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Reliability Allocation Method for Aircraft Mechanical Systems Involving Motion Performance and Failure Correlation

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Abstract

One of the most important design requirements for aircraft mechanical systems is to ensure that their motion functions can be executed smoothly. In this paper, an unconstrained reliability allocation method is proposed, taking into account the characteristics of aircraft mechanical systems. A decomposition principle for assessing the motion performance of aircraft mechanical systems has been proposed, and the contribution of each subsystem is analyzed. Weighting factors for system allocation are proposed and refined, and a failure correlation index is proposed to account for the influence of the interaction between subsystems on the potential failure rate. Furthermore, non-destructive failure events that could have a significant impact on motion performance have been taken into account in the potential improvement of subsystems. Subsequently, reliability prediction models of the systems are established using the Copula function, and a calculation method is introduced to distinguish and quantify the correlation between different subsystems. Finally, the applicability and validity of the proposed method are demonstrated through an engineering case. The results indicate that when failure correlation is considered, the reliability allocated to subsystems is significantly lower than that obtained using traditional methods, providing theoretical guidance for the reliability design of aircraft mechanical systems.

Keywords: reliability allocation; aircraft mechanical systems; motion performance; failure correlation

1. Introduction

A motion mechanism constitutes a vital subsystem of an aircraft mechanical system, aiming to realize motion functions. The reliability and accuracy of motion mechanisms in completing the prescribed tasks also become increasingly important for complex mechanical systems, such as aerospace aircraft and the control systems of weapons and equipment. It is imperative that, at the design stage of the aircraft mechanical systems, including motion mechanisms, the composition of the structure and coordination of components should be fully considered. Thereafter, the reliability of each component should be reasonably allocated [1–3].

Reliability allocation approaches fall into two main categories: constrained and unconstrained methods [4,5]. A key requirement of constrained methods is to clearly define the relationship between system reliability and related costs [6,7]. If such a functional relationship cannot be obtained during the initial stage of system design, unconstrained methods can be prioritized [8]. The essence of the unconstrained methods is to establish a

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mapping relationship between allocation indexes and various influencing factors [9]. For the unconstrained methods, the failure risk of a subsystem is typically assessed using the information provided by Failure Mode and Effects Analysis (FMEA). In [10–13], the failure occurrence and failure severity are nonlinearly transformed based on FMEA. This makes the transformed allocation factors more consistent with engineering practice requirements.

However, the failure modes' ratings based on FMEA are mainly determined through the subjective synthesis of experts or decision-makers. Moreover, at the initial stage of system design, the allocation factors are mostly vague. Therefore, many scholars have introduced fuzzy theory to reduce the influence of the subjective nature in intuitive evaluation [14,15]. Furthermore, the analytic hierarchy process (AHP) is also incorporated to enhance the practicality of reliability allocation. In research on reliability allocation methods for remanufactured equipment, Du et al. [16] conducted a fuzzy assessment of the importance and impact of each component failure, and determined the relative importance of the allocation criteria using the AHP method. Wu et al. [17] proposed a reliability allocation model suitable for wind turbine generator systems based on the improved fuzzy analytic hierarchy process (FAHP). Bai et al. [18] employed the AHP method to evaluate the weight of influencing factors and utilized the hesitant fuzzy language set to assess the various influencing factors. The above studies demonstrate once again that the application of fuzzy theory can effectively reduce the subjectivity of scoring.

Nevertheless, the aforementioned reliability allocation methods are applicable to electronic systems or simple structural mechanical systems. For aircraft mechanical systems involving motion performance [19–22], the design of such systems must not only ensure that the strength of the subsystems meets the requirements, but more importantly, ensure that the various subsystems can work collaboratively and transmit motion to fulfil their intended functions. Therefore, it is essential to take motion performance into account when assigning reliability to aircraft mechanical systems. Yadav et al. [23,24] first decomposed the system design into three dimensions: physical structure, system function and service time. Based on this, a binary mapping matrix that links the various components of the system to its functions was constructed, thereby quantifying the interrelationships between the components and functions. Moreover, for mechanical systems incorporating motion mechanisms, non-destructive failures may also lead to system failure, in addition to failures caused by structural damage to constituent units [25]. Recently, meta-action theory has been introduced to account for non-destructive failures, functional reliability and system performance in reliability allocation [26–28]. These studies demonstrate that the rational assessment and quantification of factors influencing motion functionality are crucial for the reliability allocation method of motion mechanisms. Based on the methods for analyzing motion performance in reliability allocation described in the aforementioned literature, it is necessary to integrate the characteristics of motion functions more thoroughly, extend these to aircraft mechanical systems, and conduct corresponding research into reliability allocation methods. However, these studies share a common shortcoming: none of them takes into account failure correlation within mechanical systems.

In complex mechanical systems, components and motion functions are interrelated, leading to correlations among failure modes of different functions [29–31]. Such correlations must be appropriately considered during reliability allocation. To account for failure correlation in system reliability allocation, existing approaches primarily employ two methods: utilizing copula functions to establish reliability models and designing failure correlation factors to refine the allocation factors. Cao et al. [32] proposed and quantified weighting factors for the potential impact of common-cause failures between components on systems. As an effective tool for addressing failure correlation issues, the Copula function has also gradually been applied to research into reliability allocation in mechanical systems. Yang

et al. [33] employed Copula functions for system reliability allocation and utilized fuzzy set theory to establish component failure correlation factors for modifying reliability allocation coefficients, though they did not account for variations in failure correlation among different components. Zhang et al. [4] characterized component failure correlations through relative dispersion factors, adjusting for component complexity and failure incidence. However, their modification of failure incidence requires refinement. Wu et al. [34] employed the vine Copula function to describe multi-component failure correlations, decomposing high-dimensional correlations into pairwise relationships. Nevertheless, applying this model necessitates estimating correlation coefficients using historical data. Consequently, current research on reliability allocation methods concerning failure correlation still faces two main challenges: how to account for the interaction of subsystems when modeling the allocation factors, and how to correctly distinguish and quantify the failure correlations between different components when constructing reliability prediction models. To address these issues, this paper will explore the impact of component correlations on both allocation factor models and system reliability prediction models.

In summary, this contribution aims to establish a reliability allocation method for aircraft mechanical systems that simultaneously accounts for motion performance and failure correlation. Taking into account the characteristics of aircraft mechanical systems, the structural and motion functions of the system are decomposed. Weighting factors for system allocation are proposed and refined, and a failure correlation index is proposed to account for the influence of the interaction between subsystems on the weighting factors. Then, a reliability model of the system is established using Copula function, and a calculation method is introduced to quantify the correlation between different subsystems. Finally, the applicability of the proposed method is validated through an engineering case study.

The remainder of this paper is organized as follows. Section 2 introduces the principles of system decomposition and the fuzzy language set. Section 3 first presents the allocation weighting factors and their evaluations. Subsequently, reliability models for the systems to be allocated, taking into account failure correlation, are established, and a specific allocation solution method is derived. An application example of the proposed method to an aircraft load mechanism is presented in Section 4. Section 5 summarizes the main contributions of this article.

2. Preliminaries

2.1. System Decomposition Principle of Aircraft Mechanical Systems

For aircraft mechanical systems involving motion performance, the system must be decomposed across three dimensions, as illustrated in Figure 1. From the physical structure dimension, the system design must satisfy the structural requirements of its constituent subsystems and components. From the motion performance dimension, the system must fulfil the motion function requirements at different hierarchical levels. From the service life dimension, the system design must ensure compliance with corresponding state requirements at different service intervals, establish appropriate warranty conditions, and guarantee a specified operational lifespan.

For aircraft mechanical systems, the design phase must first satisfy the structural strength requirements of components. Secondly, it must ensure the system can execute corresponding tasks with high precision and reliability. Finally, it must guarantee that the system possesses a certain service life. Based on this philosophy, when allocating reliability for aircraft mechanical systems, the approach differs from traditional structural systems. The system must be decomposed into three aspects. Firstly, starting from the system's constituent structure, analyze structural failure modes such as environmental strength failure, fatigue failure, fracture failure, and strength failure. Secondly, beginning with the

system's overall functionality, analyze motion failure modes such as starting failure, positioning accuracy failure, and unlocking failure. Thirdly, considering the system's service life, the evolution of system functionality over time is examined to establish a functional relationship model between failure modes and time. The integrated reliability allocation framework for mechanical systems, incorporating both composition and functionality, is illustrated in Figure 2.

