

Article

Electron Scattering from Superheavy Elements: Copernicium and Oganesson

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Abstract

Superheavy elements are an ideal testbed for studying relativistic, exchange, and correlation effects in scattering phenomena. In this work, we investigate electron scattering from copernicium ($Z = 112$) and oganesson ($Z = 118$) atoms. Both the relativistic Dirac and non-relativistic partial wave methods are employed to analyze the scattering dynamics, with the interaction between the projectile and target atom modeled within the framework of the optical potential approach. Our results demonstrate that relativistic, exchange, and correlation effects play a significant role in modifying the scattering cross-sections and scattering length, highlighting the influence of these interactions on the scattering processes from superheavy atomic systems. The work also attempts to identify common features of the scattering cross-section by comparing those of lighter elements in the same group.

Keywords: electron scattering; copernicium; oganesson; Dirac relativistic partial wave analysis (DRPWA)



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1. Introduction

Relativistic and correlation effects profoundly reshape the physical and chemical behavior of superheavy elements, often producing striking deviations from the trends observed in their lighter homologues [1–3]. These anomalies have motivated extensive theoretical investigations aimed at quantifying the role of relativistic and correlation effects in governing their properties [4,5]. Experimental studies are scarce, as the short half-lives of these elements preclude most direct measurements, although a few pioneering results do exist [6,7].

Among the known superheavy elements, copernicium (Cn, $Z = 112$) and oganesson (Og, $Z = 118$) stand out as the heaviest members of groups 12 and 18, respectively. Copernicium, a group-12 d-block element, was first synthesized in 1996 by bombarding a ^{208}Pb target with ^{70}Zn projectiles, resulting in the production of the isotope ^{277}Cn [8]. Among all its isotopes, ^{285}Cn is the most stable, with a half-life of approximately 29 seconds [9], long enough to allow atom-at-a-time experiments. The electronic structure of copernicium is strongly influenced by relativistic effects, which lead to the stabilization and contraction of the 7s orbital, accompanied by the destabilization and expansion of the 6d orbitals. As a result, copernicium deviates from the metallic behavior of its group 12 congeners and instead exhibits enhanced inertness, resembling a noble gas-like character [9]. A variety of theoretical methods have been used to investigate its atomic properties, including the

electric dipole polarizabilities, ionization energies, isotope shift factors, oscillator strength, atomic radius, and excitation energies of low-lying states [10–13].

While copernicium behaves more like a noble gas than a metal, oganesson (Og, $Z = 118$), a group-18 element, is predicted to deviate from the chemical inertness of its congeners [14]. Oganesson was first synthesized via the fusion reaction $^{249}\text{Cf} + ^{48}\text{Ca}$ [15], with alternative production channels such as $^{50}\text{Ti} + ^{244}\text{Cm}$ and $^{54}\text{Cr} + ^{240}\text{Pu}$ also theoretically studied [16]. Direct experimental studies of its structure are not feasible because even its most stable isotope survives for only about 0.89 ms [15]. Consequently, theoretical work provides primary insights into Og's behavior. Large spin–orbit splitting in the $7p$ shell produces a nearly uniform gas-like electronic density in the valence region, together with enhanced polarizability and Thomas–Fermi-like nucleon localization [17]. Its small predicted band gap suggests that, unlike all other noble gas solids, condensed Og may behave as a semiconductor [18].

This unusual electronic structure also motivates detailed studies of Og at the atomic level. In particular, its atomic spectrum and electric dipole transition probabilities have been examined using configuration interaction combined with perturbation theory methods, yielding predictions of excitation energies and transition strengths [19]. The electron affinity of oganesson has also been evaluated using relativistic coupled-cluster and configuration interaction methods [20].

Copernicium and oganesson represent the extremes of superheavy element behavior: a metal exhibiting noble gas-like characteristics and a noble gas showing deviations from inertness. Their anomalous properties underscore the prominent role of relativistic, exchange, and many-body correlation effects in determining the structure and dynamics of the heaviest atoms.

Although theoretical studies have extensively examined the ground- and excited-state properties of copernicium and oganesson, comparatively little effort has been devoted to their interactions with external projectiles such as electrons, positrons, or photons. An important example is the photoionization study on oganesson, which, using the relativistic random-phase approximation, demonstrated that many-electron correlations and relativistic effects strongly shape the photoionization dynamics through coupling between different photoionization channels, and this study also investigated the differences in photoionization dynamics from its lighter homologue, radon (Rn) [21]. By contrast, no studies on electron scattering from these atoms have been reported to date. Since elastic electron scattering serves as a sensitive probe of the atomic potential, such investigations could provide valuable insights into the interplay of relativistic and correlation effects in these superheavy systems. For mercury, the lighter congener of copernicium, and radon, the lighter congener of oganesson, electron and positron scattering have been extensively studied. These investigations have highlighted the prominent role of relativistic, exchange, and correlation–polarization effects in shaping scattering observables [22–26]. The results for Hg and Rn thus provide valuable benchmarks, suggesting that similar effects are expected to be even more pronounced in their superheavy analogues, Cn and Og, due to their higher nuclear charge and stronger relativistic interactions.

