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Transition Flight Trajectory Optimization for a Flapping-Wing Micro Air Vehicle with Unsteady Vortex-Lattice Method

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Abstract: Flapping-wing micro air vehicles (FWMAVs) have the capability of performing various flight modes like birds and insects. Therefore, it is necessary to understand the various flight modes of FWMAVs in order to fully utilize the capability of the vehicle. The unique flight modes of FWMAVs can be studied through the trajectory optimization. This paper proposes a trajectory optimization framework of an FWMAV. A high-fidelity simulation model is included in the framework to sufficiently consider the complicated dynamics of the FWMAV. The unsteady aerodynamics are modeled with the unsteady panel method (UPM) and the unsteady vortex-lattice method (UVLM). The effect of wing inertia is also considered in the simulation model. In this study, transition flight trajectories are searched with the proposed framework. An optimal control problem is formulated for the transition flight from hovering to forward flight and transcribed to the parameter optimization problem with the direct shooting method. The cost function is defined as energy consumption. The same converged solution can be obtained with different initial guesses. The optimization results show that the FWMAV utilizes the pitch-up maneuver to increase altitude, although the forward speed is reduced. This pitch-up maneuver is performed more actively when the target velocity of transition is low, or the wind condition is favorable to acceleration.

Keywords: flapping-wing micro air vehicle; trajectory optimization; optimal control; bio-inspired



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1. Introduction

Flapping-wing micro air vehicles (FWMAVs) have been developed to pursue the high flight performance of birds and insects. Especially, the insect-like FWMAVs have a wider range of applications because of their various flight modes and ability to freely transit each flight mode. To fully utilize the flight ability of FWMAVs, optimization studies have been conducted. Many studies have focused on the wing kinematics optimization during hovering [1–4], forward [5], and ascending [6] flights. FWMAVs can perform more various flight modes like the transition flight and dynamic soaring as well as the hovering, forward, and ascending flights. Therefore, these unique and advantageous flight modes of FWMAVs require an in-depth study.

Trajectory optimization is a technique used to find the open-loop solution for an optimal control problem [7]. The solution of trajectory optimization is the sequence of inputs to minimize the cost function while satisfying the constraints. One main advantage of trajectory optimization is the ability to handle the constraints. Therefore, feasible wing kinematics can be obtained with the constraint of inputs. Thus, the trajectory optimization is one of the best ways to study the feasible flight trajectory of FWMAVs. The sequence of inputs can be directly applied as open-loop control. In addition, guidance and control laws, which can be applied to the general situation with modeling uncertainty and disturbance, can be designed based on the optimization results. Meanwhile, the selection of the modeling method is important in the trajectory optimization of FWMAVs. The fidelity of the model must be considered because the error from the modeling cannot be canceled by a feedback loop when the open-loop solution is applied. In addition, the computational

cost of the model should be considered because numerous simulations are required in the optimization process.

The insect flight is highly characterized by unsteady and complicated aerodynamics at low Reynolds number [8–10]. To estimate the aerodynamic load, computational fluid dynamics (CFD) and quasi-steady (QS) aerodynamic model have been commonly used [11–13]. CFD is a high-fidelity approach that can give a precise prediction of the complex flow fields of a flapping-wing. However, CFD requires significant computational time, making it unsuitable for optimization problems. QS model is a low computational cost model with low fidelity. Especially, some of the aerodynamic coefficients in the QS model are obtained by experiment or CFD [14,15]. Therefore, the accuracy is dependent on the used conditions in experiment or CFD, such as wing geometry, kinematics, forward speed and so on. When this QS model is applied to different condition, the accuracy could be decreased. Therefore, the QS model has a limitation in a trajectory optimization process searching the unknown trajectories.

A few studies have been conducted on trajectory optimization for FWMVs [16,17]. Hussein et al. [16] conducted the trajectory optimization for the transition flight from forward flight to hovering. The cost was defined as the traveled distance during the transition. The optimal trajectory included a rapid pitch-up maneuver with speed decline. Dietl and Garcia [17] also performed trajectory optimization for the transition flight from hovering to forward flight. The searched input had a bang-bang nature due to the minimum time control problem with upper and lower bounds on the controllers. As the forward flight speed increased, more time was required for the transition. It is noteworthy that both studies used the QS model as the aerodynamic model and ignored the effect of wing inertia. The wing inertia has been commonly neglected in the control-oriented model [18–20], which is used to design the feedback controller. However, neglecting the wing inertia has a significant effect from a simulation standpoint [21].

Although the above-mentioned research works have been conducted on the trajectory optimization of FWMVs, the simplified modeling used in the previous research cannot consider complex aerodynamic effects and wing inertia. Therefore, the trajectory of their research does not fully use the flight capabilities of FWMVs, and this necessitates the optimization of the FWMV trajectory with the higher fidelity simulation model.

In this study, a new trajectory optimization framework for FMWVs is proposed in terms of a simulation model with a higher fidelity. The simulation model in the framework uses the UVLM and the UPM to compute the aerodynamic force. The unsteady vortex-lattice method (UVLM) and unsteady panel method (UPM) are the potential-based aerodynamic model. These methods are computationally less expensive than CFD, while they can consider the unsteady effects like the wake effect; it is one of the main aerodynamic characteristics of the FWMV, but it cannot be considered using the QS model. Especially, this wake effect should be considered for the transition flight because the advance ratio, which affects the wake effect, is varying with the accelerated forward velocity. The wing inertia effect is also considered in the simulation model.

Optimization variables are defined, considering the increased computational burden by the higher fidelity model. The optimal control problem is formulated for an insect-like FWMV to conduct an efficient transition flight from hovering to forward flight, and the cost function is defined as energy consumption. The formulated optimal control problem can be solved with two categories of methods: indirect methods and direct methods. Indirect methods introduce the Lagrange multipliers and construct the necessary conditions for optimality. Then, the solution satisfying the conditions is searched. On the other hand, the direct methods represent the control variables and/or the state variables with discretized optimization variables. Then, the problem is converted to the parameter optimization problem and can be solved by any parameter optimization solver. The direct method is used in this study because it has a wider convergence region and does not require the initial guess for the costate variables.

