

Article

Search for Super-Deformed Identical Bands in the $A = 190$ Mass Region on the Basis of Level Spin

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Abstract: The systematic study of Super-Deformed (SD) bands in the $A = 190$ mass region has been performed. We observed a large number of pairs of SD bands, with different mass numbers, having transition energies nearly equal (within 3 keV) and having identical dynamic moments of inertia. The bands having nearly equal transitions energies and other parameters are called identical bands. We have performed detailed analysis and found 16 pairs of Super-Deformed Identical Bands (SDIBs) whose E_γ energies and moment of inertia are in good agreement with each other. The modified Variable Moment of Inertia (VMI) model is applied to 16 pairs of SDIBs to estimate the band spins by fitting the two parameters $\mathcal{J}_0$ and C . We found that out of 16 pairs, the band-head spin is consistent with moment of inertia and transition energies for four pairs. For another seven pairs, the transition energies and moment of inertia are identical, but originate from levels with different spins. The remaining five pairs have the identical energy but spins are either increasing or decreasing by one unit in the pair. Secondly, the $N_p N_n$ scheme is applied to verify the existence of SDIBs. The parameters deduced from the $N_p N_n$ scheme are also in good agreement for the mentioned case. The study indicates that each pair of conjugate nuclei have nearly identical spin, moment of inertia (dynamic) and gamma transition energy.

Keywords: SD band; IB

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1. Introduction

Over the past two decades, the study of Super-Deformed (SD) nuclei has been one of the most important and exciting topics in nuclear structure studies. The discovery of multiple SD bands within one nucleus has opened up the possibility of learning the nuclear structure of SD nuclei in great details. Physicists performed a series of experiments to understand them, but only a general understanding of the Super-Deformed Rotational Bands (SDRBs) has been achieved. Although a large number of SD bands were studied in different mass regions, namely $A = 30, 60, 80, 130, 150, 160, 190, 230$ and 240 of the nuclear chart, there are still many open unexpected problems that exist. It is very surprising to note that from last decade, no new major experimental work has been reported in this area. Super deformation at high angular momentum is one of the most challenging and interesting topics for both theoreticians and experimentalists. The unexpected features observed in SD bands are used to estimate the band-head spin and level spins, which is still an open challenge. Several theoretical fitting procedures for spin assignment were proposed by many authors [1–11]. We have also developed a simple model in the same direction to predict the spins in different mass regions of SD nuclei in our previous works [12–15].

The fascinating phenomenon of Identical Bands (IBs) was first discovered in 1990 in Normal-Deformed (ND) bands, which initialized many experimental as well as theoretical studies [16–18]. The first pair of SD identical bands was an excited band in ^{151}Tb , whose γ -ray energies were identical to the yrast band of ^{152}Dy [18,19]. Later identical bands were found in Hg nuclei [20,21]. Since then, many IBs were observed at both low and high spins. In recent years, there have been a lot of work presented to observe IBs both in ND and SD nuclei [22–29]. During experimental investigation, it was noticed that the occurrence of IBs is much more frequent in SD nuclei as compared to ND nuclei, and the reason for this was attributed to the difference in pairing properties between SD and ND nuclei [30]. In 1997, an identical band in ^{194}Hg is specifically given by Ref. [31]. The cranked Nilsson–Strutinsky model was used to study the occurrence of IBs [17] in Super-Deformed nuclei, but it has certain disadvantages. Later, the description of Super-Deformed identical bands of odd- A nuclei in the $A = 190$ mass region has been investigated using a super symmetry approach [32]. Further, Reflection Asymmetric Shell Model (RASM) was used for theoretical simulation of IBs for both SD and ND nuclei [33]. Some of the identical SD band pairs are analysed using the Nuclear Softness (NS) formula [34]. The term identical band is defined using energy factor method as two different nuclei having nearly same transition energies E_γ within ± 3 keV and a nearly identical (dynamic) moment of inertia [35,36]. We have also employed the same method to define IBs in the present study. The purpose of present study is to make a theoretical simulation for a systematic study of IBs by estimating the band-head and level spins, along with γ transition energies and (dynamic) moment of inertia in the $A = 190$ mass region, which has not been obtained yet. We analysed the identical bands by using the modified Variable Moment of Inertia (VMI) model. Quantitatively good results of γ -ray energies, spins and moment of inertia are reported in this paper.