In this work, we investigate electron elastic scattering from copernicium and oganesson within the optical model potential framework. Both relativistic and non-relativistic treatments are employed, based on the Dirac and Schrödinger formalisms, respectively. In addition, electron scattering cross-section comparisons are carried out between copernicium and its lighter congeners of group 12 and between oganesson and its lighter congeners of group 18. These comparisons provide insight into the common features of scattering cross-sections within the same group while also highlighting deviations arising from relativistic effects. Further, a comprehensive discussion on the scattering length of these atoms

is also included. This analysis aims to clarify how relativistic, exchange, and correlation effects collectively influence the scattering dynamics of these superheavy atoms.

2. Methodology

2.1. Relativistic Case

The Dirac equation governing the motion of a projectile of mass m_0 and velocity v is expressed as

$$[c\boldsymbol{\alpha} \cdot \mathbf{p} + \beta m_0 c^2 + V(r)] \psi(\mathbf{r}) = E \psi(\mathbf{r}), \quad (1)$$

where $E = m_0 \gamma c^2 = E_i + m_0 c^2$ denotes the total energy of the projectile, with $\gamma = (1 - v^2/c^2)^{-1/2}$. Here, E_i represents the kinetic energy of the incident particle, c is the speed of light in vacuum, while $\boldsymbol{\alpha}$ and β are the 4×4 Dirac matrices. The incoming electron momentum is denoted by $\mathbf{p}$. The interaction between the projectile and the target, $V(r)$, is formulated within the framework of the optical model potential in this work. For the case of e^- -Cn/Og scattering, this optical potential can be expressed as

$$V(r) = V_{st}(r) + V_{ex}(r) + V_{cp}(r). \quad (2)$$

As the scattering process is purely elastic, Equation (2) does not contain an imaginary absorption term. The interaction potential $V(r)$ consists of three main components: the electrostatic potential $V_{st}(r)$, the exchange potential $V_{ex}(r)$, and the correlation–polarization potential $V_{cp}(r)$. The static potential $V_{st}(r)$ is constructed from the Fermi nuclear charge distribution $\rho_n(r)$ and the Dirac–Hartree–Fock (DHF) electron densities $\rho_e(r)$, which are generated using the GRASP92 package [27]. The exchange potential $V_{ex}(r)$ arises from the indistinguishability between the projectile electron and the target’s bound electrons, and it is described using the semi-classical exchange model proposed by Furness and McCarthy [28] in this work. Its functional form is given by

$$V_{ex}(r) = \frac{1}{2}[E_i - V_{st}(r)] - \frac{1}{2}\sqrt{\{E_i - V_{st}(r)\}^2 + 4\pi a_0 e^4 \rho_e(r)}, \quad (3)$$

with a_0 being the Bohr radius and e the elementary charge.

The polarization potential arises from the distortion of the target atom’s electron cloud induced by the incident electron, and it acts as an attractive interaction. In the present work, we employ a global correlation–polarization potential $V_{cp}(r)$, which combines the long-range polarization potential with the short-range correlation potential $V_{co}(r)$ derived within the local density approximation (LDA) [29]. This potential can be expressed as follows:

$$V_{cp}(r) = \begin{cases} \max \{V_{co}(r), V_{cps}(r)\} & \text{if } r < r_c \\ V_{cps}(r) & \text{if } r \geq r_c \end{cases}. \quad (4)$$

Here, r_c denotes the outer radius at which the correlation potential $V_{co}(r)$ intersects with the polarization potential $V_{cps}(r)$. The long-range correlation–polarization potential $V_{cps}(r)$ is approximated as

$$V_{cps}(r) = -\frac{\alpha_d e^2}{2(r^2 + d^2)^2}, \quad (5)$$

where α_d is the static dipole polarizability. The values used are $\alpha_d = 27.92$ a.u. for copernicium [30] and $\alpha_d = 57.98$ a.u. for oganesson [17]. The cutoff parameter d can be expressed as [31]

$$d^4 = \frac{1}{2}\alpha_d a_0 Z^{-1/3} b_{pol}^2, \quad (6)$$

where b_{pol} is an adjustable energy-dependent parameter $b_{pol}^2 = \max\{(E - 50 \text{ eV})/(16 \text{ eV}), 1\}$. The particular parameters in the expression for b_{pol} are chosen such that the expression provides electron elastic differential cross-sections (DCSs) that agree well with experimental measurements at small scattering angles, as observed in the cases of noble gases and the mercury atom [29].

The correlation potential $V_{co}(r)$ is formulated within the local density approximation (LDA), assuming that the correlation energy of a projectile at a distance r is equivalent to that in a homogeneous electron gas of density ρ_e equal to the local atomic electron density [29]. For convenience, the density parameter is introduced as

$$r_s \equiv \frac{1}{a_0} \left\{ \frac{3}{4\pi\rho_e(r)} \right\}^{1/3}. \quad (7)$$

In the case of electron scattering, the correlation potential is parameterized using the formulation of Perdew and Zunger [32], which can be expressed as

$$V_{co}^{(-)}(r) = -\frac{e^2}{a_0} (0.0311 \ln(r_s) - 0.0584 + 0.00133 r_s - 0.0084 r_s) \quad \text{for } r_s < 1 \quad (8)$$

and

$$V_{co}^{(-)}(r) = -\frac{e^2}{a_0} \beta_0 \frac{1 + (7/6) \beta_1 r_s^{1/2} + (4/3) \beta_2 r_s}{(1 + \beta_1 r_s^{1/2} + \beta_2 r_s)^2}, \quad \text{for } r_s \geq 1, \quad (9)$$

where $\beta_0 = 0.1423$, $\beta_1 = 1.0529$, and $\beta_2 = 0.3334$.

