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Robust Full Tracking Control Design of Disturbed Quadrotor UAVs with Unknown Dynamics

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Abstract: In this study, we develop a rigorous tracking control approach for quadrotor unmanned aerial vehicles (UAVs) with unknown dynamics, unknown physical parameters, and subject to unknown and unpredictable disturbances. In order to better estimate the unknown functions, seven interval type-2-adaptive fuzzy systems (IT2-AFSs) and five adaptive systems are designed. Then, a new IT2 adaptive fuzzy reaching sliding mode system (IT2-AFRSMS) which generates an optimal smooth adaptive fuzzy reaching sliding mode control law (AFRSMCL) using IT2-AFSs is introduced. The AFRSMCL is designed a way that ensures that its gains are efficiently estimated. Thus, the global proposed control law can effectively achieve the predetermined performances of the tracking control while simultaneously avoiding the chattering phenomenon, despite the approximation errors and all disturbances acting on the quadrotor dynamics. The adaptation laws are designed by utilizing the stability analysis of Lyapunov. A simulation example is used to validate the robustness and effectiveness of the proposed method of control. The obtained results confirm the results of the mathematical analysis in guaranteeing the tracking convergence and stability of the closed loop dynamics despite the unknown dynamics, unknown disturbances, and unknown physical parameters of the controlled system.

Keywords: quadrotor UAV; type-2 fuzzy systems; tracking control; adaptive control; sliding mode control

1. Introduction

Over the last decade, several robust approaches have been proposed for the control of unmanned aerial vehicles (UAVs), most of which use intelligent and robust control approaches such as fuzzy logic control (FLC), the H_∞ technique, sliding mode control (SMC), the backstepping technique, and adaptive control [1–8]. The quadrotor is among the most popular UAVs and is widely used in many applications such as surveillance, air mapping, inspection, aerial cinematography, rescue missions, search missions in hostile environments, etc. It presents many desirable features in comparison with other UAVs such as high maneuverability, landing ability, vertical takeoff, and low cost.

The tracking control of quadrotors plays an important role in achieving accurate operations and stable missions. However, realizing the desired objective of control is a very complicated task due to the fact that the quadrotors are underactuated systems that are subject, in general, to undesirable features such as aerodynamic friction force, high nonlinear dynamics, parameter variations, dynamics with strong coupling, gyroscopic uncertain effect, unmodeled dynamics, wind gusts, and other unpredictable and unknown disturbances which affect, in particular, the tracking response and the

stability of the control system. To deal with some of these constraints of control, several robust approaches have been developed. In [3], an adaptive tracking algorithm was developed to force a quadrotor aerial vehicle to achieve a desired task despite modeling errors and disturbance uncertainties. Additionally, in [9], the authors used both the SMC and terminal SMC to accomplish good tracking control of a small quadrotor UAV despite external disturbances.

SMC is a robust method of control that is well appreciated by many researchers due to its capability to reject the undesirable effects acting on the system dynamics such as uncertainties, unknown disturbances, etc. However, the chattering phenomenon resulting from the fact that the SMC law is discontinuous can be harmful to the actuators [10]. To eliminate or at least decrease the chattering, higher order SMC (HO-SMC) and boundary layer techniques are usually adopted in the literature to guarantee the desired objective of control [11–15]. However, these methods are only employed when the upper bounds of the different kinds of disturbances that influence the control system are known, which constrains their implementation in control systems. In [16], in order to ensure good tracking control for a small quadrotor UAV, a second-order SMC was developed. Additionally, in [17], the authors employed a second-order super twisting (SOST) algorithm to ensure the attitude tracking and robustness of a quadrotor against bounded external disturbances. The second-order super-twisting SMC (SOST-SMC) is a particular kind of HO-SMC that has been successfully implemented by several researchers in various fields of systems control [18–20]. SOST-SMC was introduced by Levant [21] in order to handle the chattering problem while simultaneously ensuring the convergence and stability of the control system. However, the SOST-SMC algorithm requires a good estimation of the gains of its control law which is one of its major problems in controlling systems.

On the other hand, due to the fact that fuzzy systems (FSs) are universal approximators [22], intelligent algorithms using FSs have been extensively employed and successfully applied for the control of uncertain nonlinear systems. In [23], a FS was used for an autonomous underwater vehicle manipulator to track a desired trajectory in the presence of uncertainties and disturbances. In [24], the authors used a fuzzy model to describe a wind turbine plant, and as a consequence, a fuzzy controller was designed to achieve the desired task of control. Additionally, in [25], a proportional integral derivative (PID) controller with a fuzzy logic mechanism was designed for a quadrotor with high nonlinear dynamics and parameter uncertainty. However, type-1 (T1)-FSs (T1-FSs) have a drawback when the linguistic information used to describe the system dynamics contains uncertainties. To cope with this constraint, another kind of FS called type-2 (T2)-FS (T2-FS) was introduced into the modeling and design of robust controllers for complex systems with uncertainties [26–30]. The capability of T2-FSs to handle the problem of inaccurate linguistic information much better than T1-FSs is due to the uncertainty property footprint incorporated into the membership functions (MFs) of T2 fuzzy sets [31–33].

The main contributions of this paper compared to the previous studies are

- (1) An intelligent and sophisticated approach to full tracking control is developed for the quadrotor UAVs, subject to the following constraints:
 - All dynamics of the quadrotor are considered entirely unknown.
 - No prior knowledge is required for the upper bounds of unknown and unpredictable disturbances acting on the quadrotor dynamics, including aerodynamic perturbations such as unpredictable wind gusts, time varying disturbances, gyroscopic effects, and other unknown disturbances.
 - The physical parameters of the quadrotor including the mass and the inertia moment are considered entirely unknown and they suffer from time varying disturbances.
- (2) By taking advantage of the properties of T2-FSs and adaptive control techniques in the design of robust controllers, seven interval T2 adaptive FSs (IT2-AFSs) and five adaptive systems are synthesized to better estimate the unknown dynamics and unknown parameters of the studied quadrotor.

- (3) A new IT2-adaptive fuzzy reaching sliding mode system (IT2-AFRSMS) is introduced in order to efficiently estimate the optimal values of the gains of a designed reaching sliding mode control law (RSMCL) online. The output of this IT2-AFRSMS is an IT2-adaptive fuzzy RSMCL (IT2-AFRSMCL), designed in such a way as to yield an optimal global control law that is capable of dealing with approximation errors and all unknown and unpredictable disturbances that perturb the quadrotor dynamics, and simultaneously coping with the chattering phenomenon. Then, to tackle the underactuated constraint of the quadrotor control system, two virtual control inputs terms are added to the control system.
- (4) The parameters of the global developed control law are adjusted online by utilizing the stability analysis theorem of Lyapunov. The proposed algorithm of control is stable in the sense of Lyapunov, and the asymptotic convergence of the system state trajectories is established.

This paper is organized as follows. In Section 2, the IT2-FSs are described, and then, the problem formulation and the proposed control design for the quadrotor UAVs are presented in Sections 3 and 4, respectively. Finally, in Section 5, the simulation results for a quadrotor system are presented to show the effectiveness of the designed control algorithm in accomplishing the desired objectives.

2. Interval Type-2 Fuzzy Systems

T2-FSs are used in control systems due to their excellent efficiency when directly handling the measurement uncertainties and inaccurate linguistic information used to synthesize T2 fuzzy rules. Thus, the fuzzy sets for a T2-FS are implemented in such a way that their associated MFs can easily incorporate the above discussed uncertainties through their footprint of uncertainty property. In this study, only IT2-FSs are adopted as approximator systems, on the one hand, because they do not require a lot of computation which makes them more convenient to use in real applications in comparison with other classes of T2-FSs, and on the other hand, due to their efficiency in capturing uncertainties.

The j th IT2 fuzzy rule of an IT2-FS which has n inputs and one output can be formulated as follows [34]:

$$\text{Rule}(j): \text{ If } x_1 \text{ is } \tilde{E}_1^j \text{ and } x_2 \text{ is } \tilde{E}_2^j \dots \text{ and } x_n \text{ is } \tilde{E}_n^j, \text{ then, } y \text{ is } \tilde{\theta}^j, j = 1, \dots, M \quad (1)$$

where $\tilde{E}_i^j$ represents antecedent IT2 fuzzy sets, and $\tilde{\theta}^j$ represents consequent IT2 fuzzy sets; $\underline{x} = [x_1 \ x_2 \ \dots \ x_n]^T \in \mathbb{R}^n$ denotes the state vector; $y \in \mathbb{R}$ is the output of the system (1); and M denotes the rule number.

In this study, the firing set can be given for the system (1), with the meet operation implemented by the t-norm product, as follows:

$$W^j(\underline{x}) = [w_l^j(\underline{x}), w_r^j(\underline{x})] \quad (2)$$

where $w_l^j(\underline{x}) = \mu_{\tilde{E}_1^j}^L(x_1) \times \mu_{\tilde{E}_2^j}^L(x_2) \times \dots \times \mu_{\tilde{E}_n^j}^L(x_n)$ and $w_r^j(\underline{x}) = \mu_{\tilde{E}_1^j}^R(x_1) \times \mu_{\tilde{E}_2^j}^R(x_2) \times \dots \times \mu_{\tilde{E}_n^j}^R(x_n)$, such that $\mu_{\tilde{E}_i^j}^L(x_i)$ and $\mu_{\tilde{E}_i^j}^R(x_i)$ denote, respectively, the left- and right-most values of the MFs associated with the IT2 fuzzy sets $\tilde{E}_i^j$.

Based on the center of sets technique, the outputs of IT2 fuzzy sets of the inference engine are reduced to an IT1 fuzzy set. Then, by adopting the center of gravity method, the crisp output of the system (1) can be given as [35]

$$y = \frac{y_l + y_r}{2} \quad (3)$$

where y_l and y_r can be expressed as

$$\begin{cases} y_l = \min_{w^j} \frac{\sum_{j=1}^M \theta_l^j w^j}{\sum_{j=1}^M w^j} = \theta_l^T \tilde{\zeta}_l \\ y_r = \max_{w^j} \frac{\sum_{j=1}^M \theta_r^j w^j}{\sum_{j=1}^M w^j} = \theta_r^T \tilde{\zeta}_r \end{cases} \quad (4)$$

where $\tilde{\zeta}_l = [\tilde{\zeta}_l^1 \ \tilde{\zeta}_l^2 \ \dots \ \tilde{\zeta}_l^M]^T$ and $\tilde{\zeta}_r = [\tilde{\zeta}_r^1 \ \tilde{\zeta}_r^2 \ \dots \ \tilde{\zeta}_r^M]^T$ are the lower and upper vectors of fuzzy basis functions, and they are obtained using Karnik–Mandel algorithm [36], with $w^j \in W^j$; $\theta_l = [\theta_l^1 \ \theta_l^2 \ \dots \ \theta_l^M]^T$ and $\theta_r = [\theta_r^1 \ \theta_r^2 \ \dots \ \theta_r^M]^T$ being the left- and right-most conclusion vectors of the system (1) (see [28] for more details).

3. Model Dynamics of the Quadrotor UAV and Problem Formulation

The quadrotor is a highly nonlinear underactuated system with multiple inputs and multiple outputs and strong dynamic coupling, which is subject to aerodynamic forces, gyroscopic effects, parameter variations, unmodelled dynamics, and unknown and unpredictable disturbances. In order to overcome these constraints, a sophisticated robust tracking control algorithm is developed in this paper for disturbed quadrotor UAVs with unknown dynamics and unknown physical parameters, including the mass and the inertia moment.

A schematic configuration of a quadrotor UAV system is depicted in Figure 1, where $E(o_e, x_e, y_e, z_e)$ denotes an inertial frame and $B(o_b, x_b, y_b, z_b)$ is a body frame fixed to the quadrotor; ϕ , θ and ψ are, respectively, the roll, the pitch, and the yaw angles, such that $-\frac{\pi}{2} < \phi < \frac{\pi}{2}$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, and $-\pi < \psi < \pi$.

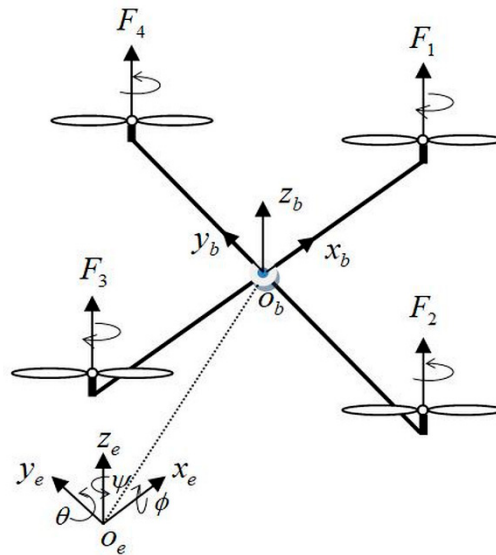


Figure 1. A schematic representation of the quadrotor unmanned aerial vehicle (UAV).

For more information about the useful structural properties of the quadrotor UAV, see e.g., [17,37].

