atoms

MDPI

Perspective

Which Atomic Level Lifetimes in Heavy Ions to Measure Next?

Elmar Träbert

Astronomical Institute (AIRUB), Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, 44780 Bochum, Germany; traebert@astro.rub.de

Abstract

Charlotte Froese Fischer and Ian Philip Grant were both eminent in the numerical treatment of atomic structure theory. How do they still guide future work in "my" research sector, experimental atomic spectroscopy and lifetime measurements, which uses fast ion beams and ion traps?

Keywords: atomic physics; atomic transition rates; MCDHF; Grant code; spectroscopic techniques; beam–foil spectroscopy; electron beam ion trap; heavy-ion storage ring

1. Introduction

Charlotte Froese Fischer (CFF) has (almost) "always" been "there" during my scientific career. A well-informed person of standing and honour (yet approachable) at many conferences, a practical institution to address with key questions on the field or to send students to in order to have them get some well-supervised practical experience with running atomic structure program packages. CFF "stayed around" well into her formal retirement, and I remember several eulogies at (decenially) round-number birthdays. Among the latest are two by Alan Hibbert, one written at the occasion of her turning 90 [1], and the other an obituary looking back at about 95 [2]. The latter article includes an informal conference photo with CFF in the middle and Ian Grant (IPG) at her side, and the scientific conference community around the two. How nice that a photo version of only the two protagonists has been taken on the same occasion and has been provided for this Special Issue! However, for Ian Grant we have another chance coincidence, my physics colleague Giulio Del Zanna happening upon Ian Grant in hospital and soliciting a collection of reminiscences for a future review of Ian's many achievements in developing relativistic atomic physics codes [3,4]. With these three recollections already available, what else should/could I contribute to this Special Issue? I am an atomic physics experimenter, with very limited insight into what CFF and IPG are being cherished for. Let those who know what they are talking about offer the praise! Let me write about how important even "weak-link", occasional contacts may be over time, and how influential and helpful. How these mostly incidental contacts with CFF and IPG still shape the prospects of my field. Let me first, however, ramble about paths crossing incidentally, before specific projects bring about further interactions.

In the mid-1970s, I joined the beam-foil spectroscopy group of Paul Henrich Heckmann at Ruhr-Universität Bochum (Germany) for graduate work in accelerator-based atomic physics. The technique of beam-foil spectroscopy had well over a decade earlier been initiated by L. Kay at Manchester (UK) and by S. Bashkin at Tucson (AZ, USA) [5–7]. At Bochum, a still-new university, a working group around H.H. Bukow had begun using a small ion accelerator a few years before my "arrival". I joined not long after a new, much more powerful and very versatile heavy-ion accelerator (a 4 MV Dynamitron tandem

Check for updates

Academic Editors: Ran Si, Harry Quiney and Jacek Bieroń

Received: 16 September 2026
Revised: 3 October 2026
Accepted: 8 October 2026
Published: 12 October 2026

Copyright: © 2026 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license.

accelerator) had been set up and was entering routine operation. All this happened in a nuclear physics environment, in which atomic physics was seen as an oddity from the past. In the magnanimous words of one of the professors, "yes, one may do something like that for a while, and then drop it again" (and return to real physics?). In such an environment, I recall as few as two (!) books of spectral tables in the institute library (the "MIT wavelength tables" and Striganov/Sventitskij [8], both practically exclusively listing data from the spectra of neutral atoms and singly charged ions. Much later we learned of the Wiese tables (from the US National Bureau of Standards (NBS), which later reconfigured as the National Institute of Standards and Technology (NIST) and moved from downtown Washington, DC, to Gaithersburg (MD); eventually the atomic spectroscopy group was forced out of NIST (and the Department of Commerce) by 2025). The move out of the city had ostensibly been ordered to reduce the density of important targets in downtown Washington, should the Cold War turn hot. The downtown location was readily grabbed by another institution, and thus one may suspect different motives underlying the move.

We eventually enjoyed Bashkin & Stoner's Grotrian diagrams [9], and years later again I came across the Kelly & Palumbo tables [10]. The tables had been issued by the Naval Research Laboratory (NRL), located in the same metropolitan area, but their lead author taught at a West Coast military academy. Those tables were excellent in their coverage and typeset rendering. The production proceeded from formatted files on magnetic tape, without human interference in the actual typesetting, which resulted in a marvellously low rate of printing errors, if any. However, a detail almost as clear as the dichotomy of theory and experiment (see me writing about CFF and IPG) became apparent: according to the introduction to his big book (the Kelly/Palumbo tables), the senior author and academic teacher Raymond Kelly selected the data entries to the tables by their authors' own quality judgement (error bars), and thus he fell foul (in a few instances) of their self-(over-)confidence.

Some historic physics journals in the Bochum physics faculty library were spare copies from Balliol College, Oxford (UK), where they probably did not much need their first copies either (Among British politicians, an education at Balliol is rather frequent). CFF and IPG were theoreticians—they were not inside my horizon at the time, and I suspect that was the same for the more advanced members of the Bochum physics faculty. By the way, both Kay and Bashkin worked in faculties dominated by nuclear physics interests and were made to feel their oddity when their perspective dared to swivel [11,12]. Kay returned to the fold, and Bashkin relocated from Iowa to a more open-minded faculty at Tucson (AZ).

The above is not to say that Bochum physics was without theory: There were theorists in nuclear, solid state and plasma physics, the three declared principal columns of wisdom (and power) in this newly formed faculty, just nobody caring much about atomic structure theory. In fact, my co-graduate Friedbert Hannebauer obtained atomic structure programs and did his own computations of various terms of multiply-excited few-electron oxygen ions [13]. My horizon widened when I visited with Josh Silver's group at Oxford (UK), and I subsequently became a postdoc there. On the experimental side, this included (among many other points) the second-order Doppler effect, which we had not cared about at Bochum, the refocusing of spectroscopic devices for a moving light source, and the value of moiré fringe length gauges in establishing spectrometer precision. From a general perspective, relativity and quantum electrodynamics (QED) suddenly sprang up as essential facts of life. Instead of merely detecting spectral lines of interest and determining atomic lifetimes more reliably, as we had tried at provincial Bochum, Josh's aims were "fundamental" measurements in a worldwide competition of excellence. Among his projects was the accurate measurement of 1s2s $^3S_1$–1s2p $^3P^o_{0,1}$ transition wavelengths in He-like ions (starting with Si^{12+} ions [14]). The basic idea was to avoid the line broadening associated with short level lifetimes, say

in the 1s-2p transition in H-like ions, by exploiting the longevity of $n = 2$ triplet levels in He-like ions. The 2s level Lamb shift is smaller than the 1s level Lamb shift, but the measurement precision of the 2s Lamb shift in a triplet level of a He-like ion is higher than that of measuring the 1s Lamb shift in an H-like ion. This idea has been pursued and refined in several laboratories.

A particularly notable competitor was Henry Gordon Berry (Chicago) (a former Oxford graduate) who tended to claim a particularly high accuracy of his results, which might raise envy in the competition as well as fear that one's own funding prospects might consequently suffer. (By the way, HGB was also remarkably quick. When he joined (in the 1960s) an international group of colleagues who at Tucson learned and developed Bashkin's new technique of beam-foil spectroscopy, he jumped into the scientific discussions, vented ideas and wrote papers at a speed that astonished his colleagues. Some claim that they had problems reigning him in at times, insisting that they would want to publish their own ideas first and under their own names.)

In order to remain competitive in the race for better experimental results, there were technical developments to consider, as well as theory. The QED contribution to the wavelength of interest is determined from the difference between a theoretical prediction and measurement. Interestingly, it is not just the QED prediction that matters, but also quantum mechanics "without QED" to apply the QED correction to. Experiment cannot switch off the QED contribution, and thus it will only yield the combination of "classical quantum mechanics" with QED. Maybe it is almost an insult to QED experts who are working on what is considered to be the most accurate theory available yet (results with an accuracy of a dozen decimals)—to test this theory alongside non-QED quantum mechanics (QM). Considering the present Special Issue, CFF is strongly associated with non-QED QM, and IPG has concentrated on relativity and QED corrections. In specific magnetic dipole ($M1$) transitions, the rate of which can be measured fairly well, a certain 0.45% correction to the transition moment is considered a possible test of the limits of the Standard Model. However, the (dominant) many-body QM part of the transition rate cannot be as reliably calculated (yet) as is necessary in order to test that correction. I will return to this point among the examples below.

Let us go back to the measurements of 1979 at Oxford. Josh Silver, of course, asked around (Oxford has experts on almost anything), discussed with David Mayers (of the IPG cloud) and others, found a study by Ermolaev & Jones [15], and thus an entry into the theory side of his QED project. At the time, relativistic atomic structure computations were in their early stages. Cheng, Kim and Desclaux [16] had just published a very useful tabulation of ground state transitions in many Li- to F-like ions, including the rates of several electric dipole ($E1$) forbidden transitions ($M1$, $E2$). Commanding rather limited computer resources, they employed rather small basis sets and de-selected transitions which they thought to be of little interest (which I later needed and then missed in the tables), but for several decades this was a major reference paper. Meanwhile, IPG and his team developed what eventually became the Grant code.

