

Article

Study on Longitudinal Dynamic Stability of a Swift-Inspired Idealized Model Considering Body Periodic Vibrations

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Abstract

This study focuses on the longitudinal dynamic stability of swifts in cruising forward flight, which is critical for their high maneuverability but remains insufficiently investigated. Understanding longitudinal dynamic stability is the essential prerequisite for revealing the physical mechanism underlying their maneuverability: it is the dynamic stability characteristics that determine how the flight state responds to disturbances and control inputs, thereby laying a foundation for subsequent flight control during agile maneuvers. Conventional studies mostly adopt steady or quasi-steady assumptions, which cannot accurately reflect the influence of periodic body vibration. This study combines CFD numerical simulation and dynamic modeling to systematically analyze the unsteady dynamic stability of swifts. A bio-inspired dynamic model is established using the BE3357B airfoil with a 5° sweep angle, and the flapping-wing motion is decomposed into three degrees of freedom: sweeping, pitching, and flapping. Numerical reliability is assessed through grid independence and time-step independence verification. Aerodynamic force and moment trimming are performed on fixed-DOF and free-DOF models, where the latter considers coupled heaving–pitching motion and adjusted trim parameters. Stability analysis is conducted using three aerodynamic derivative methods: fixed velocity, forced oscillation, and Floquet. By solving small perturbation equations, eigenvalues and eigenmodes are obtained. All three methods identify two stable modes: a short-period mode with damping coefficient 0.1236–0.1870 and oscillation period 0.1121 s–0.1380 s, and a long-period mode with damping coefficient 0.2456–0.6203 and damping half-life 3.5803 s–4.8890 s, verifying stability under periodic vibration and unsteady aerodynamic coupling. Flow field results show clear distinct dynamic pressure and drag fluctuation characteristics between the downstroke and the upstroke. The unsteady stability framework provides a theoretical reference for analyzing the longitudinal stability of biomimetic flapping-wing aircraft and offers useful insight for future bird-inspired flight dynamics studies.



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Keywords: computational fluid dynamics (CFD); dynamic modeling; forced oscillation method; Floquet method; longitudinal dynamic stability analysis

1. Introduction

The flight of swifts is characterized by strongly unsteady aerodynamic force generation associated with periodic wingbeat-induced vortex dynamics [1]; existing studies have primarily focused on their gliding state, with relatively few studies on their flapping forward flight condition. In recent years, although some studies have explored the forward flight dynamic stability of insects (e.g., bumblebees) using the CFD method [2], research on

the longitudinal dynamic stability of birds, especially swifts, during cruising forward flight remains insufficient.

In addition, the swift exhibits excellent flight performance, with various species of swifts capable of reaching flight speeds exceeding 140 body lengths per second; moreover, small birds such as the barn swallow can achieve roll rates above 5000 degrees per second, far exceeding the approximately 720 degrees per second roll rate of the highly aerobatic A-4 “Skyhawk” aircraft [3]. Wind-tunnel experiments have been successfully used to investigate the unsteady aerodynamic characteristics of bird-inspired flapping wings, including vortex evolution, aerodynamic force generation, and the effects of wing morphology or kinematic parameters [4–8]. However, for high-speed and highly maneuverable bird-inspired flight, wind-tunnel tests may still face limitations in simultaneously reproducing complex wing–body coupling, prescribed and free-body motions, three-dimensional unsteady flow structures, and systematic variations of multiple flight parameters. Therefore, numerical simulation is introduced in this study as a complementary approach to resolve the unsteady flow field and aerodynamic loads of the swift-inspired model under specified flight conditions shown in Figure 1, providing further guidance for the design of high-maneuverability flapping-wing aircraft. It should be noted that this study focuses only on the longitudinal dynamic stability of the swift in stable level cruise flight and does not cover maneuvering flight conditions. However, understanding the stability characteristics during level cruise flight is the basis for subsequent research on more complex maneuvers (such as turns and rapid deceleration), which are crucial for fully grasping the flight capabilities of the swift.



Figure 1. Flight postures of swifts during constant-speed forward flight.

The emergence and rapid development of CFD technology have made it possible to obtain key aerodynamic parameters of swifts during flapping forward flight. Studying the flight stability of birds is an important aspect of research on avian flight performance, and such research can provide useful references for aircraft stability. Stability analysis is usually divided into two categories: first, static stability analysis, which focuses on the stability characteristics of an aircraft in an equilibrium state; second, dynamic stability analysis, which studies the motion response characteristics of an aircraft after being subjected to external disturbances [9].

Existing research on stability analysis mainly focuses on two aspects. The first is the inherent stability characteristics of organisms in an equilibrium state, namely their static stability. For example, G. K. Taylor and A. L. R. Thomas et al. proposed three mechanisms for birds to maintain static stability [10]:

1. Velocity stability: when the flight speed increases, the lift generated by the bird’s aerodynamic center behind its center of gravity naturally forms a nose-down moment, which suppresses further speed increases.
2. Angle-of-attack stability: when the body pitch angle increases, the aerodynamic forces generated by the tail form a restoring nose-down moment, which returns the flight attitude to equilibrium [11].

3. Frequency-coupled dynamic stability: Birds can actively damp and suppress the body's pitching vibrations by precisely controlling the phase of forces and moments from wing flapping, and this is a unique active stability mechanism for flapping-wing flight. C. Harvey and V. B. Baliga et al. found that birds can switch between stable and unstable states by changing the shape of their wings [12].

The second aspect is exploring the effects of biological wing flexibility and dynamic modeling on flight stability. For example, Mardanpour et al. explored how wing flexion improves flight stability by analyzing the uniform flow of stresses across the wing surface [13], and further conducted experimental and theoretical research to reasonably analyze and simplify the longitudinal system matrix, exploring the longitudinal stability mechanism of desert locusts during flight [14]. Mao Sun et al. studied the role of honeybees in achieving flight stability by adjusting wing flapping frequency, flapping angle and flexible wing deformation during hovering [15]. Mistick et al. used high-speed 3D motion capture technology to quantitatively reveal the relationship between the flexibility of bumblebee flexible wings and flight stability, and established a quantitative correlation between the stiffness distribution of flapping wings and dynamic response [16]. Bierling and Patil, to consider the periodic effects and nonlinear changes in aerodynamic forces caused by the flapping process, further explored the stability of time-varying systems based on Floquet theory, on the basis of investigating the stability of linear time-averaged systems [17]. On this foundation, Mao Sun et al. applied these two methods to study two hovering states and finally concluded that when the wing mass is much smaller than the fuselage mass, the stability conclusions obtained by the two methods for time-averaged and time-varying systems are similar [18]. In addition, a review study summarized the application of various methods—including Floquet theory—in the stability analysis of flapping-wing aircraft, which provides a theoretical basis for the use of the Floquet method in this study [19]. However, previous studies have provided valuable insights into flapping-wing aerodynamics and hydrodynamics, including vortex-body interactions, tandem flapping-plate locomotion, flexible propulsion, and morphing-wing aerodynamic performance [20–23]. However, these works mainly focused on force generation, wake structures, propulsive efficiency, or prescribed kinematic optimization, while the dynamic stability of bird-like flapping-wing systems has received comparatively less attention. For flapping-wing flight, the aerodynamic forces and moments vary periodically within each wingbeat, which directly affects the trim state and longitudinal stability modes. Therefore, the present study uses CFD-resolved unsteady aerodynamic loads, combined with cycle-averaged processing, forced-oscillation analysis, trim calculation, and Floquet theory, to investigate the longitudinal dynamic stability of a swift-like flapping-wing model.

Therefore, this study aims to investigate the longitudinal dynamic stability of the idealized swift model through Computational Fluid Dynamics (CFD) numerical simulations. Previous CFD-based dynamic stability analyses have mainly focused on fixed-wing aircraft, whereas flapping-wing models generate strongly periodic aerodynamic forces and moments within each flapping cycle, making it difficult to evaluate their stability using conventional steady or periodic averaging treatments alone. To address this issue, this study establishes a unified CFD-based longitudinal stability analysis framework for the present low-frequency bird-type flapping model. First, the aerodynamic forces and moments obtained from CFD calculations within one flapping cycle are processed using a periodic averaging method to obtain the averaged aerodynamic characteristics. On this basis, the forced oscillation method commonly used in wind tunnel dynamic derivative measurements is introduced to obtain the derivatives related to pitch angular velocity. Finally, the body degrees of freedom are further released, and the Floquet method is applied to analyze the periodic time-varying stability characteristics of the system. The main

novelty of this framework lies in applying these three stability analysis methods to the same idealized swift model and the same trim condition, so that the influence of aerodynamic derivative treatment and periodic time-varying aerodynamic effects on the predicted longitudinal modes can be directly compared.

This study investigates the longitudinal dynamic stability of swifts by obtaining the aerodynamic forces and aerodynamic moments within one flapping cycle using the CFD method. The scientific contribution of this work is mainly reflected in the systematic comparison of the fixed velocity method, the forced oscillation method, and the Floquet method for a low-frequency bird-type flapping model. This comparison clarifies that different stability analysis methods lead to consistent qualitative stability conclusions, but produce noticeable differences in the predicted damping and recovery characteristics, especially for the long-period mode. The results further indicate that the long-period mode is more sensitive to the inclusion of periodic flapping dynamics than the short-period mode, which provides a useful reference for the stability analysis and control design of biomimetic flapping-wing aircraft. The structure of this study is as follows:

1. Based on the constructed swift-inspired constant-speed forward flight model, calculate the trim conditions of two swift-inspired dynamic models with fixed body degrees of freedom and free body degrees of freedom;
2. On the basis of trim with fixed body degrees of freedom, calculate different operating conditions near the trim state and the time-averaged aerodynamic derivatives, and use two different methods—the fixed velocity method and the forced oscillation method—to calculate the aerodynamic moment derivatives;
3. On the basis of trim with free body degrees of freedom, solve the nonlinear small perturbation equations using the Floquet method;
4. Calculate the relevant eigenvalues and eigenmodes obtained, explore the longitudinal dynamic stability of swifts during forward flight, and analyze the effects of two different dynamic modeling methods and three different dynamic derivative calculation methods on stability analysis.

2. Numerical Methods

2.1. Modeling

To simulate the geometric characteristics of swift wings in nature more accurately, this study comprehensively selects representative geometric parameters of swift wings. First, based on the research findings of Bokhorst et al. [24], the BE3357B airfoil is adopted as the baseline airfoil for the average airfoil profile of swifts, which is designed for low-Reynolds-number flows ($\approx 10^4$) and small angles of attack ($\approx 0\text{--}10^\circ$), matching the swift's glide regime and the present trim condition ($Re \approx 2.3 \times 10^4$, $\alpha = 3.5^\circ$). By delaying flow separation and maintaining high lift with low drag, it fits the swift's efficient cruise. Second, based on the measurement data from Lentink et al. [25], a 5° sweep angle is selected as the planform characteristic of the wing (as shown in Figure 2), and a bio-inspired geometric model of the flapping wing is then established (as shown in Figure 2).

Based on the research of Sun and Xiong [26], this study adopts the rigid body approximation hypothesis and decomposes the motion of the flapping wing into three degrees-of-freedom motions: pitch angle (δ), sweep angle (β) and flapping angle (α). Considering the research of Warrick, D.R. et al. [27] on key flight data (e.g., wing flapping frequency and flapping amplitude) of bank swifts during flight, as well as the kinematic data of swifts measured by Tobalske, B.W. et al. [28] within the speed range of 3 m/s to 15 m/s, the *flapping* (δ), *sweep* (β) and *pitching* (α) amplitudes of the swift's wing are defined as 55° ,

22.5° and 18° respectively. Thus, taking the left wing of the flapping wing as an example, the specific motion equations are expressed as follows:

$$\text{sweep}(\text{angle}) = 22.5^\circ - 22.5^\circ \cdot \cos(2\pi ft) \quad (1)$$

$$\text{sweep}(\text{angle})_{\text{dot}} = 22.5^\circ \cdot 2\pi f \cdot \sin(2\pi ft) \quad (2)$$

$$\text{flapping}(\text{angle}) = 55^\circ \cdot \sin(2\pi ft) \quad (3)$$

$$\text{flapping}(\text{angle})_{\text{dot}} = 55^\circ \cdot 2\pi f \cdot \cos(2\pi ft) \quad (4)$$

$$\text{pitching}(\text{angle}) = -18^\circ \cdot \cos(2\pi ft) \quad (5)$$

$$\text{pitching}(\text{angle})_{\text{dot}} = 18^\circ \cdot 2\pi f \cdot \sin(2\pi ft) \quad (6)$$

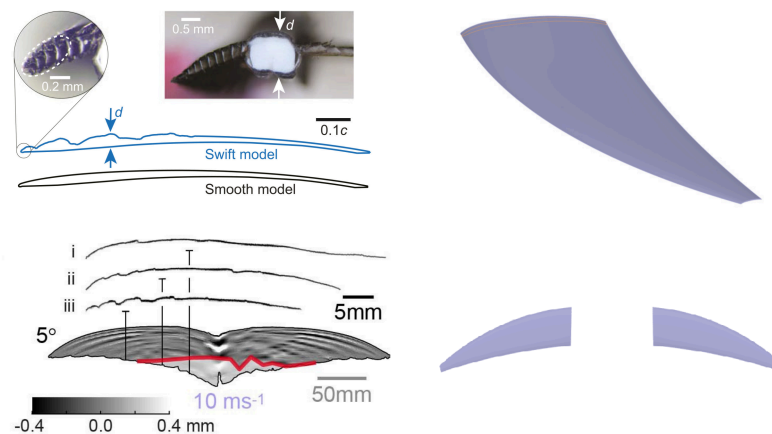


Figure 2. Schematic of swift airfoil and wing planform [25].

