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Generating Sustained Coherence in a Quantum Memory for Retrieval at Times of Quantum Revival

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Abstract: We study the time degradation of quantum information stored in a quantum memory device under a dissipative environment in a parameter range which is experimentally relevant. The quantum memory under consideration is comprised of an optomechanical system with additional Kerr nonlinearity in the optical mode and an anharmonic mechanical oscillator with quadratic nonlinearity. Time degradation is monitored, both in terms of loss of coherence, which is analyzed with the help of Wigner functions, as well as in terms of loss of amplitude of the original state, studied as a function of time. While our time trajectories explore the degree to which the stored information degrades depending upon the variation in values of various parameters involved, we suggest a set of parameters for which the original information can be retrieved without degradation. We identify a very interesting situation where the role played by the nonlinearity is insignificant, and the system behaves as if the information is stored in a linear medium. For this case, the information retrieval is independent of the coherence revival time and can be retrieved at any instant during the time evolution.



Citation: Kaur, T.; Kaur, M.; Arvind; Arora, B. Generating Sustained Coherence in a Quantum Memory for Retrieval at Times of Quantum Revival. *Atoms* **2022**, *10*, 81. <https://doi.org/10.3390/atoms10030081>

Academic Editor: Yew Kam Ho

Received: 28 June 2022

Accepted: 5 August 2022

Published: 10 August 2022

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Keywords: quantum memory; optomechanical system; quantum master equation; Wigner function; coherence revival time

1. Introduction

Quantum memory is a device that stores quantum information in the form of quantum states for later retrieval [1,2]. Due to an upsurge in the field of quantum computation and quantum communication, there is an increasing demand for devices that can store quantum information. However, the storage of quantum information is a challenging task due to the presence of quantum decoherence effects. These effects render the stored information unsuitable for further use. The need of the hour is to devise quantum memories that can not only preserve coherence, but can also store the information for longer durations. Criteria for assessing the performance of quantum memory include fidelity [3], efficiency [4], transfer coefficient [5], conditional variance [6], multimode capacity [7,8] and storage time [9]. Optomechanical systems are highly recommended to store and retrieve quantum states. They provide a promising mechanism for a high-fidelity quantum memory that is faithful to a given temporal mode structure, and can be recovered synchronously. The low dissipation rate of a mechanical oscillator allows the optomechanical oscillator to be used as a good quantum memory device. Teh et al. [10] carried out quantum simulations using a nonlinear optomechanical system to give a complete model for the storage of a coherent state and to increase the fidelity beyond the quantum threshold. Recently, quantum memory protocol for the creation, storage and retrieval of Schrödinger cat states in optomechanical system was proposed by Teh et al. [11]. Quantum state formation through system anharmonicities, as well as decoherence due to interactions with the

environment in terms of a Wigner current, was studied by Braasch et al. [12]. Chakraborty et al. [13] presented a scheme to enhance the steady-state quantum correlations in an optomechanical system by incorporating an additional cross-Kerr-type coupling between the optical and the mechanical modes. He et al. [14] proposed a dynamical approach for quantum memories using an oscillator-cavity model to overcome the known difficulties of achieving high quantum input–output fidelity with long storage times.

Quantum memories are an indispensable part of quantum information processing and long-distance communication, where the long distance is divided into shorter elementary links and the quantum state is stored independently for each link. It finds numerous applications in quantum networks [15], quantum repeaters [16,17] and linear quantum computing [18,19]. Quantum memory can be used to enhance the sensitivity of precision measurements where the sensitivity is limited by the time over which phase can be accumulated, and hence storage of quantum states is required [20].

In the present work, we study an optomechanical system with nonlinearities to make it useful for the purpose of a quantum memory. Standard quantum optical techniques make it easy to generate quantum states of light which can be stored in optical oscillator, but high dissipation rates for optical oscillators make the storage time very small. Mechanical oscillators, on the other hand, have very small dissipation rates, which make them ideal system for storage of quantum information. An optomechanical oscillator has advantage of both; the optical part of the optomechanical oscillator generates the quantum states, which can be stored in the mechanical part of it for a longer period of time.

Motivated by this fact, we consider that the quantum information is stored in the long-lived mechanical mode of the optomechanical system as a quantum state. As proposed in this work, the stored quantum state is initially chosen to be a coherent state obtained by displacing the vacuum state by a certain amount [11]. The mechanical mode interacts with the optical mode through radiation pressure in such a way that state transfer between both the modes is achievable [21–23]. We study the time evolution of our quantum state under the combined effect of nonlinearity and dissipation using the Master equation method. The dissipation is modeled by assuming that the system interacts with a constant temperature bath. During the evolution, highly non-classical superpositions of coherent states, i.e., multi-component Schrödinger cat states (Kitten states) are formed [24]. Revival of an initial quantum state occurs when it evolves over time to a state that reproduces its original coherent form. The characteristic time scale over which this phenomenon happens is called the coherence revival time [25]. The periodic revival of coherence at the coherence revival times makes retrieval of the quantum information possible at these times.

We use the Wigner function in our analysis, which provides a particularly useful geometric representation of quantum states as a real valued function in the two-dimensional system phase space [12,26–29]. The time evolution of the Wigner function gives us quantum phase space dynamics. In phase space, the classical state (coherent state) is marked by positive value of probability density, while the non-classical states formed by superposition of two or more coherent states (Schrödinger cat states or Kitten states) possess the negative value of probability density. We notice that the presence of nonlinearity causes the initial coherent state with associated positive Wigner functions to evolve into non-classical states associated with negative Wigner functions [12,30]. As expected, the coherent state resurrects itself, maintaining its coherence at coherence revival time with certain amount of degradation due to the presence of the environmental effects caused by the bath.

Further, since quantum information is stored in the form of a coherent state with certain amplitude that should be maintained at coherence revival time, we compute the amplitude of the coherent state as a function of time and study the effect of decoherence on the amplitude at different periods of time during the evolution. On varying the values of bath temperature, dissipation and nonlinearity, we gain insight into the extent to which coherences for the evolved quantum state can be sustained. In addition, we propose the optimum parameters for an optomechanical system which can be used to retrieve quantum information at longer times, with minimum loss.

