TRX	-	-	Sample is too small	-
0	MIOTA	-	-	Sample is too small	-
0	EOS	-	-	Sample is too small	-
0	XTZ	-	-	Sample is too small	-
0	ZIL	-	-	Sample is too small	-
0	RVN	-	-	Sample is too small	-
0	BAT	-	-	Sample is too small	-
0	BNT	-	-	Sample is too small	-
0	ICX	-	-	Sample is too small	-
0	VGX	-	-	Sample is too small	-
0	STORJ	-	-	Sample is too small	-
0	OMG	-	-	Sample is too small	-
0	QTUM	-	-	Sample is too small	-
0	IOST	-	-	Sample is too small	-
0	CELO	-	-	Sample is too small	-

Table A5. Hurst exponent, returns from 1 January 2016 to 26 March 2021, DFA method and 90% confidence intervals.

	Currency	Lower	Upper	Hurst	Behaviour
1908	BTC	0.5014	0.6293	0.5691	persistent
1908	ETH	0.5301	0.6580	0.5978	persistent
1336	BNB	0.4656	0.6138	0.5439	random walk
1268	ADA	0.5179	0.6694	0.5980	persistent
1908	XRP	0.4826	0.6105	0.5503	random walk
1908	LTC	0.5097	0.6376	0.5775	persistent
1160	THETA	0.4436	0.6009	0.5266	random walk

1279	LINK	0.3819	0.5328	0.4617	random walk
1338	BCH	0.4151	0.5631	0.4933	random walk
1908	XLM	0.4906	0.6185	0.5584	random walk
1908	DOGE	0.4476	0.5756	0.5154	random walk
1286	TRX	0.4556	0.6062	0.5352	random walk
1908	XMR	0.4972	0.6251	0.5649	random walk
1378	MIOTA	0.4520	0.5983	0.5293	random walk
1360	EOS	0.4679	0.6149	0.5456	random walk
1267	XTZ	0.4220	0.5736	0.5021	random walk
1908	XEM	0.5394	0.6673	0.6071	persistent
1655	NEO	0.5774	0.7129	0.6491	persistent
1867	DCR	0.5500	0.6790	0.6183	persistent
1908	DASH	0.5318	0.6597	0.5995	persistent
1152	ZIL	0.4950	0.6528	0.5783	random walk
1108	RVN	0.4636	0.6241	0.5483	random walk
1390	BAT	0.3601	0.5058	0.4371	random walk
1605	ZEC	0.4498	0.5870	0.5224	random walk
1702	ETC	0.4996	0.6336	0.5706	random walk
1373	BNT	0.4864	0.6329	0.5638	random walk
1242	ICX	0.4767	0.6295	0.5574	random walk
1754	WAVES	0.5094	0.6417	0.5794	persistent
1908	XWC	0.3855	0.5193	0.4563	random walk
157	VGX	0.3580	0.7910	0.5830	random walk
1908	DGB	0.4756	0.6035	0.5434	random walk
1359	STORJ	0.3930	0.5401	0.4707	random walk
1347	OMG	0.4729	0.6205	0.5509	random walk
1398	QTUM	0.4135	0.5588	0.4903	random walk
1161	IOST	0.4187	0.5759	0.5017	random walk
157	CELO	0.2818	0.7148	0.5068	random walk
1811	LSK	0.4885	0.6191	0.5577	random walk

Table A6. Hurst exponent, returns from 26 March 2019 to 26 March 2021, DFA method and 90% confidence intervals.

N	Currency	Lower	Upper	Hurst	Behaviour
728	BTC	0.4795	0.6731	0.5814	random walk
728	ETH	0.4585	0.6522	0.5604	random walk
728	BNB	0.5153	0.7089	0.6172	persistent
728	ADA	0.5187	0.7124	0.6206	persistent
728	XRP	0.3404	0.5340	0.4423	random walk
728	LTC	0.4482	0.6418	0.5501	random walk
728	THETA	0.4536	0.6472	0.5555	random walk
728	LINK	0.3785	0.5721	0.4804	random walk
728	BCH	0.3415	0.5351	0.4433	random walk
728	XLM	0.4213	0.6149	0.5231	random walk
728	DOGE	0.4823	0.6759	0.5842	random walk
728	TRX	0.3781	0.5717	0.4800	random walk
728	XMR	0.3852	0.5788	0.4871	random walk
728	MIOTA	0.4567	0.6503	0.5586	random walk
728	EOS	0.3344	0.5280	0.4362	random walk
728	XTZ	0.3301	0.5237	0.4320	random walk
728	XEM	0.4403	0.6339	0.5422	random walk
728	NEO	0.3727	0.5664	0.4746	random walk
728	DCR	0.5200	0.7137	0.6219	persistent
728	DASH	0.3983	0.5920	0.5002	random walk
728	ZIL	0.5120	0.7056	0.6139	persistent
728	RVN	0.5202	0.7138	0.6221	persistent