The solution of the Dirac equation can be written in terms of a four-component Dirac spinor as

$$\Psi(\mathbf{r})_{E_{\kappa m}} = \frac{1}{r} \begin{pmatrix} P_{E_{\kappa}}(r) \Omega_{\kappa m}(\hat{\mathbf{r}}) \\ i Q_{E_{\kappa}}(r) \Omega_{-\kappa m}(\hat{\mathbf{r}}) \end{pmatrix}, \quad (10)$$

where $P_{E_{\kappa}}(r)$ and $Q_{E_{\kappa}}(r)$ denote the large and small components of the radial Dirac central-field spinor, respectively, and $\Omega_{\kappa m}(\hat{\mathbf{r}})$ represents the angular component of the Dirac wavefunction. Here κ represents the relativistic angular quantum number. The coupled radial equations for the large and small components of the Dirac spinor are expressed as

$$\frac{dP_{E_{\kappa}}(r)}{dr} = \frac{E - V(r) + 2m_0c^2}{c\hbar} Q_{E_{\kappa}}(r) - \frac{\kappa}{r} P_{E_{\kappa}}(r), \quad (11)$$

$$\frac{dQ_{E_{\kappa}}(r)}{dr} = -\frac{E - V(r)}{c\hbar} P_{E_{\kappa}}(r) + \frac{\kappa}{r} Q_{E_{\kappa}}(r). \quad (12)$$

The scattering phase shift can be obtained from the asymptotic behavior of the large component of the radial Dirac spinor. In the limit $r \rightarrow \infty$, the large component $P_{E_{\kappa}}(r)$ takes the form

$$P_{E_{\kappa}}(r) \simeq \sin\left(kr - \frac{l\pi}{2} + \delta_{\kappa}(k)\right), \quad (13)$$

where $k = \sqrt{2E + \alpha^2 E^2}$ is the relativistic wave number of the projectile, with α being the fine structure constant, and $\delta_{\kappa}(k)$ denotes the scattering phase shift corresponding to a given value of κ . Since κ admits two possible values for each l , two distinct phase shifts are obtained. These give rise to two types of scattering amplitudes: the direct amplitude $f(\theta)$ and the spin-flip amplitude $g(\theta)$, which are expressed as

$$f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} [(l+1)\{\exp(2i\delta_{\kappa=-l-1}) - 1\} + l\{\exp(2i\delta_{\kappa=l}) - 1\}] P_l(\cos\theta), \quad (14)$$

and

$$g(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} [\exp(2i\delta_{\kappa=l}) - \exp(2i\delta_{\kappa=-l-1})] P_l^1(\cos\theta). \quad (15)$$

Here, $P_l(\cos\theta)$ and $P_l^1(\cos\theta)$ denote the Legendre polynomial and the associated Legendre polynomial of degree l , respectively. The DCS and the integral cross-section (ICS) can be expressed in terms of the scattering amplitudes as

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 + |g(\theta)|^2, \quad (16)$$

and

$$\sigma_{integral} = \int \frac{d\sigma}{d\Omega} d\Omega = 2\pi \int_0^\pi (|f(\theta)|^2 + |g(\theta)|^2) \sin\theta d\theta. \quad (17)$$

The electron scattering cross-sections for copernicium and oganesson are calculated using the ELSEPA package [33] in the relativistic framework.

2.2. Non-Relativistic Case

The Schrödinger equation describing the scattering state of an electron is given by

$$\left[\frac{\hbar^2}{2m} \frac{d^2}{dr^2} + \left(E - V_{total}(r) - \frac{\hbar^2 l(l+1)}{2mr^2} \right) \right] u_l(r) = 0, \quad (18)$$

where $E = \frac{k^2}{2}$ represents the kinetic energy of the incident electron, l is the orbital angular momentum quantum number, and $V_{total}(r)$ is the total potential, as defined in Equation (2), with electron density $\rho_e(r)$ calculated by solving the non-relativistic Hartree–Fock (HF) equation using the GRASP92 package [27].

Equation (18) is solved numerically using the Numerov method. In the asymptotic region ($r > r_{max}$), where the potential becomes negligible, the solution of Equation (18) reduces to a linear combination of spherical Bessel and Neumann functions:

$$u_l(r > r_{max}) \propto kr \left[\cos\delta_l j_l(kr) - \sin\delta_l n_l(kr) \right], \quad (19)$$

with δ_l denoting the scattering phase shift for the l^{th} partial wave. The scattering amplitude can be expressed in terms of the phase shift as

$$f(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin\delta_l P_l(\cos\theta), \quad (20)$$

where $P_l(\cos\theta)$ denotes the Legendre polynomial of order l and θ is the scattering angle. The DCS is then obtained as

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2, \quad (21)$$

and integration over the solid angle yields the ICS:

$$\sigma_{el} = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) \sin^2\delta_l. \quad (22)$$