The paper is organized as follows: In Section 2, we present the theory and the model used in our calculations, with the Hamiltonian and relevant quantum states and their dynamics defended in Section 2.1, the Master equation and phase space dynamics described in Section 2.2 and the numerical simulation techniques described in Section 2.3. In Section 3, we discuss our results. The results are first presented in terms of Wigner function to study the effect of decoherence on the quantum information at different periods of time during evolution; thereafter, temporal evolution of the amplitude is presented as a function of dissipation, nonlinearity, bath temperature and initial amplitude. Section 4 contains some concluding remarks.

2. Quantum Memory Model

A typical quantum optomechanical system as represented by Figure 1 consists of a Fabry–Perot cavity with one of the movable mirrors acting as a mechanical oscillator [10]. The optical mode trapped inside the Fabry–Perot cavity is coupled with the mechanical mode via a generic coupling represented by g_0 [13]. The optical mode and the mechanical mode have the frequencies ω_c and ω_m , respectively. Additionally, Kerr nonlinearity of strength k_c is present in the optical mode, and k_m is the strength of the quadratic nonlinearity in the anharmonic mechanical oscillator. Practically, the optomechanical system always interacts with its environment, resulting in decoherence through damping and fluctuations [10]. While interacting with their corresponding reservoirs, the cavity decay rate is represented by γ_c and the mechanical damping rate is given by γ_m . As a result of such dissipation, the quantum information begins to lose its quality during the time evolution.

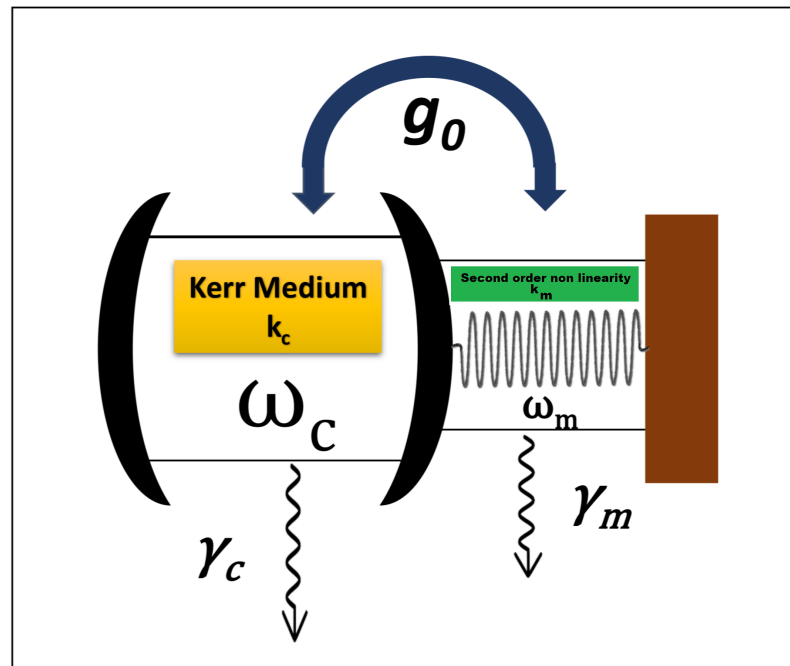


Figure 1. Schematic diagram of optomechanical system taken as quantum memory model. An optical mode (with frequency ω_c) couples with the mechanical mode (with frequency ω_m) via the radiation–pressure coupling g_0 . Additionally, Kerr nonlinearity of strength k_c and quadratic nonlinearity of strength k_m are included in optical and mechanical modes, respectively. Here, γ_c is the optical decay rate and γ_m is the mechanical damping rate of the respective modes.

2.1. Hamiltonian for Quantum Information Dynamics

With the above considerations, the total Hamiltonian of the optomechanical system can be written as

$$H_{total} = H_c + H_m + H_{coupling} \quad (1)$$

where H_c , H_m and $H_{coupling}$ are the Hamiltonian corresponding to the cavity, the mechanical oscillator and the coupling between them, respectively, and are given by the following expressions:

$$\begin{aligned} H_c &= \hbar\omega_c a^\dagger a + \hbar k_c (a^\dagger a)^2 \\ H_m &= \hbar\omega_m b^\dagger b + \hbar k_m (b^\dagger b)^2 \\ H_{coupling} &= -\hbar g_0 a^\dagger a (b + b^\dagger). \end{aligned} \quad (2)$$

Here, $a(a^\dagger)$ and $b(b^\dagger)$ are the annihilation(creation) operators for optical and mechanical modes, respectively. The terms $\hbar\omega_c a^\dagger a$ and $\hbar\omega_m b^\dagger b$ correspond to the energy of the optical cavity with frequency ω_c and mechanical oscillator with frequency ω_m [10]. $H_{coupling}$ describes the interaction between the optical mode and the mechanical mode, while $\hbar k_c (a^\dagger a)^2$ and $\hbar k_m (b^\dagger b)^2$ are the nonlinear terms present in the optical mode and the mechanical oscillator, respectively.

In quantum information processing, quantum states are prepared, stored and retrieved on demand using certain protocols. A unitary displacement operator $\mathcal{D}$ generates a coherent state $|\alpha\rangle$ via phase space displacement of the vacuum state $|0\rangle$ [31,32] as

$$|\alpha\rangle = \mathcal{D}(\alpha)|0\rangle, \quad (3)$$

where α is a complex parameter and the Displacement operator is given by

$$\mathcal{D}(\alpha) = \exp(\alpha a^\dagger - \alpha^* a) \quad (4)$$

and $|\alpha\rangle$ can be expanded in terms Fock or number states as [28,33]:

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \quad (5)$$

This initially prepared quantum coherent state is then allowed to evolve under system–environment dynamics to study the effect of dissipation and nonlinearity on the stored information. For nonlinear oscillators, after few cycles, quantum interference takes over, leading to a significant spread of the coherent state. The original state is no longer recognizable, and is said to have “collapsed”. As time advances, it resurrects itself, leading to its “revival”. The time at which this revival takes place and the initial coherent state also regains its coherence is referred to as ‘coherence revival time’ T_{rev} [34]. In the optomechanical system, we have two modes each with nonlinearity such that both the modes behave as nonlinear harmonic oscillators coupled with one another. Instead of considering these nonlinear modes individually, we work with the combined system as a whole. Therefore, the collapses and revivals occur for the two-mode system as a whole [34]. Consequently, we define coherence revival time for our system as a whole

$$T_{rev} = \frac{2\pi}{k_c + k_m}. \quad (6)$$

The entire numerical simulations under system–environment dynamics are performed to bring forth the retrieval phenomenon of stored quantum information at coherence revival time in the presence of environmental interactions.

