

Article

Shock Angle Characteristics and Test Analysis of Hypersonic Wide-Speed-Range Cruise Aircraft

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Abstract

Hypersonic aircraft represent a cutting-edge technology in aerospace engineering, where the shock angle serves as a critical aerodynamic parameter. However, existing studies remain limited by significant prediction errors for the shock angle. This study employs a combination of numerical simulation and wind tunnel test techniques to analyze the shock angle characteristics of hypersonic wide-speed-range cruise aircraft. Consequently, a numerical simulation analysis model for the shock angle of such aircraft was established. Shock angle measurement tests were conducted at various Mach numbers in a pulsed combined high-enthalpy wind tunnel. Comparing the simulation results to the wind tunnel results revealed a numerical error of 4.08%, validating the accuracy of the numerical model. Shock angles at Mach numbers 6, 7, 8, 9, 10, 12, 15 and 20 were analyzed in the numerical simulations, and a nonlinear fitting method was used to determine the functional relationship between the shock angle and Mach number. The results indicate that as the Mach number increases, the shock angle progressively decreases, and its attenuation rate diminishes. The shock angle exhibits an exponentially decreasing relationship with the Mach number, approaching 10.708° as the Mach number approaches infinity. This study provides methodological support and data references for predicting shock wave characteristics and designing aerodynamic hypersonic aircraft.

Keywords: shock angle; hypersonic; cruise aircraft; wide-speed-range; numerical simulation; wind tunnel test

1. Introduction

As a critical aerodynamic parameter, the shock angle directly influences flight stability, thermal load distribution and structural design [1,2]. Under wide-speed-range cruise conditions, the nonlinear evolution of the shock angle becomes particularly complex. Existing theoretical models are often constrained by high prediction errors, necessitating integrated approaches combining high-accuracy numerical simulation and test validation [3,4]. The hypersonic ($Ma > 5$) aircraft represents a cutting-edge aerospace technology with significant strategic value in military reconnaissance, long-range strikes, space exploration and high-speed transportation [5,6].

Rankine and Hugoniot developed the Rankine–Hugoniot relations [7,8], laying the theoretical framework for abrupt changes in thermodynamic parameters across shock fronts. Busemann [9] introduced the theory of shock polar lines, providing graphical tools for understanding shocks. Shapiro [10] established the fundamental model for shock angles in compressible fluids. The shock angle theory originates from classical gas dynamics [11].



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Anderson [1] systematically elucidated the mechanisms of hypersonic flow. Joumaa [12] derived analytical expressions for the maximum wedge angle and shock angle, discussing the application scenarios and limitations of the expressions. Prandtl and Meyer proposed the theory of supersonic expansion waves, laying the foundation for studying flow parameter relationships between the shock angle, Mach number and deflection angle [13].

When a vehicle flies at high Mach numbers, the surrounding flow field undergoes extreme aerodynamic heating and complex physicochemical processes. During this process, gas molecules undergo phenomena such as vibrational excitation, dissociation and ionization, collectively termed the real gas effect. Anderson [1] systematically expounded the fundamental principles of high-temperature gas dynamics, providing a comprehensive theoretical and knowledge framework for understanding and researching hypersonic real gas effects. Maus [14], whilst analyzing anomalous pitch moments in the Space Shuttle, observed that ground wind tunnel data aligned well with flight data when aerodynamic models accounted for air dissociation. Candler [15] developed Direct Simulated Monte Carlo (DSMC) methods and high-fidelity computational fluid dynamics programs capable of directly simulating microscopic physical processes within shock layers, providing powerful tools for understanding non-equilibrium effects. Park [16] established multiple chemical kinetic models, with his two-temperature model—separating vibrational temperature from translational–rotational temperature—significantly enhancing the prediction accuracy for vibrational relaxation and dissociation rates. Gupta [17] systematically compiled thermodynamic data and transport properties for eleven air composition models, establishing a standard database for finite-rate chemical reaction calculations in most hypersonic CFD software (Fluent V19.0+). Bertin [18] discussed the impacts of real gas effects in high-enthalpy flows on engineering design. The core challenge in physical modeling lies in accurately describing the thermochemical non-equilibrium processes within air. Horning [19] explored the complex coupling mechanism between real gas effects and transition processes. Theoretical models often neglect viscous effects and real gas effects [20], leading to insufficient reliability in engineering applications. Knight [3] assessed the impact of real gas effects on the predictive accuracy of hypersonic aerothermal heating. Stemmer [21] employed high-fidelity DNS computations to reveal thermochemical non-equilibrium effects on obstacle flow around hypersonic boundary layers. Hao [22] examined the influence of vibrational non-equilibrium effects on hypersonic shock/laminar boundary layer interactions. Rahmani [23] employed a high-order WENO scheme to investigate the interaction between real gas effects and shock disturbances around blunt bodies. Urzay [24] analyzed the interactions between real gas effects and turbulence. Zanardi [25] utilized Physically Informed Neural Networks (PINNs) for efficient simulation of hypersonic thermochemical non-equilibrium effects. Rui [26] discovered that non-Boltzmann distributions of oxygen molecular vibrational energies influence dissociation rates under high-temperature non-equilibrium conditions. As aerospace applications advance towards higher Mach numbers, future research necessitates further breakthroughs in real gas effect modeling and high-fidelity simulation capabilities.

Many scholars have conducted in-depth research on high-accuracy shock-capturing formats, adaptive mesh techniques and simulations of complex physical effects. Wilcox [27] developed the k - ω turbulence model, which is widely applied in hypersonic simulations using the Fluent software (Fluent V19.0+). Nagdewe [28] compared the performance of HLLC and AUSM schemes in simulating hypersonic flows, demonstrating their ability to more precisely capture flow features like shock waves. Addressing the thermochemical non-equilibrium effects essential in hypersonic flows, Prakash [29] proposed a fifth-order accurate shock-fitting method. Jun [30] proposed a shock-fitting technique based on unstructured moving meshes, maintaining stability even under irregular shock conditions.

Li [31] analyzed the application of high-order DG methods in hypersonic flow simulations. Ben [32] introduced an adaptive meshing technique that automatically refines the mesh near the shock wave based on flow field characteristics, yielding more accurate shock angles. Maeda [33] described an improved scheme to suppress numerical instabilities caused by strong shock computations at high Mach numbers. Huang [34] developed a shock-capturing method combining adaptive mesh refinement with positivity preservation techniques. With significant advances in numerical simulation techniques, Wang [35] enhanced the shock angle prediction accuracy using the SST k-omega model. Ren [36] conducted an in-depth analysis of the numerical stability issues in second-order Godunov-type schemes when simulating strong shocks at high Mach numbers.