Based on this, a bionic swift swept-wing model was established based on existing research on avian biomechanics, as shown in Figure 3:



Figure 3. Swift swept-wing model.

It must be acknowledged that, although the present model is constructed with reference to the actual morphology of swifts, certain specialized structural features, such as the three-segment coupling motion of the wing involving flapping, sweeping, and pitching [29], have been simplified. Additionally, the wing is assumed to be rigid. These simplifications inevitably lead to discrepancies between the simulated motion and the actual flight kinematics of real swifts. Therefore, the present study can only be regarded as an investigation based on an idealized swift-inspired model, and the results are not fully representative of the true flight of swifts.

2.2. Equations of Motion

Since this article focuses on the cruise flight state of the swift in a steady level flight, at low speed and at low altitude, and does not consider the high-speed maneuvering flight of the swift, the flight under the influence of strong gusts, or the flight during vertical climbing or rapid diving, therefore, when the flight posture of the swift model and the surrounding environment have no significant changes, the motion equations can be decoupled using the small perturbation assumption, and the small perturbation equations can be simplified by setting the climb angle to 0 and ignoring the influence of height.

Based on the longitudinal small perturbation motion equations proposed by Etkin, B. et al. in his classic flight dynamics textbook, Dynamics of Flight: Stability and Control [30], and when neglecting the effects of external forces and moments induced by altitude variations in the disturbance motion (i.e., $X_H = Z_H = \overline{M}_H = 0$), the altitude variation equation can be separated and solved independently at this time, and the matrix form of the longitudinal small-disturbance equations can be written as follows:

$$\frac{d}{dt} \begin{bmatrix} \Delta V \\ \Delta \alpha \\ \Delta q \\ \Delta \theta \end{bmatrix} = \begin{bmatrix} X_V & X_\alpha + g \cos \gamma_* & 0 & -g \\ -Z_V & -Z_\alpha + g \cos \gamma_*/V_* & 1 & -g \sin \gamma_*/V_* \\ \overline{M}_V - \overline{M}_{\dot{\alpha}} Z_V & \overline{M}_\alpha - \overline{M}_{\dot{\alpha}} (Z_\alpha - g \sin \gamma_*/V_*) & \overline{M}_q + \overline{M}_{\dot{\alpha}} & -\overline{M}_{\dot{\alpha}} g \sin \gamma_*/V_* \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta \alpha \\ \Delta q \\ \Delta \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} & X_{\delta_p} \\ -Z_{\delta_e} & -Z_{\delta_p} \\ \overline{M}_{\delta_e} - \overline{M}_{\dot{\alpha}} Z_{\delta_e} & \overline{M}_{\delta_p} - \overline{M}_{\dot{\alpha}} Z_{\delta_p} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta_e \\ \Delta \delta_p \end{bmatrix} = Ax + Bu \quad (7)$$

When the model is assumed to undergo horizontal forward flight motion $\gamma = 0^\circ$, the equations are simplified as follows:

$$\frac{d}{dt} \begin{bmatrix} \Delta V \\ \Delta \alpha \\ \Delta q \\ \Delta \theta \end{bmatrix} = \begin{bmatrix} X_V & X_\alpha + g & 0 & -g \\ -Z_V & -Z_\alpha & 1 & 0 \\ \overline{M}_V - \overline{M}_{\dot{\alpha}} Z_V & \overline{M}_\alpha - \overline{M}_{\dot{\alpha}} Z_\alpha & \overline{M}_q + \overline{M}_{\dot{\alpha}} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta \alpha \\ \Delta q \\ \Delta \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} & X_{\delta_p} \\ -Z_{\delta_e} & -Z_{\delta_p} \\ \overline{M}_{\delta_e} - \overline{M}_{\dot{\alpha}} Z_{\delta_e} & \overline{M}_{\delta_p} - \overline{M}_{\dot{\alpha}} Z_{\delta_p} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta_e \\ \Delta \delta_p \end{bmatrix} = Ax + Bu \quad (8)$$

where $x = [\Delta V \ \Delta \alpha \ \Delta q \ \Delta \theta]$ is the state vector, and the coefficient matrix is the state matrix. The parameters $X_V, X_\alpha, Z_V, Z_\alpha, \overline{M}_V, \overline{M}_\alpha, \overline{M}_{\dot{\alpha}}$ in the state matrix are the aerodynamic derivatives of the total lift force, total drag force and total pitching moment with respect to the angle of attack, inflow velocity and pitching angular velocity, respectively. The eigenvalues obtained by solving the state matrix correspond to each eigenmode reflecting the longitudinal dynamic stability of the airframe and contain the motion laws of the actual physical system (damping ratio, period, half-decay period, doubling period, phase difference, etc.). Among them, the aerodynamic derivatives related to the phugoid mode are mainly $X_V, X_\alpha, Z_V, \overline{M}_V$, referring to the rate of change in the total force along the X-axis with velocity disturbance and reflecting the recovery rate of the velocity after being disturbed; X_α , the velocity damping of the model, refers to the rate of change in the total force along the X-axis with the disturbance of the model's cruise angle of attack, corresponding to the recovery speed of the angle of attack after being disturbed in the phugoid mode and reflecting the energy exchange efficiency of the system matrix; Z_V corresponds to the rate of change in lift caused by velocity variation in the phugoid mode after

disturbance, reflecting the recovery effect of the model after altitude disturbance, i.e., the altitude damping. Z_α , M_α , M_q are the aerodynamic derivatives related to the short-period mode: Z_α corresponds to the rate of lift change caused by angle of attack disturbance in the short-period mode, which is the most direct reflection of the lift curve slope in the system matrix and reflects the stiffness of the normal force; M_α reflects the longitudinal static stability margin, which is generally a negative value, and its absolute value reflects the magnitude of the restoring moment generated by the system after an angle of attack disturbance; and M_q corresponds to the pitching damping of the short-period mode, reflecting the oscillation attenuation effect of the model's short-period mode, which is also generally a negative value, and its absolute value reflects the rate of oscillation decay of the short-period mode, corresponding to the damping ratio in aircraft flying qualities. M_V reflects the coupling effect between the phugoid mode and the short-period mode, corresponding to the static stability characteristics when the aerodynamic center shifts abruptly during transonic flight. $u = [\delta_e \ \delta_p]^T$ is the control vector; δ_e and δ_p are the deflection angle of the aerodynamic control surface (the elevator or all-moving horizontal tail for the longitudinal direction of a conventional aircraft) and the throttle lever position, respectively, and the coefficient matrix is the control matrix. The parameters X_{δ_e} , X_{δ_p} , Z_{δ_e} , Z_{δ_p} , M_{δ_e} , M_{δ_p} , $M_{\dot{\alpha}}$ in the control matrix are the aerodynamic derivatives of the total lift force, total drag force and total pitching moment with respect to the deflection angle of the aerodynamic control surface and the throttle lever position, which reflect the longitudinal controllability of the airframe.

The correlation equations for expressing the longitudinal dynamic derivatives by aerodynamic derivatives are as follows [30]:

$$\begin{aligned} X_V &= \frac{T_V \cos(\alpha_* + \varphi_T)}{m} - \frac{(C_{DV} + 2C_D^*)q \cdot S}{mV_*} & X_\alpha &= \frac{-T \sin(\alpha_* + \varphi_T) - C_{D\alpha} \cdot q \cdot S}{m} \\ Z_V &= \frac{T_V \sin(\alpha_* + \varphi)}{mV_*} + \frac{(C_{LV} + 2C_L^*)q \cdot S}{mV_*^2} & Z_\alpha &= \frac{(C_D^* + C_{L\alpha})q \cdot S}{mV_*} \\ \overline{M_V} &= \frac{(C_{mV} + 2C_{m*})q \cdot S c}{V_* I_y} & \overline{M_\alpha} &= C_{m\alpha} \frac{q \cdot S c}{I_y} \\ \overline{M_q} &= C_{mq} \left(\frac{c}{2V_*} \right) \frac{q \cdot S c}{I_y} & \overline{M_{\dot{\alpha}}} &= C_{m\dot{\alpha}} \left(\frac{c}{2V_*} \right) \frac{q \cdot S c}{I_y} \end{aligned} \quad (9)$$

2.3. Flow Field Solution Method

The Computational Fluid Dynamics (CFD) method is based on the governing equations of fluid mechanics, such as the Navier–Stokes equations, while the Reynolds-Averaged Navier–Stokes (RANS) equations are obtained from them through Reynolds averaging and represent one commonly used formulation in CFD simulations. Since the flight speed of birds in nature is less than 0.3 Mach, the flow is considered incompressible under such incoming flow conditions. Therefore, the unsteady incompressible Navier–Stokes equations are adopted as the governing equations in this study.

Conservation of Mass Equation:

$$\nabla \cdot \vec{V} = 0 \quad (10)$$

Momentum Equation:

$$\frac{\partial}{\partial t}(\rho \vec{v}) + \nabla \cdot (\rho \vec{v} \vec{v}) = -\nabla \cdot p + \nabla \cdot (\bar{\tau}) \quad (11)$$

where

ρ : density of the fluid; $\vec{v}$: velocity components; p : static pressure; and $\bar{\tau}$: stress tensor.

The stress tensor is expressed as follows:

$$\bar{\tau} = \mu[(\nabla \cdot \vec{v} + \nabla \cdot \vec{v}^T) - \frac{2}{3}\nabla \cdot \vec{v} I] \quad (12)$$

where

μ : dynamic viscosity coefficient; I : unit tensor.

In addition, based on the average aerodynamic chord length (0.04 m) and the cruising speed of 8.5 m/s, the Reynolds number is approximately $Re = 2.3 \times 10^4$, indicating that it is in the low Reynolds number region, where transition and separation may occur [31]. Although transition models such as γ - $Re\theta$ SST can predict the onset time of transition more accurately, they require reliable free-stream turbulence intensity, which is difficult to determine for flapping-wing motion [31–33].

Previous studies on low Reynolds number flapping flows, including the simulation of hummingbird wings by Lee et al. [34] and the analysis of bird wing shapes by Omar et al. [35], have shown that although the SST k- ω model assumes complete turbulence, it can still provide engineering-level accuracy [34,35]. Since this study focuses on the time-averaged longitudinal dynamic stability over multiple flapping cycles rather than the transition details at the flapping scale, the dominant flow physical processes are mainly controlled by strong non-stationary periodic disturbances. Moreover, the SST k- ω model has been fully validated in terms of separated flow and pressure gradient boundary layers [36], which are key features of the downstroke/upstroke transition process. Therefore, due to the high computational cost of high-fidelity methods (such as LES or separation vortex simulation [32]), they are not applicable for the large-scale parametric analysis required here. In light of this, this paper selects the SST k- ω model as the turbulence model for the calculations.

The transport equations of the SST turbulence model are as follows:

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j}(\Gamma_k \frac{\partial k}{\partial x_j}) + G_k - Y_k + S_k \quad (13)$$

$$\frac{\partial}{\partial t}(\rho \omega) + \frac{\partial}{\partial x_j}(\rho \omega u_j) = \frac{\partial}{\partial x_j}(\Gamma_\omega \frac{\partial \omega}{\partial x_j}) + G_\omega - Y_\omega + S_\omega \quad (14)$$

where

G_k : turbulent kinetic energy production term induced by mean velocity gradient;

G_ω : specific dissipation rate production term;

Γ_k : effective diffusion of turbulent kinetic energy;

Γ_ω : effective diffusion of specific dissipation rate;

Y_k : turbulent kinetic energy dissipation term;

Y_ω : specific dissipation rate dissipation term;

S_k : user-defined source term of turbulent kinetic energy;

S_ω : user-defined source term of specific dissipation rate;

D_ω : cross diffusion term.

2.4. Example Calculation and Independence Verification

2.4.1. Example Calculation

To validate the feasibility and accuracy of the present numerical method, a CFD simulation is conducted to reproduce the flapping-wing experiment performed by Heathcote et al. [37] in a water tunnel. The experimental model consists of a rectangular wing with a NACA0012 airfoil profile. The Reynolds number based on the chord length

is $Re = 3 \times 10^4$, and the wing undergoes heaving motion with the kinematic prescribed as follows. The experimental model and its geometric parameters are summarized in Table 1.

$$H = 0.175 \cdot c \cdot \cos(\omega t) \quad (15)$$

Table 1. The geometric and motion parameters of the experimental model by Heathcote et al. [37].