2. Materials and Methods

2.1. FWMAV Model and Kinematics

In this paper, the natural flier, hawkmoth, is the reference insect and has been generally used in research on insect-like FWMAVs [22–24]. The wing is assumed to be rigid and connected with a three degree-of-freedom (3 DOF) revolute joint to the body. The hawkmoth parameters presented in Table 1 are based on the measurements by Ellington [25] and R. O’hara and A. Palazotto [26]. In Table 1, m_w , R , and $\bar{c}$ are the mass, radius, and mean chord length of the wing, respectively. m_{bd} and L_{bd} are the mass and length of the body. L_0 is the distance from the center of mass of the body to the tip of the head, and L_1 is the distance from the center of mass to the wing-base pivot. Figure 1 shows the FWMAV and geometry parameters of the body.

Table 1. Geometry and mass parameters of the reference insect.

Parameter of the Wing	Value	Parameter of the Body	Value
m_w (mg)	46.87	m_{bd} (mg)	1485
R (mm)	48.50	L_{bd} (mm)	44.80
$\bar{c}$ (mm)	16.81	L_0 / L_{bd}	0.45
		L_1 / L_{bd}	0.25

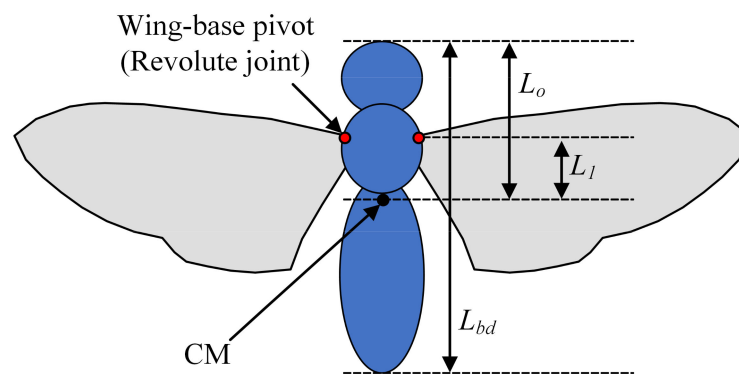


Figure 1. FWMAV and parameters of the body.

Figure 2 shows reference frames and angles to define the kinematics of the FWMAV. The body-fixed frame $[x_b \ y_b \ z_b]$ is attached to the center of the mass of the body, and the ground-fixed frame $[x_G \ y_G \ z_G]$ is the inertial frame. The wing-fixed frame $[x_w \ y_w \ z_w]$ is attached to the wing-base pivot where the y_w axis is parallel to the spanwise direction, and the x_w axis is parallel to the chord direction. Stroke plane frame $[x_{sp} \ y_{sp} \ z_{sp}]$ is used to define the wing kinematics. The y_{sp} axis is parallel to the y_b axis, and the x_{sp} axis makes an angle of $STRP_X$ and $strp_x$ with respect to the x_G – y_G and x_b – y_b planes, respectively.

The orientation of the wing is defined by three Euler angles on the stroke plane. The sequence of the rotation should be 3-1-2 corresponding to the sweep angle ϕ_w , the elevation angle θ_w , and the rotation angle α_w . The wing kinematics of the FWMAV is defined by the 3-order Fourier series as:

$$\phi_w(t) = \frac{\phi_{a0}}{2} + \sum_{k=1}^3 [\phi_{a_k} \cos(k2\pi ft) + \phi_{b_k} \sin(k2\pi ft)] \quad (1)$$

$$\theta_w(t) = \frac{\theta_{a0}}{2} + \sum_{k=1}^3 [\theta_{a_k} \cos(k2\pi ft) + \theta_{b_k} \sin(k2\pi ft)] \quad (2)$$

$$\alpha_w(t) = \frac{\alpha_{a0}}{2} + \sum_{k=1}^3 [\alpha_{a_k} \cos(k2\pi ft) + \alpha_{b_k} \sin(k2\pi ft)] \quad (3)$$

where f is the flapping frequency. a_k and b_k are the Fourier coefficients determined by fitting the measurement data [27], and prescripts ϕ , θ , and α denote that the Fourier coefficients are for the sweep, elevation, and rotation angle, respectively. Here, the wing kinematics is assumed to be symmetrical by the x_b – z_b plane.

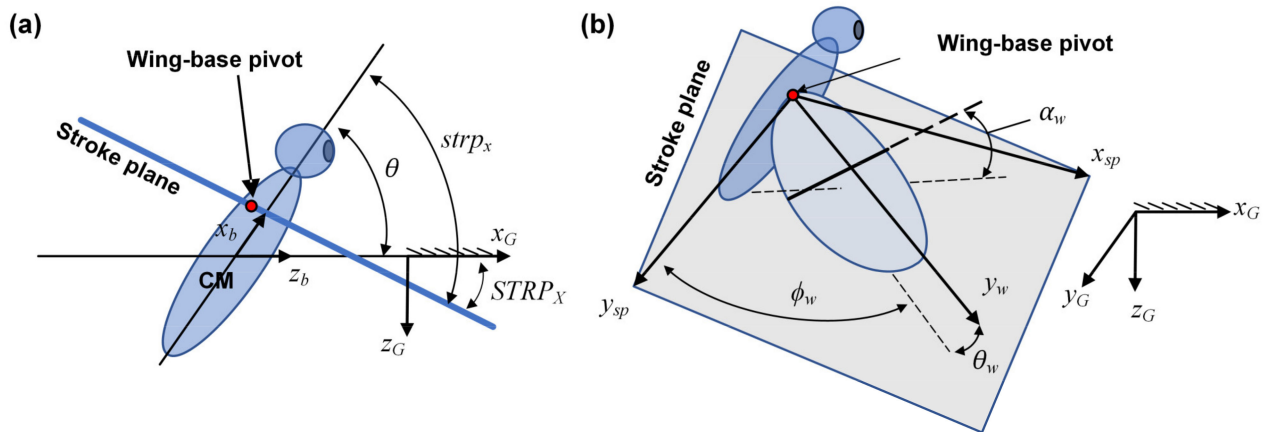


Figure 2. Reference frames and positional angle in (a) side view and (b) isometric view.

2.2. Simulation Environment

Figure 3 shows the developed simulation environment, which consists of the three parts: aerodynamic model, multibody dynamics solver, and controller. The equation of motion is time-integrated by the multibody dynamics solver. The sub-routine, which is programmed in the FORTRAN language, computes the aerodynamic load on the FWMAV. The wing kinematics of FWMAV depends on the control variables, and the control variables are computed by a controller, which will be designed in Section 2.4.

