

Article

Diagnostics of Spin-Polarized Ions at Storage Rings

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Abstract: Polarized heavy ions in storage rings are seen as a valuable tool for a wide range of research, from the study of spin effects in relativistic atomic collisions to the tests of the Standard Model. For forthcoming experiments, several important challenges need to be addressed to work efficiently with such ions. Apart from the production and preservation of ion polarization in storage rings, its measurement is an extremely important issue. In this contribution, we employ the radiative recombination (RR) of polarized electrons into the ground state of initially hydrogen-like, finally helium-like, ions as a probe process for beam diagnostics. Our theoretical study clearly demonstrates that the RR cross section, integrated over photon emission angles, is highly sensitive to both the degree and the direction of ion polarization. Since the (integrated) cross-section measurements are well established, the proposed method offers promising prospects for ion spin tomography at storage rings.

Keywords: spin-polarized ions; radiative recombination; integrated recombination cross section



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1. Introduction

Polarization has been found to be one of the key degrees of freedom in modern research, such as atomic, optical, and nuclear physics, quantum information, and quantum communications. Several methods for the generation and diagnosis of polarized electrons, photons, and protons are now well established and successfully employed in a wide variety of fundamental and applied studies. Much less progress has been achieved so far in dealing with polarized ion beams at storage ring facilities. In recent decades, however, these beams have attracted particular theoretical attention as a potentially valuable tool for studying various relativistic and spin phenomena, the fundamental symmetries of nature [1–3], as well as for the search for new physics beyond the Standard Model [4,5]. Several proposals have been put forward for the production of spin-polarized ion beams, demanded for such studies [6,7]. Hydrogen-like ions, in which both the bound electron and the (non-zero spin) nucleus can be polarized, are of special interest here.

Whereas the production of polarized ion beams is fairly well understood, at least from the viewpoint of theory, two other strongly related issues are still under debate. In particular, further experimental and theoretical studies are needed to understand whether the produced polarization is maintained during the ion motion in magnetic fields of the

ring. A more complete understanding of the issue of polarization maintenance requires, in turn, the development of efficient methods for the *diagnostics* of the ion spin states. The radiative recombination (RR) of an electron into a bound ionic state has been found to be a promising tool for such spin diagnostics. The linear polarization of photons, emitted during the RR, is known to be very sensitive to the spin states of colliding particles [8]. However, in practice, X-ray polarization measurements are quite demanding and require the use of specially designed Compton detectors [9]. In the present paper, we propose and theoretically investigate a much simpler approach based on the analysis of the cross section for the RR of polarized electrons with polarized ions, integrated over photon emission angles. To illustrate the merits of this approach, in Section 2, we briefly recall the theory of electron capture into the $1s^2\ ^1S_0$ ground state of initially a hydrogen-like (finally helium-like) heavy ion. For simplicity, the nuclear spin is assumed to be zero in this analysis, $I = 0$, implying that the ion polarization is due to the polarization of the bound electron. By making use of the density matrix approach, we derive the integrated RR cross section which depends on the components of the polarization vectors of both the incident electron and the hydrogen-like ion (i.e., of the bound electron). Detailed calculations of this cross section for the RR of Xe^{53+} , Au^{78+} and U^{91+} ions, being initially in their ground $1s_{1/2}$ state, are presented in Section 3. The results of the calculations clearly show that the RR cross section is very sensitive to both the direction and the degree of ion polarization and that polarized electrons have to be used to implement this sensitivity. The summary of the proposed ion “spin tomography” methods is given in Section 4.

Relativistic units ($\hbar = c = m_e = 1$) are used throughout the paper.