The paper is organised as follows: a brief description of our approach to assign the band-head spin of SD bands is presented in Section 2. Section 3 is devoted to explaining the identical band parameters for the selected case. In Section 4, we present the calculations, results and discussion for IBs in the $A = 190$ mass region for SD nuclei. Finally, a conclusion is given in Section 5.

2. A Brief Description of the VMI Model

The complete VMI model is described in Refs. [12,37,38]. The band energy levels of a rotational band are represented as:

$$E_I = E_0 + \frac{1}{2\mathcal{J}_I}[I(I+1) - I_0(I_0+1)] + \frac{1}{2}C(\mathcal{J}_I - \mathcal{J}_0)^2, \quad (1)$$

where E_0 is the band-head energy of the rotational band and I_0 is the band-head spin. In the plot of RMS deviation versus spin, the smallest deviation for the particular spin is considered as the band-head spin (I_0). The energy level for $I_0 = 0$, the ground-state bands in even–even nuclei, is represented as:

$$E_I(\mathcal{J}) = \frac{1}{2}C(\mathcal{J}_I - \mathcal{J}_0)^2 + \frac{1}{2}\left[\frac{I(I+1)}{\mathcal{J}_I}\right]. \quad (2)$$

The cubic equation is taken from the VMI model. This cubic equation has one real root for any finite positive value of $\mathcal{J}_0$ and C . The cubic equation is given as:

$$\mathcal{J}_I^3 - \mathcal{J}_I^2\mathcal{J}_0 - ([I(I+1)]/2C) = 0. \quad (3)$$

The transition energy for SD band is defined as [14]:

$$E_\gamma(I \rightarrow I-2) = \frac{1}{2\mathcal{J}_0}[I(I+1) - (I-2)(I-1)] + \frac{1}{8C\mathcal{J}_0^4}([I(I+1)]^2 - [(I-2)(I-1)]^2). \quad (4)$$

In this equation, the parameters $\mathcal{J}_0$ and C are determined by fitting the experimentally known transition energies by using the Best-Fit Method (BFM).

The Root Mean Square Deviation (RMSD) of the calculated transition energies for different I_0 values are calculated. The RMSD value is lowest for the correct band-head spin value of a band. If I_0 shifts away from the correct value by ± 1 , a rapid shift in RMSD (χ) is observed. The RMSD is defined as [13,14]:

$$\chi = \left[\frac{1}{n} \sum \left| \frac{E_{\gamma}^{cal}(I_i) - E_{\gamma}^{exp}(I_i)}{E_{\gamma}^{exp}(I_i)} \right|^2 \right]^{\frac{1}{2}}, \quad (5)$$

where n is the total number of transitions involved in the fitting. If I_0 is known, then all the level spin values of the SD band can be determined easily. The purpose of estimating the spins is to identify the identical bands in a new way, i.e., on the basis of spin along with transition energies and dynamic moment of inertia in the $A = 190$ mass region. A total of 16 SD pairs are reported in this paper.

For all 16 pairs, the transition energies and (dynamic) moment of inertia are identical, yet the challenge is the band-head spin, which is not identical in all 16 pairs and discussed as follows:

- (i) For the first four pairs, the band-head spin is consistent;
- (ii) For the next seven pairs, the bands originate from the levels with different spins, so band-head spin is different for two nuclei.
- (iii) For the last 5 pairs, the spins are either increasing or decreasing by one unit in the pair.

To find the rotational frequency $\hbar\omega$, the kinematic $\mathcal{J}^{(1)}$ and dynamic $\mathcal{J}^{(2)}$ moment of inertia [12,37,38], we have differentiated Equations (2) and (3) with respect to $I(I+1)$. Using the chain rule, we can extract $\hbar\omega$, $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ from their definitions as:

$$\hbar\omega = \frac{dE}{d[I(I+1)]}, \quad (6)$$

$$\mathcal{J}^{(1)} = \hbar^2 [I(I+1)]^{1/2} \left(\frac{dE}{d[I(I+1)]} \right)^{-1} \simeq \frac{2I-1}{E_{\gamma}(I \rightarrow I-2)}. \quad (7)$$

$$\mathcal{J}^{(2)} = \hbar^2 \left(\frac{d^2E}{d[I(I+1)]^2} \right)^{-1} \simeq \frac{4}{E_{\gamma}(I+2 \rightarrow I) - E_{\gamma}(I \rightarrow I-2)}. \quad (8)$$

The $\mathcal{J}^{(1)}$ moment of inertia is a direct measure of the transition energies, while $\mathcal{J}^{(2)}$ is obtained from differences in transitions energies (relative change in transition energies). The $\mathcal{J}^{(2)}$ dynamic moment of inertia is plotted with ($\hbar\omega$) rotational frequency to confirm the identical bands for SD nuclei.

