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Robust Control of Rigid-Body Attitude System with Guaranteed Cost: A Linearization-Free Method

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Abstract

This paper is concerned with the guaranteed cost control issue of nonlinear rigid-body attitude systems under parameter uncertainty. By exploiting the nonlinear attitude dynamics characterized by quaternions, a novel mechanism for system analysis and synthesis is proposed, which is different from most of the existing guaranteed cost attitude control results. The mechanism enables attitude systems to achieve robustness and guaranteed cost performance. It is highlighted that the proposed method performs controller design with linear matrix inequality constraints, independent of model linearization, which can explicitly deal with the nonlinearity in attitude systems and then avoid modeling error. A numerical example is taken to validate the efficacy and advantage of the proposed method.

Keywords: rigid-body attitude control; guaranteed cost control; robust control; nonlinear system analysis; linearization-free controller design

1. Introduction

In recent decades, rigid-body attitude systems have gradually become a key research area due to their powerful capacity for modeling unmanned vehicles, such as spacecraft [1], aircraft [2], missiles [3], and surface vessels [4]. Rigid-body attitude systems are typically a kind of nonlinear system with strong coupling between states. Moreover, due to appendage motion and complex missions such as rendezvous, separation and capturing, attitude systems are subject to detrimental uncertainty and parameter/state jump. These issues result in difficulties in stability analysis and stabilization of attitude systems [5]. Several methodologies can be adopted for system synthesis such as adaptive control [6], sliding-mode control [7], hybrid control [8], impulsive system control [9], and switched system control [10].

Beside the stability objective in robust control, system performance is also the core objective of designers. Great progress has been made in performance/time guaranteed control methods, such as optimal control [11,12], prescribed-performance control [1,13], H_∞ control [14–17], guaranteed cost control [18–23], finite-time control [24–27], prescribed-time control [28,29], and predefined-time control [30,31]. These methods have been extensively applied to rigid-body attitude systems to improve performance. For example, in [23], the guaranteed cost attitude tracking problem of uncertain UAVs with state constraints is

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investigated, in which control barrier function and disturbance attenuation function are designed to modify the reward function to obtain an optimal regulation method.

Based on the above state of the art, this paper focuses mainly on robust attitude control and guaranteed cost control, which have been extensively explored in existing results. In terms of dealing with parameter uncertainty, especially additive uncertainty, the dominant approach lies in leveraging the linear robust control theory for analysis based on a linearized attitude model [2,4]. However, it is noteworthy that the inherent strong nonlinearity in attitude systems is inevitable in system analysis and synthesis, which indicates that model linearization leads inevitably to modeling error and decreases control performance, especially in the scenario of large-angle rotation. A wide range of linearization-based performance-related methods, such as linear quadratic regulator (LQR) control [32], H_∞ control [14–16] and conventional guaranteed cost control [18–20,22], are also subject to this problem. Several improved techniques have been developed to reduce the design conservatism such as LPV control [3,33], fuzzy control [34,35], and neural network-based control [36,37]. But essentially, these methods are also conducted within linear systems control framework. An alternative approach for achieving robustness or guaranteed performance without model linearization follows the line of nonlinear system control [6,7], such as the aforementioned prescribed-performance control [1,13] and finite-time control [27,38]. But a new issue arises that the nonlinear controller design entails refined tuning of controller parameters. Moreover, several nonlinear system-based optimal methods, such as the nonlinear optimal control [17,39] and learning-based optimal control [11,12,23], cannot output analytical solutions and also suffer from the computational burden of complex iterative solving. In summary, nonlinear system-based robust or optimal control methods are not amenable in terms of tractable off-line design conditions. The comparison of existing attitude methodologies is intuitively concluded in Table 1.

Table 1. Comparison of existing attitude control methodologies.

Methodology	Controller Design	Advantage	Disadvantage
Linear robust control LQR control, H_∞ control, conventional guaranteed cost control	$u = Kx$ under LMIs or Riccati equation	Tractable design condition, off-line solving	Model linearization
Nonlinear robust control, prescribed-performance control, finite-time control, etc.	$u = f(x)$ with $f(\cdot)$ being nonlinear	Flexible structure	Parameter tuning
Nonlinear optimal control, learning-based optimal control	Numerical solution	Optimality for nonlinear model	Computational burden

Motivated by the above discussions, a research problem is posed naturally regarding whether it is feasible to construct a robust guaranteed cost attitude controller with tractable design conditions and without model linearization. To the best of the authors' knowledge, there exist extremely few results on this research problem. Although [40,41] explore the problem, time-varying parameter uncertainty has not been considered. In this paper, the guaranteed cost attitude control problem is studied for nonlinear rigid-body attitude systems with parameter uncertainty. The main contributions of this paper are stated as follows.

- A new analysis mechanism for robust attitude control is developed. Different from conventional linearization-based robust attitude control results, the proposed method is carried out by handling the inherent nonlinearity, without model linearization. Thus,

the new method eliminates the underlying modeling error and is more suitable for large-angle maneuvering tasks.

- A novel guaranteed cost attitude control method is proposed for rigid-body attitude systems. By exploring the characteristics of attitude dynamics described by quaternions, the method develops a special analysis technique for system synthesis. Thereby, the controller design in this paper is transformed as a series of bilinear/linear matrix inequalities (BMIs/LMIs), which are tractable to form an optimization problem for improving system performance.

Notations: This paper uses unified symbols. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space ($\mathbb{R}$ is $\mathbb{R}^1$), and $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ real matrices. Superscript T stands for vector or matrix transposition. I and 0 are the identical matrix and null matrix of appropriate dimensions, respectively. $*$ in matrices denotes the terms introduced by symmetry. diag$\{\cdots\}$ refers to a block-diagonal matrix. In addition, the representations $M < 0 (\leq 0); M > 0 (\geq 0)$ indicate that M is negative-definite (semi-definite) and positive-definite (semi-definite) for a symmetric matrix M of arbitrary dimension.

2. Attitude Dynamics and Problem Formulation

To represent the attitude dynamics of a rigid-body system, the classic quaternion model is considered in this paper [41], which is widely used in the field of aerospace.

$$\dot{q}_0 = -\frac{1}{2} q^T \omega \tag{1}$$

$$\dot{q} = \frac{1}{2}(q_0 I + q^\times)\omega \tag{2}$$

where $q_0 \in \mathbb{R}$, $q \triangleq [q_1, q_2, q_3]^T \in \mathbb{R}^3$ are the scalar portion and the vector portion of the attitude quaternion $q_v \triangleq \left[q_0, q^T\right]^T$ that satisfies $q_0^2 + q^T q = 1$. The cross product matrix $q^\times$ in the quaternion model is denoted by

$$q^\times = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix}.$$

Moreover, ω is the angular velocity of the rigid body with respect to the inertial frame $\mathcal{I}$ and is expressed in the body-fixed frame $\mathcal{B}$. The angular velocity evolves according to the following equation [41]

$$J\dot{\omega} = -\omega^\times J\omega + Bu \tag{3}$$

where $u \in \mathbb{R}^a$ is the control input, $J \in \mathbb{R}^{3\times 3}$ denote the inertia matrix, $B \in \mathbb{R}^{3\times a}$ represent the input matrix and $a \geq 3$ is the number of actuators. Note that Bu shows the control torque imposed on the rigid body outputted by actuators. To achieve three-axis control, the input matrix B must be of full row rank.

The model (1)–(3) is commonly used to describe many kinds of rigid-body attitude systems. An example for a spacecraft is depicted in Figure 1 to visualize the rigid-body attitude system and to characterize the underlying uncertainty problem. In aerospace engineering, a spacecraft of on-orbit service is usually equipped with liquid fuel, payloads or appendages, such as parabolic antenna for high-gain communication or solar panels for power supply. Thus, the inertia information J is usually uncertain and time-varying due to the effect of fuel consumption, payload change or appendage motion. Furthermore, to provide fully actuated torques, it requires three-axis actuators that are illustrated by reaction wheels in Figure 1. The installation deviation of the position or direction of the

wheels, as well as faults, results in the uncertainty of the input matrix B. Then, a typical form for modeling system uncertainty is taken as follows [18].

