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Research on Enhanced Orbit Prediction Techniques Utilizing Multiple Sets of Two-Line Element

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Abstract: Acquiring accurate space object orbits is crucial for many applications such as satellite tracking, space debris detection, and collision avoidance. The widely used two-line element (TLE) method estimates the position and velocity of objects in space, but its accuracy can be limited by various factors. A combination of multiple TLEs and advanced modeling techniques such as batch least squares differential correction and high-precision numerical propagators can significantly improve TLE accuracy and reliability, ensuring better space object surveillance. Previous studies analyzed additional factors that may influence TLE accuracy and evaluated the accuracy of Starlink TLE using precise ephemeris data from SpaceX. The results indicate that utilizing multiple TLEs for precise orbit determination can significantly enhance the performance of orbit prediction methods, particularly when compared to SGP4. By leveraging 10-day Starlink TLEs, the accuracy of 5-day predictions can be improved by approximately twofold. Additionally, producing two pseudo-observations within an orbital period near the TLE epoch yields the greatest effect on prediction accuracy, with this distribution of pseudo-observations increasing accuracy by approximately 10% compared to a uniform distribution. Further research can explore more data fusion and machine learning approaches to optimize operations in space.

Keywords: situation awareness; space debris; navigation; two-line element; orbit prediction

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1. Introduction

In recent years, with the increase in the number of space objects, there has been a growing trend towards using more ground-based and space-based monitoring systems to obtain more accurate orbital observation information [1]. The goal is to improve the accuracy of monitoring data and the reliability of satellite orbits. Inaccurate monitoring and prediction of space object orbits can have severe consequences, such as satellite collisions that may destroy satellites and produce additional space debris that could result in further collisions [2]. For example, the collision between Iridium 33 and deactivated Kosmos 2251 on 10 February 2009 resulted in approximately 2000 additional pieces of space debris [3]. This event marked the first publicly reported destruction of a working satellite by space debris. Similarly, in March 2021, YunHai 1-02 was struck by a space object, resulting in the loss of some functionality [4].

The continued deployment of constellation projects such as Starlink, along with an increase in launch frequency and space object re-entries, has led to a congested space environment [5]. As of March 2023, the number of objects orbiting Earth has surpassed 47,000, consisting of approximately 7000 functional payloads and the rest categorized as debris or unclassified objects. The majority of these objects (about 88%) occupy low-Earth orbit, where most have a diameter greater than 10 cm.

The increasing density of space objects has become a significant environmental challenge, which can be addressed through two methods: debris mitigation and debris removal [6,7]. A crucial element of debris mitigation is collision avoidance. Precise OD and prediction are essential for safe spacecraft operation. However, achieving accurate orbital information remains a challenging task. While recognizing the significance of obtaining precise orbits for space objects, solving this challenge poses a complex problem.

Since the 1970s, the US government has provided GP or general perturbations orbital data to the rest of the world [8]. These data are produced by fitting observations from the US SSN using the SGP4 orbit propagator to produce Brouwer mean elements. The data were originally transmitted via TLEs, which were designed to provide minimum data requirements due to limited transmission and storage capabilities.

In 2011, a method was proposed for determining and predicting orbits utilizing multiple TLEs [9]. This method, known as TLE-OD/OP, employed batch least squares differential correction and high-precision numerical propagators to fit orbits using pseudo-observations from several TLEs. By utilizing the fitted orbit, improvements in TLE OP accuracy were realized. The performance enhancements of TLE-OD/OP accuracy hinge on several factors, including the distribution of pseudo-observations and TLE accuracy. Furthermore, with the ongoing development of satellite constellations such as Starlink, further research is necessary to effectively improve the orbital accuracy of ascending and descending Starlink satellites utilizing TLE-OD/OP methods.

Firstly, the basic theory of the TLE-OD/OP method is introduced. Then, TLEs for Starlink satellites and Starlette were selected to study the performance of TLE-OD/OP under different pseudo-observation distributions or OD durations. Finally, the potential for further improving the OP accuracy of TLE in the future is discussed, and conclusions are drawn.

2. Methodology

The TLE-OD/OP method utilizes the least squares theory and numerical integrator to determine an orbit that optimally fits the pseudo-observed data generated by multi-TLE. The advanced computational technology and numerical integration algorithms used in this method enable the consideration of complex mechanical models. Consequently, TLE-OD/OP provides higher orbital accuracy than SGP4.

The TLE-OD/OP method involves determining the three-dimensional position and velocity vector of an object in space using a set of observations, including positions from TLE. Simultaneously, the parameters of the force model can be estimated, such as the drag or solar radiation coefficient. The least squares estimation method is commonly used to estimate the unknows of both the trajectory and the parameters, and minimize the sum of the squares of the differences between theoretical and observed values.

The state variable $\boldsymbol{x}$ is defined as:

$$\boldsymbol{x} = \{\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{p}\}^T \tag{1}$$

where $\boldsymbol{r}$ and $\boldsymbol{v}$ represent the position and velocity vectors of the orbit, respectively, and $\boldsymbol{p}$ represents the model parameters to be estimated. Assuming that there is a function relationship between the prior state variable $\boldsymbol{x_0}$ and the measured observables $\boldsymbol{y}$, given by

$$\boldsymbol{y} = f(\boldsymbol{x_0}) + v \tag{2}$$

where v is the measurement error. The goal of OD is to estimate the weighted difference between the TLE-based observations and the theoretical trajectory using mathematical models and statistical properties of the noise.

$$\left(y - f(\boldsymbol{x_0})\right)^T P(\boldsymbol{y} - f(\boldsymbol{x_0})) = min \tag{3}$$

$$\hat{\boldsymbol{x}}_{\boldsymbol{0}} = \boldsymbol{x_0} + \Delta\hat{\boldsymbol{x}}_{\boldsymbol{0}}, \tag{4}$$

$$\Delta\hat{\boldsymbol{x}}_{\boldsymbol{0}} = \left(\boldsymbol{A^T P A}\right)^{-1}\boldsymbol{A^T P}(\boldsymbol{y} - f(\boldsymbol{x_0})) \tag{5}$$

The estimated state $\hat{\boldsymbol{x}}_{\mathbf{0}}$ can be obtained by iterating and solving for $\hat{\boldsymbol{x}}_{\mathbf{0}}$ until the change in $\hat{\boldsymbol{x}}_{\mathbf{0}}$ is less than a given value, such as 1 m in position. The partial derivative matrix $\boldsymbol{A} = \left(\frac{\partial f}{\partial x}\cdot\right)\Big|_{\boldsymbol{x}=\boldsymbol{x_0}}$ is necessary to calculate the determined weighted residuals from the observation and modelling process.

The basic steps of the precise OD process are as follows.

(1) Data collection. The TLE-OD/OP method utilizes SGP4 with multiple TLEs to generate pseudo-observations for space objects. The precision of the generated pseudo-observations is influenced by various disturbances, which in turn impacts the accuracy of orbital determination and prediction. A commonly used approach for generating pseudo-observations in the TLE-OD/OP method involves the generation of 100 uniformly distributed positions from all successive TLEs over a specified time period. We hypothesize that greater accuracy can be achieved by generating pseudo-observations closer to the TLE epoch. Through our study, we generate positions for one or more orbital cycles centered on each TLE epoch and aim to identify the most effective strategy for distributing pseudo-observations.