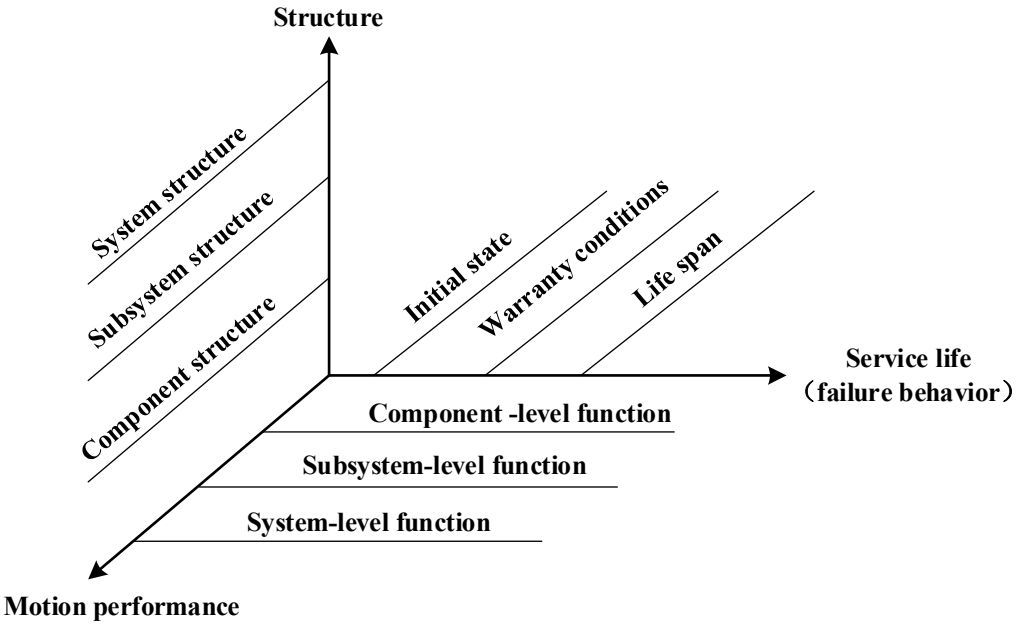


Figure 1. Three-dimensional disassembly of aircraft mechanical systems.

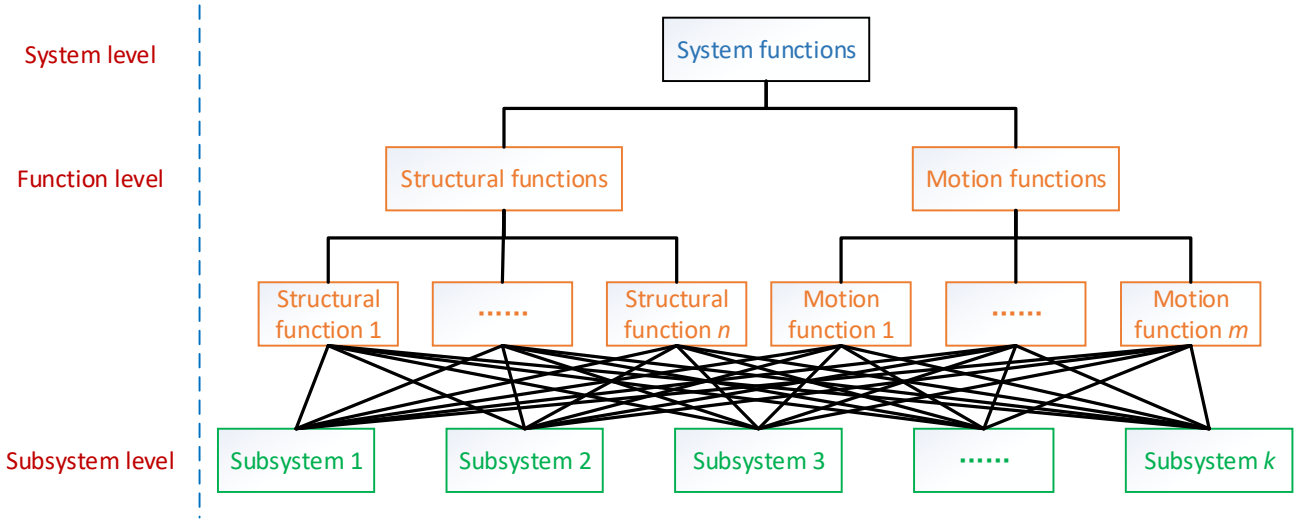


Figure 2. Functional decomposition diagram of aircraft mechanical systems considering structural function and motion function.

Consequently, when allocating reliability for aircraft mechanical systems involving motion performance, both structural function and motion function must be taken into account. As the outcome of reliability allocation is closely linked to the allocation factors, this paper employs the above decomposition principle to allocate system reliability targets by evaluating six indicators: subsystem contribution, complexity, severity, potential improvement, dynamic performance and operating time.

For ease of analysis, the decomposition of the system functions can also be presented in tabular form, as shown in Table 1.

Table 1. Statistics of the subsystem function.

	Structural Function			**Motion Function**		
	Function 1	**Function *j***	**Function *n***	**Function 1**	**Function *l***	**Function *m***
Number of subsystems	1, 2, . . ., *k*	1, 2, . . ., *k*	1, 2, . . ., *k*	1, 2, . . ., *k*	1, 2, . . ., *k*	1, 2, . . ., *k*

2.2. Hesitate Fuzzy Language Set

When evaluating the allocation factors, experts may be indecisive among several scores. To reduce the uncertainty in the subjective evaluation process, a hesitant fuzzy language set has been proposed to evaluate different influencing factors. Fuzzy numbers are introduced to describe the corresponding ratings. The most commonly used fuzzy numbers are triangular fuzzy numbers and trapezoidal fuzzy numbers. The membership functions of these numbers are defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, a \le x \le b; \\ \frac{c-x}{c-b}, b \le x \le c; \\ 0, othereise. \end{cases} \tag{1}$$

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{b-a}, a \le x \le b; \\ 1, b \le x \le c; \\ \frac{d-x}{d-c}, c \le x \le d; \\ 0, otherwise. \end{cases} \tag{2}$$

where $\mu_{\tilde{A}}(x)$ is the degree of membership of element x in fuzzy set $\tilde{A}$.

By defuzzifying the fuzzy set $\tilde{A}$, they can be converted into definite numerical values. The most widely used method is the centroid defuzzification method [35].

For triangular fuzzy numbers $\tilde{A} = (a, b, c)$, $C_{\tilde{A}} = \frac{a+b+c}{3}$, where $C_{\tilde{A}}$ is the value of the fuzzy set $\tilde{A}$ after defuzzification.

For trapezoidal fuzzy numbers $\tilde{A} = (a, b, c, d)$, the defuzzification formula can be expressed as [35]:

$$C_{\tilde{A}} = \frac{c^2 + d^2 + cd - a^2 - b^2 - ab}{3(c + d - a - b)} \tag{3}$$

3. Proposed Method for Reliability Allocation of the Aircraft Mechanical Systems

The unconstrained allocation method is utilized to allocate the reliability of the aircraft mechanical systems. To truly reflect the performance characteristics of the aircraft mechanical systems, the following six allocation factors are selected and expanded, namely contribution, complexity, severity, potential improvement, dynamic performance and operating time. Among them, complexity is a quantitative factor, which is quantified through historical data; the remaining five factors are qualitative factors, which are evaluated by experienced experts.

3.1. Reliability Allocation Factors

3.1.1. Contribution C_i

The various subsystems cooperate with each other to execute the various functions of the aircraft mechanical systems. A thorough analysis and decomposition of the system is crucial for the allocation of reliability. Therefore, based on the principles of system func-

tional decomposition outlined in Section 2.1 and utilizing the AHP method, a quantitative allocation factor for subsystem contribution has been proposed. Subsystems with higher contributions should be allocated greater reliability. The contribution of each subsystem is assessed through the following method. Firstly, assess the relative importance of each subfunction; secondly, evaluate the contribution of each subsystem to each subfunction. The specific implementation steps are as follows.

Step 1: Employ the AHP method to quantify the relative importance of each subfunction. The importance of subfunction i relative to subfunction j is denoted by v_{ij}, with the meaning of v_{ij} values listed in Table 2 [28]. The subfunction relative importance matrix V is formed:

$$V = \begin{pmatrix} v_{11}, v_{12}, \cdots v_{1(m+q)} \\ v_{21}, v_{22}, \cdots v_{2(m+q)} \\ \cdots \\ v_{(m+q)1}, v_{(m+q)2}, \cdots v_{(m+q)(m+q)} \end{pmatrix} \tag{4}$$

where m denotes the number of the structural subfunctions, and q denotes the number of the motion subfunctions.

Table 2. Scale of comparison matrix.

v_{ij}	**Definition**
1	v_i and v_j are equal to the objective
3	v_i is slightly important over v_j
5	Experience and judgment strongly important v_i over v_j
7	v_i is very strongly important over v_j
9	v_i is extremely important over v_j
2, 4, 6, 8	The median of the above two adjacent judgments

Then, the relative importance of subfunction i can be obtained by:

$$g_i = \frac{\sqrt[m+q]{\prod_{j=1}^{m+q} v_{ij}}}{\sum_{i=1}^{m+q} \left(\sqrt[m+q]{\prod_{j=1}^{m+q} v_{ij}} \right)} \tag{5}$$

Step 2: Employ fuzzy set theory to evaluate the contribution of components to subfunctions, with specific evaluation criteria as outlined in Table 3; the contribution matrix $\widetilde{U}$ for each subsystem is formed:

$$\widetilde{U} = \begin{pmatrix} \widetilde{u}_{11}, \widetilde{u}_{12}, \cdots \widetilde{u}_{1n} \\ \widetilde{u}_{21}, \widetilde{u}_{22}, \cdots \widetilde{u}_{2n} \\ \cdots \\ \widetilde{u}_{(m+q)1}, \widetilde{u}_{(m+q)2}, \cdots \widetilde{u}_{(m+q)n} \end{pmatrix} \tag{6}$$

where n represents the number of subsystems of an aircraft mechanical system.

Table 3. Fuzzy evaluation of contribution.

