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A Reformulation of the Lambert Conformal Conic Projection with Application to Bulgarian National Mapping

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Abstract

This paper revisits the Lambert conformal conic (LCC) projection and rederives its equations using a new notation, V, defined as the reciprocal of the commonly used U, which simplifies the expressions. Based on the resulting distortion formulas, conditions determining whether the projection has two, one, or no standard parallels are obtained. An optimal LCC configuration is defined by requiring equal local linear scale factors at the bounding parallels and symmetric maximum and minimum distortions about unity. Applied to the territory of Bulgaria ($\varphi_S \approx 41^\circ 14'$, $\varphi_N \approx 44^\circ 13'$), this criterion yields optimized standard parallels at $\varphi_1 \approx 41^\circ 40'$ and $\varphi_2 \approx 43^\circ 47'$. The corresponding local linear scale factors range from ca. 0.999832 to 1.000168, i.e., symmetric distortions of approximately $\pm 1.7 \times 10^{-4}$. Compared with existing implementations such as BGS2000 and BGS2005, the proposed configuration slightly reduces the distortion range and provides a more balanced distribution of scale over the country.

Keywords: map projection; Lambert conic conformal; standard parallels; Bulgaria

1. Introduction

The fundamental principles of conic conformal map projections were established by Johann Heinrich Lambert in 1772 [1]. In recognition of his contributions, one of these projections is known as the Lambert conic conformal map projection. In the third part of his Beyträge zum Gebrauche der Mathematik und deren Anwendung (Contributions to the Usage of Mathematics and Its Application, 1772), titled Anmerkungen und Zusätze zur Entwerfung der Land- und Himmelscharten (Remarks and Additions to the Establishment of Terrestrial and Celestial Maps), Lambert describes the projection of spheres and spheroids onto a plane. Consequently, according to Frischauf (1905) [2], Deetz and Adams (1934) [3], Lapaine and Kuveždić (2007) [4], and others, Lambert may be regarded as the founder of general map projection theory. Whereas his predecessors investigated only individual projections, Lambert approached the problem of mapping a surface onto a plane from a broader perspective and formulated general mathematical conditions that such mappings must satisfy. The most important of these conditions concern the preservation of angles (conformality) and the preservation of area (equivalence). Although Lambert did not develop a complete theory of these mappings, he was the first to articulate these ideas in a clear and systematic manner.

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In this paper, we focus on a conformal conic projection known as the Lambert conformal conic (LCC). For completeness, we derive the equations of this projection in Section 2. In doing so, we introduce a new variable, V, defined as the reciprocal of the commonly used quantity U (i.e., the natural exponential of the isometric latitude), to simplify the notation in the resulting formulas. Special attention is given to the distortion characteristics, enabling us to formulate, at the end of this section, the conditions that determine whether the projection has two, one, or no standard parallels.

The basic form of the LCC projection includes two parameters that influence the distribution of linear and areal distortions over the territory to which the projection is applied. In Section 3, we describe a criterion that is well known in the literature: the optimal LCC projection is obtained when the two parameters are chosen such that the local linear scale factors along the two boundary parallels are equal and the maximum local linear scale factor exceeds unity by the same amount that the minimum local linear scale factor falls below unity.

In Section 4, we apply this criterion to compute the projection parameters and determine the standard parallels for an optimal LCC projection for the territory of Bulgaria, which constitutes the principal objective of this paper.

This paper addresses two closely related aspects of the Lambert conformal conic projection. First, the projection equations are reformulated using the reciprocal variable V of the commonly used quantity U, which leads to a simpler analytical form of the distortion relations. Second, this formulation is applied to determine optimal standard parallels for the territory of Bulgaria based on a symmetric distortion criterion. The reformulation is introduced solely to facilitate the theoretical analysis and derivation of optimization conditions; implementation aspects using any GIS software are beyond the scope of this study.