(2) Precise OD. After acquiring pseudo-observations from TLEs, the next step involves performing an OD using least squares theory to match an orbit as closely as possible to the pseudo-observations. This method utilizes a sophisticated numerical integration technique with an accurate mechanical model accounting for various factors such as the gravity field, Sun–Moon gravity, Earth's solid tides and ocean tides, solar light pressure, and atmospheric drag. Notably, in the case of ultralow orbit space objects (below 800 km), the drag coefficient is considered a parameter while estimating orbital parameters in atmospheric drag calculations. The numerical integration method used in this context is Cowell's method [10].

(3) OP and OP error computation. Once the OD results and force model parameters have been obtained, the predicted orbit is generated and compared with a reference orbit to calculate the OP error. Typically, the reference orbit is derived from sources such as TLEs, ephemeris data, or future observations over a fixed time interval. The OP error requires computing the spatial three-dimensional distance between the reference orbit position and the predicted orbit position. To predict an orbit for a certain period of time, for example, predicting orbit error for one day, it is necessary to use multiple three-dimensional distance differences within a certain time range (such as one orbit cycle) near the predicted time of one day, and calculate the RMS of these differences.

$$\mathrm{RMS} = \sqrt{\frac{D_1^2 + D_1^2 + \cdots + D_n^2}{n}} \tag{6}$$

Here, D represents the difference in distance between the reference position and the computed position, while n denotes the number of positions utilized.

3. Data

To obtain a more dependable and accurate assessment of the TLE-OD/OP method, we deliberately selected space objects possessing both TLEs and high-precision reference orbits. Specifically, our study focused on evaluating the TLE-OD/OP method's performance using the Starlette and Starlink satellites. We regarded the satellite as pseudo-debris and leveraged the high-precision reference orbit to assess the TLE-OD/OP method. We utilized both CPF and precise ephemeris data as our reference orbits. For the Starlette satellite, we employed one year's worth of TLE and CPF data, while for the Starlink satellite, we were restricted to one month of data.

The Starlette satellite, launched by CNES in 1975, possesses an almost spherical shape and is equipped with an angular laser reflector for SLR [11]. For geodesy purposes, the ILRS publishes daily laser ranging data and orbital products related to this satellite. Contrarily, SpaceX has launched tens of thousands of LEO communication satellites such as Starlink [12]. In our study, we obtained TLEs and precise ephemeris data for 46 Starlink satellites, which are listed in Table 1. Additionally, Table 2 furnishes information about six selected Starlink satellites, two of which have decayed while the other two are in the ascending and descending phases, respectively.

Table 1. Information regarding two space objects.

Name	NORAD ID	Altitude/km	Inclination/Degree	Area to Mass Ratio/(m^2/kg)	Date Time
Starlette	7646	800	50	0.000962	01/01/2021–12/31/2021
Starlink	46671–46727	545	53	0.005596	26/09/2022–26/10/2022

Table 2. Information about the six Starlink satellites.

Phase	NORAD ID	Launch Date	Decay Date	Date Time	Inclination/ Degree	Apogee Height/km	Perigee Height/km
Ascent	53818	19/09/2022	--	24/09/2022–07/10/2022	53.22	350	348
Ascent	53820	19/09/2022	--	Same as above	53.22	350	348
Stable	46673	18/10/2020	--	23/09/2022–07/10/2022	53.05	549	546
Stable	46687	18/10/2020	--	Same as above	53.05	549	546
Deorbit	45209	17/02/2020	15/10/2022	Same as above	53.03	316	310
Deorbit	45231	17/02/2020	23/10/2022	Same as above	53.01	319	313

3.1. TLE

Space-Track.org is an organization that promotes the utilization of TLE data. Currently, they provide access to TLE data for over 50,000 space objects, with more than 20,000 of them being updated daily. Some of these objects have completed their intended mission and are no longer in orbit. The applications of TLE data are manifold, including aiding in space object observation mission planning, enabling space object re-entry prediction, facilitating space collision warning systems, and assisting with space debris removal efforts.

TLE data store essential orbital information for a given space object using only two lines, such as the object's catalog number, B* drag term, orbital inclination, right ascension of the ascending node, eccentricity, perigee argument, mean anomaly, and mean motion. SGP4 was developed by David Vallado and T.S. Kelso in the 1980s, building upon previous work on orbital theory conducted by Roger Bate [13]. SGP4 uses a simplified analytical method that considers perturbations induced by Earth's gravitational field, atmospheric drag, and the gravitational effects of the Moon and Sun. Inputs required by the model include satellite TLE data, specifying its position and velocity at a given time, and physical parameters such as Earth's gravitational constant and atmospheric density.

Typically, TLE data may yield position errors ranging from several hundred kilometers to a few kilometers over several days, with larger errors observed during periods of heightened solar activity or magnetic storm [14]. The position accuracy from TLE data is primarily influenced by two factors: the object's orbital altitude and eccentricity, with some variation observed between different objects [15].

3.2. Starlink TLE

The deployment of Starlink satellites has significantly increased the number of objects in Earth's orbit, raising concerns for space debris management and its impact on future space activities. To address these concerns, SpaceX has implemented measures such as designing new satellites with built-in propulsion systems for better control and maneuverability, as well as implementing deorbiting plans for satellites that are no longer in use.

Furthermore, there are increasing concerns about the potential impact of the growing number of Starlink satellites on astronomical observations, particularly those involving ground-based telescopes. The bright reflections from the satellites cause interference, disrupting scientific research. Astronomers and satellite operators are working jointly to develop strategies to minimize this disruption.

Despite these concerns, the Starlink program has enormous potential to transform internet access by providing high-speed broadband services to remote and underprivileged areas globally. With more satellites planned for deployment, it is expected that network coverage and speed will continue to improve even further.

As of 26 May 2023, there were 4125 successfully deployed Starlink satellites in orbit. On average, around 50 satellites were launched per mission, totaling 83 launches. Figure 1 depicts the launch date statistics for these Starlink satellites in orbit, demonstrating an increasing trend in both satellite numbers and launch activity over time. It is worth noting that a total of 298 Starlink satellites have decayed and are no longer in orbit, including all those launched on 24 May 2019. Decayed Starlink satellites are those that have become inoperable or lost their ability to maintain a stable orbit. Figure 2 displays the launch dates of these decayed satellites.

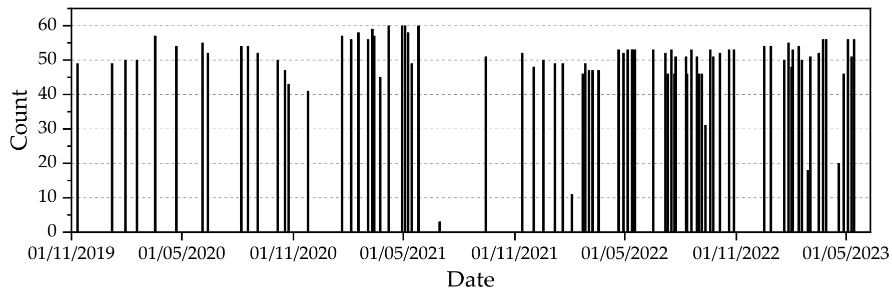

Figure 1. Launch dates of in-orbit Starlink satellites as of 26 May 2023.

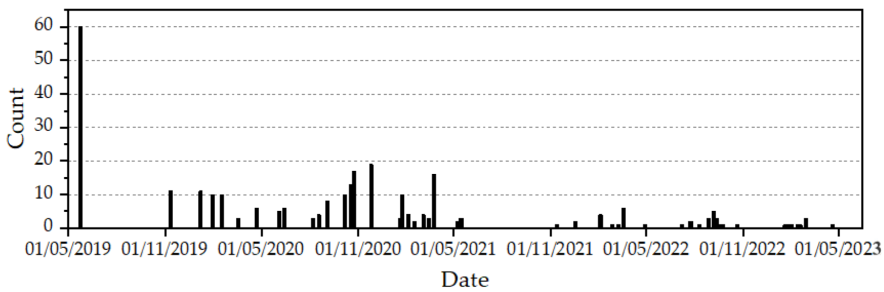

Figure 2. Launch dates of decayed Starlink satellites as of 26 May 2023.