Fuzzy Level	**Criteria for Evaluation**	**Fuzzy Number**
VL	Very low contribution	(1, 2, 3)
L	Low contribution	(3, 4, 5)
M	Moderate high contribution	(5, 6, 7)
H	High contribution	(7, 8, 9)
VH	Very high contribution	(8, 9, 10)

Step 3: The TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method [16] is introduced and combined with the relative importance of subfunctions to obtain the subsystem contribution.

According to the fuzzy number multiplication operation, the comprehensive evaluation matrix of the subsystem is $\widetilde{A} = \left(\widetilde{a}_{ij}\right)_{(m+q)\times n}$:

$$\widetilde{a}_{ij} = g_i \widetilde{u}_{ij} \tag{7}$$

Then, the intermediate values are obtained, namely the positive and negative ideal solutions:

$$\begin{cases} M^+ = \left(m_1^+, m_2^+, \cdots, m_n^+\right) \\ M^- = \left(m_1^-, m_2^-, \cdots, m_n^-\right) \end{cases} \tag{8}$$

where $m_j^+ = \max\left\{\widetilde{a}_{1j}, \widetilde{a}_{2j}, \cdots, \widetilde{a}_{(m+q)j}\right\}$, and $m_j^- = \min\left\{\widetilde{a}_{1j}, \widetilde{a}_{2j}, \cdots, \widetilde{a}_{(m+q)j}\right\}$.

Calculating the Euclidean distance between the evaluation of the contribution and the ideal solution:

$$\begin{cases} Ed_i^+ = \sqrt{\sum_{j=1}^{m+q}\left(\widetilde{a}_{ij} - m_j^+\right)^2} \\ Ed_i^- = \sqrt{\sum_{j=1}^{m+q}\left(\widetilde{a}_{ij} - m_j^-\right)^2} \end{cases} \tag{9}$$

Then, the contribution of each subsystem can be obtained by:

$$\widetilde{C}_i = \frac{Ed_i^-}{Ed_i^+ + Ed_i^-} \tag{10}$$

Finally, the normalized subsystem contribution C_i is given as:

$$C_i = \frac{\widetilde{C}_i}{\sum_{i=1}^{n}\widetilde{C}_i} \tag{11}$$

3.1.2. Complexity K_i

According to the traditional method, the complexity K_i of a subsystem is the ratio of the number of components in the subsystem to the total number of components in the entire system; that is:

$$K_i = \frac{n_i}{\sum_{i=1}^{n} n_i} \tag{12}$$

For an aircraft mechanical system, there will be interrelated loads among different subsystems. Therefore, the complexity of the subsystems is also related to the dispersion of the loads they bear. As the dispersion of the system's loads increases, the degree of failure correlation among the subsystems also gradually increases. At this point, the complexity of a subsystem cannot be solely determined by the number of components it contains.

The dispersion of the load is related to the system's environmental conditions. The dispersion of the load increases as the environmental conditions deteriorate. The dispersion of the system's performance is related to material processing techniques, part manufacturing techniques, and component assembly techniques, and can be qualitatively characterized by a technical level. Consequently, the working conditions and technical level can be used to approximately characterize the relative dispersion of the load of subsystems [4,34]. L_{sri} represents the load dispersion coefficient, which can be used to measure the failure

correlation between each subsystem and other subsystems. L_{sri} can be obtained by the following:

$$L_{sRi} = \frac{E_i}{E_i + T_i} \tag{13}$$

where E_i represents the environmental condition factor, and T_i represents the technology level factor. Both can be obtained by consulting references [4,33].

To take into account the influence of load dispersion, the subsystem complexity is modified as follows:

$$K_i^* = \frac{n_i^{1-L_{Rri}}}{\sum_{i=1}^{n} n_i^{1-L_{sRi}}} \tag{14}$$

Specifically, when L_{sri} trends to 0, that is, the failure of subsystems occurs independently, and K_i^* equals to K_i. When the L_{sri} trends to 1, the failure of subsystems is completely correlated, meaning that the complexity of the subsystem is independent of n_i.

The normalized complexity $\overline{K}_i$ is given as:

$$\overline{K}_i = \frac{K_i^*}{\sum_{i=1}^{n} K_i^*} \tag{15}$$

3.1.3. Severity S_i

For aircraft mechanical systems, both structural failure and motion failure should be considered. In other words, a motion failure occurs when the motion error exceeds the allowable limit. Although the structural function remains normal in this case, subsystems with high motion accuracy requirements can no longer meet user needs. Subsystems with a higher severity level need to be prioritized for improvement and require careful consideration in the reliability allocation process. However, the traditional 10-level ordinal scale method based on FMEA for quantifying fault levels does not accurately reflect the severity of the failure mode.

Consequently, to reflect the severity of failure modes more accurately, the severity is subjected to the following exponential transformation [10]:

$$S_{ij}^* = \exp(\theta \times S_{ij}) \tag{16}$$

where S_{ij} is the severity rating derived from the ten-point scale shown in Table 4, $\widetilde{S}_{ij}$ denotes the nonlinear transformation of the failure severity, and θ denotes the transformation coefficient.

Assuming that the system comprises n subsystems, $i = 1, 2, \ldots, n$. Compared with the ordinal scale, this nonlinear transformation amplifies the differences between severity ratings.

When subsystem i contains multiple failure modes, the most severe failure mode is selected to represent the potential severity of subsystem i:

$$S_i = \max(S_{i1}^*, S_{i2}^*, \ldots, S_{if}^*) \tag{17}$$

where f is the total number of failure modes of subsystem i.

The severity is normalized as follows:

$$\overline{S}_i = \frac{S_i}{\sum_{i=1}^{n} S_i} \tag{18}$$

Table 4. Fuzzy evaluation grades of failure severity.

Fuzzy Level	Failure Severity Level Description	Fuzzy Number
None (N)	No effect	(1, 1, 2)
Very minor (VMR)	System operable but minor discomfort	(1, 2, 3)
Minor (MR)	System operable but moderate discomfort	(2, 3, 4)
Very low (VL)	System operable but with severe discomfort	(3, 4, 5)
Low (L)	System operable but with a certain degradation of performance	(4, 5, 6)
Moderate (M)	System operable but with a significant degradation of performance	(5, 6, 7)
High (H)	System inoperable with equipment failure	(6, 7, 8)
Very high (VH)	System inoperable with serious equipment failure	(7, 8, 9)
Hazardous with warning (HWW)	Very high severity level with warning	(8, 9, 10)
Hazardous without warning (HWOW)	Very high severity level without warning	(9, 10, 10)

3.1.4. Potential Improvement P_i

Subsystems with higher failure rates are likely to possess greater potential for improvement, where further enhancements can be achieved at relatively low cost and will be assigned higher reliability. The subsystem potential improvement P_i is derived as follows [11]:

$$P_i = -\frac{ln\lambda_i}{r} \tag{19}$$

where λ_i denotes the failure rate, and r is a positive constant.

The failure rate of a subsystem can be determined based on historical failure data or test data. Where the number of failure samples is limited, the subsystem's probable failure rate can be inferred from the failure rate information provided in the FMEA [10,12]:

$$\lambda_{ij} = \exp(\beta \times O_{ij} + \gamma) \tag{20}$$

where λ_{ij} denotes the failure rate of the jth failure mode in subsystem i, and O_{ij} denotes the occurrence rating of the jth failure mode in subsystem i, as shown in Table 5.

Table 5. Fuzzy evaluation grades of occurrence rating.

Fuzzy Level	Failure Occurrence Level Description	Fuzzy Number
R	Unlikely failure	(1, 1, 1, 2)
L	Relatively rate failure	(1, 2, 3, 4)
M	Occasional failure	(3, 4, 6, 7)
H	Repeated failure	(6, 7, 8, 9)
VH	Almost inevitable failure	(8, 9, 10, 10)

The above calculation of the failure rate is under the assumption that each subsystem fails independently. The correlation among subsystems may lead to a certain concentration trend in the failures of each subsystem. Therefore, based on the calculation of load dispersion, a failure correlation index ε_i is proposed to quantitatively describe the failure correlation between the subsystems; that is:

$$\varepsilon_i = \exp(\mu \times L_{sri}) = \exp\left(\mu \times \frac{E_i}{E_i + T_i}\right) \tag{21}$$

where μ is an undetermined coefficient, $\mu > 0$.

The modified occurrence rating O_{ij}^* can be derived as:

$$O_{ij}^{*} = \varepsilon_i \times O_{ij} \tag{22}$$

For aircraft mechanical systems, the execution of various movements relies on the coordination among subsystems. Therefore, when considering the occurrence rating, not only the conventional fault events (including part fracture, fatigue, corrosion, etc.) should be taken into account, but also nondestructive fault events should be considered [4]. Nondestructive fault events include parts' assembly, lubrication, coordination, etc., all of which will have an impact on the motion performance of the aircraft mechanical systems [25,36]. Furthermore, the degradation of components will not cause the institution to cease operation, but it will have an impact on the motion performance [18]. Thus, this paper also assesses the failure rate caused by these abnormal actions.

Similarly, the abnormal actions of parts can also cause a failure correlation of subsystems. Then, the failure rate caused by abnormal cations can be expressed as:

$$\lambda_{ij}^{a} = \exp\left(\beta \times \varepsilon_i \times O_{ij}^{a} + \gamma\right) \tag{23}$$

where O_{ij}^{a} is the occurrence rating of abnormal actions.