Shock angle test analysis methods primarily rely on wind tunnel tests. Bingham [37] simulated the interaction between the bow shock wave of a hypersonic aircraft and an externally generated shock wave in a Mach 7.3 shock tunnel. White [38] investigated shock wave/turbulent boundary layer interactions near the expansion angle in a shock tunnel. Settles [39] detailed the application of a shadowgraph system in shock visualization. Murray [40] conducted a shock angle measurement test at Mach 8.9, although these experiments did not cover multiple flight conditions. Mudford [41] proposed a novel “free-flight” test method in a Mach 6 wind tunnel, enabling accurate acquisition of shock angle for models under hypersonic conditions. China’s JF-12 hypersonic shock tunnel [42,43] can reproduce Mach 5–9 flight environments under laboratory conditions, employing a shadowgraph system to obtain shock distributions on aircraft surfaces. Giepman [44] conducted shock studies at Mach 1.6–2.3, focusing on supersonic flight. Barber [45] noted that existing work has predominantly targeted narrow-speed-range systems, with limited data available for wide-speed-range systems. Saiprakash [46] utilized a high-speed shadowgraph system to visualize the flow field development around a pointed cone model at high Mach numbers and measured the shock angle. Shock angle variations across a wide speed range exhibit nonlinear trends [47]. In summary, researchers in this field face the following challenges at present: traditional shock angle prediction methods exhibit high error rates, numerical simulation models for shock angles lack precision, and a quantitative understanding of shock angle evolution across wide-speed-range aircraft remains insufficient.

Therefore, this study innovatively combines high-precision numerical simulation technology with pulsed high-enthalpy wind tunnel tests to investigate the shock angle for hypersonic wide-speed-range cruise aircraft. Consequently, a high-precision numerical simulation model for the shock angle is developed and validated through wind tunnel tests, revealing the relationship between the shock angle and Mach number. In particular, the functional relationship between the shock angle and Mach number is fitted to enable the prediction of the shock angle under extreme operating conditions. This study provides methodological support and data references for predicting shock wave characteristics, as well as for the aerodynamic design of hypersonic aircraft.

2. Model and Methodology for Shock Angle Study

2.1. Aircraft Model

A hypersonic cruise aircraft model was constructed using SolidWorks2018, as shown in Figure 1. The aircraft primarily consists of a nose section and a fuselage.

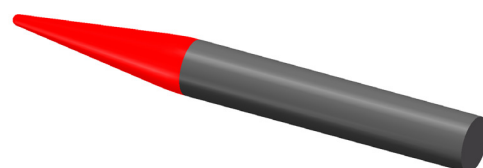


Figure 1. Hypersonic cruise aircraft.

The aerodynamic dimensions of the aircraft are illustrated in Figure 2. The aircraft diameter is $D = 80$ mm. The total axial length of the aircraft is $L1 = 9D = 720$ mm. The axial length of the nose section (conical portion) is $L2 = 3.5D = 280$ mm. The axial length of the fuselage section (constant cross-section portion) is $L3 = 5.5D = 440$ mm. The nose radius is $R1 = 0.1D = 8$ mm. The arc radius between the nose section and the fuselage is $R2 = 4D = 320$ mm.

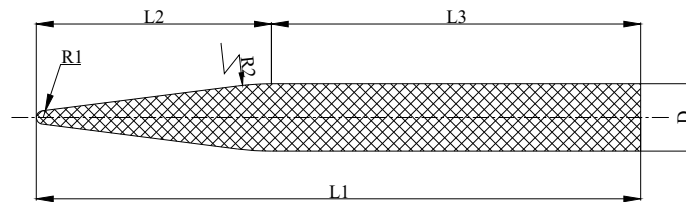


Figure 2. A schematic diagram of the aircraft's aerodynamic dimensions.

2.2. Numerical Simulation and Test Methods for Shock Angle

2.2.1. Numerical Simulation and Test Analysis Methods

First, we establish a hypersonic cruise aircraft and fluid domain model based on SolidWorks. Subsequently, we perform mesh generation on the fluid domain model using Pointwise. Then, we conduct a numerical simulation of the shock angle for the aircraft under various Mach number flight conditions using Fluent, thereby establishing a numerical simulation analysis model for the shock angle of hypersonic wide-speed-range cruise aircraft. Shock angle measurement tests at various Mach numbers were conducted in a pulsed combined high-enthalpy wind tunnel. Finally, the shock angle characteristics of the aircraft at various Mach numbers were analyzed by integrating the numerical simulation and the wind tunnel test results. The numerical simulation and test analysis method for the shock angle of hypersonic cruise aircraft is illustrated in Figure 3.

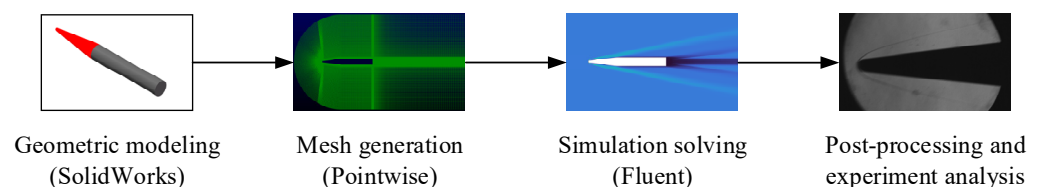


Figure 3. Schematic of shock angle numerical simulation and test analysis method.

2.2.2. Establishment of Shock Angle Numerical Simulation Analysis Model

Due to the axisymmetric structure of the cruise aircraft, an axisymmetric two-dimensional (2D) model replaces the three-dimensional (3D) model for numerical simulation to enhance computational efficiency. The boundary naming scheme is illustrated in Figure 4. The aircraft boundary consists of the nose, forepart, fuselage and tail, with all boundary types set as the wall.

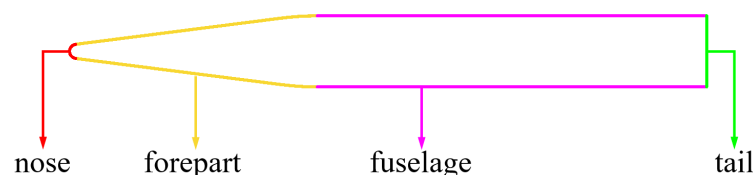


Figure 4. Aircraft boundary naming.

The external flow domain model for the aircraft was established in SolidWorks. The schematic diagram of the fluid domain dimensions is shown in Figure 5. The distance between the fluid domain inlet and the aircraft is 5 times the aircraft diameter,

i.e., $L4 = 5D = 400$ mm. The distance between the fluid domain outlet and the aircraft is 20 times the aircraft diameter, i.e., $L5 = 20D = 1600$ mm. The radial diameter of the flow domain is 17 times the aircraft diameter, i.e., $L6 = 17D = 1360$ mm.