Parameters	String Length (c)	Heaving Amplitude (H)	Heaving Angular Frequency (ω)	Inlet Velocity (u)	Density (ρ)
Parameter value	0.1 m	0.175 m	10.92 rad/s	0.303 rad/s	1000 kg/m ³

The computational grid adopts the overset grid method, with C-type structured grids used for the component grids and H-type structured grids for the background grid. The first cell height is set to $1.8 \times 10^{-4}c$ of the chord length, ensuring $y^+ < 1$. As shown in Figure 4, although the computed thrust coefficients of the flapping wing exhibit some deviations from the experimental data, the two thrust peaks are captured more effectively compared with the numerical results of Liu [38] and Lang [39], showing better agreement with the experimental observations.

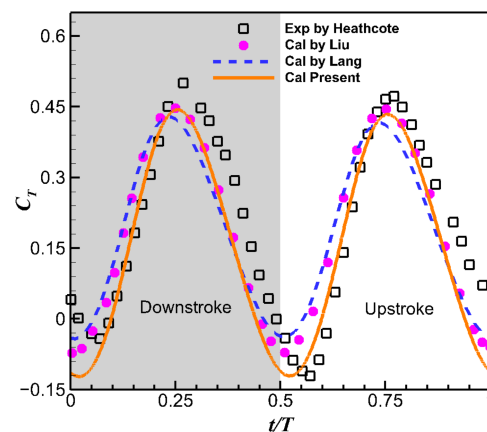


Figure 4. Comparison of the obtained computational results with literature data [37–39].

2.4.2. Independence Verification

Since the flapping wings of swifts undergo a large-amplitude motion during forward flight, the mesh size and time step length in CFD numerical simulations exert a significant influence on the information exchange of overset meshes, which in turn affects the accuracy and reliability of CFD computational results. This study adopts three sets of meshes for grid independence verification, with mesh quantities of 2.09 million (Coarse), 3.20 million (Medium), and 4.70 million (Fine). Based on this, time step $\overline{C_L}$ independence verification is conducted with 100 (Coarse), 200 (Medium), and 400 $|\Delta \overline{C_L}|$ (Fine) time steps within one flapping cycle, corresponding to time steps of 0.007134 s, 0.0035715 s, and 0.00178575 s respectively.

In the CFD simulations, the background grid is generated with a cell size 33 times that of the component grid. Mesh refinement is applied in the overlapping region between the component and background grids to ensure accurate information exchange within the overset mesh system. For boundary conditions, a Velocity Inlet is specified at the far-field inlet with a freestream velocity of 8.5 m/s, while a Pressure Outlet is applied at the far-field outlet. The remaining far-field boundaries are set as wall conditions. At the overlapping interfaces where information exchange occurs between the component and background grids, the boundary condition is also set as a wall. The convergence criterion is defined as

follows: the computation is considered converged when the error in the cycle-averaged aerodynamic forces and moments between two adjacent flapping cycles falls below 1%. Under this criterion, the obtained cycle-averaged aerodynamic forces and moments are deemed accurate.

The comparison results (Figure 5 and Table 2) show that for different numbers of grids, when the number of grids increases in multiples, the error of the average lift coefficient remains within the range of 1–4%; the influence of the time step on the lift coefficient is even more negligible. As shown in Table 3, The average lift coefficient remains almost unchanged with the increase or decrease in the time step, so it can be considered that the effects of different time steps and grid divisions on the lift coefficient can be ignored. Therefore, in order to reduce the computational cost and accelerate the calculation speed, in the subsequent calculations of this paper, 3.2 million grid units are used, and 100 calculation steps are carried out in each flapping-wing cycle.

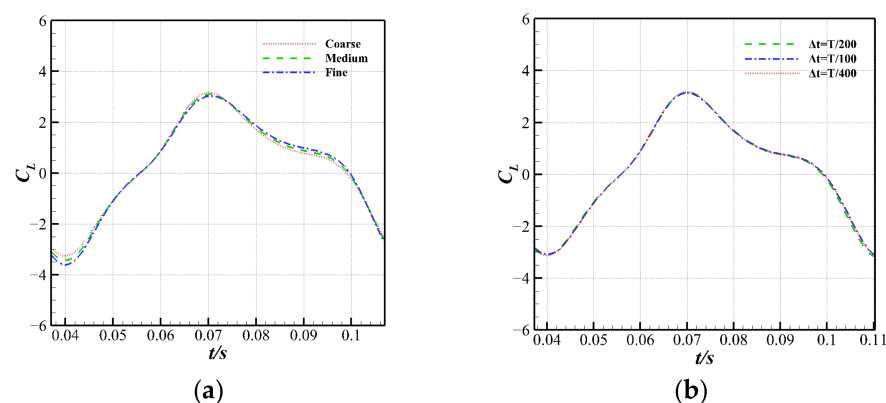


Figure 5. Grid and time step independence verification. (a) Grid independence verification. (b) Time step independence verification.

Table 2. Different grid type parameters and results.

Grid Type	Total Grid	Component Grid	Background Grid	$\overline{C_L}$	$ \Delta \overline{C_L} $
Coarse Mesh	2,110,052	1,695,813	414,239	0.23286	3.44%
Medium Mesh	3,073,547	2,440,059	633,488	0.22898	1.67%
Precision Mesh	4,595,281	3,640,093	955,188	0.22511	/

Table 3. Different time step parameters and results.

Grid Type	Time Steps	$\overline{C_L}$	$ \Delta \overline{C_L} $
Medium Mesh	$\Delta t = T/100$	0.23286	0.263%
Medium Mesh	$\Delta t = T/200$	0.23247	0.095%
Medium Mesh	$\Delta t = T/400$	0.23225	/

3. Stability Analysis of the Time-Averaged System

This chapter performs thrust–drag trim, lift–weight trim and resultant pitching moment trim on the airframe with fixed body degrees of freedom. Based on this, it calculates the aerodynamic forces corresponding to different incoming flow velocities and angles of attack near the trim point, and further computes the aerodynamic derivatives. Finally, it analyzes the longitudinal dynamic stability of the swift-inspired model.

It should be noted that in this chapter, both the trim and aerodynamic derivatives are obtained from cycle-averaged forces using the time-averaged method. This treatment attenuates the phase distribution, instantaneous peaks, and their coupling with body

perturbations, making it difficult to capture the periodic energy exchange. Consequently, the damping and attenuation of the long-period mode are notably affected.

3.1. Trim for Fixed Body Degrees of Freedom

Referring to the flapping frequency of swifts in nature [27], this chapter first keeps the fixed frequency of 14 Hz constant during the trim process, and then adopts the control variable method to explore the relationship between incoming flow velocity, angle of attack and the resultant lift and drag forces, and identify the lift-drag trim point. Based on this, it calculates the moment arm length between the center of pressure and the center of gravity, adjusts the center of gravity to align with the center of pressure, and trims the total pitching moment of the entire airframe to zero.

First, according to the research by Park et al. [40], swifts adjust their wing flapping frequency and amplitude to a relatively stable state within the cruise velocity range of 8 m/s to 12 m/s. Therefore, this study sets the cruise velocity of the swift-inspired model to 8.5 m/s and determines the model's flapping frequency as 14 Hz based on the flapping frequency of swifts in nature, with the model's weight ranging from approximately 35 g to 50 g [41]. On this premise, the effects of the incoming flow angle of attack on lift and drag forces are investigated. As shown in Figure 6, when the net drag is zero and the net lift equals the weight, the swift-inspired model achieves thrust-drag trim and lift-weight trim. The corresponding angle of attack is 3.5° , which falls within the typical cruising angle-of-attack range of swifts in nature. When the model cruises at this angle of attack, it does not change altitude due to excess or insufficient lift.

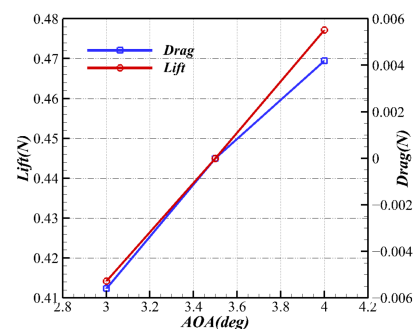


Figure 6. Trim process at flapping frequency of 14 Hz.

The pitching moment of an aircraft refers to the moment about the Y-axis of the airframe axis system generated by external forces acting on the aircraft, including the aerodynamic moment and the moment generated by the engine thrust vector that does not pass through the aircraft's center of mass. It is also known as the longitudinal moment, and its calculation formula is as follows:

$$M = F \cdot d \quad (16)$$

To achieve pitching moment trim for a flapping-wing vehicle, the center of gravity must coincide with the center of pressure. The lift force and resultant pitching moment at the trim point can be obtained through CFD numerical simulations. The distance between the center of gravity and the center of pressure can then be derived according to the above formula. As shown in Figure 7, when the center of gravity is adjusted to 99 mm from the nose based on the lift and resultant pitching moment at the trim point, the resultant pitching moment becomes zero. This indicates that, under the trimmed condition, the model exhibits no nose-up or nose-down tendency in the absence of external disturbances and is capable of maintaining level flight without changing altitude.

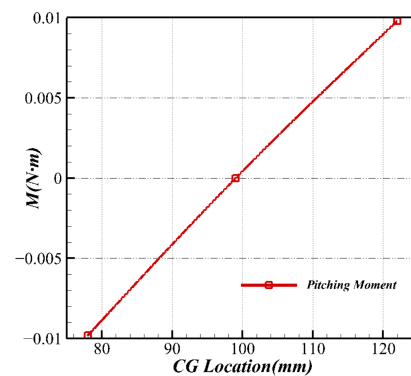


Figure 7. Variation in pitching moment with center-of-gravity position.

The center of gravity of a swift's body is usually located at the junction of the posterior end of the keel bone and the spinal column [42,43]. Its well-developed flight muscles (accounting for 8–12% of body weight) are concentratedly attached to the sternal keel bone [44]; visceral organs such as the heart and lungs are concentrated in the anterior part of the thoracic cavity [45]. In addition, swifts have a lightweight hindquarters structure: a degenerated digestive system (accounting for only 5–7% of body weight) and a short tail (accounting for 2–3% of body weight) [46]. This center of gravity configuration endows swifts with significant biomechanical advantages: the forward-shifted center of gravity forms an optimal match with the aerodynamic center of the wings [42], which not only ensures longitudinal static stability during cruise flight [47] but also meets the sharp turning maneuverability required for predation [40].

Based on this, the center of gravity position is adjusted, and the model with the adjusted center of gravity is analyzed using CATIA software, yielding an I_{yy} of $0.00015 \text{ kg}\cdot\text{m}^2$ for the model after pitching moment trim. At this point, the trim work of the swift-inspired model is basically completed. Table 4 presents the detailed parameters for the flapping-wing aircraft to achieve trim and the weight of each key part:

Table 4. Detailed Trimmed Parameters.

Trim Parameters	Trim Conditions/Swifts	Key Components	Weight/Actual Proportion
Angle of Attack (deg)	3.5	Single Wing Weight (g)	4.45 (10%)
Flapping Amplitude (deg)	$\pm 55/\pm 55$	Total Wing Weight (g)	8.9 (20%)
Incoming Flow Velocity ($\text{m}\cdot\text{s}^{-1}$)	8.5/8 ~ 12	Head Weight (g)	3.115 (7%)
Flapping Frequency (Hz)	14/8 ~ 20	Tail Weight (g)	1.335 (3%)
Weight (g)	44.5/30 ~ 50	Fuselage Weight (g)	31.15 (70%)
Moment of Inertia I_{yy} ($\text{kg}\cdot\text{m}^2$)	$0.00015/10^{-4} \sim 10^{-5}$	Total Weight (g)	44.5

The variation history of lift and drag forces of the model within one flapping cycle -after trimming is shown in Figure 8, which clearly depicts the dynamic evolution process of the model's lift and drag forces with the change in flapping phase angle during one cycle.

Analysis of the lift variation trend shows that when the flapping wing moves downward from the highest point, the wing generates positive lift due to the pressure difference between its upper and lower surfaces. When the flapping wing moves down to the neutral plane, its downward flapping velocity reaches the maximum, and the lift force peaks. When the flapping wing transitions from downward flapping to upward flapping, the lift force changes from positive to negative and reaches the maximum upward flapping velocity and the maximum negative lift at the neutral plane during upstroke.