Then the modified failure rate is:

$$\lambda_{ij}^{*} = \lambda_{ij} + \lambda_{ij}^{a} \tag{24}$$

Assuming the ith subsystem has f failure modes, its overall failure rate can be expressed as:

$$\lambda_i = \sum_{j=1}^{f} \lambda_{ij}^{*} \tag{25}$$

The normalized $\overline{P}_i$ is calculated as follows:

$$\overline{P}_i = \frac{P_i}{\sum_{i=1}^{n} P_i} \tag{26}$$

3.1.5. Dynamic Performance D_i

For aircraft mechanical systems, as the subsystems can achieve more types of movements and perform more motion functions, the possibility of failure is higher than that of a static subsystem. Subsystems with fewer moving components should be given higher reliability. Employ fuzzy set theory to evaluate the dynamic performance of each subsystem, with specific evaluation criteria as outlined in Table 6 [37]. D_i represents the weight factor for describing dynamic performance.

Table 6. Fuzzy evaluation grades of dynamic performance.

Fuzzy Level	Dynamic Performance Level Description	Fuzzy Number
R	0–20% of the system's moving components are located in this subsystem.	(1, 1, 1, 2)
L	20–40% of the system's moving components are located in this subsystem.	(1, 2, 3, 4)
M	40–60% of the system's moving components are located in this subsystem.	(3, 4, 6, 7)
H	60–80% of the system's moving components are located in this subsystem.	(6, 7, 8, 9)
VH	80–100% of the system's moving components are located in this subsystem.	(8, 9, 10, 10)

The normalized dynamic performance factor $\overline{D}_i$ is:

$$\overline{D}_i = \frac{D_i}{\sum_{i=1}^{n} D_i} \tag{27}$$

3.1.6. Operating Time T_i

Some subsystems will have a shorter operating time than that of the entire system. Consequently, a lower reliability level may be assigned to those subsystems with a shorter operating time. Because for subsystems with shorter operation times, reducing their failure rate is a more reasonable choice [35,38]. Assess the operating time of each subsystem as T_i.

Employ fuzzy set theory to evaluate the operating time of each subsystem, with specific evaluation criteria as outlined in Table 7 [18].

Table 7. Fuzzy evaluation grades of operating time.

Fuzzy Level	Operating Time Level Description	Fuzzy Number
L	Less than 25%	(1, 2, 3, 4)
M	More than 20% and less than 50%	(3, 4, 5, 6)
H	More than 50% and less than 75%	(5, 6, 7, 8)
VH	More than 75%	(7, 8, 9, 10)

The normalized operating time factor $\overline{T}_i$ is:

$$\overline{T}_i = \frac{T_i}{\sum_{i=1}^{n} T_i} \tag{28}$$

3.2. Calculation of the Overall Reliability Allocation Index

In the design phase of aircraft mechanical systems, it is necessary to assign the target reliability of the system to each subsystem, namely:

$$R_i = R_{obj}^{\omega_i} \tag{29}$$

where ω_i is the allocation weight factor, R_{obj} is the target reliability, and R_i is the reliability assigned to the ith subsystem.

It follows from Equation (29) that, when the range of ω_i is (0, 1), the smaller the value of ω_i assigned to a subsystem, the higher the value of R_i. Hence, inspired by Yadav and Zhuang [11] and combined with the definition of the above allocation factors, the contribution, severity and operating time are inversely proportional to the allocation weight, that is, $\omega_i \propto 1/C_i$, $\omega_i \propto 1/S_i$, $\omega_i \propto 1/T_i$, while the complexity, potential improvement and dynamic performance are proportional to the allocation weight, that is, $\omega_i \propto K_i$, $\omega_i \propto P_i$, $\omega_i \propto D_i$. Therefore, the allocation weight ω_i^* is calculated as:

$$\omega_i^* = \frac{K_i \times P_i \times D_i}{C_i \times S_i \times T_i} \tag{30}$$

The allocation weight can be further derived as:

$$\omega_i = \frac{\omega_i^*}{\sum_{i=1}^{n} \omega_i^*} \tag{31}$$

3.3. Reliability Models of Aircraft Mechanical Systems Considering Failure Correlation

3.3.1. Reliability Models

The reliability model aims to establish reliability mappings between a system and its constituent components; this is a key prerequisite for the rational allocation of reliability. As

shown in the system decomposition diagram in Figure 1, most subsystems are interrelated to perform the system's various functions. Consequently, traditional reliability models based on the assumptions of statistical independence, random independence and failure independence among subsystems are no longer applicable. This paper will apply Copula function to establish the reliability models for aircraft mechanical systems that take failure correlation into account.

Assume the reliability of the ith subsystem is R_i, where $R_i = \Pr(Y_i \geq 0)$ and Y_i represents the limit state function. F_i is the failure probability and is obtained by $F_i = 1 - R_i$, where Y_i can be generalized as $Y_i = \delta_i > h_i$. δ_i indicates the generalized strength of the ith subsystem and h_i indicates the generalized stress of the ith subsystem. For the motion functions of the aircraft mechanical systems, for example, δ_i can represent the allowable motion, h_i can be the actual motion error.

The failure probability F_{ser} of a series system is calculated by:

$$F_{ser} = \Pr\left\{(Y_1 \leq 0) \bigcup (Y_2 \leq 0) \bigcup \cdots \bigcup (Y_n \leq 0)\right\} \tag{32}$$

The copula function provides an effective solution for addressing failure correlation. The multiple joint cumulative distribution function (CDF) of variables $(X_1, X_2, \ldots, X_n)$ can be expressed as [39]:

$$F(x_1, \ldots, x_n) = \Pr\left\{(X_1 \leq x_1) \bigcap \ldots \bigcap (X_n \leq x_n)\right\} = C(F_1(x_1), \ldots, F_n(x_n); \alpha) \tag{33}$$

where $F(x_1, x_2, \ldots, x_n)$ presents the joint CDF of the variables $(X_1, X_2, \ldots, X_n)$, $F_i(x_i) = \Pr(X_i \leq x_i)(i = 1, 2, \ldots, n)$ is the marginal CDF of variable X_i, and α is the correlation coefficient. There are different Copula functions C to establish various dependence structures among variables.

In practical engineering applications, the various subsystems are interrelated and exhibit positive correlation. The Gumbel copula function, characterized by a high upper-tail coefficient and a low lower-tail coefficient, is well-suited to describing the positive correlation between variables exhibiting upper-tail correlation. Consequently, the Gumbel copula is particularly suitable for systems with a long service life [4,40].

The expression of the Gumbel copula is as follows:

$$C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n); \alpha) = \exp\left(-\left\{[-ln(F_1(x_1))]^\alpha + [-ln(F_2(x_2))]^\alpha + \ldots + [-ln(F_n(x_n))]^\alpha\right\}^{\frac{1}{\alpha}}\right) \tag{34}$$

where α is the correlation coefficient, $\alpha \in [1,\infty]$. When $\alpha = 1$, there is no correlation among variables. As α increases, the correction also increases.

α can be derived by the Kendall correlation coefficient τ; that is [39]:

$$\alpha = \frac{1}{1 - \tau} \tag{35}$$

At the initial stage of system design, it is a major challenge to determine the correlation coefficient α without historical data. Inspired by the load dispersion L_{sRi}, the value of α can be approximated by:

$$\alpha = \frac{1}{1 - \sqrt[n]{\prod_{i=1}^{n} L_{sRi}}} \tag{36}$$

Through Equation (34), the correlation between different subsystems can be quantified. For example, for two subsystems, the correlation coefficient can be derived as: $\alpha_{i,j} = \frac{1}{1-\sqrt[2]{L_{sRi} \times L_{sRj}}}$; for three subsystems, the correlation coefficient can be derived as: $\alpha_{i,j,l} = \frac{1}{1-\sqrt[3]{L_{sRi} \times L_{sRj} \times L_{sRl}}}$.

Applied Copula function, F_{ser} can be rewritten as

$$\begin{aligned} F_{ser}^{D} &= \sum_{i=1}^{n} F_i(0) - \sum_{1\le i<j\le n} \Pr\left(Y_i \le 0 \cap Y_j \le 0\right) - \cdots - (-1)^n \times \Pr(Y_1 \le 0 \cap \ldots \cap Y \le 0) \\ &= \sum_{i=1}^{n} F_i(0) - \sum_{1\le i<j\le n} C\left(F_i(0), F_j(0); \alpha_{i,j}\right) - \cdots - (-1)^n \times C\left(F_1(0), F_2(0), \ldots, F_n(0); \alpha_{i,j,\ldots,n}\right) \\ &= \Delta_{F_1(0)}^{1} \Delta_{F_2(0)}^{1} \cdots \Delta_{F_n(0)}^{1} C_n\left(F_1(0), F_2(0), \ldots, F_n(0); \alpha_{i,j,\ldots,n}\right) \end{aligned} \tag{37}$$

where Δ denotes the difference sign, and $\Delta_{x_1}^{x_2} f(x) = f(x_2) - f(x_1)$.

Finally, for an aircraft mechanical system in series, the reliability can be expressed as:

$$\begin{aligned} R_{ser} &= R_{ser}^{D} \times R_{ser}^{I} \\ &= \left(1 - \Delta_{F_1(0)}^{1} \Delta_{F_2(0)}^{1} \cdots \Delta_{F_{n1}(0)}^{1} C_{n1}\left(F_1(0), F_2(0), \ldots, F_{n1}(0); \alpha_{i,j,\ldots,n1}\right)\right) \times \prod_{i=n1+1}^{n-n1} R_i \end{aligned} \tag{38}$$

where R_{ser} is the reliability of a series system; R_{ser}^{D} and R_{ser}^{I} denote the reliability of the dependent subsystems and independent subsystems in this series system, respectively; and $n1$ is the total number of the dependent subsystems.