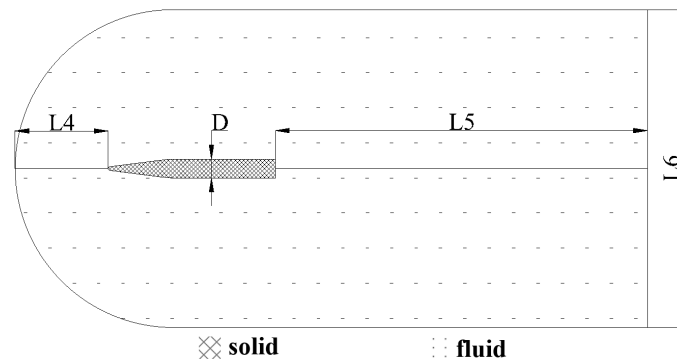


Figure 5. Schematic diagram of fluid domain dimensions.

The fluid domain boundary consists of an inlet (in), an outlet (out) and an axis of symmetry (axis). The inlet boundary is a pressure far field, the outlet boundary is a pressure outlet, and the axis of symmetry boundary is axisymmetric. Due to the significant gradient in aerodynamic characteristics of the boundary layer at hypersonic speeds, it is essential to appropriately set the height of the first mesh layer in the boundary layer on the aircraft surface. This enhances the quality of the fluid domain mesh and improves computational accuracy. The first boundary layer height in the numerical simulation is calculated using Equation (1) [13]. Here, ρ is air density, $\rho = 0.194$ kg/m³; V_c is the incoming flow velocity, $V_c = 1770$ m/s; L is the characteristic length scale, $L = 0.72$ m; μ is the dynamic viscosity coefficient, $\mu = 1.4216 \times 10^{-5}$ kg/(m · s); Re is the Reynolds number; C_f is the friction drag coefficient; τ_w is the wall shear stress; u_τ is the friction velocity; y^+ is the dimensionless wall distance, $y^+ = 4$; h^* is the boundary layer height correction factor, $h^* = 2$; and h is the first boundary layer height, which is calculated using Equation (1) (i.e., $h = 0.0096$ mm ≈ 0.01 mm). The distribution of the dimensionless wall distance on the wall surface obtained through numerical simulation is shown in Figure 6. The results indicate that the dimensionless wall distance ranges between 2.7 and 4.8, falling within the viscous sublayer. Therefore, setting the first boundary layer height to 0.01 mm in the numerical simulation is appropriate.

$$\begin{cases} Re = \frac{\rho V_c L}{\mu} \\ C_f = \frac{0.026}{Re^{1/7}} \\ \tau_w = \frac{1}{2} C_f \rho V_c^2 \\ u_\tau = \sqrt{\frac{\tau_w}{\rho}} \\ h = \frac{h^* y^+ \mu}{u_\tau \rho} \end{cases} \quad (1)$$

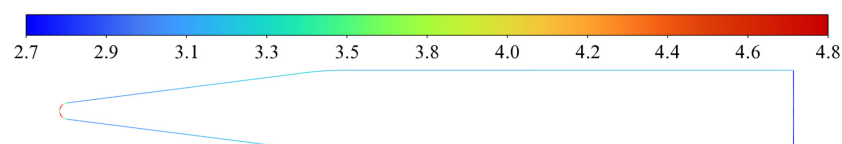


Figure 6. The distribution of the dimensionless wall distance on the wall surface.

To avoid excessive computational errors caused by overly coarse meshes while preventing increased computational resource consumption due to overly fine meshes, this study conducts mesh independence verification. Mesh independence verification demonstrates that numerical simulation results are unaffected by mesh density, validating the reliability

and stability of computational outcomes. Three meshes with varying densities—coarse, medium and fine—were generated. The influence of these meshes on simulation results was analyzed by monitoring the stagnation pressure. The parameters and results for the different mesh densities are shown in Table 1. The numerical simulation iteration results for the stagnation pressure at the different mesh densities are shown in Figure 7. The results indicate that the calculations for stagnation pressure converge for all three mesh densities, and the computational results for different meshes no longer show significant variation with increasing mesh density. Considering both computational accuracy and computational cost, the medium-density mesh is selected for numerical simulation.

Table 1. Parameters and results for different mesh densities.

Mesh Density	First Boundary Layer Height (mm)	Number of Meshes	Iteration Step Count	Stagnation Pressure (kPa)
Coarse	0.01	83,695	144	556
Medium	0.01	129,575	182	559
Fine	0.01	181,455	217	560

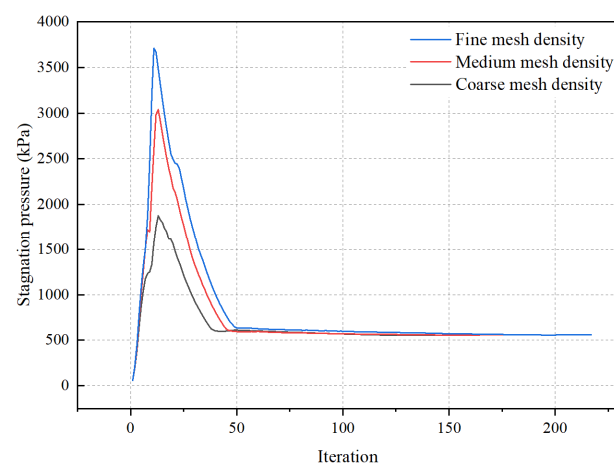


Figure 7. Numerical simulation results of stagnation pressure for different mesh densities.

A schematic diagram illustrating the naming of fluid domain boundaries and the boundary layer refinement is shown in Figure 8. The fluid domain was meshed using the Pointwise software(Pointwise V18.6), generating 129,575 structured mesh cells. The Minimum Orthogonal Quality (MOQ) of the mesh is 0.91. Generally, a MOQ greater than 0.2 indicates acceptable mesh quality. The mesh quality of this model is significantly higher than 0.2, earning it an excellent rating.

The fluid domain medium in this study is air, employing the ideal gas assumption in numerical simulations, neglecting chemical reactions such as vibration excitation, dissociation and ionization. Viscosity follows Sutherland's law. The free-stream pressure is 12,044.5 Pa, and the free-stream temperature is 216.65 K. A numerical simulation analysis model for the shock angle of hypersonic wide-speed-range cruise aircraft was established using the Fluent software(Fluent 2022R2). This model investigates the shock angle behavior under various Mach number flight conditions. The simulation employs an axisymmetric two-dimensional spatial domain. A pressure-based solver is employed. The turbulence model selection is the SST k-omega [48–50] turbulence model. Viscous heating and compressibility effects are accounted for in the steady-state calculation. The turbulence intensity and turbulence viscosity ratio are set to 1%. The pressure–velocity coupling strategy is Couple. The flux type is Rhie–Chow distance-based. The spatial discretization method is Green–Gauss cell-based. The pressure method is second order. The density, momentum

and energy methods are second-order upwind. The turbulent kinetic energy and specific dissipation rate methods are first-order upwind [51].