Let $\underline{V} = [V_x \ V_y \ V_z]^T$ and $\underline{\Omega} = [\Omega_\phi \ \Omega_\theta \ \Omega_\psi]^T$ denote the linear and angular velocities in frame B , respectively. In addition, let $\Phi = [\phi \ \theta \ \psi]^T$ and $P = [x \ y \ z]^T$ denote the Euler angles and the position of the quadrotor in frame E , respectively.

The relation between the velocities $(\dot{P}, \dot{\Phi})$ and $(\underline{V}, \underline{\Omega})$ can be expressed as follows:

$$\begin{aligned}\dot{P} &= \underline{R}\underline{V} \\ \underline{\Omega} &= \underline{N}\dot{\Phi}\end{aligned}\quad (5)$$

$$\text{where } \underline{R} = \begin{bmatrix} \cos(\theta)\cos(\psi) & \sin(\phi)\sin(\theta)\cos(\psi) - \cos(\phi)\sin(\psi) & \cos(\phi)\sin(\theta)\cos(\psi) + \sin(\phi)\sin(\psi) \\ \cos(\theta)\cos(\psi) & \sin(\phi)\sin(\theta)\sin(\psi) + \cos(\phi)\cos(\psi) & \cos(\phi)\sin(\theta)\sin(\psi) - \sin(\phi)\cos(\psi) \\ -\sin(\theta) & \sin(\phi)\cos(\theta) & \cos(\phi)\cos(\theta) \end{bmatrix}$$

$$\text{and } \underline{N} = \begin{bmatrix} -\sin(\theta) & 0 & 1 \\ \cos(\theta)\sin(\phi) & \cos(\phi) & 0 \\ \cos(\theta)\cos(\phi) & -\sin(\phi) & 0 \end{bmatrix}.$$

Using the Newton–Euler formulation, the model describing the quadrotor UAV dynamics can be given as follows:

$$\begin{aligned}\ddot{z} &= -g - \frac{k_z}{m}\dot{z} + \frac{\cos(\phi)\cos(\theta)}{m}u_1 + d_1 \\ \ddot{\phi} &= \left(\frac{I_y - I_z}{I_x}\right)\dot{\psi}\dot{\theta} + \frac{1}{I_x}u_2 + d_2 \\ \ddot{\theta} &= \left(\frac{I_z - I_x}{I_y}\right)\dot{\psi}\dot{\phi} + \frac{1}{I_y}u_3 + d_3 \\ \ddot{\psi} &= \left(\frac{I_x - I_y}{I_z}\right)\dot{\theta}\dot{\phi} + \frac{1}{I_z}u_4 + d_4 \\ \ddot{x} &= \frac{u_1}{m}u_5 - \frac{k_x}{m}\dot{x} + d_5 \\ \ddot{y} &= \frac{u_1}{m}u_6 - \frac{k_y}{m}\dot{y} + d_6\end{aligned}\quad (6)$$

where I_x , I_y and I_z denote, respectively, the inertia parameters along the x_b , y_b , and z_b axes; x , y and z denote the quadrotor's position in the earth-fixed frame $E(o_e, x_e, y_e, z_e)$; d_i ($i = 1, \dots, 6$) are the bounded unknown disturbances including gyroscopic effects, time varying disturbances, aerodynamic perturbations such as unpredictable wind gusts, and other neglected and unmodeled dynamics; g is the gravity acceleration; k_x , k_y , and k_z denote their drag coefficients along the x_e , y_e , and z_e directions, respectively; m is the mass of the quadrotor; $u_5 = \cos(\phi)\sin(\theta)\cos(\psi) + \sin(\phi)\sin(\psi)$ and $u_6 = \cos(\phi)\sin(\theta)\sin(\psi) - \sin(\phi)\cos(\psi)$; and u_i ($i = 1, \dots, 4$) are the control inputs of the system (6), and they are defined as follows:

$$\begin{aligned}u_1 &= F_1 + F_2 + F_3 + F_4 = b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\ u_2 &= L(F_4 - F_2) = Lb(\omega_4^2 - \omega_2^2) \\ u_3 &= L(F_1 - F_3) = Lb(\omega_1^2 - \omega_3^2) \\ u_4 &= \frac{c}{b}(F_2 + F_4 - F_1 - F_3) = c(\omega_2^2 + \omega_4^2 - \omega_1^2 - \omega_3^2)\end{aligned}\quad (7)$$

where L is the distance between the center of mass and the center of each rotor of the quadrotor; b and c are the thrust factor and the drag factor, respectively; F_i ($i = 1, \dots, 4$) is the force generated by the rotor i ; and ω_i ($i = 1, \dots, 4$) denotes the angular velocity of the i th rotor.

The state vector is defined to be $\underline{X} = \begin{bmatrix} X^T & \dot{X}^T \end{bmatrix}^T$ and assumed to be available for measurement, with $X = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 \end{bmatrix}^T = \begin{bmatrix} z & \phi & \theta & \psi & x & y \end{bmatrix}^T$ being the first element of the state vector. Then, system (6) can be reformulated as:

$$\begin{cases} \ddot{X}_i = f_i + g_i u_i + d_i, & i = 1, \dots, 6 \\ Y_i = X_i \end{cases}\quad (8)$$

where $f_1 = -g - \frac{k_z}{m}\dot{z}$, $f_2 = \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right)$, $f_3 = \dot{\phi}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right)$, $f_4 = \dot{\phi}\dot{\theta}\left(\frac{I_x - I_y}{I_z}\right)$, $f_5 = -\frac{k_x}{m}\dot{x}$, $f_6 = -\frac{k_y}{m}\dot{y}$ and $(g_1, g_2, g_3, g_4, g_5, g_6) = \left(\frac{\cos(\theta)\cos(\phi)}{m}, \frac{1}{I_x}, \frac{1}{I_y}, \frac{1}{I_z}, \frac{u_1}{m}, \frac{u_1}{m}\right)$; Y_i and u_i are, respectively, the i th output and the i th input of system (8), where u_5 and u_6 are virtual control inputs to be designed later.

In this study, f_i and g_i are unknown nonlinear continuous functions. In addition, it is assumed that the system (8) is controllable. So, we consider that g_i^{-1} exists.

4. Control Law Design

The main objective of control is to steer state X to a desired reference $X_d = [X_1^d \ X_2^d \ X_3^d \ X_4^d \ X_5^d \ X_6^d]^T = [z_d \ \phi_d \ \theta_d \ \psi_d \ x_d \ y_d]^T$.

As the system (8) describes a quadrotor UAV with unknown dynamics that is subject to unknown and unpredictable disturbances, a new robust IT2-AFRSMS was designed to deal with such constraints while ensuring the best tracking performance and avoiding the chattering phenomenon.

The system (8) has four independent inputs to control its six outputs; it is an underactuated system. Therefore, in order to overcome this constraint, two virtual control inputs, u_5 and u_6 , are introduced to generate the desired ϕ_d and θ_d angles to achieve the desired longitudinal and lateral position tracking.

The desired ϕ_d and θ_d angles are determined according to the following equation:

$$\begin{aligned} u_5 &= \cos(\phi_d) \sin(\theta_d) \cos(\psi) + \sin(\phi_d) \sin(\psi) \\ u_6 &= \cos(\phi_d) \sin(\theta_d) \sin(\psi) - \sin(\phi_d) \cos(\psi) \end{aligned} \quad (9)$$

After some rearrangement, we get

$$\begin{aligned} \phi_d &= \arcsin(u_5 \sin(\psi) - u_6 \cos(\psi)) \\ \theta_d &= \arcsin\left(u_5 \frac{\cos(\psi)}{\cos(\phi_d)} + u_6 \frac{\sin(\psi)}{\cos(\phi_d)}\right) \end{aligned} \quad (10)$$

In order to better estimate the unknown nonlinear functions of the system (8), the IT2-FS defined in (3) is used to substitute f_i and g_1 with their IT2-AFS approximators $\hat{f}_i$ and $\hat{g}_1$, respectively, as given in the following equation:

$$\begin{aligned} \hat{f}_i &= \zeta_f^T(i) \theta_f(i) \\ \hat{g}_1 &= \zeta_g^T(1) \theta_g(1) \end{aligned} \quad , \quad i = 1, \dots, 6 \quad (11)$$

where $\zeta_f^T(i) = \frac{1}{2}(\zeta_f(i)_l^T + \zeta_f(i)_r^T)$ and $\zeta_g^T(1) = \frac{1}{2}(\zeta_g(1)_l^T + \zeta_g(1)_r^T)$ such that $\zeta_f(i)_l = [\zeta_f(i)_l^1 \ \zeta_f(i)_l^2 \ \dots \ \zeta_f(i)_l^{M_i}]^T$, $\zeta_f(i)_r = [\zeta_f(i)_r^1 \ \zeta_f(i)_r^2 \ \dots \ \zeta_f(i)_r^{M_i}]^T$, $\zeta_g(1)_l = [\zeta_g(1)_l^1 \ \zeta_g(1)_l^2 \ \dots \ \zeta_g(1)_l^{M_g}]^T$ and $\zeta_g(1)_r = [\zeta_g(1)_r^1 \ \zeta_g(1)_r^2 \ \dots \ \zeta_g(1)_r^{M_g}]^T$ are, as described in (4), the vectors of fuzzy basis functions; $\theta_f(i) = [\theta_f^1(i) \ \theta_f^2(i) \ \dots \ \theta_f^{M_i}(i)]^T$ and $\theta_g(1) = [\theta_g^1(1) \ \theta_g^2(1) \ \dots \ \theta_g^{M_g}(1)]^T$ are adaptive parameter vectors; and M_i and M_g denote the number of fuzzy rules of the IT2-AFSs $\hat{f}_i$ and $\hat{g}_1$, respectively.

In order to estimate the rest of the unknown terms $g_i (i = 2, \dots, 6)$, the following adaptive systems are designed as

$$\begin{aligned} \hat{g}_i &= \theta_g(i) \\ \hat{g}_5 &= \theta_g(5) u_1 \quad , \quad i = 2, 3, 4 \\ \hat{g}_6 &= \theta_g(6) u_1 \end{aligned} \quad (12)$$

where $\theta_g(j)$, $j = 2, \dots, 6$ are adaptive parameters.

4.1. Sliding Mode Control Law Design

The SMC is considered to be among the most robust methods of control and is capable of steering the system state trajectories towards the desired dynamics.

Let $e = \begin{bmatrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{bmatrix}^T = X_d - X$ be the tracking error. Then, the sliding surface can be defined as [38]

$$\begin{aligned} s(\underline{X}, t) &= \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \end{bmatrix}^T \\ &= \left(\frac{\partial}{\partial t} + \lambda \right)^{(p-1)} e \\ &= \sum_{j=0}^{p-1} \frac{(p-1)!}{j!(p-j-1)!} \left(\frac{\partial}{\partial t} \right)^{(p-j-1)} \lambda^j e \end{aligned} \quad (13)$$

where $\lambda = \text{diag}(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6)$ is a matrix of diagonal slopes $\lambda_i (i = 1, \dots, 6)$, and p denotes the system order.

The quadrotor (8) is a second-order system ($p = 2$). Therefore, Equation (13) becomes

$$s_i = \dot{e}_i + \lambda_i e_i, \quad i = 1, \dots, 6. \quad (14)$$

Considering the system defined in (8), the time derivative of the sliding surface can be obtained as

$$\begin{aligned} \dot{s}_i &= \ddot{e}_i + \lambda_i \dot{e}_i \\ &= \ddot{X}_i^d - (f_i + g_i u_i + d_i) + \lambda_i \dot{e}_i. \end{aligned} \quad (15)$$

The desired dynamics are obtained when the following condition is verified: $s_i = \dot{s}_i = 0$.

The optimal parameters of $\hat{f}_i$ and $\hat{g}_i$ can be expressed as

$$\begin{aligned} \theta_f^*(i) &= \underset{\theta_f(i)}{\operatorname{argmin}} \left(\sup |\hat{f}_i - f_i| \right) \\ \theta_g^*(i) &= \underset{\theta_g(i)}{\operatorname{argmin}} \left(\sup |\hat{g}_i - g_i| \right) \end{aligned}, \quad (i = 1, \dots, 6). \quad (16)$$

The minimum approximation error of f_i and g_i is then given by

$$\varepsilon_i = f_i^* - f_i + (g_i^* - g_i)u_i \quad (17)$$

where $f_i^* = \xi_f^T(i)\theta_f^*(i)$ and $g_i^* = \theta_g^*(i)$ are the optimal approximations of f_i and g_i , respectively, with

$$\theta_c^*(i) = \begin{cases} \xi_g^T(1)\theta_g^*(1) & i = 1 \\ \theta_g^*(i) & i = 2, 3, 4 \\ \theta_g^*(i)u_1 & i = 5, 6 \end{cases}.$$

The control law synthesized to satisfy the desired objective of control is expressed as

$$u_i = \hat{g}_i^{-1} \left(\ddot{X}_i^d - \hat{f}_i + \lambda_i \dot{e}_i - u_{sm}(i) \right), \quad i = 1, \dots, 6 \quad (18)$$

where $u_{sm}(i)$ is a RSMCL.