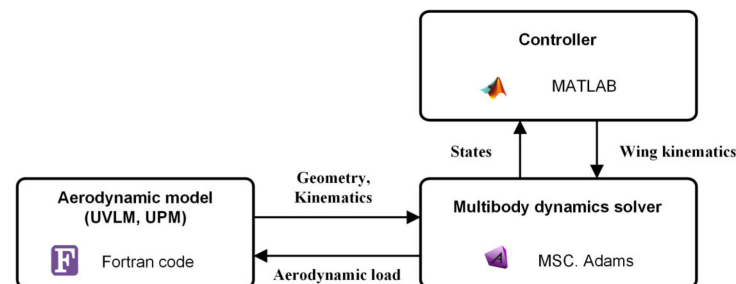


Figure 3. Simulation environment.

2.2.1. Aerodynamic Model

The unsteady panel method (UPM) and the extended unsteady vortex-lattice method (UVLM) are used to compute the aerodynamic forces. Both methods are based on the potential flow theory [28] with the assumption that the flow is assumed to be incompressible, inviscid, and irrotational.

For the potential flow, the velocity potential Φ needs to satisfy a Laplace equation:

$$\Delta\Phi = 0 \quad (4)$$

The velocity potential Φ also needs to satisfy three boundary conditions, including the far field, the Neumann, and the Dirichlet boundary conditions, as in Equations (5)–(7), respectively.

$$\lim_{|\mathbf{r}| \rightarrow \infty} \nabla\Phi = 0 \quad (5)$$

$$(\nabla\Phi - \mathbf{V}_b) \cdot \mathbf{n} = 0 \quad (6)$$

$$\Phi_i = \text{const} \quad (7)$$

Here, r , $\mathbf{V}_b$, $\mathbf{n}$, and Φ_i are the position vector, the velocity of the body, the normal vector of the body surface, and the velocity potential of the inside of the body, respectively. The solution of Φ can be expressed by the sum of source and doublet distribution on the body and wing surface [28].

The UPM is applied to the bluff object like body. The body surface is discretized into the triangular and quadrilateral panels, and each panel has a constant-strength source and doublet. The UVLM is applied to the thin object like a wing. The UVLM can be considered as a reduced version of the UPM. The source strengths on upper and lower surfaces cancel each other. The doublet elements on upper and lower surfaces are equivalent to the vortex ring [28]. The UVLM is extended by applying the leading-edge suction analogy model to consider the effect of spiral leading-edge vortices. The vortex-core growth model is used to incorporate the effect of eddy viscosity in the vortex wake and tackle the singularity problems. The other viscosity effects cannot be considered in the used aerodynamic model because the flow is assumed to be inviscid. However, viscosity effects do not have a significant contribution to the aerodynamics of insects [29]. Therefore, ignoring other viscosity effects is reasonable considering the reduced computational time compared to higher fidelity models, such as CFD. More details about the aerodynamic model are found in the study by Nguyen et al. [30,31].

2.2.2. Multibody Dynamics Solver

The nonlinear equations of motion are established and solved based on the multibody dynamics solver, MSC. Adams. Such multibody dynamics solvers have been used to study dynamics and control of FWMVs [32–34]. The aerodynamic force and moment required in the solver are computed with the UPM and the extended UVLM.

In the MSC. Adams, the equations of the motion are formulated as a set of nonlinear differential-algebraic equations (DAE) presented as:

$$\mathbf{M}\ddot{\mathbf{q}} + \varphi_{\mathbf{q}}^T \lambda - \mathbf{A}^T \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}) = 0 \quad (8)$$

$$\varphi(\mathbf{q}, t) = 0 \quad (9)$$

where $\mathbf{q}$ is the vector containing the coordinates that represent displacement, $\mathbf{M}$ represents the mass matrix of the system, φ represents the kinematic constraints, λ contains the Lagrange multipliers for the constraints, $\mathbf{F}$ is the vector containing applied forces and gyroscopic terms of the inertia forces, $\mathbf{A}^T$ represents the matrix that projects $\mathbf{F}$ in $\mathbf{q}$ direction, and $\varphi_{\mathbf{q}}$ represents the gradient of the constraints.

2.3. Trim Search

A trim condition is needed to formulate an optimal control problem and design a pitch controller. Finding the trim condition is complicated because of the oscillating aerodynamic load within a wingbeat stroke. In this study, the trim search algorithm proposed by Kim et al. [35] is used. During the trim search process, the flapping frequency, the mean sweep angle, the mean rotation angle, and the initial value of states are searched to satisfy the trim criteria as presented in Equations (10)–(12).

$$\bar{\mathbf{F}} = [\bar{F}_x \quad \bar{F}_y \quad \bar{F}_z]_G = [0 \quad 0 \quad 0] \quad (10)$$

$$\bar{\mathbf{M}} = [\bar{M}_x \quad \bar{M}_y \quad \bar{M}_z]_G = [0 \quad 0 \quad 0] \quad (11)$$

$$\bar{\mathbf{x}}_{6DOF} = [\bar{u} \quad \bar{v} \quad \bar{w} \quad \bar{p} \quad \bar{q} \quad \bar{r}]_G = [u_e \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \quad (12)$$

Here, the subscript G means that the value is represented in the ground-fixed frame. The upper bar denotes the wingbeat-cycle-averaged value. The trim of the FWMAV is obtained about the cycle-averaged states due to the oscillating characteristic of the system. $\mathbf{F}$ is applied force, $\mathbf{M}$ is applied moment, and $\mathbf{x}_{6DOF}$ is 6-DOF states. u_G , v_G and w_G are the velocity components along the x_G , y_G and z_G axes, respectively. p_G , q_G and r_G are the angular velocities about the x_G , y_G and z_G axes, respectively. u_e means given forward flight velocity. Figure 4 shows the flowchart of the trim search algorithm.