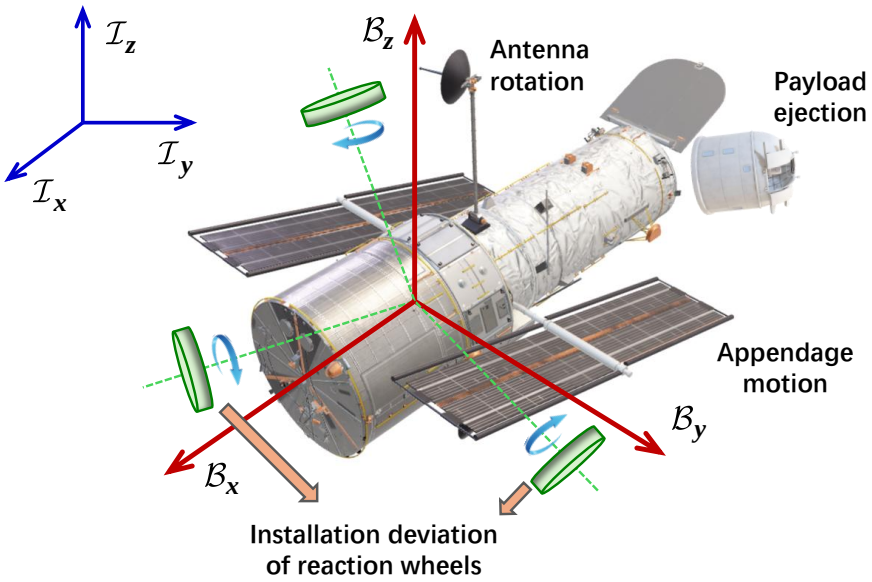


Figure 1. An example of a spacecraft attitude system with an uncertainty problem.

$$J = J_0 + \Delta J, B = B_0 + \Delta B \tag{4}$$

where the nominal parts J_0, B_0 are known matrices of appropriate dimensions, and $\Delta J, \Delta B$ describe the time-varying parameter uncertainty. The uncertainty is considered to be bounded and is modeled as

$$\begin{bmatrix} \Delta J & \Delta B \end{bmatrix} = D \begin{bmatrix} H_1(t)D^T & H_2(t)E \end{bmatrix} \tag{5}$$

where $D \in \mathbb{R}^{3\times m}$, $E \in \mathbb{R}^{n\times a}$ are known constant real matrices, and $H_1(t) \in \mathbb{R}^{m\times m}$, $H_2(t) \in \mathbb{R}^{m\times n}$ are unknown matrix-valued functions and satisfy

$$\begin{aligned} &H_k H_k^T \leq F_k,\ k = 1, 2 \\ &\dot{H}_1 \dot{H}_1^T \leq L \end{aligned} \tag{6}$$

where F_k, $L > 0$ are constant real matrices of appropriate dimensions. The constraints (6) state that some restrictions on system parameter uncertainty are necessary to carry out subsequent stability analysis and controller design. In particular, the last constraint in (6) results from the assumption of the boundedness of rate of change for J, which is mild and practical in aerospace engineering. In addition, according to the positive definiteness of J, J_0 and H_1 are symmetric, i.e., $J_0^T = J_0$, $H_1^T = H_1$.

Remark 1. *The additive uncertainty form (4)–(6) is an extensively used modeling technique for dealing with system uncertainty, which is assumed to be time-varying and upper-bounded [18,42,43]. In such a context, the corresponding system model and simulation setting adopted in this paper are consistent with [18,43]. However, as discussed in the Introduction, all of the prevailing methodologies for handling (4)–(6) depend on model linearization, which promotes the well-established linear robust control and optimal control. In this paper, a new analysis and synthesis method is developed to obtain tractable conditions for controller design without model linearization.*

To achieve the objective of stabilization and performance guarantee of the uncertain attitude system (1)–(3), we adopt the classic PD controller formed as

$$u = K_q q + K_\omega \omega \tag{7}$$

where $K_q,\ K_\omega \in \mathbb{R}^{a\times 3}$ are controller gain matrices to be determined.

The objective in this paper is using the controller (7) to stabilize the uncertain attitude system (1)–(3) and ensuring the system of given performance with respect to the following cost function:

$$F \triangleq \int_0^\infty \left[x^T Q x + u^T R u\right] dt \tag{8}$$

where $x \triangleq \left[q^T \omega^T\right]^T$, $Q \triangleq \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}, Q_{11}, Q_{22} \in \mathbb{R}^{3\times 3}$ and $R \in \mathbb{R}^{a\times a}$ are positive-definite weighting matrices.

3. Stabilization and Performance Analysis

For presenting the main results, some necessary lemmas are given below.

Lemma 1. *For all real matrices Y, Z and positive-definite matrix P of appropriate dimensions, the following inequality always holds:*

$$Y^T Z + Z^T Y \le Y^T P Y + Z^T P^{-1} Z.$$

Proof. The conclusion is a simple deduction from Lemma 1 in [33] and then the proof is omitted. □

Lemma 2 ([44])**.** *Given real matrices $X, Y, Z, P > 0$ of appropriate dimensions and matrix-valued function $W(t)$, for arbitrary positive scalar ϵ satisfying $\epsilon Z^T Z < I$ and $W(t)$ satisfying $W(t)W(t)^T \le P$, the following inequality always holds:*

$$[X + YW(t)Z][X + YW(t)Z]^T \le X(I - \epsilon Z^T Z)^{-1} X^T + \epsilon^{-1} Y P Y^T.$$

To achieve the control objective, the following conclusion shows a set of sufficient conditions for probing the existence of the controller (7).

Lemma 3. *Consider the uncertain attitude system (1)–(3), the controller (7) and the cost function (8). Given a constant $\gamma > 0$, if there exist scalars $p_2 > 0, p_1, p_3$ and matrices $K_q,\ K_\omega \in \mathbb{R}^{a\times 3}$ such that the following conditions hold,*

$$\begin{bmatrix} p_1 I & p_2 J \\ * & p_3 J \end{bmatrix} > 0 \tag{9}$$

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ * & \Phi_{22} \end{bmatrix} < 0 \tag{10}$$

$$c_1 p_1 + c_2 p_2 + c_3 p_3 < \gamma \tag{11}$$

where

$$\Phi_{11} \triangleq p_2 B K_q + p_2 K_q^T B^T + Q_{11} + K_q^T R K_q,$$
$$\Phi_{12} \triangleq p_2 B K_\omega + p_3 K_q^T B^T + \frac{1}{2} p_1 I + p_2 \Delta \dot{J} + Q_{12} + K_q^T R K_\omega$$
$$\Phi_{22} \triangleq \frac{1}{2} p_2 \left(\epsilon_1 J^2 + \frac{1}{\epsilon_1} I \right) + p_3 B K_\omega + p_3 K_\omega^T B^T + p_3 \Delta \dot{J} + Q_{22} + K_\omega^T R K_\omega,$$
$$c_1 \triangleq 2[1 - q_0(0)], c_2 \triangleq 2q^T(0) J \omega(0), c_3 \triangleq \omega^T(0) J \omega(0)$$

and $\epsilon_1 > 0$ is a given parameter, then the controller (7) can asymptotically stabilize the uncertain attitude system (1)–(3) and ensure the system of cost index γ.

Proof. The proof includes two parts, of which the first part concerns stability analysis and the second is devoted to performance analysis. The stability analysis is conducted based on the Lyapunov stability theory, for which we choose the candidate Lyapunov function as

$$V \triangleq p_1 \left[q^T q + (1 - q_0)^2 \right] + 2 p_2 q^T J \omega + p_3 \omega^T J \omega \tag{12}$$

and reformulate it as

$$V = \begin{bmatrix} q \\ \omega \end{bmatrix}^T \begin{bmatrix} p_1 I & p_2 J \\ * & p_3 J \end{bmatrix} \begin{bmatrix} q \\ \omega \end{bmatrix} + p_1 (1 - q_0)^2. \tag{13}$$

To make V a qualified Lyapunov function, a necessary condition is V being positive-definite for all non-equilibrium states. It is easily seen from (13) that (9) and $p_1 > 0$ ensure the necessary condition. Furthermore, $p_1 > 0$ is necessarily guaranteed by (9).

The other necessary condition is $\dot{V} \leq 0$. Taking the derivative of V and substituting the system dynamics (1)–(3), one has

$$\begin{aligned} \dot{V} =& 2 p_1 \left[q^T \dot{q} - (1 - q_0) \dot{q}_0 \right] + 2 p_3 \omega^T J \dot{\omega} + p_3 \omega^T \dot{J} \omega + 2 p_2 \left(q^T J \dot{\omega} + \omega^T J \dot{q} + q^T \dot{J} \omega \right) \\ =& p_1 q^T \omega + p_2 \omega^T J \left(q_0 I + q^\times \right) \omega - 2 p_2 q^T \omega^\times J \omega + 2 p_2 q^T \Delta \dot{J} \omega + p_3 \omega^T \Delta \dot{J} \omega \\ &+ \left(2 p_2 q^T + 2 p_3 \omega^T \right) B u. \end{aligned} \tag{14}$$

By virtue of the property of mixed product of vectors, i.e., $\omega^T J q^\times \omega = q^T \omega^\times J \omega$, the second and third terms in (14) are summed as $p_2 \omega^T J (q_0 I - q^\times) \omega$. Note that the properties hold $(q_0 I - q^\times)^T = (q_0 I + q^\times)$ and $(q_0 I + q^\times)(q_0 I - q^\times) \leq I$, due to the the unit constraint of quaternion and the feature of cross-product matrix $q^\times$. If $p_2 > 0$, then according to Lemma 1 with P being assigned as $\epsilon_1 I, \epsilon_1 > 0$, the sum can further be transformed as

$$\begin{aligned} & p_2 \omega^T J \left(q_0 I - q^\times \right) \omega \\ &= \frac{1}{2} p_2 \omega^T \left[J \left(q_0 I - q^\times \right) + \left(q_0 I + q^\times \right) J \right] \omega \\ &\leq \frac{1}{2} p_2 \omega^T \left[\epsilon_1 J^2 + \frac{1}{\epsilon_1} \left(q_0 I + q^\times \right) \left(q_0 I - q^\times \right) \right] \omega \\ &\leq \frac{1}{2} p_2 \omega^T \left(\epsilon_1 J^2 + \frac{1}{\epsilon_1} I \right) \omega. \end{aligned} \tag{15}$$