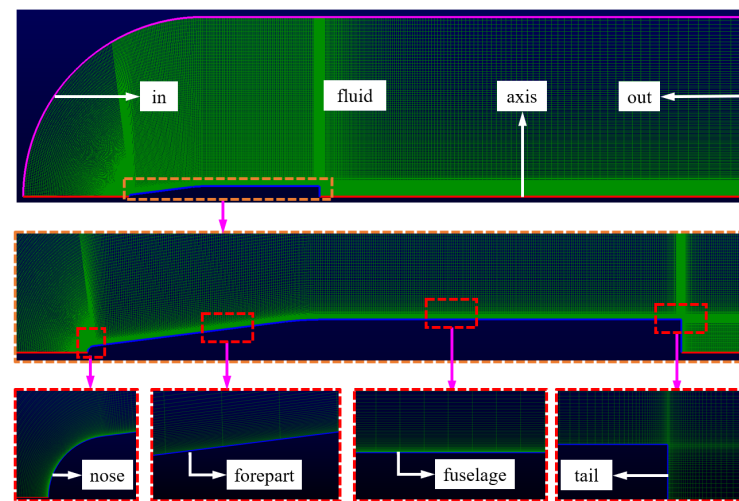


Figure 8. Schematic diagram of fluid domain boundary naming and boundary layer refinement.

2.2.3. Shock Angle Wind Tunnel Measurement Test

The FL-63 wind tunnel is a pulsed combined high-enthalpy facility featuring two operational modes: Ludwig tube and shock tube. The Ludwig tube mode simulates flight conditions at Mach numbers ranging from 3.0 to 4.5, while the shock tube mode replicates conditions at Mach numbers from 5.0 to 10.0. The FL-63 wind tunnel can be used to conduct aerodynamic thermal load measurement, engine ignition and flow visualization and measurement tests. The wind tunnel primarily consists of a Laval nozzle, test chamber, shadowgraph system and display equipment. The test chamber houses angle-of-attack and sideslip angle adjustment mechanisms along with the aircraft. This study selected the shock tube mode of the FL-63 wind tunnel to conduct tests for cruise aircraft at flight Mach numbers of 6, 7 and 8. The test plan for obtaining the aircraft shock wave diagram is illustrated in Figure 9. During testing, airflow within the wind tunnel is accelerated through the Laval nozzle to achieve hypersonic velocity. This airflow is directed onto the aircraft installed within the test chamber. Generated shock waves are continuously captured and recorded by the shadowgraph system. The resulting shock waves are captured throughout the process by the shadowgraph system, with the shock wave patterns stored on computer terminals via display equipment. Finally, the hypersonic aircraft shock angle wind tunnel measurement tests are completed through post-processing.

The FL-63 wind tunnel operates using dry air. This wind tunnel can accommodate test models measuring 1.2 m in length and 0.2 m in diameter. The model material is nickel-based superalloy Inconel 718, with dimensions within the wind tunnel's permissible range. Given the short test duration (20 ms), thermal conduction effects are negligible, meaning the model's initial thermal state does not significantly influence the shock angle. The shock angle uncertainty in the wind tunnel test is $\pm 0.1^\circ$, falling within a reasonable error range. In the shock tube operating mode of the FL-63 wind tunnel, the typical total temperature is approximately 950 K. Additionally, the short duration of the wind tunnel operation is insufficient to induce significant oxygen dissociation. Under wind tunnel test conditions, the vibration relaxation time is approximately 100 ms, with vibrations not fully stabilized. The dissociation time exceeds 10^3 s, and the dissociation reaction is completely frozen. Therefore, treating the wind tunnel flow as an ideal gas is a reasonable approximation during the experimental validation phase. The operating state parameters of the wind tunnel test are shown in Table 2.

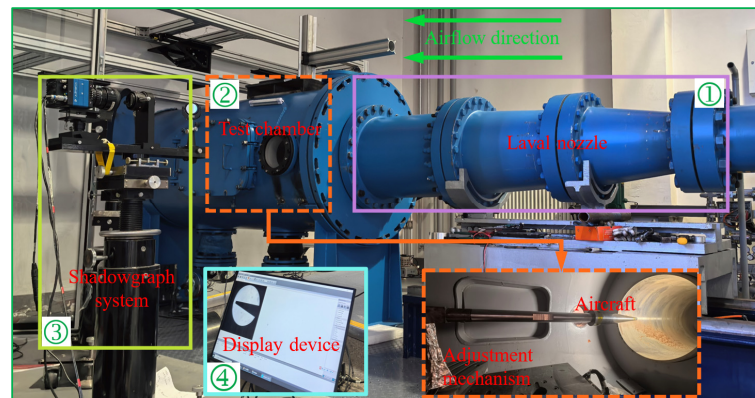


Figure 9. Test plan for shock angle wind tunnel measurement using FL-63.

Table 2. Operating parameters of the wind tunnel test.

Nominal Mach Number	Actual Mach Number	Angle of Attack (°)	Sideslip Angle (°)	Total Pressure (kPa)	Total Temperature (K)
6	6.02	0	0	2512	920
7	6.97	0	0	2690	951
8	8.03	0	0	2787	971

3. Results Analysis

3.1. Analysis of Numerical Simulation Results

The results for flight conditions at Mach numbers 6, 7, 8, 9, 10, 12, 15 and 20 were obtained using the numerical simulation analysis model for the shock angle of hypersonic wide-speed-range cruise aircraft. The Mach contour plot for the flight condition at Mach 6 is shown in Figure 10. The contour plot reveals that the Mach number undergoes a deceleration and stagnation process along the fluid domain from the inlet toward the aircraft's stagnation point, sequentially experiencing $Ma = 6$, $Ma = 5$, $Ma = 4$, $Ma = 3$, $Ma = 2$ and $Ma = 1$.

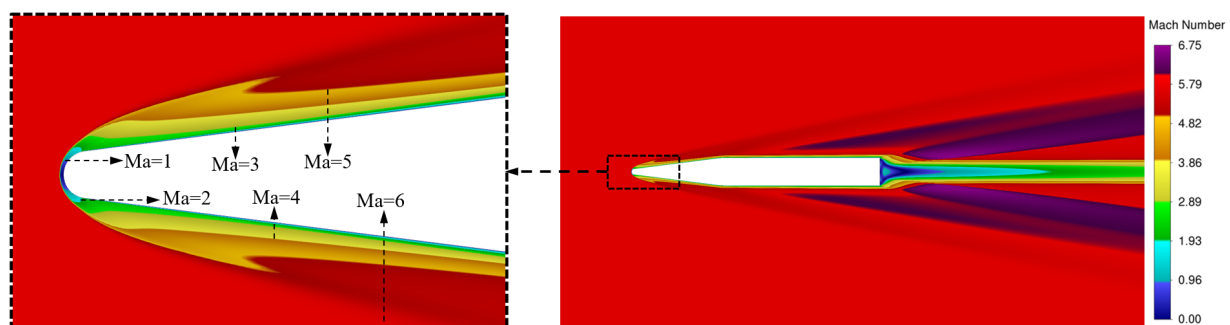


Figure 10. Mach contour plot at $Ma = 6$.