2. Lambert Conformal Conic Projection

In normal conic projections, meridians are mapped as straight lines that intersect at angles proportional to the differences in their longitudes. Parallels are represented as arcs of concentric circles centered at the point where the meridians converge. To depict the region of interest on a plane, it is necessary to establish a relationship between the geodetic coordinates φ and λ on the ellipsoid or sphere and the corresponding planar coordinates. In conic projections, the planar coordinates are expressed using the polar angle δ and the polar radius ρ (Figure 1).

The general equations of normal conic projection in polar coordinates are defined as follows (Figure 1):

$$\rho = \rho(\varphi), \tag{1}$$

$$\delta = n(\lambda - \lambda_0), \tag{2}$$

where n is proportionality constant, λ is longitude and λ_0 is longitude of the central meridian of the projected area.

The form of the function $\rho = \rho(\varphi)$ determines whether the projection is conformal, equivalent, equidistant, or neither. The relationship between polar and rectangular coordinates is:

$$x = q - \rho \cos\delta, \tag{3}$$

$$y = \rho \sin\delta, \tag{4}$$

where q is the radius of the latitude of origin (Figure 1). The x-axis of the rectangular coordinate system coincides with the polar axis, and the origin is located at the intersection of the latitude of origin and the central meridian in the projection plane (Figure 1).

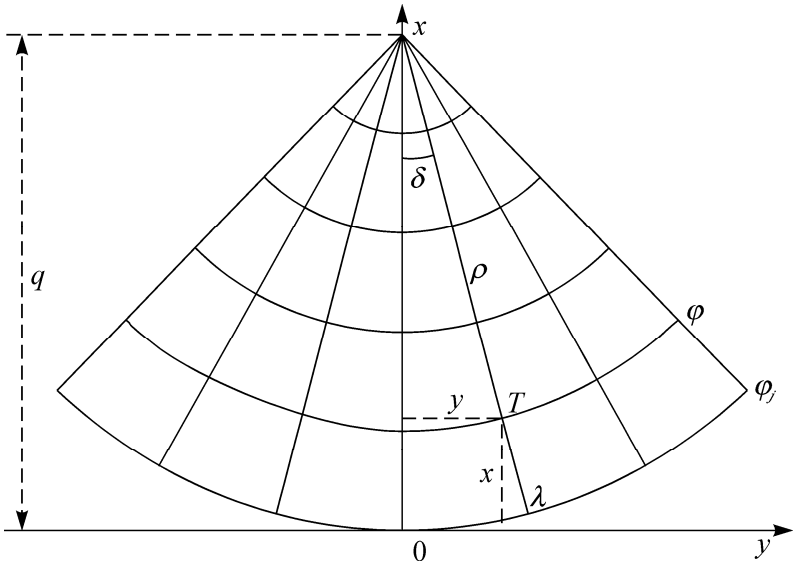


Figure 1. Coordinate system in normal conic projections.

Formulas for the linear scale factors along meridians and parallels are well established in the literature [5,6]. The local linear scale factor along a meridian is $h = \frac{\sqrt{E}}{M}$ and the local linear scale factor along a parallel is $k = \frac{\sqrt{G}}{r}$. In these expressions, E and G are the first fundamental quantities of Gauss; M is the meridional radius of curvature; N is the prime-vertical radius of curvature; and r is the radius of the parallel, computed using the following expression:

$$r = r(\varphi) = N \cos \varphi. \tag{5}$$

The required partial derivatives are

$$\frac{\partial x}{\partial \varphi} = -\frac{d\rho}{d\varphi}\cos\delta, \ \frac{\partial y}{\partial \varphi} = \frac{d\rho}{d\varphi}\sin\delta, \tag{6}$$

$$\frac{\partial x}{\partial \lambda} = n\rho \sin\delta, \ \frac{\partial y}{\partial \lambda} = n\rho \cos\delta. \tag{7}$$