3. Identical Bands Parameters

The N_{π} proton bosons and N_{ν} neutron bosons are assigned as an intrinsic quantum number on the basis of the concept of F-spin [39], $F = 1/2$, with projection $F_0 = +1/2$ for proton bosons and $F_0 = -1/2$ for neutron bosons.

Therefore, a certain nucleus is then characterized by two quantum numbers,

$$F = \Sigma F_i = \frac{1}{2}(N_{\pi} + N_{\nu}) = \frac{1}{4}(N_p + N_n),$$

and

$$F_0 = \frac{1}{2}(N_{\pi} - N_{\nu}) = \frac{1}{4}(N_p - N_n).$$

On squaring and subtracting, these yield

$$4(F^2 - F_0^2) = 4N_\pi N_\nu = N_p N_n.$$

In nuclear structure, the product ($N_p N_n$) is used for the classification of the transitional region [24,40–42]. Saha and Sen [25] also considered that the moment of inertia ($\mathcal{J}$) has a simple dependence on the product of valence proton and neutron numbers ($N_p N_n$) written in the form:

$$\mathcal{J} \propto SF.SP, \quad (9)$$

where SF and SP are known as the structure factor and saturation parameter. They are given as:

$$SF = N_p N_n (N_p + N_n), \quad (10)$$

and

$$SP = \left[1 + \frac{SF}{(SF)_{max}} \right]^{-1}, \quad (11)$$

computed by taking

$$N_p = \min[(Z - 50), (82 - Z)], \quad (12)$$

$$N_n = \min[(Z - 82), (126 - Z)]. \quad (13)$$

Here, N_p and N_n are the valence nucleons, which are counted from the nearest closed shell. The low spin dynamic moment of inertia is also defined as:

$$\mathcal{J}^{(2)} = \frac{4}{E_\gamma(I + 4 \rightarrow I + 2) - E_\gamma(I + 2 \rightarrow I)} \quad (14)$$

where $\mathcal{J}^{(2)}$ shows an approximate dependence on structure factor as,

$$\mathcal{J}^{(2)} \propto (SF)^{\frac{1}{2}}. \quad (15)$$

The structure factor is not only related to the absolute value of the ground state moment of inertia, but also to its angular momentum. Furthermore, it was given [22,43–45] that the development of collectivity and deformation in medium and heavy nuclei is very smoothly parametrized by the P-factor. It is defined as:

$$P = \frac{N_p N_n}{N_p + N_n} \quad (16)$$

The P-factor can be taken as the ratio of the number of valence n-p residual interactions to the number of valence like nucleon-pairing interactions, or, if the n-p and pairing interactions are orbit-independent, then P is proportional to the ratio of the integrated n-p interaction strength. With the help of all the above parameter calculations, the identical bands are verified for Super-Deformed nuclei.

4. Results

In the present paper, we have presented a total of 16 pairs of SDIBs bands in the $A = 190$ mass region. The experimental data for SD bands in this region have been taken from the ENSDF and XUNDL databases [46]. One challenging and interesting study is to predict the correct band-head spin and transition energies. However, physicists proposed

several theoretical procedures to estimate the band-head spin, and the reason for this is the absence of linkage with normal bands.

We have also carried out the calculations to estimate the band-head spin for the SD bands in the $A = 190$ mass region using a modified VMI model. The results are presented in Table 1. In Table 1, we reported the details of the two important parameters $\mathcal{J}_0$ and C that are obtained after fitting the VMI equations. The last two columns give the experimental band-head spin and estimated band-head spin for the 16 pairs of SD bands in the $A = 190$ mass region. It is surprising to note that the band-head spin for these 16 pairs of identical bands in SD nuclei are experimentally available.