The density contour plot for the flight condition at Mach 6 is shown in Figure 11. A bow-shaped shock wave forms along the aircraft wall, with a boundary layer developing near the wall surface. The region between the wall and the shock wave constitutes the shock layer. The shock angle is the angle between the shock wave front and the direction of the incoming flow. When the shock angle is 90° , the shock is a normal shock. When the shock angle is less than 90° , the shock is an oblique shock. At the aircraft's stagnation point, the shock is a normal shock, while at all other locations, it is an oblique shock.

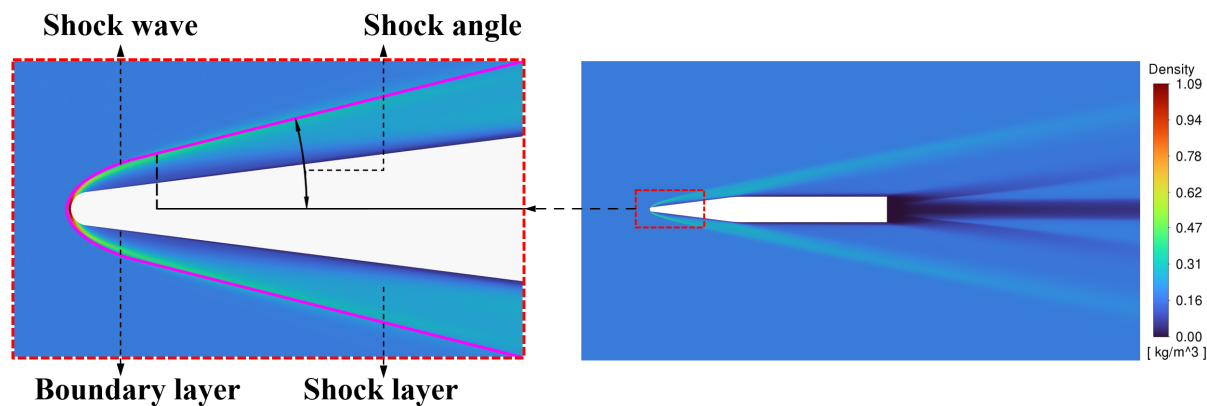


Figure 11. Density contour plot at $Ma = 6$.

The classical oblique shock theory [13] establishes the relationship between the shock angle (β), flow deflection angle (θ) and Mach number (Ma), expressed by Equation (2). This relationship is derived from the assumptions of an ideal gas, inviscid, adiabatic, steady and one-dimensional flow model.

$$\tan \theta = \frac{Ma^2 \sin^2 \beta - 1}{\left[Ma^2 \left(\frac{\gamma+1}{2} - \sin^2 \beta \right) + 1 \right] \tan \beta} \quad (2)$$

The shock angle equation is theoretically applicable to an infinitely long tapered model. In practical applications, to mitigate aerodynamic heating effects, the nose of an aircraft is often designed with a blunt-nose configuration. The blunt-nose model generates a bow shock near the stagnation point, which gradually transitions into an oblique shock. The shock intensity in the far downstream region is significantly less pronounced than in the upstream region. The upstream shock exerts a more pronounced effect on the aircraft, and the wind tunnel test captures the far downstream shock less effectively. Considering both the shock distribution characteristics and the shock capture efficiency in the wind tunnel test, the measurement reference point was therefore positioned upstream of the oblique shock. The shock angle smoothly varies with the aircraft structure. The axial distance from the measurement reference point to the aircraft's stagnation point is set as $d = 0.5D = 40$ mm. The shock angle obtained from the numerical simulation analysis model for hypersonic wide-speed-range cruise aircraft is denoted by β . A schematic diagram of the numerical simulation shock angle measurement is shown in Figure 12. The shock angle solved from Equation (2) is denoted by β_t . The absolute value of the difference between the numerical shock angle and the theoretical shock angle is denoted by δ_a , i.e., $\delta_a = |\beta - \beta_t|$.

Numerical simulation techniques yielded shock angle measurements for hypersonic cruise aircraft operating at Mach numbers 6, 7, 8, 9, 10, 12, 15 and 20. Density contour plots for various Mach numbers and flight conditions are shown in Figure 13. Shock angles at various Mach numbers are listed in Table 3. When $Ma = 6$, $\beta = 16.23^\circ$, $\beta_t = 15.04^\circ$ and $\delta_a = 1.19^\circ$. When $Ma = 7$, $\beta = 14.91^\circ$, $\beta_t = 13.76^\circ$ and $\delta_a = 1.15^\circ$. When $Ma = 8$, $\beta = 13.89^\circ$, $\beta_t = 12.83^\circ$ and $\delta_a = 1.06^\circ$. When $Ma = 9$, $\beta = 13.26^\circ$, $\beta_t = 12.14^\circ$ and $\delta_a = 1.12^\circ$. When $Ma = 10$, $\beta = 12.63^\circ$, $\beta_t = 11.61^\circ$ and $\delta_a = 1.02^\circ$. When $Ma = 12$, $\beta = 12.01^\circ$, $\beta_t = 10.85^\circ$ and $\delta_a = 1.16^\circ$. When $Ma = 15$, $\beta = 11.28^\circ$, $\beta_t = 10.16^\circ$ and $\delta_a = 1.12^\circ$. When $Ma = 20$, $\beta = 10.79^\circ$, $\beta_t = 9.57^\circ$ and $\delta_a = 1.22^\circ$. A comparison of the shock angles obtained from numerical simulations at various Mach numbers and those derived from the theory is shown in Figure 14. From Mach 6 to Mach 20, the shock angle obtained from numerical simulation consistently exceeded that derived from the theory, i.e., $\beta > \beta_t$. The maximum difference is $\delta_{a\max} = 1.22^\circ$, and the minimum

difference is $\delta_{amin} = 1.02^\circ$, indicating significant discrepancies between the two methods. From Mach 6 to Mach 20, the shock angle from numerical simulation decreased by 5.44° . From Mach 6 to Mach 9, it decreased by 2.97° . From Mach 9 to Mach 12, it decreased by 1.25° . From Mach 12 to Mach 15, it decreased by 0.73° . From Mach 15 to Mach 20, it decreased by 0.49° . During hypersonic flight, as the Mach number increases, the shock angle progressively decreases while the rate of decrease becomes increasingly gradual.