Thus,

$$E = \left(\frac{d\rho}{d\varphi}\right)^2, G = n^2\rho^2 \tag{8}$$

finally,

$$h = -\frac{d\rho}{Md\varphi}, \ k = \frac{n\rho}{r}. \tag{9}$$

Due to the conformality of the Lambert conic projection,

$$-\frac{d\rho}{Md\varphi} = \frac{n\rho}{r}. \tag{10}$$

from which the function $\rho = \rho(\varphi)$ can be determined. If the expressions for r, M and N are substituted into the above relation, it becomes

$$-\frac{d\rho}{\rho} = n\frac{M}{N}\frac{d\varphi}{\cos\varphi} = n\frac{1-e^2}{1-e^2\sin^2\varphi}\frac{d\varphi}{\cos\varphi}, \tag{11}$$

where e is the eccentricity of the ellipsoid. The integral of the term multiplying n on the right-hand side of (11) corresponds to the isometric latitude, which frequently appears in conformal mappings of the ellipsoid. After integration, we obtain

$$\ln \rho = -n \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) + ne \ln \tan\left(\frac{\pi}{4} + \frac{\psi}{2}\right) + \ln F, \tag{12}$$

where F is the constant of integration, and ψ is a function of latitude determined by the expression

$$\sin \psi = e \sin \varphi. \tag{13}$$

The previous equation can be written in a simpler form, as

$$\rho\ (\varphi) = \frac{F}{U^n(\varphi)} = FV^n(\varphi), \tag{14}$$

The constant of integration F determines the scale of the map projection. Geometrically, it acts as a proportionality factor controlling the radii of the mapped parallels in the plane and therefore influences the number and position of the standard parallels. We note that uniform scaling does not influence conformality, so such a parameter is expected during the derivation of all conformal mappings.

In the map projection literature, the designation U is commonly used. Its descriptive meaning is the natural exponential of the isometric latitude.

For convenience, we introduce the reciprocal of U, denoted by V, as defined in the following equation. Introducing this variable simplifies the analytical form of the projection equations and, in particular, the expressions for the local linear scale factor and its extrema, which are used later in the optimization procedure.

$$V(\varphi) = \frac{1}{U(\varphi)} = \frac{\tan^e\left(\frac{\pi}{4} + \frac{\psi}{2}\right)}{\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)} = \tan\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)\tan^e\left(\frac{\pi}{4} + \frac{\psi}{2}\right). \tag{15}$$

Accordingly, the equations for conformal conic projections in the normal aspect are given by

$$\delta\ (\lambda) = n(\lambda - \lambda_0), \rho(\varphi) = F\ V^n(\varphi), \tag{16}$$

$$V\ (\varphi) = \tan\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)\tan^e\left(\frac{\pi}{4} + \frac{\psi}{2}\right), \quad \sin \psi = e \sin \varphi \tag{17}$$

$$x = q - \rho \cos \delta, y = \rho \sin \delta, \tag{18}$$

$$h = k = \frac{n\rho}{r}, \quad p = k^2, \quad \omega = 0. \tag{19}$$

In the preceding formulae, p denotes the local areal scale factor, and ω represents the maximal angular deviation. These formulae involve two constants, n and F, which can be determined under various conditions. Together, they define the Lambert conformal conic projection, hereafter referred to as LCC.

For $\varphi = 0$, we have $V(0) = 1$ and $\rho(0) = F$; thus, F corresponds to the radius of the Equator's image in the projection plane.