2. Theoretical Background

2.1. Density Matrix Approach

Since density matrix theory has been extensively used in the past to describe radiative recombination [10,11], we will restrict ourselves here to a brief description of its basic expressions, which are necessary for further discussion. In our particular case, we consider the RR of an initially hydrogen-like ion, being in a magnetic substate $|1s_{1/2}\ \mu_0\rangle$, where μ_0 is a projection of the total angular momentum $j_0 = 1/2$ of a bound electron, with a free electron $|pm_s\rangle$ characterized by an asymptotic linear momentum $\mathbf{p}$, energy $\varepsilon = \sqrt{p^2 + 1}$, and spin projection m_s onto the propagation direction (i.e., helicity). By assuming, moreover, that a final helium-like ion is produced in the ground $1s^2\ ^1S_0$ state, one can derive the density matrix of recombination photons

$$\begin{aligned} \langle k\lambda | \hat{\rho}_f | k\lambda' \rangle &= \sum_{m_s m'_s \mu_0 \mu'_0} \mathcal{T}_{m_s \mu_0, \lambda} \mathcal{T}_{m'_s \mu'_0, \lambda'}^* \\ &\times \langle pm_s | \hat{\rho}_i^{(\text{el})} | pm'_s \rangle \langle 1s_{1/2}\ \mu_0 | \hat{\rho}_i^{(\text{ion})} | 1s_{1/2}\ \mu'_0 \rangle \end{aligned} \quad (1)$$

emitted with wave vector $\mathbf{k}$, energy ω and helicity $\lambda \pm 1$. In this expression, we use a short-hand notation for the (two-electron) transition matrix element

$$\mathcal{T}_{m_s \mu_0, \lambda} = \langle 1s^2\ ^1S_0 | \hat{R}_\lambda | 1s_{1/2}\ \mu_0, pm_s \rangle, \quad (2)$$

where $\hat{R}_\lambda = -e/\sqrt{2\omega(2\pi)^3} \sum_{q=1,2} \boldsymbol{\alpha}_q \boldsymbol{\epsilon}_{\mathbf{k}\lambda}^* e^{-ikr_q}$ is the electron–photon interaction operator, which is characterized by the circular polarization vector $\boldsymbol{\epsilon}_{\mathbf{k}\lambda}$, and $\boldsymbol{\alpha}_q$ is the vector of Dirac matrices; see Refs. [10–13] for further details. Within the framework of the independent

particle model (IPM), which is well justified for the analysis of the RR of high- Z ions [10], this matrix element can be rewritten:

$$\mathcal{T}_{m_s\mu_0,\lambda} = (-1)^{j_0-\mu_0} \tau_{m_s\lambda-\mu_0} \quad (3)$$

in terms of its one-electron counterpart

$$\tau_{m_s\lambda\mu_0} = -\frac{e}{\sqrt{2\omega(2\pi)^3}} \langle 1s\mu_0 | \boldsymbol{\alpha} \boldsymbol{\epsilon}_{k\lambda}^* e^{-ikr} | \boldsymbol{p}m_s \rangle \quad (4)$$

whose evaluation has been discussed in detail before [10,14].

For further evaluation of Equation (1), it is convenient to express (the elements of) density matrices of the incident electron $\hat{\rho}_i^{(\text{el})}$ and of the hydrogen-like ion $\hat{\rho}_i^{(\text{ion})}$ in terms of the corresponding statistical tensors [15–17]:

$$\langle \boldsymbol{p}m_s | \hat{\rho}_i^{(\text{el})} | \boldsymbol{p}m'_s \rangle = \sum_{k=0,1} \sum_{q=-k}^k (-1)^{1/2-m'_s} C_{1/2 m_s, 1/2 -m'_s}^{k q} \rho_{kq}^{(\text{el})} \quad (5a)$$

$$\langle 1s_{1/2} \mu_0 | \hat{\rho}_i^{(\text{ion})} | 1s_{1/2} \mu'_0 \rangle = \sum_{K=0,1} \sum_{Q=-K}^K (-1)^{1/2-\mu'_0} C_{1/2 \mu_0, 1/2 -\mu'_0}^{K Q} \rho_{KQ}^{(\text{ion})}, \quad (5b)$$

where $C_{1/2 m_s, 1/2 -m'_s}^{k q}$ and $C_{1/2 \mu_0, 1/2 -\mu'_0}^{K Q}$ are the Clebsch–Gordan coefficients. In addition to well-defined symmetry properties, a great advantage of these tensors is their direct relation to the components $(P_x^{(\text{el,ion})}, P_y^{(\text{el,ion})}, P_z^{(\text{el,ion})})$ of electron and ion polarization vectors:

$$\rho_{00}^{(\text{el,ion})} = \frac{1}{\sqrt{2}}, \quad \rho_{10}^{(\text{el,ion})} = \frac{P_z^{(\text{el,ion})}}{\sqrt{2}}, \quad \rho_{1\pm 1}^{(\text{el,ion})} = \mp \frac{1}{2} (P_x^{(\text{el,ion})} \mp i P_y^{(\text{el,ion})}). \quad (6)$$

