

Article

Navigation Service Performance Assessment for Cislunar Transfer Spacecraft Using BDS-3 and DRO/NRHO-Augmented Constellations

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Abstract

This study assesses the navigation-service availability and observation geometry of a layered cislunar architecture using the BeiDou Navigation Satellite System Phase III (BDS-3) as the Earth-side primary source and distant retrograde orbit (DRO) and near-rectilinear halo orbit (NRHO) nodes as lunar-side augmentation sources. The assessment distinguishes geometrically visible links, beam-accessible links, receiver-detectable signals, and usable navigation observations. Four configurations—BDS-3, BDS-3+DRO, BDS-3+NRHO, and BDS-3+DRO+NRHO—are compared along a representative transfer trajectory and over a grid in the principal Earth–Moon plane. BDS-3 alone provides intermittent support that decreases sharply beyond the near-Earth region. DRO extends coverage and supplies usable observations, whereas the present two-node NRHO configuration provides localized geometric supplementation. The combined configuration yields the largest improvement and reaches 59.7% qualified navigation availability during the late-transfer stage, although the middle-transfer corridor remains weak. The study evaluates service access and geometry rather than end-to-end positioning accuracy.

Keywords: cislunar navigation; BDS-3; distant retrograde orbit; near-rectilinear halo orbit; navigation service availability; signal detectability; position dilution of precision; service continuity

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1. Introduction

With the continuous advancement of space technology, cislunar space has become a new frontier for human space exploration and applications. Lunar exploration is gradually shifting from individual short-duration missions toward long-term operations, support from lunar orbital platforms, and sustained activities on the lunar surface [1,2]. Several countries and space agencies have incorporated crewed lunar exploration, lunar resource utilization, and long-term lunar activities into their future deep-space exploration plans [1].

The capability for round-trip transportation between the Earth and the Moon is an important foundation for crewed lunar landing and lunar return missions, while continuous autonomous navigation throughout the entire mission is essential for flight safety and successful mission execution. Compared with conventional near-Earth missions, cislunar missions generally include multiple phases, such as Earth departure, cislunar transfer, lunar approach, lunar orbital operations, and return. These missions are characterized by large spatial scales, long durations, and complex dynamical environments, which impose higher requirements on the autonomy, continuity, and robustness of spacecraft navigation systems [3]. Traditional ground-based and deep-space tracking systems determine spacecraft orbits mainly through range, range-rate, and angular measurements and remain important components of deep-space mission navigation. However, ground-based navigation depends on tracking-station networks, communication links, and ground-processing resources. As the number of deep-space missions increases, constraints related to network scheduling and communication resources will become more significant. Therefore, improving onboard autonomous navigation can reduce the sustained dependence on ground-tracking systems [3]. Vision-based navigation and terrain-relative navigation estimate spacecraft position and motion using lunar terrain, impact craters, and other optical features, and have become important techniques for autonomous navigation during lunar approach, descent, and landing [4–6]. However, these methods generally rely on optical sensors such as cameras, identifiable terrain features, suitable imaging conditions, and prior maps or feature databases. Their applicability can therefore be affected by illumination changes, terrain distinguishability, and sensor field-of-view limitations [4–6]. Consequently, vision-based navigation is more suitable for lunar approach, descent and landing, and local operational phases, but it is difficult for such methods alone to provide continuous absolute navigation throughout the complete cislunar transfer.

Compared with orbit determination methods that rely entirely on ground-based tracking, onboard GNSS receivers can directly provide cislunar spacecraft with position, velocity, and time information, thereby improving navigation autonomy and reducing continuous dependence on ground tracking resources and ground-based data processing [7]. With advances in high-sensitivity receivers, weak-signal acquisition and tracking techniques, and multi-constellation interoperability, the use of Earth-based GNSS signals for autonomous navigation during cislunar transfer and in the lunar vicinity has become an important research direction in cislunar navigation [7,8]. Delépaut et al. analyzed GNSS signal visibility, link conditions, and PDOP near lunar altitude using realistic three-dimensional transmit antenna patterns for GPS and Galileo. Their results showed that a certain number of GNSS observations can still be obtained near lunar altitude when high-gain receiving antennas and low acquisition and tracking thresholds are used. They also emphasized the importance of sidelobe antenna patterns and multi-constellation interoperability for navigation performance [7]. Yang et al. systematically investigated signal coverage, signal strength, visibility, observation geometry, GDOP, and Doppler shift in high-altitude regions above the GNSS constellations. Their results indicated that the combined use of main-lobe and sidelobe signals, together with multi-constellation reception, can increase the number of beam-accessible satellite signals, improve observation geometry, and reduce GDOP [8]. Guan et al. further evaluated the navigation performance of BDS-3, GPS, Galileo, and GLONASS sidelobe signals in geostationary Earth orbit, highly elliptical orbit, and cislunar transfer trajectories. They found that most signals received by cislunar transfer spacecraft originate from antenna sidelobes and that higher receiver sensitivity and multi-GNSS integration can significantly improve navigation continuity [9].

For missions in the lunar vicinity, Bernelli-Zazzera and Forte used the LUMIO mission as a case study to establish GPS and Galileo constellation, link-budget, and visibility models, and evaluated the feasibility of autonomous GNSS navigation near an Earth–

Moon L2 halo orbit [10]. Wang et al. investigated an integrated navigation method combining GNSS, an inertial navigation system, and star sensors for spacecraft operating in cislunar space. Their results indicated that GNSS can compensate for the limited real-time capability and autonomy of ground-based tracking, although weak signal power and poor observation geometry remain major challenges in the lunar vicinity [11]. The LuGRE mission was designed to demonstrate the reception and navigation use of GPS and Galileo signals at lunar distances. Its mission design included a weak-signal receiver, a high-gain antenna, pseudorange, carrier-phase, and Doppler measurements, as well as onboard least-squares and Kalman-filter-based navigation solutions [12]. During cislunar transfer and at lunar distances, Earth-based GNSS signals are mainly limited by the transmit antenna patterns, sidelobe power, and extremely long propagation distances [7–9]. As the spacecraft approaches the Moon, the further reduction in signal power, celestial-body occultation, and degradation of observation geometry place additional constraints on autonomous navigation performance [10,11]. Therefore, relying solely on Earth-based GNSS is still insufficient to ensure continuous and independent navigation throughout the entire cislunar mission, and lunar-side augmentation sources or other autonomous measurements are required.

In addition to studies on Earth-based navigation sources, the deployment of dedicated radio-navigation nodes in cislunar space and the lunar vicinity to provide communication, navigation, and timing services for lunar transfer, orbital, and surface users has become an important research direction for international lunar infrastructure [13]. Gao and Hou analyzed the coverage performance of lunar communication and navigation constellations based on halo orbits and distant retrograde orbits (DROs). Their results showed that properly configured halo-orbit and DRO nodes can provide a high level of lunar-surface coverage, while a combined configuration can achieve continuous single-fold coverage over the entire lunar surface [14]. Wang et al. further proposed a multi-orbit lunar navigation constellation composed of DRO and halo-orbit satellites and evaluated the navigation performance of different configurations in terms of the number of geometrically visible satellites, lunar-surface coverage, and position dilution of precision [15]. Conti and Circi designed a lunar global positioning, navigation, timing, and communication constellation based on halo orbits near the Earth–Moon L1, L2, and L3 points, demonstrating the feasibility of an integrated architecture for continuous Earth–Moon communication and lunar PNT services [16]. Zhang et al. used a lunar digital elevation model to analyze the effects of terrain blockage in the lunar south polar region on the visibility and positioning performance of a lunar navigation satellite system, indicating that local terrain occultation should also be considered in lunar-surface navigation service assessments [17]. Meanwhile, the LunaNet Interoperability Specification proposed a lunar communication and PNT framework composed of multiple cooperative service-provider nodes, with coverage extending to users in transit to the Moon, in lunar orbit, and on the lunar surface. Under the LunaNet framework, Dafesh et al. further investigated interoperable navigation signals for future lunar missions and presented spreading-code, data-synchronization, and acquisition designs for lunar augmented forward signals [18]. The CAPSTONE mission was designed to validate operations in a near-rectilinear halo orbit (NRHO) and demonstrate autonomous navigation technologies in cislunar space, providing an engineering reference for the use of NRHO platforms in future lunar communication and navigation architectures [19].

These studies indicate that future cislunar communication and navigation systems may consist of Earth-based GNSS, lunar-orbit navigation nodes, relay satellites, and integrated Earth–Moon networks to improve autonomous PNT services for users during cislunar transfer and in the lunar vicinity. Earth-based GNSS can provide weak-signal observations during the early cislunar transfer stage and in some deep-space regions,

whereas lunar-side navigation nodes in DRO, halo, and NRHO can provide supplementary observations when Earth-based signals are significantly attenuated or the observation geometry becomes unfavorable [15]. However, the coverage capability and geometric contribution of different orbital nodes vary considerably across regions, and a single-orbit configuration is generally unable to simultaneously meet the continuous service requirements of the lunar polar regions, equatorial regions, and the cislunar transfer corridor [15]. Among the existing global navigation satellite systems, the space segments of GPS and Galileo are mainly composed of medium Earth orbit satellites, whereas BDS-3 adopts a hybrid constellation consisting of geostationary Earth orbit, inclined geosynchronous orbit, and medium Earth orbit satellites [20]. Studies on high-altitude space services have shown that GNSS signal availability is jointly affected by the transmit antenna pattern, Earth occultation, propagation distance, receiver threshold, and observation geometry. The use of sidelobe signals and multi-constellation integration can increase the number of beam-accessible satellite signals and improve the observation geometry for high-altitude users [8].