The RSMCL $u_{sm}(i)$ is introduced in order to maintain the desired dynamics ($s_i = \dot{s}_i = 0$) by ensuring that the effects of the approximation errors and all disturbances that affect the quadrotor dynamics are eliminated or at least reduced. Therefore, to guarantee the sliding mode, the expression of $u_{sm}(i)$ is given as

$$u_{sm}(i) = -\eta_i h_i(s_i) - \alpha_i \int_0^{t_i^r} \operatorname{sign}(s_i) \, dt - \mu_i s_i, \quad i = 1, \dots, 6 \quad (19)$$

where $h_i(s_i) = \begin{cases} \rho_i \text{sign}(s_i), & s_i \in \Omega \\ \frac{\text{sign}(s_i)}{\log^2|s_i|}, & s_i \notin \Omega \end{cases}$, with $\Omega = \{s_i | |s_i| \geq \frac{N_i}{2}, 0 < N_i \leq 1\}$, and $\rho_i = \frac{1}{\log^2(\frac{N_i}{2})}$ to make sure that the $h_i(s_i)$ function is continuous everywhere; η_i , α_i , and μ_i are positive reaching control gains; $t_i^r = \begin{cases} t & |s_i| > \vartheta_i \\ t_{\vartheta_i} & |s_i| \leq \vartheta_i \end{cases}$ is the convergence time of the sliding surface s_i to a vicinity ϑ_i of $s_i = 0$.

The adaptation laws of the IT2-AFSs defined in (11) and the adaptive systems given by (12) are expressed as

$$\begin{cases} \dot{\theta}_f(i) = -\gamma_f(i)s_i\tilde{\zeta}_f(i) & , i = 1, \dots, 6 \\ \dot{\theta}_g(1) = -\gamma_g(1)s_1u_1\tilde{\zeta}_g(1) \\ \dot{\theta}_g(i) = -\gamma_g(i)s_iu_i & , i = 2, 3, 4 \\ \dot{\theta}_g(i) = -\gamma_g(i)s_iu_iu_1 & , i = 5, 6 \end{cases} \quad (20)$$

where $\gamma_f(i)$ and $\gamma_g(i)$ are positive learning parameters.

Theorem 1. Using the IT2-FS approximators defined in (11), the adaptive systems presented in (12) and the adaptation laws expressed in (20), the control law (18) developed for the underactuated quadrotor (8) is stable in the sense of Lyapunov and the asymptotic convergence of the tracking error is established despite unknown dynamics, unknown physical parameters, and all unknown and unpredictable disturbances that affect the control system.

Proof 1. Use the following augmented Lyapunov function candidate:

$$v_i = \frac{1}{2}s_i^2 + \frac{1}{2}\tilde{\theta}_f^T(i)\tilde{\theta}_f(i) + q_i, \quad i = 1, \dots, 6 \quad (21)$$

where $\tilde{\theta}_f(i) = \theta_f(i) - \theta_f^*(i)$; $q_i = \frac{1}{2\gamma_g(i)} \begin{cases} \tilde{\theta}_g^T(1)\tilde{\theta}_g(1) & , i = 1 \\ \tilde{\theta}_g^2(i) & , i = 2, \dots, 6 \end{cases}$, such that $\tilde{\theta}_g(j) = \theta_g(j) - \theta_g^*(j)$, $j = 1, \dots, 6$;

The time derivative of the above equation is

$$\dot{v}_i = s_i \left(\ddot{X}_i^d - f_i - g_i u_i - d_i + \lambda_i \dot{e}_i \right) + \frac{1}{\gamma_f(i)} \dot{\theta}_f^T(i) \tilde{\theta}_f(i) + \dot{q}_i \quad (22)$$

where $\dot{q}_i = \frac{1}{\gamma_g(i)} \begin{cases} \dot{\theta}_g^T(1)\tilde{\theta}_g(1) & , i = 1 \\ \dot{\theta}_g(i)\tilde{\theta}_g(i) & , i = 2, \dots, 6 \end{cases}$.

From (18), we get

$$\ddot{X}_i^d = \hat{f}_i + \hat{g}_i u_i + u_{sm}(i) - \lambda_i \dot{e}_i. \quad (23)$$

Substituting (23) into (22) gives

$$\begin{aligned} \dot{v}_i &= s_i \left(\hat{f}_i - f_i + (\hat{g}_i - g_i)u_i + u_{sm}(i) - d_i \right) + \frac{1}{\gamma_f(i)} \dot{\theta}_f^T(i) \tilde{\theta}_f(i) + \dot{q}_i \\ &= s_i \left((\hat{f}_i - f_i^*) + (f_i^* - f_i) + (\hat{g}_i - g_i^*)u_i + (g_i^* - g_i)u_i \right) + s_i(u_{sm}(i) - d_i) + \frac{1}{\gamma_f(i)} \dot{\theta}_f^T(i) \tilde{\theta}_f(i) + \dot{q}_i \\ &= s_i \tilde{\zeta}_f^T(i) \tilde{\theta}_f(i) + s_i \tilde{\theta}_c u_i + s_i(u_{sm}(i) - \Delta_i) + \frac{1}{\gamma_f(i)} \dot{\theta}_f^T(i) \tilde{\theta}_f(i) + \dot{q}_i \\ &= \left(s_i \tilde{\zeta}_f^T(i) + \frac{1}{\gamma_f(i)} \dot{\theta}_f^T(i) \right) \tilde{\theta}_f(i) + \Phi_g(i) \tilde{\theta}_g(i) + s_i(u_{sm}(i) - \Delta_i) \end{aligned} \quad (24)$$

where $\Delta_i = d_i - \varepsilon_i$ such that $|\Delta_i| \leq \delta_i$ and $\delta_i \geq 0$; $\tilde{\theta}_c = \theta_c - \theta_c^*$, with $\theta_c(i) = \begin{cases} \tilde{\zeta}_g^T(1)\theta_g(1) & i = 1 \\ \theta_g(i) & i = 2, 3, 4 \\ \theta_g(i)u_1 & i = 5, 6 \end{cases}$;

$$\text{and } \Phi_g(i) = \begin{cases} s_i u_i \tilde{\zeta}_g^T(1) + \frac{1}{\gamma_g(i)} \dot{\theta}_g^T(1) & , i = 1 \\ s_i u_i + \frac{1}{\gamma_g(i)} \dot{\theta}_g(i) & , i = 2, 3, 4 \\ s_i u_i u_1 + \frac{1}{\gamma_g(i)} \dot{\theta}_g(i) & , i = 5, 6 \end{cases}.$$

Substituting (20) into (24) gives

$$\dot{v}_i = s_i(u_{sm}(i) - \Delta_i). \quad (25)$$

Substituting $u_{sm}(i)$ by its expression gives

$$\dot{v}_i = -s_i \left(\eta_i h_i(s_i) + \alpha_i \int_0^{t_i^r} \text{sign}(s_i) dt + \mu_i s_i \right) - s_i \Delta_i. \quad (26)$$

Consider the following inequality:

$$-s_i \left(\eta_i h_i(s_i) + \alpha_i \int_0^{t_i^r} \text{sign}(s_i) dt + \mu_i s_i \right) \leq s_i \Delta_i. \quad (27)$$

If the following condition is assured:

$$\mu_i |s_i| + \alpha_i t_i^r + \eta_i |h_i(s_i)| \geq \delta_i, \quad i = 1, \dots, 6 \quad (28)$$

Then the inequality (27) is verified, and therefore, the $\dot{v}_i$ functions defined in (26) are negative. Thus, Proof 1 is established. $\square$

In order to ensure the above inequality (28), the values of parameters μ_i , α_i , and η_i should be well chosen. However, in practice, choosing the right values of these parameters, which ensures the desired tracking objective while simultaneously avoiding the chattering, remains one of the major problems in systems control. The large values generate a large amount of chattering, and the small ones affect the robustness of the controlled system against uncertainties and disturbances and deteriorate the performance of the tracking control. Therefore, in order to overcome this control constraint, in this study, we propose the introduction of a rigorous IT2-AFRSMS in order to efficiently estimate the optimal values of parameters μ_i , α_i and η_i online to ensure both the desired performance of tracking control of the quadrotor (8) by guaranteeing the condition shown in (28), and avoiding the chattering phenomenon.

4.2. Proposed Adaptive Fuzzy Sliding Mode Control Design Method

In order to efficiently estimate the optimal gains of the RSMCL $u_{sm}(i)$ defined in (19), a new IT2-AFRSMS similar to the IT2-FS defined in (3) and characterized by the following properties is introduced:

- The sliding surface $s(\underline{X}, t)$ is the input vector of the IT2-AFRSMS;
- The outputs of the IT2-AFRSMS are the online estimations of the terms $u_\eta(i) = -\eta_i h_i(s_i)$, $u_\alpha(i) = -\alpha_i \int_0^{t_i^r} \text{sign}(s_i) dt$, and $u_\mu(i) = -\mu_i s_i$ of the RSMCL $u_{sm}(i)$.

Thus, $u_\eta(i) = -\eta_i h_i(s_i)$, $u_\alpha(i) = -\alpha_i \int_0^{t_i^r} \text{sign}(s_i) dt$ and $u_\mu(i) = -\mu_i s_i$ are substituted, respectively, by their IT2-AFS estimators, as follows:

$$\begin{aligned}\hat{u}_\eta(i) &= \xi_\eta^T(i) \theta_\eta(i) |h_i(s_i)| \\ \hat{u}_\alpha(i) &= \xi_\alpha^T(i) \theta_\alpha(i) t_i^r \\ \hat{u}_\mu(i) &= \xi_\mu^T(i) \theta_\mu(i) |s_i|\end{aligned}, \quad i = 1, \dots, 6 \quad (29)$$

where $\xi_\alpha(i) = \frac{1}{2}(\xi_\alpha(i)_l + \xi_\alpha(i)_r) = [\xi_\alpha^1(i) \quad \xi_\alpha^2(i) \quad \dots \quad \xi_\alpha^M(i)]^T$, $\xi_\eta(i) = \frac{1}{2}(\xi_\eta(i)_l + \xi_\eta(i)_r) = [\xi_\eta^1(i) \quad \xi_\eta^2(i) \quad \dots \quad \xi_\eta^M(i)]^T$ and $\xi_\mu(i) = \frac{1}{2}(\xi_\mu(i)_l + \xi_\mu(i)_r) = [\xi_\mu^1(i) \quad \xi_\mu^2(i) \quad \dots \quad \xi_\mu^M(i)]^T$ are, as described in (4), the vectors of fuzzy basis functions; $\theta_\alpha(i) = [\theta_\alpha^1(i) \quad \theta_\alpha^2(i) \quad \dots \quad \theta_\alpha^M(i)]^T$, $\theta_\eta(i) = [\theta_\eta^1(i) \quad \theta_\eta^2(i) \quad \dots \quad \theta_\eta^M(i)]^T$ and $\theta_\mu(i) = [\theta_\mu^1(i) \quad \theta_\mu^2(i) \quad \dots \quad \theta_\mu^M(i)]^T$ are the online adjustable parameter vectors; M is the number of rules.

We define the optimal parameters of the IT2-AFSs $\hat{u}_\eta(i)$, $\hat{u}_\alpha(i)$, and $\hat{u}_\mu(i)$ as

$$\begin{aligned}\theta_\eta^*(i) &= \underset{\theta_\eta(i)}{\operatorname{argmin}} \left(\sup_{s_i} |\hat{u}_\eta(i) - u_\eta(i)| \right) \\ \theta_\alpha^*(i) &= \underset{\theta_\alpha(i)}{\operatorname{argmin}} \left(\sup_{s_i} |\hat{u}_\alpha(i) - u_\alpha(i)| \right) \\ \theta_\mu^*(i) &= \underset{\theta_\mu(i)}{\operatorname{argmin}} \left(\sup_{s_i} |\hat{u}_\mu(i) - u_\mu(i)| \right)\end{aligned} \quad (30)$$

The global proposed control law is designed as follows:

$$u_i = \hat{g}_i^{-1} \left(\ddot{X}_i^d - \hat{f}_i + \lambda_i \dot{e}_i - \hat{u}_{sm}(i) \right), \quad i = 1, \dots, 6 \quad (31)$$

where $\hat{u}_{sm}(i) = \hat{u}_\eta(i) + \hat{u}_\alpha(i) + \hat{u}_\mu(i)$ is the designed IT2-AFRSMCL.

The adaptation laws designed for the estimators defined in (29) are given as:

$$\begin{aligned}\dot{\theta}_\eta(i) &= -\gamma_\eta(i) s_i |h_i(s_i)| \xi_\eta(i) \\ \dot{\theta}_\alpha(i) &= -\gamma_\alpha(i) s_i t_i^r \xi_\alpha(i) \\ \dot{\theta}_\mu(i) &= -\gamma_\mu(i) s_i^2 \text{sign}(s_i) \xi_\mu(i)\end{aligned} \quad (32)$$

where $\gamma_\eta(i)$, $\gamma_\alpha(i)$ and $\gamma_\mu(i)$ are positive learning parameters.