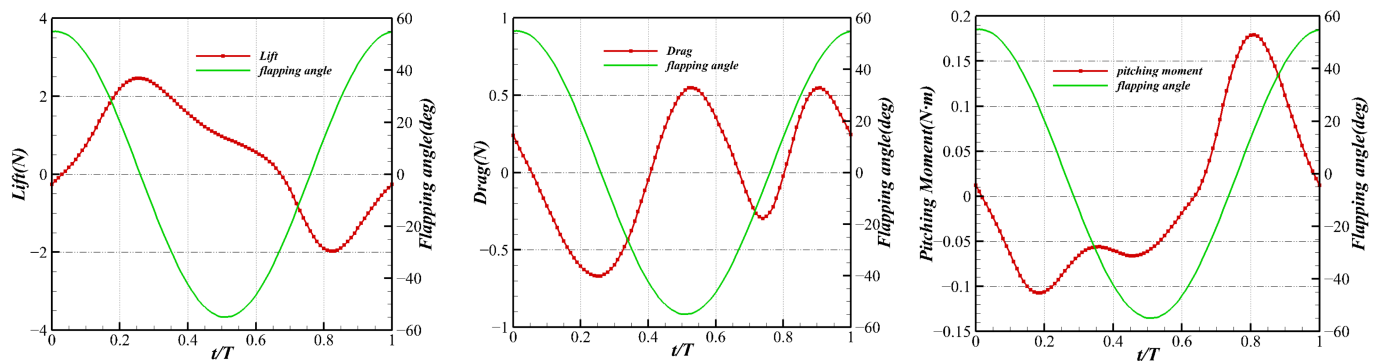


Figure 8. Variation Trends of Lift, Drag and Pitching Moment Within One Cycle Under Trimmed State.

Analysis of the drag variation trend shows that when the flapping wing moves downward from the highest point, the aircraft's thrust increases and drag decreases. When the flapping wing moves down to the neutral plane, the downward flapping velocity is the fastest, the thrust reaches a peak, while the drag reaches a trough, showing a negative peak. When the flapping wing continues to move downward from the neutral plane, the flapping velocity slows down, leading to a decrease in thrust and an increase in drag; when it moves to the lowest point, the thrust reaches a trough and the drag reaches a peak, showing a positive peak. Similarly, when the flapping wing moves upward, the thrust first increases and the drag decreases; when it reaches the M_q neutral plane with the maximum flapping velocity during upstroke, a second negative peak appears where thrust peaks and drag troughs. Then, as it moves toward the highest point with a decreasing velocity, the drag increases and the thrust decreases, forming a second positive peak. It can be seen from this that the thrust–drag aerodynamic effect of the flapping-wing aircraft presents a typical bimodal curve.

Analysis of the pitching moment variation trend shows that when the wing flaps downward, the pitching coefficient is negative and the aircraft noses down; when the wing flaps upward, the pitching coefficient is positive and the aircraft noses up.

Since the motion imposed on the rigid wing in the present study is not purely a flapping motion, but rather a multi-degree-of-freedom coupled motion involving flapping, sweeping, and pitching, the flapping angle shown in the figure serves only as a phase reference for the cycle period. The actual aerodynamic response is also influenced by the sweeping and pitching motions, their velocity terms, and the wake history. In the first half of the cycle, the lift reaches a positive peak while the drag becomes negative, indicating a favorable coupled aerodynamic effect with concurrent enhancement of lift and thrust for flight. In the second half of the cycle, the opposite occurs: the lift becomes negative while the drag increases; however, the pitching moment rises rapidly to its peak during this phase, suggesting that the pitching or torsional aerodynamic loading becomes more prominent within this phase interval. According to the equivalent moment-arm relation ($M = F \cdot d$), the pitching moment is affected by both the aerodynamic force and the effective moment arm. As shown in Figure 9, the equivalent center of pressure varies during the flapping cycle rather than remaining fixed. Although local sharp changes appear near several transition phases due to the high sensitivity of the equivalent moment-arm calculation, the overall variation confirms the dynamic migration of the center of pressure during the periodic flapping cycle.

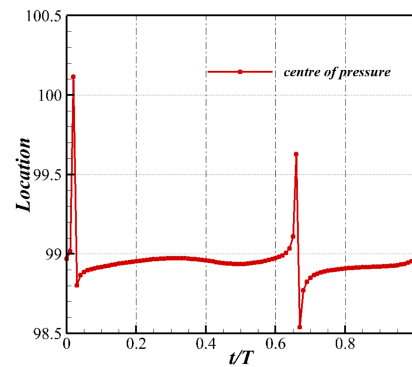


Figure 9. Variation in the equivalent center-of-pressure location during one flapping cycle.

Figure 10 shows the surface pressure distribution and streamlines of the trimmed model within one flapping cycle. During the downstroke, the wing reaches a high velocity near the neutral position, increasing the local dynamic pressure and the pressure difference between the upper and lower surfaces; therefore, the aerodynamic forces and pitching moment increase, accompanied by an increase in lift. During the upstroke, the aerodynamic loading weakens, the pressure is redistributed over the wing surface, and the lift decreases. The coupled flapping, sweeping, and pitching motions further promote smooth surface-flow and load redistribution, so no abrupt change is observed in the pressure distribution or streamline patterns.

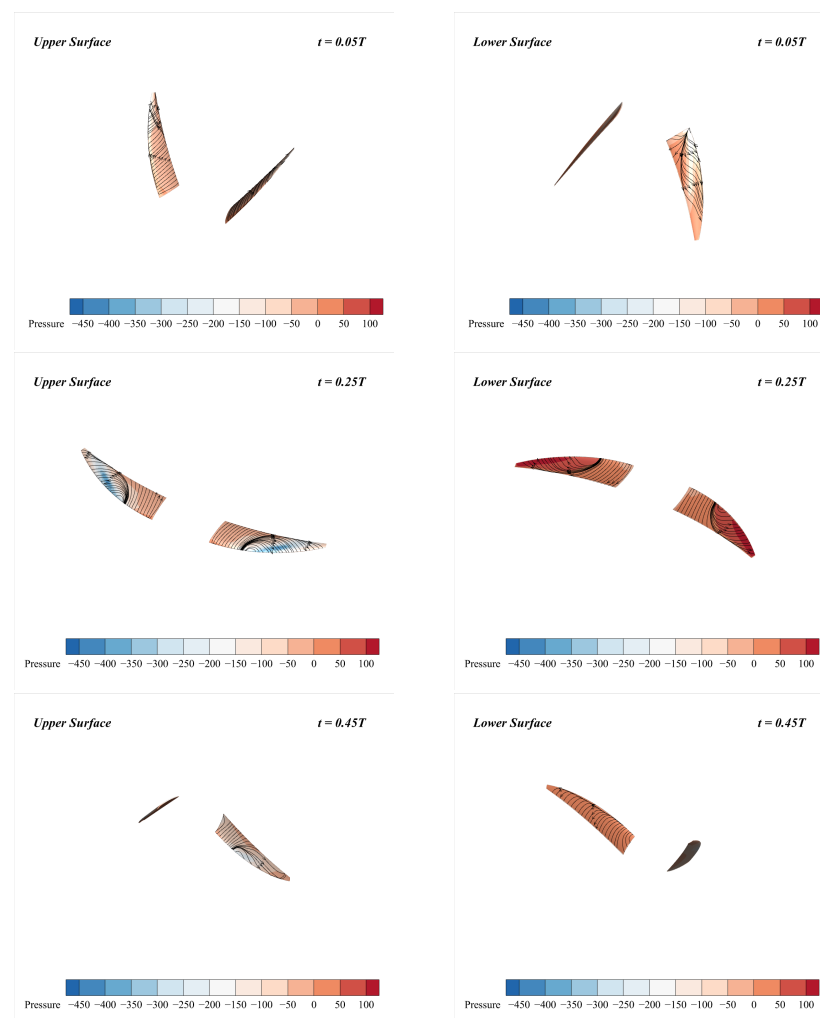


Figure 10. Cont.

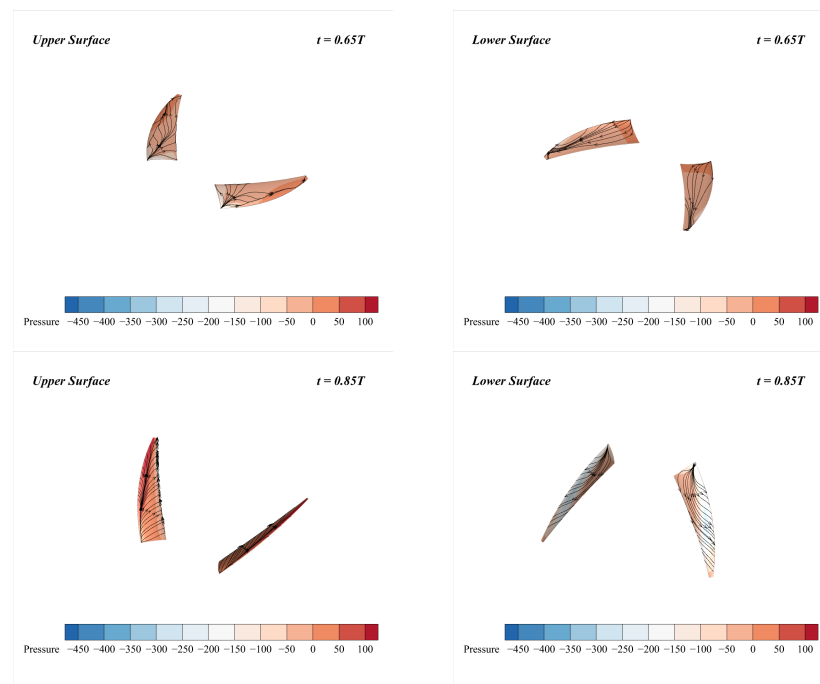


Figure 10. Variation Trends of CP Distribution and Streamlines Within One Cycle Under Trimmed State.

Combining Figure 8 with the aerodynamic-force and pitching-moment variations in Figure 10, it can be seen that although local changes in streamline direction occur near the upper and lower stroke-reversal positions, no sudden aerodynamic-force variation, obvious large-scale separation, or abrupt pressure redistribution is observed. Therefore, the pitching-moment variation within one flapping cycle is mainly related to the redistribution of aerodynamic loads and dynamic pressure, rather than abrupt vortex formation, vortex shedding, or large-scale flow separation.

3.2. Acquisition of Aerodynamic Derivatives and Dynamic Stability Analysis via the Fixed Velocity Method

After completing the longitudinal trim of the swift-inspired model, small disturbances were applied to the incoming flow velocity (u) and angle of attack (AOA) near the trim point. Based on the small disturbance theory, the aerodynamic derivatives of the aerodynamic forces and moments for the full-airframe 3D model were derived.

The results of the CFD numerical simulations are presented in Figures 11 and 12. It can be clearly observed that when small-amplitude perturbations are applied to the model near the trim angle of attack and freestream velocity, the variations in lift, drag, and resultant pitching moment all exhibit a linear trend. This further confirms that the present model is suitable for the small-disturbance equations of motion with highly linear characteristics, as given in Section 2.2 of this paper, and that the aerodynamic derivatives can be further obtained through linear fitting using the least squares method.

In addition, it can be derived from the motion equation that to obtain the state matrix that determines the dynamic stability of the system, it is also necessary to calculate the dynamic derivatives related to the pitching angular velocity $\overline{M}_q, \overline{M}_a$. According to the research by Mao Sun et al. [18], when the pitching angular velocity is sufficiently small, the change in the pitch angle caused by the pitching angular velocity is negligible in the first few flapping cycles at the start of motion. In this case, the applied pitching angular velocity can be considered to have no effect on the model's angle of attack, and the variation in

the resultant pitching moment is only related to the pitching angular velocity applied to the model.

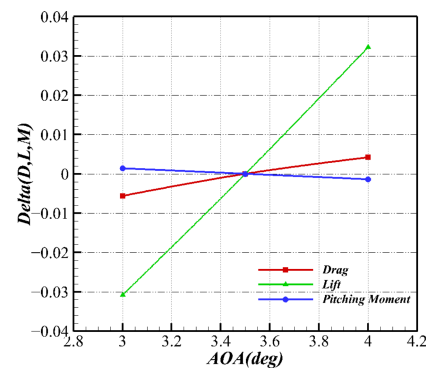


Figure 11. Variations in Drag, Lift and Resultant Pitching Moment with Small Disturbances Applied to the Angle of Attack.

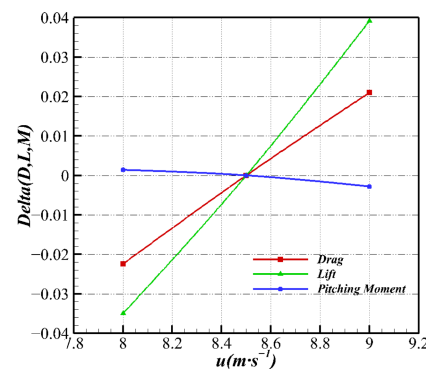


Figure 12. Variations in Drag, Lift and Resultant Pitching Moment with Small Disturbances Applied to the Incoming Flow Velocity.

Based on this, two fixed pitch angular velocity values of 0.01 rad/s and −0.01 rad/s were applied to the model in this chapter, and the corresponding CFD numerical results are presented in Figure 13. It can be observed that the resultant pitching moment of the model also varies linearly with the pitch angular velocity, which is consistent with the small-disturbance assumption.

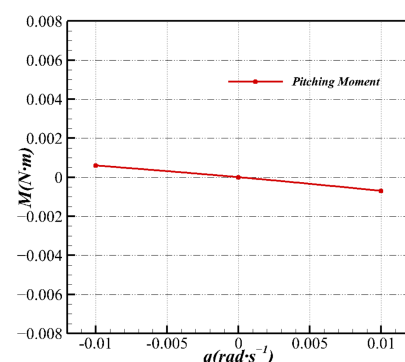


Figure 13. Variation in the Resultant Pitching Moment via the Fixed Velocity Method.