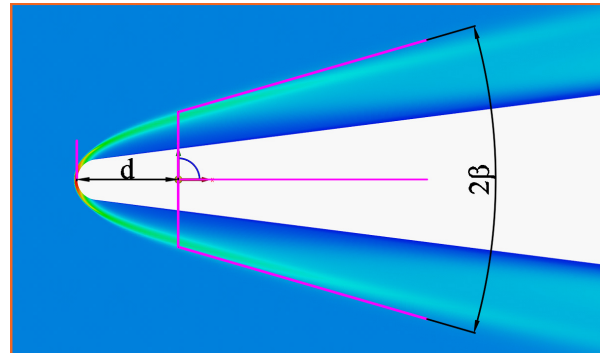


Figure 12. Schematic diagram of shock angle measurement in numerical simulation.

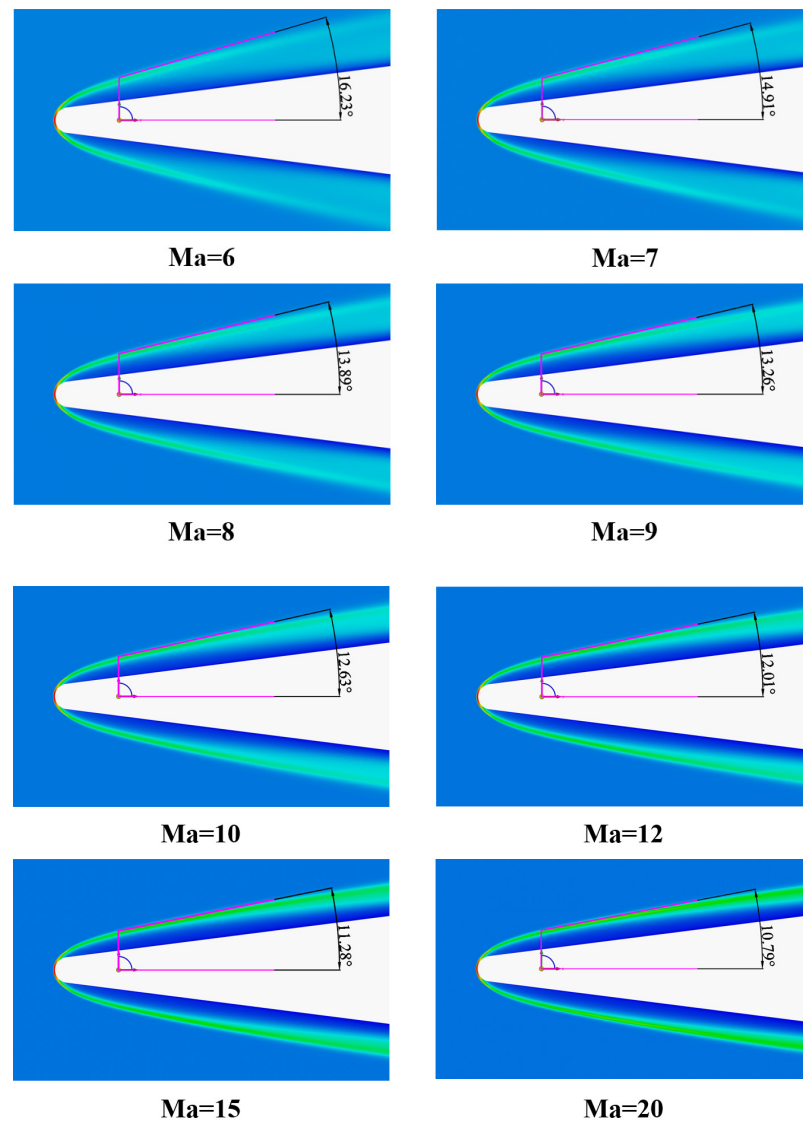
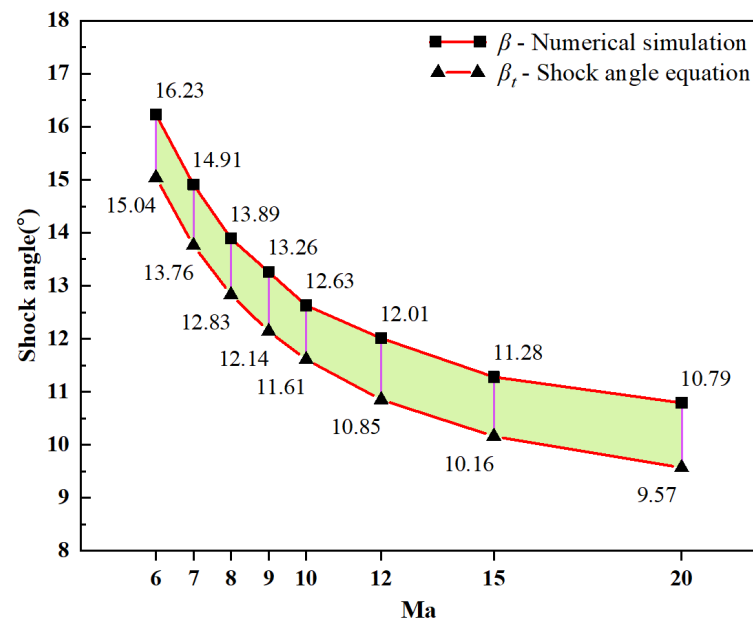


Figure 13. Density contour plots and shock angles at various Mach numbers.

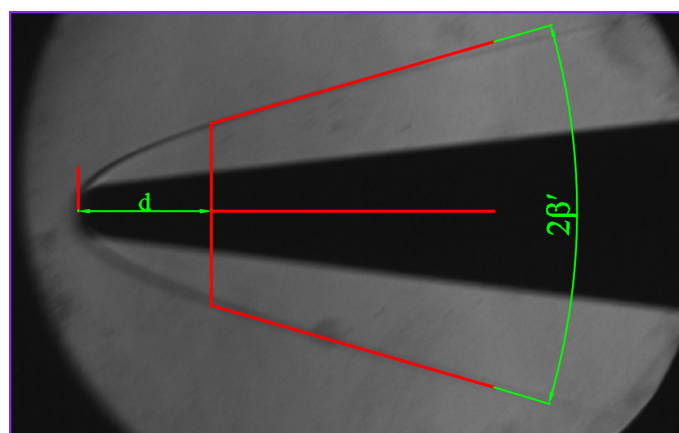
Table 3. Shock angles from numerical simulation and theory at various Mach numbers.

Ma	6	7	8	9	10	12	15	20
Numerical β	16.23	14.91	13.89	13.26	12.63	12.01	11.28	10.79
Theoretical β_t	15.04	13.76	12.83	12.14	11.61	10.85	10.16	9.57
Discrepancy δ_a	1.19	1.15	1.06	1.12	1.02	1.16	1.12	1.22

**Figure 14.** Comparison of shock angles between numerical simulation and equation at various Mach numbers.

3.2. Comparison Analysis of Wind Tunnel Test and Numerical Simulation

The axial distance from the measurement reference point to the aircraft's stagnation point in the wind tunnel test is set as $d = 0.5D = 40$ mm. The shock angle derived from wind tunnel analysis is denoted by β' . The schematic diagram of shock angle measurement in a wind tunnel test is shown in Figure 15.