The high frequency of TLE updates (1.7 per day on average) during 10–12 November 2022 suggests a need for frequent adjustments to the orbital path and position. The statistics displayed in Figure 3 indicate that the majority of Starlink satellites underwent at least one TLE update during the aforementioned three-day period, with some receiving up to 10 updates. This highlights the dynamic and ongoing nature of monitoring and adjusting the orbits of Starlink satellites to reflect constantly changing conditions in space.

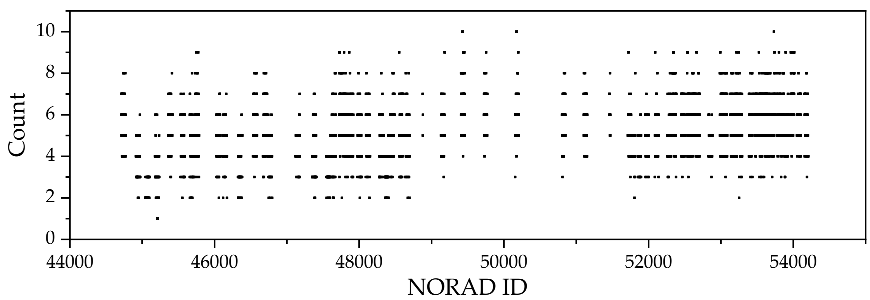

Figure 3. Number of Starlink two-line element sets released in a three-day period.

3.3. Starlink TLE Accuracy

To evaluate the accuracy of Starlink TLEs, two methods were employed: TLE intercomparison and comparison with precision ephemeris. The former involves comparing the positions predicted by different TLEs at the same time, while the latter entails comparing TLE positions with a high-precision reference orbit.

The TLE intercomparison method is commonly used for assessing LEO debris OP accuracy and precision. By comparing TLE-predicted positions at the same time, differences between the predicted positions can be evaluated.

In contrast, the precision ephemeris method generates a high-precision OP using the precise tracking of an object's position and velocity over time. This reference standard can then be used to compare the accuracy and precision of TLEs.

Both methods have their advantages and limitations. The TLE intercomparison method is simple to implement but limited in accuracy due to errors inherent in TLE data. The precision ephemeris method provides a highly accurate reference standard but requires sophisticated tracking equipment and telemetry data.

We describe orbit errors by means of three-dimensional positional distance errors, which accurately capture differences between predicted and actual positions, enable quantification of the magnitude of these differences, and provide insight into sources of error in the model.

Figure 4 shows Starlink TLE OP errors obtained by the TLE intercomparison method for half-day, one-day, two-day, and three-day predictions. The results indicate that the OP accuracy of Starlink TLEs exhibits small differences within one day, and the OP errors of newly launched satellites with larger NORAD ID dissipate faster after two days.

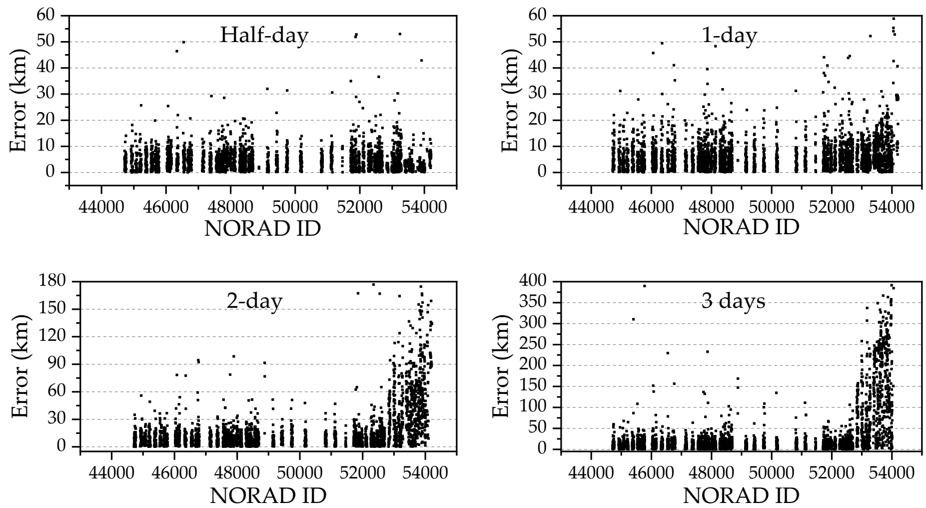

Figure 4. Orbit prediction errors for a Starlink satellite using two-line elements.

The findings highlight the value of employing the TLE intercomparison method for assessing OP, particularly for newly launched satellites. Although the reliability of this method may be questioned, it still provides valuable insights into the dependability of TLE predictions that are crucial for successful space missions.

In addition to the TLE intercomparison method, precision ephemeris can also be used to assess the accuracy of Starlink TLE. This type of ephemeris, available for download from SpaceX and Space-track.org, provides vital accurate orbital data to evaluate the accuracy of Starlink orbits. Combing this approach with the TLE intercomparison method can provide a more thorough assessment of the accuracy and reliability of Starlink TLE.

The on-board GNSS receiver plays an important role in generating precise ephemeris by providing measurements of the satellite's position and velocity relative to GPS satellites. These measurements, along with other variables such as atmospheric drag, solar radiation pressure, and gravitational perturbations, accurately predict satellite positions and velocities at various future times.

The precision ephemeris of Starlink allows SpaceX to continuously monitor critical parameters such as location and velocity in real-time, ensuring each satellite maintains its designated orbit and maximizing the constellation's efficiency while minimizing the risk of collisions with other space objects. Published three times daily, the Starlink ephemeris contains information on over 2000 satellites providing accurate position, velocity, and covariance data in the J2000 coordinate system for approximately three days.

Although the short-term (the first half of the day) position errors of the Starlink ephemeris vary based on several factors, they typically fall within a range of a few meters to a few kilometers. A common method to assess the accuracy of Starlink TLE is by comparing it with the Starlink precision ephemeris. Results of this comparison indicate that the one-day OP error of the TLE is around 76.8 km, with the maximum error exceeding 1300 km and the minimum error being 428 m. The number of errors less than 100 km, 50 km, 20 km, 15 km, 10 km, 5 km, and 1 km are 99%, 98%, 89%, 79%, 59%, 28%, and 1%, respectively.

The precision ephemeris provides much more accuracy in OP compared to TLE. However, TLE is still widely used because of its convenience and low cost. Despite its limitations, TLE is suitable for some low-precision applications where higher accuracy is not necessary. However, to obtain precise satellite locations, it is recommended to use the Starlink precision ephemeris.

3.4. CPF Data

The CPF product offered by the ILRS has been vital in acquiring accurate orbital positions for over 100 SLR satellites. In addition to supplying precise orbital position data, CPF also serves as a tracking reference for SLR by providing accurate positional information that ensures the accurate laser ranging of satellites. Through monitoring and measuring distances to the satellite, this tracking process can refine both the OP and CPF files to enhance accuracy.

A CPF file comprises highly precise and detailed positional data expressed in x, y, and z coordinates of the geocentric coordinate system, covering specific time intervals spanning several days. Ground-based observations and modeling techniques are utilized to generate this data, considering celestial body gravity, atmospheric drag, solar radiation pressure, and other perturbations affecting the satellite's orbit.