728	BAT	0.4048	0.5984	0.5067	random walk
728	ZEC	0.4074	0.6010	0.5093	random walk
728	ETC	0.3794	0.5730	0.4813	random walk
728	BNT	0.5142	0.7078	0.6161	persistent
728	ICX	0.4439	0.6375	0.5458	random walk
728	WAVES	0.4769	0.6705	0.5788	random walk
728	XWC	0.2935	0.4871	0.3954	anti-persistent
157	VGX	0.3580	0.7910	0.5830	random walk
728	DGB	0.4214	0.6150	0.5233	random walk
728	STORJ	0.3801	0.5737	0.4820	random walk
728	OMG	0.4051	0.5987	0.5070	random walk
728	QTUM	0.3762	0.5698	0.4781	random walk
728	IOST	0.4338	0.6274	0.5357	random walk
157	CELO	0.2818	0.7148	0.5068	random walk
728	LSK	0.4399	0.6335	0.5418	random walk

Table A7. Hurst exponent, returns from 25 March 2017 to 25 March 2019, DFA method and 90% confidence intervals.

N	Currency	Lower	Upper	Hurst	Behaviour
731	BTC	0.5132	0.7065	0.6149	persistent
731	ETH	0.5248	0.7180	0.6265	persistent
608	BNB	0.4731	0.6837	0.5838	random walk
540	ADA	0.5333	0.7562	0.6504	persistent
731	XRP	0.4555	0.6487	0.5572	random walk
731	LTC	0.4764	0.6696	0.5781	random walk
432	THETA	0.2769	0.5257	0.4073	random walk
551	LINK	0.3601	0.5808	0.4760	random walk
610	BCH	0.4262	0.6365	0.5367	random walk
731	XLM	0.4961	0.6894	0.5979	random walk
731	DOGE	0.4636	0.6568	0.5653	random walk
558	TRX	0.5008	0.7202	0.6160	persistent
731	XMR	0.4728	0.6661	0.5745	random walk
650	MIOTA	0.4572	0.6613	0.5645	random walk
632	EOS	0.5176	0.7244	0.6263	persistent
539	XTZ	0.3740	0.5972	0.4912	random walk
731	XEM	0.5341	0.7273	0.6358	persistent
731	NEO	0.5362	0.7295	0.6379	persistent
731	DCR	0.4395	0.6327	0.5412	random walk
731	DASH	0.4939	0.6872	0.5956	random walk
424	ZIL	0.4118	0.6629	0.5434	random walk
380	RVN	0.3763	0.6418	0.5154	random walk
662	BAT	0.3743	0.5766	0.4807	random walk
731	ZEC	0.4439	0.6371	0.5456	random walk
731	ETC	0.4705	0.6638	0.5722	random walk
645	BNT	0.3995	0.6044	0.5072	random walk
514	ICX	0.4404	0.6687	0.5602	random walk
731	WAVES	0.4904	0.6836	0.5921	random walk
731	XWC	0.5181	0.7113	0.6198	persistent
731	DGB	0.4883	0.6816	0.5900	random walk
631	STORJ	0.3483	0.5552	0.4571	random walk
619	OMG	0.4839	0.6927	0.5936	random walk
670	QTUM	0.4160	0.6172	0.5219	random walk
433	IOST	0.2886	0.5371	0.4188	random walk
731	LSK	0.5035	0.6967	0.6052	persistent
0	VGX	-	-	Sample is too small	-
0	CELO	-	-	Sample is too small	-

Table A8. Hurst exponent, returns from 1 January 2016 to 24 March 2017, DFA method and 90% confidence intervals.