Our students at Bochum saw the success of fine structure computations in few-electron ions by followers of the Grant school. Their interest was on intercombination transition rates, and the Grant code worked reasonably well for these, too, but not spectacularly so. When one of our students presented his work to a seminar in Theoretical Chemistry, there were people in the audience who were convinced that their techniques worked much better—but they could not corroborate that claim. Different communities have different laboratory languages and concepts. Therefore, our experimental physics student, Holger Blanke, for his next-level thesis, changed fields and joined Burkhard Fricke's group at Kassel, where other people studied nuclear collisions at relativistic velocities. They tried

to find out by theoretical means whether merged heavy-ion nuclei at $Z = 137$ or $Z = 173$ would result in new phenomena (spontaneous positron or e^+-e^- pair production and the like). It would be difficult to reach a higher value of Z in experiments on Earth, indeed. At Kassel, Blanke used the (relativistic) Desclaux code for the structure of multiply excited few-electron ions, which at the time was considered to be more advanced and more widely accepted. At the same time, Jiro Hata and IPG performed excellent computations of core-excited three- and four-electron ions [17,18]. Thus, concurrent with some skepticism, there were indications of the great capabilities of the new code.

Another attempt by our group members using the Grant code remained disappointingly inconclusive. Eventually it transpired (by gossip) that a large number of QED-related integrals wanted in the Grant code had not been calculated before and would take much time and effort to provide. "Somebody" (gossip mentioned Marcel Klapisch) therefore "guess-timated" a number of these integrals in order to reach a provisional working version of the code. A few years later the estimates were replaced by computed values, and since then the results all seemed fine. Profiting from many international collaborations, the code has evolved into GRASP and its siblings and has become one of the backbones of atomic structure computations worldwide. If not so many authors nowadays would like to appear as experts by writing theory sections to their tables of computed data in which they recall specifics that are merely extracts from the descriptions in the original publications of the code versions ...

With so many achievements on the relativistic atomic structure computation trail by IPG and his cloud of collaborators, what has CFF done, who for decades has concentrated on non-relativistic computations? CFF had studied with Hartree. Thus it comes as little surprise that she extensively worked with and on Hartree–Fock codes. Computers were very limited in capacity in those days, and non-relativistic wave functions are simpler than relativistic ones, making it easier to cope with atomic structure complexity when avoiding relativity. Nevertheless, users sought to introduce relativistic corrections (Cowan's HXR code). While the Cowan code might not produce accurate spectra on its own, there are adjustable parameters which can be scaled, resulting in a massively improved agreement with experiment. This renders the Cowan code highly useful: The quickly applied code yields a "complete" level scheme that can be fitted to match experimental data, and then the differences between predicted and measured levels reveal details about atomic structure [19]. The predictive power can be improved by more complex wave functions (Multi-Configuration Hartree–Fock MCHF, MCHF-BP with Breit–Pauli correction, Multi-Configuration Dirac–Hartree–Fock MCDHF, and so on). CFF excelled in driving these computations to ever higher accuracy, and eventually her tabulations of transitions and rates in ions up to the Ar isoelectronic sequence (with G. Tachiev and A. Irimia) [20,21] partly superseded the much earlier scaled HXR computations performed by Brian Fawcett [22–24].

She worked with many visitors, and she tutored students. Very occasionally, CFF enquired about options for experimental support. However, she had worked with experimenters since the early beam-foil days when most work was done on visible spectra and then had pursued those problems with improved computations, while we at Bochum had started later, with the EUV spectroscopy of more highly charged ions. Consequently, we did not command spectroscopic equipment for the visible range that would have helped her. Anyway, thanks for the trust in our capabilities! We did co-operate with CFF and her group remotely and by sending a student for a tutoring visit, nevertheless, as is mentioned below. It is interesting to see that many theoreticians (see, for example, Stephan Fritzsche (who "grew up" in the Fricke group at Kassel) and Per Jönsson (Malmö)) collaborated with

both the CFF and the IPG groups. This qualified the visitors, and it helped to bring both major atomic structure code packages onto an equal quality level.

In short, there are two aspects to the combination of CFF and IPG discussed here: CFF started out from non-relativistic Hartree-Fock computations and added complexity (Breit operator, many configurations) and relativistic effects, for example in the Multi-Configuration Dirac–Hartree–Fock approach. The non-relativistic approach is best used for low-charge ions, whereas for high-charge ions, relativistic effects become more and more important. IPG started out from a relativistic approach and tried to make the program package more practical to handle. This approach works best in an atomic system dominated by the nuclear Coulomb field, that is, for highly charged ions. "Unfortunately", our environment is predominantly electrically neutral (neutral atoms and very low charge state ions), which poses a particular accuracy challenge for relativistic wave functions. Both major approaches need to reach high accuracy in an ab initio process, starting from basic principles and proceeding without semi-empirical adjustments. Otherwise, they do not fulfil the promise of atomic structure theory having predictive power. The present discussion condenses around the kernel of atomic-level lifetimes/radiative transition rates.

It has been suggested to take the title of my essay literally and to provide a list of which individual cases to measure next. However, each experiment requires research funding, access to suitable facilities, specific research equipment, manpower, helpers and supporters, and whatever else. The research options discussed in the following are meant to help with recognising research opportunities worth pursuing, but each individual experiment will likely be limited by local conditions.

2. Samples of Transition Rates of Interest

What is "of interest" may change over time. I am picking examples that have been of interest at one time. For example, when, after World War II, computers were permitted to try some non-military work, Bates and Damgaard implemented the Numerical Coulomb Approximation (NCA) on such a machine (the American Physical Society has reported their 1949 paper as one of the most-cited physics publications ever). Initially, it dealt with a single valence electron in a central Coulomb field, which implied that the results could be compared with those for the hydrogen atom (with the help of mechanical calculators) after applying some scaling factors. When in the early 1970s lasers were employed to selectively excite the np resonance levels in alkali atoms, the expectation was to find out which theoretical computation did best for the transition rate. Experiments used fast ion beams of Li and Na (that were then neutralised; work by Hartmetz in the Schmoranzer group at Kaiserslautern, Wittmann, Gaupp, Kuske in the Andrä group at Berlin) or vapour cells (Carlsson and Sturesson at Lund). Hartmetz and Carlsson reported uncertainties of about 1% [25], while the Berlin group reported uncertainties of 0.25%, which they shrank even further over time [26]. The seemingly less accurate data agreed with theoretical results (that also scattered), whereas the more ambitious experiments (considered to represent the leading edge of science) agreed only with the NCA result, which was surprising. Experimenters, of course, assumed that theorists needed to catch up. CFF tried and found that her results would not move towards the experimental numbers. She even said so audibly at conferences. Quite a few years later, some of those experimenters of the "timid" fraction had improved their study of systematic error so much [27] that they suggested an alignment problem in the earlier work of the slightly overconfident colleagues. Their own results agreed with advanced theory. Meanwhile, a very different approach (modeling cold atomic collisions [28,29]) had determined the decisive oscillator strengths independently and with even higher accuracy—and showed that CFF had been right. A point for Ms. Fischer and theory.

However, her (and other) computations did not all succeed with similar glory (see some samples of $E1$-forbidden fine structure transitions by Froese Fischer & Liu [30] and by Bhatia, Doschek, and Kastner [31,32], as discussed elsewhere [33]). This needs some explanation and discussion. However, different transitions pose different problems—there is no "one atomic structure theory (numerical code package) fits all". I am picking examples and discussing some of the particular points of interest next.

2.1. Resonance Transitions

The hydrogen atom with its single electron outside a single proton is a standard item of atomic physics and quantum mechanics that can be computed to high accuracy. Of course, it is much more complex than it may seem; if one wants to declare Dirac's treatment as relativistically perfect, one has overlooked that it considers the nucleus as infinitely heavy and thus not moving. Many corrections have been formulated to overcome the shortcomings of already amazingly accurate descriptions of the hydrogen atom. This field has become a cottage industry of its own. CFF was not particularly interested in this two-particle system (if one neglects the particle properties of the photon . . .), but she cared for atomic systems with many more electrons. Of these, the relatively simple ones have a single electron outside of closed shells. Still, it is a challenge to compute the spectrum of the valence electron including the interactions of the electrons in those purportedly closed shells, with their numerous virtual excitations.

I have already mentioned the ns–np resonance transitions in the alkali atoms, but there is another branch of study as well, the isoelectronic sequences, that is, ions with the same electron structure as the name-sake atoms, but with higher nuclear charges. There are isoelectronic trends of atomic parameters, as have first been described by Egil Hylleraas in the $1/Z$ series expansions of various atomic entities in the late 1920s. Bengt Edlén in his Handbook of Physics article [34] has given many examples, usually showing that the isoelectronic trends are clearly seen for wide ranges of Z, but are often not yet applicable to the first few members of an isoelectronic sequence.