Over 30 organizations across different regions, including Europe, Asia, the United States, and Australia, have partnered with ILRS to provide CPF data for SLR satellites. This collective effort facilitates the sharing of data and expertise, leading to significant advancements in SLR measurements and orbital positions. Beyond offering CPF data, many of these organizations operate their SLR stations, contributing to the global network of ground-based observatories used to track satellites. The collaboration through ILRS enables a broad range of scientific studies, such as investigating climate change and sea level rise, monitoring Earth's magnetic field dynamics, and studying near-Earth asteroids' behavior. With improved SLR technology, international collaboration and data-sharing initiatives through platforms such as ILRS will become even more crucial.

Although a typical CPF file's accuracy level falls within several meters, this value varies based on the specific satellite and parameters related to CPF file generation. While the format of a CPF ephemeris may differ across satellites, it commonly features a header segment, a positional information record section, and an endpoint marker to signify the dataset's conclusion. Reference [16] offers further insights into CPF file accuracy. Upon analyzing the Starlette CPF data from 2021, discrepancies between various CPF files were below 2 m, underscoring the importance of precise CPF files for TLE accuracy evaluations.

4. Results of TLE-OD/OP

This section is divided into three parts, where we describe the methodology adopted and present results for Starlette and Starlink. The aim of the research is to evaluate the effectiveness of the TLE-OD/OP method using TLE data under different pseudo-observation distributions. Presently, uniformly distributed pseudo-observations are employed during the OD process. However, we propose that pseudo-observations generated near the TLE epoch exhibit a higher efficiency as position accuracy tends to decline with increasing time between the TLE epoch and pseudo-observation.

The study followed a seven-step methodology to assess the TLE-OD/OP method's performance using TLE data with varying pseudo-observation distributions:

(1) Data Preparation: We downloaded Starlette and Starlink TLE data along with their respective reference orbit data. Starlette's reference orbit was CPF data from ILRS, while SpaceX provided ephemeris data for Starlink.
(2) Pseudo-Observation Generation: Pseudo-observations were generated for TLE data using the SGP4 algorithm at various positions to evaluate the method under different distributions.
(3) OD Configuration: The ballistic coefficient (or area-to-mass ratio), force models, and spatial object information (gravity force, third-body gravity, atmospheric drag, and solar radiation pressure) were integrated into the OD process.
(4) OD: The least square differential correction theory was applied to determine an accurate orbit with minimal deviation from all pseudo-observations.
(5) OP: The obtained orbit was used to predict orbital trajectory.
(6) Reference Orbital Generation: The Starlink ephemeris or Starlette CPF files were updated daily, containing predicted orbits for several days. Differences between predicted orbits were corrected through orbital fitting procedures.
(7) Calculation of OP Errors and Statistics: The differences between the predicted orbit and reference orbit were computed to obtain one-dimensional OP error. We obtained a set of cases every 10 days for each distribution of pseudo-observations using Starlette data from 2021, and the average OP errors were used to evaluate the method's performance.

To maintain consistency, we used the same coordinate and time systems throughout the study. The inertial coordinate system had the Earth's center as the origin, the equatorial plane as the basic plane, and the XYZ axes forming a right-handed coordinate system. UTC was employed as the time system.

4.1. Starlette Results

Regarding the Starlette result, an OD duration of 10 days was utilized and different distributions of pseudo-observations were employed for Figure 5 and as follows.

(1) Uniform distribution.
(2) Close to the epoch of TLE.
(3) Before the TLE epoch.
(4) After the TLE epoch.

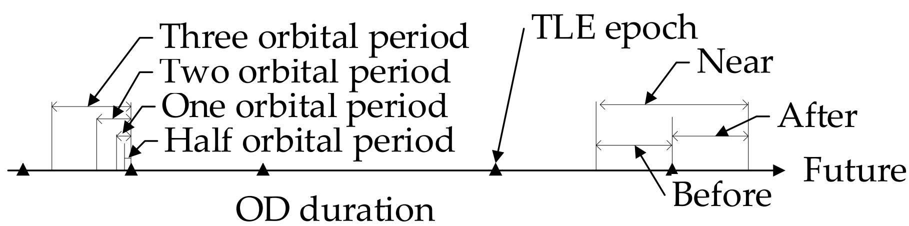

Figure 5. The diagram illustrates the distribution of pseudo-observations.

There are 100 pseudo-observations in each distribution. The uniform distribution indicates that pseudo-observations are evenly spaced in time. For the TLE epoch, the distribution of pseudo-observations was investigated for half, one, two, and three orbital cycles before, near, and after each TLE epoch. The distribution diagram of pseudo-observations is illustrated in Figure 5.

We calculated results every 10 days, starting from 1 January 2021, and obtained 35 distinct sets of results. The lines shown in Figure 6 represent the averaged OP errors across these 35 sets.

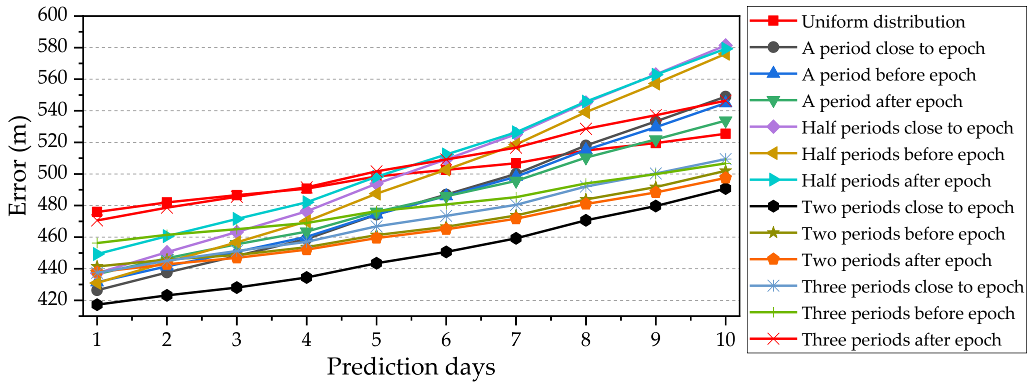

Figure 6. Orbit prediction errors for Starlette with various pseudo-observation distributions.

The findings reveal that distribution of two orbital period pseudo-observations closer to the TLE epoch generated the lowest OP errors. Compared to the uniform distribution, this arrangement displayed an average improvement of approximately 10% in OP over 10 days. The half-orbital-cycle pseudo-observation distribution recorded the highest OP errors from day 5 to 10, while the distribution of pseudo-observations closest to the TLE epoch generally performed better (except for one-orbital-cycle results—best performance before TLE epoch), while the worst outcomes occurred after the TLE epoch. The study further indicated a 15% variation between maximum and minimum OP errors in various pseudo-observation distributions.

This study underscores the importance of carefully selecting the pseudo-observation distribution when predicting satellite orbit paths. The evidence suggests that generating pseudo-observations closer to the TLE epoch can improve accuracy. With historical TLEs of Starlette having several kilometers of OP errors for many days, the 2021 TLEs were more precise and produced OP errors below 600 m using the TLE-OD/OP approach. This could be due to improved equipment performance and data processing capabilities, as well as relatively subdued solar activity in 2021.

4.2. Starlink Results

The results of the Starlink study can be classified into two parts. The first portion analyzes the TLE-OD/OP method's performance under different pseudo-observation distributions. The second segment evaluates the TLE-OD/OP method's impact on Starlink during various orbit phases, such as ascension, descent, and stable orbit stages.