2.2. Master Equation and Phase Space Representation of Quantum States

We develop the theoretical description to model the decoherence effects on our two-mode open system. The environment–system interactions are assumed to be Markovian. In

this formalism, the time derivative of the density operator ρ describing the quantum state of the nonlinear optomechanical system is given by the Master equation

$$\frac{d\rho}{dt} = \mathcal{L}\rho \quad (7)$$

where $\mathcal{L}$ is the Lindblad operator acting on the density matrix ρ . It is convenient to decompose $\mathcal{L}$ into

$$\mathcal{L} = \mathcal{L}_c + \mathcal{L}_m + \mathcal{L}_{int}, \quad (8)$$

where the operators $\mathcal{L}_c$ and $\mathcal{L}_m$ act on the optical mode and mechanical mode, respectively, while the operator $\mathcal{L}_{int}$ describes coupling in the optomechanical system.

On the account of H_c in Equation (2), the total Lindblad operator for the optical cavity is given as:

$$\mathcal{L}_c\rho = -\frac{\Gamma}{\hbar}[\omega_c a^\dagger a + k_c(a^\dagger a)^2, \rho] + L_c\rho, \quad (9)$$

where L_c describes the collapse operator as

$$\begin{aligned} L_c\rho = & \gamma_c(n_c + 1)(\rho a a^\dagger - \frac{1}{2}(a^\dagger a \rho + \rho a^\dagger a)) \\ & + \gamma_c(n_c)(a^\dagger \rho a - \frac{1}{2}(a \rho a^\dagger + \rho a a^\dagger)). \end{aligned} \quad (10)$$

Here, n_c is the mean photon occupation number of the surrounding thermal reservoir kept at temperature T associated with optical mode, and is given by

$$n_c = \frac{1}{\exp(\frac{\hbar\omega_c}{k_B T}) - 1}. \quad (11)$$

In analogy to Equation (9), for the mechanical mode b associated with the mechanical oscillator, the total Lindblad operator is given by

$$\mathcal{L}_m\rho = -\frac{\Gamma}{\hbar}[\omega_m b^\dagger b + k_m(b^\dagger b)^2, \rho] + L_m\rho, \quad (12)$$

where

$$\begin{aligned} L_m\rho = & \gamma_m(n_m + 1)(\rho b b^\dagger - \frac{1}{2}(b^\dagger b \rho + \rho b^\dagger b)) \\ & + \gamma_m(n_m)(b^\dagger \rho b - \frac{1}{2}(b \rho b^\dagger + \rho b b^\dagger)) \end{aligned} \quad (13)$$

and the mean thermal phonon occupation number of the bath (at temperature T) corresponding to mechanical mode is

$$n_m = \frac{1}{\exp(\frac{\hbar\omega_m}{k_B T}) - 1}. \quad (14)$$

Eventually, the coupling Hamiltonian $H_{coupling}$ in Equation (2) couples the number of photons $\hat{n} = a^\dagger a$ of the optical mode with the position $\hat{x} = b + b^\dagger$ of the mechanical mode. Its Lindblad operator is denoted as

$$\mathcal{L}_{int}\rho = -\frac{\Gamma}{\hbar}[-g_0 a^\dagger a (b^\dagger + b), \rho]. \quad (15)$$

As the Lindblad Master equation (Equation (7)) describes the quantum dynamics of the system state characterized by density operator $\rho(t)$, we use the Wigner function to describe the quantum state in phase space [10]. Wigner function is a phase space representation whose integration with respect to the system position coordinate x gives

the marginal probability density in the momentum coordinate p (and vice versa) [12]. The Wigner function representation of the quantum state $\rho(t)$ on phase space is defined as [12,26,27,29,35]

$$\begin{aligned} W(x, p, t) &= \frac{1}{\pi\hbar} \int_{-\infty}^{+\infty} dy e^{-2ipy/\hbar} \langle x+y|\rho(t)|x-y\rangle \\ &= \frac{1}{\pi\hbar} \int_{-\infty}^{+\infty} dp' e^{+2ip'x/\hbar} \langle p+p'|\rho(t)|p-p'\rangle, \end{aligned} \quad (16)$$

where the normalized wave function in momentum space is proportional to Fourier transform of the wave function in position space.

Further, to locate the time instant at which it is most appropriate to retrieve the stored information, we would like to calculate the expectation values of amplitude $\langle a(t) \rangle$ for the stored coherent states as a function of time. The solution of Equation (7) for optomechanical system can be written as

$$\rho(t) = \exp[\mathcal{L}(t)]\rho(t_0). \quad (17)$$

From the above equation, one can compute the time-evolved density operator $\rho(t)$, which is further used to calculate the expectation value of the amplitude via the relation given by

$$\langle a(t) \rangle = \langle a|\rho(t)|a \rangle \quad (18)$$

We calculate and display the snapshots of Wigner function at different times and plots of the expectation of the amplitude as two indicators in our analysis.

2.3. Details of Numerical Simulations

All numerical simulations were carried out using QuTiP [36], which is an open source software package written in Python. The numerical results are obtained by solving the Lindblad master equation (Equation (7)) to get the time-evolved coherent state density matrix. The density evolution is further extended to Wigner function evolution after adopting the complete parameters of the corresponding phase space. Further, the expectation value of the amplitude, representing quantum information is obtained by solving Equation (18). The evolution of the stored information was observed until $2T_{rev}$, where T_{rev} is given by Equation (6).

To choose the parameters for our system that show experimental consistency, we follow the work carried out by [13]. They numerically illustrated the effect of cross-Kerr coupling on the steady-state behavior and stability condition of the optomechanical system by considering the experimentally accessible frequencies of optical and mechanical modes as $\omega_c = 2\pi \times 370$ THz and $\omega_m = 2\pi \times 10$ MHz, respectively. To embody the strong coupling regime between the two modes, the value of g_0 is taken as 1.347 kHz. All the physical quantities are evaluated in atomic units (a.u.) by substituting $\hbar$ and K_B equal to 1. Accordingly, our corresponding parameters in a.u. are $\omega_c = 2\pi \times 0.056233$, $\omega_m = 2\pi \times 0.151983 \times 10^{-8}$ and $g_0 = 0.20472 \times 10^{-2}$.