N	Currency	Lower	Upper	Hurst	Behaviour
449	BTC	0.2644	0.5084	0.3924	random walk
449	ETH	0.5028	0.7468	0.6307	persistent
449	XRP	0.2634	0.5074	0.3914	random walk
449	LTC	0.2460	0.4900	0.3740	anti-persistent
449	XLM	0.1647	0.4087	0.2926	anti-persistent
449	DOGE	0.2078	0.4518	0.3357	anti-persistent
449	XMR	0.3782	0.6222	0.5061	random walk
449	XEM	0.4262	0.6702	0.5542	random walk
196	NEO	0.1555	0.5355	0.3534	random walk
408	DCR	0.4916	0.7476	0.6257	random walk
449	DASH	0.5165	0.7606	0.6445	persistent
146	ZEC	0.4544	0.9069	0.6894	random walk
243	ETC	0.2308	0.5674	0.4064	random walk
295	WAVES	0.3808	0.6838	0.5391	random walk
449	XWC	0.1370	0.3810	0.2650	anti-persistent
449	DGB	0.2753	0.5193	0.4032	random walk
352	LSK	0.1073	0.3834	0.2518	anti-persistent
0	BNB	-	-	Sample is too small	-
0	ADA	-	-	Sample is too small	-
0	THETA	-	-	Sample is too small	-
0	LINK	-	-	Sample is too small	-
0	BCH	-	-	Sample is too small	-
0	TRX	-	-	Sample is too small	-
0	MIOTA	-	-	Sample is too small	-
0	EOS	-	-	Sample is too small	-
0	XTZ	-	-	Sample is too small	-
0	ZIL	-	-	Sample is too small	-
0	RVN	-	-	Sample is too small	-
0	BAT	-	-	Sample is too small	-
0	BNT	-	-	Sample is too small	-
0	ICX	-	-	Sample is too small	-
0	VGX	-	-	Sample is too small	-
0	STORJ	-	-	Sample is too small	-
0	OMG	-	-	Sample is too small	-
0	QTUM	-	-	Sample is too small	-
0	IOST	-	-	Sample is too small	-
0	CELO	-	-	Sample is too small	-

Table A9. DFA Hurst exponent estimates' inference on the prices of cryptocurrencies.

Currency	From	1 January 2016	1 January 2016	25 March 2017	26 March 2019
	To	26 March 2021	24 March 2017	25 March 2019	26 March 2021
BTC		persistent	persistent	persistent	persistent
ETH		persistent	persistent	persistent	persistent
BNB		persistent	-	persistent	persistent
ADA		persistent	-	persistent	persistent
XRP		persistent	persistent	persistent	persistent
LTC		persistent	persistent	persistent	persistent
THETA		persistent	-	persistent	persistent
LINK		persistent	-	persistent	persistent
BCH		persistent	-	persistent	persistent
XLM		persistent	persistent	persistent	persistent
DOGE		persistent	persistent	persistent	persistent
TRX		persistent	-	persistent	persistent

XMR	persistent	persistent	persistent	persistent
MIOTA	persistent	-	persistent	persistent
EOS	persistent	-	persistent	persistent
XTZ	persistent	-	persistent	persistent
XEM	persistent	persistent	persistent	persistent
NEO	persistent	persistent	persistent	persistent
DCR	persistent	persistent	persistent	persistent
DASH	persistent	persistent	persistent	persistent
ZIL	persistent	-	persistent	persistent
RVN	persistent	-	persistent	persistent
BAT	persistent	-	persistent	persistent
ZEC	persistent	persistent	persistent	persistent
ETC	persistent	persistent	persistent	persistent
BNT	persistent	-	persistent	persistent
ICX	persistent	-	persistent	persistent
WAVES	persistent	persistent	persistent	persistent
XWC	persistent	persistent	persistent	persistent
VGX	persistent	-	-	persistent
DGB	persistent	persistent	persistent	persistent
STORJ	persistent	-	persistent	persistent
OMG	persistent	-	persistent	persistent
QTUM	persistent	-	persistent	persistent
IOST	persistent	-	persistent	persistent
CELO	persistent	-	-	persistent
LSK	persistent	persistent	persistent	persistent

Note

^{1.} Detailed results can be found in Appendix A.