Accordingly, this study develops an integrated navigation architecture composed of BDS-3 as the primary navigation source and DRO/NRHO nodes as lunar-side augmentation sources, and evaluates its service capability in a representative cislunar transfer scenario. Geometrically visible, beam-accessible, detectable, and usable links, together with formal solvability, qualified navigation availability, PDOP, and service continuity of different configurations are systematically analyzed. The results provide a reference for cislunar navigation-source configuration and the design of multi-source collaborative PNT architectures.

2. Navigation Service Availability and Geometry Assessment Model

The assessment separates link access from navigation-solution quality. For each source and epoch, the model first tests Earth/Moon occultation, then transmitter beam access, receiver detectability, and observation usability. Only usable observations enter the geometry matrix. Formal solvability, PDOP, qualified navigation availability, and continuity are evaluated from that set. This ordering prevents the terms visible, detectable, usable, and navigation-available from being used interchangeably.

2.1. Geometric Visibility Assessment

Geometric visibility is used to determine whether the line-of-sight propagation path between a navigation source and the cislunar transfer spacecraft is blocked by the Earth or the Moon. In cislunar navigation scenarios, navigation signals propagate over long distances, and the relative positions of the spacecraft with respect to the Earth and the Moon vary significantly during the near-Earth departure, middle cislunar transfer, and late cislunar transfer stages. Therefore, celestial-body occultation is the primary constraint in determining whether a navigation source can provide service.

At epoch t_k , let the position vector of the spacecraft be $\mathbf{r}_s(t_k)$ and that of the i -th navigation source be $\mathbf{r}_i(t_k)$. The line-of-sight vector between them can then be expressed as:

$$\boldsymbol{\rho}_i(t_k) = \mathbf{r}_s(t_k) - \mathbf{r}_i(t_k) \quad (1)$$

The propagation distance is:

$$\rho_i(t_k) = \|\mathbf{r}_s(t_k) - \mathbf{r}_i(t_k)\| \quad (2)$$

The segment connecting the navigation source and the spacecraft is parameterized as:

$$\mathbf{r}(\lambda) = \mathbf{r}_i(t_k) + \lambda[\mathbf{r}_s(t_k) - \mathbf{r}_i(t_k)], \quad 0 \leq \lambda \leq 1 \quad (3)$$

Let the position vector of the center of the occulting body b be $\mathbf{r}_b(t_k)$, with radius R_b , where b denotes either the Earth or the Moon. If the line segment $\mathbf{r}(\lambda)$ intersects the spherical body, the line of sight between the navigation source and the spacecraft is considered occulted. The corresponding criterion is written as:

$$\min_{0 \leq \lambda \leq 1} \|\mathbf{r}(\lambda) - \mathbf{r}_b(t_k)\| \leq R_b \quad (4)$$

When the minimum distance from the source-to-spacecraft line segment to the center of either occulting body is less than or equal to the corresponding body radius, the link is regarded as occulted. Accordingly, the geometric visibility indicator is defined as:

$$V_i^{geo}(t_k) = \begin{cases} 1, & \text{not blocked by Earth or Moon} \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

where $V_i^{geo}(t_k) = 1$ indicates that the i -th navigation source satisfies the geometric visibility condition at time t_k , whereas $V_i^{geo}(t_k) = 0$ indicates that its line of sight is occulted. Geometric visibility only indicates whether an unobstructed propagation path exists between the navigation source and the spacecraft; it does not guarantee that the received signal strength is sufficient for signal reception or navigation solution computation. Therefore, after identifying the set of geometrically visible navigation sources, beam accessibility and detectability must then be evaluated using the transmit antenna pattern, propagation loss, and receiver thresholds.

2.2. Link Screening and Signal Usability

After the occultation test, a geometrically visible link is classified as beam-accessible only if the transmitter model supports radiation toward the spacecraft. The received power and carrier-to-noise-density ratio (C/N_0) are then evaluated for every beam-accessible link. A beam-accessible signal is detectable at $C/N_0 \geq 12$ dB-Hz and usable at $C/N_0 \geq 18$ dB-Hz when it also satisfies the adopted beam-strength criterion.

Let the unit vector along the transmission boresight of the i -th navigation source at epoch t_k be $\mathbf{e}_{a,i}(t_k)$, and let the unit line-of-sight vector from the navigation source to the cislunar transfer spacecraft be $\mathbf{e}_{\rho,i}(t_k)$. Then,

$$\mathbf{e}_{\rho,i}(t_k) = \frac{\mathbf{r}_s(t_k) - \mathbf{r}_i(t_k)}{\|\mathbf{r}_s(t_k) - \mathbf{r}_i(t_k)\|} \quad (6)$$

The corresponding off-boresight angle is:

$$\theta_i(t_k) = \arccos[\mathbf{e}_{a,i}^T(t_k) \mathbf{e}_{\rho,i}(t_k)] \quad (7)$$

Here, $\mathbf{r}_s(t_k)$ and $\mathbf{r}_i(t_k)$ are the spacecraft and navigation-source position vectors in the Earth-centered inertial frame.

For BDS-3 satellites, the transmit boresight is approximated as Earth-pointing. For DRO and NRHO nodes, an equivalent isotropic-radiation model is used. The adopted 13 dBW per-node EIRP defines a common engineering comparison scenario rather than a finalized payload design; no additional directional transmit gain or off-boresight antenna-pattern constraint is applied.

Because a public full-space measured BDS-3 antenna pattern is unavailable, this study uses a one-dimensional, azimuth-averaged equivalent pattern whose main-, side-, and back-lobe behavior is consistent with published GNSS space-service-volume analyses [7–9,21]. The model is an engineering approximation and is not presented as an official

BDS-3 three-dimensional flight pattern. On-orbit BDS measurements further indicate that BDS-3 MEO signals beyond approximately 26° can remain observable and that typical sidelobe gains are 20–30 dB below the main-lobe peak [22].

$$G_{t,i}(t_k) = G_t[\theta_i(t_k)] \quad (8)$$

For each off-boresight angle, the transmit gain is interpolated from the equivalent pattern. Beam class is determined by the gain drop relative to the pattern maximum: a drop no greater than 3 dB denotes the main lobe, a drop greater than 3 dB but no greater than 20 dB denotes a side lobe, and a larger drop denotes the back lobe.

$$\Delta G_i(t_k) = G_{\max} - G_{t,i}(t_k)$$

$$C_i(t_k) = \begin{cases} \text{main,} & \Delta G_i(t_k) \leq 3 \text{ dB} \\ \text{side,} & 3 < \Delta G_i(t_k) \leq 20 \text{ dB} \\ \text{back,} & \Delta G_i(t_k) > 20 \text{ dB} \end{cases} \quad (9)$$

For BDS-3, beam accessibility is determined from the adopted equivalent transmit pattern. For the isotropic DRO/NRHO augmentation nodes, every geometrically unobstructed link is treated as beam-accessible.

The angular samples near 23° and 60° are engineering-selected interpolation knots used to reproduce representative main-lobe roll-off and sidelobe coverage in the equivalent azimuth-averaged curve; they are not measured BDS-3 beam boundaries. Because no public full-space BDS-3 calibration pattern was available, the study does not attribute these two angles to an official antenna specification. Beam class is determined only by the relative gain-drop criteria, and the resulting sensitivity to antenna and receiver assumptions is stated as a modeling limitation. The equivalent azimuth-averaged BDS-3 transmit antenna pattern adopted in this study is illustrated in Figure 1.

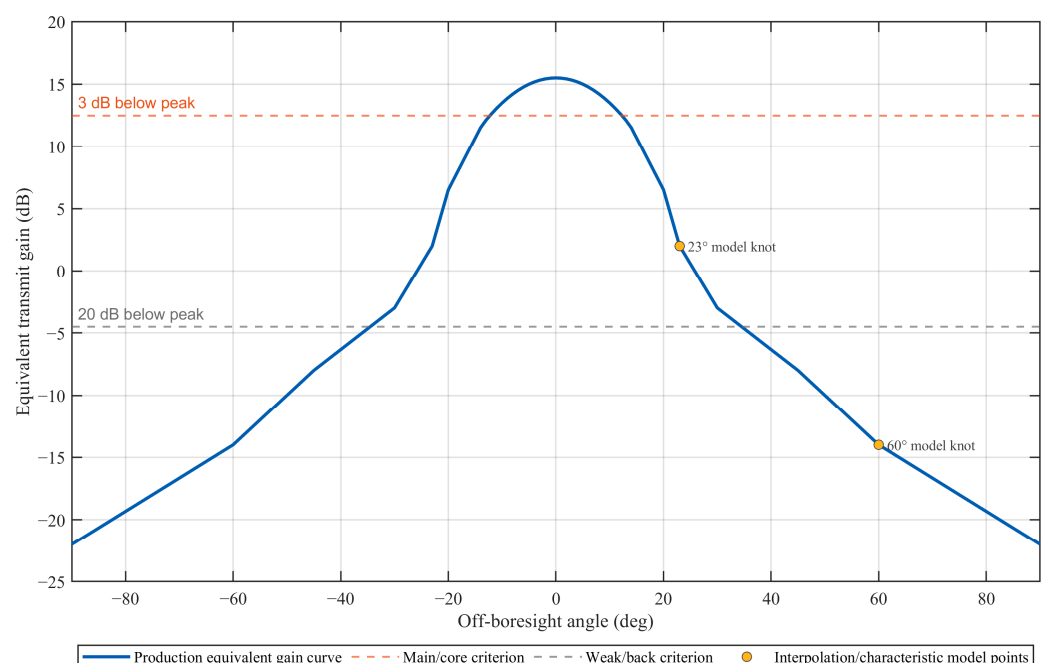


Figure 1. Equivalent azimuth-averaged BDS-3 transmit antenna pattern used in the link assessment [7–9,21]. The 23° and 60° markers are interpolation knots or characteristic model points, not physical beam boundaries; production beam-region classification uses relative gain drop.