In the following, both polarization vectors $\boldsymbol{P}^{(\text{ion})}$ and $\boldsymbol{P}^{(\text{el})}$ are defined in the ion rest frame, the z -(quantization) axis is chosen along the momentum of the incident electron $\boldsymbol{p}$ and together with the x -axis defines the reaction plane. Moreover, the degree of polarization $P^{(\text{el,ion})} = \sqrt{(P_x^{(\text{el,ion})})^2 + (P_y^{(\text{el,ion})})^2 + (P_z^{(\text{el,ion})})^2} \leq 1$ is equal to *unity* for a completely polarized beam, and $P^{(\text{el,ion})} < 1$ if the electrons or ions are partially polarized.

2.2. RR Cross Section

By making use of the photon density matrix (1), one can obtain the cross section of the radiative recombination:

$$\sigma_{\text{RR}} = \sum_{\lambda} \int \langle k\lambda | \hat{\rho}_f | k\lambda \rangle d\Omega, \quad (7)$$

where an integration over the photon emission angles and summation over the photon helicity λ need to be performed. In the following, we refer to this cross section as the *integrated* one to emphasize that it still depends on the polarization of both the initial ion and the incident electron.

To further evaluate the cross section σ_{RR} , we insert the elements of the photon (1), electron (5a), and ion (5b) density matrices into Equation (7), and express two-electron matrix elements (2) in terms of the one-electron ones:

$$\sigma_{RR} = \sum_{\lambda} \sum_{m_s m'_s \mu_0 \mu'_0} \sum_{kq KQ} \left[(-1)^{1-m'_s-\mu_0} C_{1/2 m_s, 1/2 -m'_s}^{kq} C_{1/2 \mu_0, 1/2 -\mu'_0}^{KQ} \rho_{kq}^{(el)} \rho_{KQ}^{(ion)} \right. \\ \left. \times \int \tau_{m_s \lambda - \mu_0} \tau_{m'_s \lambda - \mu'_0}^* d\Omega \right]. \quad (8)$$

As the next step, the standard multipole expansions of the electron–photon interaction operator $\hat{R}_\lambda$ as well as of the continuum electron wave function $|pm_s\rangle$ [10,14] have to be employed to rewrite the matrix elements $\tau_{m_s \lambda \mu_0}$ as

$$\tau_{m_s \lambda \mu_0} = -\frac{e}{\sqrt{4\omega(2\pi)^3 \varepsilon p}} \sum_{LM\mathfrak{p}} \sum_{\kappa} \left[i^{l-L} e^{i\Delta_\kappa} \frac{[L, l]^{1/2}}{[j]^{1/2}} (-i\lambda)^{\mathfrak{p}} \langle l0 \, 1/2 m_s | j m_s \rangle \right. \\ \left. \times \langle j_0 \mu_0 \, LM | j m_s \rangle D_{M\lambda}^{L*}(\hat{\mathbf{k}}) \langle 1s \| \mathbf{a} \mathbf{a}_L^{(\mathfrak{p})\dagger} \| \varepsilon \kappa \rangle \right]. \quad (9)$$

Here, the Wigner–Eckart theorem was used to obtain the reduced matrix elements $\langle 1s \| \mathbf{a} \mathbf{a}_L^{(\mathfrak{p})*} \| \varepsilon \kappa \rangle$ that describe the RR of a continuum electron with energy ε and Dirac’s quantum number $\kappa = \pm(j + 1/2)$ for $l = j \pm 1/2$, under the emission of an electric ($\mathfrak{p} = 1$) or magnetic ($\mathfrak{p} = 0$) photon with total angular momentum L . In Equation (9), moreover, $[L] = 2L + 1$, $\hat{\mathbf{k}} = \mathbf{k}/k$ is the propagation vector of the emitted photon, and Δ_κ is the Coulomb phase of the electron.