3. Results and Discussion

This section presents the calculated electron scattering cross-sections for copernicium and oganesson within the optical potential framework. To assess the influence of relativistic effects, the results are obtained using both the Dirac relativistic partial wave analysis (DRPWA) and non-relativistic partial wave approximation (PWA). Henceforth in

the manuscript, DRPWA refers to the relativistic partial wave calculations and PWA to the non-relativistic partial wave calculations. Before proceeding with the scattering best to examine the differences in the electronic structures of these atoms due to these effects. Table 1 presents the valence shell energies of Cn and Og. In the non-relativistic case, no splitting occurs for the $6d$ orbital in Cn or the $7p$ orbital in Og. When spin–orbit coupling is taken into account, however, both Cn and Og have spin–orbit split channels and the corresponding orbital energies deviate from the non-relativistic values. Since the spin–orbit interaction scales approximately as Z^4 , the energy difference between the $7p_{1/2}$ and $7p_{3/2}$ states in Og is larger than that between the $6d_{3/2}$ and $6d_{5/2}$ states in Cn, owing to the higher atomic number (Z) of Og.

Table 1. Valence shell energies of copernicium (Cn) and oganesson (Og) in eV from relativistic and non-relativistic calculations.

Atom	Relativistic Subshells	Energy (eV)	Non-Relativistic Subshells	Energy (eV)
Cn	$6d_{3/2}$	15.311	$6d$	18.941
	$6d_{5/2}$	12.027	$7s$	6.478
	$7s$	12.274		
Og	$7s$	35.289	$7s$	21.058
	$7p_{1/2}$	20.119	$7p$	10.732
	$7p_{3/2}$	8.315		

The comparisons of the radial probability distributions obtained using the DHF and HF methods for Cn and Og atoms are presented in Figure 1. Figure 1a,b illustrate the probability distribution of the outermost subshell of electrons of both atoms, whereas Figure 1c,d display the total electronic contributions. It is evident that the DHF orbitals are more compact compared to the HF orbitals, indicating that they are more tightly bound to the nucleus, which is clearly a relativistic effect. This observation is supported by the valence shell energies listed in Table 1. For Cn, the bound-state energy of the $7s$ orbital is $E = -12.274$ eV in the DHF case, while in the HF case, it is $E = -6.478$ eV. For Og, the DHF $7p_{1/2}$ orbital has an energy of $E = -20.119$ eV, whereas the HF $7p$ orbital is less bound with $E = -10.732$ eV; however, the $7p_{3/2}$ orbital shows energy values close to those of the HF case. This stronger binding of the DHF orbitals is also reflected in the total probability density plots in Figure 1c,d for Cn and Og, where the distinct peaks correspond to the localization of electrons in different atomic orbitals.

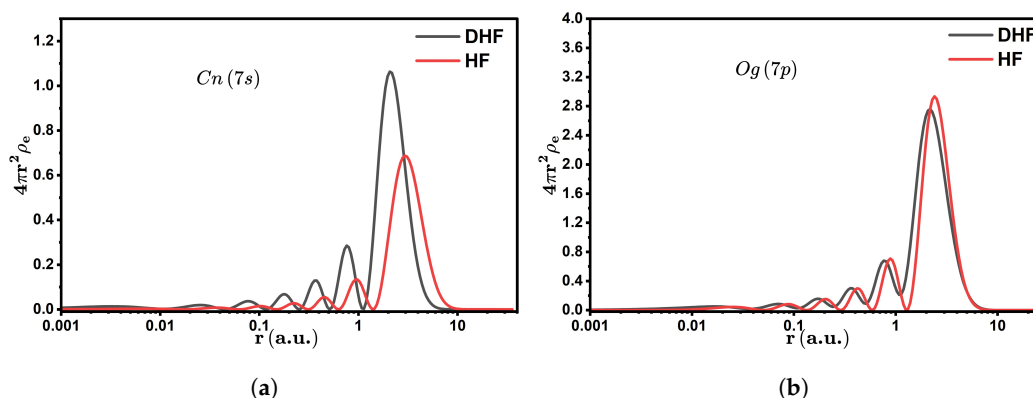


Figure 1. Cont.

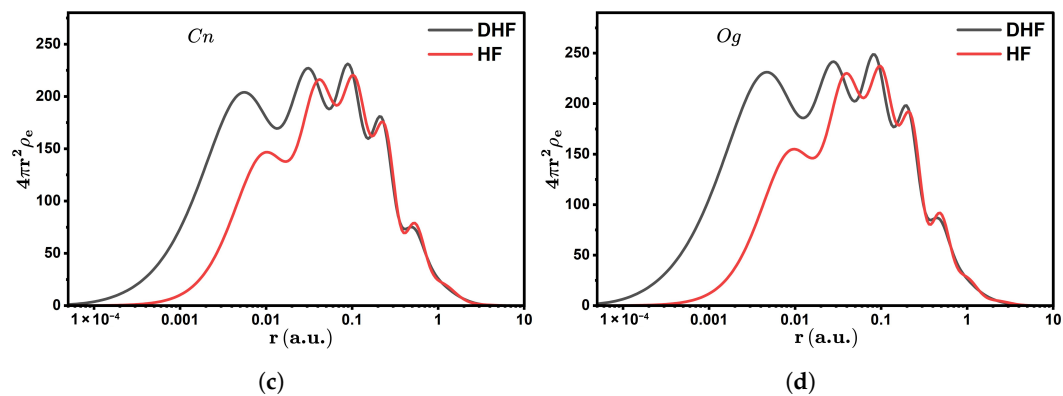


Figure 1. Comparison of radial probability distributions obtained from DHF and HF approaches: (a) outermost subshell contribution for Cn, (b) outermost subshell contribution for Og, (c) total electronic density contribution for Cn, and (d) total electronic density contribution for Og.