For propagation distance $\rho_i(t_k)$ and carrier wavelength λ_c , the free-space path loss is:

$$L_{fs,i}(t_k) = 20 \log_{10} \left(\frac{4\pi\rho_i(t_k)}{\lambda_c} \right) \quad (10)$$

The received power at the spacecraft is:

$$P_{r,i}(t_k) = P_{t,i} + G_{t,i}(\theta_i(t_k)) + G_r - L_{fs,i}(t_k) \quad (11)$$

where $P_{r,i}$ is transmit power, $G_{t,i}(\theta_i(t_k))$ is transmit gain in the spacecraft direction, and G_r is receiving-antenna gain. Atmospheric, ionospheric, and tropospheric losses are neglected relative to the dominant free-space loss at cislunar distances.

The received carrier-to-noise-density ratio is then:

$$\left(\frac{C}{N_0} \right)_i(t_k) = P_{r,i}(t_k) - N_0 \quad (12)$$

The equivalent noise spectral density can be expressed using receiver noise parameters as:

$$N_0 = k_{dB} + 10 \log_{10}(T_{sys}) + NF \quad (13)$$

where $k_{dB} = 10 \log_{10}(k)$ is Boltzmann's constant expressed in dBW/K/Hz, T_{sys} is the receiver reference noise temperature used in the receiver noise model in kelvin, and NF is the receiver noise figure in decibels.

Two receiver thresholds are used. A beam-accessible signal is classified as detectable when $C/N_0 \geq 12$ dB-Hz. Detectability alone does not admit the signal to the navigation solution.

$$D_i(t_k) = \begin{cases} 1, & V_i^{beam}(t_k) = 1, \left(\frac{C}{N_0} \right)_i(t_k) \geq \gamma_d \\ 0, & \text{others} \end{cases} \quad (14)$$

Here, $\gamma_d = 12$ dB-Hz, $D_i(t_k) = 1$ indicates that the signal from the i -th navigation source satisfies the detection condition at epoch t_k , whereas $D_i(t_k) = 0$ indicates that the signal does not satisfy the detection condition.

Furthermore, to prevent extremely weak sidelobe or backlobe signals from being directly included as usable navigation observations, an antenna-pattern constraint is introduced in addition to the carrier-to-noise-density-ratio threshold. Let the antenna-pattern criterion be denoted by $B_i(t_k)$. Then,

$$B_i(t_k) = \begin{cases} 1, & \text{DRO/NRHO isotropic link} \\ 1, & \text{BDS-3 main-lobe/core signal} \\ 1, & \text{BDS-3 sidelobe signal with } G_{t,i} \geq G_{strong} \\ 0, & \text{otherwise} \end{cases} \quad (15)$$

where G_{strong} is the minimum transmit-gain threshold for a side-lobe signal to be admitted as a navigation observation.

The usability indicator is therefore:

$$U_i(t_k) = \begin{cases} 1, & V_i^{beam}(t_k) = 1, \left(\frac{C}{N_0} \right)_i(t_k) \geq \gamma_u, B_i(t_k) = 1 \\ 0, & \text{others} \end{cases} \quad (16)$$

Here, $\gamma_u = 18$ dB-Hz, $U_i(t_k) = 1$ indicates that the i -th navigation source can be used as a valid navigation observation at epoch t_k , whereas $U_i(t_k) = 0$ indicates that

the navigation source does not satisfy the conditions required for use in the navigation solution.

The four screening levels are reported separately. Geometrically visible counts include links blocked by neither Earth nor Moon. Beam-accessible counts additionally apply the transmitter-pattern constraint. Detectable counts additionally require $C/N_0 \geq 12$ dB-Hz. Usable counts require $C/N_0 \geq 18$ dB-Hz and the adopted strong-beam condition. Only the usable set is used for formal solvability, PDOP, and qualified navigation availability.

2.3. Position Dilution of Precision

After link screening, the spatial geometry of the usable observations is evaluated. The ability to form a navigation solution depends not only on the number of observations but also on their directional distribution. PDOP is used to characterize the geometric quality of each configuration.

Let S_k denote the usable observation set at epoch t_k . The unit line-of-sight vector associated with observation is:

$$\mathbf{u}_i(t_k) = \frac{\mathbf{r}_i(t_k) - \mathbf{r}_s(t_k)}{\|\mathbf{r}_i(t_k) - \mathbf{r}_s(t_k)\|} \quad (17)$$

The observation geometry matrix is constructed as:

$$\mathbf{H}(t_k) = \begin{bmatrix} -\mathbf{u}_1^T(t_k) & 1 \\ -\mathbf{u}_2^T(t_k) & 1 \\ \vdots & \vdots \\ -\mathbf{u}_m^T(t_k) & 1 \end{bmatrix} \quad (18)$$

where m denotes the number of usable observations at epoch t_k . In this service-performance assessment, BDS-3, DRO, and NRHO observations are assumed to be referred to a common time reference. Transmitter clock offsets and inter-system time biases are assumed to be known and corrected before the geometry matrix is formed. Therefore, only one common receiver clock-bias state is included in the positioning model, and estimation of three-dimensional position and receiver clock bias requires at least four independent observations.

The normal matrix and geometry covariance matrix are:

$$\begin{aligned} \mathbf{M}(t_k) &= \mathbf{H}^T(t_k)\mathbf{H}(t_k) \\ \mathbf{Q}(t_k) &= \left(\mathbf{H}^T(t_k)\mathbf{H}(t_k)\right)^{-1} \end{aligned} \quad (19)$$

PDOP is defined as:

$$PDOP(t_k) = \sqrt{Q_{11}(t_k) + Q_{22}(t_k) + Q_{33}(t_k)} \quad (20)$$

where Q_{11} , Q_{22} and Q_{33} are the position-related diagonal elements of the covariance matrix. A smaller PDOP indicates a more favorable spatial distribution, whereas a large PDOP indicates geometric degradation.

Because the sets of epochs at which a formally solvable navigation solution can be obtained are not identical among the different configurations, a comparison based solely on the median PDOP cannot fully characterize geometric service capability. When at least four usable observations are available and the normal matrix satisfies the conditioning criterion, the geometry covariance matrix and PDOP are computed. An epoch is classified as formally solvable only when the resulting PDOP is finite. Qualified navigation availability further requires $PDOP \leq 20$. Therefore, the median PDOP, 95th-percentile PDOP, and navigation availability are jointly considered in the performance analysis. The median

PDOP represents the typical geometric quality of formally solvable epochs, whereas the 95th-percentile PDOP characterizes the risk of high-percentile geometric degradation.

2.4. Navigation Availability and Service Continuity

The four link-screening levels are quantified separately. Let N_s denote the total number of navigation sources. At epoch t_k , the number of geometrically visible sources is:

$$N_{geo}(t_k) = \sum_{i=1}^{N_s} V_i^{geo}(t_k) \quad (21)$$

Here, $V_i^{geo}(t_k) = 1$ indicates that the line of sight from source i is blocked by neither the Earth nor the Moon. The number of beam-accessible links is:

$$N_{beam}(t_k) = \sum_{i=1}^{N_s} V_i^{beam}(t_k) \quad (22)$$

Here, $V_i^{beam}(t_k) = 1$ indicates that the link is geometrically visible and supported by the transmitter beam model. The number of detectable signals is:

$$N_{det}(t_k) = \sum_{i=1}^{N_s} D_i(t_k) \quad (23)$$

$$N_{use}(t_k) = \sum_{i=1}^{N_s} U_i(t_k) \quad (24)$$

where $D_i(t_k)$ and $U_i(t_k)$ are shown in the above equation respectively. Only these usable observations enter the geometry matrix. Formal solvability is defined as:

$$I_{sol}(t_k) = \mathbf{1}\{N_{use}(t_k) \geq 4 \wedge \text{rcond}(H_k^T H_k) > 10^{-10} \wedge PDOP(t_k) < \infty\} \quad (25)$$

where $\mathbf{1}\{\cdot\}$ denotes the indicator function, which equals 1 when the enclosed condition is satisfied and 0 otherwise.

Thus, $I_{sol}(t_k) = 1$ requires at least four usable observations, a sufficiently well-conditioned normal matrix, and a finite PDOP. Qualified navigation availability is then defined as:

$$I_{nav}(t_k) = I_{sol}(t_k) \cdot \mathbf{1}\{PDOP(t_k) \leq 20\} \quad (26)$$

Here, $I_{nav}(t_k) = 1$ indicates a formally solvable epoch with finite PDOP not exceeding 20; otherwise, $I_{nav}(t_k) = 0$. The qualified navigation availability ratio over the analysis interval is:

$$R_{nav} = \frac{1}{N_t} \sum_{k=1}^{N_t} I_{nav}(t_k) \quad (27)$$

where N_t is the total number of epochs. Continuity metrics are evaluated from the binary qualified-availability sequence $I_{nav}(t_k)$. Together with the four link-count levels, formal-solvability ratio, and PDOP statistics, they describe how persistently a configuration provides a qualified geometric solution during the transfer.