Similarly, the failure probability F_{par} of a parallel system is:

$$F_{par} = \Pr\left\{(Y_1 \le 0) \bigcap (Y_2 \le 0) \bigcap \cdots \bigcap (Y_n \le 0)\right\} \tag{39}$$

Considering the failure correlation, F_{par} can be derived as:

$$F_{par}^{D} = C\left(F_1(0), F_2(0), \ldots, F_n(0); \alpha_{ij\ldots n}\right) \tag{40}$$

For an aircraft mechanical system in parallel, the reliability can be calculated as:

$$R_{par} = 1 - F_{par}^{D} \times F_{par}^{I} \tag{41}$$

where R_{par} is the reliability of a parallel system, F_{ser}^{D} and F_{ser}^{I} denote the reliability of the dependent subsystems and independent subsystems in this parallel system, respectively. F_{par}^{I} can be obtained by $F_{par}^{I} = \prod_{i=1}^{n-n1} F_i$, where $n1$ is the total number of the dependent subsystems.

For an aircraft mechanical system comprising both series and parallel subsystems, it can be decomposed into series modules and parallel modules based on the logical relationships between the subsystems. By combining the above formulas, a reliability model for this hybrid system can then be derived.

3.3.2. Reliability Allocation

Reliability allocation can be conducted based on the reliability models of aircraft mechanical systems. To facilitate the calculation, the relative weighting method is adopted. Assume the relative weight ω_l' of the lth $(1 \le l \le n)$ subsystem can be expressed by $\omega_l' = \omega_l/\omega_k$, where ω_k of the kth $(1 \le k \le n)$ subsystem is the standard weight. The reliability assigned to the lth subsystem is $R_l = R_k^{\omega_l'}$. Thus, the reliability of the system R_s can be expressed as a function of a variable R_k, where R_s can be obtained based on the aforementioned reliability model. Let R_s be equal to R_s^{obj}, where R_s^{obj} is the target reliability to be allocated. Consequently, R_k can be obtained. R_l can also be derived by $R_l = R_k^{\omega_l'}$.

In particular, the allocation process of the parallel system is conducted from the perspective of failure probability. The reliability allocated to subsystem i is $R_i = 1 - \left(F_s^{obj}\right)^{\omega_i}$. Consequently, the relationship between the allocation weights and the allocated reliability differs from that in series systems. Consequently, for parallel systems, the allocation weight should be modified to $\omega_i = \frac{1/\omega_i^*}{\sum_{i=1}^{n} 1/\omega_i^*}$.

4. Engineering Case

As a key actuator in an aircraft rudder control system, the aircraft load mechanism is capable of performing complex tasks with high precision [41–43]. The aircraft load mechanism is a typical series mechanical system involving motion characteristics, as shown in Figure 3. This system requires each moving component to work in concert to perform tasks accurately and stably; consequently, the reliability of these components must be exceptionally high. Particularly in the case of modern fighter aircraft, the motion performance of the aircraft load mechanism directly impacts the aircraft's combat capabilities. Therefore, it is essential to allocate reliability within the aircraft load system accurately and appropriately.

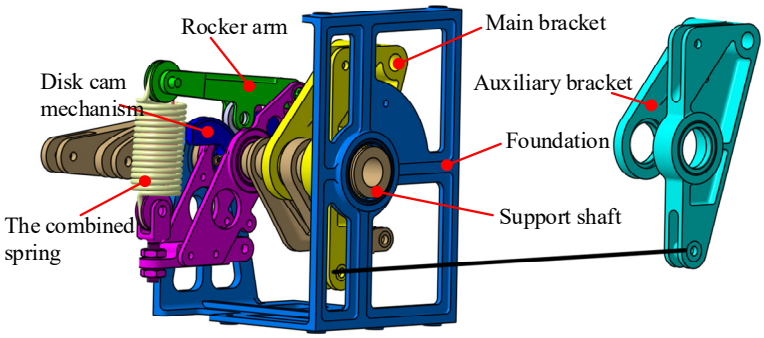


Figure 3. Composition of the aircraft load mechanism.

According to the design requirements, the task of the aircraft load mechanism is to provide stable and reliable resistance for the rudder control system. There are mainly five functions of the aircraft load mechanism, namely, the loading function, cyclic operation function, starting function, transmission function, and reset function. Among them, the first two subfunctions are caused by the fracture of components, while the latter three subfunctions are triggered by the motion faults of the subsystems.

In detail, the primary function of the aircraft load mechanism is to convert the input angle into torque output. Consequently, it is necessary to ensure that the transmission error of the mechanism remains within a defined range. Once the range is exceeded, the transmission function will fail. Simultaneously, considering the presence of joint clearance and wear during the motion process, a certain empty travel is allowed at the starting stage of the mechanism. However, when the empty travel is too large, the starting function will also fail. When there is no motion input to the mechanism, under the action of the pre-tightening force, the mechanism will automatically move to the balanced state. Therefore, when the pre-tightening force provided by the springs is insufficient to overcome the return resistance, the reset function of the mechanism will fail.

There are seven main subsystems in the system: auxiliary bracket, foundation, main bracket, support shaft, disk cam mechanism, rocker arm, and combined spring. Next, considering this case in its earlier design stage and using the proposed method, the reliability target value $R_s^{obj} = 0.9$ for the aircraft load mechanism is allocated to these seven subsystems. The system decomposition of the aircraft load mechanism is shown in Figure 4.

The mapping relationship between the system level and the subsystem level of the aircraft load mechanism is also shown in Table 8. The index numbers in Table 8 are listed in ascending order, corresponding to the subsystems shown in Figure 4 from left to right.

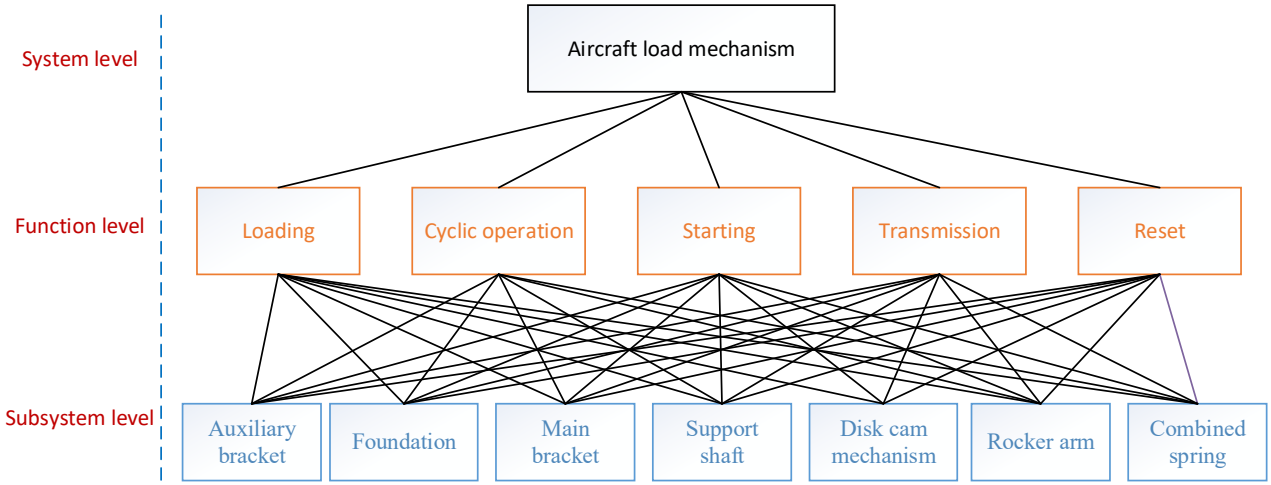


Figure 4. System decomposition of the aircraft load mechanism.

Table 8. Statistics of the subsystem function of the aircraft load mechanism.

	Structural Function		**Motion Function**		
	Loading	**Cyclic Operation**	**Starting**	**Transmission**	**Reset**
Number of subsystems	1, 2, 3, 4, 5, 6, 7	1, 2, 3, 4, 5, 6, 7	1, 2, 3, 4, 5, 6, 7	1, 2, 3, 4, 5, 6, 7	1, 2, 3, 4, 5, 6, 7

4.1. Calculation of Allocation Factors

During the motion transmission process of the aircraft load mechanism, the actions of each subsystem are triggered by the previous subsystem. Therefore, the motion functions of the system are mainly influenced by the motion subsystems. Therefore, based on the system decomposition principle in Figure 1, the subsystem contribution analysis is performed. Firstly, apply the AHP model to deal with the relative importance of each subfunction, as shown in Table 9. Then, by applying Equation (5), the relative importance index g_j of each subsystem is obtained, and the results are shown in the last column of Table 9.

Table 9. Subsystem relative importance matrix.