The aerodynamic derivatives are obtained using a dynamic derivative analysis method that combines the periodic time-averaged approach with CFD numerical simulations. As shown in Figure 13, under the premise of the control variable method, small perturbations are applied to the angle of attack or velocity at the trim state, and the resulting variations

in aerodynamic forces and moments are obtained. Upon examining the CFD results, it is found that within the small perturbation range, both the aerodynamic forces and moments exhibit a linear variation with respect to the perturbed variables. Therefore, the approximate dynamic derivatives are derived by linear fitting using the least squares method to determine the slopes of the linear force and moment coefficients with respect to the perturbed quantities. The final aerodynamic derivatives are then calculated according to the expressions given in Section 2.2, and the results are presented in Table 5.

Table 5. Aerodynamic Derivatives of the swift-Inspired Model.

Dynamic Derivatives	X_V	X_α	Z_V	Z_α	M_V	M_α	M_q	$M_{\dot{\alpha}}$
FMAV	−0.9753	−12.76	0.257	9.543	−28	−1069.5	−1.12	−1.12

As can be seen from Table 5, the values of the dynamic derivatives and $M_{\dot{\alpha}}$ are consistent. However, they should be two different physical quantities representing the sensitivity of the pitching moment to the pitch angular velocity q of the body axis system and the sensitivity of the pitching moment to the rate of change in the angle of attack $\dot{\alpha}$. The reason for their values being the same here is merely a coincidence of the author's assumption: under the assumption of small disturbance longitudinal motion and ignoring lateral coupling, in the straight-level flight state with track angle $\gamma = 0^\circ$, the pitching angle perturbation is equal to the angle of attack perturbation ($\theta = \alpha$), so the pitch angular velocity q of the body axis system θ is equivalent to the rate of change in the angle of attack α of the wind axis system in the linearized sense. At this time, the body axis system and the wind axis system rotate synchronously, and the two derivatives naturally show the same form in numerical terms. If lateral motion or non-straight-level flight states are considered, they will no longer be consistent.

Based on the fitted derivative values, a trial-and-error analysis was further conducted. It was found that the aerodynamic derivative located in the second row and first column of the system matrix (Z_V) has a critical influence on the system stability characteristics. Specifically, when this derivative decreases from 0.27 to 0.2, the system transitions from stable to unstable. This indicates that the sensitivity of lift generation to velocity perturbations substantially governs the overall stability of the system.

According to the linearly simplified motion equation based on the small disturbance theory in Section 2.2, the state matrix processed after substituting the aerodynamic derivatives is as follows:

$$A = \begin{bmatrix} -0.9753 & -12.76 & 0 & -9.8 \\ -0.26 & -9.543 & 1 & 0 \\ -28 & -1069.5 & -2.24 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The longitudinal system eigenvalues of the swift-inspired model solved by the fixed velocity method were obtained as shown in Table 6. The system modes consist of two pairs of complex conjugate roots with negative real parts $\lambda = n \pm iw$, which indicate dynamic stability.

Table 6. System Eigenvalues Obtained by the Fixed Velocity Method.

Mode	Eigenvalue
Mode1 λ_1	$-6.1856 \pm 32.4950j$
Mode2 λ_2	$-0.1936 \pm 0.2448j$

From the eigenvalues, the motion system solved by the fixed velocity method is composed of two modes, both of which are complex conjugate roots with negative real parts, representing stable motion modes. Since the disturbed motion is a linear superposition of each eigenmode, the system exhibits overall dynamic stability. A detailed analysis of each mode is presented below:

Mode 1 corresponds to the short-period stable mode. As shown in Table 7, the negative real part indicates that the mode is dynamically stable, while the relatively large imaginary part shows that the mode is characterized by high-frequency oscillation; the damping coefficient $\zeta = 0.1870 < 1$, indicating that this mode is an underdamped oscillatory mode with an oscillation period of 0.1934 s. The oscillation amplitude will be reduced to half of its initial value after approximately 0.1121 s. This means that when the swift-inspired model is subjected to a small disturbance, the inherent dynamic characteristics of the model will generate a restoring moment rapidly, driving the system back to a stable state, while an obvious oscillatory response will occur during this restoration process.

Table 7. Dynamic Characteristics of the Short-Period Stable Mode.

Mode	Eigenvalue	Oscillation Period T	Damping Coefficient ζ	Half-Amplitude Time
Mode1 λ_1	$-6.1856 \pm 32.4950j$	0.1934 s	0.1870 (underdamped)	0.1121 s

Mode 2 corresponds to the long-period stable mode. As shown in Table 8, the damping coefficient $\zeta = 0.6203 < 1$, so this mode is also an underdamped oscillatory mode, with an oscillation period of 25.6666 s. The oscillation amplitude will be reduced to half of its initial value after approximately 3.5803 s. This indicates that the attenuation response of this mode is extremely slow, corresponding to the low-frequency oscillation characteristics brought about by the small imaginary part. When the system is subjected to a small disturbance, this mode will not generate a significant restoring response immediately, but instead produce a slow, small-amplitude and long-duration oscillatory response, and the system will return to the equilibrium state slowly after a long time.

Table 8. Dynamic Characteristics of the Long-Period Stable Mode.

Mode	Eigenvalue	Oscillation Period T	Damping Coefficient ζ	Half-Amplitude Time
Mode2 λ_2	$-0.1936 \pm 0.2448j$	25.6666 s	0.6203 (underdamped)	3.5803 s

To quantitatively describe the eigenvectors of each mode in the system, normalization processing was performed on each eigenmode in this chapter, with the results shown in Table 9.

Table 9. Amplitude and Phase Angle of Eigenvectors After Normalization.

Mode	$\delta u/u_1$	$\delta w/u_1$	$\delta q/q_1$	$\delta \theta$
Mode1 λ_1	$0.68e^{j90.4^\circ}$	$1.01e^{j16.7^\circ}$	$33.08e^{j100.8^\circ}$	1.00
Mode2 λ_2	$19.22e^{j151.3^\circ}$	$0.50e^{-j28.7^\circ}$	$0.31e^{j128.3^\circ}$	1.00

The normalization results show that: In the short-period mode, the amplitude of $\delta q/q_1 = 33.08e^{j100.8^\circ}$ is the largest, indicating that the short-period mode is mainly dominated by the roll angle velocity. At the same time, the absolute value of the amplitude of $\delta w/u_1 = 1.01e^{j16.7^\circ}$ is also relatively large. Therefore, the short-period mode

mainly exhibits rapid coupling oscillation between the roll angle velocity and the vertical velocity perturbation, which is in line with the characteristics of short-period modes being high-frequency and rapidly decaying.

For the long-period mode, the amplitude of $\delta u/u_1 = 19.22e^{j151.3^\circ}$ is the largest, indicating that the long-period mode is mainly dominated by the forward velocity perturbation. The amplitude of the roll angle velocity is only $\delta q/q_1 = 0.31e^{j128.3^\circ}$, accounting for a relatively small proportion. The response is relatively weak. Therefore, the long-period mode mainly exhibits a low-frequency coupling oscillation between velocity and roll attitude.

The analysis of the swift-inspired model via the fixed velocity method shows that the system has good dynamic stability. Both eigenmodes are complex conjugate roots with negative real parts, and the system can ultimately return to a stable state through self-adjustment after superposition of the modes. Among them, the short-period mode provides the system with the ability of rapid attitude recovery in a short time after being disturbed; although the long-period mode has a long half-amplitude period, it still presents a slow attenuation response in the long-term response, thus ensuring the overall stability of the system.

3.3. Acquisition of Aerodynamic Derivatives and Dynamic Stability Analysis via the Forced Oscillation Method

For the acquisition of dynamic derivatives related to pitching angular velocity, traditional wind tunnel tests do not adopt the method in Section 3.1—applying a constant velocity to the model to make it perform pitching motion at an extremely small constant pitching angular velocity. Instead, a small-amplitude periodic forced oscillation is applied to the model to explore the variation trend of the resultant pitching moment of the model under forced oscillation.

In recent years, the method of investigating dynamic derivatives via forced oscillation has also been applied to the calculation of aerodynamic derivatives through CFD numerical simulation. B. Mi and H. Zhan et al. studied the aerodynamic derivatives of pitching combinations using the unsteady method of small-amplitude forced oscillation based on sliding mesh technology [48]; C. Ye and D. Ma et al. obtained the sum of dynamic derivatives of the pitching moment coefficient with respect to the rate of angle of attack change and pitching angular velocity by simulating the forced pitching oscillation of aircraft [49]. Existing studies on calculating aerodynamic derivatives via the forced oscillation method through CFD numerical simulation have provided new ideas for researching the dynamic stability of birds in nature.

On the basis of these studies, the forced-oscillation method is applied to the flapping-wing model for aerodynamic-derivative calculation. The selection of the pitching period is based on the time-scale separation commonly observed in flapping-flight dynamics. For birds, Iosilevskii showed that the rigid-body longitudinal motion and the wingbeat motion generally occur on different time scales [50]. For rigid-wing flapping models, Taylor and Thomas suggested that the rigid-body approximation requires the wingbeat frequency to be at least one order of magnitude higher than the highest natural-mode frequency [8]. In addition, Sun and Xiong reported that, for a hovering bumblebee, the oscillatory-mode period is approximately 50 wingbeat periods [27]. Therefore, in the present study, the pitching period is conservatively set to 100 times the flapping period to reduce direct coupling between the imposed pitching oscillation and the rapid flapping-induced aerodynamic fluctuations. The pitching moment is then averaged over each flapping cycle to extract the slow-varying aerodynamic response associated with the long-period pitching motion.

First, a periodic forced intermittent pitching oscillation about the center of gravity MCG is applied to the model, with the pitching angle ranging from -5° to 0° , and the variation trend of the pitching moment coefficient with time $C_m(t)$ is obtained.

$$\theta(t) = \theta_0 + \theta_m \cdot \sin(\omega t) \quad (17)$$

The average C_m for each period is calculated via the periodic time-averaging method to explore the variation trend of C_m within one flapping period. The results are fitted and decoupled using the high-order Fourier series method, with the decoupling principle as follows: the influence of M is decomposed into three parts: the initial C_{m0} , a trigonometric term in the phase with angular displacement, and a trigonometric term in the phase with angular velocity.

$$C_m = C_{m0} + C_{m\alpha}\alpha(t) + C_{m\dot{q}}\dot{q}(t) \quad (18)$$

The fitting results are shown in Figure 14, and the calculated value $C_{m\alpha}$ is -2.25 , which is very close to the quasi-static assumption. The average resultant pitching moment for each period is obtained via the time-averaging method to further explore the variation trend of the time-averaged resultant pitching moment over 100 flapping periods within one pitching period. It can be seen from the fitting curve that the resultant pitching moment presents a typical quasi-trigonometric variation trend within one pitching period, which is consistent with the motion law of the periodic forced simple harmonic pitching oscillation about the center of gravity set for the model in this section.

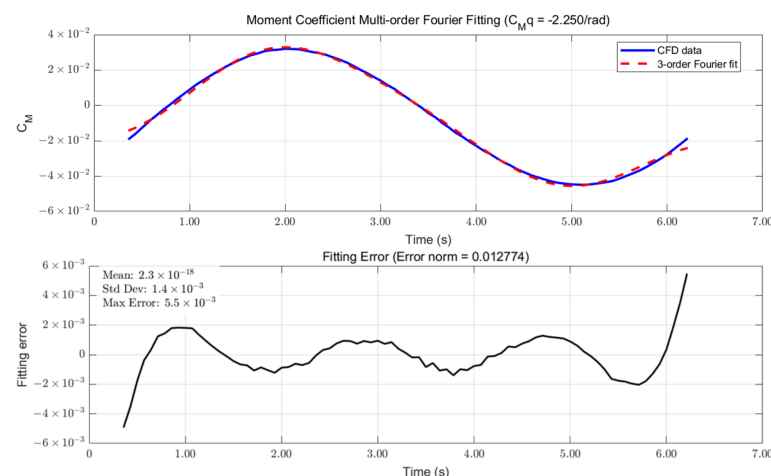


Figure 14. Comparison Between High-Order Fourier Series Fitting Results and CFD Calculation Results.

Fitting the CFD numerical simulation results with a high-order Fourier series shows that the fitting error is relatively large at the start and end of the period, while it remains small in the middle of the calculation process. This phenomenon is mainly caused by the Gibbs-like boundary effect. Although the CFD data are continuous within the calculated time interval, the fitting is performed on a finite-length data segment. When this segment is periodically extended for Fourier fitting, an implicit discontinuity may appear between the start and end boundaries, leading to relatively large local errors near both ends. Therefore, the high-order Fourier series can achieve good fitting accuracy in the middle of the data segment but may produce relatively large errors at the two boundaries. To further evaluate the influence of this boundary effect, the first and last 5% and 10% of the data were removed, respectively, and the fitting was repeated. The results were then compared with those obtained using the full dataset, as shown in Table 10. It can be observed that the values of the extracted aerodynamic derivatives do not change significantly after removing the

boundary data, while the fitting error decreases by approximately one order of magnitude. This indicates that the Gibbs-like boundary effect is mainly localized near the endpoints and does not have a significant influence on the overall fitting results or the final aerodynamic-derivative extraction.