**Figure 15.** Schematic diagram of shock angle measurement in wind tunnel test.

The absolute value of the difference between the numerical shock angle and the experimental shock angle is denoted by δ . The numerical simulation shock angle error ε is defined as the ratio of the absolute value of the difference between the numerical shock angle and the experimental shock angle at the same Mach number condition to the experimental shock angle, as given by Equation (3).

$$\varepsilon = \frac{\delta}{\beta'} = \frac{|\beta - \beta'|}{\beta'} \quad (3)$$

The absolute difference between the theoretical shock angle and the experimental shock angle is denoted by δ_t . The theoretical shock angle error ε_t is defined as the ratio of the absolute value of the difference between the theoretical shock angle and the experimental shock angle at the same Mach number condition to the experimental shock angle, as given by Equation (4).

$$\varepsilon_t = \frac{\delta_t}{\beta'} = \frac{|\beta_t - \beta'|}{\beta'} \quad (4)$$

The wind tunnel test yielded shock angle measurements for the hypersonic cruise aircraft operating at Mach numbers 6, 7 and 8. When $Ma = 6$, $\beta' = 16.92^\circ$ and $\beta = 16.23^\circ$. A comparison of the shock angle between the numerical simulation and the wind tunnel test at Mach 6 flight conditions is shown in Figure 16. When $Ma = 7$, $\beta' = 15.52^\circ$ and $\beta = 14.91^\circ$. A comparison of the shock angle between the numerical simulation and the wind tunnel test at Mach 7 flight conditions is shown in Figure 17. When $Ma = 8$, $\beta' = 14.29^\circ$ and $\beta = 13.89^\circ$. A comparison of the shock angle between the numerical simulation and the wind tunnel test at Mach 8 flight conditions is shown in Figure 18. The comparison results indicate good consistency between the shock distributions analyzed via numerical simulation and the wind tunnel test.

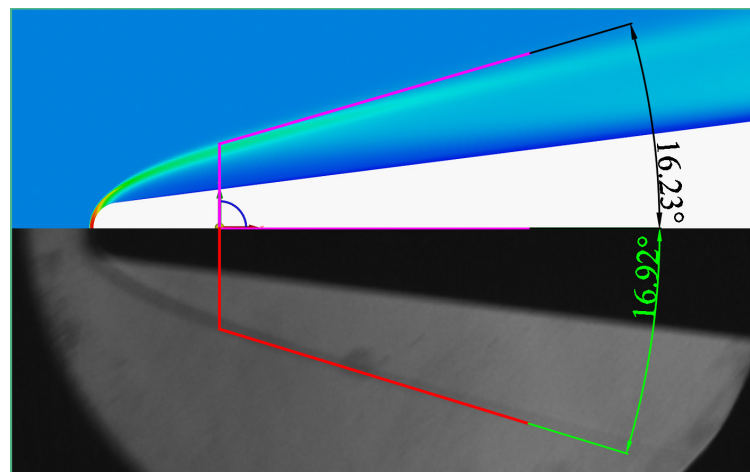


Figure 16. Comparison of shock angle between numerical simulation and wind tunnel test at $Ma = 6$.

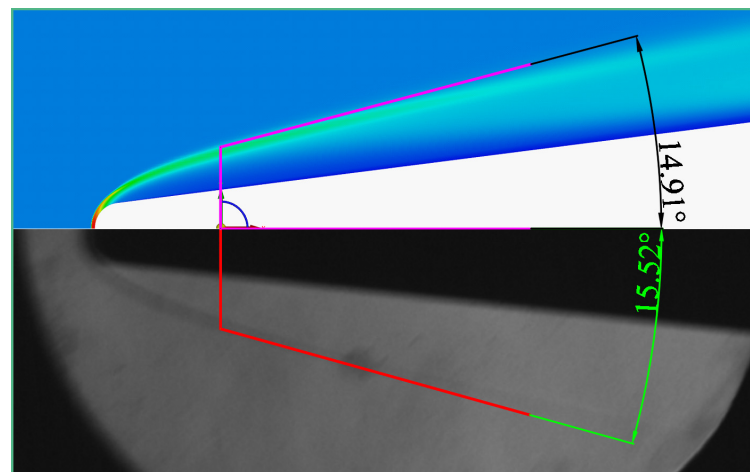


Figure 17. Comparison of shock angle between numerical simulation and wind tunnel test at $Ma = 7$.

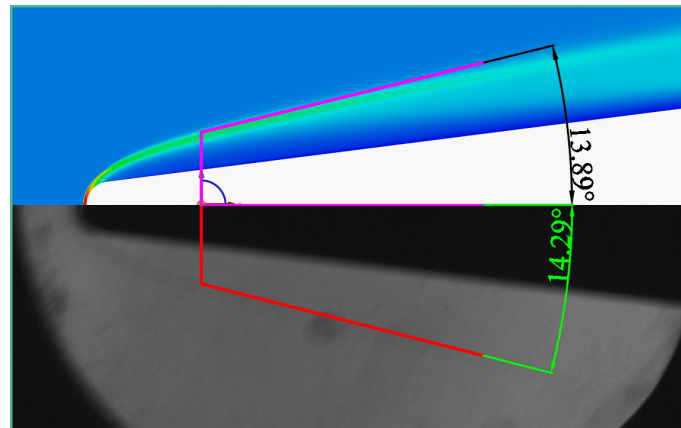


Figure 18. Comparison of shock angle between numerical simulation and wind tunnel test at $Ma = 8$.

The comparison of results from the theory, numerical simulation and wind tunnel test is detailed in Table 4, and the comparison between the shock angle error in the numerical simulation and that in the theory is shown in Figure 19. When $Ma = 6$, $\delta = 0.69^\circ$, $\delta_t = 1.88^\circ$, $\varepsilon = 4.08\%$ and $\varepsilon_t = 11.11\%$, the numerical simulation method improves the shock angle accuracy by 7.03% compared to the shock angle equation. When $Ma = 7$, $\delta = 0.61^\circ$, $\delta_t = 1.76^\circ$, $\varepsilon = 3.93\%$ and $\varepsilon_t = 11.34\%$, the numerical simulation method improves the shock angle accuracy by 7.41% compared to the theory. When $Ma = 8$, $\delta = 0.40^\circ$, $\delta_t = 1.46^\circ$, $\varepsilon = 2.80\%$ and $\varepsilon_t = 10.22\%$, the numerical simulation method improves shock angle accuracy by 7.42% compared to the theory. The results indicate that the maximum error in the numerical shock angle is 4.08%, while the maximum error in the theoretical shock angle is 11.34%. Thus, at the same Mach number, the numerical simulation method achieved at least 7.03% higher accuracy in the shock angle compared to the theoretical analysis. In summary, the numerical simulation analysis model for the shock angle of hypersonic wide-speed-range cruise aircraft presented in this paper demonstrates high accuracy and reliability.