To enhance the accuracy of OP for Starlink satellites, we utilized a set of selected Starlink TLEs and their ephemeris as the reference orbit, with OD periods of 3, 5, 7, 10, and 15 days. Figure 7 displays the average OP errors of the 46 Starlink satellites, depicting varied results with different OD intervals. The OP errors exhibited an inverse relationship with prediction time length, and the difference between maximum and minimum errors gradually increased as OP time increased, ranging from about 93% on the first day to up to 500% on the fifth day. To achieve lower OP errors than those derived from TLEs, the TLE-OD/OP method required an OD duration of greater than five days.

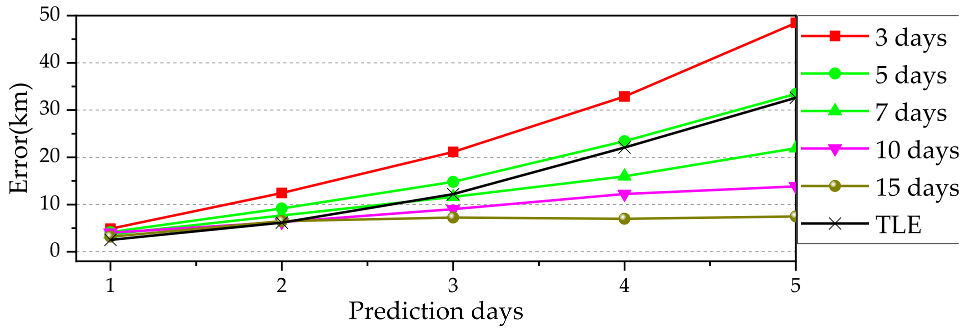

Figure 7. Average prediction error for Starlink satellites under different orbital detection periods.

Figures 8–11 illustrate the results when employing pseudo-observation distributions similar to those in the previous section, comparing the effectiveness of the TLE-OD/OP technique under diverse pseudo-observation generation approaches with varying orbital determination periods of 3, 5, 7, and 10 days.

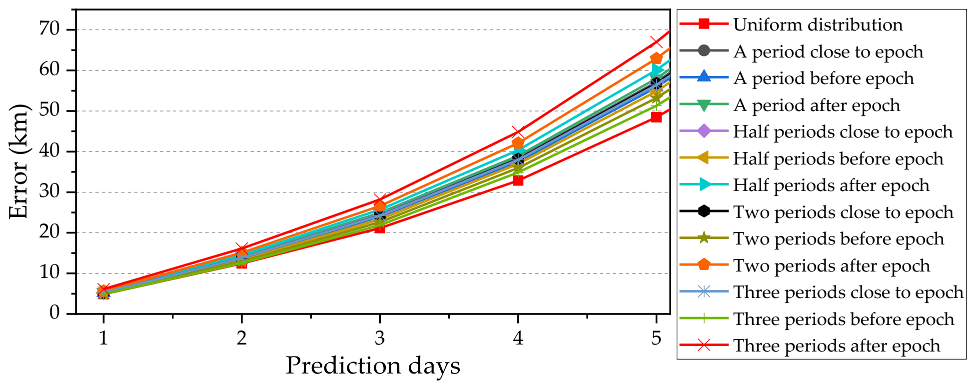

Figure 8. Variation of Starlink's OP error with different observation distribution in a 3-day OD.

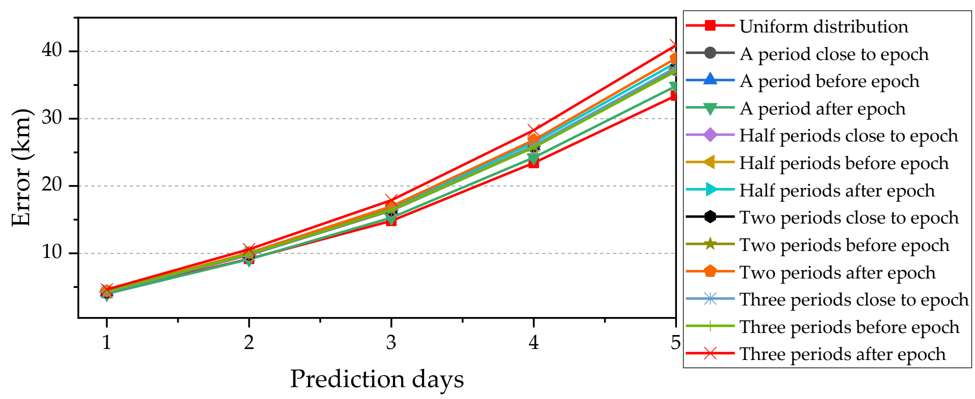

Figure 9. Variation of Starlink's OP error with different observation distribution in a 5-day OD.

The results indicate that a uniform distribution of pseudo-observations is optimal for 3-day and 5-day OD, while TLE performed superiorly to TLE-OD/OP. For a 10-day OD interval, the best pseudo-observation distribution was close to the TLE epoch in half orbital cycles. As the OD period increases, differences in OP error from different pseudo-observation distributions decline. Setting the OD period to 3 days yields a maximum and minimum OP error difference of over 18 km, whereas for 10 days, it is less than 1 km. The 15-day OP errors using various pseudo-observation distributions were very similar, warranting no figure presentation in the paper.

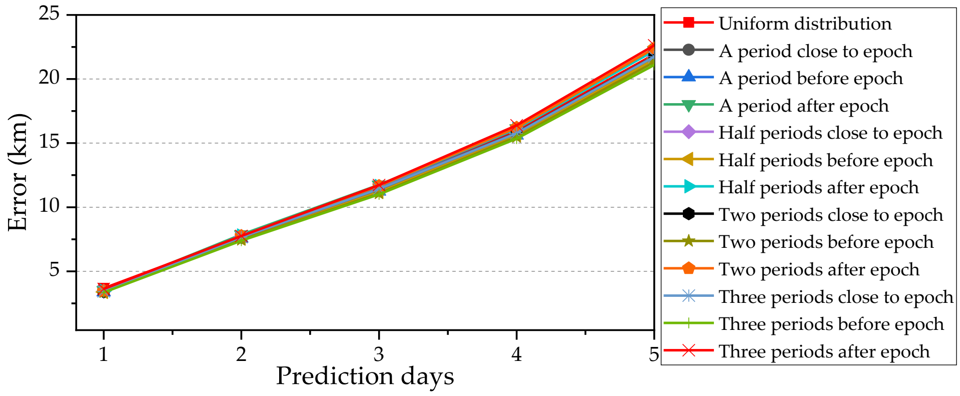

Figure 10. Variation of Starlink's OP error with different observation distribution in a 7-day OD.

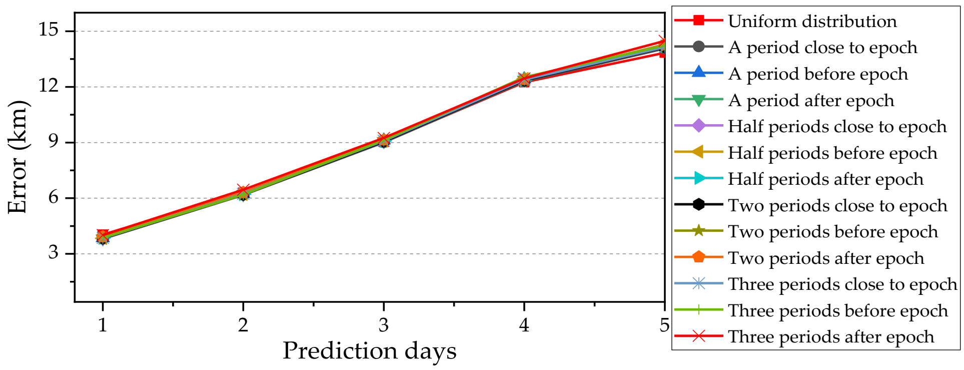

Figure 11. Variation of Starlink's OP error with different observation distribution in a 10-day OD.