Properties of the neutral atom can be measured very precisely, but the information cannot necessarily be extended along the sequence. The high-Z trend may be modified by relativistic effects which cannot be computed well by non-relativistic wave functions. "Classical relativity" is seen in high-Z ions with higher electron velocities and electron-electron interactions; nowadays it is also of interest to treat low-charge ions with relativistic programs, competitive accuracy, and ab initio approaches, with the goal in mind of eventually tackling neutral many-electron systems by just one "universally applicable" program package. High-Z, few-electron ions are enhancing QED effects (high-field); low-Z (low-ζ, the charge seen by the valence electron) atomic systems feature electron-electron interactions in a non-dominant central Coulomb field.

Intra-shell transitions in low-n configurations are prominent in many light sources. The transition energies as well as the transition rates scale linearly with Z. Thus, the same spectrometers can be used for wide sections of an isoelectronic sequence, for example grating spectrometers from Li I (2s-2p resonance lines in the visible range) to the soft-X-ray range for the 2s-2p$_{1/2}$ transition in U XC (U^{89+}), with 2p$_{1/2}$ level lifetimes from the multi-nanosecond range to hundreds of picoseconds (measured up to U ($Z = 92$, see also [35,36])). However, the 2p$_{1/2,3/2}$ fine structure splitting is a relativistic effect; the splitting rises with Z^4. Hence the 2s-2p$_{3/2}$ transition energy rises faster than linearly with Z, and the components of the low-Z fine structure level doublet differ by more than a factor of two at high Z. Consequently, the 2p$_{3/2}$ level lifetime reaches the range of a few picoseconds (and a practical limit for beam-foil spectroscopic lifetime measurements) already in the middle of the periodic table. At high Z, one would like to measure that level

lifetime in order to test the relativistic computations, but one can't practically do so with high precision. At low Z, one may try, but the measurement tests the computation only for one ion species at the time, because the long-range systematic trend is established only at slightly higher ionisation stages (higher Z). There, the present measurement techniques are not as accurate as one would need for interesting investigations, say, better than 1%. The presently accumulated lifetime measurements (practically only beam-foil experiments) with uncertainties mostly in the range from 3 to 8% confirm the expected isoelectronic trend as well as the onset of a notable relativistic contribution, but this was more than a quarter of a century ago. Can't we do better?

The precise atomic lifetime measurements on alkali atoms use laser excitation, not because laser light is something special, but because narrow-band laser light may selectively populate the level of interest. Since Bennett & Kindlmann [37] we know that any population of higher levels will result in multi-exponential decay curves which usually are afflicted by systematic error in the evaluation. Apparently nobody has tried out an excitation of the $2p_{1/2}$ level of Be^+ or B^{2+} ions using, for example, frequency-doubled light from a dye laser or by synchrotron radiation, although that should be feasible. Actually, Ansbacher et al. [38] have demonstrated such a measurement on Na-like Mg^+ ions and have reached a lifetime uncertainty near 1% for both fine structure levels. About half a dozen measurements on higher Z members of the Na isoelectronic sequence (up to Au, $Z = 79$) have used beam-foil excitation and reached uncertainties on the order of 5 to 8%. The main problem there is that the non-selective ion-foil interaction excites not only the $3p_{1/2,3/2}$ levels, but also the $3d_{3/2,5/2}$ levels (a little shorter in lifetime than the levels of interest, but not drastically different), as well as the many higher-lying levels that decay towards the yrast line of levels of maximum angular momentum ℓ for any given principal quantum number n, and thus all these cascades feed the levels of interest. This increases the signal of the lines of interest, which helps to identify them in spectra, but it severely hampers the decay curve analysis that is supposed to reveal the atomic level lifetime (the inverse of the sum of all decay rates). One can produce the ions with excited 3p levels by other means, for example in an electron beam ion trap, but then one needs fast switching of drift tube voltages on the picosecond scale in order to then apply electronic timing of the decay signal. Alternatively, one might consider ion beam bunches in a heavy-ion storage ring or fast-switching laser light pulses, but those options are very expensive to pursue experimentally. Considering the level of accuracy that may be reached, these further experiments are of limited interest. The resonance transitions are all closely related to the well-calculable transitions in H-like ions, and theory can be expected to reach a higher level of accuracy than experiment for a small fraction of the cost. It would seem much more fruitful to let theory and experiment address atomic systems in which atomic structure is more challenging to explain reliably and in detail. Most of these systems will feature levels that cannot (or not simply) decay by $E1$ radiation.

2.2. Intercombination Transitions

The fact of two electrons (in He-like ions) interacting introduces the phenomenon of total spin and results in two seemingly separate term systems, at first identified as belonging to two different atomic species, ortho- and parahelium. In due course, intercombination (spin-changing) transitions were recognised, which raised the question of how spin conservation may be violated. The phenomenon is nowadays called singlet-triplet (more generally: multiplet) mixing. A small admixture of singlet wave functions to the dominant triplet wave functions that describe a triplet level ties the decay of the latter to a spin-conserving decay in the singlet system. The latter is closely related to the transitions in a H-like system (and thus of no particular interest); the point of interest is the multi-

plet mixing of the wave functions, which can be enumerated from the intercombination transition rate in relation to the computed resonance line transition rate. The resonance line energy and transition rate are dominated by the large n = 1–2 transition energy, and small corrections are difficult to evaluate on the large-scale backdrop. Relatively larger are similar corrections compared to energies that are not that large, for example, transitions in the $n = 2$ shell as discussed above for Li-like systems. We take Be-like ions as specimens.

The $2s^2\ {}^1S_0$–$2s2p\ {}^{1,3}P^o_1$ resonance and intercombination transitions of Be-like ions have been discussed systematically in [36]. The present discussion touches on different aspects and the little experimental work that has been added since. The rates of spin-conserving intra-shell transitions scale linearly with Z, whereas the rates of spin-changing intra-shell transitions feature the additional scaling of the multiplet mixing (Z^3), which enters via the square of the mixing matrix element; hence, this amounts to a total scaling proportional to Z^7. In the B^+ ion, the $2s2p\ {}^1P^o_1$ resonance level lifetime amounts to $\tau \approx 0.83$ ns [39], and the resonance level lifetime changes but relatively little along the isoelectronic sequence. In the same ion the $2s2p\ {}^3P^o_1$ level features a level lifetime of $\tau \approx 100$ ms, which varies drastically along the isoelectronic sequence and which, at low Z, requires rather different measurement techniques.

The (roughly) nanosecond lifetime of the resonance level should be easily measurable by beam-foil spectroscopy, but the measurement situation is made complicated by the cascade scheme. There are $2p^2$ singlet and triplet levels, and of these, the ${}^1S_0$ level is some 20% shorter lived than the resonance level to which it decays. This combination produces a so-called growing-in cascade in which at the end of the sudden excitation by the beam-foil interaction the $2s2p\ {}^1P^o_1$ resonance level first rises in population before it then drops. There also is a slow cascade from the $2p^2\ {}^1D_2$ level, which is about a factor of five longer-lived than the resonance level and can seemingly be disentangled easily in the decay curve analysis—but the $2p^2\ {}^1D_2$ level collects the yrast cascade tail with many fast and slow cascade components. Multi-exponential fits to the decay curves, or population model fits, have confirmed the significant systematic errors that occur if one neglects the cascade situation, but the lifetime of interest has usually been estimated to carry an uncertainty on the order of 10% (see [36]). This must be an overestimate; the measurements for many ions agree so well with a straightforward isoelectronic trend that a mean error of 3% appears applicable.

The situation is much more challenging for the corresponding triplet level, $2s2p\ {}^3P^o_1$. The level in C^{2+} and its intercombination decay rate have been of astrophysical interest, but theory was uncertain by $\pm$20% in the 1970s. In the mid 1990s, theory suddenly reached uncertainties of around 1%, and about the same time experiment used a heavy-ion storage ring as a large ion trap and obtained the 10-ms level lifetime with an uncertainty of a small fraction of 1% [40]. Large-scale computations [41] thereafter reached a comparable low uncertainty—but with a significant slight offset. Measurements on the neighbouring low-Z ions established a consistent isoelectronic trend with uncertainties below 1%. The measurements at a heavy-ion storage ring represent an extension of the beam-foil technique. However, they do not suffer from the multi-exponential decay problem, because the level lifetime of interest is so extremely long compared to all higher levels that might cause cascades, that all levels can decay in their time, and the decay curve of interest can be established even after that, when all cascades have settled down. Why do I call the situation challenging? The challenge lies in the range of data in the isoelectronic sequence (presently with data from $Z = 5$ to $Z = 54$), and for most of the range the experimental data are not satisfactory in quality and partly outright inconsistent.