3.1. Integral Cross-Section for Electron Scattering from Copernicium and Oganesson

The electron scattering ICS from copernicium atom is presented in Figure 2, calculated using both DRPWA and PWA over the energy range 0.1–1000 eV. The electronic densities of both atoms are generated in the non-relativistic and relativistic regimes using the GRASP92 package [27], and these densities are employed in the formation of scattering potential. At very low energies ($E < 1$ eV), the relativistic and non-relativistic results deviate markedly. The relativistic ICS for e^- -Cn starts from a comparatively lower value and rises to form a broad resonance near $E \approx 1$ eV due to relativistic effects, whereas the non-relativistic ICS begins from a higher value and decreases monotonically. This highlights that the inclusion of relativistic orbital contraction and spin-orbit interaction modifies the low-energy scattering dynamics, giving rise to structures that are absent in the non-relativistic description. At such low energies, the electron spends more time within the potential region, making it more sensitive to the differences introduced by relativistic effects.

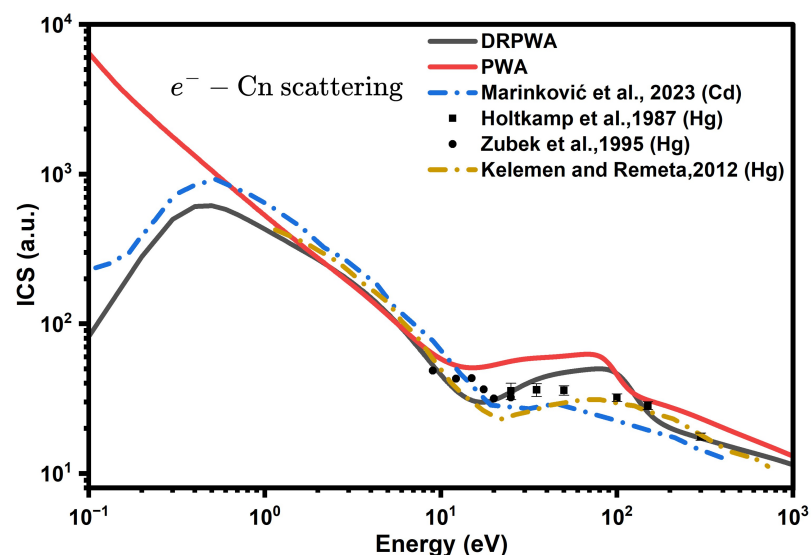


Figure 2. Integral cross-section for electron scattering from copernicium. The present results are compared with e^- -Cd cross section reported by Markinković et al. [34] (blue dash-dot line), and with e^- -Hg cross sections reported by Holtkamp et al. [35] (black squares), Zubek et al. [36] (black circles), and Kelemen and Remeta [37] (yellow dash-dot line).

In the intermediate energy region (10–100 eV), both approaches show oscillatory behavior, but with different amplitudes: the DRPWA results exhibit deeper minima and a broader peak compared with the PWA. A meaningful comparison can be made with the lighter group 12 congeners, cadmium and mercury. In this energy range, the relativistic Cn cross-section closely follows the experimental and theoretical e^- -Hg cross-section [35–37], whereas the theoretical e^- -Cd cross-section [34] remains lower above 20 eV. A minimum near $E \approx 20$ eV is a universal feature observed across the elements of this group. Overall, in this energy domain, the scattering cross-sections of Cd, Hg, and Cn are broadly comparable, consistent with their periodic group relationship.

At higher energies ($E > 200$ eV), the relativistic and non-relativistic ICSs for Cn gradually converge. This behavior is consistent with the expectation that relativistic effects become less pronounced once the projectile wavelength is much smaller than the atomic dimensions. In this energy range, the electron effectively senses only the averaged electrostatic potential, without resolving the finer contributions from exchange and correlation–polarization potentials where relativistic modifications occur, and the scattering is therefore dominated by electrostatic interaction.

Figure 3 presents the ICS for electron scattering from oganesson. The DRPWA results lie above the PWA cross-section at low energies, highlighting the enhanced role of relativistic $7s$ orbital contraction and the large spin–orbit splitting of the $7p$ shell. The relativistic contraction of the $7s$ orbital reshapes the electron density near the nucleus, thereby modifying the atomic potential. This altered potential affects the scattering phase shifts, resulting in an enhanced scattering amplitude. The DRPWA and PWA curves intersect near 1 eV. Beyond this point, the PWA cross-section slightly exceeds the relativistic result, displaying two shallow minima, whereas in the DRPWA case, the first minimum is shifted to a different energy. In the intermediate energy range (10–100 eV), both approaches exhibit oscillatory structures due to interference between partial waves, although the amplitudes and positions differ. A sharp resonance near 30 eV appears in the DRPWA calculation, arising from the lowering of the angular momentum barrier compared to the non-relativistic case, which results from relativistic modifications to the potential.