By combining (15) and inserting the controller (7) into (14), we have

$$\begin{aligned}\dot{V} \leq & p_1 q^T \omega + \frac{1}{2} p_2 \omega^T \left(\epsilon_1 J^2 + \frac{1}{\epsilon_1} I \right) \omega + 2 p_2 q^T \Delta \dot{J} \omega + p_3 \omega^T \Delta \dot{J} \omega + 2 p_2 q^T B K_q q \\ & + 2 p_2 q^T B K_\omega \omega + 2 p_3 \omega^T B K_q q + 2 p_3 \omega^T B K_\omega \omega \\ = & \begin{bmatrix} q \\ \omega \end{bmatrix}^T \left\{ \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ * & \Phi_{22} \end{bmatrix} - \begin{bmatrix} Q_{11} + K_q^T R K_q & Q_{12} + K_q^T R K_\omega \\ * & Q_{22} + K_\omega^T R K_\omega \end{bmatrix} \right\} \begin{bmatrix} q \\ \omega \end{bmatrix}. \end{aligned} \tag{16}$$

Note that

$$\begin{bmatrix} Q_{11} + K_q^T R K_q & Q_{12} + K_q^T R K_\omega \\ * & Q_{22} + K_\omega^T R K_\omega \end{bmatrix} = Q + \begin{bmatrix} K_q^T \\ K_\omega^T \end{bmatrix} R \begin{bmatrix} K_q & K_\omega \end{bmatrix} > 0$$

which, combined with the condition (10), ensures the negative definiteness of the matrix in the last line of (16). Then, we get $\dot{V} < 0$ that implies the asymptotic stability of the attitude system (1)–(3). In addition, according to the designed Lyapunov function, we get q_0 converging to 1 as $q \to 0$, which indicates that the stable equilibrium is $(1, 0, 0, 0)^T$, and another equilibrium $(-1, 0, 0, 0)^T$ of the quaternion-based attitude control system is not considered in this paper [45]. However, the two equilibria refer to the same physical location. Then, the above conclusion does not influence the system stability, and the rigid body can be driven to the right attitude.

What remains is the performance analysis. Since the states q_v, ω are asymptotically convergent, it holds that $V(\infty) = 0$. Then, by considering the conditions (10) and (16), the cost function (8) satisfies

$$\begin{aligned} F = & \int_0^\infty \begin{bmatrix} q \\ \omega \end{bmatrix}^T \begin{bmatrix} Q_{11} + K_q^T R K_q & Q_{12} + K_q^T R K_\omega \\ * & Q_{22} + K_\omega^T R K_\omega \end{bmatrix} \begin{bmatrix} q \\ \omega \end{bmatrix} dt \\ < & \int_0^\infty \begin{bmatrix} q \\ \omega \end{bmatrix}^T \left\{ \begin{bmatrix} Q_{11} + K_q^T R K_q & Q_{12} + K_q^T R K_\omega \\ * & Q_{22} + K_\omega^T R K_\omega \end{bmatrix} - \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ * & \Phi_{22} \end{bmatrix} \right\} \begin{bmatrix} q \\ \omega \end{bmatrix} dt \\ \leq & - \int_0^\infty \dot{V} dt \\ = & V(0) - V(\infty) = V(0). \end{aligned}$$

Moreover, by virtue of the quaternion constraint $q_0^2 + q^T q = 1$, the designed Lyapunov function (12) can further be transformed as $V = 2p_1(1 - q_0) + 2p_2 q^T J \omega + p_3 \omega^T J \omega$, which indicates that $V(0) = c_1 p_1 + c_2 p_2 + c_3 p_3$. Therefore, the condition (11) ensures that the value of the cost function is less than the given index γ.

In summary, if the condition set (9)–(11) holds, then the closed-loop attitude system (1)–(3) with the designed controller (7) is asymptotically stable and can be guaranteed with the cost index γ. □

It is worth noting that the controller existence conditions (9)–(11) provided by Lemma 3 cannot be solved directly, since the conditions include uncertain matrices J, B and $\dot{H}_1$. Next, we incorporate the parameter uncertainty (4)–(6) to present solvable existence conditions for controller design.

Theorem 1. *Consider the uncertain attitude system (1)–(3), the controller (7) and the cost function (8). Given a scalar $\gamma > 0$, if for all uncertain parameters $H_k, k = 1, 2$ satisfying (6), there exist scalars $p_2 > 0, p_1, p_3$ and matrices K_q, $K_\omega \in \mathbb{R}^{a \times 3}$ such that the following conditions hold, then the uncertain attitude system (1)–(3) under the controller (7) is asymptotically stable and can be guaranteed with the prescribed cost γ.*

$$\begin{bmatrix} p_1 I & p_2 J_0 & \epsilon_2 p_2 D & p_2 D \\ * & p_3 J_0 & 0 & 0 \\ * & * & 2\epsilon_2 p_3 F_1^{-1} & 0 \\ * & * & * & 2\epsilon_2 p_3 I \end{bmatrix} > 0 \tag{17}$$

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & K_q^T E^T \\ * & \Psi_{22} & K_\omega^T E^T \\ * & * & -\epsilon_6 I \end{bmatrix} < 0 \tag{18}$$

$$c_1 p_1 + \bar{c}_2 p_2 + \bar{c}_3 p_3 < \gamma \tag{19}$$

where

$$\begin{aligned}
\Psi_{11} &\triangleq p_2 B_0 K_q + p_2 K_q^T B_0^T + p_2 \epsilon_4 D L D^T + \epsilon_6 p_2^2 D F_2 D^T + Q_{11} + K_q^T R K_q \\
\Psi_{12} &\triangleq p_2 B_0 K_\omega + p_3 K_q^T B_0^T + \frac{1}{2} p_1 I + \epsilon_6 p_2 p_3 D F_2 D^T + Q_{12} + K_q^T R K_\omega \\
\Psi_{22} &\triangleq \frac{1}{2\epsilon_1} p_2 I + \frac{1}{2\epsilon_3} p_2 \epsilon_1 D F_1 D^T + p_3 B_0 K_\omega + p_3 K_\omega^T B_0^T + \frac{1}{2} p_2 \epsilon_1 J_0 \left(I - \epsilon_3 D D^T \right)^{-1} J_0 \\
&\quad + \epsilon_6 p_3^2 D F_2 D^T + \frac{1}{\epsilon_4} p_2 D D^T + \frac{1}{2} p_3 \epsilon_5 D L D^T + \frac{1}{2\epsilon_5} p_3 D D^T + Q_{22} + K_\omega^T R K_\omega \\
\bar{c}_2 &\triangleq q^T(0) \left(J_0 + D F_1 D^T \right) q(0) + \omega^T(0) \left(J_0 + D D^T \right) \omega(0) \\
\bar{c}_3 &\triangleq \omega^T(0) \left(J_0 + \frac{1}{2} D F_1 D^T + \frac{1}{2} D D^T \right) \omega(0)
\end{aligned}$$

and $\epsilon_j > 0$, $j = 1, 2, 4, 5, 6$ *are arbitrarily given parameters, and* $\epsilon_3 > 0$ *is a given parameter satisfying* $\epsilon_3 D D^T < I$.

Proof. The following proof focuses on giving sufficient conditions for satisfying (9)–(11) and shows that the sufficient conditions are expressed in the form of (17)–(19).

(1) We first consider the fulfillment of condition (9). According to the Schur complement lemma [45], (9) is equivalent to $p_3 > 0, p_1 I - \frac{p_2^2}{p_3} J > 0$. By applying Lemma 1 with setting $P = \epsilon_2 I$ where $\epsilon_2 > 0$ is arbitrarily given, the following inequality holds:

$$\begin{aligned}
J &= J_0 + \frac{1}{2} D H_1 D^T + \frac{1}{2} D H_1^T D^T \\
&\leq J_0 + \frac{\epsilon_2}{2} D F_1 D^T + \frac{1}{2\epsilon_2} D D^T.
\end{aligned}$$

Then, an obvious sufficient condition for (9) can be given as

$$p_3 > 0, \frac{p_2^2}{p_3} \left(J_0 + \frac{\epsilon_2}{2} D F_1 D^T + \frac{1}{2\epsilon_2} D D^T \right) < p_1 I \tag{20}$$

which can be transformed as (17) by applying the Schur complement lemma. It indicates that (9) can be ensured by (17).