We also examined the OP errors of Starlink satellites during their ascending, stabilizing, and descending phases. The study employed pseudo-observation distributions near, before, and after the TLE epoch in two orbital cycles, together with uniformly distributed pseudo-observations during OD duration evaluation. We also compared various OD durations on the performance of the TLE-OD/OP method.

(1) Orbital insertion phase

We used data from Starlink 53818 and Starlink 53820. We used 3-day TLE data from 30 September 2022. We also implemented 5-day TLEs after 28 September 2022. Figure 12 displays the results of 53818 and 53820. Our findings indicate that, during the ascent stage of Starlink, errors can exceed 100 km for up to 3 days OP. We observed minor variations in the results of the TLE-OD/OP method across different pseudo-observation distributions throughout the 3-day OD period. Specifically, for the 53818, the TLE-OD/OP method demonstrated lower OP errors within 1.5 days, while for the 53820 Starlink, it showed significant OP errors within 3 days, which could be attributed to lower TLE accuracy. Additionally, we evaluated the 5-day, 7-day, and 9-day OD durations but did not achieve satisfactory outcomes.

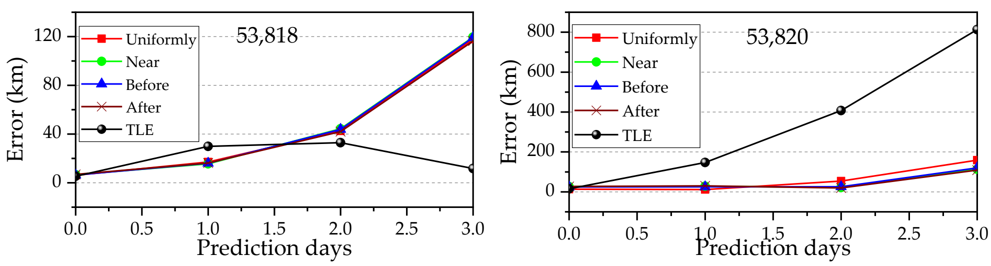

Figure 12. Predicted orbital error for two ascending Starlink satellites during a 3-day OD period.

(2) Stable orbit phase

The research focused on the stable orbit phase of Starlink 46673 and 46687. All OD was based on a common end date of 2 October 2022. Figure 13 depicts the outcomes of two studies on the stable orbit phase of Starlink satellites, utilizing the TLE-OD/OP and SGP4 methods. The research reveals that SGP4 achieved OP errors of less than 20 km for stable Starlink. However, TLE-OD/OP resulted in significantly varied OP errors depending on the chosen pseudo-observation distribution scheme. Among the four distributions tested, the largest OP errors were observed to be more than 1.5 times for Starlink 46673 and approximately 1.2 times for Starlink 46687. Uniform distribution was found to be the most effective of the four pseudo-observation schemes. Notably, using TLE-OD/OP with 3-day TLEs led to greater OP errors compared to TLE results.

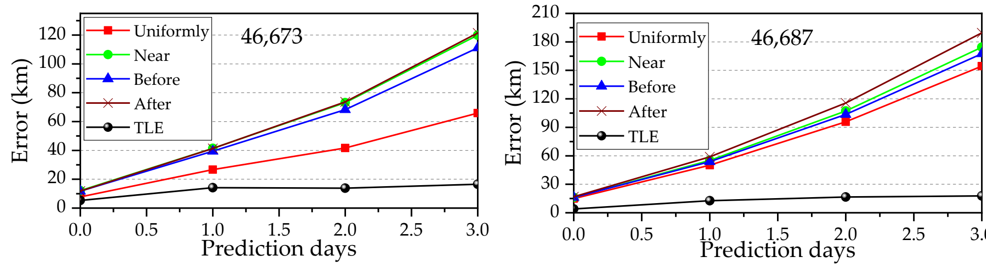

Figure 13. OP errors for two stable orbit Starlink satellites using a 3-day OD.

In Figure 14, we present the results obtained using different OD durations. We utilized the TLE-OD/OP method and employed pseudo-observations distributed uniformly. It is evident from our study's findings that the OD duration can significantly affect the OP errors associated with this method. The maximum to minimum ratio of OP error is observed to be more than 7. For Starlink 46673, the most favorable outcomes were achieved with a 7-day OD duration, while a 10-day duration yielded the best results for Starlink 46687. Interestingly, the OP errors of both Starlink utilizing an OD duration of more than 5 days were smaller than those attained using SGP4. Moreover, the disparity between various pseudo-observation distributions gradually diminished as the OD duration increased.

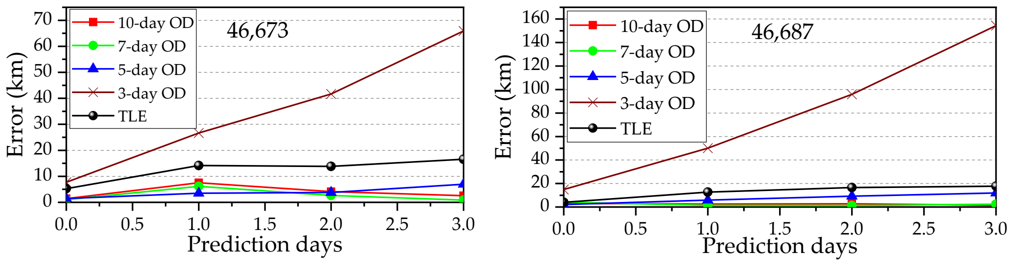

Figure 14. OP error of two stable orbit Starlink satellites with different OD time periods.

(3) Deorbit phase

In the deorbit phase of our study, we utilized data from 45209 and 45231, which decayed on 15 and 23 of October 2022. All OD was based on a common end date of 2 October 2022.

Figure 15 illustrates our study on two Starlink satellites during their deorbiting periods. Our research shows that the OP errors of SGP4 for a 3-day prediction were below 200 km. However, we observed significant differences in the ratio, with the largest being more than 1.3 for 45209 and around 1.1 for 45231 across the four pseudo-observation distributions. We found that uniformly distributed pseudo-observations resulted in fewer OP errors compared to the other three distributions, except for Starlink 45209 after a 2-day prediction. Moreover, we discovered that the OP errors produced by the TLE-OD/OP method using a 3-day OD duration were greater than those generated by SGP4.

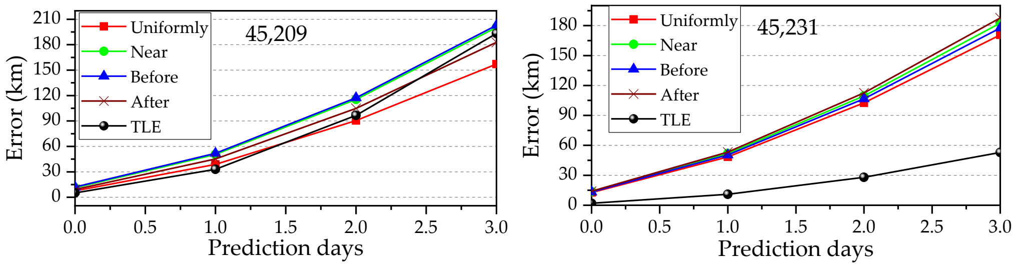

Figure 15. OP errors for two deorbiting orbit Starlink satellites using a 3-day OD.