3. Results and Discussion

3.1. Visualization of the Time Evolution through Wigner Representation

We plot the Wigner function corresponding to the quantum state at different times in order to see the transformations caused in the state due to nonlinearity and system–environment coupling. We consider an initial bit of quantum information embedded in a coherent state of amplitude $\alpha = 1.5$. The effective nonlinearity of both the optical and mechanical modes is taken as 0.01 a.u. The bath is maintained at 0K, such that $n_c = n_m = 0$. The overall dissipation resulting from this system–bath interaction is selected to be $\gamma_c = \gamma_m = 0.00001$ a.u. It is observed that during the time evolution, the coherent state undergoes a series of alterations, as is obvious in the contour plots of the Wigner function displayed at different times in Figure 2. In these plots, (Figure 2a–o) regions color coded blue depict a positive Wigner function, which corresponds to conventional

probability density, while red regions depict a negative Wigner function, which corresponds to a non-classical states. [12].

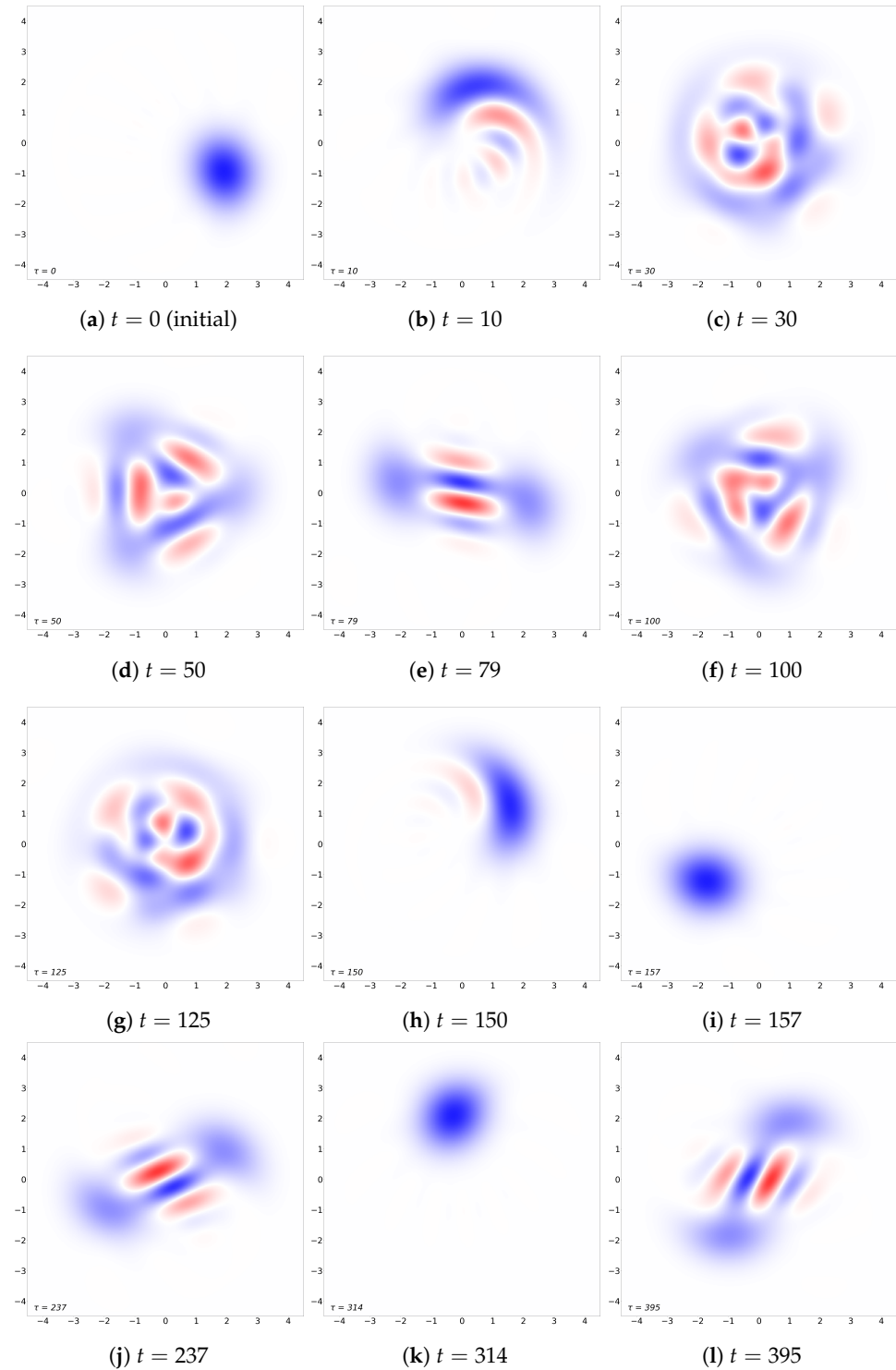


Figure 2. Cont.

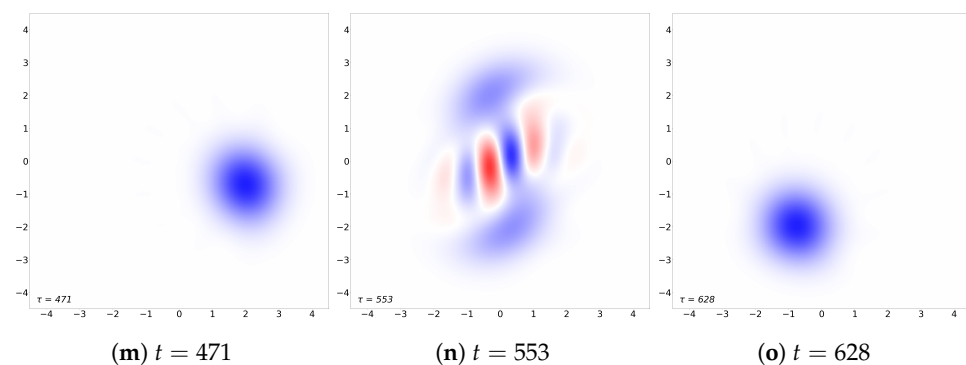


Figure 2. Snapshots in term of contour plot of the Wigner function of the evolving coherent state prepared inside optomechanical quantum memory for $N = 10$. The optical cavity decay rate γ_c and the mechanical damping rate $\gamma_m = 0.00001$ a.u. where the bath temperature corresponding to each mode is $T = 0$. Nonlinearities present in both the modes take the value $k_c = k_m = 0.01$ a.u.