References

- Aggarwal, Divya. 2019. Do bitcoins follow a random walk model? *Research in Economics* 73: 15–22. <https://doi.org/10.1016/j.rie.2019.01.002>.
- Aharon, David Yechiam, and Mahmoud Qadan. 2019. Bitcoin and the day-of-the-week effect. *Finance Research Letters* 31: 415–424. <https://doi.org/10.1016/j.frl.2018.12.004>.
- Alvarez-Ramirez, Jose, Jesus Alvarez, Eduardo Rodriguez, and Guillermo Fernandez-Anaya. 2008. Time-Varying Hurst Exponent for US Stock Markets. *Physica A* 387: 6159–6169.
- Amirat, Amina. 2021. Exploring the random walk in cryptocurrency market. *Indian Journal of Economics and Business* 20: 735–742.
- Arouxet, M. Bélen, Aurelio F. Bariviera, Verónica E. Pastor, and Victoria Vampa. 2022. COVID-19 impact on cryptocurrencies: Evidence from a wavelet-based Hurst exponent. *Physica A: Statistical Mechanics and Its Applications* 596: 127170.
- Balci, Mehmet Ali, Larissa M. Batrancea, Ömer Akgüller, Lucian Gaban, Mircea-Iosif Rus, and Horia Tulai. 2022. Fractality of Borsa Istanbul during the COVID-19 Pandemic. *Mathematics* 10: 2503. <https://doi.org/10.3390/math10142503>.
- Bariviera, Aurelio F. 2021. One model is not enough: Heterogeneity in cryptocurrencies' multifractal profiles. *Finance Research Letters* 39: 101649.
- Bryce, Robert M., and Kevin B. Sprague. 2012. Revisiting detrended fluctuation analysis. *Scientific Reports* 2: 315.
- Carbone, Anna, Giuliano Castelli, and Eugene H. Stanley. 2004. Time-Dependent Hurst Exponent in Financial Time Series. *Physica A* 344: 267–271.
- Cohen, Gil, and Mahmoud Qadan. 2022. The Complexity of Cryptocurrencies Algorithmic Trading. *Mathematics* 10: 2037.
- Czarnecki, Łukasz, Dariusz Grech, and Grzegorz Pamula. 2008. Comparison Study of Global and Local Approaches Describing Critical Phenomena on the Polish Stock Exchange Market. *Physica A* 387: 6801–6811.
- Di Matteo, Tiziana. 2007. Multi-Scaling in Finance. *Quantitative Finance* 7: 21–36.
- Di Matteo, Tiziana, Tomaso Aste, and Michel M. Dacorogna. 2005. Long-Term Memories of Developed and Emerging Markets: Using the Scaling Analysis to Characterize Their Stage of Development. *Journal of Banking and Finance* 29: 827–851.
- Engle, Robert F., and Clive W. J. Granger. 1987. Cointegration and error correction: Representation, estimation and testing. *Econometrica* 55: 251–76.
- Grech, Dariusz K., and Zygmunt Mazur. 2004. Can One Make any Crash Prediction in Finance Using the Local Hurst Exponent Idea? *Physica A* 336: 133–145.
- Hsieh, A. David. 1991. Chaos and Nonlinear Dynamics: Application to Financial Markets. *Journal of Finance* 46: 1839–1877. <https://doi.org/10.1111/j.1540-6261.1991.tb04646.x>.