Subfunctions	**Loading**	**Cyclic Operation**	**Starting**	**Transmission**	**Reset**	g_i
Loading	1	1	1/4	1/3	1/2	0.090909091
Cyclic operation	1	1	1/4	1/3	1/2	0.090909091
Starting	4	4	1	4/3	2	0.363636364
Transmission	3	3	3/4	1	3/2	0.272727273
Reset	2	2	1/2	2/3	1	0.181818182

Secondly, the contribution matrix $\widetilde{U}$ is obtained and listed in Table 10.

Finally, apply Equations (7)–(11) and utilize the TOPSIS method to obtain the value of the subsystem contribution. The Euclidean distance and the normalized contribution index C_i are obtained, as shown in Table 11.

Collect the environmental conditions, technology level and the number of components of each subsystem; then the normalized complexity $\overline{K}_i$ is calculated by using Equations (13)–(15). The results are presented in the last column of Table 12.

Table 10. Contribution matrix for each subsystem.

Contribution $\widetilde{u}_{ij}$	Loading	Cyclic Operation	Starting	Transmission	Reset
Auxiliary bracket	M	H	R	R	L
Foundation	VH	M	R	R	L
Main bracket	M	M	R	R	L
Support shaft	H	M	L	R	R
Disk cam mechanism	L	L	M	M	R
Rocker arm	L	R	R	L	L
Combined spring	L	L	H	M	H

Table 11. Subsystem contribution index.

Subsystems	Euclidean Distance Ed_i^+	Euclidean Distance Ed_i^-	Contribution $\widetilde{C}_i$	Normalized Contribution C_i
Auxiliary bracket	3.630677372	1	0.215951128	0.084159457
Foundation	3.608987862	1.044465936	0.22444962	0.087471449
Main bracket	3.62953905	0.862439362	0.191995438	0.074823558
Support shaft	3.012371187	1.317397886	0.304265161	0.118576786
Disk cam mechanism	1.997932816	2.617487282	0.567117884	0.221014512
Rocker arm	3.44736095	1.004123729	0.225570523	0.087908283
Combined spring	0.744122979	3.81059853	0.83662602	0.326045954

Table 12. Subsystem complexity index.

Subsystems	Environment Condition E_i	Technology Level T_i	Load Dispersion L_{sri}	Number of Components n_i	Modified Complexity K_i^*	Normalized Complexity K_i
Auxiliary bracket	0.45	0.45	0.5	3	1.732050808	0.130422936
Foundation	0.54	0.54	0.5	3	1.732050808	0.130422936
Main bracket	0.55	0.54	0.504587156	5	2.219620452	0.167136793
Support shaft	0.52	0.39	0.571428571	6	2.155228856	0.162288124
Disk cam mechanism	0.69	0.73	0.485915493	4	2.039434275	0.153568825
Rocker arm	0.32	0.48	0.4	2	1.515716567	0.114133029
Combined spring	0.49	0.67	0.422413793	3	1.886160544	0.142027356

Based on the FMEA information and the characteristics of the system, fuzzy evaluations are conducted on the failure modes, severity, occurrence, dynamic performance and operating time of each subsystem, as shown in Table 13.

The evaluation results are defuzzied by using the method in Section 2.2. Assuming that the transformation coefficient $\theta = 0.8$, the normalized severity $\overline{S}_i$ can be obtained by using Equations (16)–(18). The results are presented in the last column of Table 14.

For the aircraft load mechanism, fracture, joint stuck, joint wear, loose assembly, and stress relaxation are the primary failure modes. Among them, the last three failure modes are mainly affected by the abnormal action between the components. The evaluation of occurrence in Table 13 is firstly converted into fuzzy numbers, and after defuzzification by Equation (3), the score of the occurrence can be obtained, which is shown in Table 15.

The subsystems of the aircraft load mechanism are influenced by the same environmental variables, and execute the system functions corporately, so there exists failure correlation between the subsystems. By applying Equation (21) and assuming $\mu = 0.2$, the failure correlation index ε_i can be obtained. Then, according to Equations (19)–(25),

assuming the constant parameter $r = 100$, the potential reliability improvement index can be obtained, which is shown in Table 16.

Table 13. Fuzzy evaluation of each subsystem.

Subsystems	Failure Modes	Severity S_{ij}	Occurrence O_{ij}	Occurrence O_{ij}^{a}	Dynamic Performance D_i	Operating Time T_i
Auxiliary bracket	FM11—fracture	M	R	--	M	M
	FM12—joint stuck	VL	--	R		
Foundation	FM21—fracture	L	R	--	R	M
	FM22—joint stuck	VL	--	R		
Main bracket	FM31—fracture	VH	L	--	L	M
	FM32—joint wear	L	--	L		
Support shaft	FM41—fracture	HWW	M	--	H	VH
	FM42—loose assembly	H	--	H		
Disk cam mechanism	FM51—fracture	M	H	--	VH	H
	FM52—joint wear	HWW	--	M		
	FM53—loose assembly	H	--	R		
Rocker arm	FM61—fracture	H	M	--	M	M
	FM62—joint wear	M	--	R		
Combined spring	FM71—fracture	L	R	--	M	VH
	FM72—stress relaxation	VH	--	R		

Table 14. Subsystem severity index.

Subsystems	Failure Modes	Fuzzy Numbers of S_{ij}	Defuzzied S_{ij}	Transformed Severity S_{ij}^{*}	Normalized Severity S_i
Auxiliary bracket	FM11	(5, 6, 7)	6	121.5104175	0.028068373
	FM12	(3, 4, 5)	4		
Foundation	FM21	(4, 5, 6)	5	54.59815003	0.012611933
	FM22	(3, 4, 5)	4		
Main bracket	FM31	(7, 8, 9)	8	601.8450379	0.139023562
	FM32	(4, 5, 6)	5		
Support shaft	FM41	(8, 9, 10)	9	1339.430764	0.309402628
	FM42	(6, 7, 8)	7		
Disk cam mechanism	FM51	(5, 6, 7)	6	1339.430764	0.309402628
	FM52	(8, 9, 10)	9		
	FM53	(6, 7, 8)	7		
Rocker arm	FM61	(6, 7, 8)	7	270.4264074	0.062467313
	FM62	(5, 6, 7)	6		
Combined spring	FM71	(4, 5, 6)	5	601.8450379	0.139023562
	FM72	(7, 8, 9)	8		

The evaluation of dynamic performance rating and operating time rating in Table 13 is firstly converted into fuzzy numbers, and after defuzzification by Equation (3), the dynamic performance index and the operating time index can be obtained, which are shown in Table 17.

Finally, Table 18 summarizes the six weighting factors. The allocation weight ω_i can be obtained by Equations (30) and (31). Apply the relative weighting method, and the relative allocation weight can be derived as $\omega_i' = \omega_i/\omega_1$, which is shown in the last column of Table 18.

As the aircraft load mechanism is a series system, under the independent assumption, the reliability of the system can be derived as $R_{sys}^{I} = \prod_{i=1}^{7} R_i$. The reliability of each subsystem can be expressed as $R_i^{*} = \left(R_s^{obj}\right)^{\omega_i}, i = 1, 2, 3, 4, 5, 6, 7$.

Table 15. Subsystem occurrence index.

Subsystems	Failure Modes	Fuzzy Numbers of O_{ij}	Fuzzy Numbers of O_{ij}^{a}	O_{ij}	O_{ij}^{a}
Auxiliary bracket	FM11	(1, 1, 1, 2)	--	1.333333333	--
	FM12	--	(1, 1, 1, 2)	--	1.333333333
Foundation	FM21	(1, 1, 1, 2)	--	1.333333333	--
	FM22	--	(1, 1, 1, 2)	--	1.333333333
Main bracket	FM31	(1, 2, 3, 4)	--	2.5	--
	FM32	--	(1, 2, 3, 4)	--	2.5
Support shaft	FM41	(3, 4, 5, 6)	--	5	--
	FM42	--	(6, 7, 8, 9)	--	7.5
Disk cam mechanism	FM51	(6, 7, 8, 9)	--	7.5	--
	FM52	--	(3, 4, 6, 7)	--	5
	FM53	--	(1, 1, 1, 2)	--	1.333333333
Rocker arm	FM61	(3, 4, 6, 7)	--	5	--
	FM62	--	(1, 1, 1, 2)	--	1.333333333
Combined spring	FM71	(1, 1, 1, 2)	--	1.333333333	--
	FM72	--	(1, 1, 1, 2)	--	1.333333333

Table 16. Potential reliability improvement index of subsystems.

Subsystems	Correlation Index ε_i	Occurrence O_{ij}^{*}	Occurrence O_{ij}^{a*}	Failure Rate λ_i	Improvement P_i	Normalized $\overline{P_i}$
Auxiliary bracket	1.105170918	1.473561224	1.473561224	0.000284461	0.081649215	0.184825238
Foundation	1.105170918	1.473561224	1.473561224	0.000284461	0.081649215	0.184825238
Main bracket	1.106185302	2.765463254	2.765463254	0.000769405	0.071698978	0.162301387
Support shaft	1.121072386	5.605361928	8.408042891	0.033112741	0.034078394	0.077141555
Disk cam mechanism	1.102062141	8.265466059	6.979726894	0.036477781	0.033110542	0.074950675
Rocker arm	1.083287068	5.416435338	1.444382757	0.003102933	0.057754112	0.130735092
Combined spring	1.088154083	1.45087211	1.45087211	0.000279533	0.081823966	0.185220815

Table 17. Dynamic performance and operating time of subsystems.