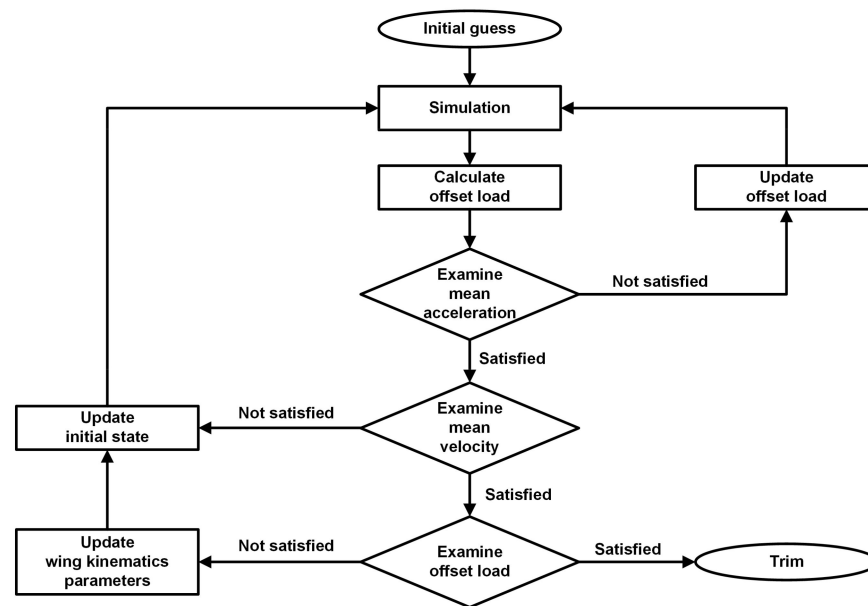


Figure 4. Flowchart of a trim search algorithm.

After searching the trim, the equations of motion can be linearized at each trim condition. The eigenvalues of the system matrix are compared with those of Nguyen et al. [30], who used the same aerodynamic model. Figure 5 shows that the eigenvalues of longitudinal dynamics are similar to each other. The states of the longitudinal dynamics are the velocity components along the x_b and z_b axes, the angular velocity about the y_b axis, and the pitch angle of the body. The power consumption for the trim flights is also calculated (Figure 6), where the calculation for the power consumption will be presented in Section 2.4.2.

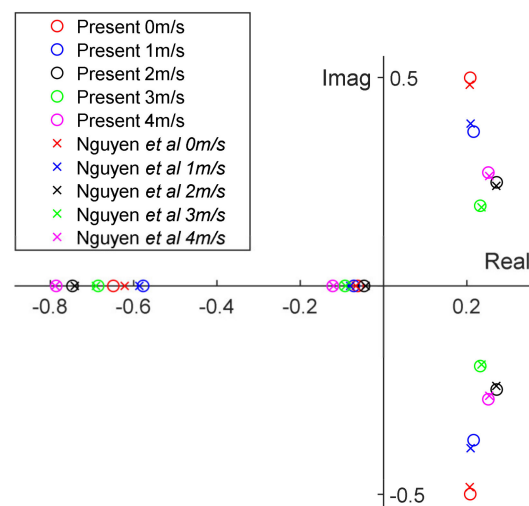


Figure 5. Eigenvalues comparison of the longitudinal dynamics from the present model and Nguyen et al. [30].

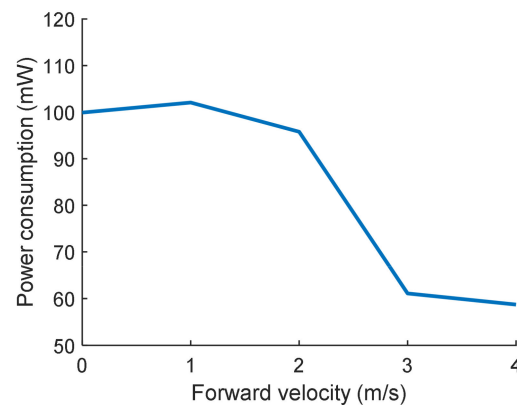


Figure 6. Cycle-average power consumption of the trim flight.

2.4. Optimal Control Problem Formulation

In this section, an optimal control problem is formulated for the FWMAV to conduct transition flight from hovering to the forward flight with a target velocity. Since the reference insect, hawkmoth, can fly up to 5 m/s [36], the target velocities u_{target} of the transition flight are set as 2, 3, and 4 m/s in the range. In addition to accelerating to the target velocities, there should be no altitude loss because the FWMAV consumed additional energy to recover the altitude loss after the transition. In this section, the control variables, cost function, and constraints of the optimal control problem are defined.

2.4.1. Control Variables

More control variables lead to a more complex optimization problem. Therefore, some appropriate wing kinematics parameters should be defined as the control variables, and the other parameters need to be fixed or defined as a function of state and control variables. In this paper, two control variables are used: flapping frequency f and sweeping bias ϕ_0 . The flapping frequency mainly affects the magnitude of the aerodynamic force [37] while the sweeping bias is used to control the pitch axis. The two control variables have been commonly used in the control of FWMAVs [38–40]. The two control variables determine the wing kinematics as:

$$\phi_w(t) = \frac{\phi a_0}{2} + \sum_{k=1}^3 \{ \phi a_k \cos[k2\pi\Omega(t)] + \phi b_k \sin[k2\pi\Omega(t)] \} + \phi_{trim} + \phi_0(t) \quad (13)$$

$$\theta_w(t) = \frac{\theta a_0}{2} + \sum_{k=1}^3 \{ \theta a_k \cos[k2\pi\Omega(t)] + \theta b_k \sin[k2\pi\Omega(t)] \} \quad (14)$$

$$\alpha_w(t) = \frac{\alpha a_0}{2} + \sum_{k=1}^3 \{ \alpha a_k \cos[k2\pi\Omega(t)] + \alpha b_k \sin[k2\pi\Omega(t)] \} + \alpha_{trim} \quad (15)$$

$$\Omega(t) = \int_0^t f(t) dt \quad (16)$$

where α_{trim} and ϕ_{trim} are functions of the forward velocity determined by interpolation between the values obtained by the trim search.

ϕ_0 needs to be frequently changed in order to stabilize the FWMAV, which requires many nodes for the transcription method. Even though the used aerodynamic model is more computationally efficient than CFD, it is not as fast as the QS model. Therefore, too many nodes need to be avoided because more nodes require more time for optimization.

The pitch controller is designed to reduce the number of nodes. ϕ_0 is determined by the pitch controller as follows:

$$\phi_0 = k_1(\dot{\theta}_{cmd} - \dot{\theta}) + k_2(\theta_{cmd} - \theta) + k_3 \int (\theta_{cmd} - \theta) dt \quad (17)$$

Here, the gains k_1 , k_2 and k_3 were respectively determined as -0.08 , -0.74 , and -5.14 by trial and error to maximize the pitch command tracking performance of the FWMAV. Eventually, the pitch command of the controller θ_{cmd} and flapping frequency f become inputs of the total system, which includes the system dynamics and the pitch controller. The inputs (θ_{cmd} and f) are the optimization variables, which are obtained from the optimization.