By inserting the multipole expansion of the matrix element (9) into Equation (8), performing integration over the photon emission angle and making some angular momentum algebra, we derive the integrated RR cross section in the form

$$\sigma_{RR} = \sum_{k=0,1} \sum_{K=0,1} \sum_{t=0,1,2} \left\{ \rho_k^{(el)} \otimes \rho_K^{(ion)} \right\}_{t0} B_{kKt}(\varepsilon, Z), \quad (10)$$

where $\left\{ \rho_k^{(el)} \otimes \rho_K^{(ion)} \right\}_{t0}$ is the (irreducible tensor) product of electron and ion statistical tensors [15,18], and the function $B_{kKt}(\varepsilon, Z)$ is given by

$$B_{kKt}(\varepsilon, Z) = \frac{e^2 \pi}{\omega(2\pi)^3 \varepsilon p} \sum_{L\mathfrak{p}} \sum_{\kappa \kappa'} \left[i^{l-l'} (-1)^{L+K-j'-l-1/2} e^{i(\Delta_\kappa - \Delta_{\kappa'})} [l, l', j, j', k, K]^{1/2} \right. \\ \times C_{l'0, l0}^{t0} \left\{ \begin{matrix} 1/2 & 1/2 & K \\ j' & j & L \end{matrix} \right\} \left\{ \begin{matrix} 1/2 & j & l \\ 1/2 & j' & l' \\ k & K & t \end{matrix} \right\} \\ \left. \times \langle 1s \| \mathbf{a} \mathbf{a}_L^{(\mathfrak{p})\dagger} \| \varepsilon \kappa \rangle \langle 1s \| \mathbf{a} \mathbf{a}_L^{(\mathfrak{p})\dagger} \| \varepsilon \kappa' \rangle^* \right]. \quad (11)$$

This function, whose calculation requires summation over the reduced RR matrix elements, depends on the electron energy ε , the ion’s nuclear charge Z , as well as on the ranks k and K of electron and ion statistical tensors.

The parity selection rules embedded in the reduced matrix elements $\langle 1s \| \mathbf{a} \mathbf{a}_L^{(\mathfrak{p})\dagger} \| \varepsilon \kappa \rangle$ and the symmetry properties of the Clebsch–Gordan coefficient and the 9-j symbol in

Equation (11) cause the functions $B_{kk't}(\varepsilon, Z)$ to be *non-zero* for $t = 0$ and 2. This allows further simplification of the integrated cross section σ_{RR} to

$$\begin{aligned} \sigma_{RR} = & \left\{ \rho_0^{(el)} \otimes \rho_0^{(ion)} \right\}_{00} B_{000}(\varepsilon, Z) \\ & + \left\{ \rho_1^{(el)} \otimes \rho_1^{(ion)} \right\}_{00} B_{110}(\varepsilon, Z) + \left\{ \rho_1^{(el)} \otimes \rho_1^{(ion)} \right\}_{20} B_{112}(\varepsilon, Z). \end{aligned} \quad (12)$$

Here, the expression on the right-hand side of the first line is obtained for zero-rank statistical tensors $\rho_{00}^{(el)} = 1/\sqrt{2}$ and $\rho_{00}^{(ion)} = 1/\sqrt{2}$, i.e., when $k = q = K = Q = 0$, and, hence, represents the total cross section for the radiative recombination of *unpolarized* electrons and ions:

$$\sigma_{RR}^{(unp)} = \frac{1}{2} B_{000}(\varepsilon, Z) = \frac{1}{4} \sum_{\lambda} \sum_{m_s \mu_0} \int |\tau_{m_s \lambda \mu_0}|^2 d\Omega, \quad (13)$$

that has been extensively discussed in the literature [3,10]. On the other side, the terms in the second line of Equation (13) contain the products of the first-rank tensors $\rho_{1q}^{(el)}$ and $\rho_{1Q}^{(ion)}$, which are directly related to the components of the electron and ion polarization vectors. In order to make this polarization dependence more obvious, we employ Equation (6) and derive the final formula for the integrated RR cross section:

$$\sigma_{RR} = \sigma_{RR}^{(unp)} + \sum_{i=x,y,z} P_i^{(el)} P_i^{(ion)} \Delta\sigma_{RR}^{(i)}. \quad (14)$$