A fast-ion beam (beam-foil) lifetime measurement on Be-like Xe^{50} ions [42] has found a lifetime $\tau \approx 470$ ps with some 10% uncertainty. In between, from Ne^{6+} to Kr^{32+}, other

measurements show uncertainties ranging from 20 to 50%, and in ions with several measurements, several results differ from each other by more than their combined error bars. The Ne^{6+} data are based on an estimate from plasma spectra, and their factor-of-two uncertainty might suggest leaving this value out of the discussion. Why is there no proper measurement yet? A recent experiment on Ar^{14+} has avoided the beam-foil method; instead, a laser selectively transferred the population of a long-lived $J = 2$ level (populated in an electron beam ion trap) to the 2s2p $^3P^o_1$ level [43]. As much as such a new experimental approach is being welcomed, the disappointment about an uncertainty not significantly smaller than that from the beam-foil data in the middle-Z range is just as great. The extent of the cloud of scattered data and their error bars from Ne to Kr does not help much (yet) to discriminate among several theoretical attempts to describe the transition rate from low to high nuclear charge Z. My favourite is the computation that involves Charlotte Froese Fischer [44], while others favour the MZ code or the Many-Body Perturbation Theory (MBPT) employed by Ulyana Safronova and colleagues [45]. There have been quite a number of such treatments. Interestingly, the laser excitation result for Be-like Ar^{14+} [43] lies outside the range of scatter of practically all computed predictions. This may be meaningful, but at the present level of experimental uncertainty, nobody can tell.

Clearly, reliable and precise lifetime measurements of the 2s2p $^3P^o_1$ level are dearly wanted. The predicted lifetimes range from about 54 μs (Ne^{6+}) via 350 ns (Ar^{14+}) to 20 ns (Fe^{22+}), too short for ion traps (unless supported by pulsed lasers) and rather too long for beam-foil experiments. Maybe a relatively slow beam of ions extracted from an electron cyclotron ion source (ECRIS) or from an EBIT can be used in time-of-flight measurements employing a large position-sensitive detector (as has been demonstrated with a beam of recoil ions measuring a 200 ns lifetime in He-like Ar^{16+} ions [46]).

2.3. E1-Forbidden Transitions

After our group had succeeded (better than expected) with an experiment at the Heidelberg heavy-ion storage ring TSR, measuring transition rates of several intercombination transitions with high accuracy (because the atomic physics situation permitted to do so), I chanced to meet CFF at a conference and asked for her advice. What lifetimes to measure next? Is it worth the effort to measure the rates of electric-dipole forbidden transitions? Her answer was, "Probably not, because the computations are so good these days." Really? Looking back at many excellent results obtained by CFF—and a few others—revealed that some computations had gone astray. This had not always been noted by the theoreticians in charge.

I have already mentioned that the Cowan code permits the adjustment of certain parameters (Slater integrals) in order to improve the agreement of computed spectra with reality. The same problem afflicts other codes as well, of course. Transition rates are computed from transition energies and other contributing factors, and the computed transition energies do not necessarily fully agree with the precise information available from spectroscopy. Sometimes a researcher may be intrigued to find out whether his/her calculations of the other contributing factors might not lead to better results if combined with known (measured) transition energies. In some people's view that spoils the integrity of the theoretical approach. Others might argue with results that come closer to the experimental findings (but do not represent "pure" theory, tainting the claim of predictive power of theory), which may be informative. Another fraction might claim that the correction method matters (the euphemism used often is "semi-empirical adjustment"). For example, one might adjust the sample of wave functions used, or blame radial wave functions, which are not orthogonal and thus incompletely defined, or introduce a scaling factor. CFF has on various occasions demonstrated which corrections applied in which

way had which results. Other colleagues have discussed their intricate calculations, but somehow have forgotten to mention that—when the agreement of transition energies with experiment was not as good as hoped for—they just replaced their own energy values with experimental ones. This is not always easy to spot.

However, there are computations that seem easy to factorise. $E1$-forbidden transitions are often discussed as a Racah algebra factor ($6j$ symbols etc.) combined with a transition energy. Since computation has problems with obtaining accurate fine structure intervals, one is tempted to use the experimental data instead of the personally computed ones. The results are tables with theoretical transition rates that hardly scatter, because the cause of notable scatter, the transition energy which enters the calculation with a high power, has been removed before the act. Halt. Among the many look-alike results, one set reported by CFF and her student [30] sticks out by some 20%. It has been obtained by theory only, and the authors say so. Good. This also shows why certain atomic lifetime measurements even on $E1$ transitions are necessary. In other examples, of collision strength computations of various ions, there are many $E1$ transitions that on average probably have decent results, but an $E1$-forbidden transition between two low-lying levels carries a rate value that is wrong by some 30%, without any cautionary remark. To enhance the confusion, may I mention that some theorists recommend using the combination of theoretical energies and whatever other contributions purely, without any correction. And not to forget that most atomic structure details are more complex than initially pretended: the usual examples of Racah algebra in the computation of $E1$-forbidden transition rates imply a single-configuration picture, whereas that is not a given. In fact, it is likely that several configurations might be involved, and then the seeming exactness of straightforward mathematics with a few coefficients that are fractions of small numbers evaporates.

The multipole expansion of the radiation field is a basic concept in electrodynamics. Monopole radiation has often been searched but not found. Thus the lowest order (and usually most common) transition is the electric dipole ($E1$) transition that connects atomic levels of opposite parity ($(-1)^{\ell}$, with the electron angular momentum quantum number ℓ), with total angular momentum quantum numbers J that differ by no more than one unit. Higher-order radiation may be admixed if parity and total angular momenta fit, but it is expected to be much weaker. However, parity and J values may block $E1$ transitions (and/or higher multipole orders), and then $M1$, $E2$, $M2$, $E3$, $M3$ and so on may be the lowest order available, usually resulting in much longer level lifetimes than common $E1$ transitions of a similar transition energy would lead to. The transition rates of the higher multipole order transitions scale more steeply with Z than does $E1$ radiation (well, multiplet mixing also causes a steep increase, as mentioned above), which challenges theory to provide accurate transition energies. Out of the zoo of possibilities (see, for example, [47]) I am discussing just a few options in the specific context of this text.

2.4. Fine Structure Transitions

The fine structure levels of a given term arise from the same electron configuration and consequently they have the same parity. $E1$ transitions between these levels are forbidden, and $M1$ and $E2$ transitions should exist. If the levels can decay to other terms with opposite parity, those (possibly $E1$) transitions would usually dominate. Anyway, the fine structure (the reason for its name) is a complication that is relatively small compared to the main transition energy and thus difficult to resolve from that. Studying fine structure is best done in terms that have no competing decay channels, that is, in the ground terms of a term system. With increasing number of electrons, B-like ions are the first that feature fine structure in the ground term ($1s^2 2s^2 2p\ ^2P^o$). The dominant transition amplitude for the $j = 1/2 - 3/2$ transition is $M1$, with an $E2$ admixture on the order of 1%. This spectral line

has been observed in many dilute plasmas. The low density of the plasma is necessary for making the line visible; otherwise, the (rather long-lived) upper level might be not only collisionally excited but also depleted, possibly faster than by the radiative decay. In fact, this level is the first excited level, and thus its decay is a clear sign that the respective charge state of a given element is present in the plasma. If one can evaluate the line intensity or the relative intensities (line ratios) of specific other transitions (see [48] and references therein) that vary with the relative populations of the ground term fine structure levels, this serves as a density diagnostic. The wavelengths of these fine structure transitions (in particular in B- and F-like ions, but also in analogous atomic spectra) have been measured for many elements with high accuracy, much better than theory can be expected to achieve. In the following case, the transition rate is of primary interest for fundamental physics reasons. This has come as a surprise, since very few atomic lifetime measurements have reached a precision well below 1%, nor have the computations of transition rates.

A progression of lifetime measurements has addressed the fine structure transition in the ground term of the Ar^{13+} ion, beginning with the electrostatic (Kingdon) ion trap at Reno (NV, USA) [49] and followed by the electron beam ion traps (EBIT) at Livermore (CA, USA) [50]. It was an achievement to measure atomic lifetimes on the order of 10 ms with a precision of a few percent. The Heidelberg (D) EBIT group continued on this path and reached a new domain, 0.05% [51,52] (which has not yet been duplicated elsewhere). This jump in error reduction immediately raised the interest of theorists. For example, Walter Johnson was visiting Heidelberg at the time and remembered a seemingly forgotten textbook of many decades earlier in which a correction for the electron anomalous magnetic moment (EAMM) had been proposed, a 0.45% correction to the $M1$ transition moment (for details, see [53]). This QED correction is considered to be at the fringe of the Standard Model. A correction like that would usually be expected to require a much higher measurement precision. Here the experiment seemed to test this effect with an experimental result already ten times more precise than the amount of the effect itself. Wouldn't that be great? Unfortunately, QED undoubtedly is highly precise, but the QED effects are added to the predictions by non-QED many-body quantum mechanics. The various QM predictions available at the time scattered by some 5% of the total, which is much too poor to judge the EAMM contribution. Of course, the local theory experts have tried to improve on the precision of the QM result, but they have not yet reached the necessary range of less than 0.5%. Here is a key point in the legacy of both CFF and IPG: how to significantly improve on the predictive power of computations for a many-body problem.