3. Simulation Results and Discussion

3.1. Simulation Scenario Setup

All source and user states are expressed in the same geocentric inertial frame and processed at a 60 s step. The simulation reports geometrically visible, beam-accessible,

detectable, and usable counts, together with formal solvability, PDOP, qualified navigation availability, and continuity.

At each epoch, Earth/Moon occultation is evaluated first. Beam access and the link budget are then evaluated for unobstructed links. The 12 dB-Hz threshold defines detectable signals; the 18 dB-Hz threshold and beam-strength condition define usable observations. When at least four usable observations are available and the normal matrix satisfies the conditioning criterion, PDOP is computed. Epochs with finite PDOP are classified as formally solvable, while qualified navigation availability additionally requires $\text{PDOP} \leq 20$. Table A2 lists the numerical assumptions.

3.1.1. Cislunar Transfer Spacecraft Trajectory

A representative outward cislunar transfer is used as the common service-user trajectory. The simulation uses a 60 s step and provides the spacecraft position and velocity and the Moon position at every epoch. Figure 2 shows the complete spacecraft and Moon trajectories; the displayed Moon, spacecraft marker, and navigation-source locations form a common snapshot at $t = 2185.9$ h.

All four configurations use the same spacecraft trajectory, time sequence, and assessment model. At each epoch, the model evaluates Earth/Moon occultation, transmitter off-boresight angle and gain, free-space loss, received power, and C/N_0 before computing formal solvability, PDOP, and qualified navigation availability from the usable observations.

As shown in Figure 2, propagation distance and observation geometry change substantially from Earth departure to lunar approach. BDS-3 provides its strongest support near Earth; weak-signal reception and geometry degradation dominate the middle transfer; and lunar occultation and local geometry become increasingly important near the Moon. The stage-based analysis therefore isolates the main service limitations along the transfer.

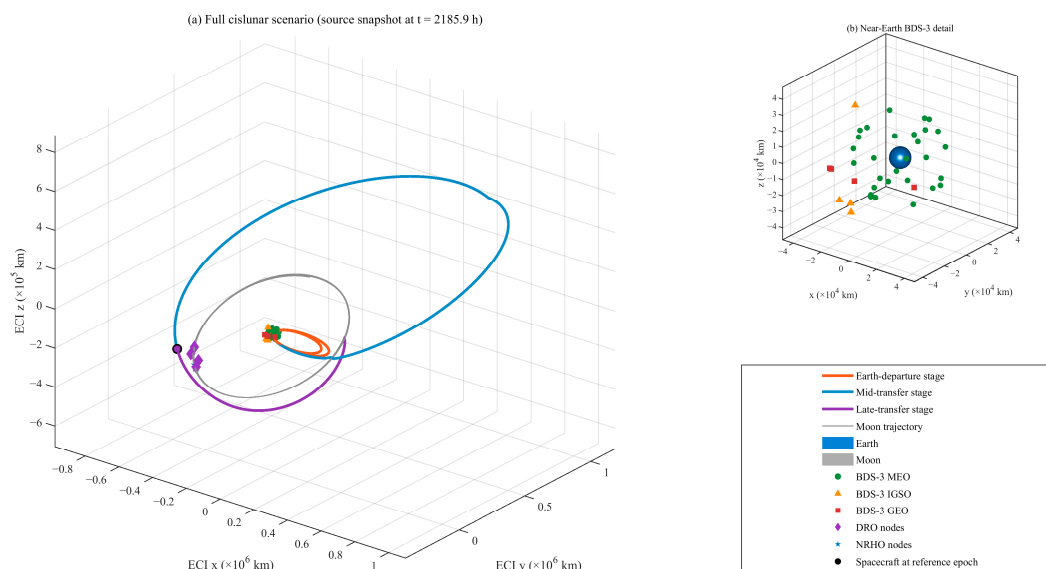


Figure 2. Integrated three-dimensional cislunar scenario. The spacecraft and Moon trajectories cover the full analysis interval, whereas the Moon sphere, spacecraft marker, BDS-3, DRO, and NRHO locations are shown at the common reference epoch $t = 2185.9$ h. Panel (a) shows the three trajectory stages, and panel (b) enlarges the same-epoch near-Earth BDS-3 configuration so that the MEO, IGSO, and GEO spacecraft can be distinguished.

To reflect the stage-dependent variations in the navigation environment throughout the mission, the flight trajectory is divided into three stages: the near-Earth departure stage, the middle cislunar transfer stage, and the late cislunar transfer stage. The time ranges and orbital characteristics of each stage are listed in Table 1.

Table 1. Stage division and time intervals of the cislunar transfer.

Stage	Time Interval (h)	Duration (d)	Orbital Characteristics
Earth departure	0–371.2333	15.4681	Near-Earth phasing and trajectory adjustment
Mid-transfer	371.2333–2185.8667	75.6097	Outward transfer after departure from the near-Earth environment
Late transfer	2185.8667–2482.8833	12.3757	Approach to the lunar vicinity with increasing lunar occultation and local geometry effects

3.1.2. BDS-3, DRO, and NRHO Configurations

The navigation sources considered in the simulation consist of two parts: Earth-side primary navigation sources and lunar-side augmentation sources. BDS-3 is adopted as the Earth-side primary navigation source. The BDS-3 orbital states are obtained by propagating two-line element data in STK. Based on the BDS-3-related satellites available during the simulation period and their corresponding NORAD catalog numbers, a total of 38 satellites are included, comprising 30 medium Earth orbit (MEO), 4 inclined geosynchronous orbit (IGSO), and 4 geostationary Earth orbit (GEO) satellites. The satellite names, pseudorandom noise codes, NORAD catalog numbers, and orbit types are listed in **Table A1** in Appendix A.

The lunar-side augmentation layer consists of four DRO navigation sources and two NRHO navigation sources. The DRO configuration adopts a four-satellite phased arrangement on a periodic orbit generated under the circular restricted three-body problem. The reference NRHO state is obtained from a NASA/JPL reference ephemeris in SPICE format, and two satellites are placed on the same nominal orbit with a phase offset. To compare the service improvements provided by different augmentation schemes, four navigation configurations are considered: BDS-3, BDS-3+DRO, BDS-3+NRHO, and BDS-3+DRO+NRHO. Figure 2 illustrates the spatial layouts of the BDS-3, DRO, and NRHO navigation sources.

The BDS-3 navigation sources are mainly distributed in near-Earth space, whereas the DRO and NRHO navigation sources are located in the lunar vicinity and dynamically sensitive regions of the Earth–Moon system. BDS-3 provides basic Earth-side navigation support. The four-satellite DRO configuration has a relatively large spatial scale and a more continuous orbital distribution, allowing it to provide additional observations along the cislunar transfer corridor and in the outer lunar region. The two-satellite NRHO configuration exhibits pronounced three-dimensional orbital characteristics near the Moon and can provide geometric supplementation during specific periods and within local regions. All navigation-source states are transformed into the geocentric inertial coordinate system, and the same celestial-body occultation, link-budget, availability, and observation-geometry criteria are applied. This ensures a consistent comparison among the four navigation configurations.

3.2. Results and Discussion

To establish the baseline service capability of the Earth-side primary navigation sources, the service capability of the BDS-3-only configuration is first analyzed for the different flight stages. This configuration represents the basic level of navigation service available to the cislunar transfer spacecraft without lunar-side augmentation.

3.2.1. BDS-3-Only Navigation-Service Availability and Observation Geometry

(1) Earth-Departure Stage

During Earth departure, BDS-3 provides a mean of 3.1150 beam-accessible signals and 0.92224 usable observations. Qualified navigation availability is 1.7689%. The median and 95th-percentile PDOP values over 945 formally solvable epochs are 31.306 and 876.91, respectively; the longest continuous service outage is 201.98 h.

The maximum usable-observation peak occurs at $t = 0.300000$ h, when the spacecraft geocentric distance is 10,247.768 km. At this epoch, Earth blocks five BDS-3 satellites and the Moon blocks none; the remaining 33 satellites are geometrically visible, beam-accessible, detectable, and usable. The usable observations span off-boresight angles of 11.580° – 21.562° and C/N0 values of 33.303–44.829 dB-Hz. This confirms that the >30 count is a brief initial near-Earth geometry event rather than persistent transfer-corridor service.

Over the full trajectory, a sensitivity test was also performed to assess antenna-modeling uncertainty without changing the production baseline. Moderate imposed attenuation of 3–6 dB beyond 15° did not eliminate the initial peak, whose maximum remained 32–33 usable observations. In the stress test, all three >30 peaks fell below 30 only after a severe artificial 9 dB attenuation beyond 15° . That stress case also reduced the number of formally solvable epochs from 1138 to 307 and qualified navigation availability from 0.30945% to 0.13627%. These stress-test curves are not asserted to be physical BDS-3 antenna patterns. The temporal variations in signal accessibility, usable observations, PDOP, and navigation availability during the Earth-departure stage are presented in **Figure 3**.