Table 10. Gibbs boundary effect error analysis.

Data Length	Aerodynamic Derivatives (C_{Mq})	Std Dev	Max Error
100%	−2.25	1.41×10^{-3}	5.48×10^{-3}
90%	−2.268	3.30×10^{-4}	1.04×10^{-3}
80%	−2.274	1.57×10^{-4}	4.48×10^{-4}

Accordingly, the aerodynamic derivatives of the swift-inspired model obtained via the forced oscillation method are presented in Table 11.

Table 11. Aerodynamic Derivatives of the swift-Inspired Model.

Dynamic Derivatives	X_V	X_α	Z_V	Z_α	M_V	M_α	M_q	$M_{\dot{\alpha}}$
FMAV	−0.9753	−12.76	0.26	9.543	28	1069.5	−0.993	−0.993

In terms of sensitivity analysis, the value of (M_q) obtained from the forced-oscillation method has a significant influence on the modal characteristics of the system. A comparison of the aerodynamic derivatives obtained by the fixed-velocity method and the forced-oscillation method shows that the main difference lies in the calculated value of (M_q), which is −1.12 for the fixed-velocity method and −0.993 for the forced-oscillation method. As an important parameter governing short-period pitching damping, the smaller absolute value of (M_q) obtained from the forced-oscillation method indicates weaker damping, resulting in a longer recovery time and a weaker attenuation response after disturbance. Further sensitivity analysis shows that as (M_q) increases, the short-period mode gradually transitions into two negative real roots with relatively large magnitudes, indicating that the system response changes from oscillatory decay to rapid non-oscillatory attenuation. This difference arises because the fixed-velocity method cannot fully capture the coupling between pitching oscillation and flapping-induced flow, whereas the forced-oscillation method accounts for this coupling through imposed pitching motion, making the obtained aerodynamic derivatives more consistent with the actual unsteady flow characteristics. Subsequently, according to the linearized small-disturbance motion equations in Section 2.2, the state matrix obtained after substituting the aerodynamic derivatives is expressed as follows:

$$A = \begin{bmatrix} -0.9753 & -12.76 & 0 & -9.8 \\ -0.26 & -9.543 & 1 & 0 \\ -28 & -1069.5 & -1.986 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The longitudinal system eigenvalues of the swift-inspired model solved by the forced oscillation method are obtained as shown in Table 12. The system modes consist of two pairs of complex conjugate roots with negative real parts, which indicate dynamic stability.

Table 12. System Eigenvalues Obtained by the Forced Oscillation Method.

Mode	Eigenvalue
Mode1 λ_1	$-6.0588 \pm 32.4793j$
Mode2 λ_2	$-0.1934 \pm 0.2454j$

It can be seen that the system eigenvalues obtained by the forced oscillation method are very close to those from the fixed velocity method. Consistent with the fixed velocity method, Mode 1 corresponds to the short-period mode. As shown in Table 13, the damping coefficient $\zeta = 0.1834 < 1$, indicating that this mode is an underdamped oscillatory mode with an oscillation period of 0.1935 s. The oscillation amplitude will be reduced to half of its initial value after approximately 0.1144 s. The dynamic characteristics of this short-period mode ensure that the system generates a restoring moment that can quickly return to the equilibrium state after being subjected to a small disturbance.

Table 13. Dynamic Characteristics of the Short-Period Stable Mode.

Mode	Eigenvalue	Oscillation Period T	Damping Coefficient ζ	Half-Amplitude Time
Mode1 λ_1	$-6.0588 \pm 32.4793j$	0.1935 s	0.1834 (underdamped)	0.1144 s

Mode 2 corresponds to the long-period stable mode. As shown in Table 14, the damping coefficient $\zeta = 0.6190 < 1$, so this mode is also an underdamped oscillatory mode, with an oscillation period of 25.6039 s. The oscillation amplitude will be reduced to half of its initial value after approximately 3.5840 s. This indicates that the attenuation response of this mode is extremely weak. When the system is subjected to a small disturbance, the long-period mode will generate a slow, low-amplitude and long-lasting oscillatory response, and it takes a considerable amount of time to slowly return to the equilibrium state.

Table 14. Dynamic Characteristics of the Long-Period Stable Mode.

Mode	Eigenvalue	Oscillation Period T	Damping Coefficient ζ	Half-Amplitude Time
Mode2 λ_2	$-0.1934 \pm 0.2454j$	25.6039 s	0.6190 (underdamped)	3.5840 s

As shown in Figure 15, a comparison of the eigenvalues obtained by the fixed velocity method and the forced oscillation method shows that both yield two pairs of complex conjugate roots with negative real parts, corresponding to the stable long-period and short-period modes respectively. However, compared with the fixed velocity method, the short-period mode from the forced oscillation method has a smaller absolute value of the real part and a nearly identical absolute value of the imaginary part. This means that the half-amplitude time of the short-period characteristics of the system obtained by the fixed velocity method is shorter than that by the forced oscillation method, the pitching oscillation can be damped more rapidly, and the system recovers to stability more quickly after being disturbed.

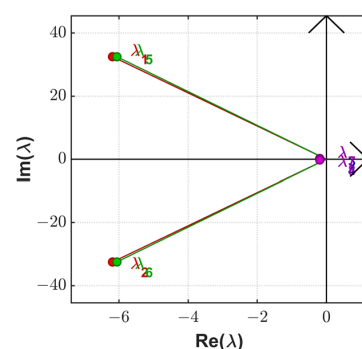


Figure 15. Eigenvalue Distribution of the Fixed Velocity Method ($\lambda_1, \lambda_2, \lambda_3, \lambda_4$) and the Forced Oscillation Method ($\lambda_5, \lambda_6, \lambda_7, \lambda_8$).

For the long-period mode, the half-amplitude time obtained by the forced oscillation method is longer than that by the fixed velocity method, which means the restoring response corresponding to the long-period characteristics solved by the forced oscillation method is longer, and the system slowly returns to the equilibrium state after a long time. This long-time characteristic of the long-period mode is also consistent with the characteristic of swifts constantly adjusting their flight attitudes during flight.

4. Stability Analysis of Time-Varying Systems

To describe the longitudinal dynamic characteristics of the periodic time-varying flapping system more appropriately, this study adopts the Floquet method to conduct an in-depth analysis of dynamic stability. This method achieves periodic trim of the flight state by gradually releasing the body degrees of freedom and performing repeated iterations, thus allowing the dynamic response of the system to retain the time-varying characteristics within the flapping cycle. Based on this, this study further explores the specific performance and variation law of dynamic stability in the time-varying system with released degrees of freedom, providing theoretical support and analytical basis for understanding the longitudinal motion characteristics of the present swift-inspired model.

Compared with the averaging method within the cycle, from the perspective of periodic time-varying system analysis, the Floquet method takes into account the periodic up-and-down and pitch motions of the present swift-inspired model. At the same time, the Floquet method incorporates the periodic flapping motion into the stability analysis, retaining the time information within the flapping cycle and capturing the influence of periodic aerodynamic-load variation on the dynamic response. Previous studies have applied the Floquet method to the stability analysis of biological and biomimetic flapping systems. For example, Taylor et al. [10] studied the stability of locust flight and found that the stability conclusions obtained from the Floquet method were consistent with experimental flight-dynamic observations. These studies support the methodological applicability of Floquet analysis to periodic flapping systems, but they do not replace direct validation of the present swift-inspired model.

4.1. Trim for Free Body Degrees of Freedom

The core of the Floquet method lies in studying the dynamic evolutionary behavior of a time-varying system over a complete cycle. Correspondingly, the motion equations described in Section 2.2 should be adjusted accordingly, transforming from the original form suitable for constant-coefficient systems to the form applicable to time-varying systems.

$$\begin{aligned}
 \frac{d}{dt} \begin{bmatrix} \Delta V \\ \Delta \alpha \\ \Delta q \\ \Delta \theta \end{bmatrix} &= \begin{bmatrix} X_V(t) & X_\alpha(t) + g \cos \gamma_* & 0 & -g \\ -Z_V(t) & -Z_\alpha(t) + g \cos \gamma_* / V_* & 1 & -g \sin \gamma_* / V_* \\ \overline{M_V(t)} - \overline{M_\alpha(t)} Z_V(t) & \overline{M_\alpha(t)} - \overline{M_\alpha(t)} (Z_\alpha(t) - g \sin \gamma_* / V_*) & \overline{M_q(t)} + \overline{M_\alpha(t)} & -\overline{M_\alpha(t)} g \sin \gamma_* / V_* \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta \alpha \\ \Delta q \\ \Delta \theta \end{bmatrix} \\
 + \begin{bmatrix} X_{\delta_e}(t) & X_{\delta_p}(t) \\ -Z_{\delta_e}(t) & -Z_{\delta_p}(t) \\ \overline{M_{\delta_e}(t)} - \overline{M_\alpha(t)} Z_{\delta_e}(t) & \overline{M_{\delta_p}(t)} - \overline{M_\alpha(t)} Z_{\delta_p}(t) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta_e \\ \Delta \delta_p \end{bmatrix} &= Ax + Bu
 \end{aligned} \quad (19)$$

When the model's motion is defined as horizontal forward flight, the equations are simplified as follows:

$$\begin{aligned}
 \frac{d}{dt} \begin{bmatrix} \Delta V \\ \Delta \alpha \\ \Delta q \\ \Delta \theta \end{bmatrix} &= \begin{bmatrix} X_V(t) & X_\alpha(t) + g & 0 & -g \\ -Z_V(t) & -Z_\alpha(t) & 1 & 0 \\ \frac{M_V(t) - \overline{M_{\dot{\alpha}}(t)}Z_V(t)}{0} & \frac{M_\alpha(t) - \overline{M_{\dot{\alpha}}(t)}Z_\alpha(t)}{0} & \frac{M_q(t) + \overline{M_{\dot{\alpha}}(t)}}{1} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta \alpha \\ \Delta q \\ \Delta \theta \end{bmatrix} \\
 + \begin{bmatrix} X_{\delta_e}(t) & X_{\delta_p}(t) \\ -Z_{\delta_e}(t) & -Z_{\delta_p}(t) \\ \frac{M_{\delta_e}(t) - \overline{M_{\dot{\alpha}}(t)}Z_{\delta_e}(t)}{0} & \frac{M_{\delta_p}(t) - \overline{M_{\dot{\alpha}}(t)}Z_{\delta_p}(t)}{0} \end{bmatrix} \begin{bmatrix} \Delta \delta_e \\ \Delta \delta_p \end{bmatrix} &= Ax + Bu
 \end{aligned} \quad (20)$$

Since the amplitude of the resultant force acting on swifts along the X-axis of the body axis system is small during cruising forward flight, and the vibration velocity along the X-axis is extremely minor compared with the flight velocity, this study only considers the heaving motion of the swift-inspired model along the Z-axis of the body axis system and the pitching motion about the Y-axis of the body axis system in the process of trim for free body degrees of freedom, ensuring the airframe undergoes periodic heaving and pitching motions.

4.1.1. Heaving Motion Trim

Based on Newton's second law and the airframe motion equations, the motion equation for heaving motion is derived as follows:

$$ma = F_L \quad (21)$$

where m is the mass of the swift-inspired model, a is the acceleration along the Z-axis of the body axis system, and F_L is the resultant aerodynamic force acting on the model along the Z-axis of the body axis system.

The left panel of Figure 16 shows the iterative evolution process of the swift-inspired model after the release of heaving degrees of freedom. It can be seen from the figure that in the 1st to 2nd flapping cycles immediately after the initial release of the heaving constraint, the heaving motion of the model exhibits intense unstable fluctuations, and the motion state has not yet reached equilibrium. With the progress of iteration, after about five flapping cycles, the amplitude of heaving motion gradually stabilizes and finally reaches a convergent state, indicating that the system gradually enters a stable motion mode.

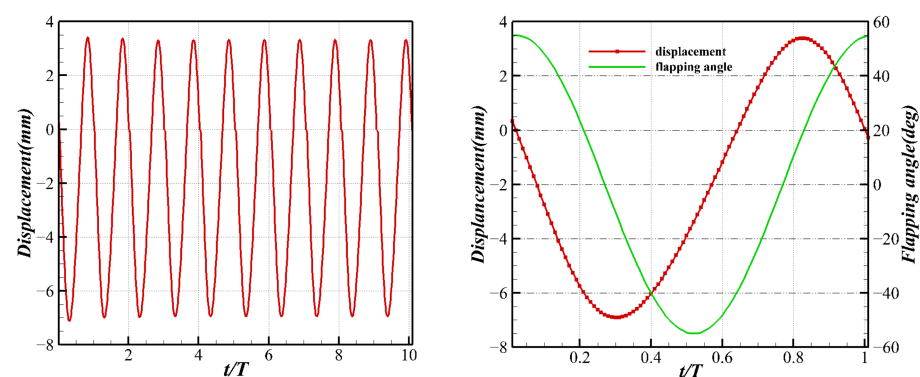


Figure 16. Calculation convergence process of the model's heaving displacement (**left**) and schematic diagram of heaving displacement in the last cycle (**right**).