The Wigner plot for the initial coherent state at $t = 0$ a.u., an elliptical blue region, is shown in Figure 2a. As time advances, the coherent state loses its classicality and the non-classical red regions appear. The resulting state is marked by a tail of interference fringes as portrayed by snapshot at $t = 10$ a.u. (Figure 2b). Further evolution is marked by the appearance of highly non-classical, multi-component Schrödinger cat states, which are also called kitten states, as shown in Figure 2c,d. Two component Schrödinger cat states are also observed at $t = \frac{T_{rev}}{4} = 79$ a.u. as depicted in Figure 2e. During the evolution, appearance of non-classicality in an evolving coherent state is due to natural dispersion. However, the presence of nonlinearities compensates the dispersion and leads to the revival of coherent state. Eventually, the initial classical coherent state reappears at $t = \frac{T_{rev}}{2} = 157$ a.u. given in Figure 2i, with nonclassicality disappearing! As we further observe the time evolution of the Wigner function, we notice that the coherence revival states (coherent states) arise at multiples of $\frac{T_{rev}}{2}$ time, as illustrated by Figure 2i,k,m,o. The analysis above shows that the coherences underlying the coherent state collapse and revive periodically, which plays an important role in deciding when to retrieve the information.

3.2. Evolution of the Expectation Value of the Amplitude

In order to elucidate the loss of amplitude at coherence collapse time (cat states) and regained amplitude at coherence revival time (coherent states), we calculate the expectation value of coherent state amplitude using the Master equation. The time evolution of $\langle a(t) \rangle$ shown in Figure 3 endorses that collapsed state presents loss in amplitude at odd multiples of $\frac{T_{rev}}{4}$ time, while the revived state presents regain in amplitude at multiples of $\frac{T_{rev}}{2}$. A similar time evolution plot was generated only for an optical and a mechanical oscillator. Since the revival time is inversely proportional to the nonlinearity, it was found that the stored quantum state revives after a longer period of time if only an optical or mechanical oscillator is used in comparison to using an optomechanical oscillator. This implies that the quantum memory can be retrieved at more number of revival times within a given period of real time scale for an optomechanical oscillator.

Further, to study the extent of degradation of quantum information, we extend our results to find the evolution of the expectation value of the amplitude under various circumstances. In particular, we study (1) The effect of environment; (2) The effect of nonlinearities in the system; (3) The effect of bath temperature; (4) The effect of the initial amplitude of the coherent state. After this study, we will be able to draw out the best-suited combinations for information retrieval.

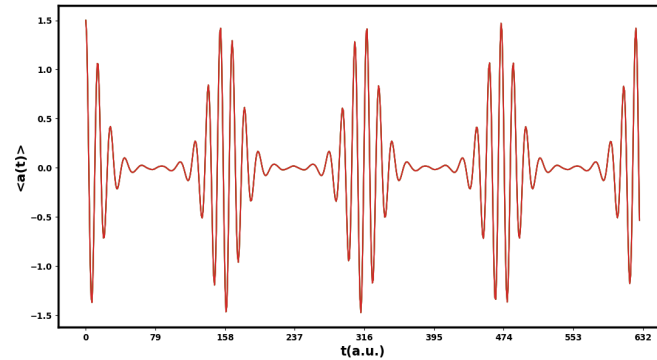


Figure 3. The expectation value $\langle a(t) \rangle$ plotted as a function of time for coherent quantum state in the presence of dissipation $\gamma_c = \gamma_m = 0.00001$ a.u. and nonlinearities $k_c = k_m = 0.01$ a.u. The bath temperature corresponding to each mode is $T = 0$.

3.2.1. The Effect of Environment

We begin with a coherent quantum state and study the effect of the environment on the expectation value of its amplitude as a function of time in the presence of nonlinearity. We analyze the system in the parameter region, in which the strength of nonlinearity in the optical mode (k_c) as well as mechanical mode (k_m) is 0.01 a.u. The results are displayed in Figure 4, where we have plotted $\langle a(t) \rangle$ as a function of time and the effect of the environment is studied for different values of dissipation, denoted in terms of γ_c and γ_m values, which are 0.00001 a.u., 0.0001 a.u., 0.001 a.u. and 0.01 a.u. The dissipation caused by the environment is evident through the damping and ultimate decay of the expectation value of amplitude $\langle a(t) \rangle$ as time evolves. As we scan through different values of γ_s , we find there is a threshold value of dissipation up to which revivals survive, as illustrated below.