2.4.2. Cost Function

The cost function J is defined as the mechanical energy consumption for the transition flight as follows:

$$J = \int_0^{t_f} P_{\text{mechanical}} dt \quad (18)$$

where

$$P_{\text{mechanical}} = P_{\phi_w} + P_{\theta_w} + P_{\alpha_w} \quad (19)$$

$$\begin{aligned} P_j &= \tau_j \frac{dj}{dt} \quad (\tau_j \frac{dj}{dt} > 0) \\ P_j &= 0 \quad (\tau_j \frac{dj}{dt} \leq 0) \\ (j &= \phi_w, \theta_w, \alpha_w) \end{aligned} \quad (20)$$

In this paper, it is assumed that the FWMAV does not store the energy in the elastic elements during a period of negative work like the reference insect [41]. The final transition time t_f is given as 2 s.

2.4.3. Constraints

For the complete transition flight, the final forward velocity should be higher than the target velocity. The direction of the vertical velocity should not be downward, and the altitude should not decrease. In addition, the initial states are set to be the values of hovering obtained by the trim search. Equations (21)–(24) show the constraints on states. The initial and final inputs are set to be the inputs of hovering and forward flight with the target velocity, respectively. Equations (25)–(28) show the constraints on inputs.

$$\bar{w}_G(t_f) \leq \varepsilon_{tol} \quad (21)$$

$$\bar{u}_G(t_f) \geq u_{\text{target}} - \varepsilon_{tol} \quad (22)$$

$$\bar{z}_G(t_f) \leq 0 \quad (23)$$

$$\mathbf{x}_0 = \mathbf{x}_{\text{trim}} \text{ (0 m/s)} \quad (24)$$

$$f(0) = f_{\text{trim}} \text{ (0 m/s)} \quad (25)$$

$$\theta_{cmd}(0) = \theta_{\text{trim}} \text{ (0 m/s)} \quad (26)$$

$$f(t_f) = f_{\text{trim}}(u_{\text{target}}) \quad (27)$$

$$\theta_{cmd}(t_f) = \theta_{\text{trim}}(u_{\text{target}}) \quad (28)$$

Here, $\mathbf{x}$ is the longitudinal state vector $[u \ w \ q \ \theta]_G$. As mentioned earlier, u_{target} is set as 2, 3, and 4 m/s. ε_{tol} is the tolerance for the velocity constraints, which is set to be 0.3 m/s.

Equations (29)–(32) show the other constraints on inputs where T is the flapping period during hovering flight. These constraints on the inputs and the rates of the input are for the feasible flapping kinematics. It can also prevent the numerical simulation from diverging. The input ranges are set not to be more than twice the range of the measurement

values of the real hawkmoth [27]. The bound on the inputs rates are set so that more than 10 cycles of flapping are required to change the inputs from the upper and the lower bound or vice versa.

$$-2 \leq \Delta f (= f - f_{trim} \text{ (0 m/s)}) \leq 2 \quad (29)$$

$$\frac{3}{180}\pi \leq \theta_{cmd} \leq \frac{52}{180}\pi \quad (30)$$

$$|\dot{f}| \leq \frac{4}{10T} \quad (31)$$

$$|\dot{\theta}_{cmd}| \leq \frac{49\pi/180}{10T} \quad (32)$$

2.5. Transcription and Parameter Optimization Problem

The formulated optimal control problem is transcribed to the parameter optimization problem. Here, the used transcription method is the direct shooting method. The advantage of this technique is that there are fewer optimization variables because only the inputs, not the states, become the optimization variables to be searched.

With the direct shooting method, each input is discretized into a set of nodes. Continuous inputs are obtained by the linear interpolation of the discretized input and then used in the simulation to calculate the cost and constraints. Now, the optimal control problem is transcribed to the parameter optimization problem by searching a limited number of variables. The parameter optimization problem is solved using FMINCON, a MATLAB solver, and the sequential quadratic programming (SQP) is used as an algorithm of the solver.

3. Results and Discussions

The SQP is the gradient-based method requiring an initial guess, and the main concern of the method is that it can be sensitive to the initial guess. Therefore, the effect of the initial guess is investigated. After that, the optimization results are introduced then followed by the optimization results in the presence of the wind.

3.1. Initial Guess for SQP

If the optimized results are changed with the initial guess, the results are likely to be a local minimum. On the other hand, if the optimized results are not changed with the initial guess, the results have a high probability of becoming a global minimum, and the problem is not sensitive to the initial guess. In order to check how sensitive the formulated problem is to the initial values, optimization is performed with three different initial guesses in the case of $u_{target} = 3 \text{ m/s}$.

Figure 7 shows the used initial guesses. The guess #1 is shaped like a ramp function, and the guess #2 is given by linear interpolation between the initial and final input constraints as presented in Equations (25)–(28). The guess #3 has an arbitrary shape. Figure 8 shows that the results with the three initial guesses are the same as each other. This shows that the formulated problem can be considered not to be sensitive to the initial guess. The linear interpolated guess will be used in this study.

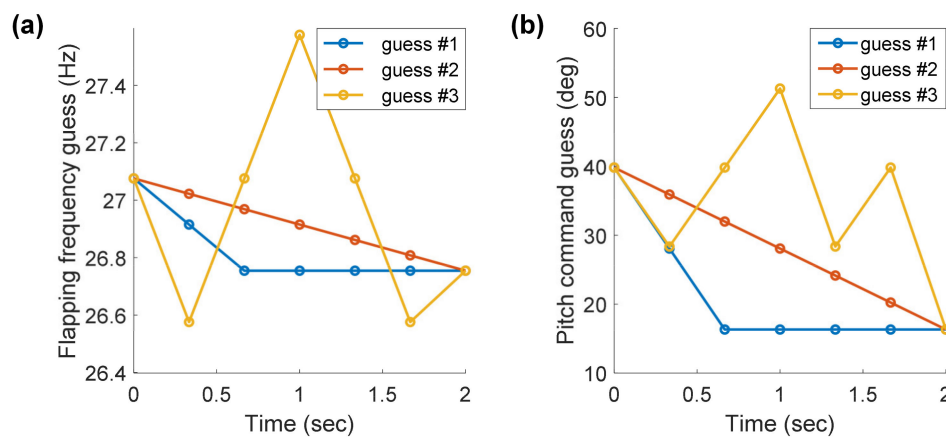


Figure 7. Three different initial guesses for SQP: (a) flapping frequency; (b) pitch command.