In Figure 16, we present the results of our investigation into two Starlink descent periods using different OD durations. Our findings show that the OD duration has a significant impact on the TLE-OD/OP method's results, as evidenced by the maximum to minimum ratio of OP error exceeding 3. We found that using a 7-day OD duration yielded the lowest OP error compared to other durations when using the 3-day mean OP error as the standard. Specifically, for Starlink 45209, the OP errors for 5-day, 7-day, and 10-day durations were smaller than those generated by the SGP4 method. For Starlink 45231, only the OP errors produced by 5-day and 7-day OD durations were smaller than those obtained via the SGP4 approach.

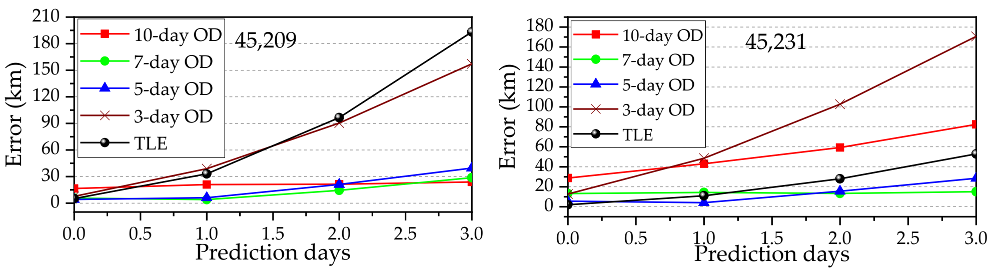

Figure 16. OP error of two deorbiting orbit Starlink satellites with different OD time periods.

5. Discussion

5.1. Optimization of the TLE-OD/OP Methodology

Various factors impacted the accuracy and efficiency of TLE-OD/OP methods beyond OD duration and pseudo-observation distribution. These factors include the quantity of pseudo-observations, TLE data precision, and orbital fitting techniques. By considering and analyzing these factors, we can improve the TLE-OD/OP model's precision, enhancing our ability to remotely sense and obtain critical information. Therefore, comprehensive research is crucial in evaluating each of these factors and developing more reliable and robust OD/OP methodologies.

We conducted a study to analyze how the number of pseudo-observations affects TLE-OD/OP method OP errors. We generated 50, 100, 200, 500, and 1000 pseudo-observations from all TLEs of Starlink and Starlette satellites within 10 days. The pseudo-observations were distributed uniformly or within two orbital periods close to the TLE epoch. Our research found that, given a constant distribution of pseudo-observations, any variation in OP error resulting from differences in the quantity of pseudo-observations was less than 0.1%. This indicates that increasing the number of pseudo-observations has minimal impact on the TLE-OD/OP method OP errors' accuracy.

Our study also evaluated how TLE accuracy affects TLE-OD/OP method performance. We used precise orbits as reference orbits to determine TLE accuracy. If the RMS between a TLE-derived orbit using SGP4 and the reference orbit was significant, the associated pseudo-observations had lower weights. In contrast, if the RMS was small, the corresponding pseudo-observations had greater weightings. These weights were determined quantitatively based on the average RMS value of all TLEs or the RMS of a particular TLE employed as a benchmark for obtaining the weights of other TLEs. Pseudo-observations were created and weighted based on their corresponding TLE. Our thorough testing demonstrated that accounting for TLE accuracy can reduce OP error by approximately 1%.

However, our observations indicate that the vast majority of space debris do not have precise orbits suitable as reference orbits. This resulted in an unreliable reference orbit when relying solely on TLE-generated orbits, leading to inadequate accuracy assessment of TLEs and weighting of pseudo-observations. We only used TLEs to generate reference orbits and assess the accuracy of other TLEs for Starlink and Starlette satellites. Our results showed that considering TLE accuracy instead of equal weights decreased OP error by less than 1%.

To achieve high computational accuracy, we incorporated all perturbation forces into both numerical integrations for OD and OP, despite the resulting time consumption. We investigated the impact of different perturbation forces on the OP errors of the TLE-OD/OP method and determined the most appropriate force model to balance computational time and accuracy. Our tests revealed that using more precise force models did not significantly reduce OP errors. For example, switching from 60*60 to 360*360 EGM 2008 for the gravity field resulted in less than a 0.1% reduction in OP errors, while increasing computation time fourfold.

We used star catalogs for calculating the third-body gravitational force and compared the effects of using two versions, DE200 and DE 406, on OP error. Our findings indicated that using different star catalogs resulted in OP errors smaller than one-thousandth. This highlights that the choice of DE200 or DE 406 has a negligible impact on the OP error when computing the third-body gravitational force due to their high precision, expansive coverage, and utilization of advanced celestial measurement data and numerical calculation techniques.

Moreover, atmospheric drag is a crucial factor affecting LEO OP. Atmospheric mass density is a vital variable when calculating atmospheric drag, and we compared the effects of using DTM78 and NRLMSISE-00 models on OP by calculating atmospheric density. Our findings indicated that using different atmospheric density models resulted in OP errors of less than 1%. One reason for this is that we computed the drag coefficient during OD, which absorbed any errors caused by the atmospheric density model.

Furthermore, we compared the impact of Earth radiation pressure and ocean tides on orbit propagation. The findings demonstrate that the difference in OP error between considering and not considering these two factors in TLE-OD/OP is less than one ten-thousandth.

The TLE-OD/OP approach offers a significant improvement in OP accuracy compared to SGP4. However, the effectiveness of this method is limited by the low quality and sparse distribution of TLEs. Theoretically, longer OD durations would lead to smaller OP errors. Nonetheless, our study's results indicate that the OP errors resulting from a 10-day OD duration are not as satisfactory as those of 7 days. This may be due to the low-quality TLEs or large atmospheric model errors. To optimize the TLE-OD/OP process, possible strategies include utilizing a more accurate atmospheric model and incorporating attitude measurement data of space objects. We anticipate that using more accurate atmospheric models will further improve OP accuracy. Additionally, since ascending orbital periods contain small thrusts, we could introduce thrust estimation or prediction techniques to enhance the OP accuracy for maneuvering objects.

The TLE-OD/OP method adopts the least squares theory to fit pseudo-observations. However, the least squares approach is highly susceptible to gross errors. If one or more abnormal TLEs are used to generate a set of pseudo-observations, they are inclined to be gross errors, making the TLE-OD/OP method ineffective, or even worse than the SGP4 algorithm in enhancing OP accuracy. Therefore, it is necessary to detect and remove abnormal TLEs before employing the TLE-OD/OP method. Because TLE quality often varies by several hundred meters or kilometers, developing efficient techniques for detecting abnormal TLE remains an ongoing research subject.

5.2. Improved OP for Space Debris or NCT

TLE data, widely used for space debris and other objects, are not a direct observation of the object's orbit but rather an orbital product derived from observations, resulting in limited accuracy. Meanwhile, the SGP4 algorithm utilizes only general perturbations, leading to lower accuracy compared to satellites equipped with GNSS receivers. However, equipping all space debris with GNSS receivers is impractical, making continuous improvement of available orbital data quality and processing essential for enhancing OP accuracy.

Various techniques, including radar and optical angular measurements, laser ranging, and others, can be utilized during OD to achieve greater accuracy and denser distribution of orbital measurement data. The more accurate the measurement data, the more beneficial it would be for OD and OP accuracy. Additionally, uniform and denser distribution of data over an orbital period could significantly enhance OD and prediction accuracy. Hence, precise OD could enable reduced intrinsic errors of SGP4 by estimating accurate TLE, finally contributing to generating low bias error in RMSE.