Here, $\sigma_{RR}^{(unp)}$ is given by Equation (13), and the cross-section corrections

$$\begin{aligned} \Delta\sigma_{RR}^{(x)} &= \Delta\sigma_{RR}^{(y)} = -\frac{1}{2\sqrt{3}} \left(B_{110}(\varepsilon, Z) + \frac{B_{112}(\varepsilon, Z)}{\sqrt{2}} \right), \\ \Delta\sigma_{RR}^{(z)} &= -\frac{1}{2\sqrt{3}} \left(B_{110}(\varepsilon, Z) - \sqrt{2} B_{112}(\varepsilon, Z) \right), \end{aligned} \quad (15)$$

are expressed in terms of functions (11) and, therefore, depend on the incident electron energy and the nuclear charge of the ion.

As seen from Equations (12) and (14), the integrated RR cross section contains a term $\sigma_{RR}^{(unp)}$ that is completely independent of the ion and electron polarization, and a sum of terms proportional to the *product* of the polarization components $P_i^{(el)} P_i^{(ion)}$. There are no contributions to the cross section that are proportional to a single polarization component $P_i^{(el)}$ or $P_i^{(ion)}$. This is due to the fact that all contributions to σ_{RR} , proportional to the product of the statistical tensors of zero- and first ranks, i.e., $\rho_{1q}^{(el)} \rho_{00}^{(ion)}$ or $\rho_{00}^{(el)} \rho_{1Q}^{(ion)}$, are identically zero by virtue of the symmetry properties, discussed above.

3. Results and Discussion

It follows from Equation (14) that the capture of polarized electrons by polarized ions can lead to a modification of the cross section σ_{RR} . The effect occurs if the polarization vectors of electrons and ions are non-orthogonal to each other, i.e., when $\mathbf{P}^{(el)} \cdot \mathbf{P}^{(ion)} \neq 0$, and its strength is determined by the functions $\Delta\sigma_{RR}^{(x,y,z)}$. In Figure 1, these functions as well as the “unpolarized” total cross section $\sigma_{RR}^{(unp)}$ are displayed for the electron kinetic energies in the range from 1 keV to 110 keV and for the RR of hydrogen-like Xe^{53+} , Au^{78+} and U^{91+} ions. As seen from the figure, the corrections $\Delta\sigma_{RR}^{(i)}$ are negative, and their absolute values are rather close, although always smaller than the cross section $\sigma_{RR}^{(unp)}$ for all three ions and for the entire range of electron kinetic energies. The difference between $|\Delta\sigma_{RR}^{(x,y,z)}|$ and $\sigma_{RR}^{(unp)}$

is below 1.5% for Xe^{53+} and reaches about 5–10% for heavier ions. To understand why $\Delta\sigma_{\text{RR}}^{(i)} \approx -\sigma_{\text{RR}}^{(\text{unp})}$, we refer to the Pauli exclusion principle. Indeed, the RR of a hydrogen-like ion and an electron, having co-directed polarization $\mathbf{P}^{(\text{ion})} \uparrow\uparrow \mathbf{P}^{(\text{el})}$, is significantly suppressed since (i) the spin flipping of a recombining electron is unlikely [3,19], and (ii) a bound state with the particular spin projection is already “occupied” by a spectator electron. This suppression by the Pauli principle is reflected by Equation (14), which can be written as

$$\sigma_{\text{RR}}(P_i^{(\text{ion})} = P_i^{(\text{el})} = 1) = \sigma_{\text{RR}}^{(\text{unp})} + \Delta\sigma_{\text{RR}}^{(i)} \approx 0, \quad (16)$$

for the case $P_i^{(\text{ion})} = P_i^{(\text{el})} = 1$. A slight deviation from a zero value of the cross section σ_{RR} in Equation (16) arises from the spin-flip electron transition. Although in general, the probability of such a transition is small, it increases with the nuclear charge Z of an ion [3], thus leading to about a ten percent difference between $\sigma_{\text{RR}}^{(\text{unp})}$ and $|\Delta\sigma_{\text{RR}}^{(x,y,z)}|$ for U^{92+} .