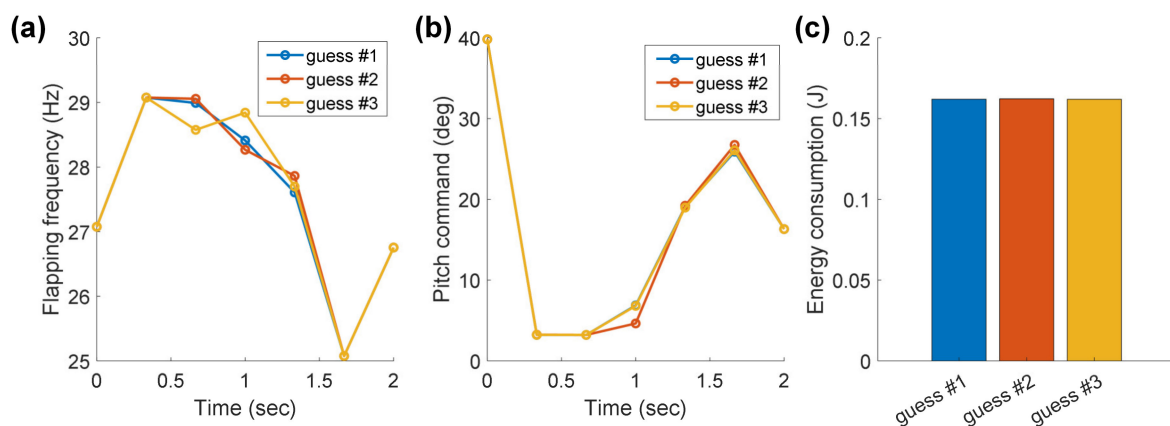


Figure 8. Optimized (a) flapping frequency, (b) pitch command, and (c) energy consumption with the three different initial guesses.

3.2. Optimization Results

With 7 nodes (See Appendix A for a discussion of the node number) and a linear interpolated initial guess, the optimization is conducted for the transition flight with the three target velocities. The optimization converges to the feasible solution except in the case of $u_{target} = 4$ m/s. To obtain a feasible solution for this case, the frequency range or the final time can be increased. The result with twice the frequency range (± 4 Hz) is used in the comparison analysis with the same time duration.

Figure 9 shows the optimization results of the flapping frequency and pitch command. A higher target velocity leads to an overall higher flapping frequency and energy consumption. For all cases, the pitch command decreases to the lower bound in the initial stage, and then a pitch-up maneuver is conducted. In Figure 10c, all of the transition flights are completed without the additional increase in altitude. The cases of $u_{target} = 2$ and 3 m/s have similar trajectories, which are characterized by a down-up shape, while the trajectory of $u_{target} = 4$ m/s is slightly different. Because the given time is not enough to accelerate to 4 m/s, an overall high flapping frequency is maintained for acceleration and leads to more lift, which eliminates the altitude loss. In order to observe how the trajectory changes for a sufficient time like 4 s, an additional optimization is conducted. As shown in Figure 11c, the down-up trajectory can be obtained like in the cases of $u_{target} = 2$ and 3 m/s.

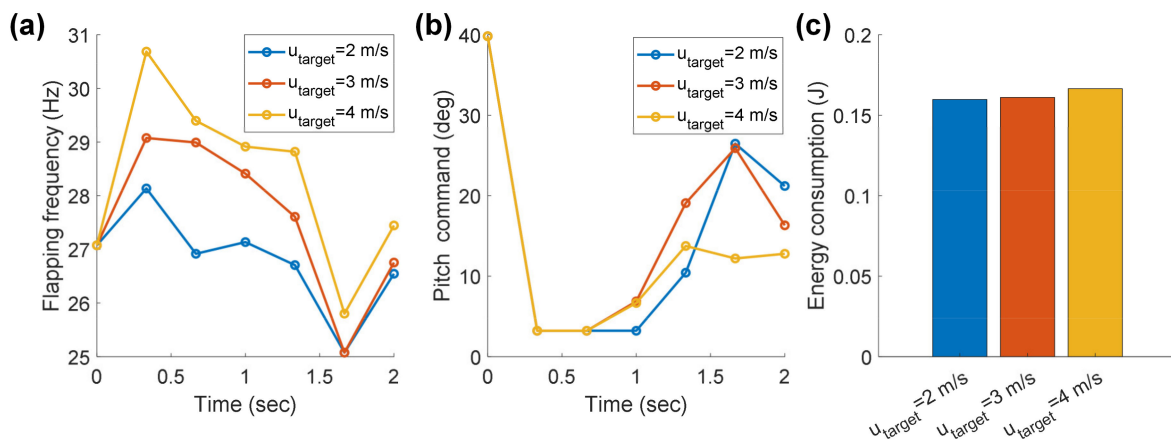


Figure 9. Optimized (a) flapping frequency, (b) pitch command, and (c) energy consumption.

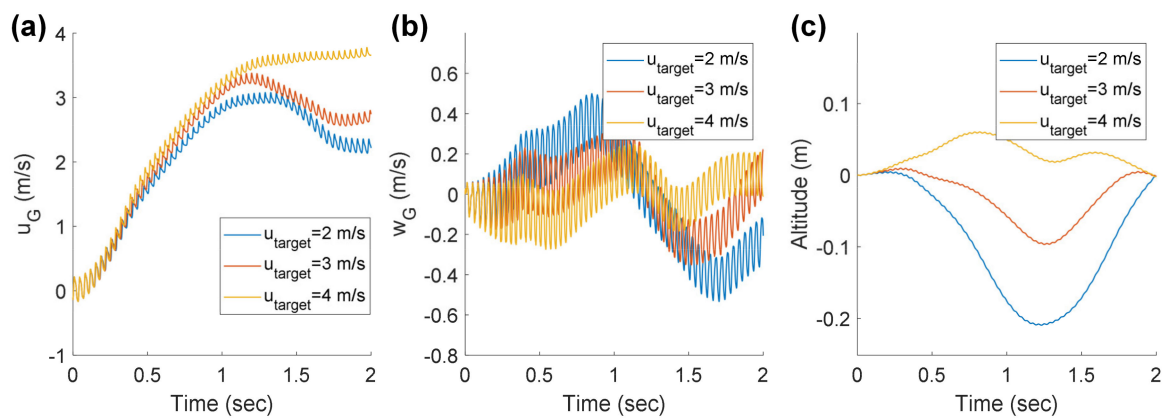


Figure 10. (a) Forward velocity, (b) vertical velocity, and (c) altitude of optimal trajectory.