The right panel of Figure 16 details the heaving motion of the swift-inspired model in the last stable iterative cycle. It can be seen from the curve variation that the model starts to move downward from the initial position at the beginning of the cycle; as time

elapses to 0.3 T, the heaving motion reaches the lowest point, i.e., the maximum sinking displacement. The motion direction then reverses and enters the ascending phase; when the motion proceeds to about 0.85 T, the model reaches the highest point of the ascending motion. It then turns to sinking again and returns to the initial position at the end of the cycle, completing a closed motion cycle. The amplitude of heaving motion is stably maintained at approximately 11 mm throughout the cycle, reflecting good periodicity and repeatability.

4.1.2. Pitching Motion Trim

Based on Newton's second law and the airframe motion equations, the motion equation for pitching motion is derived as follows:

$$I_{yy}\dot{q}(t) = M(t) \quad (22)$$

where I_{yy} is the moment of inertia of the swift-inspired model about the Y-axis of the body axis system, $\theta, \dot{q}(t)$ is the angular acceleration, and $M(t)$ is the resultant pitching moment acting on the swift-inspired model.

The left panel of Figure 17 shows the trim results of the swift-inspired model during pitching motion. It can be clearly observed that in the first three motion cycles after the initial release of the pitching degrees of freedom, the amplitude variation of the model's pitching motion is significant, with obvious fluctuations and differences. With the progress of time and motion iteration, after about five cycles of adjustment and adaptation, the pitching amplitude of the swift-inspired model gradually becomes consistent, showing a good convergence trend, and finally reaches a stable state in the subsequent cycles, achieving kinematic balance.

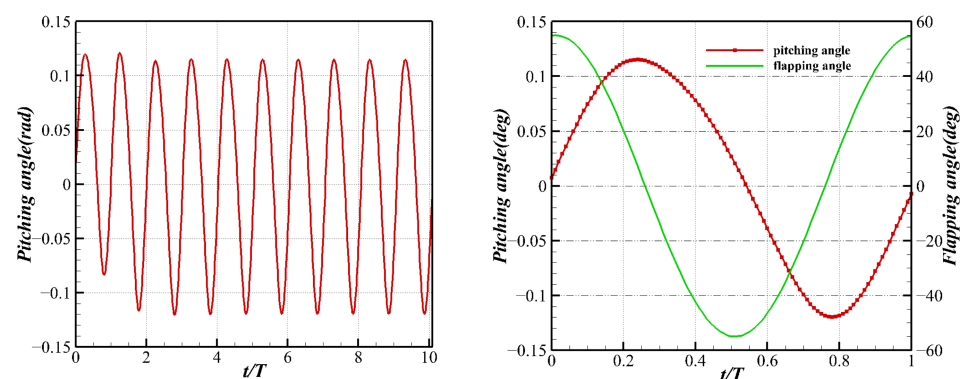


Figure 17. Calculation convergence process of the model's pitching motion (left) and schematic diagram of pitching angular displacement in the last cycle (right).

The right panel of Figure 17 specifically depicts the detailed process of angular displacement changing with time in the last complete cycle of pitching motion iteration. From the start of the cycle to approximately 0.25 T, the model gradually pitches up from the initial equilibrium position to the maximum pitch angle; the motion direction then reverses and enters the pitching down phase until it reaches the lowest pitch position at about 0.75 T; and the model then turns to pitching up again and accurately returns to the initial position at the end of the cycle, thus completing a full pitching motion cycle. Throughout the motion process, the pitching amplitude is basically stable at around 0.25 rad with small fluctuations, showing excellent periodicity and repeatability, which is consistent with the design expectation of bionic motion.

Releasing the degrees of freedom of pitching motion and heaving motion separately and conducting coupling analysis on this basis can more accurately simulate the flight

attitude adjustments of swifts caused by external factors such as airflow changes and obstacle avoidance during flight. This coupling method can accurately reflect the dynamic adjustments and adaptive changes of swifts to multiple flight characteristics such as altitude and angle of attack in actual flight.

4.1.3. Trim Results

Trim was performed based on the CFD numerical simulation results of the iterative coupling of heaving motion and pitching motion. The gravity–thrust trim of the model was achieved by adjusting the incoming flow angle of attack and flapping frequency of the swift-inspired model, with the trim results shown in Table 15. Compared with the state with fixed body degrees of freedom, the lift of the model slightly increases and the drag slightly rises after the release of degrees of freedom; therefore, trim needs to be achieved by increasing the incoming flow angle of attack and appropriately raising the flapping frequency of the model.

Table 15. Detailed Trimmed Parameters.

Trim Parameters	Trim Conditions
Angle of Attack (deg)	4.5
Flapping Amplitude (deg)	55 sin(2 π ft)
Incoming Flow Velocity (m·s ^{−1})	8.5
Flapping Frequency (Hz)	14.5
Weight (g)	45.22
Moment of Inertia I_{yy} (kg · m ²)	0.00015

4.2. Acquisition of Aerodynamic Derivatives and Dynamic Stability Analysis via the Floquet Method

By applying small disturbances of incoming flow velocity and angle of attack to the trimmed model near the equilibrium point, the variations in aerodynamic forces and resultant pitching moment over one motion cycle can be obtained, and then the periodically varying system matrix is derived. For such linear differential equations with periodically varying coefficients, this section adopts the Floquet method for solution and judges the dynamic stability of the system by obtaining the Floquet multipliers.

4.2.1. Acquisition of Dynamic Derivatives

Figures 18–20 show the unsteady response curves of the aerodynamic forces and aerodynamic moments of the swift-inspired model over one flapping cycle near the equilibrium point after applying small disturbances of incoming flow velocity (u) and angle of attack (AOA) to the model near the equilibrium point. It can be seen that under different disturbances, the aerodynamic forces and aerodynamic moments exhibit the same variation characteristics over one flapping cycle: with the increase in incoming flow velocity, the absolute values of lift, drag and moment at the same time instant all increase, reaching the maximum increment at the position where the wing transitions from downstroke to upstroke; with the increase in angle of attack, the absolute values of lift and drag at the same time instant increase, while the influence on the moment is small throughout the flapping cycle. During the entire flapping cycle, the relative dynamic pressure is high in the wing downstroke phase, and the amplitude of drag fluctuation is significantly larger than that in the upstroke phase; the lift shows a trend of increasing significantly first, then decreasing continuously, and finally rising. In the wing downstroke phase, the drag shows a trend of decreasing first and then increasing; in the wing upstroke phase, the drag shows a trend of decreasing first, then increasing, and finally decreasing; the resultant moment shows a trend of decreasing first, then increasing, and finally decreasing.

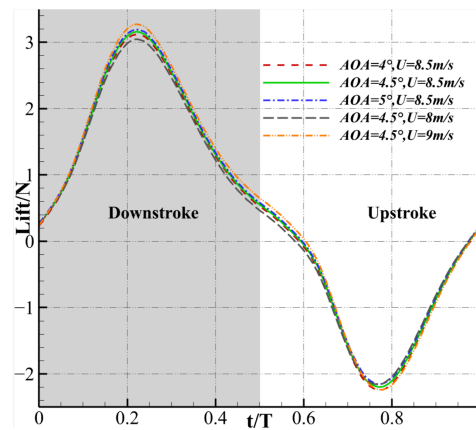


Figure 18. Variation in Lift Over One Cycle Under Different Disturbances.

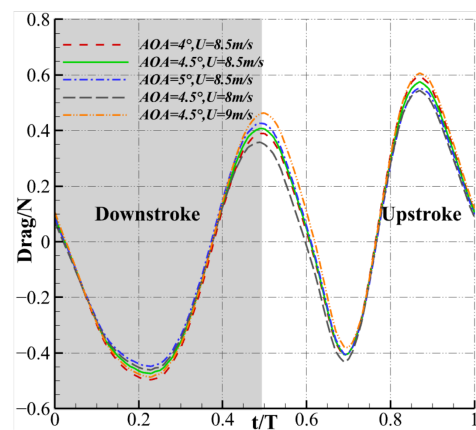


Figure 19. Variation in Drag Over One Cycle Under Different Disturbances.

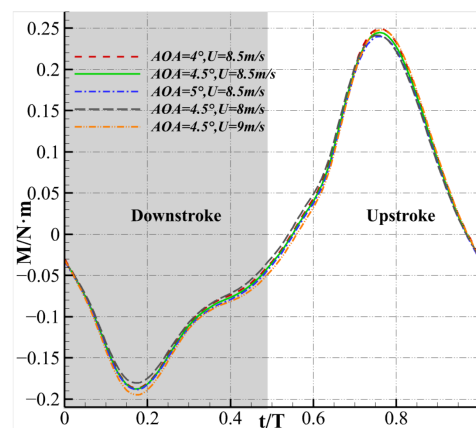


Figure 20. Variation in Resultant Pitching Moment Over One Cycle Under Different Disturbances.

Thus, the dynamic derivative relationships of the aerodynamic coefficients with respect to the incoming velocity and angle-of-attack disturbances within one flapping cycle can be obtained for the streamlined-inspired model. As shown in Figures 21 and 22, all aerodynamic derivatives return to their initial values after one complete flapping cycle, indicating that the aerodynamic response of the system is periodic and satisfies the fundamental requirement of Floquet theory for a linear periodic system, namely $A(t + T) = A(t)$. Therefore, it is reasonable and feasible to further apply the Floquet method to analyze the longitudinal dynamic stability of the present model.

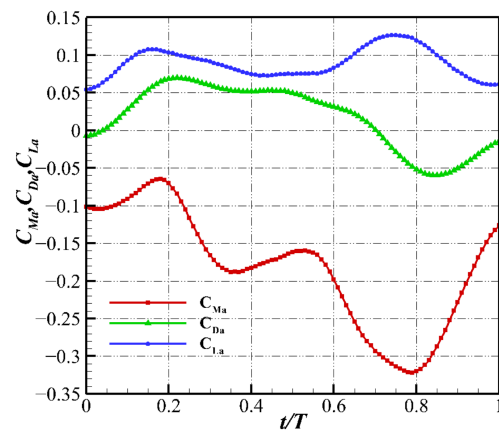


Figure 21. Aerodynamic Derivatives with Respect to Incoming Flow Angle of Attack.

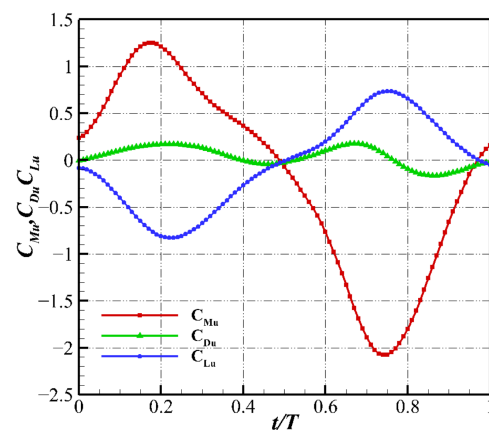


Figure 22. Aerodynamic Derivatives with Respect to Incoming Flow Velocity.

4.2.2. The Floquet Method

Let $x(t) = [\Delta V \quad \Delta \alpha \quad \Delta q \quad \Delta \theta]^T$; then, the motion equations in Section 2.2 can be rewritten as:

$$\frac{d}{dt}x(t) = A(t)x(t) \quad (23)$$

where $A(t)$ is the system matrix for judging dynamic stability. It can be obtained from the acquired dynamic derivatives that the matrix $A(t)$ is a periodically varying system matrix; thus:

$$A(t) = A(T + t) \quad (24)$$

According to Floquet theory, for a system with periodically varying parameters, the solution of the motion equation of its linear differential system can be expressed as:

$$x(t) = \sum_{i=1}^4 \alpha_i p_i(t) e^{\lambda_i t} \quad (25)$$

where $p_i(t)$ is a vector function with a period of T and λ_i is the Floquet exponent, which determines the stability behavior of the system in long-term operation. If $\Psi(t)$ is the fundamental solution matrix of the system, there exists a constant matrix B , such that:

$$\psi(t + kT) = \psi(t) B^k \quad (26)$$

It can be simplified as:

$$\psi(t) = P(t) e^{Rt} \quad (27)$$

where $P(t)$ is a periodic function, R is a constant matrix, and $B = e^{Rt}$. The eigenvalues ρ_i of matrix B are called Floquet multipliers, and the eigenvalues λ_i of matrix A are the eigenvectors for judging the dynamic stability of the system. The transformation relationship between them is:

$$\lambda_i = (\ln|\rho_i| + i\theta) / T \quad (28)$$

Accordingly, the complete calculation procedure is as follows. First, the aerodynamic derivatives obtained at different phases within one flapping cycle are substituted into the longitudinal small-perturbation equations to construct the time-periodic system matrix ($A(T)$), where ($A(t+T) = A(t)$). Then, the state transition equation ($\dot{\Phi}(t) = A(t)\Phi(t)$) is integrated over one complete flapping period using the fourth-order Runge–Kutta method, with the initial condition ($\Phi(0) = I$). After integration to ($t = T$), the monodromy matrix is obtained as ($B = \Phi(T)$). The eigenvalues of (B) are defined as the Floquet multipliers (ρ_i). The stability criterion is determined by their magnitudes: the system is stable when all ($|\rho_i| < 1$), unstable when any ($|\rho_i| > 1$), and critically stable when ($|\rho_i| = 1$). For comparison with the modal eigenvalues obtained from the time-averaged methods, the Floquet multipliers are further converted into Floquet exponents as ($\lambda_i = (\ln|\rho_i| + i\theta) / T$).