Table 1. Experimental and estimated band-head spin (I_0) for identical bands in SD nuclei, along with the stiffness constant (C) and band-head moment of inertia ($\mathcal{J}_0$) used in the fitting procedure, are given. A total of 16 SD band pairs for *Hg*, *Pb*, *Tl*, and *Bi* have been fitted by using the VMI equation, where b1, b2, b3, b4, b5 and b6 represent band1, band2, band3, band4, band5 and band6, respectively. The fitted parameters within a pair are the same, except for one or two cases.

Pair	SD Band	Stiffness Constant ($C \times 10^6$) (keV ³)	Band-Head Moment of Inertia ($\mathcal{J}_0$) (keV ⁻¹)	Band-Head Spin (I_0) Exp. Data	Band-Head Spin (I_0) VMI
1	¹⁹¹ Hg(b2)	7.88	0.0944	10.5	10.5
	¹⁹³ Pb(b3)	7.80	0.0946	10.5	10.5
2	¹⁹¹ Hg(b2)	7.88	0.0944	10.5	10.5
	¹⁹³ Hg(b2)	8.15	0.0936	10.5	10.5
3	¹⁹³ Hg(b2)	8.15	0.0936	10.5	10.5
	¹⁹³ Hg(b4)	8.15	0.0936	10.5	10.5
4	¹⁹¹ Hg(b3)	7.87	0.0944	11.5	11.5
	¹⁹³ Pb(b4)	7.98	0.0941	11.5	11.5
5	¹⁹² Hg(b2)	7.93	0.0942	10	10
	¹⁹⁴ Hg(b2)	8.00	0.0940	8	8
6	¹⁹³ Hg(b1)	5.46	0.0935	9.5	9.5
	¹⁹³ Hg(b3)	8.25	0.0933	9.5	9.5
7	¹⁹³ Hg(b1)	5.46	0.0935	9.5	9.5
	¹⁹⁵ Hg(b2)	8.09	0.0937	11.5	11.5
8	¹⁹¹ Tl(b2)	8.31	0.0931	12.5	12.5
	¹⁹³ Pb(b5)	8.32	0.0931	8.5	8.5
9	¹⁹⁵ Pb(b1)	13.16	0.0987	7.5	7.5
	¹⁹⁷ Pb(b4)	13.14	0.0988	9.5	9.5
10	¹⁹⁴ Pb(b1)	5.28	0.0877	4	4
	¹⁹² Hg(b1)	6.93	0.0877	8	8
11	¹⁹⁵ Tl(b1)	15.02	0.0955	5.5	5.5
	¹⁹⁷ Bi(b1)	14.82	0.0958	7.5	7.5
12	¹⁹¹ Au(b1)	3.48	0.1158	9.5	9.5
	¹⁹¹ Hg(b1)	6.97	0.1157	16.5	16.5
13	¹⁹¹ Au(b2)	8.39	0.0929	17.5	17.5
	¹⁹¹ Tl(b1)	8.44	0.0928	11.5	11.5
14	¹⁹¹ Au(b2)	8.39	0.929	17.5	17.5
	¹⁹³ Pb(b6)	8.43	0.0928	9.5	9.5

Table 1. Cont.

Pair	SD Band	Stiffness Constant ($C \times 10^6$) (keV ³)	Band-Head Moment of Inertia ($\mathcal{J}_0$) (keV ⁻¹)	Band-Head Spin (I_0) Exp. Data	Band-Head Spin (I_0) VMI
15	¹⁹⁸ Pb(b3)	7.17	0.0873	8	8
	¹⁹⁶ Pb(b1)	7.19	0.0872	6	6
16	¹⁹⁶ Pb(b1)	7.19	0.0872	6	6
	¹⁹² Pb(b1)	5.45	0.0870	8	8

We have plotted the RMS deviation versus spin, showing the smallest deviation for the particular spin called band-head spin (I_0). The results are encouraging and reflect the suitability of model in predicting the band-head spin for SD bands in the $A = 190$ mass region. The plots are shown in Figure 1. In Figure 1, we have shown only two pairs as illustration one [¹⁹¹Hg(b2), ¹⁹³Pb(b3)] in which both the bands in the pair have same spin value, i.e., $I_0 = 10.5$, which means that the band-head spin in a particular pair has identical value. The same is performed for another pair [¹⁹¹Au(b2), ¹⁹¹Tl(b1)] where we found $I_0 = 17.5$ for [¹⁹¹Au(b2)] and $I_0 = 11.5$ for [¹⁹¹Tl(b1)], which depicts that band-head spins are in good agreement with the experimental value, although are inconsistent with each other within the pair. The band-head spin for all 16 pairs are thus calculated and verified with experimental values.