- Hurst, Harold Edwin 1951. Long-Term Storage Capacity of Reservoirs. *Transactions of the American Society of Civil Engineers* 116: 770–799.
- Jach, Agnieszka, Tucker McElroy, and Dimitris N. Politis. 2011. Subsampling inference for the mean of heavy-tailed long-memory time series. *Journal of Time Series Analysis* 33: 96–111. <https://doi.org/10.1111/j.1467-9892.2011.00742.x>.
- Keshari Jena, Sangram, Aviral Kumar Tiwari, Buhari Doğan, and Shawkat Hammoudeh. 2022. Are the top six cryptocurrencies efficient? Evidence from time-varying long memory. *International Journal of Finance & Economics* 27: 3730–3740.
- Krištoufek, Ladislav. 2010. Rescaled Range Analysis and Detrended Fluctuation Analysis: Finite Sample Properties and Confidence Intervals. *Czech Economic Review* 4: 315–329.
- Kyriazis, Nikolaos A. 2019. A Survey on Efficiency and Profitable Trading Opportunities in Cryptocurrency Markets. *Journal of Risk and Financial Management* 12: 67.
- Magner, Nicolás, and Nicolás Hardy. 2022. Cryptocurrency forecasting: More evidence of the Meese-Rogoff puzzle. *Mathematics* 10: 2338. <https://doi.org/10.3390/math10132338>.
- Matos, Jose A. O., Sílvia M. A. Gama, Heather J. Ruskin, Adel Al Sharkasi, and Martin Crane. 2008. Time and scale Hurst exponent analysis for financial markets. *Physica A: Statistical Mechanics and its Applications* 387: 3910–3915.
- Palamalai, Srinivasan, Krishna K. Kumar, and Bipasha Maity. 2021. Testing the random walk hypothesis for leading cryptocurrencies. *Borsa Istanbul Review* 21: 256–268. <https://doi.org/10.1016/j.bir.2020.10.006>.
- Papadimitriou, Theofilos, Periklis Gogas, and Fotios Gkatzoglou. 2020. The evolution of the cryptocurrencies market: A complex networks approach. *Journal of Computational and Applied Mathematics* 376: 112831.
- Peters, Edgar, E. 1991. A Chaotic Attractor for the S&P 500. *Financial Analysts Journal* 47: 55–62.
- Peters, Edgar E. 1994. *Fractal Market Analysis—Applying Chaos Theory to Investment and Analysis*. New York: John Wiley & Sons, Inc.
- Qadan, Mahmoud, David Y. Aharon, and Ron Eichel. 2022. Seasonal and calendar effects and the price efficiency of cryptocurrencies. *Finance Research Letters* 46: 102354.
- Qureshi, Saba, Muhammad Aftab, Elie Bouri, and Tareq Saeed. 2020. Dynamic interdependence of cryptocurrency markets: An analysis across time and frequency. *Physica A: Statistical Mechanics and its Applications* 559: 125077.
- Schwert, G. William 1989. Tests for UNIT ROOTS: A Monte Carlo Investigation. *Journal of Business & Economic Statistics* 7: 147–159.
- Siriopoulos, Costas 1996. Investigating the behaviour of mature and emerging capital markets. *Indian Journal of Quantitative Economics* 11: 76–98.
- Siriopoulos, Costas, and Maria Skaperda. 2020. Investing in mutual funds: Are you paying for performance or for the ties of the manager? *Bulletin of Applied Economics* 7: 153–164. <https://doi.org/10.47260/bae/7212>.
- Sirlantzis, Konstantinos, and Costas Siriopoulos. 1993. Deterministic chaos in stock markets: Empirical results from monthly returns. *Neural Network World* 3: 855–864.
- Valencia, Franco, Alfonso Gómez-Espinosa, and Benjamin Valdés-Aguirre. 2019. Price Movement Prediction of Cryptocurrencies Using Sentiment Analysis and Machine Learning. *Entropy* 21: 589.
- Vandewalle, Nicolas, Marcel Ausloos, and Pierre Boveroux. 1997. *Detrended Fluctuation Analysis of the Foreign Exchange Market*. Budapest: Econophysics Workshop.
- Verma, Ruchita, Dhanraj Sharma, and Shiney Sam. 2022. Testing of random walk hypothesis in the cryptocurrency market. *FIIB Business Review* 231971452211012. <https://doi.org/10.1177/23197145221101238>.
- Wendler, Martin, and Annika Betken. 2018. Subsampling for general statistics under Long Range Dependence. *Statistica Sinica* 28: 1199–1224. <https://doi.org/10.5705/ss.202015.0435>.
- Weron, Rafal. 2002. Estimating long-range dependence: Finite sample properties and confidence intervals. *Physica A: Statistical Mechanics and Its Applications* 312: 285–299.
- Yang, Boyu, Yuying Sun, and Shouyang Wang. 2020. A novel two-stage approach for cryptocurrency analysis. *International Review of Financial Analysis* 72: 101567.
- Zhang, Wei, Pengfei Wang, Xiao Li, and Dehua Shen. 2018. Some stylized facts of the cryptocurrency market. *Applied Economics* 50: 5950–5965.
- Zhang, Yuanyaun, Stephen Chan, Jeffrey Chu, and Saralees Nadarajah. 2019. Stylized facts for high frequency cryptocurrency data. *Physica A: Statistical Mechanics and Its Applications* 513: 598–612.
- Zhengyang, Wang, Li Xingzhou, Ruan Jinjin, and Kou Jiaqing. 2019. Prediction of cryptocurrency price dynamics with multiple machine learning techniques. Paper presented at the 2019 4th International Conference on Machine Learning Technologies, Nanchang, China, June 21–23; pp. 15–19.

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