Theorem 2. Using the IT2-AFSs defined in (11) and (29), the adaptive systems defined in (12), the adaptation laws given by (20) and (32), the global control law (31) developed for the underactuated quadrotor (8) is stable in the sense of Lyapunov, and the asymptotic convergence of the tracking error is established despite the unknown dynamics, unknown physical parameters, and all of the unknown and unpredictable disturbances that affect quadrotor dynamics.

Proof 2. Use the following new augmented Lyapunov function candidate:

$$v_i = \frac{1}{2} s_i^2 + \frac{1}{2} \tilde{\theta}_f^T(i) \tilde{\theta}_f(i) + q_i + \frac{1}{2} \tilde{\theta}_\eta^T(i) \tilde{\theta}_\eta(i) + \frac{1}{2} \tilde{\theta}_\alpha^T(i) \tilde{\theta}_\alpha(i) + \frac{1}{2} \tilde{\theta}_\mu^T(i) \tilde{\theta}_\mu(i), \quad i = 1, \dots, 6 \quad (33)$$

where $\tilde{\theta}_\eta = \theta_\eta - \theta_\eta^*$, $\tilde{\theta}_\alpha = \theta_\alpha - \theta_\alpha^*$, $\tilde{\theta}_\mu = \theta_\mu - \theta_\mu^*$.

Considering Equations (21), (25), and (31), the time derivative of (33) gives:

$$\dot{v}_i = s_i(u_{sm}(i) - \Delta_i) + \frac{1}{\gamma_\eta(i)} \dot{\theta}_\eta^T(i) \tilde{\theta}_\eta(i) + \frac{1}{\gamma_\alpha(i)} \dot{\theta}_\alpha^T(i) \tilde{\theta}_\alpha(i) + \frac{1}{\gamma_\mu(i)} \dot{\theta}_\mu^T(i) \tilde{\theta}_\mu(i). \quad (34)$$

Let $\hat{u}_\eta^*(i) = \zeta_\eta^T(i) \theta_\eta^*(i) |h_i(s_i)| = -\eta_i^* h_i(s_i)$, $\hat{u}_\alpha^*(i) = \zeta_\alpha^T(i) \theta_\alpha^*(i) t_i^r = -\alpha_i^* \int_0^{t_i^r} \text{sign}(s_i) dt$, and $\hat{u}_\mu^*(i) = \zeta_\mu^T(i) \theta_\mu^*(i) |s_i| = -\mu_i^* s_i$ be, respectively, the online optimal estimations of $u_\eta(i)$, $u_\alpha(i)$ and $u_\mu(i)$ that ensure the best tracking control performance of the quadrotor (8) by providing optimal gains in η_i^* , α_i^* , and μ_i^* for the RSMCL $u_{sm}(i)$, which allows the perturbations Δ_i to be efficaciously rejected through verification of the condition shown in (28) while simultaneously avoiding the undesired chattering. Then, by introducing the optimal IT2-AFRSMCL $u_{sm}^*(i) = u_\eta^*(i) + u_\alpha^*(i) + u_\mu^*(i)$ into (34), we get:

$$\begin{aligned} \dot{v}_i &= s_i((\hat{u}_{sm}(i) - u_{sm}^*(i) + u_{sm}^*(i) - \Delta_i) + \frac{1}{\gamma_\eta(i)} \dot{\theta}_\eta^T(i) \tilde{\theta}_\eta(i) + \frac{1}{\gamma_\alpha(i)} \dot{\theta}_\alpha^T(i) \tilde{\theta}_\alpha(i) + \frac{1}{\gamma_\mu(i)} \dot{\theta}_\mu^T(i) \tilde{\theta}_\mu(i)) \\ &= s_i \left(\zeta_\eta^T(i) \tilde{\theta}_\eta(i) |h_i(s_i)| + \zeta_\alpha^T(i) \tilde{\theta}_\alpha(i) t_i^r + \zeta_\mu^T(i) \tilde{\theta}_\mu(i) |s_i| \right) + s_i(u_{sm}^*(i) - \Delta_i) + \\ &\quad \frac{1}{\gamma_\eta(i)} \dot{\theta}_\eta^T(i) \tilde{\theta}_\eta(i) + \frac{1}{\gamma_\alpha(i)} \dot{\theta}_\alpha^T(i) \tilde{\theta}_\alpha(i) + \frac{1}{\gamma_\mu(i)} \dot{\theta}_\mu^T(i) \tilde{\theta}_\mu(i) \\ &= \left(s_i |h_i(s_i)| \left(\zeta_\eta^T(i) + \frac{1}{\gamma_\eta(i)} \dot{\theta}_\eta^T(i) \right) \tilde{\theta}_\eta(i) + (s_i t_i^r \zeta_\alpha^T(i) + \frac{1}{\gamma_\alpha(i)} \dot{\theta}_\alpha^T(i)) \tilde{\theta}_\alpha(i) + \right. \\ &\quad \left. \left(s_i^2 \text{sign}(s_i) \zeta_\mu^T(i) + \frac{1}{\gamma_\mu(i)} \dot{\theta}_\mu^T(i) \right) \tilde{\theta}_\mu(i) + s_i(u_{sm}^*(i) - \Delta_i) \right) \end{aligned} \quad (35)$$

Substituting $\dot{\theta}_\eta(i)$, $\dot{\theta}_\alpha(i)$, and $\dot{\theta}_\mu(i)$ by their expressions defined in (32) gives

$$\begin{aligned} \dot{v}_i &= s_i(u_{sm}^*(i) - \Delta_i) \\ &= s_i \left(-\eta_i^* h_i(s_i) - \alpha_i^* \int_0^{t_i^r} \text{sign}(s_i) dt - \mu_i^* s_i \right) - s_i \Delta_i \\ &= -|s_i|(\eta_i^* |h_i(s_i)| + \alpha_i^* t_i^r + \mu_i^* |s_i|) - s_i \Delta_i \end{aligned} \quad (36)$$

If the following inequality is guaranteed,

$$\mu_i^* |s_i| + \alpha_i^* t_i^r + \eta_i^* |h_i| \geq \delta_i, \quad i = 1, \dots, 6. \quad (37)$$

Then, the values of $\dot{v}_i$ defined in (36) are negative.

Additionally, since η_i^* , α_i^* , and μ_i^* are the online optimal estimations of η_i , α_i , and μ_i that ensure the condition shown in (28) is satisfied while simultaneously avoiding chattering. Thus, the condition shown in (37) is verified. Therefore, Proof 2 is established. $\square$

The control design method developed in this paper is represented in Figure 2.

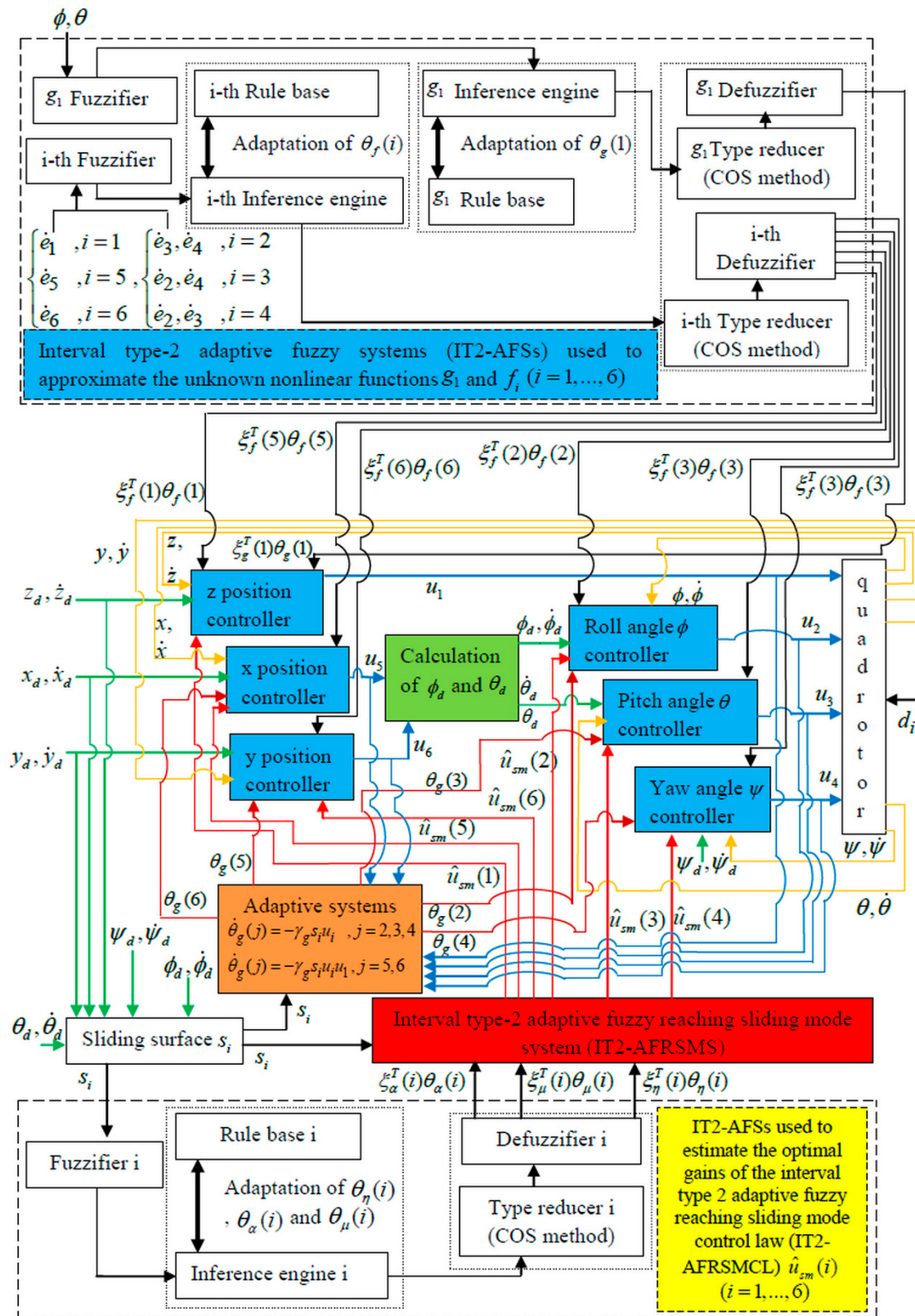


Figure 2. Developed control design scheme.

5. Simulation Results

In order to validate the effectiveness of the developed tracking control method for the quadrotor system (8), we present the simulation results in this section.

The quadrotor parameters used for simulation are listed in Table 1 below.

Table 1. Physical parameters of the quadrotor UAV.

Parameters	Values	Units
m	2.8	kg
L	0.25	m
I_x	1.98×10^{-2}	$\text{kg}\cdot\text{m}^2$
I_y	1.92×10^{-2}	$\text{kg}\cdot\text{m}^2$
I_z	2.94×10^{-2}	$\text{kg}\cdot\text{m}^2$
k_x, k_y, k_z	0.24	$(\text{N}\cdot\text{s})/\text{m}$
g	9.8	m/s^2

All unknown disturbances that affect the quadrotor system (8), including gyroscopic effects and aerodynamic perturbations, are represented by

$$\begin{aligned}
 d_1 &= 1.5 \sin(t/2) \\
 d_2 &= D(t) - 0.15 \dot{\phi} \\
 d_3 &= D(t) - 0.15 \dot{\theta} \\
 d_4 &= 1.5 \sin(t/2) - 0.15 \dot{\psi} \\
 d_5 &= D(t) \\
 d_6 &= D(t)
 \end{aligned}$$

With the function $D(t)$ being represented in Figure 3 below.

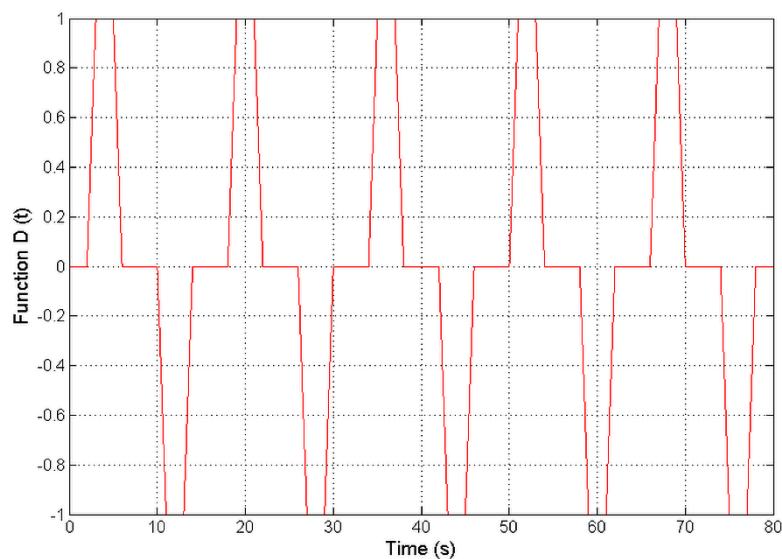


Figure 3. Representative curve of the $D(t)$ function.