However, securing greater accuracy and denser distribution of orbital measurement data is a long-term process. Thus, improving the technique of orbital determination currently remains the most effective approach to enhance OP accuracy for space debris. Several methods, such as calibration of the Analytical Multiobject Dynamic Model (AMDM) using orbital measurement data from LEO space objects within a short time interval to calibrate the model, can improve OD and OP accuracy. Moreover, introducing AI to OD and prediction also presents a potential research area that could lead to significant improvements in OP accuracy [17].

6. Conclusions

As the number of space objects continues to grow, traditional TLE with SGP4 algorithm is no longer sufficient for applications such as remote sensing, collision warning, and space traffic management. The method uses a series of successive TLEs within a short period to enhance OP accuracy, but its effectiveness depends on several factors such as pseudo-observation distribution.

To investigate this, we analyzed Starlette and Starlink data using various pseudo-observation generation strategies. These strategies were divided into four groups based on TLE epoch relationship, generating pseudo-observations using each TLE over OD duration. The difference lay in their distribution, with possibilities including distribution before, after, or near TLE epoch during one, two, three or half orbital circles, along with the uniform distribution of pseudo-observations over OD duration.

We evaluated the improvement in OP performance using high-precision orbits as reference data. For Starlette, we used CPF data from ILRS while employing precise ephemeris predictions from SpaceX for Starlink satellites. We determined the accuracy of TLEs for all operational Starlink satellites, calculating OP errors over a period of three days, and analyzed OP error of the SGP4 algorithm with precise ephemeris predictions.

Statistical analysis revealed that using all TLEs to generate two orbital periods near the TLE epoch within a 10-day period resulted in the best TLE-OD/OP method performance for Starlette. For Starlink, an OD duration of greater than 5 days was necessary to achieve superior performance over SGP4. As the OD duration increased, the impact of pseudo-observation distribution on TLE-OD/OP method performance decreased. This TLE-OD/OP method's application also covered different phases such as ascending, stable, and descending for comprehensive observations.

These findings provide valuable insights to improve LEO constellations' precision that depend on TLE data, generating more accurate orbit information for long-distance space object observations, and space traffic management.

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Data Availability Statement: The TLE data and precise orbit ephemeris for Starlink can be downloaded from https://www.space-track.org/ (accessed on 26 May 2023). The CPF data can be obtained from https://edc.dgfi.tum.de/pub/slr/cpf_predicts_v2/website (accessed on 26 May 2023).

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Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

AI	Artificial Intelligence
AMDM	Atmospheric Mass Density Model
CNES	French National Centre for Space Studies
CPF	Consolidated Prediction Format
DTM78	Drag temperature model 1978
GNSS	Global Navigation Satellite System
GPS	Global Positioning System
ILRS	International Laser Ranging Service
J2000	Julian year 2000, is also known as the J2000.0 celestial reference system or the ICRF (International Celestial Reference Frame) of epoch J2000.0
LEO	Low-Earth Orbit
NCT	Non-Cooperative Target
NRLMSISE-00	Naval Research Laboratory Mass Spectrometer and Incoherent Scatter radar 2000—Extended
NORAD	North American Aerospace Defense Command
OD	Orbit Determination
OP	Orbit Prediction
RMS	Root Mean Square
RMSE	Root Mean Square Error
SGP4	Simplified General Perturbations Version 4
SLR	Satellite Laser Ranging
SSN	Space Surveillance Network
TLE	Two-Line Element
TLE-OD/OP	Orbit Determination and Orbit Prediction method using multiple TLE
UTC	Universal Time Coordinated

References

1. Klinkrad, H. *Space Debris: Models and Risk Analysis*; Springer: Berlin/Heidelberg, Germany, 2006.
2. Puente, C.; Saenz-Nuno, M.A.; Villa-Monte, A.; Olivas, J.A. Satellite Orbit Prediction Using Big Data and Soft Computing Techniques to Avoid Space Collisions. *Mathematics* **2021**, *9*, 14. [CrossRef]
3. Kelso, T.S. Analysis of the iridium 33 cosmos 2251 collision. In Proceedings of the AAS Space Flight Mechanics Meeting, Savannah, GA, USA, 8–12 February 2009.
4. NASA. Accidental Collision of YunHai 1-02. *Orbital Debris Quarterly News*, 20 December 2021; 1.
5. Kocifaj, M.; Kundracik, F.; Barentine, J.C.; Bara, S. The proliferation of space objects is a rapidly increasing source of artificial night sky brightness. *Mon. Not. R. Astron. Soc.* **2021**, *504*, L40–L44. [CrossRef]
6. Brainard, J. Space junk removal planned. *Science* **2019**, *366*, 1293.

7. Murtaza, A.; Pirzada, S.J.H.; Xu, T.G.; Liu, J.W. Orbital Debris Threat for Space Sustainability and Way Forward (Review Article). *IEEE Access* **2020**, *8*, 61000–61019. [CrossRef]
8. Hoots, F.R.; Roehrich, R.L. *Space Track Report No. 3—Models for Propagation of NORAD Element Sets*; Aerospace Defense Command, United States Air Force: Peterson, CO, USA, 1980; pp. 1–79.
9. Levit, C.; Marshall, W. Improved orbit predictions using two-line elements. *Adv. Space Res.* **2011**, *47*, 1107–1115. [CrossRef]
10. Lyddane, R.; Cohen, C. Numerical comparison between Brouwer's theory and solution by Cowell's method for the orbit of an artificial satellite. *Astron. J.* **1962**, *67*, 176. [CrossRef]
11. Pearlman, M.; Arnold, D.; Davis, M.; Barlier, F.; Biancale, R.; Vasiliev, V.; Ciufolini, I.; Paolozzi, A.; Pavlis, E.C.; Sosnica, K.; et al. Laser geodetic satellites: A high-accuracy scientific tool. *J. Geod.* **2019**, *93*, 2181–2194. [CrossRef]
12. McDowell, J.C. The Low Earth Orbit Satellite Population and Impacts of the SpaceX Starlink Constellation. *Astrophys. J. Lett.* **2020**, *892*, 18. [CrossRef]
13. Vallado, D.A.; Crawford, P.; Hujsak, R.; Kelso, T. Revisiting spacetrack report# 3. In Proceedings of the AIAA/AAS Astrodynamics Specialist Conference, Keystone, CO, USA, 21–24 August 2006; p. AIAA 2006-6753.
14. Kovar, P.; Puricer, P.; Kovarova, K. Study of the Two-Line Element Accuracy by 1U CubeSat with a GPS Receiver. *Sensors* **2022**, *22*, 2902. [CrossRef] [PubMed]
15. Dong, W.; Changyin, Z. Analysis on the Accuracy of the SGP4/SDP4 Model. *Acta Astron. Sin.* **2009**, *50*, 332–339. [CrossRef]
16. Najder, J.; Sosnica, K. Quality of Orbit Predictions for Satellites Tracked by SLR Stations. *Remote Sens.* **2021**, *13*, 1377. [CrossRef]
17. Li, B.; Huang, J.; Feng, Y.M.; Wang, F.H.; Sang, J.Z. A Machine Learning-Based Approach for Improved Orbit Predictions of LEO Space Debris with Sparse Tracking Data From a Single Station. *IEEE Trans. Aerosp. Electron. Syst.* **2020**, *56*, 4253–4268. [CrossRef]