(2) Now consider the condition (10), of which the uncertainty results from the terms of J, B and $\dot{H}_1$. Note first that, according to Lemma 2, for a given parameter $\epsilon_3 > 0$ satisfying $\epsilon_3 D D^T < I$, one has

$$\begin{aligned}
J^2 &= \left(J_0 + D H_1 D^T \right) \left(J_0 + D H_1 D^T \right)^T \\
&\leq J_0 \left(I - \epsilon_3 D D^T \right)^{-1} J_0 + \frac{1}{\epsilon_3} D F_1 D^T.
\end{aligned} \tag{21}$$

Then, as an extension of Lemma 3 in [46], the following inequality can be easily obtained:

$$\begin{bmatrix} 0 & Y^T Z \\ Z^T Y & 0 \end{bmatrix} \leq \begin{bmatrix} Y^T P Y & 0 \\ 0 & Z^T P^{-1} Z \end{bmatrix}. \tag{22}$$

For dealing with the terms about $\dot{H}_1$, we extract the related matrix terms in Φ_{12} and Φ_{22} to form a new matrix, and apply Lemma 1 and the inequality (22) while introducing given parameters $\epsilon_4, \epsilon_5 > 0$. As a result, under the conditions $p_2 > 0,\ p_3 > 0$, we have

$$\begin{bmatrix} 0 & p_2 D\dot{H}_1 D^T \\ * & p_3 D\dot{H}_1 D^T \end{bmatrix} \leq \begin{bmatrix} p_2\epsilon_4 DLD^T & 0 \\ * & \aleph \end{bmatrix} \tag{23}$$

where $\aleph = p_2\epsilon_4^{-1}DD^T + \frac{1}{2}p_3\epsilon_5 DLD^T + \frac{1}{2\epsilon_5}p_3 DD^T$, and $p_3 > 0$ can be ensured by (17).

To further handle the terms regarding B, we substitute (4)–(6) into (10) and combine the inequalities (21) and (23); thus, a sufficient condition of (10) can be derived as

$$\begin{bmatrix} \bar{\Phi}_{11} & \bar{\Phi}_{12} \\ * & \bar{\Phi}_{22} \end{bmatrix} + \begin{bmatrix} p_2 D \\ p_3 D \end{bmatrix} H_2 \begin{bmatrix} EK_q & EK_\omega \end{bmatrix} + \begin{bmatrix} EK_q & EK_\omega \end{bmatrix}^T H_2^T \begin{bmatrix} p_2 D \\ p_3 D \end{bmatrix}^T < 0$$

where $\bar{\Phi}_{11} \triangleq p_2 B_0 K_q + p_2 K_q^T B_0^T + p_2\epsilon_4 DLD^T + Q_{11} + K_q^T R K_q$, $\bar{\Phi}_{12} \triangleq p_2 B_0 K_\omega + p_3 K_q^T B_0^T + \frac{1}{2}p_1 I + Q_{12} + K_q^T R K_\omega$, $\bar{\Phi}_{22} \triangleq \frac{1}{2\epsilon_1}p_2 I + \frac{1}{2\epsilon_3}p_2\epsilon_1 DF_1 D^T + p_3 K_\omega^T B_0^T + p_3 B_0 K_\omega + \frac{1}{\epsilon_4}p_2 DD^T + \frac{1}{2}p_2\epsilon_1 J_0\left(I - \epsilon_3 DD^T\right)^{-1} J_0 + Q_{22} + \frac{1}{2}p_3\epsilon_5 DLD^T + \frac{1}{2\epsilon_5}p_3 DD^T + K_\omega^T R K_\omega$. By applying Lemma 1 to the above inequality, it can be known that, for all H_2 satisfying $H_2 H_2^T \leq F_2$, the above inequality holds if, for a given parameter $\epsilon_6 > 0$, it holds that

$$\begin{bmatrix} \bar{\Phi}_{11} & \bar{\Phi}_{12} \\ * & \bar{\Phi}_{22} \end{bmatrix} + \epsilon_6 \begin{bmatrix} p_2 D \\ p_3 D \end{bmatrix} F_2 \begin{bmatrix} p_2 D \\ p_3 D \end{bmatrix}^T + \frac{1}{\epsilon_6} \begin{bmatrix} EK_q & EK_\omega \end{bmatrix}^T \begin{bmatrix} EK_q & EK_\omega \end{bmatrix} < 0$$

which, by applying the Schur complement lemma, is equivalent to

$$\begin{bmatrix} \bar{\Phi}_{11} + \epsilon_6 p_2^2 DF_2 D^T & \bar{\Phi}_{12} + \epsilon_6 p_2 p_3 DF_2 D^T & K_q^T E^T \\ * & \bar{\Phi}_{22} + \epsilon_6 p_3^2 DF_2 D^T & K_\omega^T E^T \\ * & * & -\epsilon_6 I \end{bmatrix} < 0$$

i.e., (18). That is to say, the original condition (10) can be ensured by (17) and (18).

(3) To obtain a sufficient condition for (11), we invoke Lemma 1 by setting $P = I$ and get the following conclusion:

$$\begin{aligned} c_2 &= q^T(0)DH_1 D^T\omega(0) + \omega^T(0)DH_1^T D^T q(0) + q^T(0)J_0\omega(0) + \omega^T(0)J_0 q(0) \\ &\leq q^T(0)DF_1 D^T q(0) + \omega^T(0)DD^T\omega(0) + q^T(0)J_0 q(0) + \omega^T(0)J_0\omega(0) \\ &= \bar{c}_2 \\ c_3 &= \omega^T(0)J_0\omega(0) + \omega^T(0)DH_1 D^T\omega(0) \\ &\leq \omega^T(0)J_0\omega(0) + \frac{1}{2}\omega^T(0)DF_1 D^T\omega(0) + \frac{1}{2}\omega^T(0)DD^T\omega(0) \\ &= \bar{c}_3. \end{aligned}$$

Then, by replacing $c_2,\ c_3$ in (11) with $\bar{c}_2,\ \bar{c}_3$, a sufficient condition for (11) is obtained as (19). □

Remark 2. *It can be seen from Theorem 1 that the controller design is posed in the form of matrix inequality based on the nonlinear attitude model. This is the distinct difference between the proposed method and conventional attitude control methods. To achieve robust guaranteed cost performance and obtain tractable matrix inequality conditions, the proposed method first considers the attitude dynamics described by the quaternion rather than Euler angles, which are commonly used in*

traditional guaranteed cost attitude control [19,20,23]. Accordingly, some special quaternion-based techniques are developed for tackling nonlinear terms to obtain state-irrelevant matrix inequalities, as shown in the proof of Lemma 3. Furthermore, by matrix scaling technologies, the underlying uncertain terms are eliminated to obtain matrix inequalities that can be used for solving controller gains, as shown in the proof of Theorem 1.

Remark 3. *The derived results are more generalized than conventional results of guaranteed cost attitude control [19,20,23], since the considered time-varying uncertainty (4)–(6) not only includes the system matrix uncertainty that arises from inertia parameters but also considers the input matrix uncertainty introduced by installation deviation or lose-efficacy faults of actuators. In addition, the proposed method is more applicable than linear optimal control methods, for example the classic LQR method [32], since most of the linear optimal methods require nominal system information for controller design and then cannot deal properly with parameter uncertainty.*

4. Performance Improvement and Discussion

In Theorem 1, controller design is posed as a feasibility problem of conditions (17)–(19), which can be solved off-line. However, it should be noted that although (17)–(19) do not include time-varying terms, (18) incorporates many bilinear terms of decision variables, i.e., the products between p_2, p_3 and K_q, K_ω, and nonlinear terms like $\frac{1}{2}p_2\epsilon_1 J_0\left(I-\epsilon_3 DD^T\right)^{-1}J_0$. Such conditions are referred to as BMIs that are essentially nonlinear and increase the complexity of solving. In contrast to BMI, LMI is more tractable for solving and its convexity is helpful to form optimization problems for searching for optimal solutions and improving control performance. In addition, value assignment of the free parameters $\epsilon_i > 0$ indeed influences the solvability of conditions (17)–(19) and affects the feasible region of decision variables. For the goals of reducing parameter tuning and finding optimal solution, some free parameters could be regarded as decision variables, and some matrix manipulations can be taken to transform BMIs into LMIs.

To handle time-varying uncertainty better and obtain more tractable solving conditions, we prove in the following that the conditions given by Theorem 1 can be ensured by a set of LMI conditions.