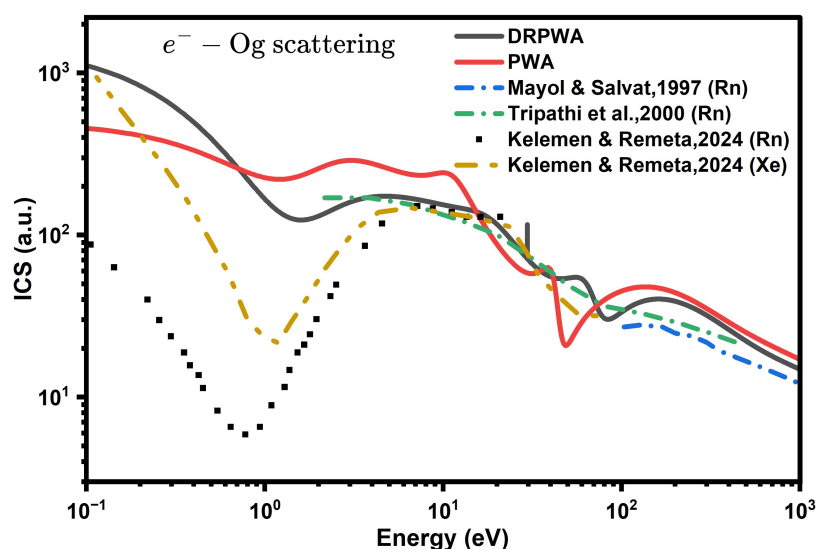


Figure 3. Integral cross-section for electron scattering from oganesson. The result are also compared with the e^- -Rn cross sections reported by Mayol & Salvat [38] (blue dash-dot line), Tripathi et al. [39] (green dash-dot line), and Kelemen & Remeta [40] (black squares), and with the e^- -Xe cross section reported by Kelemen & Remeta [40] (yellow dash-dot line).

Comparison with available scattering cross-sections for the lighter homologues—namely, theoretical e^- –Rn results [38–40] and experimental e^- –Xe cross-sections [40]—shows that the electron scattering cross-sections of oganesson are significantly larger in the low-energy region, gradually approaching comparable values in the intermediate energy range, reflecting its higher nuclear charge and its status as a superheavy noble gas. At low energies, a very shallow dip can be seen in the e^- –Og cross-section, which may be regarded as the analogue of a Ramsauer–Townsend (RT) minimum. However, unlike Xe and Rn, where the RT minima are pronounced and well developed, this feature in Og is strongly suppressed due to relativistic effects. At higher energies ($E > 200$ eV), the DRPWA and PWA results gradually converge, as the electron spends very little time near the scattering region and is thus unable to resolve the relativistic modifications in the potential. In this energy region, scattering is primarily governed by the overall electrostatic field.

Deviations between the DRPWA and PWA cross-sections are observed for both Cn and Og. However, the differences are more pronounced in Og, reflecting the stronger relativistic effects associated with its higher atomic number ($Z = 118$). This trend highlights the increasing influence of relativistic orbital contraction and spin–orbit interactions as one moves to heavier superheavy elements, which substantially modify the scattering dynamics at low and intermediate energies.

3.2. Scattering Length

The scattering length is a key parameter in the scattering process that provides insight into the nature and strength of the interaction at energies approaching zero. It can be obtained from the scattering phase shifts using the relation [41]

$$a = -\lim_{k \rightarrow 0} \frac{\tan \delta(k)}{k}, \quad (23)$$

where δ refers to the low-energy phase shift for the $\kappa = -1$ (relativistic) and $l = 0$ (non-relativistic) partial waves, as the scattering process is dominated by the s -wave contribution near zero energy. In principle, a negative value of the scattering length indicates an attractive interaction capable of supporting a bound state, whereas a positive value corresponds to a repulsive or weakly attractive interaction that does not support any bound state [42]. Scattering lengths for Cn and Og were determined using both DRPWA and PWA. The values corresponding to $k \rightarrow 0$ were obtained by extrapolating the scattering lengths calculated at several lower energies in the range $E \approx 0.005$ – 0.05 eV for both atoms. For Cn, the scattering length comes out to be $a \approx -1.3$ a.u. using DRPWA and $a \approx 15$ a.u. using PWA. The large difference between these two values can be attributed to strong relativistic effects, which also lead to significant variations in the elastic cross-sections at near-zero energies. Furthermore, the negative sign of the scattering length for Cn indicates that DRPWA supports a bound state, which could lead to a resonance, whereas PWA does not render any quasi-bound states. This contrasting behavior is evident from Figure 2, in which there is a shape resonance at lower electron energies in the DRPWA cross-section. For Og, the scattering lengths are approximately $a \approx 10$ a.u. and $a \approx 6$ a.u. from DRPWA and PWA, respectively. The reported scattering length for electron–Og scattering is $a \approx 15.1$ a.u. [43,44]. The deviation from the present result arises from the use of the multi-configuration Dirac–Hartree–Fock (MCDHF) formalism in Ref. [43], which allows for a more detailed treatment of electron correlation effects on the target potential, whereas Fedus and Karwasz [44] employed the Modified Effective Range Theory (MERT) in combination with Bayesian statistical inference.