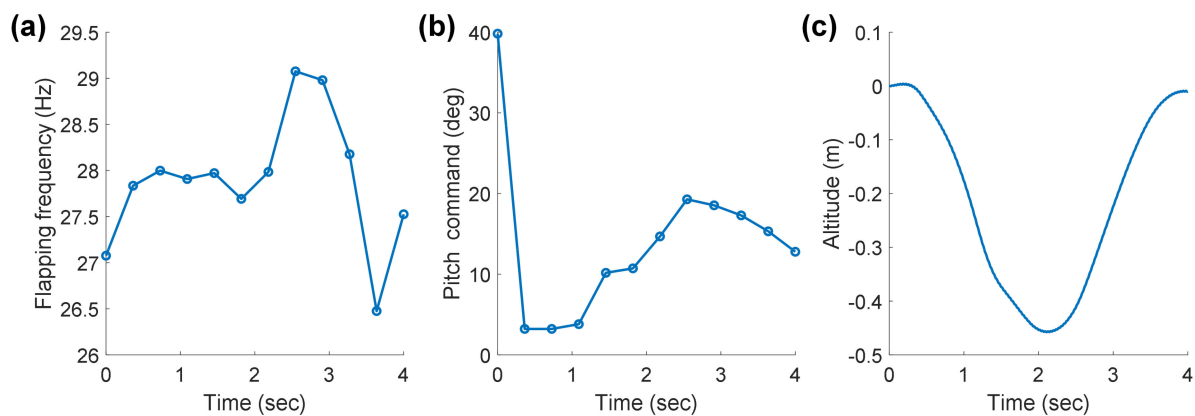


Figure 11. (a) Forward velocity, (b) vertical velocity, and (c) altitude of the optimal trajectory ($u_{target} = 4$ m/s and $t_f = 4$ s).

In Figure 9b, the low pitch command is necessary in the initial stage for the rapid acceleration even though this maneuver causes altitude loss, as seen in Figure 10c. After low pitch command, for $u_{target} = 2$ and 3 m/s, the pitch command goes beyond the trim pitch angle of the target velocity, which is set by Equation (28). Although the pitch-up maneuver decreases the forward velocity (Figure 10a), the maneuver helps FWMVs to gain much altitude (Figure 10c). Considering the low flapping frequency during the ascending, the pitch maneuver has a greater effect on the altitude gain than the frequency. Therefore, these

optimal trajectories can be understood as conducting additional acceleration over the target velocity and utilizing the extra forward velocity to gain potential energy through a pitch-up maneuver. This pitch-up maneuver is actively performed in the low-velocity transition where enough forward velocity can be sacrificed to increase the altitude.

3.3. Wind Effect

The wind can affect the trajectories of natural fliers. Birds are known to perform the dynamic soaring to utilize the wind. Therefore, it is necessary to consider the wind effect in the trajectory optimization framework. In this section, additional optimization in the presence of the wind is conducted for the 3 m/s transition. Four wind direction namely headwind, tailwind, updraft, and downdraft, are considered, and their wind speed is given as 0.3 m/s.

The optimization converges to the feasible solution except for the downdraft. For the downdraft, a feasible solution can be obtained after the frequency range increased to ± 4 Hz. Figures 12 and 13 show the optimized result for the different wind directions. In Figure 12b, the highest pitch-up maneuver is observed for the tailwind. The reason is that the tailwind is favorable to the acceleration, and this can be used to gain altitude via pitch-up maneuver. This maneuver results in the largest altitude change, as shown in Figure 13. On the other hand, the pitch-up maneuver is the lowest in the presence of the headwind, which is not favorable to acceleration. In Figure 12a, the flapping frequency for the downdraft and updraft is distinctively different from other wind conditions. A lower flapping frequency is maintained to save energy in the presence of the updraft, while a higher flapping frequency is needed to satisfy the altitude constraint for the downdraft. Figure 12c shows that the vertical direction of the wind mainly affects the energy consumption of the transition flight. In addition, headwind and updraft decrease the energy consumption compared to the no wind condition.

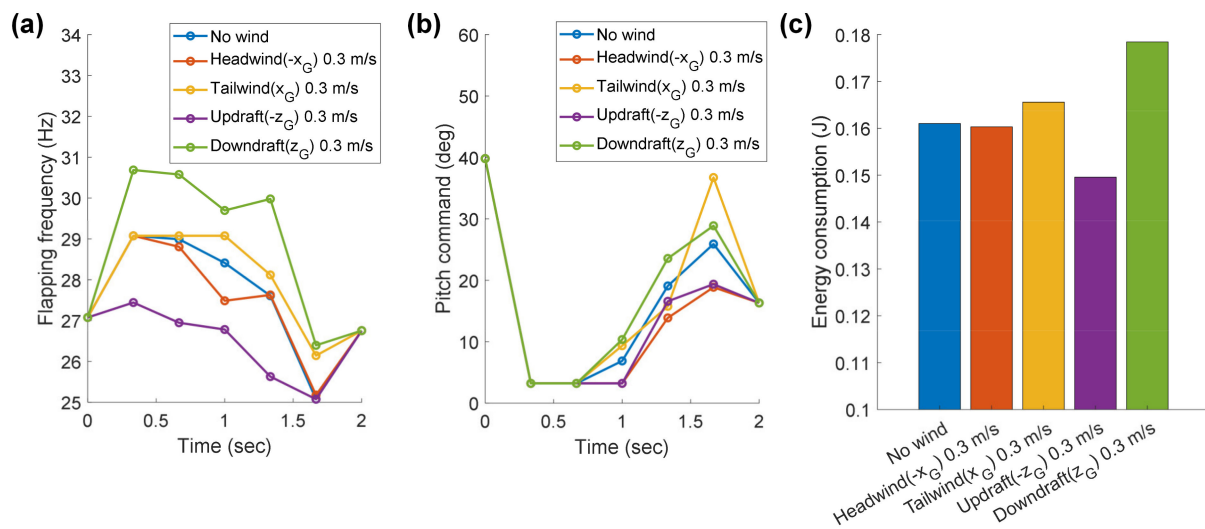


Figure 12. Optimized (a) flapping frequency, (b) pitch command, and (c) energy consumption in the presence of different wind directions.