The mass and the inertia moment of the quadrotor (8) are unknown and present the following time varying disturbances:

$$\begin{aligned}
 dm &= 1.2 \sin(t) (\text{kg}) \\
 dI_x &= 0.7 \sin(t) \cdot 10^{-2} (\text{kg}\cdot\text{m}^2) \\
 dI_y &= 0.7 \sin(t) \cdot 10^{-2} (\text{kg}\cdot\text{m}^2) \\
 dI_z &= \sin(t) \cdot 10^{-2} (\text{kg}\cdot\text{m}^2)
 \end{aligned}$$

The main objective of control is to steer the state X to the desired reference, $X_d = \begin{bmatrix} z_d & \phi_d & \theta_d & \psi_d & x_d & y_d \end{bmatrix}^T = \begin{bmatrix} 0.5 + \frac{t}{7} & \phi_d & \theta_d & 0 & 2\cos(\frac{\pi}{20}t) & 2\sin(\frac{\pi}{20}t) \end{bmatrix}^T$. The angles ϕ_d and θ_d are determined according to Equation (10).

Since the studied quadrotor (8) has unknown dynamics and unknown physical parameters and because it is subject to unknown and unpredictable disturbances and suffers from time varying perturbations, we can use the developed control law (31) to obtain the intended objectives of control.

The initial positions and angle values are $X(0) = \begin{bmatrix} 0.4 & 0 & 0 & 0.2 & 1.8 & 0.2 \end{bmatrix}^T$.

The sliding surfaces are set to $s_1 = \dot{e}_1 + \lambda_1 e_1$, $s_2 = \dot{e}_2 + \lambda_2 e_2$, $s_3 = \dot{e}_3 + \lambda_3 e_3$, $s_4 = \dot{e}_4 + \lambda_4 e_4$, $s_5 = \dot{e}_5 + \lambda_5 e_5$, and $s_6 = \dot{e}_6 + \lambda_6 e_6$, such that $e_1 = z_d - z$, $e_2 = \phi_d - \phi$, $e_3 = \theta_d - \theta$, $e_4 = \psi_d - \psi$, $e_5 = x_d - x$, and $e_6 = y_d - y$ are the tracking errors.

It is assumed that ϕ and θ belong to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and ψ belongs to $[-\pi, \pi]$.

The IT2-AFSs $\hat{f}_1$, $\hat{f}_5$ and $\hat{f}_6$ have, respectively, the inputs $\dot{e}_1$, $\dot{e}_5$ and $\dot{e}_6$; $\hat{f}_2$ has two inputs, $\dot{e}_3$ and $\dot{e}_4$; $\hat{f}_3$ has two inputs, $\dot{e}_2$ and $\dot{e}_4$; and $\hat{f}_4$ has two inputs, $\dot{e}_2$ and $\dot{e}_3$. All of these inputs are defined by three MFs, as depicted in Figures 4–7.

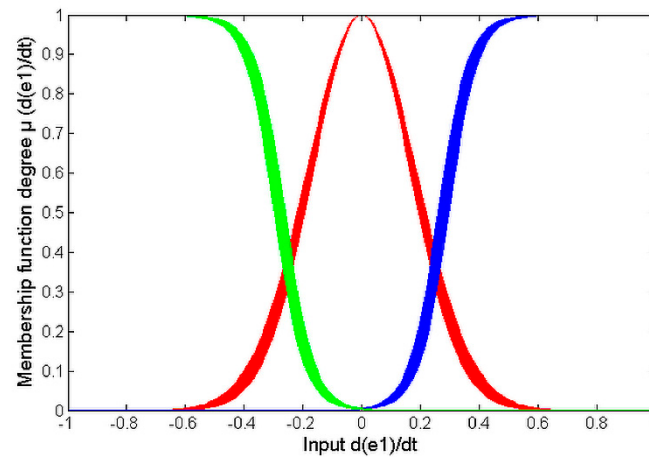


Figure 4. Membership functions of the interval T2 adaptive fuzzy system (IT2-AFS) $\hat{f}_1$.

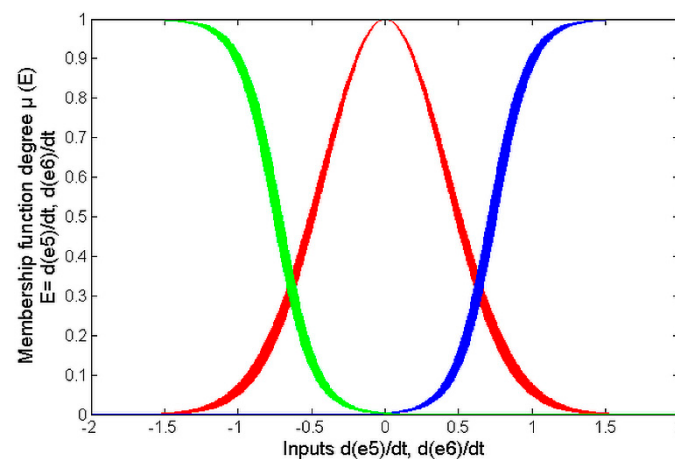


Figure 5. Membership functions of the IT2-AFSs $\hat{f}_5$ and $\hat{f}_6$.

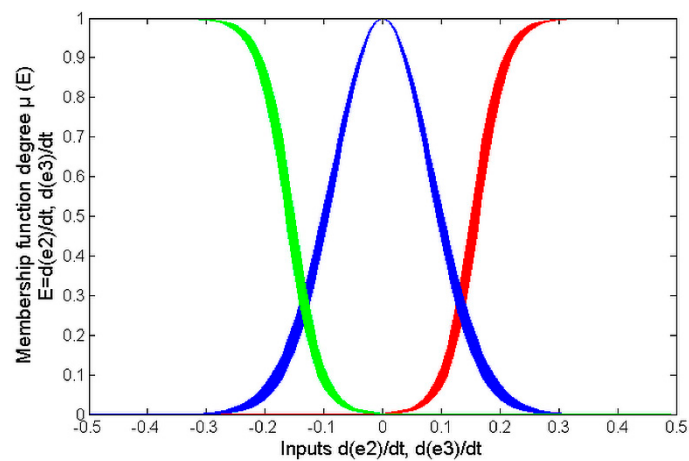


Figure 6. Membership functions used by the inputs $\dot{e}_2$ and $\dot{e}_3$.

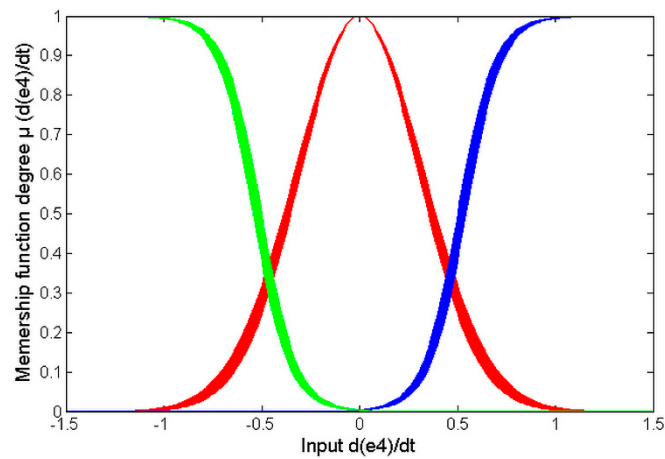


Figure 7. Membership functions used by the input $\dot{e}_4$.

The MFs designed for $\hat{f}_1$ are depicted in Figure 4; the MFs used by $\hat{f}_5$ and $\hat{f}_6$ are represented in Figure 5; the MFs used for the inputs $\dot{e}_3$ and $\dot{e}_2$ are shown in Figure 6; and the MFs designed for $\dot{e}_4$ are depicted in Figure 7.

On the other hand, the IT2-AFS $\hat{g}_1$ has two inputs, ϕ and θ . Then, three MFs are used for each input of $\hat{g}_1$, as shown in Figure 8.

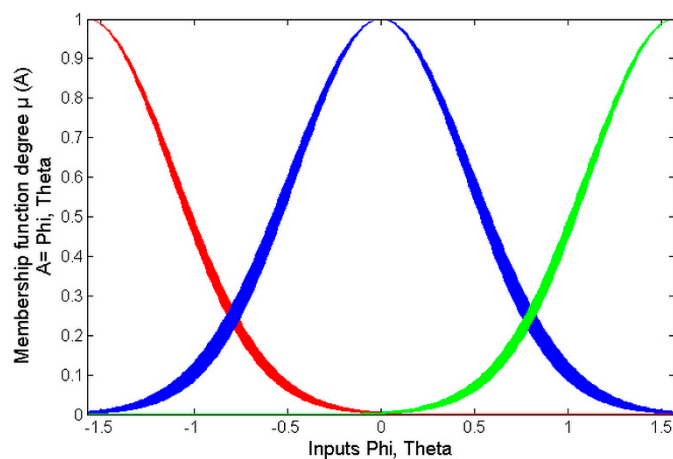


Figure 8. Membership functions of the IT2-AFS $\hat{g}_1$.

The MFs used by the IT2-AFRSMCL $\hat{u}_{sm}(i)$, ($i = 1, \dots, 6$) are represented in Figure 9 below.

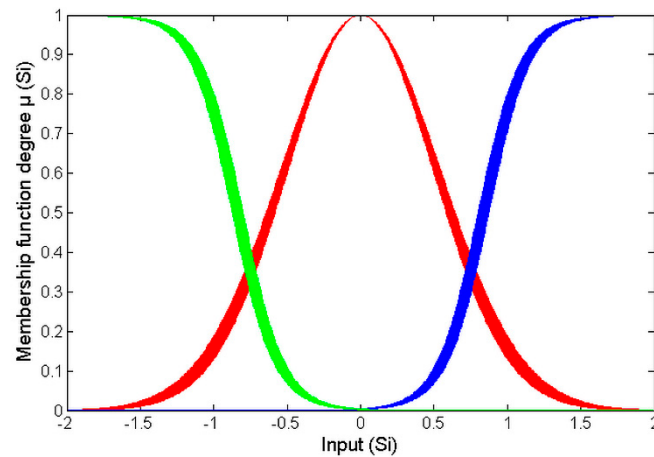


Figure 9. Membership functions of the IT2 adaptive fuzzy reaching sliding mode system (IT2-AFRSMS) $\hat{u}_{sm}(i)$.

In order to confirm the effectiveness of the proposed tracking control method (PTCM), a comparison was carried out with its counterpart method that uses T1-AFSs instead of T2-AFSs and uses a SOST-SMC to reject the undesired effects caused by unknown disturbances and approximation errors. Henceforward, the abbreviation CMTC refers to this method of tracking control.

The control law used by CMTC is given by Equation (38):

$$V_i = \bar{\hat{g}}_i^{-1} \left(\ddot{X}_i^d - \bar{\hat{f}}_i + \lambda_i \dot{e}_i - v_{ST}(i) \right), \quad i = 1, \dots, 6 \quad (38)$$

where $\bar{\hat{f}}_i$ and $\bar{\hat{g}}_1$ are T1-AFSs, and $\bar{\hat{g}}_j$ ($j = 2, \dots, 6$) are adaptive systems and they are designed in the same way as those defined in (12), $v_{ST}(i) = -\alpha_i \int_0^{t_i^r} \text{sign}(s_1) dt - \beta_i |s_i|^{0.5}$, such that α_i and β_i denote the gains of the reaching SOST control term $v_{ST}(i)$.

The MFs used by the T1-AFSs $\bar{\hat{f}}_i$ and $\bar{\hat{g}}_1$ are depicted in Figures 10–14.

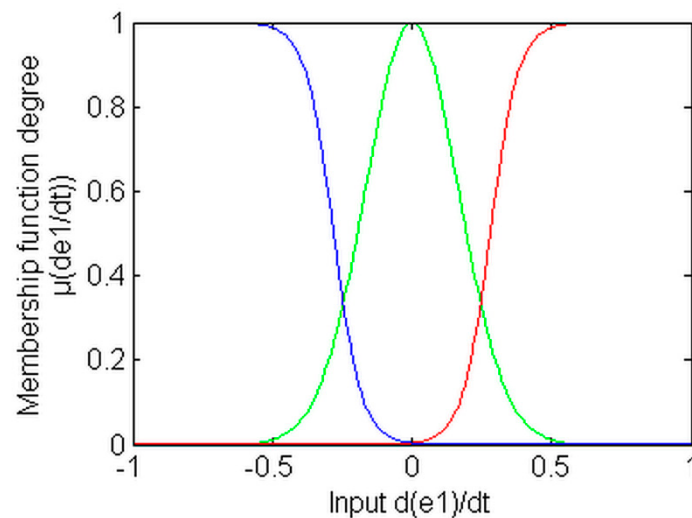


Figure 10. Membership functions used by $\bar{\hat{f}}_1$.