3.3. Differential Cross-Section for e^- –Cn and e^- –Og Scattering

In this section, the DCSs for electron scattering from copernicium and oganesson are presented at four energies: 10 eV, 50 eV, 100 eV, and 500 eV. This energy-resolved analysis illustrates how the angular distribution of scattered electrons changes with energy, as the incident electron experiences different extents of interaction with the target atom. For each energy, the DCS is plotted to show the contributions from different components of the total potential in the relativistic case, namely electrostatic (V_{st}), electrostatic plus exchange ($V_{st} + V_{ex}$), and electrostatic plus exchange plus correlation–polarization ($V_{st} + V_{ex} + V_{cp}$). In addition, comparisons with the non-relativistic DCS (V_{total}) are presented for each energy, considering all components of the potential as in the final relativistic case.

Figure 4 represents the DCS for electron elastic scattering from copernicium. At 10 eV, the electrostatic potential alone gives rise to two minima near $\theta = 50^\circ$ and $\theta = 125^\circ$, reflecting partial wave interference. Including the exchange interaction suppresses the overall magnitude, removes the first minimum, and shifts the second minimum from $\theta = 125^\circ$ to 130° . The addition of correlation–polarization enhances the forward scattering ($\theta < 40^\circ$), consistent with the increased long-range attraction. The non-relativistic DCS begins at a higher value and exhibits three minima, in contrast to the relativistic case. At 50 eV, exchange mainly changes at the intermediate angles (40° – 150°), while correlation–polarization boosts the DCS at small angles ($\theta < 20^\circ$). The PWA DCS, built from non-relativistic densities, yields a minimum at $\theta = 122^\circ$ and deviates strongly from the DPWA beyond $\theta = 70^\circ$. At 100 eV, relativistic and non-relativistic results converge up to $\theta = 80^\circ$, though differences persist at larger angles. By 500 eV, the curves nearly coincide, as the projectile effectively averages over the fine details of the relativistically modified potential.

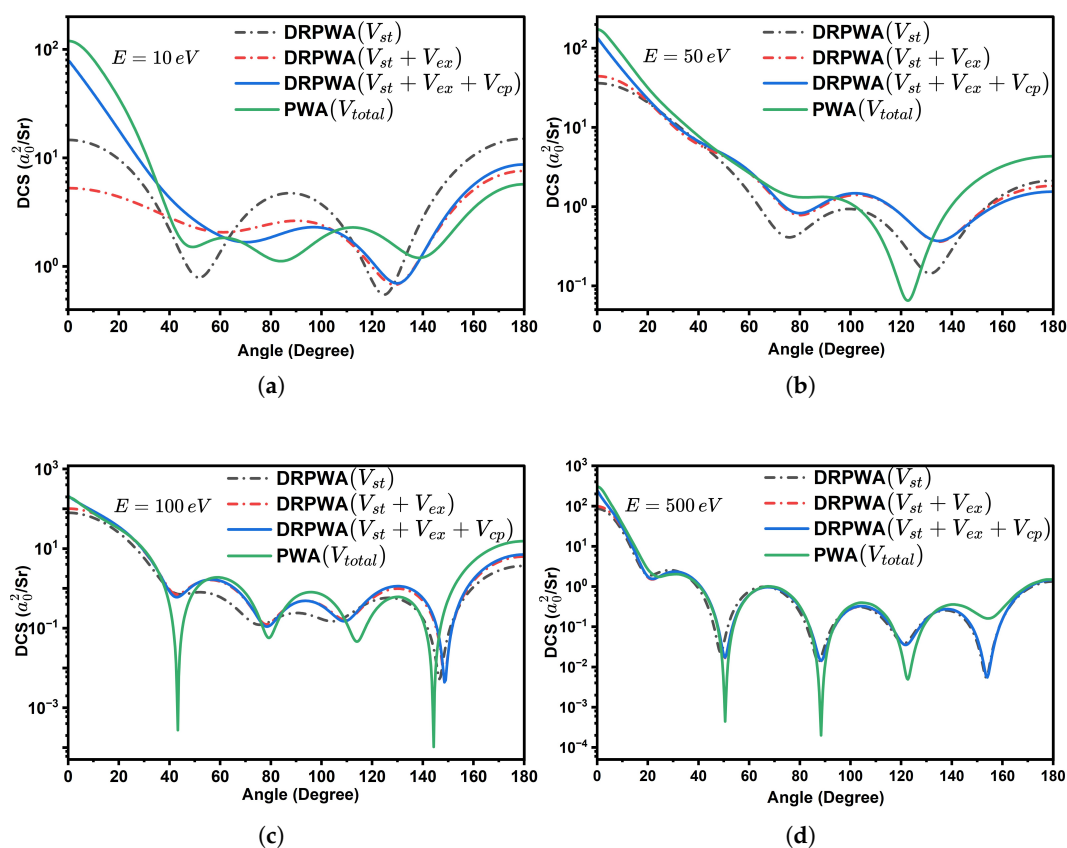


Figure 4. e^- –Cn scattering DCS at (a) 10 eV (b), 50 eV (c), 100 eV (d), and 500 eV.