The practically stationary cloud of trapped ions in an EBIT has proven to provide a clean and controlled environment for precision measurements. Further experiments are not limited to Ar ions, but Ar is easy to handle, the spectral range (the visible spectrum) is technically convenient, as is the lifetime range near 10 ms. The B-like term system seems not to harbour particular obstacles to precision lifetime measurements, whereas the seemingly so similar Al-like ions do, as I will discuss next.

2.5. Long-Lived 3d Levels-Indirect Influences and Direct Measurements

About half a century ago, beam-foil spectroscopy began measuring atomic lifetimes on the order of nanoseconds, and my own (later) measurements on levels with picosecond lifetimes addressed short-lived levels near the practical limit of the technique. Then, I recognised that in many term systems there are longer-lived levels than could be measured at the time, and apparently they were not even discussed in the literature. Brian Fawcett kindly ran the HXR code for me and provided theoretical evidence for such long-lived levels in the same multiply charged Si ion spectra as I worked on [54]. At the time this was just a conceptual confirmation of long-lived cascades to watch out for, as the computation

only comprised $E1$ transitions. Incidentally, there were a number of 3d levels involved in terms that had n = 3–3 decay channels only, which in later steps would repopulate the n = 2 levels I observed. Over the decades, my experience added slow ion beams (and a lifetime measurement of about 200 ns [46], on an $M1$ transition in Ar^{16+}) and ion traps from a radiofrequency ion trap to a heavy-ion storage ring and to electron beam ion traps (see samples and references in [55–58]). The lifetimes to be measured comprised the order of a few milliseconds up to a minute. Other people managed to store an ion beam for hours [59]. Eventually, I have been discussing schemes for measuring multi-hour atomic lifetimes, while astrophysics would like to have reliable information on even longer atomic lifetimes, in the range of days or months for the density diagnostics of planetary nebulae [60]. At the same time, a level lifetime of more than a year has been measured in a tabletop ion trap experiment [61]. This subsection is planned as a quick guide to some aspects of this field.

Fe ions are rather abundant in the solar corona. Consequently, research on ions with an open $n = 2$ shell has been expanded to Fe ions with an open $n = 3$ shell, profiting from a fair number of similarities with the former. For example, the Al-like ion Fe^{13+} has a $3s^2 3p\ ^2P^o$ ground term that resembles the $2s^2 2p\ ^2P^o$ in the B-like ion Ar^{13+}. The $E1$-forbidden fine structure transition is also in the visible spectral range, and the level lifetimes are similar within a factor of two. The lifetime measurements have been pursued in the same ion traps in the same laboratories [33,62], yet the mutual agreement of the results is somewhat poorer and has instigated further thought whether the particle density might play a role and whether other levels might have to be taken into account. Indeed, it was found that in the 3s3p3d configuration a not very high lying $^4F^o_{9/2}$ level exists that has no $E1$ decay channels, but an $M1$ decay channel to another level of the same term. Incidentally, the level lifetime of the Fe XIV 3s3p3d $^4F^o_{9/2}$ level is in the millisecond range, very close to that of the $3s^2 3p\ ^2P^o_{3/2}$ level in the ground term. All other levels of these configurations are some six orders of magnitude shorter lived.

This particular long-lived level may be the origin of a cascade that could (slightly) spoil the evaluation of the decay curve of the ground term fine structure transition—if the upper level is sufficiently populated at all. The level and its decay have not yet been observed directly, but in Fe ions with additional 3d electrons, such cascade tails have been recorded in experiments at a heavy-ion storage ring. The origin of the 3d level population in those experiments is the production and excitation of the ions of interest in the stripper terminal of a tandem accelerator that provides the fast ion beam to be stored in the heavy-ion storage ring. Atomic structure computations meanwhile predict one to about eight such long-lived 3d levels in each of the Al- through Ar-like ions. Their cascade tails are prominently seen in the decay curves of 3p levels, but there are too many cascade components to disentangle the decay curves by a multi-exponential analysis. Before the 3d level decays can be measured directly, they will have to be observed and identified. These tasks are awaiting help from theory to predict the wavelengths with sufficient reliability (somewhere in the infrared or the EUV range). Moreover, only then can detection systems be planned that eventually will be able to record the 3d level decay curves. Problems lie in the fact that the 3p–3d transitions spread out over a wide spectral range. At heavy-ion storage rings, matters of the large light source extension and of cumbersome geometry make it difficult to provide a sufficiently large solid angle of detection. New ideas for optical systems and relatively large, low-noise, position-sensitive (or tunable) UV/vis/NIR detectors are called for (and some are in the making in rather different fields of applied physics). An alternative might be the search for the 3d decays in electron beam ion traps, and the first reports on such work have recently been published. Precise lifetime measurements on those 3d levels are some way off, but improved atomic structure computations guided by experimental wavelength data would be able to yield good transition rate estimates.

In Ar-like ions, the ground state has a $3s^2 3p^6$ closed shell configuration, which entails that most (relatively low-lying) singly excited levels will be 3d levels. Of these, only a few $J = 1$ levels can decay by $E1$ transition to the $^1S_0$ ground level; their level lifetimes are in the nanosecond range. All other transitions in the first excited configuration are $E1$-forbidden, and thus their upper levels are long lived, with typical lifetimes in the millisecond range. Half a century ago, Wagner & House [63,64] computed spectra and transition rates, and a number of the transitions have since been identified in spectra of the solar corona and at electron beam ion traps. However, in spite of the CHIANTI project collecting and computing data on the solar corona, the knowledge is far from complete.

In the subsequent isoelectronic sequences (K, Ca, Sc, and so on), the ground configurations all feature open 3d shell levels. Thus, there are only $E1$-forbidden transitions, and the typical level lifetimes (for the first few ionization stages) are in the millisecond to second range. The wavelengths range from the infrared to the vacuum-UV. In short, there are dozens to hundreds of lines without clearly different timing properties. Only with very good computations can a sensible spectrum simulation be achieved that enables line identifications. The FERRUM project at Stockholm (S) [65–70] has addressed a number of singly charged ions of the iron group for their level lifetimes. Laser light was used to excite trapped ions from some long-lived level to a short-lived one, the decay of which could then be monitored as a signal of the remaining level population. Within several years, a handful of lines could be measured. This was a significant achievement, but a larger fraction of the ground configuration levels would provide a more representative sample of the predicted level lifetimes as well as a consistency check. However, the CRYRING heavy-ion storage ring, at which the Stockholm experiments have taken place, has moved to GSI Darmstadt (D) (and is operational there). New experiment designs and devices will be necessary.

For producing relatively low charge state ions of the elements of the iron group, electron cyclotron resonance ion sources or compact electron beam ion traps should be sufficient. The long lifetimes to be measured call for an ion storage vessel with ultrahigh vacuum. Long ion lifetimes usually go along with very low data rates. It is rather easy to suppress noise pulses in an X-ray detector, but the VUV/vis/NIR photons expected for the above low charge state ions result in detector signals that are very difficult to distinguish from mostly thermal noise. The recent improvement of multi-pixel NIR detectors for space telescopes (such as *James Webb)* feeds hopes that suitable detectors for the laboratory (preferably large area, finely structured, single-photon detection, fast-time resolution, affordable) can be developed as well.

2.6. Ions in Very Low Charge States

Over the last 150 years, the development of light sources has begun with flames for the study of atoms, electric arc and spark discharges for multiply charged ions, plasma discharges, laser-produced plasmas, energetic electron and ion beams for ever more highly ionised atoms in order to magnify hyperfine splittings and QED effects. Heavy elements, highly charged ions, relativistic effects all seem to be more on the IPG side than on the CFF side. However, the matter of the universe as we know it is largely in a plasma state (that is what plasma physicists like to boast). The dominant contribution to astrophysical spectra is from singly charged ions of many elements (and our environment is dominated by neutral atoms . . .). In neutral atoms and singly charged ions, the central Coulomb field is not much stronger than the electron-electron interaction, and thus not dominating the overall symmetries. This complication challenges any type of atomic structure computation. Moreover, the universe has time, much more time than the life span of any human researcher. The cooling of a planetary nebula may take centuries, millennia, whatever; collision rates may be once every few hours or days or weeks—but the detrimental collisions in laboratory

plasmas will happen much more frequently. In short, planetary nebulae (remainders of catastrophic stellar events) are being eagerly observed, and their spectra dominated by a few features are originally attributed to the hypothetical element nebulium. A hundred years ago, Ira S. Bowen explained that the nebulium hypothesis was no longer needed because the real emitter was ordinary matter in a very-low density environment, much lower in particle density than any vacuum on Earth [71]. Then astrophysicists wanted to exploit the line ratios of the prominent spectral features as a density diagnostic, and they did, based on calculated transition rates. Alas, the rates obtained by different atomic theory approaches differed from each other. In some tests, a 10% uncertainty of the rates resulted in a 50% uncertainty of the deduced densities.