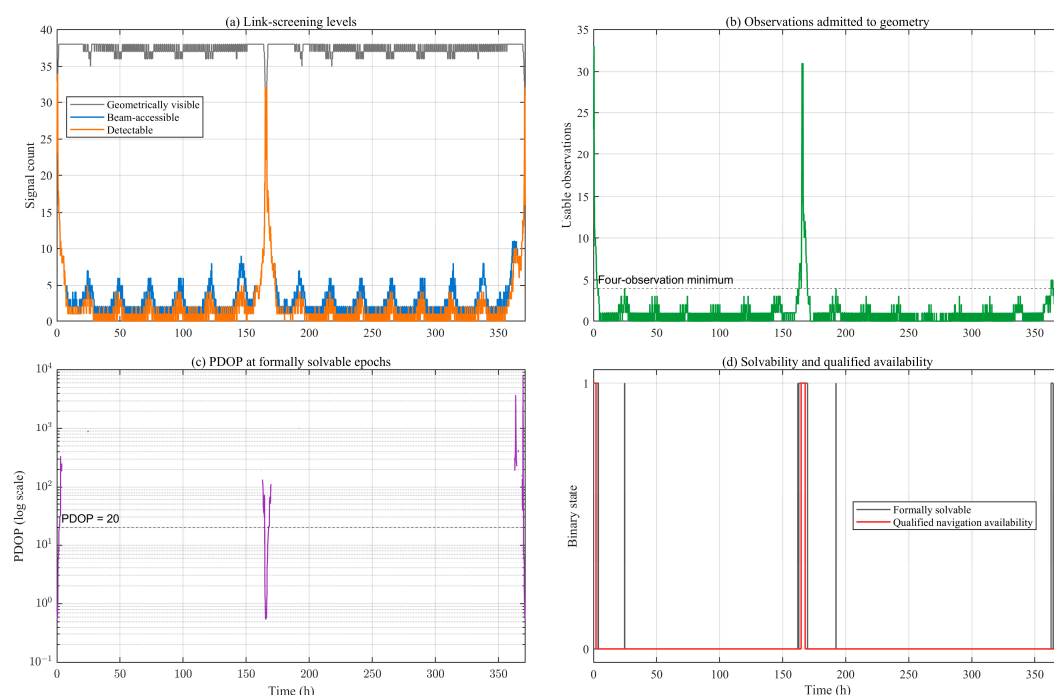


Figure 3. BDS-3 service geometry during the Earth-departure stage (0–371.2333 h): (a) geometrically visible, beam-accessible, and detectable signals; (b) usable observations and the four-observation threshold; (c) PDOP at formally solvable epochs with the PDOP = 20 qualification threshold; and (d) formal solvability and qualified navigation availability.

(2) Mid-Transfer Stage

In mid-transfer, BDS-3 provides a mean of 3.4752 beam-accessible signals and 0.04415 usable observations. Qualified navigation availability falls to 0.06154%. The 95th-percentile PDOP over formally solvable epochs is 1298.57.

Figure 4 uses the same mid-transfer time interval on every panel. Beam-accessible and usable counts fall rapidly after the stage boundary. PDOP is plotted only at formally solvable epochs, whereas the qualified-availability indicator additionally applies $\text{PDOP} \leq 20$; the two traces therefore represent different epoch sets.

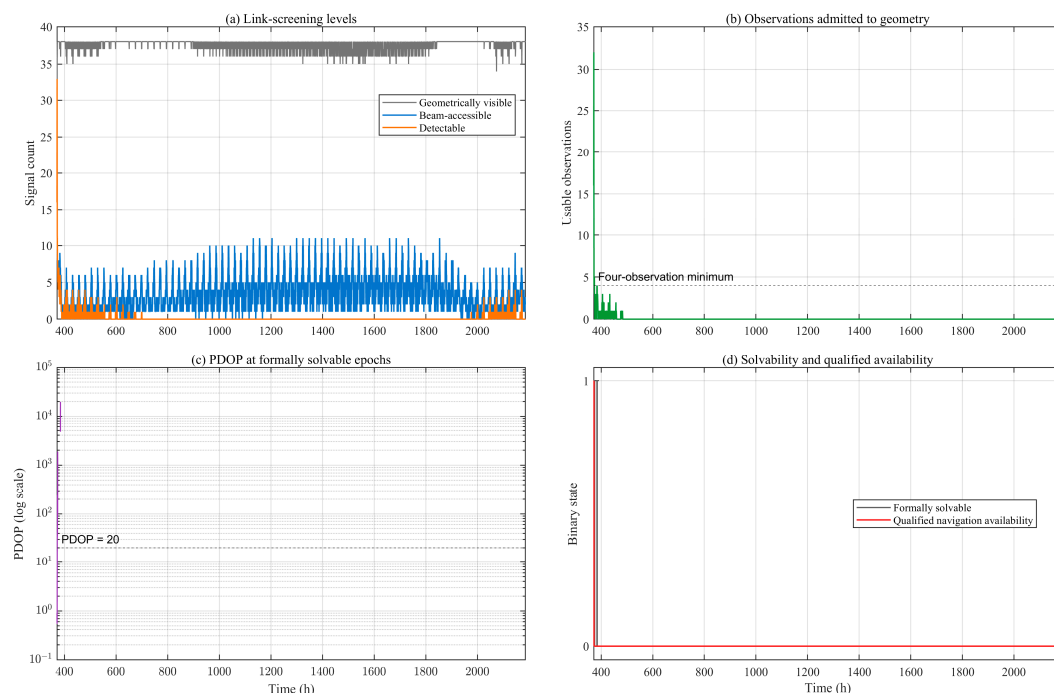


Figure 4. BDS-3 service geometry during the mid-transfer stage. All panels use the same 371.2333–2185.8667 h interval: (a) geometrically visible, beam-accessible, and detectable signals; (b) usable observations; (c) PDOP at formally solvable epochs; and (d) formal solvability and qualified navigation availability ($\text{PDOP} \leq 20$).

(3) Late-Transfer Stage

During late transfer, BDS-3 provides a mean of 3.484 beam-accessible signals and 0.01352 usable observations. No epoch is formally solvable, so qualified navigation availability is 0% and no PDOP statistic is reported.

Figure 5 shows that weak BDS-3 signals remain detectable during late transfer, but no epoch satisfies the formal-solvability criterion. BDS-3 therefore provides only supplementary weak-signal access in this stage and cannot independently provide qualified navigation service.

Table 2 summarizes the verified stage-level median C/N_0 values for detectable BDS-3 signals and usable BDS-3 observations.

Table 2. Stage-level median C/N_0 values for the BDS-3-only configuration.

Stage	Median C/N_0 of Detectable Signals (dB-Hz)	Median C/N_0 of Usable Observations (dB-Hz)
Earth departure	17.54	20.54
Mid-transfer	14.63	20.03
Late transfer	14.34	18.40

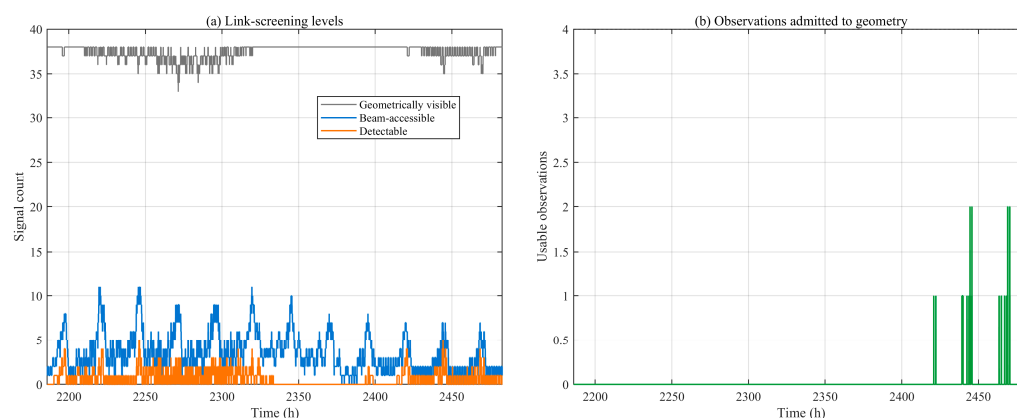


Figure 5. BDS-3 service access during the late-transfer stage: (a) geometrically visible, beam-accessible, and detectable signals; (b) usable observations. No PDOP panel is shown because no epoch is formally solvable.

3.2.2. Comparison of BDS-3 and Lunar-Augmented Configurations

Based on the BDS-3-only baseline configuration, DRO and NRHO lunar-side augmentation sources are further introduced to evaluate the improvements in navigation service capability provided by different combined configurations for the cislunar transfer spacecraft.

(1) Near-Earth Departure Stage

The stage statistics are summarized in Table 3.

As shown in Figure 6, DRO and NRHO increase beam-accessible counts during Earth departure, but the mean usable count, qualified navigation availability, and PDOP statistics remain unchanged. The lunar-side links do not meet the receiver thresholds at this distance, so the early-stage usable observations still come from BDS-3.

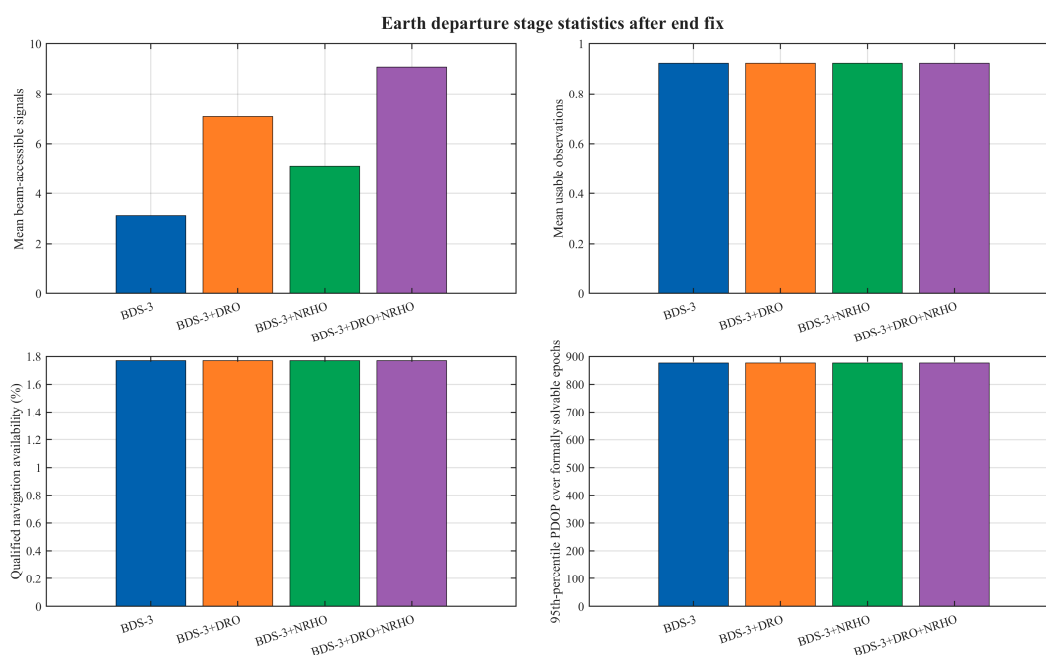


Figure 6. Comparison of the four configurations during the Earth-departure stage.