We now examine the distribution of the local linear scale factor, $k = \frac{n\rho}{r}$ in greater detail, beginning with the determination of its extreme values.

$$k = k(\varphi) = \frac{n\rho(\varphi)}{r(\varphi)}, \quad \varphi \in (-\pi/2, \pi/2). \tag{20}$$

From the condition

$$\frac{dk}{d\varphi} = n\frac{M\rho}{r^2}(\sin\varphi - n) = 0, \tag{21}$$

it follows that the stationary point φ_0 satisfies

$$n = \sin\varphi_0. \tag{22}$$

Because a stationary point may correspond to either a minimum or a maximum, we now determine which case applies. To this end, we compute the second derivative of the function $k = k(\varphi)$. At the stationary point, we obtain

$$\frac{d^2k}{d\varphi^2} = n\frac{\rho(\varphi_0)M(\varphi_0)}{r(\varphi_0)N(\varphi_0)} > 0, \tag{23}$$

which indicates that the stationary point is a minimum. The derivation up to this point is well known in the literature on map projections [3,6–10]. We now extend this result. In particular, we ask how many standard parallels the LCC projection can possess. To answer this question, we examine the corresponding condition: for a parallel to be standard, it is necessary that at every point on it satisfies

$$h(\varphi) = k(\varphi) = 1. \tag{24}$$

Since in conformal projections, $h(\varphi) = k(\varphi)$ at every point, and since in the LCC projection the local linear scale factors depend only on the geodetic latitude φ, it is sufficient to examine the condition

$$k(\varphi) = 1. \tag{25}$$

Using Equation (20), we may first write

$$\frac{n\rho(\varphi)}{r(\varphi)} = 1, \tag{26}$$

and then, by applying Equations (16) and (17),

$$\frac{nFV^n(\varphi)}{r(\varphi)} = 1, \tag{27}$$

From this, it follows that

$$F = \frac{r(\varphi)}{nV^n(\varphi)}. \tag{28}$$

Now, if we wish the parallel corresponding to the geodetic latitude φ_0 from relation (22) to be standard—i.e., for the local linear scale factor to equal 1 along that parallel—then we must choose

$$F = F_0 = \frac{r(\varphi_0)}{nV^n(\varphi_0)}. \tag{29}$$

In this case, the local linear scale factor attains its minimum value of 1 at φ_0 and, therefore, the scale factor along all other parallels is greater than 1 (blue curve in Figure 2). This corresponds to the situation with a single standard parallel.

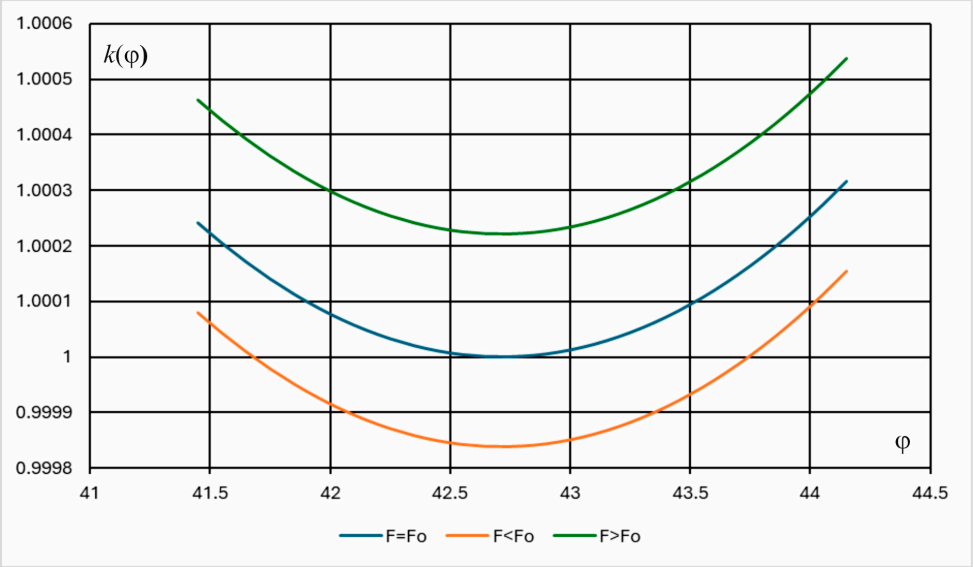


Figure 2. The local linear scale factor $k = k(\varphi)$ of the LCC projection as a function of the parameter F. Blue: one standard parallel; red: two standard parallels; green: no standard parallels.