Table 4. Comparison of shock angle analysis via different methods.

Ma	6	7	8
Experimental β'	16.92	15.52	14.29
Numerical β	16.23	14.91	13.89
Theoretical β_t	15.04	13.76	12.83
Numerical discrepancy δ	0.69	0.61	0.40
Theoretical discrepancy δ_t	1.88	1.76	1.46
Numerical error ε	4.08%	3.93%	2.80%
Theoretical error ε_t	11.11%	11.34%	10.22%

3.3. Shock Angle Prediction Analysis

The numerical shock angles at various Mach numbers and flight conditions are shown in Table 3. A functional relationship between the numerical simulation shock angle and Mach number was derived through nonlinear data fitting. The fitting results indicate that the shock angle exhibits an exponential monotonic decreasing relationship with Mach number. The shock angle fitting curve is depicted in Figure 20. The shock angle fitting function is given as Equation (5):

$$\beta = \beta(Ma) = 10.708 + 25.453e^{-0.256Ma} \quad (5)$$

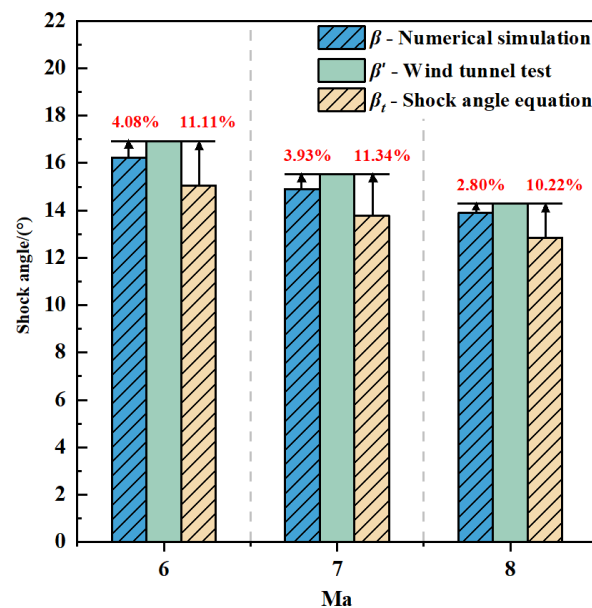


Figure 19. Comparison of shock angle error between numerical simulation and equation analysis.

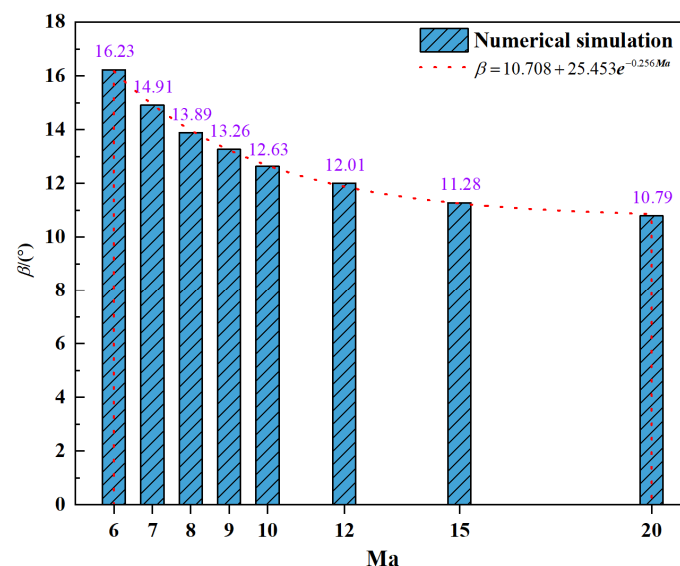


Figure 20. Nonlinear fitting curve of shock angle.

As the Mach number approaches infinity, taking the limit of the fitted shock angle function yields the aircraft's limiting shock angle of 10.708° . From Mach 6 to infinity, the shock angle decreases by 5.522° , while from Mach 20 to infinity, it decreases by only 0.082° . The analysis results indicate that as the Mach number increases, the shock angle diminishes progressively, whilst the rate of shock angle decay also diminishes. Beyond Mach 20, the shock angle remains essentially unchanged.

$$\lim_{Ma \rightarrow \infty} \beta(Ma) = \lim_{Ma \rightarrow \infty} (10.708 + 25.453e^{-0.256Ma}) = 10.708^\circ \quad (6)$$

The fitting function proposed in this study is based on specific geometric configurations and the ideal gas assumption and is applicable for predicting shock angle trends within the Mach number range of 6 to 20. This fitting function can be applied to blunt-nosed conical configuration vehicles and blunt-nosed conical-cylindrical composite configuration vehicles. For hypersonic vehicles with similar cone angles ($14^\circ \sim 15^\circ$) or similar aspect

ratios, this function provides preliminary design references. Under actual high-enthalpy or extremely high Mach number conditions, modifications using real gas models are required.

4. Conclusions

This study combined numerical simulations and a wind tunnel test to simulate the characteristics and patterns of shock angle variation at various Mach numbers for hypersonic wide-speed-range cruise aircraft and predicted the shock angle under extremely high-Mach-number conditions. As this study is based on specific geometric configurations and the ideal gas assumption, modifications using real gas models are required for analyses under actual high-enthalpy or extremely high-Mach-number conditions. The key conclusions are as follows.

- (1) A numerical simulation analysis model for the shock angle of hypersonic wide-speed-range cruise aircraft was established. Through comparison with wind tunnel test results, the maximum error in the shock angle from the numerical simulation analysis was 4.08%, validating the accuracy and reliability of the numerical simulation;
- (2) Numerical simulation enabled the analysis of shock angles at Mach numbers 6, 7, 8, 9, 10, 12, 15 and 20, revealing the variation in the shock angle at various Mach numbers. During hypersonic flight, a bow shock forms on the aircraft's surface. As the Mach number increases, the shock angle diminishes progressively, and the rate of shock angle decay also decreases;
- (3) Compared to the wind tunnel test, the numerical simulation method achieved 7.03% higher accuracy in shock angle prediction than the theory at the same Mach number;
- (4) Nonlinear data fitting yielded a functional relationship between the shock angle and Mach number, showing an exponentially decreasing trend. As the Mach number approaches infinity, the shock angle converges to 10.708° .

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Abbreviations

The following abbreviations are used in this manuscript:

2D	Two-dimensional
3D	Three-dimensional
MOQ	Minimum Orthogonal Quality

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