The case actually is somewhat more complicated. In the 1930s, line ratios were "somehow" estimated [72,73]. In the 1940s, Pasternack [74] demonstrated how Racah algebra (6j symbols and the like) might yield more specific line ratios. Since the 1960s, Czyzak et al. [75,76] have discussed the line ratio physics and various applications to density or temperature diagnostics. Theoreticians offered better atomic structure computations—and the results for a given atomic system scattered by up to 50% (see [60,77]). Of course one may claim that this or that corrective effort will surely reduce the scatter, and one may want to prefer results that agree with others—until one recognises that some publications have only cited others. Some astronomers have sought help in heaven, trying to deduce from sky objects which line ratios might be correct (see the literature collection in [78]). In some well-researched cases, N II and O III, the line ratios appear to scatter by only a few percent. However, the various transition rates (from which the line ratios are calculated by simple division) at the same time scatter by 10 to 20%. Why? Indications of problems with the radial wave functions, perhaps? Anyway, these examples concern line ratios with an identical upper level, that are not affected by the environment. (In the astrophysical observations, differential effects such as extinction may slightly modify the observed line ratios from the produced ones, over cosmologically relevant distances. Nevertheless, the data scatter persists.) If these level decay branching ratios can be computed, but not the transition rates, something is lacking. And this is only a test on the way towards eventually applying line ratios for plasma diagnostics that originate from different levels (the population of which depends on density and temperature). The application has been established more than half a century ago, but a quality measure provided by atomic structure theory is lacking.

I am not aware of any accurate laboratory measurements of such line ratios of purely atomic physics origin. What ought to be measured is also the transition rate, which in singly charged ions may well correspond to hours or days. If one could store the ions of interest for weeks and months without perturbation (in extremely high vacuum), optical observations of the decays would still suffer from a small solid angle of detection, and from the dark rate of typical detectors for visible or infrared light. In order to reach a meaningful statistical reliability of the detector signal (counting statistics, signal-to-noise) one would have to provide for data accumulation on the scale of months for a single decay curve [60]. A significant improvement in detection efficiency and thus in the time needed for data accumulation may be expected from a detection that does not involve capturing the occasionally emitted photons, but from weighing the (single!) trapped ion and in this way monitoring whether it is in its excited or in the ground state. If the decay has taken place, a new excitation can be initiated. Recent experiments at the Heidelberg Pentatrap, a highly sophisticated multiple Penning ion trap, have reached the necessary mass discrimination to "see" the total energy difference as a mass difference [79–81]. For the most urgently wanted astrophysics case of S II (S^+ ions), with level lifetimes of half an hour and an hour and a half, the necessary mass resolution is $\Delta m/m \approx 10^{-10}$. Who would have thought that $E = mc^2$ resurfaces in the context of discussing CFF's and IPG's legacy?

3. Discussion

What I have called theory is basically the translation of atomic structure theory into a numerical recipe for a machine to crunch numbers on, while true "Theory" is much loftier. My experimental work is at the receiving end of that practical process called atomic structure theory. If the reliability of numerical predictions by atomic structure theory computations can be improved, this might mean an improvement in one or more aspects of theory as well as better planning conditions for future measurements. If the astrophysicists get access to more reliable computations, they can more specifically interpret their observations of far-away locations and phenomena. The goal is physics insight, and theory, experiment, and astrophysical observation and interpretation are mere facets of the same.

The aforementioned examples illustrate atomic spectroscopy topics that ought to be pursued at the high level of atomic structure computational insight, which has been reached by and under the guidance of Charlotte Froese Fischer and Ian Philip Grant. CFF has developed and applied a variety of atomic structure computer packages, and the Grant code has mutated through a series of versions such as GRASP, GRASP90, GRASP2K, and their derivatives. As an experimenter I am impressed by how extremely well the predictions of EUV wavelengths of S-like ions of iron group elements [82] match observation, in part to better than 0.1%. I have eagerly looked at the predictions, some by the same authors using the same atomic structure program packages, of the neighbouring isoelectronic sequences—and "reality struck again": deviations between computation and observation lie in the 1% range for some spectral ranges, and near 3% for others. Also, the levels in the ground shell and in higher ones are predicted with different deviations from measurement.

Evidently there are brilliant successes, but for each isoelectronic sequence and spectrum, the status of achievement has to be checked again. The state of the art has not yet reached the level of "have that expert use this atomic structure package, and their ab initio computations will guide you through your measured spectra". No, experiment has many wide data gaps to fill or bridge, and it cannot (yet) rely on theory to provide a "push button" check on internal consistency and data quality. Correspondingly, for the ubiquitous spectral lines and line ratios associated with ions in very low charge states, theory offers almost complete information for many diagnostic purposes, but experiment will be struggling for quite a while before it can filter out which of the predictions should have been relied on, and which better not. The above percentages refer to atomic energy levels and to some differences between them. Transition rates are usually (much) less precisely predicted and, alas, measured. CFF and IPG have done well, but there is so much more to do!

Funding: This research received no external funding. The APC was partly covered by Ruhr-Universität Bochum.

Data Availability Statement: All the data presented are from the open literature cited.

Acknowledgments: Over the years and decades, I have profited again and again from help and advice by capable theorists.

Conflicts of Interest: The author declares no conflicts of interest.

References

1. Hibbert, A. Charlotte Froese Fischer—Her work and her impact. *Atoms* **2019**, *7*, 107. [CrossRef]
2. Hibbert, A.; Godefroid, M.; Brage, T.; Jönsson, P.; Bartschat, K.; Drake, G. Charlotte Froese Fischer (1929–2024): Her life in and contributions to science. *Phys. Can.* **2025**, *80*, 31–38.
3. Grant, I.P.; Del Zanna, G. Dirac's equation, quantum electrodynamics, atomic and molecular structure. *Phil. Mag.* **2026**, *106*, 1–17. [CrossRef]
4. Del Zanna, G. Ian Philip Grant (1930–2025): A legacy in relativistic atomic physics. *Atoms* **2026**, *14*, 45. [CrossRef]