If we choose $F > F_0$, so that $k(\varphi) > 1$, then the projection will have no standard parallels (green curve in Figure 2). Conversely, if we choose $F < F_0$, then there will exist two distinct latitudes φ_1 and φ_2 such that $k(\varphi_1) = k(\varphi_2) = 1$. In this case, the projection possesses two standard parallels (red curve in Figure 2). Furthermore, if we choose $F = F_0$, only one standard parallel exists.

3. The Optimal LCC Projection

In principle, every map projection can be considered optimal in some sense. However, map projection theory offers a variety of criteria for defining what "optimal" means [3,6–26]. Each previously cited paper defines "optimality" in a slightly different way, indicating that there is no consensus on the definition of the "optimal" map projection.

Because any projection may be regarded as "optimal" depending on the chosen criterion, it is important to specify what "optimal projection" means in our context. Since we are considering conformal projections—those free of angular distortion and with an areal distortion equal to the square of the linear distortion—it suffices to formulate a criterion based solely on linear distortions.

If there is no particular reason for the local linear scale factors along the southernmost and northernmost parallels of the mapped region to differ, it is natural to require them to be equal. This idea roots back to Chebyshev, who presumed that linear scales of a conformal mapping of a spherical region have minimal range, if and only if its linear scale is constant along its boundary [13]. This assumption is proven by Gravé [27]. Later, Bermejo & Otero [28] demonstrated that this criterion is also valid for the ellipsoid of revolution. A detailed study of the problem of finding the best and ideal projections is presented by Bugayevskiy and Snyder [6] (pp. 193–228). Denote the geodetic latitudes of these two boundary parallels by φ_S and φ_N, respectively. This requirement can be expressed mathematically as

$$k(\varphi_S) = k(\varphi_N). \tag{30}$$

If there is no particular reason for the minimum and maximum values of linear distortion to differ in absolute magnitude, it is natural to require that they be equal. This condition can be expressed mathematically as

$$k(\varphi_S) - 1 = 1 - k(\varphi_0) = k(\varphi_N) - 1, \tag{31}$$

where φ_0 is the geodetic latitude of the parallel at which the local linear scale factor attains its minimum. Accordingly, we propose the following criterion for selecting the optimal LCC projection for an area bounded by the parallels φ_S and φ_N.

The optimal LCC projection is the one in which the parameters n and F from Equation (16) are chosen such that the local linear scale factors along the two boundary parallels are equal, and the maximum local linear scale factor exceeds 1 by the same amount that the minimum local linear scale factor falls below 1.

This criterion is well established in the literature on map projections [3,6,7,9,10,12,15,16,18,22,29]; here, we present a clearer formulation of it. For convenience, let us introduce the notation

$$r_s = r(\varphi_S), r_N = r(\varphi_N), V_S = V(\varphi_S), V_N = V(\varphi_N) \tag{32}$$

$$\rho_S = FV_S^n, \rho_N = FV_N^n \; k_S = k(\varphi_S), k_N = k(\varphi_N), \tag{33}$$

If the scale factors along the two boundary parallels are equal (30), the expression for the constant n becomes

$$n = \frac{\ln r_N - \ln r_S}{\ln V_N - \ln V_S} = \frac{\ln(r_N/r_S)}{\ln(V_N/V_S)}. \tag{34}$$

The expression for the constant F follows from the second requirement—that the maximum local linear scale factor exceeds 1 by the same amount that the minimum local linear scale factor falls below 1:

$$F = \frac{2}{n\left(V_N^n/r_N + V_0^n/r_0\right)} = \frac{2}{n\left(V_S^n/r_S + V_0^n/r_0\right)}, \tag{35}$$

where

$$V_0 = V(\varphi_0), \;\; r_0 = r(\varphi_0). \tag{36}$$

Starting from the given geodetic latitudes φ_S and φ_N, we thus determine the projection constants n and F. Once these parameters are known, the geodetic latitudes φ_1 and φ_2 of the standard parallels (where the scale factor equals 1) can be found by solving the transcendental equation:

$$nFV^n(\varphi) = r(\varphi), \tag{37}$$

which may be solved using an appropriate iterative method.