Table 3. Navigation-service statistics for the Earth-departure stage.

Configuration	Mean Beam-Ac- cessible Signals	Mean Usable Ob- servations	Qualified Nava- tion Availability	Median PDOP	95th-Percentile PDOP
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BDS-3	3.1150	0.92224	1.7689%	31.306	876.91
BDS-3+DRO	7.0804	0.92224	1.7689%	31.306	876.91
BDS-3+NRHO	5.1098	0.92224	1.7689%	31.306	876.91
BDS-3+DRO+NRHO	9.0752	0.92224	1.7689%	31.306	876.91

(2) Middle Cislunar Transfer Stage

The comparison results for the four configurations during the middle cislunar transfer stage are shown in Figure 7.

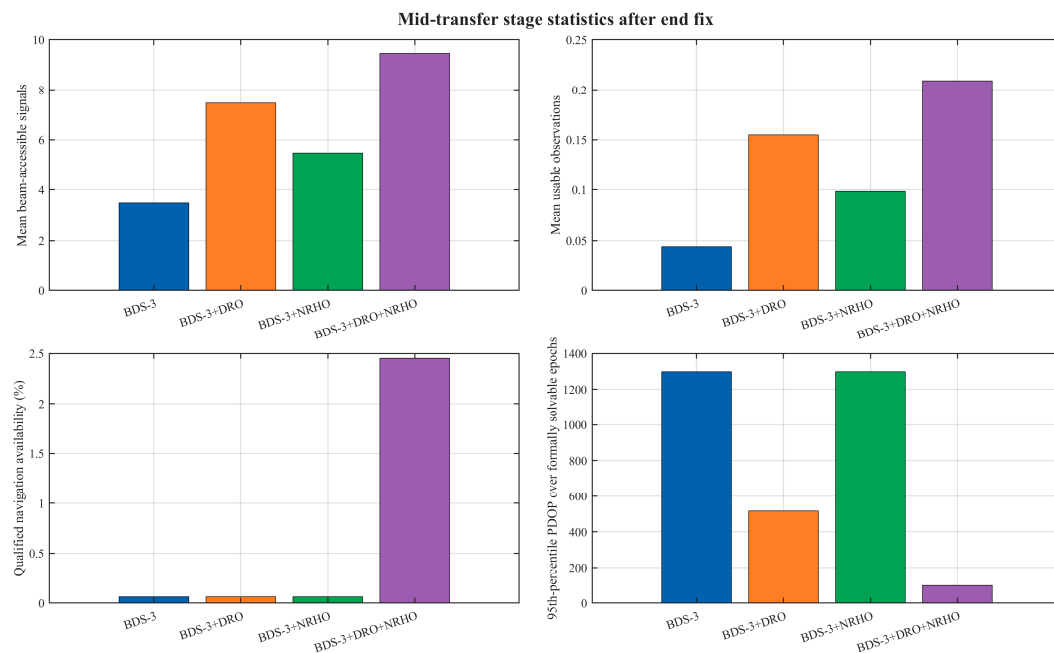


Figure 7. Comparison of the four configurations during the mid-transfer stage.

In mid-transfer, BDS-3 provides means of 3.4752 beam-accessible signals and 0.04415 usable observations. Qualified navigation availability is 0.06154%, and the 95th-percentile PDOP over formally solvable epochs is 1298.57.

Adding DRO increases the mean beam-accessible and usable counts to 7.4657 and 0.1550. The 95th-percentile PDOP decreases to 516.93, but qualified navigation availability remains 0.06154%; DRO alone improves some formally solvable geometries without adding epochs that pass the $\text{PDOP} \leq 20$ criterion.

Adding the two NRHO nodes increases the mean beam-accessible and usable counts to 5.4732 and 0.0982. Qualified navigation availability remains 0.06154%, and the 95th-percentile PDOP remains 1298.57, indicating only limited supplementation in this stage.

With DRO and NRHO combined, the mean beam-accessible and usable counts reach 9.4637 and 0.2091. Qualified navigation availability rises to 2.4523%, and the 95th-percentile PDOP decreases to 99.37. The improvement results from the additional DRO coverage and the complementary NRHO directions.

(3) Late Cislunar Transfer Stage

The comparison results for the four configurations during the late cislunar transfer stage are shown in Figure 8.

During late transfer, BDS-3 provides means of 3.484 beam-accessible signals and 0.01352 usable observations. It yields no formally solvable or qualified navigation epochs.

With DRO, the mean beam-accessible and usable counts increase to 7.482 and 2.382. Qualified navigation availability reaches 10.3%, and the 95th-percentile PDOP over formally solvable epochs is 13.66.

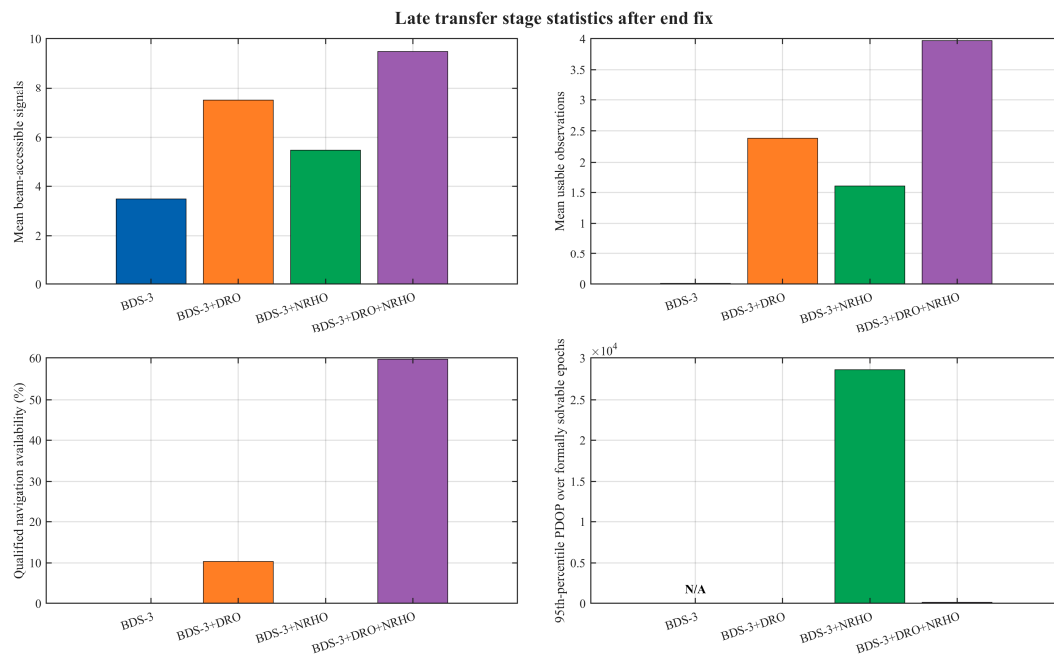


Figure 8. Comparison of the four configurations during the late-transfer stage.

With the two NRHO nodes, the mean beam-accessible and usable counts increase to 5.479 and 1.601. Qualified navigation availability remains 0%, while the 95th-percentile PDOP over formally solvable epochs is 2.86×10^4 . The additional observations are therefore poorly conditioned when NRHO is used without DRO.

The combined configuration provides means of 9.477 beam-accessible signals and 3.970 usable observations and reaches 59.7% qualified navigation availability. Its 95th-percentile PDOP is 141.6. The combined set creates many more formally solvable epochs, although some newly admitted geometries remain weak.

DRO supplies the largest standalone late-stage improvement. NRHO alone adds observations but does not produce qualified availability under the present two-node phasing. Used together, the two augmentation types provide the highest qualified availability, with a remaining high-percentile geometry limitation.

4. Grid-Based Navigation-Service Geometry and Formal-Solvability Mapping

4.1. Cislunar Spatial Grid Generation Method

The along-track analysis evaluates the reference transfer mission, whereas the grid analysis treats each grid point as a candidate user location at selected epochs. The two analyses are linked by projecting the reference trajectory and its stage-transition locations onto the grid maps. The same occultation, antenna-pattern, link-budget, observation-usability, and geometry models are adopted in both analyses.

A two-dimensional regular grid is defined in the instantaneous principal Earth–Moon plane with its origin at the Earth’s center. At each selected epoch t_k , the local x -axis is aligned with the Earth-to-Moon direction, while the local y -axis is defined by the component of the lunar velocity orthogonal to the Earth–Moon direction. The corresponding ECI position of grid point is

$${}^{ECI}\mathbf{r}_{g,ij}(t_k) = x_i \mathbf{e}_x(t_k) + y_j \mathbf{e}_y(t_k) \quad (28)$$

Thus, a fixed grid index (i, j) represents a fixed location in the instantaneous Earth–Moon local frame, while its ECI position varies with epoch as the Earth–Moon geometry evolves. The spatial extent and resolution of the grid are summarized in Table 4.