Subsystems	Fuzzy Number of D_i	Dynamic Performance D_i	Normalized $\overline{D_i}$	Fuzzy Number of T_i	Operating Time T_i	Normalized $\overline{T_i}$
Auxiliary bracket	(3, 4, 6, 7)	5	0.140625	(1, 2, 3, 4)	4.5	0.108433735
Foundation	(1, 1, 1, 2)	1.333333333	0.0375	(3, 4, 5, 6)	4.5	0.108433735
Main bracket	(1, 2, 3, 4)	2.5	0.0703125	(3, 4, 5, 6)	4.5	0.108433735
Support shaft	(6, 7, 8, 9)	7.5	0.2109375	(7, 8, 9, 10)	8.5	0.204819277
Disk cam mechanism	(8, 9, 10, 10)	9.222222222	0.259375	(5, 6, 7, 8)	6.5	0.156626506
Rocker arm	(3, 4, 6, 7)	5	0.140625	(3, 4, 5, 6)	4.5	0.108433735
Combined spring	(3, 4, 6, 7)	5	0.140625	(7, 8, 9, 10)	8.5	0.204819277

However, according to the working principle of the aircraft load mechanism, the main function of the system is to achieve motion and torque transmission, which is accomplished through the coordinated operation of different subsystems. For example, the disk cam mechanism, rocker arm and combined spring all play crucial roles in the system's transmission function and reset function. Therefore, there is a failure correlation among these three

subsystems. Similarly, there is a failure correlation between auxiliary bracket and main bracket. Based on Equation (38), the reliability model of the aircraft load mechanism can be derived as:

$$R_s = R^I \times R^D = R_2 \times R_4 \times R_{1,3} \times R_{5,6,7} \tag{42}$$

where $R_{1,3} = (1 - F_1 - F_3 + C_{Gum}(F_1, F_3; \alpha_{1,3}))$ and $R_{5,6,7} = R_5 * R_6 * R_7 = 1 - \sum_{i=5}^{7} F_i + C_{Gum}(F_5, F_6; \alpha_{5,6}) + C_{Gum}(F_5, F_7; \alpha_{5,7}) + C_{Gum}(F_6, F_7; \alpha_{6,7}) - C_{Gum}(F_5, F_6, F_7; \alpha_{5,6,7})$.

Table 18. Summary of the weighting factors.

Subsystems	C_i	$\bar{K}_i$	$\bar{S}_i$	$\bar{P}_i$	$\bar{D}_i$	$\bar{T}_i$	ω_i^*	ω_i	ω_i'
Auxiliary bracket	0.084159457	0.130422936	0.028068373	0.184825238	0.140625	0.108433735	13.23406312	0.489529417	0.084159457
Foundation	0.087471449	0.130422936	0.012611933	0.184825238	0.0375	0.108433735	7.556733758	0.279524394	0.087471449
Main bracket	0.074823558	0.167136793	0.139023562	0.162301387	0.0703125	0.108433735	1.690969011	0.062549125	0.074823558
Support shaft	0.118576786	0.162288124	0.309402628	0.077141555	0.2109375	0.204819277	0.351426428	0.012999301	0.118576786
Disk cam mecha-nism	0.221014512	0.153568825	0.309402628	0.074950675	0.259375	0.156626506	0.278738318	0.01031056	0.221014512
Rocker arm	0.087908283	0.114133029	0.062467313	0.130735092	0.140625	0.108433735	3.523863012	0.130348072	0.087908283
Combined spring	0.326045954	0.142027356	0.139023562	0.185220815	0.140625	0.204819277	0.398461413	0.014739131	0.326045954

By applying Equation (36), the correlation coefficient α of different subsystems can be obtained. Based on the relative weight method, that is $\omega_i' = \omega_i/\omega_1$, then R_s can be expressed as a function of R_1. Making R_s equal to $R_s^{obj} = 0.9$, the fminbnd function in MATLAB R2023a software is utilized to solve it. The calculation principle by applying the fminbnd function is as follows:

$$\begin{aligned} &Find \quad R_1 \\ &min \quad f(R_1) = R_s^{obj} - R_s \\ &s.t. \begin{cases} R_s^{obj} \le R_s; \\ R_s = R^I \times R^D = R_2 \times R_4 \times R_{1,3} \times R_{5,6,7}; \\ R_{1,min} \le R_1 \le R_{1,max}; \\ R_i = R_1^{\omega_i'}, i = 2,3,4,5,6,7 \end{cases} \end{aligned} \tag{43}$$

The equation is solved for the optimal value using the golden-section search and parabolic interpolation method [44]. The reliability assigned to each subsystem is listed in Table 19. In addition, Table 19 presents the allocation results under failure independence, as well as the relative errors in the allocation results obtained from these two models.

Table 19. Reliability allocation results of the aircraft load mechanism.

Subsystems	Reliability of Subsystems			
	Independent	Dependent	Relative Difference (%)	Ranking Index
Auxiliary bracket	0.94973045	0.948169345	0.16464404	7
Foundation	0.970978615	0.97006695	0.093979614	6
Main bracket	0.99343146	0.993222663	0.021022131	4
Support shaft	0.998631324	0.9985877	0.004368568	2
Disk cam mechanism	0.998914264	0.998879653	0.003464969	1
Rocker arm	0.986360335	0.985928363	0.04381368	5
Combined spring	0.998448283	0.998398829	0.004953282	3

The reliability allocation values for the aircraft load mechanism, ranked from highest to lowest, are shown in the final column of Table 19. So, the reliability allocation values of the aircraft load system are ranked as follows: disk cam mechanism > support shaft > combined spring > main bracket > rocker arm > foundation > auxiliary bracket. The severity and dynamic performance ratings of the disk cam mechanism are higher, while the potential improvement rating is lower. Thus, the disk cam mechanism has been assigned the highest level of reliability, which means that in the initial stage of system design, the design requirements for the disk cam mechanism are the most stringent.

Since support shaft is an important connecting subsystem of the system and is connected with the disk cam mechanism and the main bracket, it has many design parameters. Therefore, the reliability assigned to support shaft is second only to that of the disk cam mechanism. Also, because the foundation and auxiliary bracket contribute less to the subfunctions of the system and are composed of mature technical parts, they are assigned a lower level of reliability.

The relative difference shown in Table 19 is defined as: $relative\ difference = \frac{result\ of\ dependent\ model - result\ of\ independent\ model}{result\ of\ independent\ model} \times 100\%$. It can be concluded from the results that the relative difference is greatest for the subsystem auxiliary bracket, reaching 0.165%. It is therefore essential to take failure correlation into account when allocating reliability within an aircraft's mechanical systems.

The allocation results indicate that, when the failure correlation among subsystems is taken into account, the reliability levels assigned to the subsystems are lower than those in the case where subsystem failures are independent. Furthermore, the reliability level of the entire system remains unchanged and continues to meet the design requirements. From a practical perspective, reducing the reliability levels of the subsystems also reduces the design and manufacturing costs, thereby making it easier for the subsystems to meet the standards.

This conclusion can also be explained from an engineering perspective. As the reliability of a series aircraft mechanical system depends on the reliability of its weakest unit, which may be enhanced by the positive influence of other units that have not yet failed. Consequently, the reliability of the weakest unit is higher than when the units are independent.

4.2. Parameter Sensitivity Analysis

In order to investigate the influence of the parameters μ and α on the reliability allocation results, different cases are set to analyze their sensitivity.

μ is a parameter used to describe the interaction between subsystems, which influences the allocation factor for the potential improvement. The reliability ratios for different μ values have been derived, with the allocation results for $\mu = 0.1$ serving as the reference baseline. The results are shown in Figure 5.

It can be concluded from Figure 5 that, as the value of μ increases, the reliability assigned to auxiliary bracket, foundation and combined spring decreases accordingly, with the change in the auxiliary bracket being the most pronounced; at the same time, the reliability assigned to main bracket, support shaft, disc cam mechanism and rocker arm increases accordingly, with the rocker arm showing the most significant increase. As the current reliability of auxiliary bracket and rocker arm is relatively low, the changes in these two subsystems are particularly pronounced. Furthermore, as both subsystems interact with other subsystems, they are significantly affected by the value of μ.

α is a parameter of the Copula function that describes the correlation of failures between subsystems. Reliability ratios for different values of α have been derived, with

reference to the reliability allocation results calculated using the value of α obtained from Equation (36). These results are shown in Figure 6.

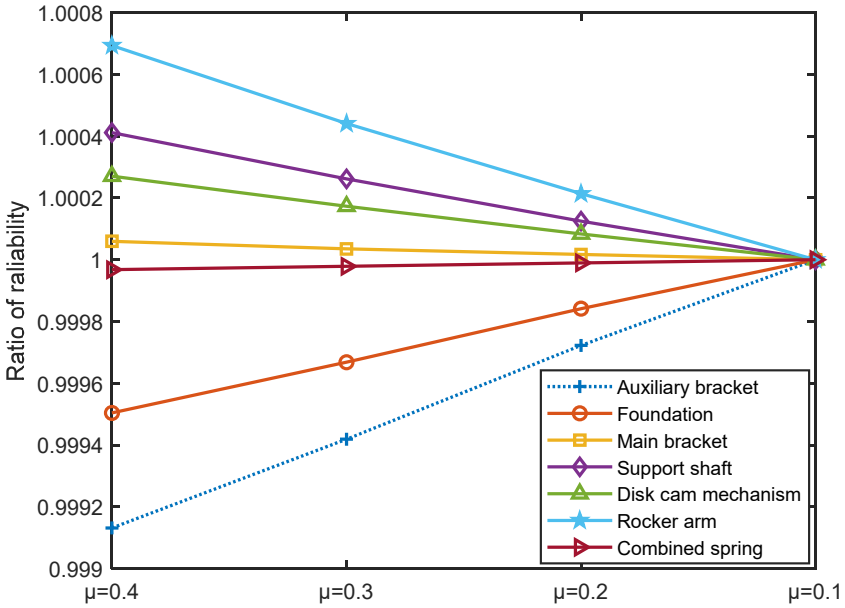


Figure 5. Reliability sensitivity with parameter μ.