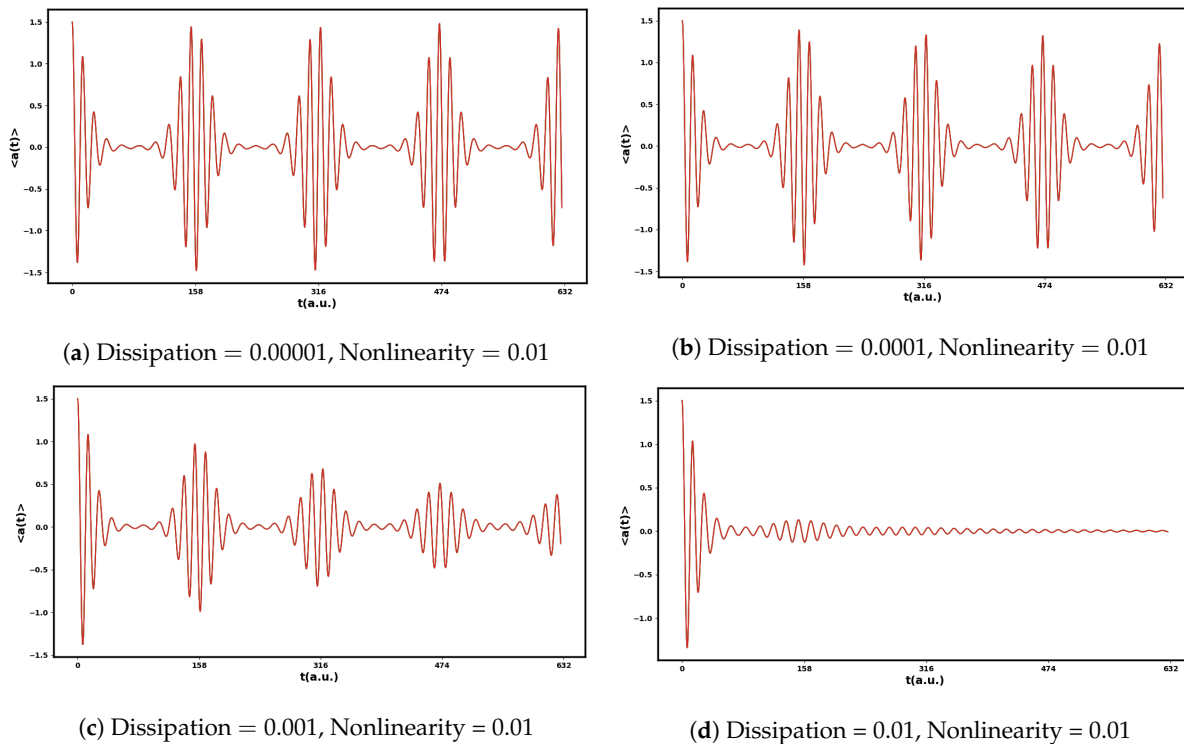


Figure 4. Collapse and revival of a coherent quantum state inside an optomechanical oscillator with nonlinearities $k_c = k_m = 0.01$ a.u. for four different values of dissipation. (a) $\gamma_c = \gamma_m = 0.00001$ a.u., (b) $\gamma_c = \gamma_m = 0.0001$ a.u., (c) $\gamma_c = \gamma_m = 0.001$ a.u. and (d) $\gamma_c = \gamma_m = 0.01$ a.u.

As shown in Figure 4a, for finite but small γ_c and γ_m values (both equal to 0.00001 a.u.), revivals are clearly noticeable, although, due to dissipation, a reduction in revived amplitude does take place. Hence, although the revivals are never complete in the presence of dissipation, their signatures are present, and can be experimentally identified. As the value of γ_c and γ_m increases, the revivals become weaker and weaker, as seen in Figure 4b,c. From Figure 4d, one can conclude that for a certain value of γ_c and γ_m equal to 0.01 a.u., the revivals disappear altogether. Hence, beyond this threshold value we do not expect to see revivals in our quantum state.

3.2.2. The Effect of Nonlinearity in System

Next, we study the effect of strong and weak nonlinearity of the revival behavior of the quantum state. In this case, the dissipation parameters γ_c and γ_m for optical and mechanical modes are chosen to be 0.00001 a.u., and the magnitude of Kerr nonlinearity k_c of the optical mode, as well as quadratic anharmonicity k_m of the mechanical mode, is varied. For Figure 5a, the value of the nonlinearity parameter is chosen to be large ($k_c = k_m = 0.5$ a.u.). The value of revival time for this nonlinearity, as calculated from Equation (6), is 6.4 a.u., and indicates that revivals should appear at these times. However, from Figure 5a, one can notice that the revival of quantum state is absent at this value of nonlinearity, and the pattern is irregular. Similar behavior is observed for nonlinearity values of 0.05 a.u. (Figure 5b). Hence, we conclude that high values of nonlinearity do not support the phenomenon of collapse and revival of a quantum state. For a comparatively smaller value of nonlinearity, 0.005 a.u. (Figure 5c), the constructive and destructive interference among multi-component Schrödinger cat states make the collapse and revival occur at a fixed time, without irregularities. A further decrease in nonlinearity value by a factor of 10 (Figure 5d) decreases the successive amplitude of revived states. This happens due to the large value of revival time, i.e., 6400 a.u., which slows down the process of collapse and revival. As a consequence, the interaction time with the dissipative environment increases, leading to a substantial decrease in amplitude. Hence, we conclude that information can be successfully retrieved when the nonlinearities in both the modes are of the order of 10^{-3} a.u.

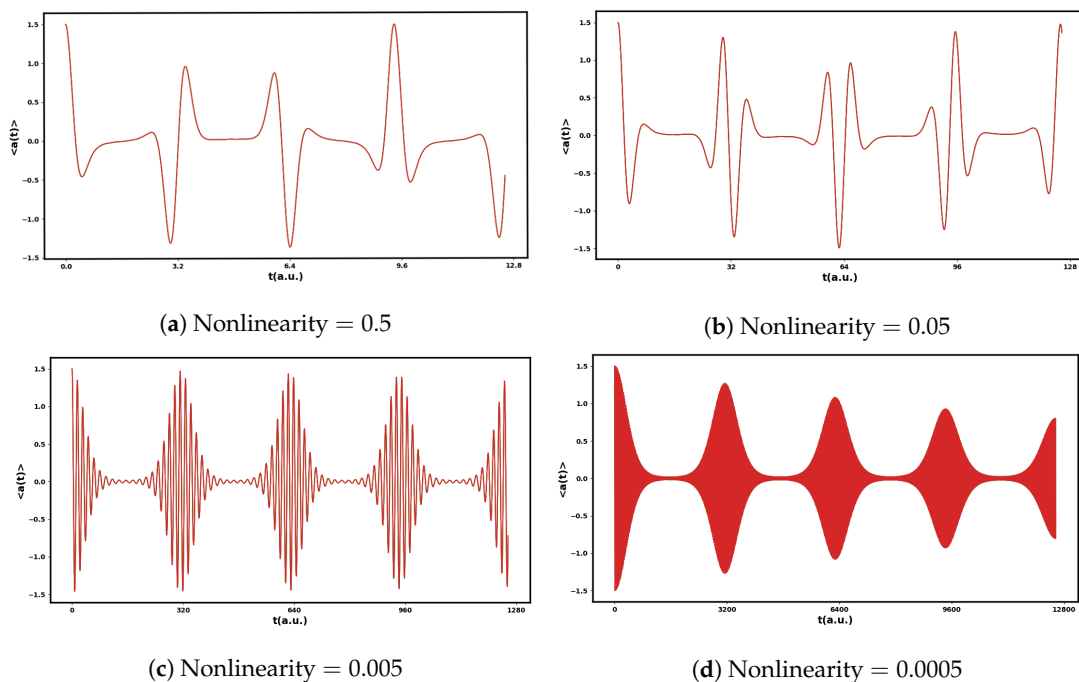


Figure 5. Effect of strong and weak nonlinearities in the optical and mechanical mode represented by k_c and k_m , respectively, on the revival behavior for a coherent quantum state. The dissipations inside the optomechanical oscillator are $\gamma_c = \gamma_m = 0.00001$ a.u. The values of nonlinearity are (a) $k_c = k_m = 0.5$ a.u., (b) $k_c = k_m = 0.05$ a.u., (c) $k_c = k_m = 0.005$ a.u. and (d) $k_c = k_m = 0.0005$ a.u.