Table 4. Grid ranges and resolution.

Parameter	Symbol	Value
x range	x	−20,000 to 420,000 km
y range	y	−180,000 to 180,000 km
Number of x grid points	N _x	121
Number of y grid points	N _y	81

At each selected epoch and valid grid point, the model records the numbers of geometrically visible sources, beam-accessible links, detectable signals, and usable observations. PDOP is computed when at least four usable observations are available and $\text{rcond}(H^T H) > 10^{-10}$. Formal solvability additionally requires the resulting PDOP to be finite, while qualified navigation availability further requires PDOP not exceeding 20. The complete 121×81 grid is evaluated for all four navigation configurations using 1242 uniformly sampled epochs from the reference transfer.

For a given grid point, if N_t representative epochs t_k are selected over the statistical interval, the statistic of a service metric I at that grid point can be defined as:

$$\bar{I}_{ij} = \frac{1}{N_t} \sum_{k=1}^{N_t} I_{ij}(t_k) \quad (29)$$

where $I_{ij}(t_k)$ point. Equation (29) is therefore used to obtain the mean geometrically visible, beam-accessible, detectable, and usable counts, as well as the formal-solvability ratio and qualified navigation availability.

PDOP statistics are evaluated separately and only at epochs satisfying the formal-solvability criterion. For each valid grid point, the temporal median and 95th percentile are defined as:

$$PDOP50_{ij} = \text{median} \{ PDOP_{ij}(t_k) : k \in S_{sol,ij} \} \quad (30)$$

$$PDOP95_{ij} = \text{percentile}_{95} \{ PDOP_{ij}(t_k) : k \in S_{sol,ij} \} \quad (31)$$

where $S_{sol,ij}$ denotes the set of formally solvable epochs at grid point (i, j) .

4.2. Grid-Based Service Geometry of the BDS-3-Only Configuration

Figure 9 provides the BDS-3 baseline. Beam-accessible links persist beyond the near-Earth region, but usable observations and formal solvability decrease sharply along the projected transfer path toward the lunar vicinity.

Figure 9a shows that BDS-3 beam access is strongest near Earth and declines outward. The trajectory overlay makes clear that the spacecraft leaves this favorable region during Earth departure and crosses a weak middle-transfer corridor.

Figure 9b shows a faster decrease in usable observations than in beam-accessible signals. Thus, deep-space service is limited mainly by receiver-level usability rather than by line-of-sight geometry alone.

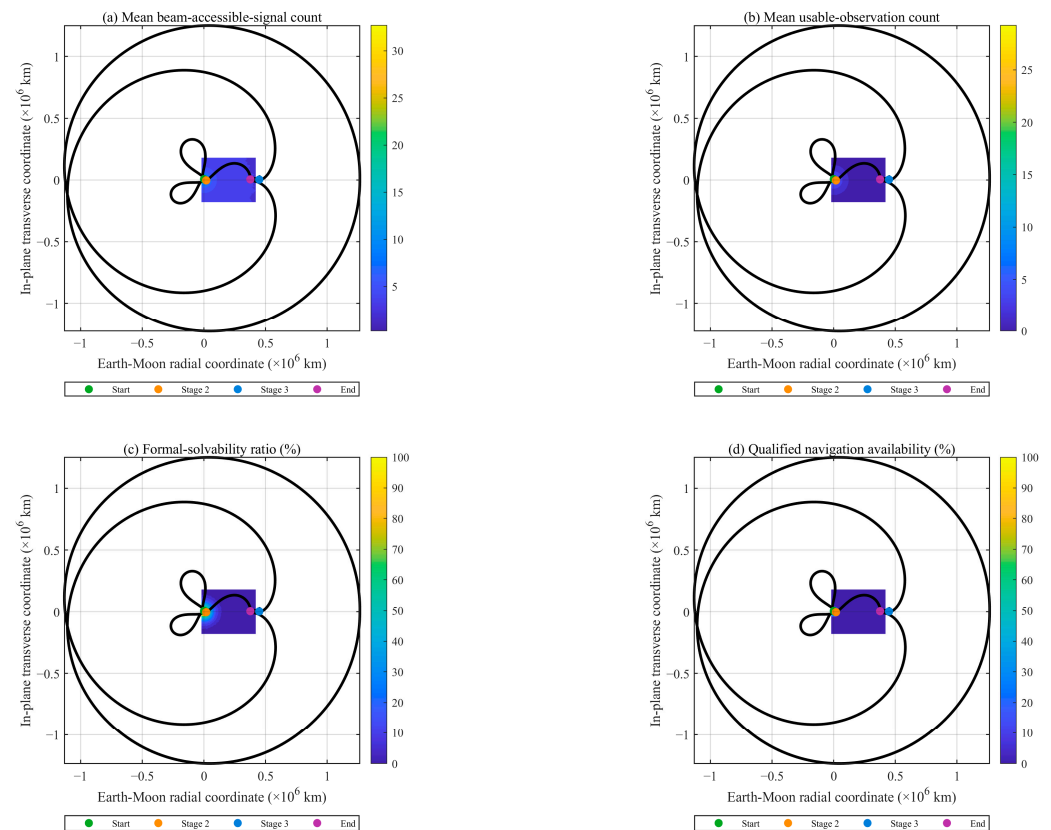


Figure 9. Full-grid BDS-3 service results with the reference transfer trajectory projected into the time-varying Earth–Moon plane: (a) mean beam-accessible-signal count; (b) mean usable-observation count; (c) formal-solvability ratio; and (d) qualified navigation availability (PDOP ≤ 20). Stage-transition locations are marked on the trajectory.

The formal-solvability map in Figure 9c concentrates its nonzero values near Earth and approaches zero toward the lunar vicinity. Figure 9d applies the additional PDOP ≤ 20 requirement and therefore shows a substantially smaller qualified-service region. For BDS-3 alone, the spatially averaged formal-solvability ratio is 6.5197%, whereas the corresponding qualified navigation availability is 1.7423%.

The difference between Figure 9c,d shows that a substantial portion of formally solvable epochs does not satisfy the additional PDOP ≤ 20 qualification criterion. Thus, formal solvability alone does not guarantee qualified navigation service.

Together, the maps identify a near-Earth BDS-3 advantage, a weak middle-transfer corridor, and little independent support near the Moon.

4.3. Grid-Based Qualified Navigation Availability of the Lunar-Augmented Configurations

Figure 10 compares the qualified navigation availability obtained after adding DRO and/or NRHO sources. The projected trajectory directly relates the spatial maps to the service encountered in each mission stage.

BDS-3+DRO extends the qualified-service region outward along the transfer direction, consistent with DRO acting primarily as a coverage-extension layer.

BDS-3+NRHO provides localized geometric supplementation, but the present two-node NRHO configuration does not create a continuous high-qualified-availability corridor.

The combined BDS-3+DRO+NRHO configuration provides the broadest qualified-service region, especially near the Moon, because DRO coverage and NRHO directional diversity are complementary.

A weak region nevertheless remains along the middle portion of the projected transfer, consistent with the stage-based along-track statistics.

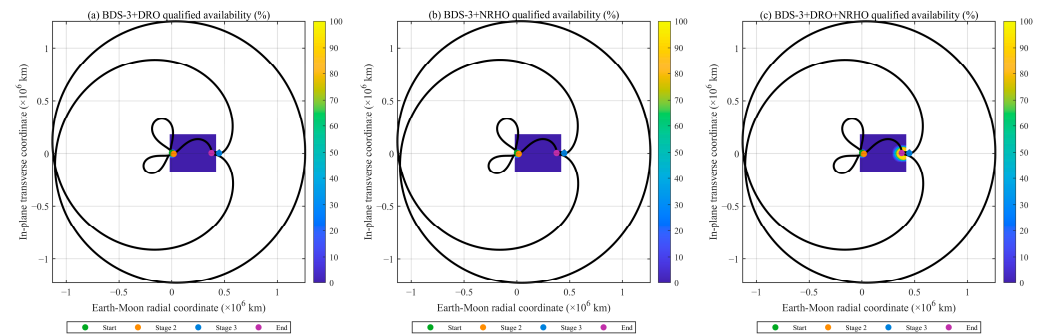


Figure 10. Qualified navigation availability ($\text{PDOP} \leq 20$) of the lunar-augmented configurations with the projected reference transfer trajectory and stage transitions: (a) BDS-3+DRO; (b) BDS-3+NRHO; and (c) BDS-3+DRO+NRHO.

Table 5 summarizes the full-grid beam-accessible and usable counts, mean formal-solvability ratio, mean qualified navigation availability, spatial extent with formal solvability of at least 10%, and conditional PDOP statistics over valid grid points.

Table 5. Full-grid statistics for the four navigation configurations.

Configuration	Mean Beam-Accessible Signals	Mean Usable Observations	Mean Formal-Solvability Ratio (%)	Mean Qualified Navigation Availability (%)	Area with Formal Solvability $\geq 10\%$ (%)	Median Spatial PDOP50	Median Spatial PDOP95
BDS-3	4.4553	1.0377	6.5197	1.7423	15.4413	1829.7206	6781.7510
BDS-3+DRO	8.3995	1.5662	7.0906	1.7423	16.3002	731.3781	5382.7823
BDS-3+NRHO	6.4509	1.2093	6.5475	1.7423	15.4413	1412.4696	6059.0787
BDS-3+DRO+NRHO	10.3951	1.7378	14.5403	8.9886	26.8944	638.6389	4500.1642

The spatial PDOP statistics use formally solvable epochs only and are therefore conditional geometry statistics rather than unconditional availability measures. Qualified navigation availability is reported separately and requires $\text{PDOP} \leq 20$.