Theorem 2. *Consider the attitude control system (1)–(3) with parameter uncertainty (4)–(6). For a given scalar $\gamma > 0$, if there exist scalars $\epsilon_6 > 0$, $\bar{s}_{2,3}, \bar{s}_{2,4}, \bar{s}_{3,5}$, s_i, $i = 1,2,3$ and matrices $\bar{K}_q$, $\bar{K}_\omega \in \mathbb{R}^{a\times 3}$ satisfying the following LMIs,*

$$\begin{bmatrix} s_1 I & s_3 J_0 & \epsilon_2 s_3 D & s_3 D \\ * & s_2 J_0 & 0 & 0 \\ * & * & 2\epsilon_2 s_2 F_1^{-1} & 0 \\ * & * & * & 2\epsilon_2 s_2 I \end{bmatrix} > 0 \tag{24}$$

$$\bar{s}_{2,3} DD^T < s_2 I \tag{25}$$

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} \\ * & \Pi_{22} & 0 & 0 \\ * & * & \Pi_{33} & 0 \\ * & * & * & \Pi_{44} \end{bmatrix} < 0 \tag{26}$$

$$s_1 < \epsilon_7 s_2 \tag{27}$$

$$\begin{bmatrix} \gamma & c_1\epsilon_7 & \bar{c}_2 & \bar{c}_3 \\ * & c_1\epsilon_7 s_3 & 0 & 0 \\ * & * & \bar{c}_2 s_2 & 0 \\ * & * & * & \bar{c}_3 s_3 \end{bmatrix} > 0 \tag{28}$$

where $\epsilon_1, \epsilon_2, \epsilon_7 > 0$ *are given parameters,*

$$\Pi_{11} \triangleq \begin{bmatrix} \Pi_{11}^{11} & \Pi_{11}^{12} \\ * & \Pi_{11}^{22} \end{bmatrix}, \Pi_{11}^{11} \triangleq B_0\bar{K}_q + \bar{K}_q^T B_0^T + \epsilon_6 DF_2D^T + \bar{s}_{2,4}DLD^T$$

$$\Pi_{11}^{12} \triangleq B_0\bar{K}_\omega + \bar{K}_q^T B_0^T + \frac{1}{2}s_1 I + \epsilon_6 DF_2D^T, \Pi_{11}^{22} \triangleq B_0\bar{K}_\omega + \bar{K}_\omega^T B_0^T + \epsilon_6 DF_2D^T + \frac{1}{2}\bar{s}_{3,5}DLD^T$$

$$\Pi_{12} \triangleq \begin{bmatrix} \bar{K}_q^T E^T & s_2 I & 0 \\ \bar{K}_\omega^T E^T & 0 & s_3 I \end{bmatrix}, \Pi_{13} \triangleq \begin{bmatrix} \bar{K}_q^T & 0 & 0 \\ \bar{K}_\omega^T & D & s_3 D \end{bmatrix}, \Pi_{14} \triangleq \begin{bmatrix} 0 & 0 & 0 \\ s_3 I & \epsilon_1 s_3 D & \epsilon_1 s_3 J_0 \end{bmatrix}$$

$$\Pi_{22} \triangleq \begin{bmatrix} -\epsilon_6 I & 0 & 0 \\ * & -M_{11} & -M_{12} \\ * & * & -M_{22} \end{bmatrix}, \Pi_{33} \triangleq \begin{bmatrix} -R^{-1} & 0 & 0 \\ * & -2\bar{s}_{3,5} I & 0 \\ * & * & -\bar{s}_{2,4} I \end{bmatrix}$$

$$\Pi_{44} \triangleq \begin{bmatrix} -2\epsilon_1 s_2 I & 0 & 0 \\ * & -2\epsilon_1 \bar{s}_{2,3} F_1^{-1} & 0 \\ * & * & -2\epsilon_1 s_2 I + 2\epsilon_1 \bar{s}_{2,3} DD^T \end{bmatrix}$$

and $\begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}^{-1}$ *with* $M_{11}, M_{22} \in \mathbb{R}^{3\times 3}$, *then (17)–(19) are feasible and the uncertain attitude system (1)–(3) is asymptotically stable with the prescribed cost* γ *under the controller (7) that is determined by*

$$K_q = s_2^{-1}\bar{K}_q, K_\omega = s_3^{-1}\bar{K}_\omega. \tag{29}$$

Proof. The proof is based on matrix manipulation techniques by introducing slack variables. To streamline the main results, the detailed proof is relegated to Appendix A. □

By Theorem 2, the proposed controller is parameterized as the solution of (24)–(28), which are formulated in LMIs and then are tractable for solving and forming convex optimization problems. In addition, note that if the given index γ is regarded as a decision variable, the LMI feature remains unchanged. Thus, to search for the minimal cost index and optimize state trajectory, γ can be considered as an optimizer variable in (24)–(28) to form a convex optimization problem as follows.

Theorem 3. *Consider the attitude system (1)–(3) with parameter uncertainty (4)–(6) and the cost function (8). If the following optimization problem has a solution* $\gamma^*, s_i^*, \bar{K}_q^*, \bar{K}_\omega^*$

$$\min_{\substack{\bar{s}_{2,3}, \bar{s}_{2,4}, \bar{s}_{3,5}, \epsilon_6 \\ s_i, \bar{K}_q, \bar{K}_\omega, \gamma}} \gamma, \quad s.t. \quad (24)\text{–}(28) \tag{30}$$

then the uncertain attitude system is asymptotically stable with the optimized performance index γ^*, *and the corresponding controller is designed as (7) with the optimized controller gains being* $K_q^* = \frac{1}{s_2^*}\bar{K}_q^*, K_\omega^* = \frac{1}{s_3^*}\bar{K}_\omega^*$.

Remark 4. *It can be seen from Theorem 2 that, besides the constraint transformation to LMIs, the number of scaling parameters is reduced from 6 to 3 for ease of implementation. For given* $\epsilon_1, \epsilon_2, \epsilon_7$, *the parameterized LMIs (24)–(28) enforce the convexity of the optimization problem (30) that can be easily solved. It is noted that, however, the solution of (30) in this case is conservative since it rests with the values of* $\epsilon_1, \epsilon_2, \epsilon_7$. *To reduce the conservatism and obtain an improved solution, we can resort to the so-called gridding technique, a commonly used technique in the field of LPV control [33]. Also, the linear search technique used in switched system control [47] offers an alternative for searching for a suboptimal solution with less computational complexity. An algorithm*

for calculating the subminimum γ^* with a group of feasible maximal values $\bar{\epsilon}_1, \bar{\epsilon}_2, \bar{\epsilon}_7$ is provided in Algorithm 1.

Algorithm 1: Computation of the subminimum γ^*

Input: $Q, R, J_0, B_0, D, E, F_k, L, q(0), \omega(0), \Delta\epsilon$
Initialization: $n = 1, \epsilon_1 = \bar{\epsilon}_1, \epsilon_2 = \bar{\epsilon}_2, \epsilon_7 = \bar{\epsilon}_7$
Step 1:
  while $\epsilon_1 > 0$ **do**
    Solve the optimization (30)
    if *feasible* **then**
      $(\check{\epsilon}_1(n), \check{\gamma}(n)) \leftarrow (\epsilon_1, \gamma)$
      $n \leftarrow n + 1$
    end if
    $\epsilon_1 \leftarrow \epsilon_1 - \Delta\epsilon$
  end while
  $s = \arg\{\min(\check{\gamma}(i), i \leq n - 1)\}$
  Update $\epsilon_1 = \check{\epsilon}_1(s), n = 1$
Step 2:
  while $\epsilon_2 > 0$ **do**
    Solve the optimization (30)
    if *feasible* **then**
      $(\tilde{\epsilon}_2(n), \tilde{\gamma}(n)) \leftarrow (\epsilon_2, \gamma)$
      $n \leftarrow n + 1$
    end if
    $\epsilon_2 \leftarrow \epsilon_2 - \Delta\epsilon$
  end while
  $t = \arg\{\min(\tilde{\gamma}(i), i \leq n - 1)\}$
  Update $\epsilon_2 = \tilde{\epsilon}_2(t), n = 1$
Step 3:
  while $\epsilon_7 > 0$ **do**
    Solve the optimization (30)
    if *feasible* **then**
      $(\hat{\gamma}(n), \hat{s}_2(n), \hat{s}_3(n), \hat{K}_q(n), \hat{K}_\omega(n)) \leftarrow (\gamma, s_2, s_3, \bar{K}_q, \bar{K}_\omega)$
      $n \leftarrow n + 1$
    end if
    $\epsilon_7 \leftarrow \epsilon_7 - \Delta\epsilon$
  end while
  $v = \arg\{\min(\hat{\gamma}(i), i \leq n - 1)\}$
Compute $K_q^* = \hat{s}_2^{-1}(v)\hat{K}_q(v), K_\omega^* = \hat{s}_3^{-1}(v)\hat{K}_\omega(v), \gamma^* = \hat{\gamma}(v)$
Output: $\gamma^*, K_q^*, K_\omega^*$

Remark 5. *Since the scheduling parameters $\epsilon_1, \epsilon_2, \epsilon_7$ are decoupled in the optimization problem (30), the search for the three suboptimal values can be carried out individually. This means using different search orders, for example $\epsilon_1 \to \epsilon_2 \to \epsilon_7$ shown in Algorithm 1. However, it is noteworthy that the results for different orders may be different since the algorithm may converge to different locally optimal solutions. To improve the result solved by Algorithm 1, it suffices to simply verify different search orders and choose the best one.*

Remark 6. *In [48], the topic of guaranteed cost attitude control is also studied for quaternion-based attitude dynamics. However, the proposed method is essentially different from that in [48], due to the following facts. This paper deals mainly with time-varying additive uncertainty, which has not been included in the theoretical analysis of [48]. Since this paper adopts a different control scheme, cost function and Lyapunov function, the stability analysis, related matrix techniques and optimization solving in this paper differ significantly from [48]. In addition, this paper develops a more generalized methodology with better robustness and applicability in other scenarios.*

5. Simulation Results

To verify the proposed guaranteed cost attitude control method, we consider a practical example of spacecraft in this section, i.e., a small low-orbit meteorological satellite ROCSAT-3 [45]. We apply the proposed method to the attitude control of the satellite, and compare it with the BMI-based stabilizing method [45], the classic LQR optimal method and the dynamic-gain-based guaranteed cost control method (DGCC) [48]. The necessary inertia information is taken from [45] and presented below.

$$J_0 = \begin{bmatrix} 5.5384 & -0.0276 & -0.0242 \\ * & 5.6001 & -0.0244 \\ * & * & 4.2382 \end{bmatrix} \text{kg} \cdot \text{m}^2.$$

The actuators are considered as reaction wheels, the installation of which adopts a pyramid configuration. Thus, the number of reaction wheels is four, i.e., $a = 4$, and the nominal input matrix is

$$B_0 = \begin{bmatrix} 1 & 0 & 0 & 0.5774 \\ 0 & 1 & 0 & 0.5774 \\ 0 & 0 & 1 & 0.5774 \end{bmatrix}.$$

The initial quaternion is assigned as $q_v(0) = [0.2, 0.5, -0.4, -0.742]^T$ and the initial angular velocity $\omega(0) = [-\pi/6, \pi/9, 2\pi/9]^T$ rad/s. The cost function is parameterized as $Q_{11} = Q_{22} = I_3, Q_{12} = 0, R = I_4$.