In Tables 2–4, we presented the detailed results of a model where all the known γ -transition energies, calculated transition energies, estimated spins along with moment of inertia of SD bands have been listed. We shall now present the main findings from our calculations:

(i) In Table 2, we first mention four pairs of SDIBs in which moment of inertia, spins and γ -transition energies, all are identical to each other within the pair. The important finding is that in our cases, the spins for the conjugate nuclei is also identical, not just the moment of inertia and γ -transition energies as mentioned by authors in earlier works to define identical bands [32,34]. Furthermore, it is noticeable that the experimental γ -energies are in good agreement with the calculated γ -energies;

(ii) In Table 3, next, seven pairs of SDIBs are reported, in which the moment of inertia and the γ -transition energies have good consistency with the same spin, although the band-head is different within the pair. For example, in ¹⁹²Hg(b2), the estimated spin $I = 12$ has energy 241 keV, which is compatible with the energy 242 keV at spin $I = 12$ for ¹⁹⁴Hg(b2), but the band-head spin for ¹⁹⁴Hg(b2) is 10 while band-head spin for ¹⁹²Hg(b2) is 12. These bands do not originate from the same spin value. On comparison of the transition energy from the identical spin, they are in good agreement with each other within the pair. For an identical band, the two different nuclei must have nearly same transition energies E_γ within ± 3 keV, while the cases mentioned in Table 3 show that the transition energies E_γ for the two different nuclei are within ± 3 keV initially, and the difference keeps on increasing as we go towards a higher spin in most cases;

(iii) In Table 4, we have presented the remaining five pairs of SDIBs in which the moment of inertia and experimental transition energies have good agreement with each other within the pair, but the band-head spins are not identical for the transition energies. For example, in the case of ¹⁹¹Au(b2), the band-head spin is at $I = 20.5$, while in ¹⁹¹Tl(b1) the band-head spin is at $I = 13.5$. There are two different band-head spin values, but if we compare the energies of these two bands, the energy at spin $I = 20.5$ is 398 keV tentatively matches with the energy of 399 keV at spin $I = 19.5$, as shown in Figure 1. Thus, these are the cases where the spin values are not identical within the pair and are either increasing or decreasing by one unit; however, they can be considered identical on the basis of transition energies and moment of inertia.

Table 2. A total of 4 pairs of SDIB are presented. Band-head moment of inertia, estimated spins, experimental transition energies and calculated transition energies for the $A = 190$ mass region are given for the SDIB in which all the parameters are identical.

$^{191}\text{Hg}(b2)$																			
$\mathcal{J}_0$	0.0944																		
Estimated Spin	12.5	14.5	16.5	18.5	20.5	22.5	24.5	26.5	28.5	30.5	32.5	34.5	36.5	38.5	40.5	42.5			
$E_\gamma \text{Exp.}(keV)$	252	293	333	373	412	450	488	525	562	597	632	666	700	733	765	796			
$E_\gamma \text{Cal.}(keV)$	251.46	292.2	332.41	372	410.9	449.04	486.32	522.69	558.06	592.36	625.5	657.41	688.02	717.24	745.01	771.23			
$^{193}\text{Pb}(b3)$																			
$\mathcal{J}_0$	0.0946																		
Estimated Spin	12.5	14.5	16.5	18.5	20.5	22.5	24.5	26.5	28.8	30.5	32.5	34.5	36.5						
$E_\gamma \text{Exp.}(keV)$	252	292	332	372	412	451	489	527	563	600	637	672	709						
$E_\gamma \text{Cal.}(keV)$	250.92	291.57	331.69	371.19	410.00	448.04	485.24	521.52	556.79	590.99	624.04	655.86	686.37						
$^{191}\text{Hg}(b2)$																			
$\mathcal{J}_0$	0.0944																		
Estimated Spin	12.5	14.5	16.5	18.5	20.5	22.5	24.5	26.5	28.5	30.5	32.5	34.5	36.5	38.5	40.5	42.5			
$E_\gamma \text{Exp.}(keV)$	252	293	333	373	412	450	488	525	562	597	632	666	700	733	765	796			
$E_\gamma \text{Cal.}(keV)$	251.46	292.2	332.41	372	410.9	449.04	486.32	522.69	558.06	592.36	625.5	657.41	688.02	717.24	745.01	771.23			
$^{193}\text{Hg}(b2)$																			
$\mathcal{J}_0$	0.0936																		
Estimated Spin	12.5	14.5	16.5	18.5	20.5	22.5	24.5	26.5	28.5	30.5	32.5	34.5	36.5	38.5	40.5	42.5	44.5	46.5	48.5
$E_\gamma \text{Exp.}(keV)$	254	295	335	374	413	451	488	525	560	595	629	662	694	726	757	787	818	848	876
$E_\gamma \text{Cal.}(keV)$	253.63	294.74	335.31	375.26	414.52	453.02	490.67	527.39	563.12	597.77	631.28	663.55	694.51	724.09	752.22	778.8	803.76	827.06	848.57
$^{193}\text{Hg}(b2)$																			
$\mathcal{J}_0$	0.0936																		
Estimated Spin	12.5	14.5	16.5	18.5	20.5	22.5	24.5	26.5	28.5	30.5	32.5	34.5	36.5	38.5	40.5	42.5	44.5	46.5	48.5
$E_\gamma \text{Exp.}(keV)$	254	295	335	374	413	451	488	525	560	595	629	662	694	726	757	787	818	848	876
$E_\gamma \text{Cal.}(keV)$	253.63	294.74	335.31	375.26	414.52	453.02	490.67	527.39	563.12	597.77	631.28	663.55	694.51	724.09	752.22	778.8	803.76	827.06	848.57