For the full-grid summary, the link-count and binary service-state metrics are first averaged over the selected epochs at each valid grid point and are then spatially averaged over all valid grid points. PDOP statistics are treated separately: PDOP50 and PDOP95 are first evaluated over formally solvable epochs at each grid point, after which their spatial medians are reported across grid points with finite values.

The area fraction with formal solvability $\geq 10\%$ is defined as the percentage of valid grid points whose temporal formal-solvability ratio is at least 0.10; grid points excluded by the Earth/Moon body masks are not included in the denominator.

The results indicate that DRO primarily increases beam-accessible and usable observations, while NRHO provides a smaller localized contribution. The combined BDS-3+DRO+NRHO configuration yields the largest mean formal-solvability ratio (14.5403%) and qualified navigation availability (8.9886%), compared with 6.5197% and 1.7423%, respectively, for BDS-3 alone.

Although the combined configuration provides the strongest improvement, its spatially averaged qualified availability remains below 10% under the adopted node count, antenna pattern, link budget, and PDOP threshold. The remaining middle-transfer gap motivates additional nodes, phasing optimization, higher link margin, or complementary observation sources.

4.4. Comparison of Cislunar Navigation Service Capability Among Different Configurations

The grid comparison confirms distinct engineering roles: BDS-3 supplies the near-Earth baseline, DRO extends observation coverage, NRHO provides localized near-lunar geometry, and their combination gives the strongest qualified service. The numerical separation between formal solvability and qualified availability also shows that observation count alone does not guarantee acceptable geometry.

Table 6 summarizes these spatial roles. Beam-accessible and usable counts quantify observation resources, formal solvability identifies numerically valid solutions, and qualified availability indicates where those solutions also satisfy $\text{PDOP} \leq 20$.

Table 6. Comparison of Cislunar Navigation Service Capability among Different Configurations.

Configuration	Spatial Performance Characteristics	Primary Functional Role
BDS-3	Strong near-Earth capability with rapid degradation toward the lunar vicinity	Earth-side baseline navigation source
BDS-3+DRO	Expanded service region along the cislunar transfer corridor	Coverage-extension augmentation
BDS-3+NRHO	Limited overall improvement with local geometric supplementation	Local geometric augmentation
BDS-3+DRO+NRHO	Simultaneous improvement in coverage and near-lunar geometry	Synergistic coverage and geometry augmentation

When considered together with the stage-based along-track statistics, the combined configuration clearly outperforms the other configurations in overall service capability. Its advantages are reflected not only in the increased mean numbers of beam-accessible signals and usable observations, but also in the qualified navigation availability of 59.7% achieved during the late cislunar transfer stage. In comparison, the standalone DRO configuration provides a certain degree of coverage enhancement, whereas the standalone NRHO configuration under the current two-satellite arrangement does not provide sustained qualified navigation service under the present two-node configuration.

In summary, the trajectory overlays connect the spatial maps to the actual mission path. The full-grid analysis confirms that combined DRO/NRHO augmentation offers the largest improvement, while the middle-transfer corridor remains the principal service gap and the absolute qualified availability remains sensitive to the link-budget and geometry thresholds.

5. Conclusions

This study evaluates service access and observation geometry for BDS-3 with DRO/NRHO augmentation along a representative cislunar transfer and over the principal Earth–Moon plane. The analysis separates geometrically visible, beam-accessible, detectable, and usable links, and applies a two-level distinction between formal solvability and qualified navigation availability.

- (1) BDS-3 support is concentrated near Earth. During Earth departure, its qualified navigation availability is 1.7689%; it falls to 0.06154% in mid-transfer and 0% in late transfer. A source-by-source audit confirms that the >30 count peaks are brief initial-geometry events: the geometric-visible, beam-accessible, detectable, and usable maxima are 38, 34, 34, and 33, respectively.
- (2) DRO is the stronger standalone augmentation source. In late transfer, BDS-3+DRO reaches 10.3% qualified navigation availability, whereas the present BDS-3+NRHO configuration remains at 0%. Their combination reaches 59.7%, showing that DRO coverage and NRHO directional diversity are complementary, although the 95th-percentile PDOP of 141.6 indicates weak geometry at some formally solvable epochs.

- (3) The full-grid recomputation shows that the combined configuration increases the mean formal-solvability ratio from 6.5197% to 14.5403%, and the mean qualified navigation availability from 1.7423% to 8.9886%. The middle-transfer corridor remains the principal gap, indicating that additional nodes, optimized phasing, or improved link margin are required for continuous qualified service.

These findings concern navigation-service availability and geometry, not end-to-end positioning accuracy. Positioning-error assessment requires measurement-error models and a navigation estimator and is reserved for future work.

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Data Availability Statement: The BDS-3 orbital data and NASA/JPL SPICE ephemerides used in this study are publicly available. The cislunar spacecraft trajectory data were provided by one of the co-authors and are not publicly available because of project-related restrictions. The processed simulation data and source code are not publicly available due to intellectual property restrictions.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

Table A1. Navigation sources used in the simulation.

No.	Satellite Name	PRN	NORAD ID	Orbit Type
1	BEIDOU-3SIGSO-1S	C31	40549	IGSO
2	BEIDOU-3SM2S	C58	40748	MEO
3	BEIDOU-3SM1S	C57	40749	MEO
4	BEIDOU-3M1	C19	43001	MEO
5	BEIDOU-3M2	C20	43002	MEO
6	BEIDOU-3M7	C27	43107	MEO
7	BEIDOU-3M8	C28	43108	MEO
8	BEIDOU-3M4	C22	43207	MEO
9	BEIDOU-3M3	C21	43208	MEO
10	BEIDOU-3M9	C29	43245	MEO
11	BEIDOU-3M10	C30	43246	MEO
12	BEIDOU-3M5	C23	43581	MEO
13	BEIDOU-3M6	C24	43582	MEO
14	BEIDOU-3M12	C26	43602	MEO
15	BEIDOU-3M11	C25	43603	MEO
16	BEIDOU-3M13	C32	43622	MEO
17	BEIDOU-3M14	C33	43623	MEO
18	BEIDOU-3M16	C35	43647	MEO
19	BEIDOU-3M15	C34	43648	MEO
20	BEIDOU-3G1	C59	43683	GEO
21	BEIDOU-3M17	C36	43706	MEO
22	BEIDOU-3M18	C37	43707	MEO

No.	Satellite Name	PRN	NORAD ID	Orbit Type
23	BEIDOU-3IGSO-1	C38	44204	IGSO
24	BEIDOU-3IGSO-2	C39	44337	IGSO
25	BEIDOU-3M24	C46	44542	MEO
26	BEIDOU-3M23	C45	44543	MEO
27	BEIDOU-3IGSO-3	C40	44709	IGSO
28	BEIDOU-3M22	C44	44793	MEO
29	BEIDOU-3M21	C43	44794	MEO
30	BEIDOU-3M19	C41	44864	MEO
31	BEIDOU-3M20	C42	44865	MEO
32	BEIDOU-3G2	C60	45344	GEO
33	BEIDOU-3G3	C61	45807	GEO
34	BEIDOU-3G4	C62	56564	GEO
35	BEIDOU-3M28	C50	58654	MEO
36	BEIDOU-3M26	C48	58655	MEO
37	BEIDOU-3M25	C47	61186	MEO
38	BEIDOU-3M27	C49	61187	MEO

Table A2. Key simulation and link-budget parameters used in the service-availability assessment.

Parameter	Symbol	Value	Role in the Assessment
Time step	Δt	60 s	Epoch-by-epoch simulation interval
Carrier frequency	f	1575.42 MHz	Common link-budget frequency
Earth radius	R_E	6378.1363 km	Spherical occultation model
Moon radius	R_M	1737.4 km	Spherical occultation model
BDS-3 reference transmit power	P_t,BDS	13 dBW	Reference power; equivalent off-boresight pattern gain is applied separately
DRO/NRHO equivalent isotropic radiated power	EIRP_L	13 dBW per node	Common engineering comparison assumption; no additional directional transmit gain is applied
DRO/NRHO transmit gain and beam access	G_t,L	0 dB; isotropic	Every unocculted lunar-side link is beam-accessible before C/N0 screening
Receiver antenna gain	G_r	0 dBi	Omnidirectional receiving approximation
Receiver system temperature	T_sys	290 K	Receiver noise model
Receiver noise figure	NF	3 dB	Receiver noise model
Detection threshold	C/N0_det	12 dB-Hz	Defines detectable signals
Usability threshold	C/N0_use	18 dB-Hz	C/N0 requirement for usable observations; applied jointly with the beam-strength criterion.
Main-lobe relative gain drop	ΔG_{main}	≤ 3 dB	Beam classification from the equivalent pattern
Sidelobe relative gain drop	ΔG_{side}	>3 dB and ≤ 20 dB	Beam classification from the equivalent pattern
Strong-sidelobe condition	G_t	≥ 0 dB	Additional usability condition for sidelobe links
Minimum usable observations	N_min	4	Formal-solvability requirement
Geometry conditioning	rcond(H ^T H)	$>10^{-10}$	Formal-solvability requirement
Qualified-availability limit	PDOP_max	20	Additional criterion beyond formal solvability

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