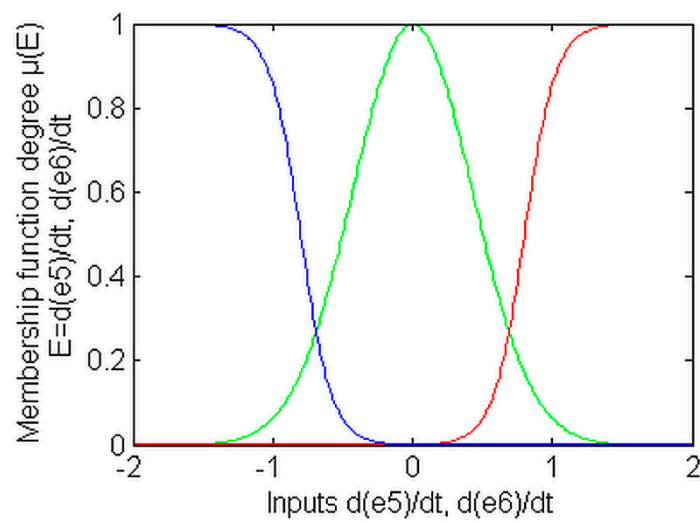


Figure 11. Membership functions used by $\bar{f}_5$ and $\bar{f}_6$.

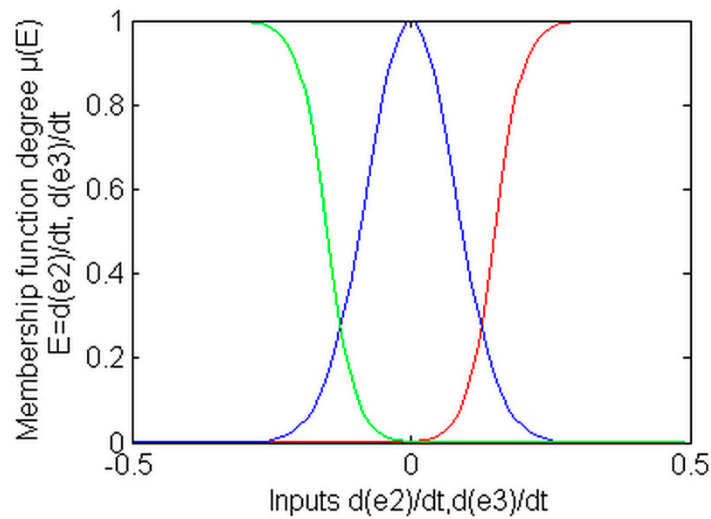


Figure 12. Membership functions used by $\bar{f}_3$ and $\bar{f}_4$.

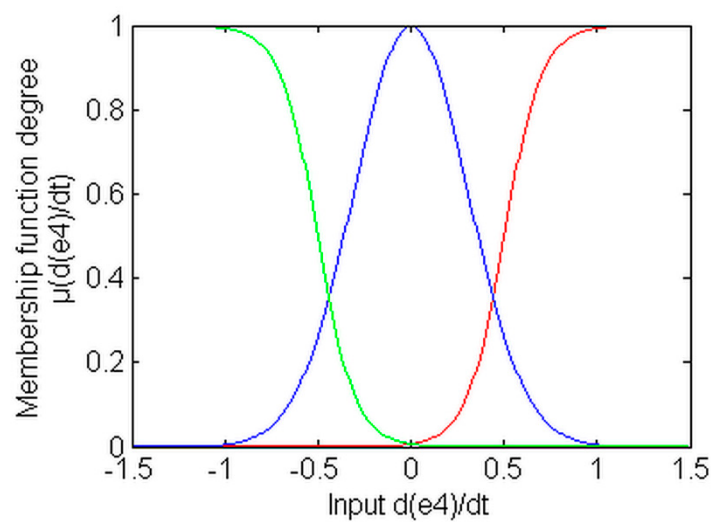


Figure 13. Membership functions used by $\bar{f}_2$.

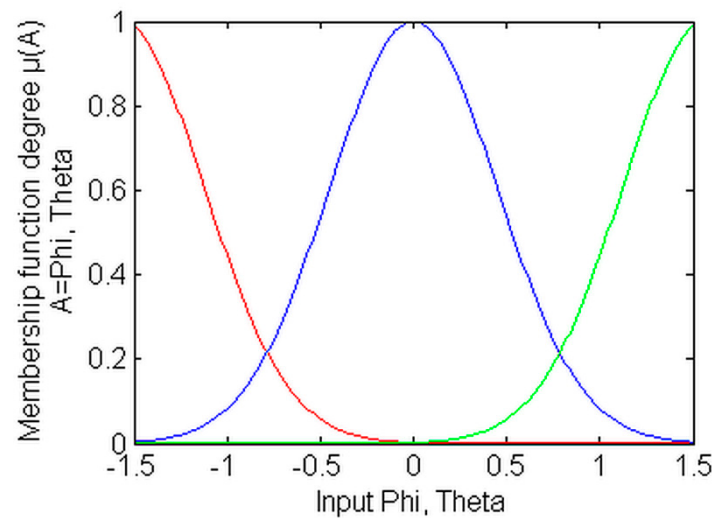


Figure 14. Membership functions of the IT2-AFS $\bar{\xi}_1$.

The constant parameters of the two compared tracking control approaches are given in Table 2.

Table 2. Constant parameters of both proposed control approaches.

Parameters	PTCM	CMTC
$\lambda_1; \lambda_2; \lambda_3$	48; 20; 20	14; 4; 4
$\lambda_4; \lambda_5; \lambda_6$	10; 5; 5	14; 16; 16
$\gamma_\alpha(1); \gamma_\alpha(2); \gamma_\alpha(3)$	140; 20; 15	-
$\gamma_\alpha(4); \gamma_\alpha(5); \gamma_\alpha(6)$	80; 20; 10	-
$\gamma_\mu(1); \gamma_\mu(2); \gamma_\mu(3)$	178; 145; 110	-
$\gamma_\mu(4); \gamma_\mu(5); \gamma_\mu(6)$	64; 92; 80	-
$\gamma_\eta(1); \gamma_\eta(2); \gamma_\eta(3)$	70; 20; 25	-
$\gamma_\eta(4); \gamma_\eta(5); \gamma_\eta(6)$	65; 10; 5	-
$\gamma_g(1); \gamma_g(2)$	0.0011; 0.0014	0.0011; 0.0014
$\gamma_g(3); \gamma_g(4)$	0.0016; 0.0012	0.0016; 0.0012
$\gamma_g(5); \gamma_g(6)$	0.0015; 0.0015	0.0015; 0.0015
$N_1; N_2; N_3$	0.11; 0.78; 0.18	-
$N_4; N_5; N_6$	0.12; 0.08; 0.15	-
$\alpha_1; \alpha_2; \alpha_3$	-	1.5; 0.2; 0.5
$\alpha_4; \alpha_5; \alpha_6$	-	0.6; 0.1; 0.6
$\beta_1; \beta_2; \beta_3$	-	2; 0.4; 0.5
$\beta_4; \beta_5; \beta_6$	-	0.5; 0.3; 0.4
$\gamma_f(1); \gamma_f(2); \gamma_f(3)$	5; 6; 8	5; 5; 10
$\gamma_f(4); \gamma_f(5); \gamma_f(6)$	6; 2; 2	5; 2; 2

PTCM: proposed tracking control method; CMTC: refers to the tracking control method defined by Equation (38).

The simulation results obtained from the comparison that was carried out between the two approaches of control are illustrated in Figures 15–28. Figures 15 and 16 depict the position tracking errors; Figures 17 and 18 represent the attitude tracking errors; Figures 19 and 20 show the position tracking evolution and its reference trajectory; the evolution of the attitude tracking and its desired reference are shown in Figures 21 and 22; Figures 23–26 represent, for both tracking control methods, the 3D position of the quadrotor and its reference trajectory in the time intervals [0, 80] s and [0, 4] s; and finally, Figures 27 and 28 represent the position and the attitude control laws of the two compared approaches of tracking control.

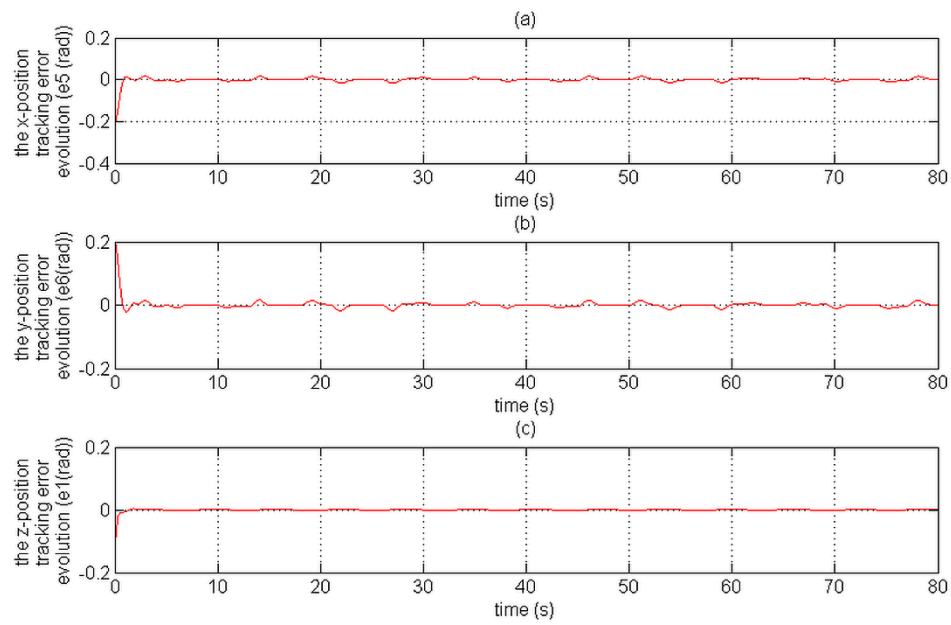


Figure 15. The position tracking errors obtained by the PTCM: (a) x-position error (m); (b) y-position error (m); (c) z-position error (m).

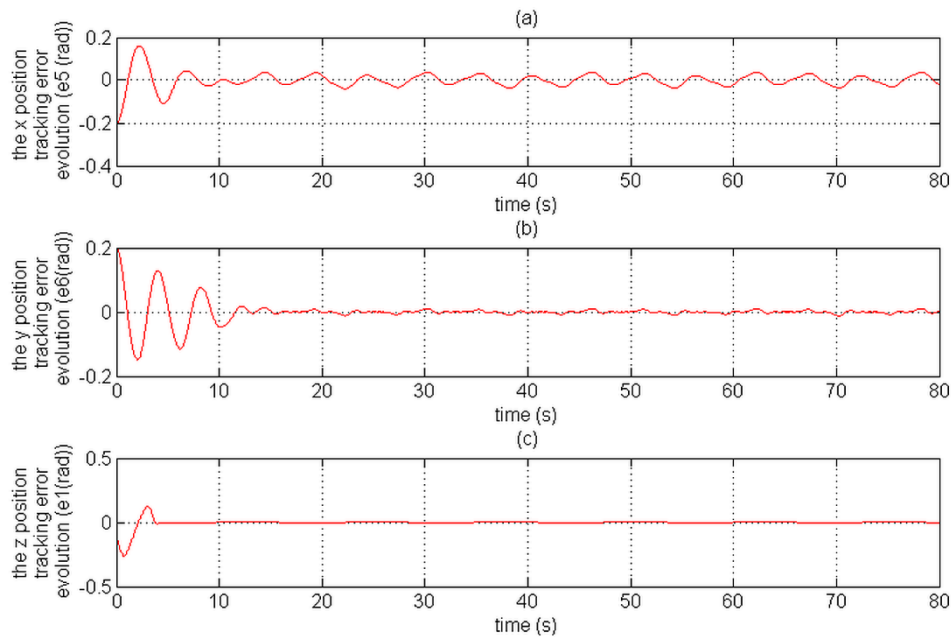


Figure 16. The position tracking errors obtained by the CMTc: (a) x-position error (m); (b) y-position error (m); (c) z-position error (m).