Table 2. Cont.

¹⁹³ Hg(<i>b4</i>)																			
$\mathcal{J}_0$	0.0936																		
Estimated Spin	12.5	14.5	16.5	18.5	20.5	22.5	24.5	26.5	28.5	30.5	32.5	34.5	36.5	38.5	40.5	42.5	44.5	46.5	48.5
$E_{\gamma}Exp.(keV)$	254	295	335	375	413	451	488	525	560	595	629	662	694	726	757	787	818	848	876
$E_{\gamma}Cal.(keV)$	253.63	294.74	335.31	375.27	414.52	453.02	490.67	527.39	563.12	597.77	631.28	663.55	694.51	724.09	752.22	778.8	803.76	827.06	848.57
¹⁹¹ Hg(<i>b3</i>)																			
$\mathcal{J}_0$	0.0944																		
Estimated Spin	13.5	15.5	17.5	19.5	21.5	23.5	25.5	27.5	29.5	31.5	33.5	35.5	37.5	39.5	41.5	43.5			
$E_{\gamma}Exp.(keV)$	272	313	353	392	430	467	504	540	575	609	643	676	709	740	771	801			
$E_{\gamma}Cal.(keV)$	271.89	312.38	352.29	391.55	430.07	467.79	504.63	540.51	575.35	609.07	641.61	672.88	702.81	731.31	758.32	783.75			
¹⁹³ Pb(<i>b4</i>)																			
$\mathcal{J}_0$	0.0941																		
Estimated Spin	13.5	15.5	17.5	19.5	21.5	23.5	25.5	27.5	29.5	31.5	33.5	35.5	37.5						
$E_{\gamma}Exp.(keV)$	273	313	353	392	430	467	504	540	575	610	645	676	707						
$E_{\gamma}Cal.(keV)$	272.77	313.4	353.45	392.84	431.51	469.37	506.35	542.37	577.35	611.22	643.91	675.32	705.4						

Table 3. A total of 7 SDIB pairs are reported. The transition energies are identical for all the pairs, but the pairs originate from levels with different band-head spins. Furthermore, the difference in transition energies keeps on increasing with spins.