3.2.3. The Effect of Bath Temperature

The fact that collapse and revival of a quantum state show dependence on bath temperature (Equations (11) and (14)) provides an opportunity to explore the revival pattern for various values of T . The results of this study are shown in Figure 6a–d. Temperatures in the μK and mK range are insignificant for optical mode; however, these temperature ranges are notable for the mechanical mode of the optomechanical system. These very small temperatures affect the collapse, and a revival pattern arises from the mechanical mode due to variation in the number of phonons involved in the interaction. As shown in Figure 6a, at μK range, the system behaves in a similar manner as it does at 0 K (Figure 3). At mK range (Figure 6b), the revival amplitude starts decreasing, with a subsequent revival period. This successive decrease in revival amplitude is due to the increase in the mean thermal phonon number of the bath, which creates a hindrance for complete a revival of amplitude while interacting with the system. A further increase in the value of temperature to 0.3 K, as indicated in Figure 6c, shows exceedingly large dissipation of the amplitude of quantum state. At 3 K, collapses and revivals in the quantum state disappear entirely due to noticeable effects of the bath temperature.

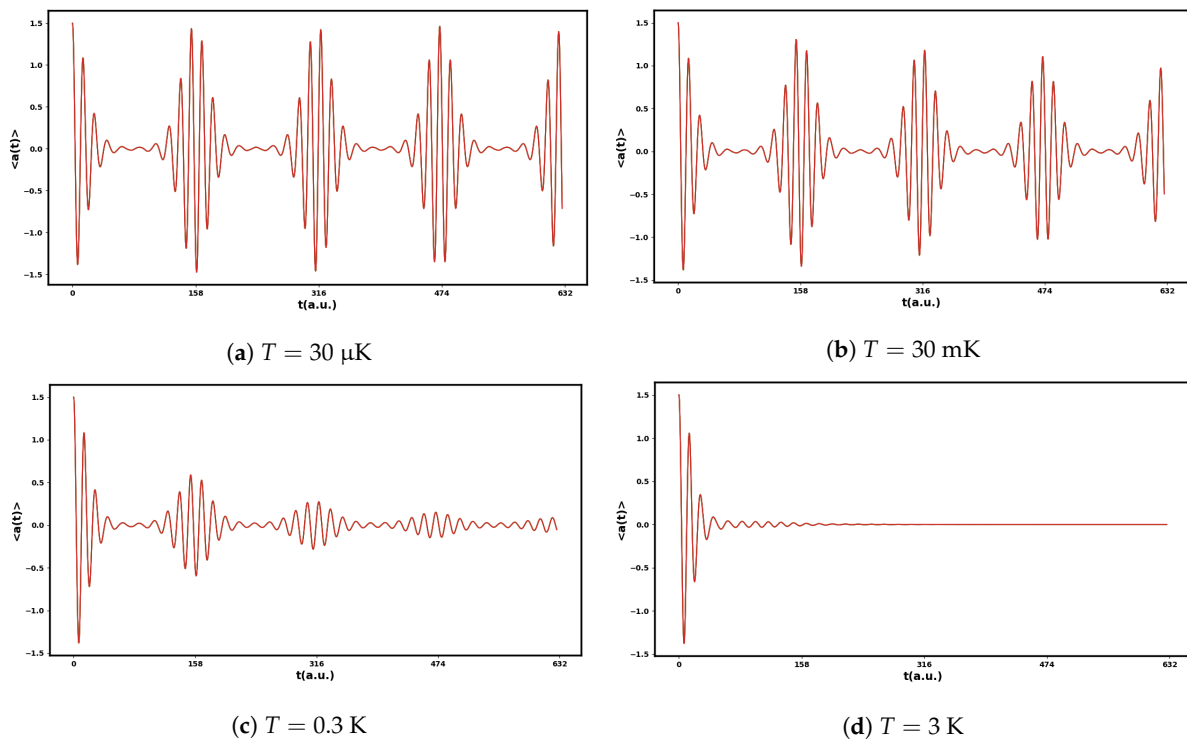


Figure 6. Effect of bath temperature on quantum state stored inside optomechanical oscillator with nonlinearities $k_c = k_m = 0.01$ a.u. and dissipation $\gamma_c = \gamma_m = 0.00001$ a.u. The amplitudes of revived states at their revival time are recorded for various values of bath temperature T .

3.2.4. The Effect of Initial Amplitude of Coherent State

The initial amplitude of coherent state measures the extent to which the displacement operator displaces the vacuum state while constructing the initial coherent state. Our findings highlight the effect of initial amplitude on the behavior of collapses and revivals in Figure 7a–d. Here, the value of nonlinearities in both the modes is 0.01 a.u. and dissipation is 0.00001 a.u. The bath temperature is maintained at 0K. The behavior of the quantum information is studied at various values of initial amplitude, taken as approximately 0.1, 0.5, 1.0, and 2.0. For very small values of amplitude, there is no perceptible effect on the pattern, as seen in Figure 7a, where we have chosen the amplitude to be 0.1. For such a small amplitude, the system–environment interaction and nonlinearity do not affect the quantum state inside the optomechanical system, and its time evolution is harmonic in nature. We

note that the revival behavior of the quantum state in this situation is similar to the revival behavior of a simple harmonic oscillator without dissipation. We term such revivals as perfect revivals, which support retrieval of quantum state without any loss of amplitude. As the value of amplitude increases to 0.5 (Figure 7b), the process of collapses and revivals is seen in its budding state. A further surge in the amplitude to 1.0 (Figure 7c) results in an erratic energy spectrum, leading to well-distinguished collapse and subsequent revival of the quantum state. At an amplitude of 1.5 (Figure 3), the collapses are pre-eminent. With a further increase in amplitude, the collapses and revivals are accompanied by a widened collapsed phase, as shown by a loss in amplitude of the quantum state for a longer period of time (Figure 7d). Thus, the retrieval of the information must be carried out at the time instant where revivals are prominent.