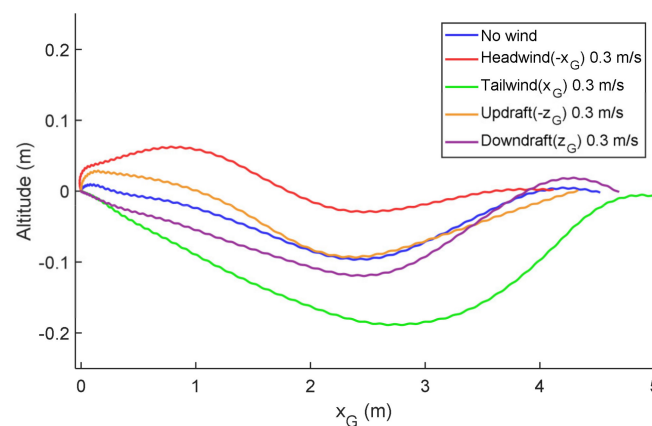


Figure 13. Flight trajectories for different wind directions.

4. Conclusions

In this paper, the trajectory optimization framework was proposed, and an efficient transition flight trajectory of the FWMAV was obtained. The multibody dynamics solver, extended UVLM, and UPM were used to ensure the feasibility of the open-loop solution, which is dependent on the fidelity of the simulation model. The optimal control problem was formulated as the minimum energy transition flight without altitude loss. To reduce the optimization time of the computationally expensive simulation model, the number of nodes was reduced using the pitch controller. As a result, the pitch command and flapping frequency were defined as the optimization variables. The formulated optimal control problem was transcribed to the parameter optimization problem with the direct shooting method, and the parameter optimization problem was solved with SQP.

The optimization results with three different initial guesses were similar, showing the reliability of the optimized solution and that the formulated problem is not sensitive to the initial guess.

The energy-optimal trajectory for the transition flight was investigated. The results can be summarized as follows:

- (1) When the target velocity is low (2 and 3 m/s), the FWMAV performs additional acceleration over the target velocity. After that, the FWMAV sacrifice the forward energy to gain the potential energy via pitch-up maneuver.
- (2) The pitch-up maneuver is performed more actively when the condition is favorable to acceleration: the target velocity is low, or the wind direction is tailwind.
- (3) When the wind direction is up and downward, the flapping frequency and energy consumption are distinctively different from the other wind conditions.

The endurance of a FWMAV is expected to be increased by the proposed framework and the trajectory which can be obtained using the framework. Moreover, the proposed trajectory optimization framework, and the developed simulation environment can be used to design the closed-loop guidance and control law, which can be more generally applied.

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Nomenclature

a_k, b_k	Fourier coefficients for the wing kinematics,
$\bar{c}$	mean chord length of the wing,
$\mathbf{F}$	applied force vector,
f	flapping frequency,
L_0	distance from the center of mass of the body to the tip of the head,
L_1	distance from the center of mass to the wing-base pivot,
L_{bd}	length of the body,
$\mathbf{M}$	applied moment vector,
m_{bd}	mass of the body,
m_w	mass of the wing,
$\mathbf{n}$	normal vector of the body surface,
$P_{\text{mechanical}}$	mechanical power consumption,
p_G, q_G, r_G	angular velocity components in ground-fixed frame,
R	radius of the wing,
$STRP_X$	stroke plane angle with respect to the ground-fixed frame,
$strp_x$	stroke plane angle with respect to the body-fixed frame,
T	flapping period during hovering flight,
t_f	final transition time,
u_e	given forward flight velocity for the trim search,
u_G, v_G, w_G	velocity components in ground-fixed frame,
$\mathbf{V}_b$	velocity vector of the body,
$\mathbf{x}$	longitudinal state vector,
$\mathbf{x}_{6DOF}$	6-degree of freedom state vector,
x_b, y_b, z_b	axes for the body-fixed frame,
x_G, y_G, z_G	axes for the ground-fixed frame,
x_w, y_w, z_w	axes for the wing-fixed frame,
α_w	rotation angle,
ε_{tol}	tolerance for the velocity constraints,
θ_{cmd}	pitch command,
θ_w	elevation angle,
τ	required torque for the given wing kinematics,
Φ	velocity potential,
Φ_i	velocity potential of the inside of body,
ϕ_0	sweeping bias,
ϕ_w	sweep angle.

Appendix A

The number of nodes can affect the optimization results. Therefore, the effect of the number of nodes is studied by additional optimization in the case of $u_{target} = 3$ m/s with three different numbers of nodes. Figure A1 shows that the selected nodes give similar results. Therefore, the selected node number has sufficient design space.

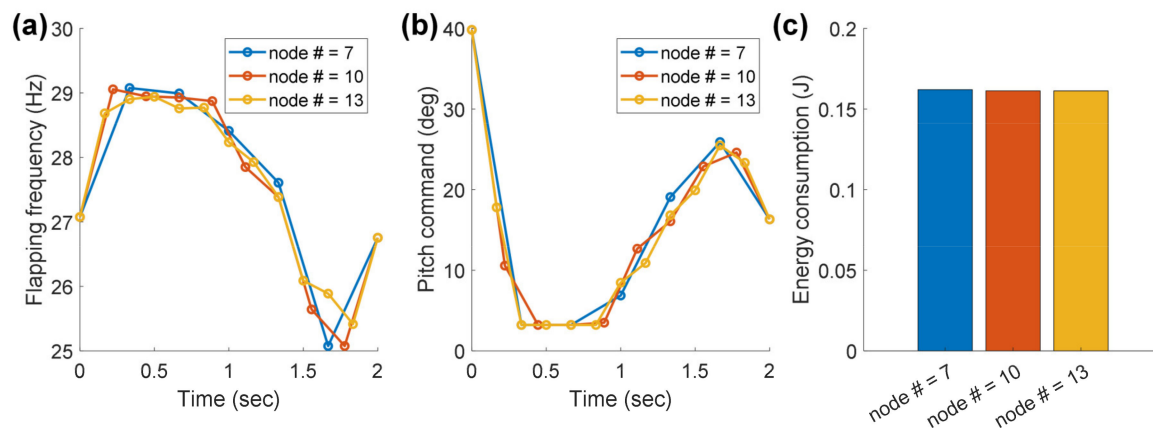


Figure A1. Optimized (a) flapping frequency, (b) pitch command, and (c) energy consumption with different numbers of nodes.

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