¹⁹² Hg(<i>b2</i>)																			
$\mathcal{J}_0$	0.0942																		
Estimated Spin	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	
$E_\gamma Exp.(keV)$	241	282	322	361	400	438	475	511	547	579	604	624	652	684	718	750	783	819	
$E_\gamma Cal.(keV)$	241.71	282.67	323.10	362.95	402.11	440.53	478.13	514.82	550.53	585.18	618.70	651.00	682.02	711.68	739.89	766.59	791.69	815.12	
¹⁹⁴ Hg(<i>b2</i>)																			
$\mathcal{J}_0$	0.0940																		
Estimated Spin	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46
$E_\gamma Exp.(keV)$	201	242	283	323	363	402	440	478	514	550	585	619	652	685	716	747	778	808	837
$E_\gamma Cal.(keV)$	200.74	242.23	283.29	323.81	363.74	403.00	441.51	479.19	515.98	551.78	586.52	620.13	652.53	683.65	713.40	741.71	768.50	793.69	817.22
¹⁹³ Hg(<i>b1</i>)																			
$\mathcal{J}_0$	0.0935																		
Estimated Spin	11.5	13.5	15.5	17.5	19.5	21.5	23.5	25.5	27.5	29.5	31.5	33.5	35.5	37.5	39.5	41.5	43.5	45.5	47.5
$E_\gamma Exp.(keV)$	233	274	314	353	392	429	464	497	528	559	590	623	656	689	721	753	785	817	848
$E_\gamma Cal.(keV)$	232.08	272.79	312.74	351.83	389.95	426.78	462.78	497.27	530.32	561.82	591.65	619.70	645.85	669.99	692.00	711.77	729.18	744.11	756.46
¹⁹³ Hg(<i>b3</i>)																			
$\mathcal{J}_0$	0.0933																		
Estimated Spin	11.5	13.5	15.5	17.5	19.5	21.5	23.5	25.5	27.5	29.5	31.5	33.5	35.5	37.5	39.5	41.5	43.5	45.5	47.5
$E_\gamma Exp.(keV)$	234	275	315	355	394	432	470	506	542	577	611	644	678	710	742	771	802	832	861
$E_\gamma Cal.(keV)$	233.66	275.14	316.13	356.54	396.29	435.32	473.54	510.88	547.26	582.60	616.83	649.87	681.64	712.07	741.08	768.59	794.52	818.81	841.37
¹⁹³ Hg(<i>b1</i>)																			
$\mathcal{J}_0$	0.0935																		
Estimated Spin	11.5	13.5	15.5	17.5	19.5	21.5	23.5	25.5	27.5	29.5	31.5	33.5	35.5	37.5	39.5	41.5	43.5	45.5	47.5
$E_\gamma Exp.(keV)$	233	274	314	353	392	429	464	497	528	559	590	623	656	689	721	753	785	817	848
$E_\gamma Cal.(keV)$	232.08	272.79	312.74	351.83	389.95	426.78	462.78	497.27	530.32	561.82	591.65	619.70	645.85	669.99	692.00	711.77	729.18	744.11	756.46
¹⁹⁵ Hg(<i>b2</i>)																			
$\mathcal{J}_0$	0.0937																		
Estimated Spin	13.5	15.5	17.5	19.5	21.5	23.5	25.5	27.5	29.5	31.5	33.5	35.5	37.5	39.5	41.5	43.5	45.5	47.5	49.5
$E_\gamma Exp.(keV)$	274	314	354	392	430	467	503	538	571	604	636	668	698	728	758	787	816	845	874
$E_\gamma Cal.(keV)$	273.94	314.74	354.96	394.53	433.36	471.39	508.53	544.71	579.85	613.88	646.71	678.27	708.48	737.27	764.56	790.27	814.33	836.66	857.17

Table 3. Cont.