For control performance comparison, parameter uncertainty is included in simulation and constructed as

$$\begin{aligned} D &= I_3, E = 0.8B_0 \\ H_1(t) &= \begin{bmatrix} \sin(0.5t+1) & 0 & 0 \\ * & 2\cos(0.3t+0.5) & 0 \\ * & * & 2\sin(0.4t+2) \end{bmatrix} \text{kg} \cdot \text{m}^2 \\ H_2(t) &= \begin{bmatrix} -0.5+0.4\sin(t) & 0 & 0 \\ * & -0.7+0.2\cos(2t) & 0 \\ * & * & -0.6+0.3\cos(0.5t) \end{bmatrix}. \end{aligned} \tag{31}$$

According to the above time-varying uncertainty, the upper bound information is computed as

$$F_1 = \begin{bmatrix} 1 & 0 & 0 \\ * & 4 & 0 \\ * & * & 4 \end{bmatrix}, F_2 = 0.81I_3, L = \begin{bmatrix} 0.25 & 0 & 0 \\ * & 0.36 & 0 \\ * & * & 0.64 \end{bmatrix}.$$

For controller design in the proposed method, the solving procedure of the optimization problem (30) is first conducted by the linear search algorithm. As shown in Figure 2, three search orders with the same starting point are tested to check the convergence of the algorithm. An important fact can be seen that search order plays a major role in the convergent values of $\epsilon_1, \epsilon_2, \epsilon_7$ but a minor role in the subminimum γ^*. Furthermore, the order $\epsilon_1 \to \epsilon_2 \to \epsilon_7$ ensures a set of convergent values $\epsilon_1 = 0.5, \epsilon_2 = 8.5, \epsilon_7 = 1$ (red pentagram point), which is the optimal solution verified by an exhaustive-like search within the cube-shaped feasible region $[0.5, 10] \times [0.5, 10] \times [0.5, 10]$. In such a case, the optimal cost index of the proposed method is computed as 351.9406, and the corresponding controller gains are solved as

$$K_q = \begin{bmatrix} -1.0470 & 0.1699 & 0.1699 \\ 0.1699 & -1.0472 & 0.1698 \\ 0.1699 & 0.1700 & -1.0470 \\ -0.4084 & -0.4084 & -0.4084 \end{bmatrix}, K_\omega = \begin{bmatrix} -3.3146 & 0.5377 & 0.5379 \\ 0.5378 & -3.3143 & 0.5382 \\ 0.5376 & 0.5373 & -3.3154 \\ -1.2929 & -1.2929 & -1.2930 \end{bmatrix}.$$

For comparison, the BMI controller design directly follows the results in [45]. The LQR controller is solved by the technique in [32] with the optimal cost index computed as 6.0523, and the DGCC controller is solved by following the setting in [48] with the optimal cost index computed as 19.7208. The corresponding controller gains of the latter two methods are presented below.

$$K_q^{\mathrm{LQR}} = -\begin{bmatrix} 0.9038 & -0.0968 & -0.0837 \\ -0.0955 & 0.9039 & -0.0850 \\ -0.1132 & -0.1119 & 0.8989 \\ 0.4014 & 0.4014 & 0.4216 \end{bmatrix}, K_\omega^{\mathrm{LQR}} = -\begin{bmatrix} 2.2309 & -0.3337 & -0.2747 \\ -0.3301 & 2.2421 & -0.2734 \\ -0.3577 & -0.3600 & 1.9926 \\ 0.8910 & 0.8940 & 0.8341 \end{bmatrix}.$$

$$K_q^{\mathrm{DGCC}} = diag\{-2.6198, -2.6198, -2.6198\}, K_\omega^{\mathrm{DGCC}} = diag\{-2.8233, -2.8233, -2.8233\}.$$

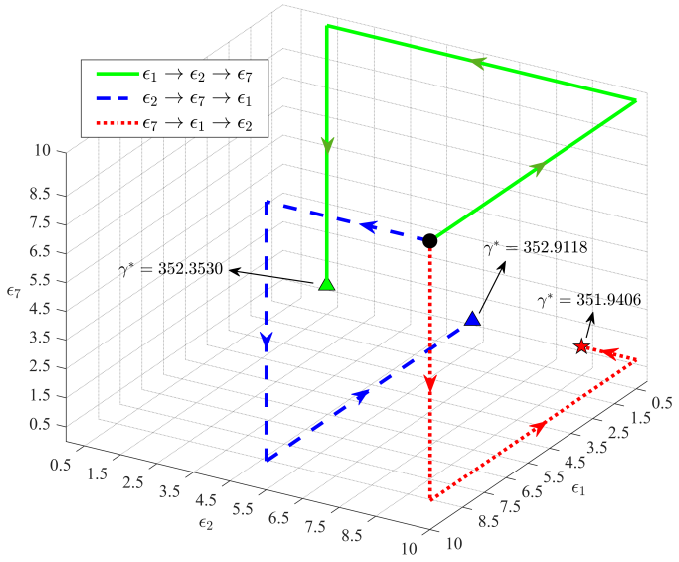
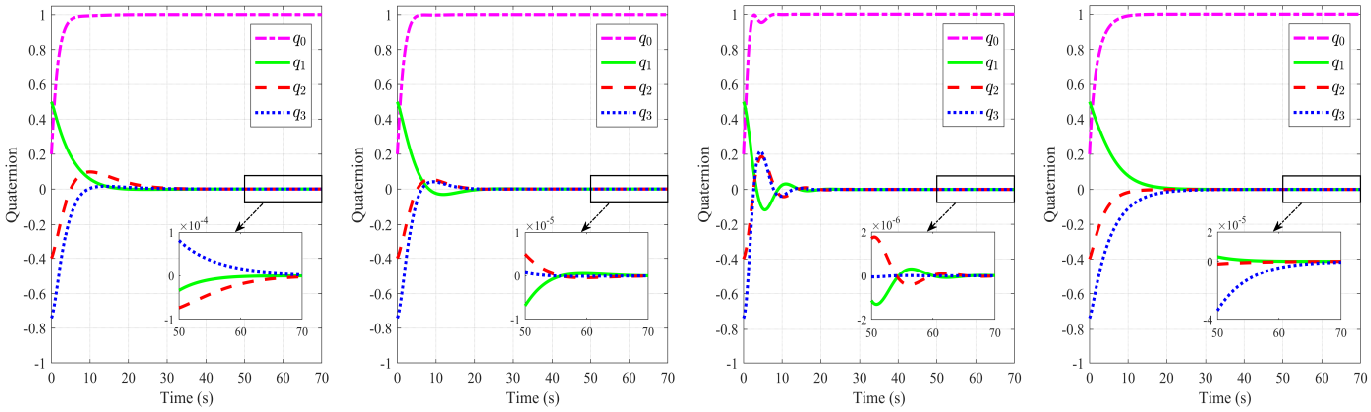


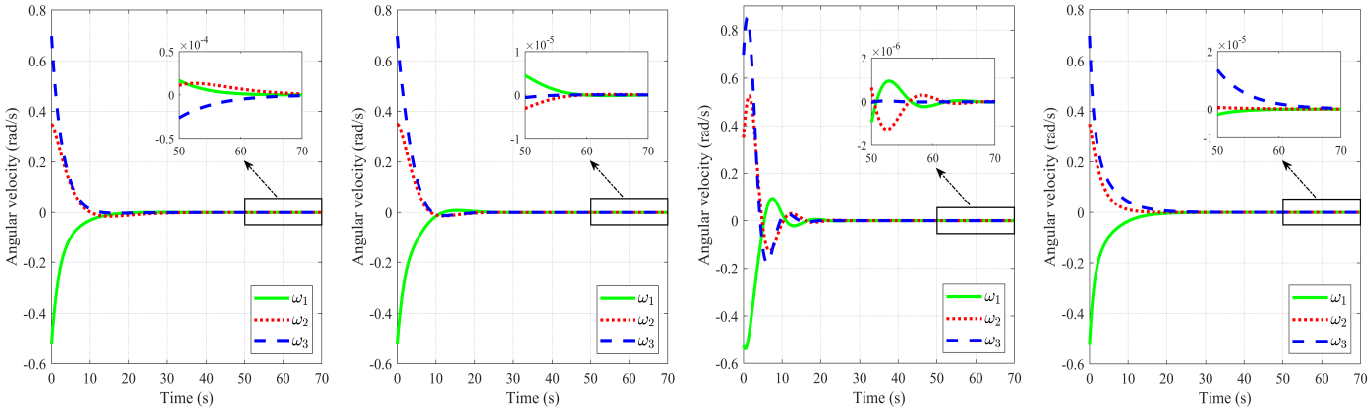
Figure 2. Convergence of linear search algorithm in different orders.