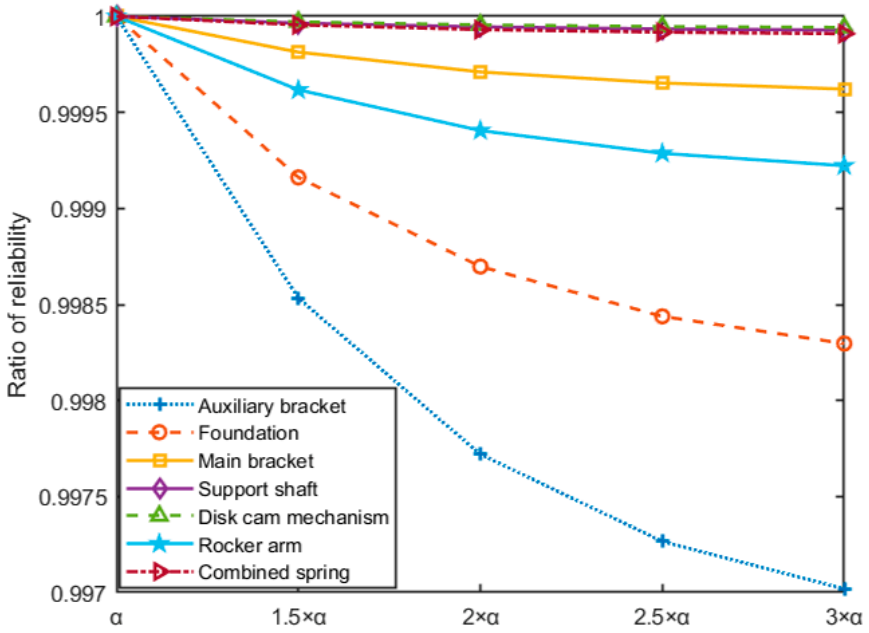


Figure 6. Reliability sensitivity with parameter α.

It can be concluded from Figure 6 that, as the value of α increases, the reliability assigned to each subsystem decreases. This indicates that as the correlation of subsystem failures increases, it becomes easier for each subsystem to meet the expected reliability targets, provided that the overall system reliability target is satisfied. The trends for the auxiliary bracket, foundation and rocker arm are particularly pronounced; this may be due to the relatively low reliability currently allocated to these subsystems. Consequently, the ranking of sensitivity is also related to the current reliability of the subsystems.

Overall, the sensitivity analysis described above demonstrates that the interactions between subsystems cannot be overlooked; they must be taken into account not only

when modeling system reliability but also when assigning factors. It also highlights the importance of correctly quantifying failure-related relationships.

4.3. Comparison and Discussion

To further verify the validity of the proposed method, the engineering example is also applied to compare the proposed method with those proposed by Yadav and Zhuang [11], Zhang [4], Yang [33], and Chen [37] and the integrated factors method (IFM) [45]. According to the method of Yadav [11], the nonlinearity severity and failure rate were considered. In the methods of Zhang [4], Yang [33] and Chen [37], failure correlation is considered. The IFM is a traditional allocation method that considers more comprehensive weighted factors. To facilitate a more intuitive comparison, the results are presented through graphics, as shown in Figure 7.

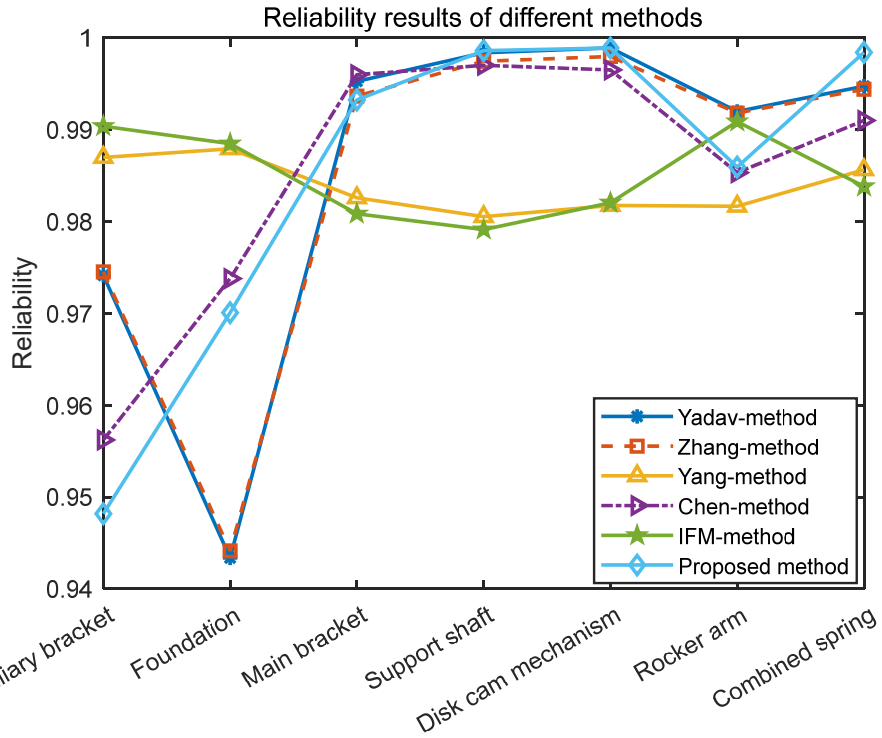


Figure 7. Comparison plots of the results for different allocation methods.

It can be obviously concluded from Figure 7 that the reliability allocation value of the auxiliary bracket in the proposed method is lower than that in the previous methods, because the motion performance and subsystem contribution are considered. Similarly, in the methods proposed by Yadav [11] and Zhang [4], the reliability allocation value of the foundation is the smallest, which does not correspond to engineering practice. This is because they have not taken into account the influence of the foundation on the loading and cyclic operation of the aircraft load mechanism. IFM [45] fails to account for the interaction between subsystems caused by load dispersion, meaning that the allocation results are significantly influenced by the number of components, which is inconsistent with engineering practice. Additionally, neither the method proposed by Yadav [11] nor the IFM [45] takes into account the failure correlation, and consequently, the overall allocation results obtained are higher.

Furthermore, Yang's method [33] employed an averaging approach to handle the influencing factors of the subsystems, causing the allocation weights of each subsystem to become concentrated. This led to similar allocation results, thereby weakening the

impact of the weighting factors and failing to adequately reflect the differences between the subsystems.

An analysis of the results obtained using Chen's method [37] reveals that they are relatively close to those of the method proposed in this paper. However, Chen's method [37] fails to account for interactions between subsystems in the allocation factors, resulting in little difference in the reliability allocated to the three subsystems: main bracket, support shaft and disc cam mechanism. In the sensitivity analysis results in Section 4.2, it can also be observed that the reliability allocated to support shaft and disc cam mechanism increases as the interaction between components increases. Conversely, the reliability of auxiliary bracket and foundation decreases as the interaction between subsystems increases. Additionally, when modeling the system reliability, there is a lack of effective computational methods to distinguish the correlations between different subsystems in Chen's method [37], which further validates the comprehensiveness of the proposed method.

In summary, it is essential to establish a correct functional decomposition model of the system, quantify the influence of each subsystem on the overall motion characteristics of the system, and take into account the interactions between subsystems in both the factor allocation model and the system reliability prediction model. All of the above are of great significance in enhancing the validity of the allocation results.

5. Conclusions

This paper aims to address the issue of reliability allocation in aircraft mechanical systems concerning motion performance and proposes a reliability allocation method. The main contributions are as follows:

(1) To comprehensively assess the motion performance of the aircraft mechanical systems, a system decomposition model is established and expanded, thereby establishing a mapping relationship between subfunctions and subsystems. Furthermore, the AHP method is employed to quantify the contribution of each subsystem to both the structural function and motion function of the system.
(2) Weighting factors for system allocation are proposed and refined. The complexity of each subsystem is modified according to the load dispersion, whilst taking into account the interactions between subsystems. A failure correlation index is also proposed to highlight the impact of these interactions on the failure occurrence rating.
(3) Non-destructive failure events that could have a significant impact on motion performance have been taken into account in the potential failure rate. Additionally, the dynamic performance index and operating time index are also developed.
(4) A reliability prediction model for the system to be allocated is established using Copula functions, which can be applied to both series and parallel systems. A method for calculating the correlation coefficient is introduced to distinguish and quantify the correlation between different subsystems.

Finally, the method proposed in this paper can also be applied to complex systems such as an aircraft flap mechanism, aircraft gear door lock system, industrial robots, etc. By considering motion reliability, failure correlation and system performance in reliability allocation, the method can better reflect actual conditions, thereby providing an important reference for reliability engineering.

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