Finally, we examine a rule—reported by several authors, including Hinks, Kavrayskiy, Deetz and Adams—for the approximate determination of the positions of the standard parallels. The rule is given by

$$\varphi_1 = \varphi_S + \frac{\varphi_N - \varphi_S}{K}, \;\; \varphi_2 = \varphi_N - \frac{\varphi_N - \varphi_S}{K}, \tag{38}$$

where K = 3, 4, 5, 6, or 7, depending on the shape of the mapped region (e.g., circular, square, etc.) [7].

According to Hinks, it is generally sufficient to place the standard parallels at approximately one-seventh of the total latitudinal extent inward from the bounding parallels [11].

Deetz and Adams [3] state that, for an approximately equal distribution of scale error, the standard parallels should be selected at one-sixth and five-sixths of the length of the portion of the central meridian being represented.

In 1962, the projection used for the International Map of the World (IMW) at a scale of 1:1,000,000 was changed from a modified Polyconic to the LCC projection for latitudes

between 84° N and 80° S. A polar stereographic projection is used for the remaining areas. Each sheet covers approximately 6° of longitude by 4° of latitude. Within each 4° latitudinal zone, the standard parallels are placed at one-sixth and five-sixths of the latitude span. The reference ellipsoid adopted was the International ellipsoid. This specification was subsequently used by the United States Geological Survey (USGS) in producing several maps for the IMW series.

The USGS also employs the LCC projection for topographic purposes in the middle latitudes for the 1:1,000,000-scale geologic series of the Moon, as well as for selected maps of Mercury, Mars, and the satellites of Jupiter [5].

Using relation (38), the parameter K can be expressed as follows:

$$K = \frac{\varphi_N - \varphi_S}{\varphi_1 - \varphi_S} \text{ or } K = \frac{\varphi_N - \varphi_S}{\varphi_N - \varphi_2}. \tag{39}$$

For a sphere rather than an ellipsoid, all of the above formulas simplify, as one simply sets $e = 0$.

4. The Optimal LCC Projection for the Territory of Bulgaria

The LCC projection has become a standard choice for official cartographic products worldwide, rivaled in prominence only by the Transverse Mercator projection [30]. Its significance is illustrated, for instance, by its adoption in Croatia, as outlined in the Decree on Establishing New Official Geodetic Datums and Planar Map Projections of the Republic of Croatia. On 4 August 2004, the Government of the Republic of Croatia established a coordinate reference system based on a normal LCC projection with standard parallels at 43°05′ and 45°55′ N for use in large-scale official mapping. The decision was motivated by the projection's low linear distortions, its favorable preservation of areal and shape characteristics, and its broad applicability. The Croatian implementation of the LCC projection is based on the GRS80 ellipsoid. In accordance with ISO 19111 [31], the official abbreviation assigned to this projection system is HTRS96/LCC [32,33].

The aforementioned standard parallels were determined for the terrestrial portion of the Republic of Croatia. Rajaković and Lapaine examined the determination of optimal standard parallels for the entire national territory, which includes not only the mainland but also a section of the Adriatic Sea [29]. Based on their analysis, they proposed the standard parallels 42°20′ and 45°50′ N.

The Lambert Conformal Conic (LCC) projection has been employed in Bulgaria across all historically established and currently operational coordinate systems. Its configuration has varied, using either one or two standard parallels, along with differing parameter values, central meridians, and latitudes of origin. In the 1970 Coordinate System, the territory of Bulgaria was divided into four zones, each defined by an LCC projection with a single standard parallel. In the BGS2000 and BGS2005 coordinate systems, the projection is defined using two standard parallels, with the latter system primarily applied for cadastral purposes.