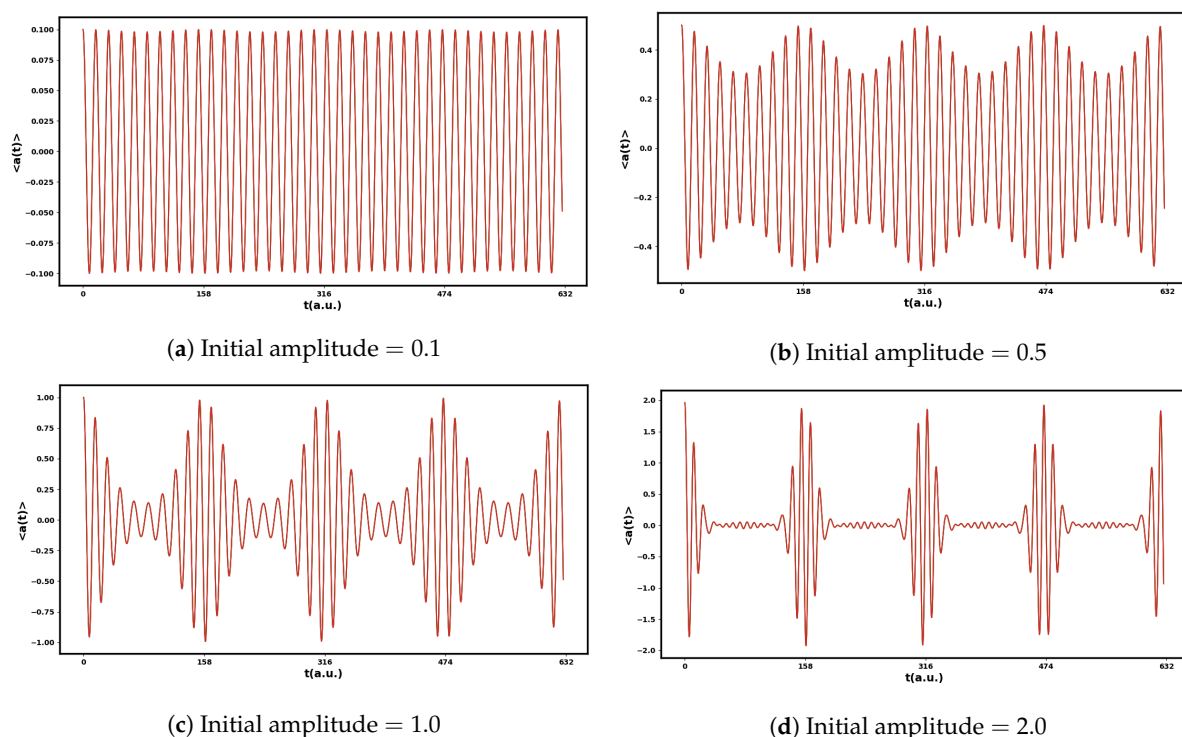


Figure 7. Collapse and revival pattern of $\langle a(t) \rangle$ of quantum state being plotted as a function of time with nonlinearities (k_c and k_m) and dissipation (γ_c and γ_m) of optical and mechanical modes equal to 0.01 a.u. and 0.00001 a.u., respectively. The pattern is shown for states with initial amplitude = 0.1, 0.5, 1.0 and 2.0.

4. Conclusions

We considered a quantum memory device comprising of an optical mode interacting with a mechanical oscillator, with nonlinearities present both in the optical as well the mechanical mode. The time degradation of quantum information stored in this quantum memory device was studied under a dissipative environment and with respect to non-linearity parameters. The quantum master equation approach was used to analyze the dissipative dynamics of coherent quantum states which were used for information storage. The parameters chosen to monitor the time degradation of the quantum states were the loss of coherence, as well as the loss of amplitude in an originally coherent quantum state with a well-defined amplitude. The nature of coherence was analyzed in terms of Wigner function, which yields useful visual insights into the progression of quantum states through system nonlinearities, as well as decoherences due to interaction with the environment. We further evaluated the expectation value of the amplitude of the stored quantum information as a function of time to mark the extent of overall sustainability of information. The time evolution of amplitude was studied for a wide range of parameters. The effects of the

environment, nonlinearity, bath temperature and initial amplitude on the revival behavior of the quantum state were determined. We also presented the most suitable parameters for quantum state retrieval. It was found that for a system to revive completely at certain periods of time, the strength of dissipation should be of the order of 10^{-5} a.u. at 0 K temperature of the bath. Information can be successfully retrieved when the nonlinearity in both the modes is of the order of 10^{-3} . The amplitude of 1.5 and 2.0 shows perfect collapses and revivals, where the amplitude during evolution entirely reduces to zero, and thereafter it completely revives at revival time. This work has potential applications in quantum information processing and long-distance communication.

Author Contributions: B.A. conceived the idea of generating sustained coherence in a quantum memory for retrieval at times of quantum revival which motivated T.K. to carry out this study. T.K. monitored time degradation, both in terms of loss of coherence, which is analyzed with the help of Wigner functions, as well as in terms of loss of amplitude of the original state, studied as a function of time and finally drafted the manuscript. M.K. verified the results and contributed in the editing of the manuscript. A. provided critical feedback and helped shape the research, analysis and manuscript. He also aided in interpretation and discussion of the results and commented on the manuscript. With the help of B.A., M.K. and A., T.K. modified this manuscript to the final version. All authors have read and agreed to the published version of the manuscript.

Funding: The work is supported by SERB-TARE (TAR/2020/000189), New Delhi, India.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data underlying this article are available in the article.

Acknowledgments: Research at Perimeter Institute is supported in part by the Government of Canada through the Department of Innovation, Science and Economic Development and by the Province of Ontario through the Ministry of Colleges and Universities.

Conflicts of Interest: The authors declare no conflict of interest.

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