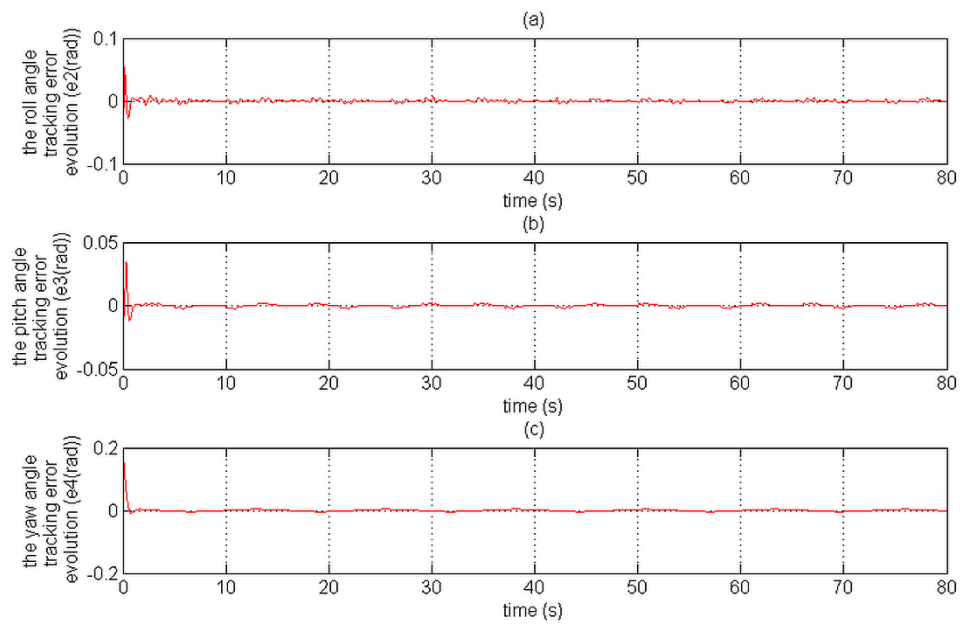


Figure 17. The attitude tracking errors obtained by the PTCM: (a) roll angle error $e_2(rad)$; (b) pitch angle error $e_3(rad)$; (c) yaw angle error $e_4(rad)$.

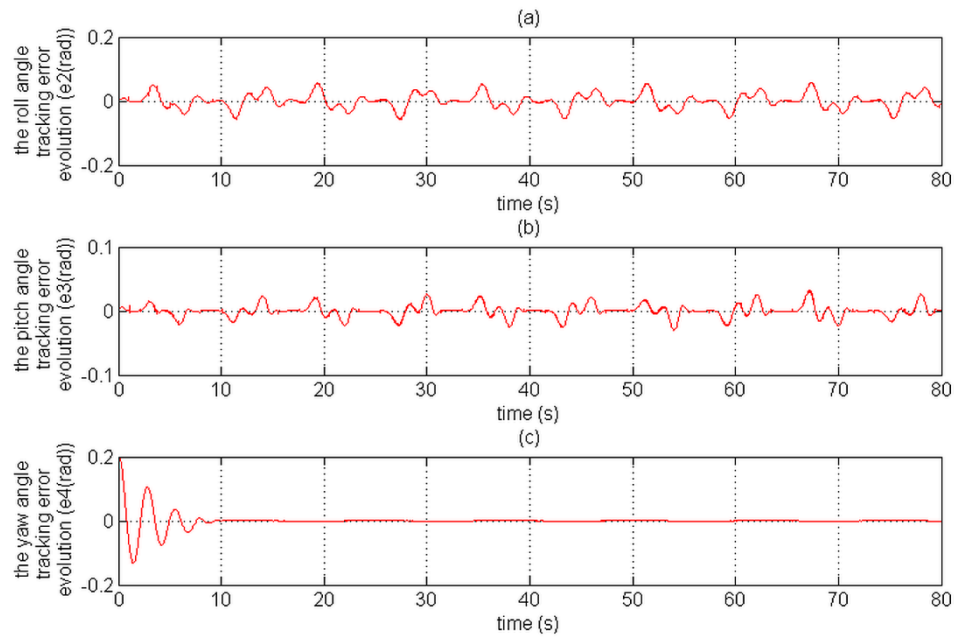


Figure 18. The attitude tracking errors obtained by the CMTC: (a) roll angle error $e_2(rad)$; (b) pitch angle error $e_3(rad)$; (c) yaw angle error $e_4(rad)$.

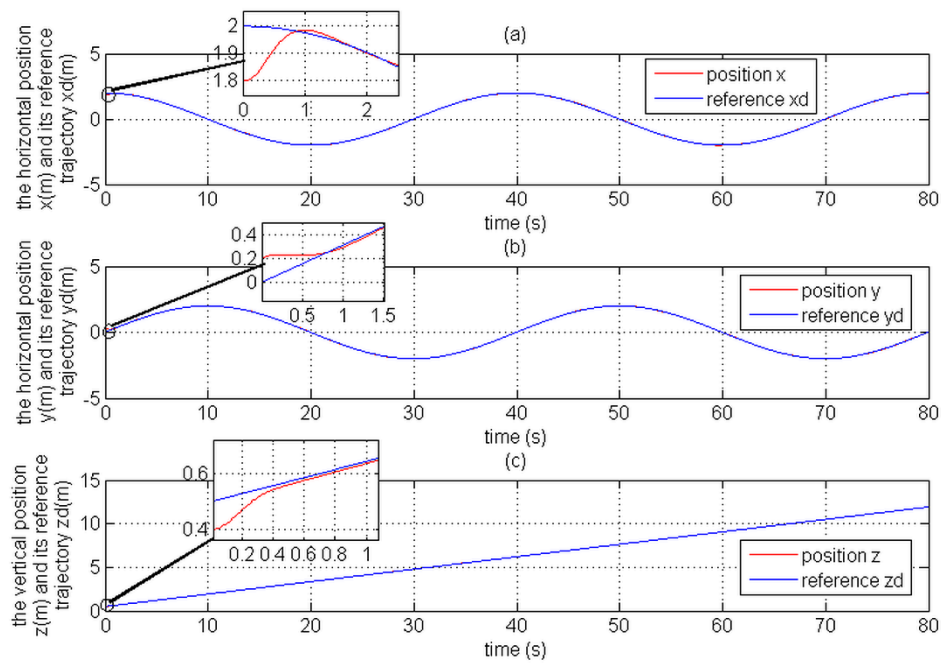


Figure 19. The position tracking evolution obtained by the PTCM: (a) x -position (m) and its reference trajectory x_d (m); (b) y -position (m) and its reference trajectory y_d (m); (c) z -position (m) and its reference trajectory z_d (m).

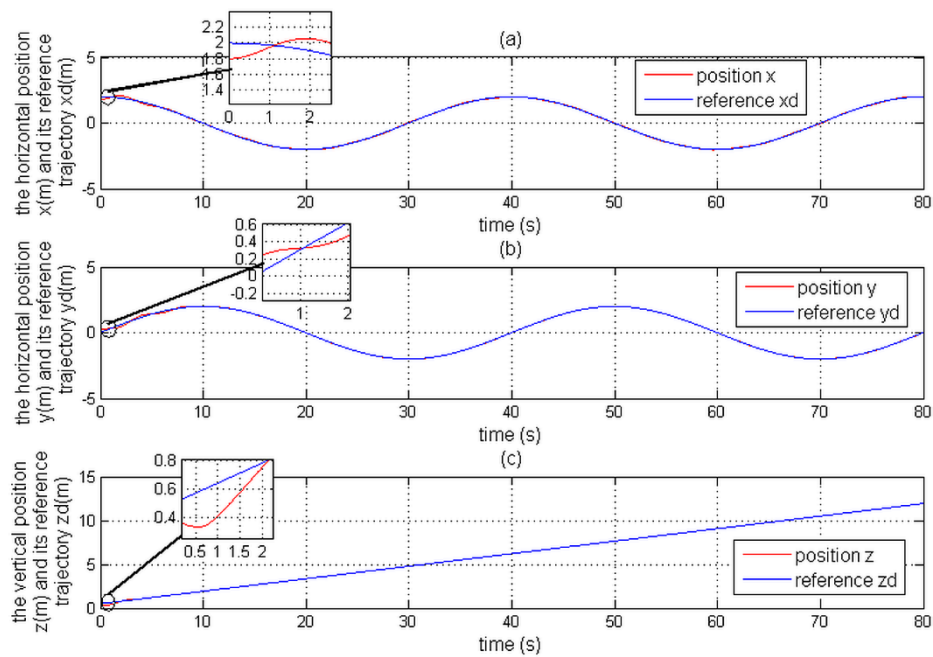


Figure 20. The position tracking evolution obtained by the CMTC: (a) x -position (m) and its reference trajectory x_d (m); (b) y -position (m) and its reference trajectory y_d (m); (c) z -position (m) and its reference trajectory z_d (m).

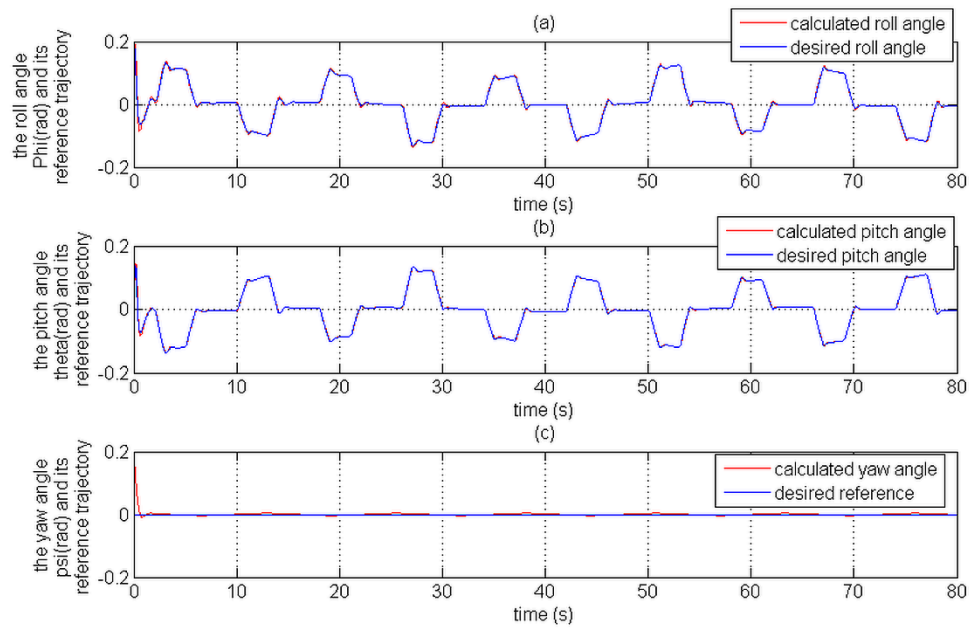


Figure 21. The attitude tracking evolution obtained by the PTCM: (a) roll angle evolution $\phi(rad)$ and its reference trajectory $\phi_d(rad)$; (b) pitch angle evolution $\theta(rad)$ and its reference trajectory $\theta_d(rad)$; (c) yaw angle evolution $\psi(rad)$ and its reference trajectory $\psi_d(rad)$.

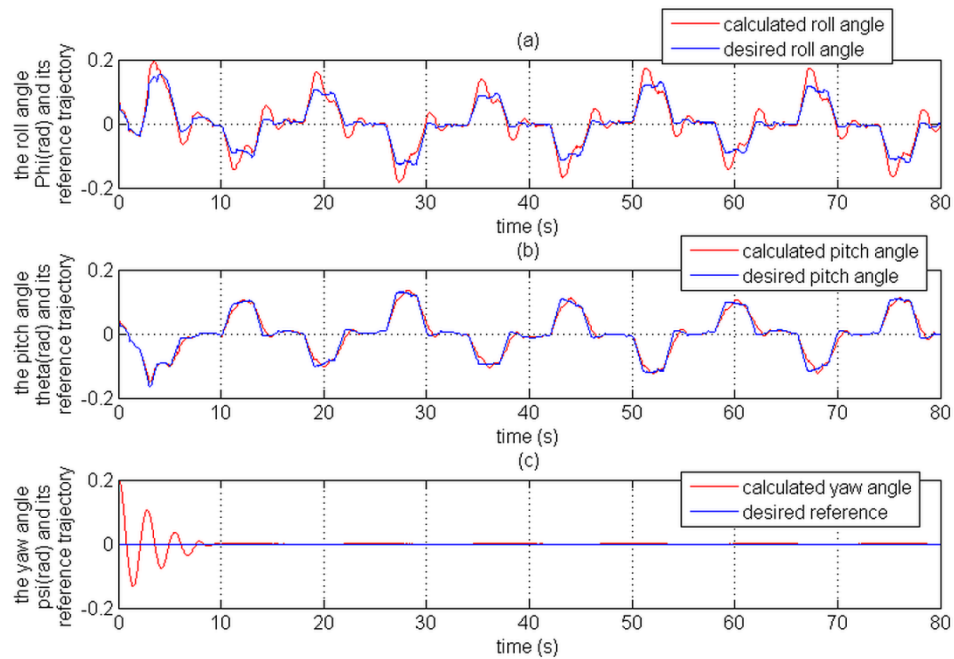


Figure 22. The attitude tracking evolution obtained by the CMTC: (a) roll angle evolution $\phi(rad)$ and its reference trajectory $\phi_d(rad)$; (b) pitch angle evolution $\theta(rad)$ and its reference trajectory $\theta_d(rad)$; (c) yaw angle evolution $\psi(rad)$ and its reference trajectory $\psi_d(rad)$.