To show the robustness of the proposed method, the time responses of the attitude quaternion and angular velocity in the nominal case are first presented in Figures 3 and 4. It can be seen that without system uncertainty, all of the four methods show satisfactory and similar efficacy, where the DGCC method presents the fastest convergence rate and the highest steady accuracy. However, when the system uncertainty (31) is imposed in simulation, the comparison of the four methods shows different results, as in Figures 5 and 6. The control performances of the BMI method, LQR method and DGCC method degrade obviously, compared to the ones in the nominal case. These results show that the three methods require nominal system information for controller design and therefore cannot deal with system uncertainty effectively. In contrast, the proposed method incorporates explicitly the parameter uncertainty in controller design and then reduces the influence of performance degradation. Thus, under parameter uncertainty, the proposed method enables the least overshoot, the fastest convergence rate and the highest steady accuracy among the comparative methods.



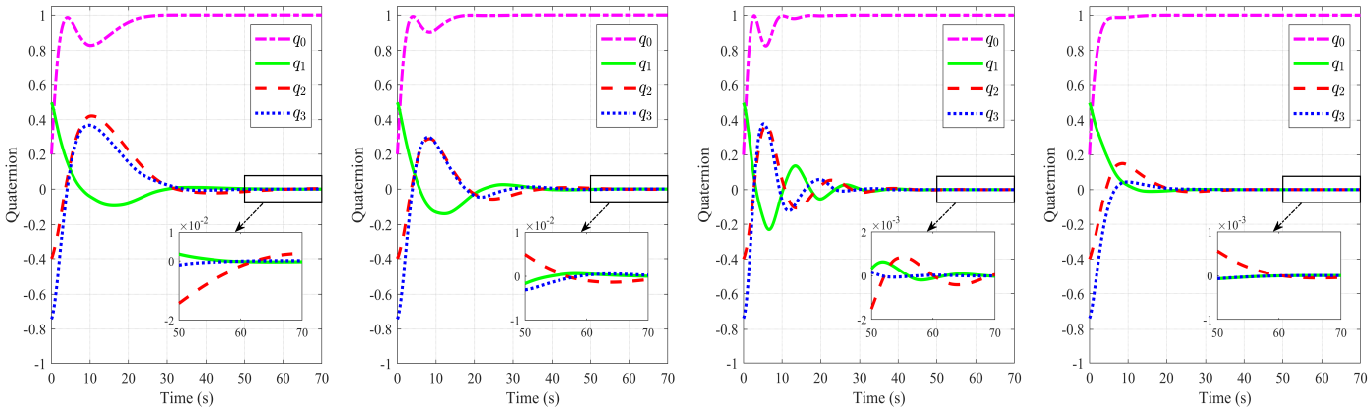
(**a**) BMI method (**b**) LQR method (**c**) DGCC method (**d**) Proposed method

Figure 3. Time responses of attitude quaternions in the nominal case.



(**a**) BMI method (**b**) LQR method (**c**) DGCC method (**d**) Proposed method

Figure 4. Time responses of angular velocities in the nominal case.

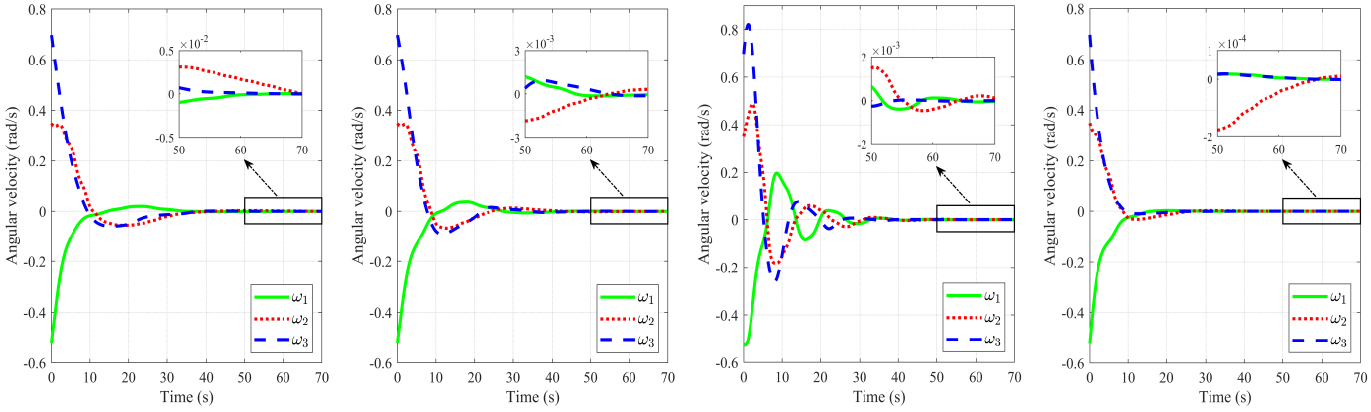


(**a**) BMI method (**b**) LQR method (**c**) DGCC method (**d**) Proposed method

Figure 5. Time responses of attitude quaternions under parameter uncertainty.

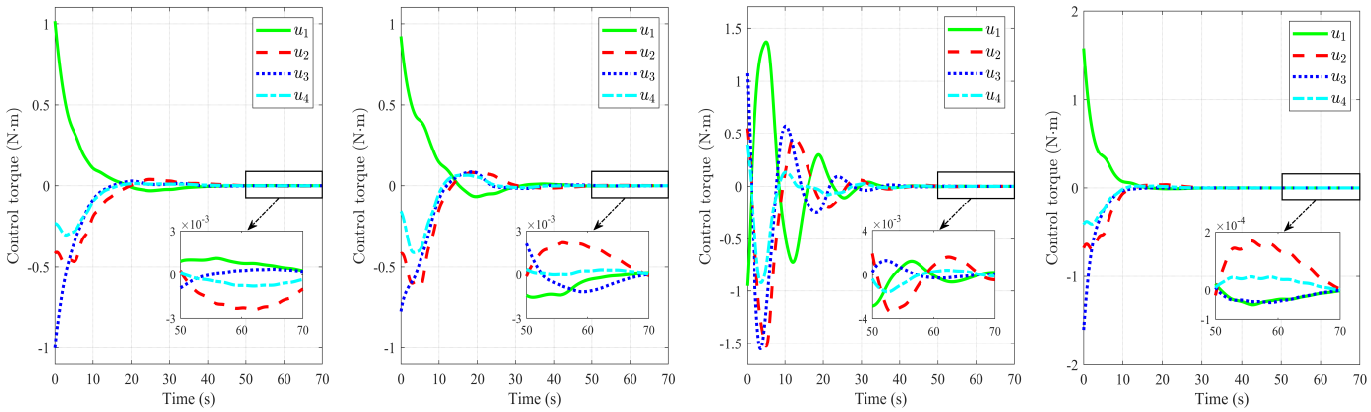
The control torque results are depicted in Figure 7, which shows similar control performance to Figures 5 and 6. The actual cost indices of the four methods are displayed in Figure 8. Due to the greatest overshoot and signal oscillation, the actual index of the DGCC method is substantially larger than that of the other three methods. In addition, owing to the larger amplitude of command torques, the actual index of the proposed method is slightly worse than that of the LQR method. However, the actual index of the

proposed method is far lower than the theoretical value, but the case is the opposite for the LQR and DGCC methods. This means that the proposed theoretical result is of better practical guiding significance. In summary, the simulation demonstrates the advantage of the proposed method over the classic methods in terms of achieving guaranteed cost and robustness simultaneously.



(**a**) BMI method (**b**) LQR method (**c**) DGCC method (**d**) Proposed method

Figure 6. Time responses of angular velocities under parameter uncertainty.



(**a**) BMI method (**b**) LQR method (**c**) DGCC method (**d**) Proposed method

Figure 7. The command torques of the four reaction wheels under parameter uncertainty.

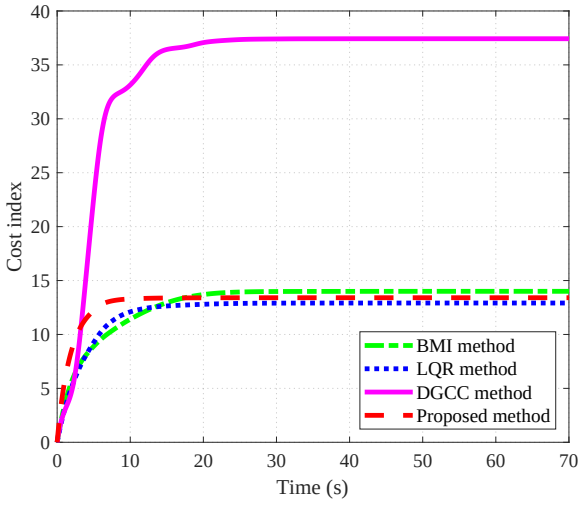


Figure 8. Comparison of actual cost indices under parameter uncertainty.