The use of this projection is determined by Bulgaria's geographic location in the mid-latitudes and by the country's predominantly east–west orientation along the parallels. Owing to these characteristics, the LCC projection has historically been regarded as the most appropriate choice for supporting national economic activities and land administration.

4.1. Bulgarian Geodetic System 2000

The Bulgarian Geodetic System 2000 (BGS2000) was officially introduced in 2002 [34]. Its specifications are detailed in an instruction issued by the Directorate Geodesy and

Cartography of the Ministry of Regional Development and Public Works [35]. The LCC projection used in BGS2000 is defined by the following parameters:

The GRS80 ellipsoid is given by

a = 6,378,137 m, $f = 1/298.257222101$, $e^2 = 6.69438002290 \times 10^{-3}$

$\varphi_S = 41^\circ 15'$; $\varphi_N = 44^\circ 10'$; $\varphi_1 = 41^\circ 51' 11.2153''$; $\varphi_2 = 43^\circ 28' 35.8786''$ [33].

The method used to determine the standard parallels is not documented.

4.2. Bulgarian Geodetic System 2005

The Bulgarian Geodetic System 2005 (BGS2005) was introduced in 2012 [36,37]. It is employed for geodetic computations throughout the country and serves as the principal coordinate system for cartographic applications. The LCC projection in BGS2005 is defined by the following parameters:

The GRS80 ellipsoid is specified by

a = 6,378,137 m, $f = 1/298.257222101$, $e^2 = 6.69438002290 \times 10^{-3}$

$\varphi_1 = 42^\circ 00' 00''$, $\varphi_2 = 43^\circ 20' 00''$, $\varphi_0 = 42^\circ 40' 04.35246''$ [36].

The geodetic latitudes of the southernmost φ_S and northernmost φ_N parallels are not explicitly defined. In the official Instruction [36], φ_0 is identified as the central parallel. However, the formulas provided in the Instruction show that φ_0 is in fact the geodetic latitude of the parallel along which the local linear scale factor k attains its minimum value. It would be more accurate to state that this latitude only approximately corresponds to the central parallel. The method used to determine the standard parallels is not documented.

4.3. Proposal by Bandrova and Gyurov

Bandrova and Gyurov define the optimal projection as the one in which the distortions at the mid-latitude and at the endpoints differ the least—that is, they are closest in magnitude [38]. They concluded that the most appropriate choice of standard parallels follows the 1/6 rule; however, in their article, they present standard parallels derived according to the 1/5 rule.

4.4. New Proposal

In contrast to the criterion proposed by Bandrova and Gyurov [38], the present study defines the optimal projection as the one for which the distortions at the midpoint latitude and at the endpoint latitudes do not differ in absolute value (Table 1). Applying this criterion to determine the optimal LCC projection for Bulgaria, using the GRS80 ellipsoid, and adopting the southernmost and northernmost parallels as follows:

$$\varphi_S = 41^\circ 14', \varphi_N = 44^\circ 13' \tag{40}$$

according to the Statistical Yearbook 2024 [39], which cites the Geodesy, Cartography and Cadastre Agency of the Ministry of Regional Development and Public Works as its data source, and

$$\varphi_S = 41^\circ 14' 05'', \varphi_N = 44^\circ 12' 45'' \tag{41}$$

according to Wikipedia [40], after substituting these latitudes into Formulas (32)–(37), we obtained the values presented in Table 2.

We note that the linear scale factors presented in Table 1 are the same as the maximal (the greater number from the two middle columns) and minimal (last column) linear distortion of each mapping within the territory of Bulgaria. This enables the reader to assess the gain resulting from optimization.