For electron scattering from oganesson (Figure 5), the behavior of the DCS is qualitatively different from that of the lighter superheavy copernicium atom. At 10 eV, the DCS exhibits a single shallow minimum at $\theta = 130^\circ$ when only the electrostatic interaction is included. The inclusion of exchange increases the overall magnitude, while adding the global correlation–polarization potential enhances the forward-angle scattering. At 50 eV, the electrostatic potential induces mild oscillations in the DCS, with the exchange interaction primarily shifting the positions of maxima and minima and the correlation–polarization term further amplifying the forward peak. The PWA predicts more pronounced structures, showing noticeable deviations from the relativistic DCS at both these energies. At 100 eV, oscillatory features are already supported by the electrostatic term, with exchange affecting the positions of maxima and minima and correlation–polarization influencing predominantly the forward angles. The PWA and relativistic results begin to converge at 100 eV and are almost identical by 500 eV. It can be seen that correlation and polarization interactions primarily affect the forward scattering region, whereas the exchange interaction influences all scattering angles at low energies. The differences between the relativistic and non-relativistic cases are more pronounced at low energies, because at these energies the exchange, correlation, and polarization potentials play a dominant role in shaping the DCS, and these components of the interaction potential are significantly modified by relativistic effects. For both copernicium and oganesson, the angular dependence of the DCS is strongly influenced by the interplay of these interactions, emphasizing the essential role of exchange and correlation–polarization effects in accurately describing electron scattering from superheavy atoms.

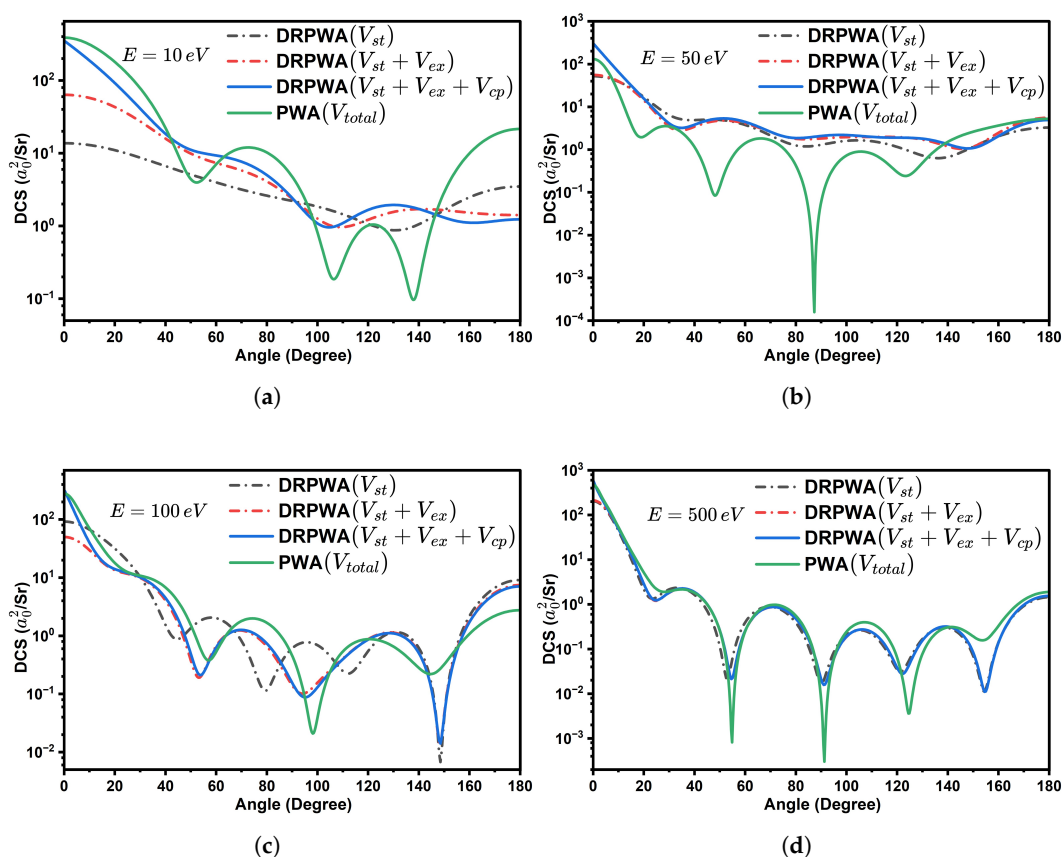


Figure 5. e^- –Og scattering DCS at (a) 10 eV, (b) 50 eV (c), 100 eV (d), and 500 eV.

4. Conclusions

We have investigated the electron scattering dynamics from the superheavy elements copernicium and oganesson using the optical potential approach. The scattering cross-sections were computed by solving the Dirac equation for the relativistic case and the Schrödinger equation for the non-relativistic case. To construct the total interaction potential for the e^- -Cn/Og system, different electronic densities were employed: for the relativistic case, the DHF equations were solved, and for the non-relativistic case, the HF equations were solved using the GRASP92 package [27].

This study reveals both quantitative and qualitative differences between the relativistic and non-relativistic treatments, highlighting the critical role of relativistic effects in shaping the scattering behavior of superheavy atoms. Additionally, comparisons of the electron scattering cross-sections for these atoms with those of their lighter congeners were presented to examine the expected trends within their respective groups and to identify features arising specifically from relativistic effects. Furthermore, the DCSs were analyzed by decomposing the contributions from different components of the total interaction potential, emphasizing the importance of exchange, correlation, and polarization potentials in low-energy electron scattering, where relativistic effects significantly influence the scattering dynamics.

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