5. Kay, L. A van de Graaff beam as a source of atomic emission spectra. *Phys. Lett.* **1963**, *5*, 36–37. [CrossRef]
6. Bashkin, S.; Meinel, A.B. Laboratory excitation of the emission spectrum of a Nova. *Astrophys. J.* **1964**, *139*, 413–416. [CrossRef] [PubMed]
7. Bashkin, S. Optical spectroscopy with van de Graaff accelerators. *Nucl. Instrum. Meth. Phys. Res.* **1964**, *28*, 88–96. [CrossRef]
8. Striganov, A.R.; Sventitskij, N.S. *Tables of Spectral Lines of Neutral and Ionized Atoms*; Atomizdat, IFI/Plenum: New York, NY, USA, 1966.
9. Bashkin, S.; Stoner, J.O., Jr. *Atomic Energy Levels and Grotrian Diagrams: Hydrogen I-Phosphorus XV*; North-Holland: Amsterdam, The Netherlands, 1975.
10. Kelly, R.L.; Palumbo, L.J. *Atomic and Ionic Emission Lines Below 2000 Å, Hydrogen through Krypton*; NRL: Washington, DC, USA, 1978.
11. Kay, L. On the origins of beam-foil spectroscopy-I. *Nucl. Instrum. Meth. Phys. Res. B* **1985**, *9*, 544–545. [CrossRef]
12. Bashkin, S. On the origins of beam-foil spectroscopy-II. *Nucl. Instrum. Meth. Phys. Res. B* **1985**, *9*, 546–548. [CrossRef]
13. Hannebauer, F.; von Buttlar, H.; Heckmann, P.H. The quartet term system of doubly excited O VI. *Phys. Scr.* **1978**, *17*, 479–482. [CrossRef]
14. O'Brien, R.; Silver, J.D.; Jelley, N.A.; Bashkin, S.; Träbert, E.; Heckmann, P.H. Observations of 1s2s ^{3}S–1s2p ^{3}P transitions in helium-like Si^{12+}. *J. Phys. B At. Mol. Phys.* **1979**, *12*, L41–L44.
15. Ermolaev, A.M.; Jones, M. A perturbation analysis of current methods of calculating relativistic energies. *J. Phys. B At. Mol. Opt. Phys.* **1973**, *6*, 1–22. [CrossRef]
16. Cheng, K.T.; Kim, Y.-K.; Desclaux, J.P. Electric dipole, quadrupole, and magnetic dipole transition probabilities of ions isoelectronic to the first-row atoms, Li through F. *At. Data Nucl. Data Tables* **1979**, *24*, 111–189. [CrossRef]
17. Hata, J.; Grant, I.P. Fine structure of the 1s2s2p ^{4}P^o and 1s2p^2 ^{4}P terms of the lithium isoelectronic sequence and wavelengths of transitions between them. *J. Phys. B At. Mol. Phys.* **1983**, *16*, 915–925. [CrossRef]
18. Hata, J.; Grant, I.P. The 1s2s2p^2 ^{5}P-1s2p^3 ^{5}S transitions in the beryllium isoelectronic sequence. *J. Phys. B At. Mol. Phys.* **1983**, *16*, L125–L128. [CrossRef]
19. Kramida, A. Cowan code: 50 years of growing impact on atomic physics. *Atoms* **2019**, *7*, 64. [CrossRef]
20. Froese Fischer, C.; Tachiev, G. Breit-Pauli energy levels, lifetimes, and transition probabilities for the beryllium-like to neon-like sequences. *At. Data Nucl. Data Tables* **2004**, *87*, 1–184. [CrossRef]
21. Froese Fischer, C.; Tachiev, G.; Irimia, A. Relativistic energy levels, lifetimes, and transition probabilities for the sodium-like to argon-like sequences. *At. Data Nucl. Data Tables* **2006**, *92*, 607–812. [CrossRef]
22. Fawcett, B.C. Wavelengths and classifications of emission lines due to 2s^{2}2p^n–2s2p^{n+1} and 2s2p^n–2p^{n+1} transitions, $Z \leq 28$. *At. Data Nucl. Data Tables* **1975**, *24*, 135–164.
23. Fawcett, B.C. Theoretical oscillator strengths for 2s22p^n–2s2p^{n+1} and 2s2p^{n+1}–2p^{n+2} transitions and for 2s^{2}2p^n "forbidden" transitions, Be I, B I, C I, N I, O I series, $Z \leq 26$. *At. Data Nucl. Data Tables* **1978**, *22*, 473–489. [CrossRef]
24. Fawcett, B.C. Calculated oscillator strengths and wavelengths for allowed transitions within the third shell for ions in the Al-like isoelectronic sequence between Cl V and Ni XVI. *At. Data Nucl. Data Tables* **1983**, *28*, 557–578. [CrossRef]
25. Carlsson, J.; Sturesson, L. Accurate time-resolved laser spectroscopy on lithium atoms. *Z. Phys. D* **1989**, *14*, 281–287. [CrossRef]
26. Gaupp, A.; Kuske, P.; Andrä, H.J. Accurate lifetime measurements of the lowest $^2P_{1/2}$ states in neutral lithium and sodium. *Phys. Rev. A* **1982**, *26*, 3351–3359. [CrossRef]
27. Volz, U.; Majerus, M.; Liebel, H.; Schmitt, A.; Schmoranzer, H. Precision lifetime measurements on Na I 3p $^2P_{1/2}$ and 3p $^2P_{3/2}$ by beam-gas-laser spectroscopy. *Phys. Rev. Lett.* **1996**, *76*, 2862–2865. [PubMed]
28. McAlexander, W.I.; Abraham, E.R.I.; Ritchie, N.W.M.; Williams, C.J.; Stoof, H.T.C.; Hulet, R.G. Precise atomic radiative lifetime via photoassociative spectroscopy of ultracold lithium. *Phys. Rev. A* **1995**, *51*, R871–R874. [CrossRef] [PubMed]
29. McAlexander, W.I.; Abraham, E.R.I.; Hulet, R.G. Radiative lifetime of the ^{2}P state of lithium. *Phys. Rev. A* **1996**, *54*, R5–R8. [CrossRef] [PubMed]
30. Fischer, C.F.; Liu, B. Energy levels, lifetimes, wavelengths, and transition probabilities of Fe XIV. *At. Data Nucl. Data Tables* **1986**, *34*, 261–277. [CrossRef]
31. Bhatia, A.K.; Kastner, S.O. Atomic data for a five-configurational model of Fe XIV. *J. Quant. Spect. Rad. Transf.* **1993**, *49*, 609–625. [CrossRef]
32. Bhatia, A.K.; Doschek, G.A. Atomic data and spectral line intensities for Fe X. *At. Data Nucl. Data Tables* **1995**, *60*, 97–143. [CrossRef]
33. Träbert, E. On the transition rates of the Fe X and Fe XIV coronal lines. *Astron. Astrophys.* **2004**, *415*, L39–L42. [CrossRef]
34. Edlén, B. Atomic Spectra. In *Handbuch der Physik*; Flügge, S., Ed.; Springer: Berlin, Germany, 1964; Volume 27, pp. 80–220.
35. Träbert, E.; Doerfert, J.; Granzow, J.; Büttner, R.; Brauckhoff, J.; Nicolai, M.; Schartner, K.-H.; Folkmann, F.; Mokler, P.-H. Lifetimes of $n = 2$ levels in foil-excited Kr^{32+} and Kr^{33+} ions. *Phys. Lett. A* **1995**, *202*, 91–98. [CrossRef]
36. Träbert, E.; Curtis, L.J. Isoelectronic trends of line strength data in the Li and Be isoelectronic sequences. *Phys. Scr.* **2006**, *74*, C46–C54. [CrossRef]

37. Bennett, W.R., Jr.; Kindlmann, P.J. Radiative and collision-induced relaxation of atomic states in the $2p^5 3p$ configuration of neon. *Phys. Rev.* **1966**, *149*, 38–51. [CrossRef]
38. Ansbacher, W.; Li, Y.; Pinnington, E.H. Precision lifetime measurement for the 3p levels of Mg II using frequency-doubled laser radiation to excite a fast ion beam. *Phys. Lett. A* **1989**, *139*, 165–169. [CrossRef]
39. Kramida, A.; Ralchenko, Y.; Reader, J.; NIST ASD Team. *NIST Atomic Spectra Database*; Version 5.7.1; National Institute of Standards and Technology: Gaithersburg, MD, USA, 2019. Available online: https://physics.nist.gov/asd (accessed on 1 June 2024).
40. Doerfert, J.S.; Träbert, E.; Wolf, A.; Schwalm, D.; Uwira, O. Precision measurement of the electric dipole intercombination rate in C^{2+}. *Phys. Rev. Lett.* **1997**, *78*, 4355–4358. [CrossRef]
41. Chen, M.H.; Cheng, K.T.; Johnson, W.R. Large-scale relativistic configuration-interaction computation of the $2s^2\ ^1S_0$–$2s2p\ ^3P_1$ intercombination transition in C III. *Phys. Rev.* A **2001**, *64*, 042507.
42. Möller, G.; Träbert, E.; Lodwig, V.; Wagner, C.; Heckmann, P.H.; Blanke, J.H.; Livingston, A.E.; Mokler, P.H. Experimental transition probability for the *E*1 intercombination transition in Be-like Xe^{50+}. *Z. Phys. D-At. Mol. Clust.* **1989**, *11*, 333–334. [CrossRef]
43. Kimura, N.; Miya, Y.; Ito, D.; Priti; Kato, D.; Baba, M.; Kuma, S.; Azuma, T.; Nakamura, N. Laboratory transition-rate measurement of the coronal intercombination line of Ar XV by time-resolved laser spectroscopy. *Astrophys. J.* **2024**, *972*, 12. [CrossRef]
44. Jönsson, P.; Froese Fischer, C.; Träbert, E. On the status and perspectives of MCDF calculations and measurements of transition data in the Be isoelectronic sequence. *J. Phys. B At. Mol. Opt. Phys.* **1998**, *31*, 3497–3511. [CrossRef]
45. Safronova, U.I.; Shlyaptseva, A.; Kato, T.; Masai, K.; Vainshtein, L. Cross sections and rate coefficients for excitation of $\Delta n = 0$ transitions in Be-like ions with $6 \leq Z \leq 54$. *At. Data Nucl. Data Tables* **1995**, *60*, 1–36. [CrossRef]
46. Hubricht, G.; Träbert, E. The Ar^{16+} $2\ ^3S_1$ lifetime from a measurement on recoil ions. *Z. Phys. D-At. Mol. Clust.* **1987**, *7*, 243–250. [CrossRef]
47. Jitrik, O.; Bunge, C.F. Transition probabilities for hydrogen-like atoms. *J. Phys. Chem. Ref. Data* **2004**, *33*, 1059–1070. [CrossRef]
48. Chen, H.; Beiersdorfer, P.; Gu, M.-F.; Heeter, L.A.; Lepson, J.K.; Liedahl, D.A.; Naranjo-Rivera, K.L.; Träbert, E. Experimental and theoretical evaluation of density-sensitive N VI, Ar XIV, and Fe XXII line ratios. *Astrophys. J.* **2004**, *611*, 598–604.
49. Moehs, D.P.; Church, D.A. Magnetic dipole transition rates from measured lifetimes of levels of Be-like and B-like argon ions. *Phys. Rev. A* **1998**, *58*, 1111–1114. [CrossRef]
50. Träbert, E.; Beiersdorfer, P.; Utter, S.B.; Brown, G.V.; Chen, H.; Harris, C.L.; Neill, P.A.; Savin, D.W.; Smith, A.J. Experimental M1 transition rates of coronal lines from Ar X, Ar XIV, and Ar XV. *Astrophys. J.* **2000**, *541*, 506–511.
51. Lapierre, A.; Jentschura, U.D.; Crespo López-Urrutia, J.R.; Braun, J.; Brenner, G.; Bruhns, H.; Fischer, D.; González Martínez, A.J.; Harman, Z.; Johnson, W.R.; et al. Relativistic electron correlation, quantum electrodynamics, and the lifetime of the $1s^2 2s^2 2p\ ^2P^o_{3/2}$ level in boronlike argon. *Phys. Rev. Lett.* **2005**, *95*, 183001. [CrossRef] [PubMed]
52. Lapierre, A.; Crespo López-Urrutia, J.R.; Braun, J.; Brenner, G.; Bruhns, H.; Fischer, D.; González-Martínez, A.J.; Mironov, V.; Osborne, C.J.; Sikler, G.; et al. Lifetime measurement of the Ar XIV $1s^2 2s^2 2p\ ^2P^o_{3/2}$ metastable level at the Heidelberg electron-beam ion trap. *Phys. Rev. A* **2006**, *73*, 052507. [CrossRef]
53. Nemouchi, M.; Godefroid, M.R. Irreducible tensor form of the relativistic corrections to the *M*1 transition operator. *J. Phys. B At. Mol. Opt. Phys.* **2009**, *42*, 175002. [CrossRef]
54. Träbert, E. Slow out.of-shell cascades in few-electron heavy ions. *Phys. Scr.* **1981**, *23*, 253–259. [CrossRef]
55. Träbert, E. Atomic lifetime measurements with ion traps of many sizes. *Phys. Scr.* **2000**, *61*, 257–286. [CrossRef]
56. Träbert, E. Precise measurements of long atomic lifetimes using stored ion beams and ion traps. *Can. J. Phys.* **2002**, *80*, 1481–1503. [CrossRef]
57. Mannervik, S. Experimental lifetime studies of metastable levels. *Phys. Scr. T* **2003**, *105*, 67–75.
58. Träbert, E. Atomic lifetime measurements using electron beam ion traps. *Can. J. Phys.* **2008**, *86*, 73–97. [CrossRef]
59. Bäckström, E.; Hanstorp, D.; Hole, O.M.; Kamińska, M.; Nascimento, R.F.; Blom, M.; Björkhage, M.; Källberg, A.; Löfgren, P.; Reinhed, P.; et al. Storing keV negative ions for an hour: The lifetime of the metastable $^2P^o_{1/2}$ level in $^{32}S^-$. *Phys. Rev. Lett.* **2015**, *114*, 143001. [CrossRef] [PubMed]
60. Träbert, E. A laboratory astrophysics problem: The lifetime of very long-lived levels in low-charge ions. *Atoms* **2020**, *8*, 21. [CrossRef]
61. Lange, R.; Peshkov, A.A.; Huntemann, N.; Tamm, C.; Surzhykov, A.; Peik, E. Lifetime of the $^2F_{7/2}$ level in Yb^+ for spontaneous emission of electric octupole radiation. *Phys. Rev. Lett.* **2021**, *127*, 213001. [CrossRef] [PubMed]
62. Brenner, G.; Crespo López-Urrutia, J.R.; Harman, Z.; Mokler, P.H.; Ullrich, J. Lifetime determination of the Fe XIV $3s^2 3p\ ^2P^o$ metastable level. *Phys. Rev.* A **2007**, *75*, 032504. [CrossRef]
63. Wagner, W.J.; House, L.L. Hartree-Fock calculations of coronal forbidden lines in the argon I isoelectronic sequence. *Astrophys. J.* **1969**, *155*, 677–686. [CrossRef] [PubMed]
64. Wagner, W.J.; House, L.L. Empirically corrected calculations of coronal visible lines from the $3p^5 3d$ configuration. *Astrophys. J.* **1971**, *166*, 683–698. [CrossRef]