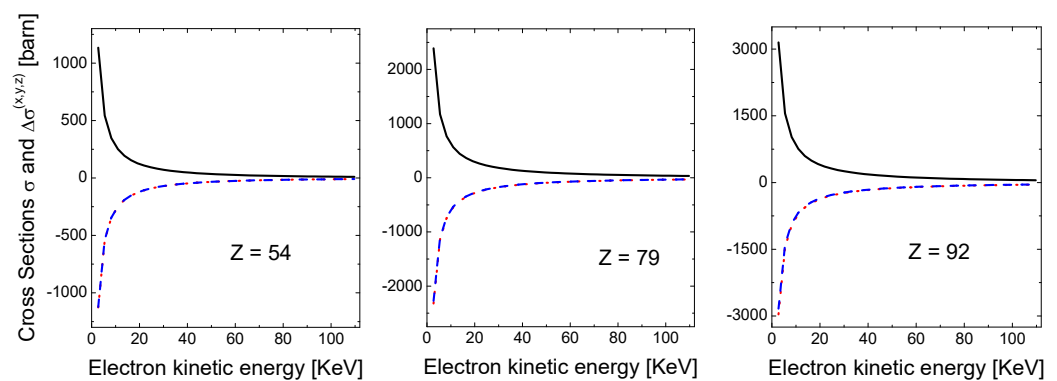


Figure 1. The total cross section $\sigma_{\text{RR}}^{(\text{unp})}$ for the radiative recombination of unpolarized electrons and ions (solid) as well as polarization corrections $\Delta\sigma_{\text{RR}}^{(x)} = \Delta\sigma_{\text{RR}}^{(y)}$ (red dotted line) and $\Delta\sigma_{\text{RR}}^{(z)}$ (blue dashed line) as functions of electron kinetic energy. The calculations were performed for the electron capture into the ground state of initially hydrogen-like xenon Xe^{53+} (left panel), gold Au^{78+} (middle panel) and uranium U^{91+} (right panel) ions.

The fact that the (absolute values of) polarization-induced corrections $\Delta\sigma_{\text{RR}}^{(x,y,z)}$ are comparable to $\sigma_{\text{RR}}^{(\text{unp})}$ makes cross-section measurements a powerful tool for ion spin diagnostics. In order to illustrate the potential of such diagnostics, we display in Figure 2 the integrated cross section (14) for the RR of initially polarized U^{91+} ions. As an example, it is assumed that ions are partially or fully polarized along the x - or z -axis. The left panel of this figure shows the results for $P_y^{(\text{ion})} = P_z^{(\text{ion})} = 0$ and $P_x^{(\text{ion})} = 0.3$ (green dash-dotted line), 0.7 (blue dashed line), 1.0 (black solid line), while the right panel shows the results for $P_x^{(\text{ion})} = P_y^{(\text{ion})} = 0$ and $P_z^{(\text{ion})} = 0.3$ (green dash-dotted line), 0.7 (blue dashed line), 1.0 (black solid line). In addition, the total “unpolarized” cross section $\sigma_{\text{RR}}^{(\text{unp})}$ is marked for reference by the horizontal dotted red line. To “read out” the ion polarization, we consider in Figure 2 incident electrons with kinetic energy $T_{\text{kin}} = 20$ keV and with the polarization vector rotating in the xz -plane as $P_x^{(\text{el})} = \sin\phi$, $P_y^{(\text{el})} = 0$, $P_z^{(\text{el})} = \cos\phi$. As seen from this figure, the RR cross section σ_{RR} is strongly sensitive to ϕ and reaches its minimum for $\phi = 0$ deg on the right panel and $\phi = 90$ deg on the left panel, i.e., when the polarization vectors of electrons and ions are co-directional, $\mathbf{P}^{(\text{ion})} \uparrow\uparrow \mathbf{P}^{(\text{el})}$. This behavior can be attributed to the Pauli exclusion principle and to the low probability of the spin-flip (free-bound) electron transition as discussed already above. If, in contrast, $\mathbf{P}^{(\text{ion})}$ and $\mathbf{P}^{(\text{el})}$ are oppositely directed, the electron can be captured without changing its spin projection that leads to the remarkable enhancement of the RR cross section σ_{RR} .