Table 1. Comparison of local linear scale factors for Bulgaria using different methods for determining the standard parallels in the LCC projection, as presented in Bandrova and Gyurov [38] and in this study.

Coordinate System or Rule	Local Linear Scale Factor at φ_N	Local Linear Scale Factor at φ_S	Local Linear Scale Factor at φ_0
BGS 2000	1.00026747	1.00021011	0.99990497
BGS 2005	1.00029966	1.00024340	0.99993773
1/6 rule	1.00018967	1.00018659	0.99985515
1/5 rule	1.00021850	1.00021495	0.99988373
Airy Criteria	1.00023959	1.00022132	0.99989734
[40]	1.00016887	1.00016887	0.99983114
[41]	1.00016824	1.00016824	0.99983176

Table 2. Proposed parameters for the optimal LCC of Bulgaria.

φ_S	φ_N	φ_1	φ_2	φ_0	n	K
41°14′	44°13′	41°40′23.16″	43°46′58.08″	42°43′51.84″	0.67856	6.88 6.78
41°14′05″	44°12′45″	41°40′25.32″	43°46′46.20″	42°43′46.76″	0.67854	6.88 6.78

5. Conclusions

The Lambert Conformal Conic (LCC) projection was examined in detail in this study. The projection equations were derived, and instead of the commonly used notation U, a new notation V—defined as the reciprocal of U—was introduced to simplify the expression of the formulas. It might be mentioned that the standard implementations of multiplication are slightly more effective than those of division, so this change in notation might result in a slightly faster evaluation. Particular attention was given to the analysis of distortions, which made it possible to formulate conditions governing the number of standard parallels, whether two, one, or none.

The equations of the LCC projection contain two parameters that determine the spatial distribution of linear and areal distortions over the territory to which the projection is applied. In this paper, we develop a criterion according to which the optimal LCC projection is defined as one in which these two parameters are selected such that the local linear scale factors along the boundary parallels are equal, and the maximum local linear scale factor exceeds 1 by the same amount that the minimum local linear scale factor falls below 1. We applied this criterion to compute the projection parameters and to determine the standard parallels for the optimal LCC projection for the territory of Bulgaria, which constitutes the primary objective of this study.

Table 2 shows that the local linear scale factors are nearly identical across all examined methods for selecting standard parallels. If we adopt the criterion that defines the optimal projection as the one in which the distortions at the stationary point and at the endpoint latitudes do not differ in absolute value, then the optimal LCC projection for Bulgaria is the one whose standard parallels are

$$\varphi_1 = 41^\circ 40', \ \varphi_2 = 43^\circ 47'. \tag{42}$$

The maximum value of the local linear scale factor at the endpoint parallels is 1.00016824, while the minimum value is 0.99983176. For practical purposes, these values can be rounded to 1.0002 and 0.9998, respectively.

The average value of the parameter K was calculated as 6.92, which is approximately 7 and aligns with the general recommendation of Hinks [11]. Accordingly, the distance between the standard parallels and the extreme parallels corresponds to roughly one-seventh of the width of the 3° latitude band defined by the northernmost and southernmost parallels of Bulgaria.

Similar map projections with exactly the same formulas can also be used for the conformal representation of other regions. It is recommended for areas with a greater east–west extent, roughly following a latitude line. One should only replace figures for the northern and southern bounding latitudes φ_S and φ_N in the derivation above. It was shown, for example, that such an LCC projection would also perform better for the area of Hungary than its current state plane coordinate system [41].

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Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

BGS2000	Bulgarian Geodetic System 2000
BGS2005	Bulgarian Geodetic System 2005
GRS80	Geodetic Reference System 1980
HTRS96/LCC	LCC with standard parallels 43°05′ N and 45°55′ N was determined in 2004 as the projection coordinate system of the Republic of Croatia for the overview state cartography
IMW	International Map of the World
ISO	International Organization for Standardization
LCC	Lambert Conformal Conic
USGS	United States Geological Survey

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