To judge the stability of the system, the Floquet method selects the initial disturbance as the identity matrix, then solves the state matrix $X(T)$ after one cycle by numerical method (Runge–Kutta method), and further solves to obtain the matrix B . The dynamic stability of the system is determined by the Floquet multipliers, with the judgment criteria shown in Table 16:

Table 16. Criteria for Judging System Dynamic Stability via the Floquet Method.

Floquet Multiplier	Characteristic of Disturbance Solution
$ \rho_i > 1$	$t \rightarrow \infty, x \rightarrow \infty$, the system is unstable
$ \rho_i < 1$	$t \rightarrow \infty, x \rightarrow 0$, the system is stable
$ \rho_i = 1$	the system is critically stable

4.2.3. Dynamic Stability Analysis

Since solving the state transition matrix by the Runge–Kutta method is affected by the time step, it is necessary to verify whether the state transition matrix finally converges without being affected by the time step when obtaining the final state transition matrix. Table 17 shows the final state transition matrices (B) obtained under the influence of different time steps. It can be seen from the table that with the decrease in time step, each term in the state transition matrix tends to converge; when the time step is reduced to 1.38×10^{-6} , all terms in the state transition matrix converge to 10^{-5} , and the state transition matrix can then be deemed convergent with respect to the time step.

$$B = \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{13} \\ B_{21} & B_{22} & B_{23} & B_{33} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{bmatrix} \quad (29)$$

Through calculation, the eigenvalues of the longitudinal system of the swift-inspired model can be solved, as shown in Table 18. The system modes consist of two pairs of complex conjugate roots with negative real parts, which indicate dynamic stability.

The solution results show that the system eigenvalues solved by the Floquet method are consistent with those obtained by the fixed velocity method and the forced oscillation method, all exhibiting obvious long-period and short-period characteristics. Among them, Mode 1 ($-5.022861 \pm 40.336851j$) corresponds to the short-period mode, with its dynamic

characteristics shown in Table 19. The damping coefficient $\zeta = 0.1236 < 1$, indicating that this mode is an underdamped oscillatory mode with an oscillation period of 0.1558 s. The oscillation amplitude will be reduced to half of its initial value after approximately 0.1380 s, corresponding to a moderate attenuation oscillatory mode. The dynamic characteristics of this short-period mode ensure that the system generates a restoring moment that can quickly return to the equilibrium state after being subjected to a small disturbance.

Table 17. The influence of time step on the state transition matrix.

Time Step	1.38×10^{-4}	1.38×10^{-5}	1.38×10^{-6}
B ₁₁	0.986723474653526	0.986724335529374	0.986724421590626
B ₁₂	−0.111208543602601	−0.111204199439583	−0.111203764934018
B ₁₃	2.498897883934347	2.498942889294083	2.498947389863034
B ₁₄	−0.016957163285338	−0.016956849959458	−0.016956818614894
B ₂₁	−0.066425245963205	−0.066402492190328	−0.066400216664092
B ₂₂	−0.382780120042752	−0.382733547305093	−0.382728889591642
B ₂₃	1.471644635614724	1.472137086463192	1.472186330667891
B ₂₄	−0.156366053925541	−0.156362501548462	−0.156362146270437
B ₃₁	−0.037293720147030	−0.037296834531219	−0.037297145954948
B ₃₂	−0.076736712115950	−0.076744719382623	−0.076745520069319
B ₃₃	−0.970630532964603	−0.970601739104241	−0.970598859729762
B ₃₄	−0.002079728431857	−0.002080489782218	−0.002080565913442
B ₄₁	−0.659611874142105	−0.659635758206305	−0.659638146621099
B ₄₂	0.075895068259843	0.075900516809594	0.075901061592364
B ₄₃	−0.156839963553126	−0.156910775145135	−0.156917856456122
B ₄₄	1.021758683644076	1.021759870665525	1.021759989359933

Table 18. System Eigenvalues Obtained by the Floquet Method.

Mode	Eigenvalue
Mode1 λ_1	$-5.022861 \pm 40.336851j$
Mode2 λ_2	$-0.141776 \pm 0.559078j$

Table 19. Dynamic Characteristics of the Short-Period Stable Mode.

Mode	Eigenvalue	Oscillation Period T	Damping Coefficient ζ	Half-Amplitude Time
Mode1 λ_1	$-5.022861 \pm 40.336851j$	0.1558 s	0.1236 (underdamped)	0.1380 s

Mode 2 corresponds to the long-period stable mode. As shown in Table 20, the damping coefficient $\zeta = 0.2458 < 1$, so this mode is also an underdamped oscillatory mode with an oscillation period of 11.2385 s. The oscillation amplitude will be reduced to half of its initial value after approximately 4.8890 s. This indicates that the attenuation response of this mode is extremely weak. When the system is subjected to a small disturbance, the long-period mode will generate a slow, low-amplitude and long-lasting oscillatory response, and it takes a considerable amount of time to slowly return to the equilibrium state.

Table 20. Dynamic Characteristics of the Long-Period Stable Mode.

Mode	Eigenvalue	Oscillation Period T	Damping Coefficient ζ	Half-Amplitude Time
Mode2 λ_2	$-0.141776 \pm 0.559078j$	11.2385 s	0.2458 (underdamped)	4.8890 s

As shown in Figure 23, a comparison of the short-period eigenvalue distributions calculated by the forced oscillation method and the Floquet method shows that compared with the eigenvalues calculated by the forced oscillation method, the eigenvalues calculated by the Floquet method have a further reduced absolute value of the real part and a further increased half-amplitude time while maintaining stability, which indicates that the damping of the swift-inspired model calculated by the Floquet method is smaller and the attenuation speed is slower. However, the increased absolute value of the imaginary part indicates a higher natural frequency of the short-period mode, leading to a faster response to disturbances. At the same time, a comparison of the eigenroots calculated by the forced oscillation method and the Floquet method shows that, since the Floquet method considers the time-varying characteristics of periodic vibration, captures the phase coupling between flapping vortices and pitching motion, and accounts for the coupled pitch and heave response of the present model by releasing the two corresponding degrees of freedom, the short-period mode frequency calculated by this method is higher and the oscillation attenuation speed is slower. This indicates that the long-period response of the present idealized swift model has a weaker passive recovery capability when periodic flapping dynamics are considered.

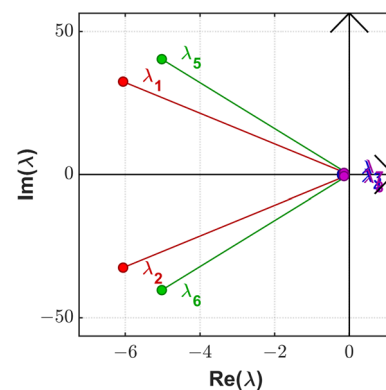


Figure 23. Eigenvalue Distribution of the Forced Oscillation Method ($\lambda_1, \lambda_2, \lambda_3, \lambda_4$) and the Floquet Method ($\lambda_5, \lambda_6, \lambda_7, \lambda_8$).

For the long-period mode, the damping coefficient calculated by the Floquet method decreased from 0.6190 in the forced oscillation method to 0.2458, indicating that the periodic aerodynamic force weakened the damping capacity of this mode. Although the real part of its eigenvalue is still negative, the system remains stable. However, the reduction in the absolute value of the real part slows down the attenuation rate of the disturbance, and the half-period time increases from 3.5840 s to 4.8890 s, suggesting that the long-period oscillation has stronger persistence. Therefore, the long-period mode should receive particular attention in the stability evaluation and control design of biomimetic flapping-wing aircraft.

In summary, the main advantage of the Floquet method lies in its ability to retain the periodic time information of flapping motion, thereby capturing the time periodic variation of aerodynamic loads that traditional averaging methods are unable to resolve. The comparison results show that the damping coefficient of the short-period mode only decreases from 0.1834 to 0.1236, and the half-period time increases from 0.1144 s to 0.1380 s, with relatively limited changes; the damping coefficient of the long-period mode significantly decreases from 0.6190 to 0.2458, and the half-period time increases from 3.5840 s to 4.8890 s. In classical flight dynamics, the long-period mode, also known as the phugoid mode, is generally associated with a slow exchange between kinetic energy and potential energy [30]. Therefore, the stronger variation in the long-period mode indicates that the

method based on the average value within the cycle may underestimate the influence of periodic unsteady aerodynamic forces on this slow energy exchange process. Since the Floquet method retains the phase-dependent variation in aerodynamic loads within the flapping cycle, it can reflect the weakened recovery capability of the long-period mode more clearly. Although the periodic oscillation does not disrupt the system stability, this result indicates that long-period damping should receive particular attention in the stability evaluation and control strategy design of biomimetic flapping-wing aircraft.

5. Conclusions

This paper systematically investigated the longitudinal dynamic stability of a swift-inspired model in steady cruising forward flight by combining computational fluid dynamics (CFD) simulations with dynamic modeling. To address the limitations of conventional steady or quasi-steady assumptions, the unsteady aerodynamic forces and moments within the flapping cycle were considered, and a multi-method framework was established to analyze the coupling between unsteady aerodynamics and body periodic vibrations.

First, for the model with fixed body degrees of freedom, the fixed-velocity and forced-oscillation methods were used to derive the dynamic derivatives, whereas the Floquet method was applied to the free-DOF model. The comparative analysis revealed two stable longitudinal modes: a rapidly convergent short-period mode with a damping ratio of 0.1236 and a half-amplitude time of 0.1380 s, and a slowly attenuating long-period mode with a damping ratio of 0.2458 and a half-amplitude time of 4.8890 s, indicating that the system remains dynamically stable under the present flight condition. This differs from the desert locust studied by Taylor and Thomas [10], which exhibits overall dynamic instability due to the coexistence of stable and unstable modes. Such a difference may be attributed to fundamental differences in morphology and flight-control mechanisms between birds and insects. Since birds can actively adjust their stability characteristics by modifying wing and tail configurations to meet different flight demands [13], and the present study focuses on steady level cruising flight rather than high-agility maneuvers, it is reasonable that the swift-inspired model exhibits a dynamically stable configuration.

Second, the main novelty of this work lies in the establishment of an unsteady longitudinal dynamic stability analysis framework that accounts for the periodic vibrations of the swift-inspired physical model. By comparing the fixed-velocity, forced-oscillation, and Floquet methods, the reliability of the stability conclusions was improved through cross-validation, and the influence of different aerodynamic derivative acquisition methods on modal characteristics was clarified.

Third, similar to insects, periodic flapping does not change the qualitative stability characteristics of birds. However, because insects usually flap at much higher frequencies, approximately (10^3) – (10^4) wingbeat cycles can occur within one long-period pitching cycle, whereas birds have lower flapping frequencies, with only about (10) – (10^2) wingbeat cycles within a comparable cycle. Therefore, the influence of periodic flapping on the modal characteristics of birds cannot be regarded as identical to that of insects. By introducing the Floquet method into the stability analysis of a bird-inspired model, this study quantitatively compared the non-uniform weakening effect of periodic time-varying dynamics on the short- and long-period modes. The results show that although the introduction of periodic dynamics does not alter the overall stability of the model, it weakens the damping characteristics of the long-period mode more significantly than those of the short-period mode. More importantly, compared with time-averaged methods, the Floquet method retains the periodic aerodynamic variation within the flapping cycle and reflects the weakened passive recovery ability of the long-period mode in the present swift model. This finding indicates

that long-period damping should be carefully considered in the stability evaluation and control strategy design of biomimetic flapping-wing aircraft.

The research outcomes deepen the understanding of the intrinsic stability mechanism of avian-inspired flapping flight and provide useful references for aerodynamic layout optimization and control strategy design of biomimetic flapping-wing aircraft. Nevertheless, several limitations remain in the present study. The model captures only the external form resemblance of a swift, without reproducing the actual joints, bones, or muscles, and the wing is assumed to be rigid. Therefore, this work should be regarded as an exploration based on an idealized swift-inspired model, and the results cannot fully represent the actual flight behavior of swifts. Future work will refine the model by incorporating skeletal representation and flexible wings, and will extend the analysis to lateral-directional stability to further improve the theoretical framework of biomimetic flapping flight.

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