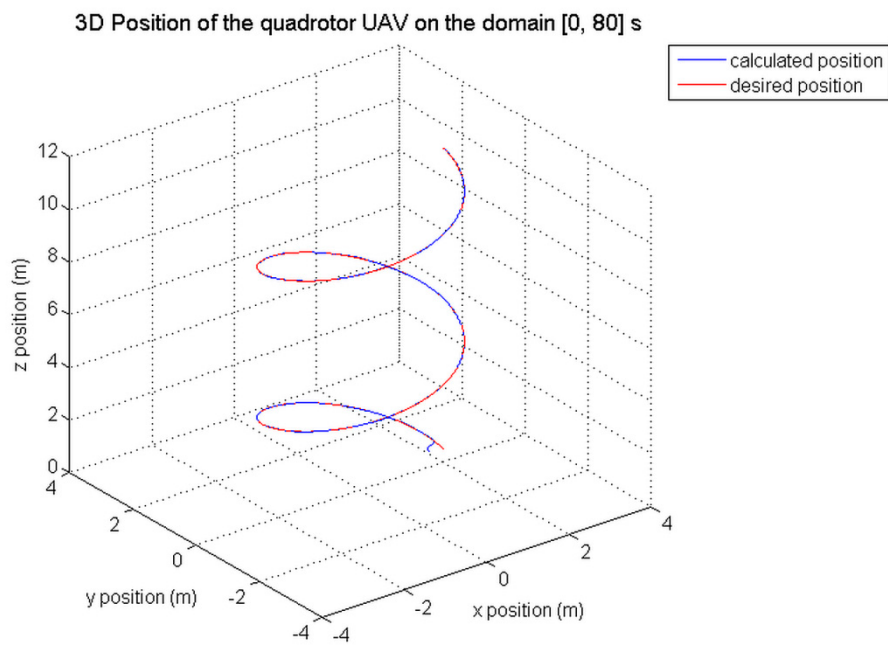


Figure 23. The 3D position of the quadrotor UAV obtained by the PTCM in the time interval $[0, 80]$ s.

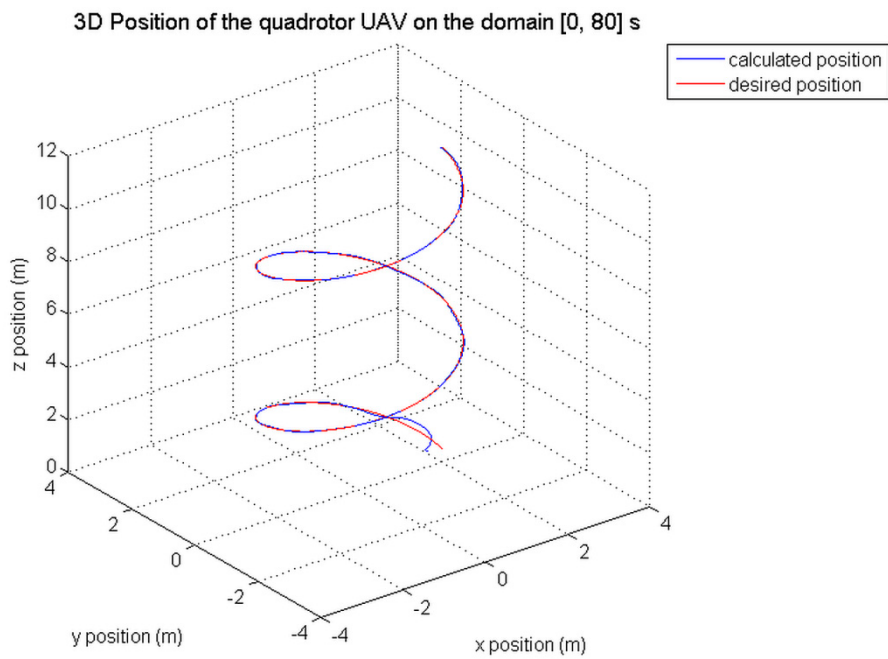


Figure 24. The 3D position of the quadrotor UAV obtained by the CMTC in the time interval $[0, 80]$ s.

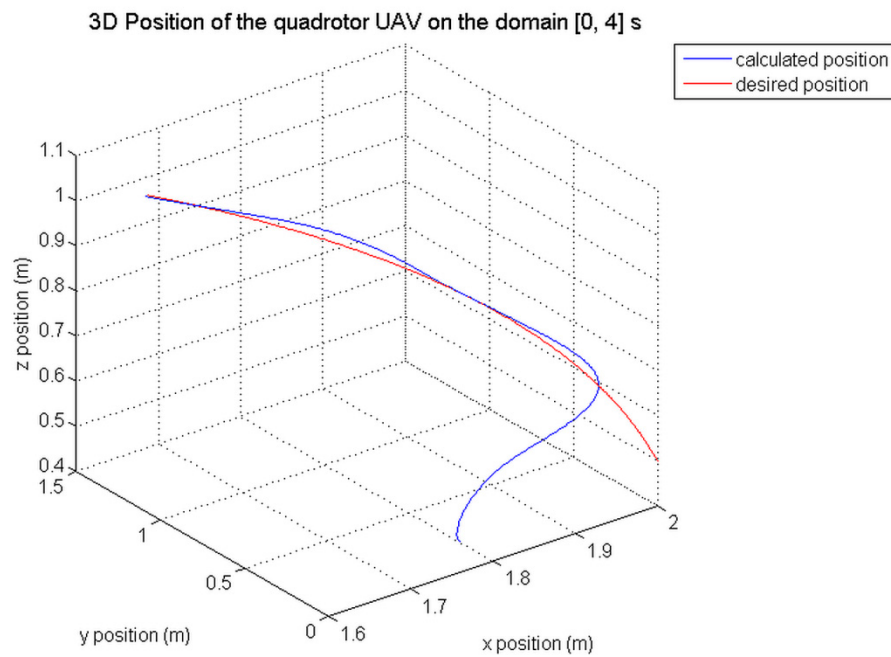


Figure 25. The 3D position of the quadrotor UAV obtained by the PTCM in the time interval [0, 4] s.

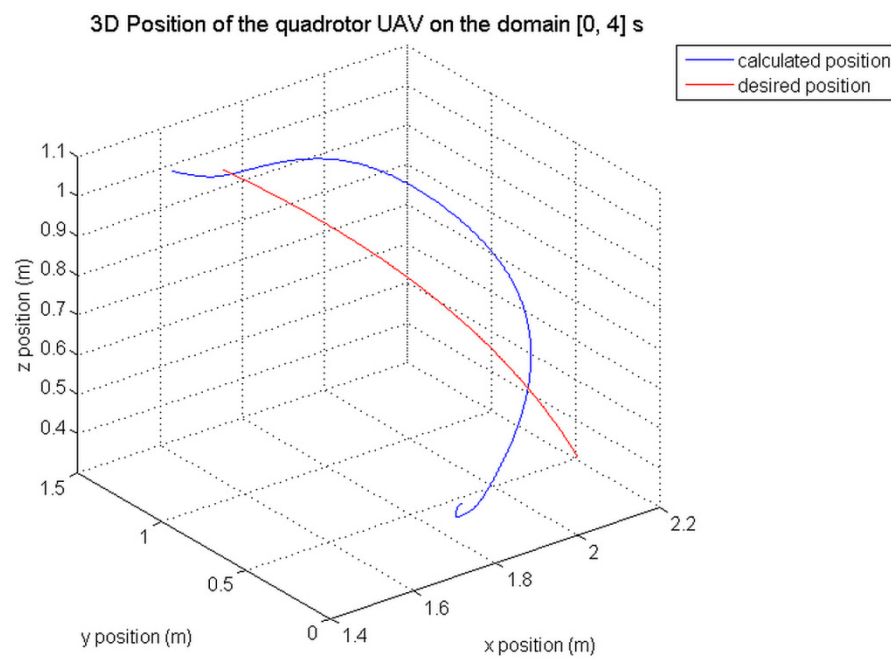


Figure 26. The 3D position of the quadrotor UAV obtained by the CMTC in the time interval [0, 4] s.

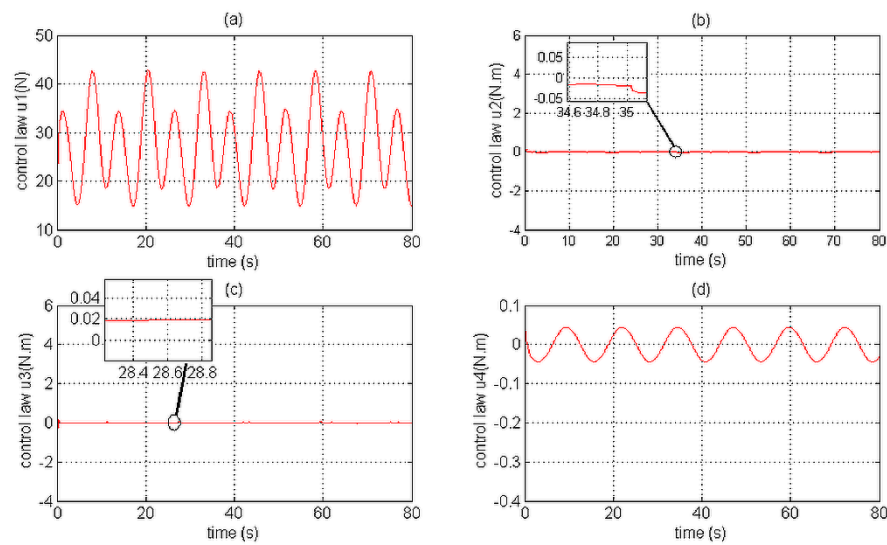


Figure 27. The applied control laws of the PTCM: (a) control law u_1 (N); (b) control law u_2 (N·m); (c) control law u_3 (N·m); (d) control law u_4 (N·m).

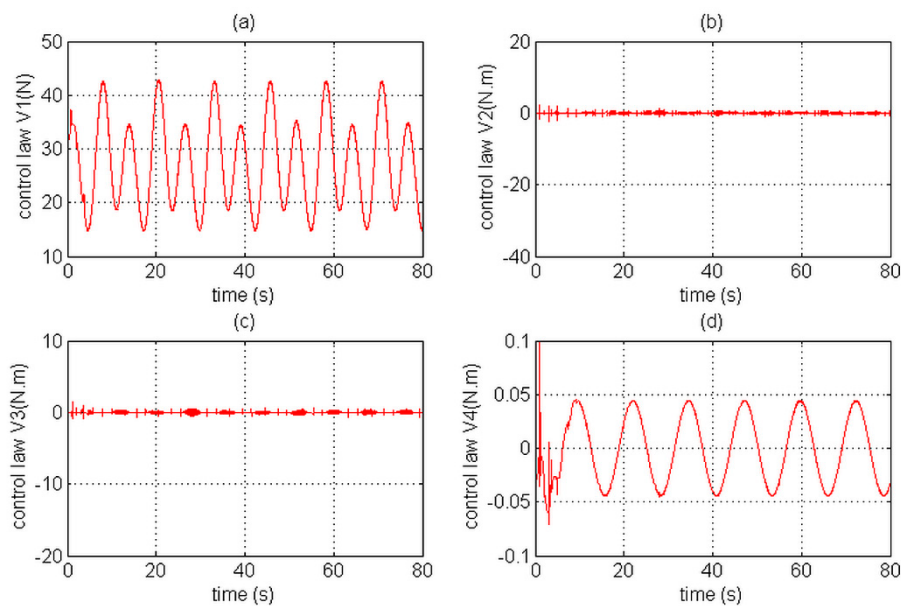


Figure 28. The applied control laws of the CMTC: (a) control law V_1 (N); (b) control law V_2 (N·m); (c) control law V_3 (N·m); (d) control law V_4 (N·m).

In comparison with the CMTC, the PTCM shows the best tracking control performance. This superiority of the PTCM in ensuring the desired objective of control despite unknown dynamics, parameter variations, unknown disturbances, and unknown physical parameters of the studied quadrotor is due to both of its features, namely (1) optimal estimation of unknown dynamics, and (2) a great efficiency in rejecting all disturbances that influence the system robustness. Also, we noticed that the control laws of the PTCM are smooth and do not present any variations which is not the case for the CMTC. Thus, the undesired chattering phenomenon is avoided, and the tracking accuracy is preserved.

6. Conclusions

In this study, we developed a robust full tracking control design method for quadrotor UAVs with unknown dynamics and unknown physical parameters that are subject to unknown and unpredictable

disturbances. In order to efficaciously estimate the unknown functions, seven IT2-AFSs and five adaptive systems were designed. Then, based on IT2-AFSs, an optimal IT2-AFRSMCL was added to the global control law in order to deal with the approximation errors and unknown and unpredictable disturbances that influence the quadrotor dynamics while simultaneously avoiding the chattering phenomenon. The underactuated problem of the quadrotor UAVs was resolved by introducing two virtual control inputs to the control system. A mathematical analysis showed that the proposed algorithm of control is stable in the sense of Lyapunov and can establish asymptotic convergence of the system state trajectories to desired references. The obtained results confirmed the mathematical analysis, ensuring the predetermined objective of control.

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