65. Mannervik, S. Studies of metastable levels in singly charged ions by laser techniques in an ion storage ring. *Phys. Scr. T* **2002**, *100*, 81–87.
66. Mannervik, S.; Ellmann, A.; Lundin, P.; Norlin, L.-O.; Rostohar, D.; Royen, P.; Schef, P. Lifetime measurements by laser probing in CRYRING—Recent experimental developments. *Phys. Scr. T* **2005**, *119*, 49–54. [CrossRef]
67. Rostohar, D.; Derkatch, A.; Hartman, H.; Johansson, S.; Lundberg, H.; Mannervik, S.; Norlin, L.-O.; Royen, P.; Schmitt, A. Lifetime measurements of metastable states in Fe^{+}. *Phys. Rev. Lett.* **2001**, *86*, 1466–1469. [CrossRef] [PubMed]
68. Hartman, H.; Derkatch, A.; Donnelly, M.P.; Gull, T.; Hibbert, A.; Johansson, S.; Lundberg, H.; Mannervik, S.; Norlin, L.-O.; Rostohar, D.; et al. The FERRUM Project: Experimental transition probabilities of [Fe II] and astrophysical applications. *Astron. Astrophys.* **2003**, *397*, 1143–1149. [CrossRef]
69. Hartman, H.; Rostohar, D.; Derkatch, A.; Lundin, P.; Schef, P.; Johansson, S.; Lundberg, L.; Mannervik, S.; Norlin, L.-O.; Royen, P. The FERRUM project: An extremely long radiative lifetime in Ti II measured in an ion storage ring. *J. Phys. B At. Mol. Opt. Phys.* **2003**, *36*, L197–L202. [CrossRef]
70. Lundin, P.; Gurell, J.; Mannervik, S.; Royen, P.; Norlin, L.-O.; Hartman, H.; Hibbert, A. Metastable levels in Sc II: Lifetime measurements and calculations. *Phys. Scr.* **2008**, *78*, 015301. [CrossRef]
71. Bowen, I.S. The origin of the nebular lines and the structure of the planetary nebulae. *Astrophys. J.* **1928**, *67*, 1B–15B. [CrossRef] [PubMed]
72. Stevenson, A.F. The intensities of certain nebular lines and the mean lives of atoms emitting them. *Proc. R. Soc. A* **1932**, *137*, 298–325. [CrossRef]
73. Boyce, J.C.; Menzel, D.H.; Payne, C.H. Forbidden lines in astrophysical sources. *Proc. Nat. Acad. Sci. USA* **1933**, *19*, 581–591. [CrossRef] [PubMed]
74. Pasternack, S. Transition probabilities of forbidden lines. *Astrophys. J.* **1940**, *92*, 129–155. [CrossRef] [PubMed]
75. Czyzak, S.J.; Krueger, T.K. Forbidden transition probabilities for some P, S, Cl, and A ions. *Mon. Not. R. Astron. Soc.* **1963**, *126*, 177–194. [CrossRef]
76. Czyzak, S.J.; Keyes, C.D.; Aller, L.H. Atomic structure calculations and nebular diagnostics. *Astrophys. J. Suppl.* **1986**, *61*, 159–175. [CrossRef] [PubMed]
77. Morisset, C.; Luridiana, V.; García-Roja, J.; Gómez-Llanos, V.; Bautista, M.; Mendoza, C. Atomic data assessment with PyNeb. *Atoms* **2020**, *8*, 66. [CrossRef]
78. Dreisbach, J.R.; Bomans, D.; Träbert, E. Line ratio in the C-like ion spectrum O III: Testing atomic theory predictions by observation of galaxies. *Astronomy* **2025**, *4*, 3. [CrossRef]
79. Schüssler, R.X.; Bekker, H.; Braß, M.; Cakir, H.; Crespo López-Urrutia, J.R.; Door, M.; Filianin, P.; Harman, Z.; Haverkort, M.W.; Huang, W.J.; et al. Detection of metastable electronic states by Penning-trap mass spectrometry. *Nature* **2020**, *581*, 42–46. [CrossRef] [PubMed]
80. Kromer, K.; Lyu, C.; Door, M.; Filianin, P.; Harman, Z.; Herkenhoff, J.; Indelicato, P.; Keitel, C.H.; Lange, D.; Novikov, Y.N.; et al. Observation of a low-lying metastable electronic state in highly charged lead by Penning-trap mass spectrometry. *Phys. Rev. Lett.* **2023**, *131*, 223002. [CrossRef] [PubMed]
81. Tu, B.; Si, R.; Shen, Y.; Wang, J.; Wei, B.; Chen, C.-Y.; Yao, K.; Zou, Y.-M. Experimental access to observing decay from extremely long-lived metastable electronic states via Penning trap spectrometry. *Phys. Rev. Res.* **2023**, *5*, 043014. [CrossRef]
82. Wang, K.; Song, C.X.; Jönsson, P.; Del Zanna, G.; Schiffmann, S.; Godefroid, M.; Gaigalas, G.; Zhao, X.H.; Si, R.; Chen, C.Y.; et al. Benchmarking atomic data from large-scale multiconfiguration Dirac-Hartree-Fock calculations for astrophysics: S-like Ions from Cr IX to Cu XIV. *Astrophys. J. Suppl. Ser.* **2018**, *239*, 30. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.