Measurements of the RR cross sections can be used to determine not only the direction but also the degree of spin polarization of hydrogen-like ions. In the most naive way, the values of the components $P_{x,y,z}^{(\text{ion})}$ and, hence, the degree of polarization, can be simply found by fitting Equation (14) to the available experimental data. A more systematic approach relies on the measurements of RR cross sections $\sigma_{\text{RR}}(P_i^{(\text{el})})$ for six different polarization states of incident electrons: $P_x^{(\text{el})} = \pm 1$, $P_y^{(\text{el})} = \pm 1$ and $P_z^{(\text{el})} = \pm 1$. From the dichroism of the pairs of corresponding cross sections, one can immediately find

$$P_i^{(\text{ion})} = \frac{1}{2} \frac{\sigma_{\text{RR}}(P_i^{(\text{el})} = +1) - \sigma_{\text{RR}}(P_i^{(\text{el})} = -1)}{\Delta\sigma_{\text{RR}}^{(i)}}, \quad i = x, y, z. \quad (17)$$

Of course, this “spin tomography” approach relies on accurate theoretical predictions for the functions $\Delta\sigma_{\text{RR}}^{(i)}$. By making use of the fully relativistic independent particle model, one can achieve an accuracy of 1–2%, which is completely sufficient for proof-of-principle tomographic studies. Even more precise calculations of the functions $\Delta\sigma_{\text{RR}}^{(i)}$ are possible employing the multi-configuration Dirac–Fock approach [10], which will be discussed in forthcoming publications.

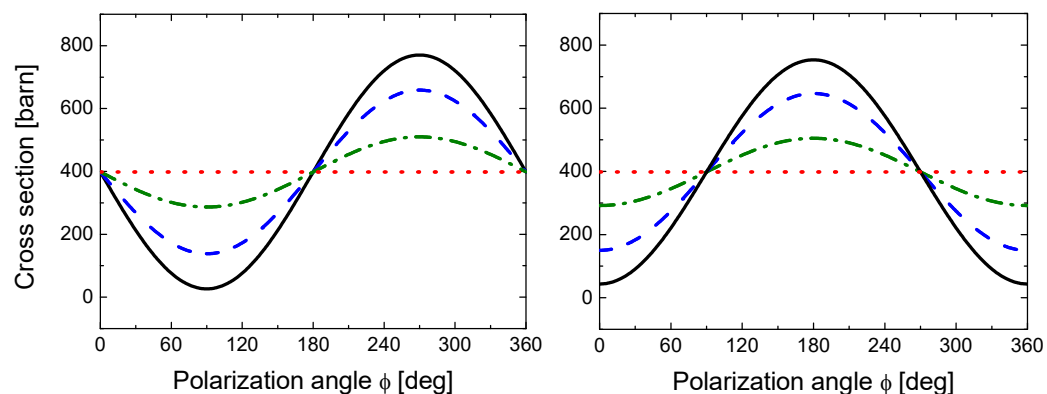


Figure 2. The (integrated) cross section σ_{RR} for the radiative recombination of polarized electrons with polarized hydrogen-like uranium ions U^{91+} , being in the $1s_{1/2}$ state. The results are presented for incident electrons with kinetic energy $T_{\text{kin}} = 20$ keV and polarization vector $\mathbf{P}^{(\text{el})} = (\sin \phi, 0, \cos \phi)$. The ions are assumed to be partially or fully polarized along either the x -axis (**left column**) or the z -axis (**right column**): $P_{x,z}^{(\text{ion})} = 0.3$ (green dash-dotted line), 0.7 (blue dashed line), 1.0 (black solid line). The red dotted line shows the total “unpolarized” cross section $\sigma_{\text{RR}}^{(\text{unp})}$.

4. Summary

The radiative recombination of polarized electrons with polarized hydrogen-like heavy ions has been studied theoretically. Particular attention has been paid to the integrated cross section for the capture of an electron into the $1s^2\ ^1S_0$ ground state of a final helium-like ion. Using the relativistic Dirac theory and the density matrix approach, we have shown that this cross section is very sensitive to the mutual orientation of the electron and the ion polarization vectors. Moreover, it varies significantly if the hydrogen-like ion is partially polarized at the time of electron capture. We argue, therefore, that cross-section measurements can provide information about both the direction and the degree of ion polarization. Such “spin tomography” of stored hydrogen-like ions is of paramount importance for planned experimental activities at GSI and FAIR facilities, aimed at searching for new physics beyond the Standard Model.

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