6. Conclusions

This paper investigates the guaranteed cost attitude control problem for rigid-body systems with time-varying parameter uncertainty. The paper deals with not only the inertial uncertainty from fuel consumption and load variation but also the input uncertainty from actuator installation deviation or failure. The time-varying uncertainty is handled by a new method, which differs from the traditional robust attitude control. The proposed method also ensures attitude systems with guaranteed cost based on a nonlinear attitude model, which is different from the existing guaranteed cost control and linear optimal control.

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Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

Proof of Theorem 2. The following proof shows that (17)–(19) can be ensured by the LMI conditions (24)–(28).

(1) We first prove that $p_2 > 0$. According to the property of the positive-definite matrix and the fact that $J_0,\ \bar{c}_3 > 0$, it can be seen from (24), (28) that $s_i > 0,\ i = 1, 2, 3$. Define

$$s_2 \triangleq p_2^{-1}, s_3 \triangleq p_3^{-1}, s_1 \triangleq \frac{p_1}{p_2 p_3} \tag{A1}$$

and then we get $p_i > 0$.

For ensuring (17), we consider its equivalent condition (20). Based on the definition (A1) and the fact that $s_2,\ s_3 > 0$, an equivalent condition of (17) is derived as

$$\frac{s_3^2}{s_2}\left(J_0 + \frac{\epsilon_2}{2} D F_1 D^T + \frac{1}{2\epsilon_2} D D^T\right) < s_1 I. \tag{A2}$$

Applying the Schur complement lemma to the above inequality, one has (24). It indicates that (17) can be ensured by (24) and (28).

(2) For ensuring (18), an equivalent condition of (18) is first gained by the Schur complement lemma

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} \\ * & \Theta_{22} & 0 & 0 \\ * & * & \Theta_{33} & 0 \\ * & * & * & \Theta_{44} \end{bmatrix} < 0 \tag{A3}$$

where

$$\Theta_{11} \triangleq \begin{bmatrix} \Theta_{11}^{11} & \Theta_{11}^{12} \\ * & \Theta_{11}^{22} \end{bmatrix}, \Theta_{11}^{11} \triangleq p_2 B_0 K_q + p_2 K_q^T B_0^T + \epsilon_6 p_2^2 D F_2 D^T + p_2 \epsilon_4 D L D^T$$
$$\Theta_{11}^{12} \triangleq p_2 B_0 K_\omega + p_3 K_q^T B_0^T + \epsilon_6 p_2 p_3 D F_2 D^T + \frac{1}{2} p_1 I$$
$$\Theta_{11}^{22} \triangleq p_3 B_0 K_\omega + p_3 K_\omega^T B_0^T + \epsilon_6 p_3^2 D F_2 D^T + \frac{1}{2} p_3 \epsilon_5 D L D^T$$
$$\Theta_{12} \triangleq \begin{bmatrix} K_q^T E^T & I & 0 \\ K_\omega^T E^T & 0 & I \end{bmatrix}, \Theta_{13} \triangleq \begin{bmatrix} K_q^T & 0 & 0 \\ K_\omega^T & p_3 D & p_2 D \end{bmatrix}, \Theta_{14} \triangleq \begin{bmatrix} 0 & 0 & 0 \\ p_2 I & p_2 \epsilon_1 D & p_2 \epsilon_1 J_0 \end{bmatrix}$$
$$\Theta_{22} \triangleq \begin{bmatrix} -\epsilon_6 I & 0 & 0 \\ * & -M_{11} & -M_{12} \\ * & * & -M_{22} \end{bmatrix}, \Theta_{33} \triangleq \begin{bmatrix} -R^{-1} & 0 & 0 \\ * & -2 p_3 \epsilon_5 I & 0 \\ * & * & -p_2 \epsilon_4 I \end{bmatrix}$$
$$\Theta_{44} \triangleq \begin{bmatrix} -2 p_2 \epsilon_1 I & 0 & 0 \\ * & -2 p_2 \epsilon_1 \epsilon_3 F_1^{-1} & 0 \\ * & * & -2 p_2 \epsilon_1 I + 2 p_2 \epsilon_1 \epsilon_3 D D^T \end{bmatrix}.$$

Performing congruent transformation on the matrix in (A3) with diag$\left\{ p_2^{-1} I, p_3^{-1} I, I, I, I, I, p_3^{-1} I, p_2^{-1} I, p_2^{-1} I, p_2^{-1} I, p_2^{-1} I \right\}$ and further combining the definition (A1) and the following denotation

$$\bar{K}_q \triangleq p_2^{-1} K_q, \bar{K}_\omega \triangleq p_3^{-1} K_\omega \tag{A4}$$

we have an equivalent condition of (A3) as

$$\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & \Upsilon_{14} \\ * & \Upsilon_{22} & 0 & 0 \\ * & * & \Upsilon_{33} & 0 \\ * & * & * & \Upsilon_{44} \end{bmatrix} < 0 \tag{A5}$$

where

$$\Upsilon_{11} \triangleq \begin{bmatrix} \Upsilon_{11}^{11} & \Upsilon_{11}^{12} \\ * & \Upsilon_{11}^{22} \end{bmatrix}, \Upsilon_{22} \triangleq \Theta_{22}, \Upsilon_{11}^{11} \triangleq B_0 \bar{K}_q + \bar{K}_q^T B_0^T + \epsilon_6 D F_2 D^T + s_2 \epsilon_4 D L D^T$$
$$\Upsilon_{11}^{12} \triangleq B_0 \bar{K}_\omega + \bar{K}_q^T B_0^T + \epsilon_6 D F_2 D^T + \frac{1}{2} s_1 I$$
$$\Upsilon_{11}^{22} \triangleq B_0 \bar{K}_\omega + \bar{K}_\omega^T B_0^T + \epsilon_6 D F_2 D^T + \frac{1}{2} s_3 \epsilon_5 D L D^T$$
$$\Upsilon_{12} \triangleq \begin{bmatrix} \bar{K}_q^T E^T & s_2 I & 0 \\ \bar{K}_\omega^T E^T & 0 & s_3 I \end{bmatrix}, \Upsilon_{13} \triangleq \begin{bmatrix} \bar{K}_q^T & 0 & 0 \\ \bar{K}_\omega^T & s_3 D & s_3 D \end{bmatrix}$$
$$\Upsilon_{14} \triangleq \begin{bmatrix} 0 & 0 & 0 \\ s_3 I & s_3 \epsilon_1 D & s_3 \epsilon_1 J_0 \end{bmatrix}, \Upsilon_{33} \triangleq \begin{bmatrix} -R^{-1} & 0 & 0 \\ * & -2 s_3 \epsilon_5 I & 0 \\ * & * & -s_2 \epsilon_4 I \end{bmatrix}$$
$$\Upsilon_{44} \triangleq \begin{bmatrix} -2 s_2 \epsilon_1 I & 0 & 0 \\ * & -2 s_2 \epsilon_1 \epsilon_3 F_1^{-1} & 0 \\ * & * & -2 s_2 \epsilon_1 I + 2 s_2 \epsilon_1 \epsilon_3 D D^T \end{bmatrix}.$$

In (A5), the parameters $\epsilon_1, \epsilon_4, \epsilon_5, \epsilon_6 > 0$ can be given arbitrarily by designers, but $\epsilon_3 > 0$ must meet the constraint $\epsilon_3 D D^T < I$. Note that $\epsilon_3, \epsilon_4, \epsilon_5$ are always coupled with

the decision variables s_2 and s_3. To reduce the number of scheduling parameters, we introduce the following slack variables:

$$\bar{s}_{2,4} > 0, \bar{s}_{2,3} > 0, \bar{s}_{3,5} > 0 \tag{A6}$$

and set $\epsilon_4 = \frac{\bar{s}_{2,4}}{s_2}, \epsilon_5 = \frac{\bar{s}_{3,5}}{s_3}, \epsilon_3 = \frac{\bar{s}_{2,3}}{s_2}$. Then, substituting (A6) into $\epsilon_3 DD^T < I$ and (A5) results in the LMI conditions (25) and (26). As for condition (A6) of the new decision variables, it can be ensured naturally by (26). Thus, it can be concluded that if the LMI conditions (25) and (26) are feasible, then the original condition (18) is guaranteed to be feasible.

(3) By utilizing the notation (A1), a sufficient condition for (19) can be derived as $\gamma - \frac{c_1 s_1}{s_2 s_3} - \frac{\bar{c}_2}{s_2} - \frac{\bar{c}_3}{s_3} > 0$. When (27) holds, the above sufficient condition can be ensured by

$$\gamma - \frac{c_1 \epsilon_7}{s_3} - \frac{\bar{c}_2}{s_2} - \frac{\bar{c}_3}{s_3} > 0$$

which, by applying the Schur complement, is equivalent to (28). This indicates that the original condition (19) can be ensured by (27) and (28). □

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