¹⁹¹ Tl(b2)																	
$\mathcal{J}_0$	0.0931																
Estimated Spin			14.5	16.5	18.5	20.5	22.5	24.5	26.5	28.5	30.5	32.5	34.5	36.5	38.5		
$E_\gamma Exp.(keV)$			296	337	378	417	456	493	530	566	600	633	666	698	728		
$E_\gamma Cal.(keV)$			296.34	337.14	377.32	416.81	455.53	493.40	530.36	566.31	601.18	634.91	667.40	698.59	728.39		
¹⁹³ Pb(b5)																	
$\mathcal{J}_0$	0.0931																
Estimated Spin	10.5	12.5	14.5	16.5	18.5	20.5	22.5	24.5	26.5	28.5	30.5	32.5	34.5	36.5			
$E_\gamma Exp.(keV)$	213	255	296	336	375	414	451	489	527	562	596	631	667	701			
$E_\gamma Cal.(keV)$	213.21	255.00	296.35	337.14	377.33	416.82	455.55	493.43	530.39	566.35	601.24	634.97	667.48	698.68			
¹⁹⁵ Pb(b1)																	
$\mathcal{J}_0$	0.0987																
Estimated Spin	9.5	11.5	13.5	15.5	17.5	19.5	21.5	23.5	25.5	27.5	29.5	31.5	33.5	35.5			
$E_\gamma Exp.(keV)$	182	222	262	302	341	381	419	458	497	536	575	613	651	689			
$E_\gamma Cal.(keV)$	181.78	221.83	261.66	301.25	340.55	379.52	418.13	456.33	494.10	531.38	568.15	604.37	639.99	674.97			
¹⁹⁷ Pb(b4)																	
$\mathcal{J}_0$	0.0988																
Estimated Spin		11.5	13.5	15.5	17.5	19.5	21.5	23.5	25.5	27.5	29.5	31.5	33.5	35.5	37.5		
$E_\gamma Exp.(keV)$		222	262	302	341	381	419	459	498	536	573	610	648	684	721		
$E_\gamma Cal.(keV)$		221.60	261.39	300.93	340.18	379.11	417.67	455.83	493.54	530.77	567.49	603.65	639.21	674.13	708.39		
¹⁹⁴ Pb(b1)																	
$\mathcal{J}_0$	0.0877																
Estimated Spin	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
$E_\gamma Exp.(keV)$	125	170	213	256	298	340	380	420	458	496	533	568	603	638	672	706	740
$E_\gamma Cal.(keV)$	124.88	169.67	213.88	257.36	299.96	341.52	381.89	420.91	458.44	494.31	528.38	560.49	590.48	618.21	643.52	666.26	686.26

Table 4. A total of 5 SDIBs are given in which band-head spins are not identical with each other. They differ by either one or two spins. The transition energies are consistent with each other in a pair along with band-head moment of inertia, but the energies matches either at one increasing or one decreasing spin.

¹⁹¹ Au(b1)																			
$\mathcal{J}_0$	0.1158																		
Estimated Spin	11.5	13.5	15.5	17.5	19.5	21.5	23.5	25.5	27.5	29.5	31.5	33.5	35.5	37.5	39.5	41.5	43.5	45.5	47.5
$E_{\gamma}Exp.(keV)$	187	229	271	312	352	392	430	468	505	541	576	611	645	678	710	743	774	806	837
$E_{\gamma}Cal.(keV)$	192.12	228.05	264.48	301.48	339.14	377.52	416.71	456.78	497.81	539.87	583.05	627.42	673.05	720.02	768.42	818.30	869.77	922.88	977.71
¹⁹¹ Hg(b1)																			
$\mathcal{J}_0$	0.1157																		
Estimated Spin	18.5	20.5	22.5	24.5	26.5	28.5	30.5	32.5	34.5	36.5	38.5	40.5	42.5	44.5					
$E_{\gamma}Exp.(keV)$	311	352	392	431	470	508	546	582	619	654	688	722	756	789					
$E_{\gamma}Cal.(keV)$	315.87	352.15	388.84	425.96	463.55	501.63	540.25	579.45	619.26	659.73	700.89	742.78	785.44	828.91					
¹⁹¹ Au(b2)																			
$\mathcal{J}_0$	0.0929																		
Estimated Spin	20.5	22.5	24.5	26.5	28.5	30.5	32.5	34.5	36.5	38.5	40.5								
$E_{\gamma}Exp.(keV)$	398	437	476	513	549	584	618	652	684	716	746								
$E_{\gamma}Cal.(keV)$	398.04	437.25	475.66	513.18	549.74	585.27	619.68	652.9	684.85	715.46	744.64								
¹⁹¹ Tl(b1)																			
$\mathcal{J}_0$	0.0928																		
Estimated Spin	13.5	15.5	17.5	19.5	21.5	23.5	25.5	27.5	29.5	31.5	33.5	35.5	37.5						
$E_{\gamma}Exp.(keV)$	277	318	359	399	438	477	515	551	588	622	656	690	721						
$E_{\gamma}Cal.(keV)$	276.65	317.87	358.51	398.5	437.77	476.22	513.8	550.42	586	620.47	653.75	685.77	716.47						
¹⁹¹ Au(b2)																			
$\mathcal{J}_0$	0.0929																		
Estimated Spin	20.5	22.5	24.5	26.5	28.5	30.5	32.5	34.5	36.5	38.5	40.5								
$E_{\gamma}Exp.(keV)$	398	437	476	513	549	584	618	652	684	716	746								
$E_{\gamma}Cal.(keV)$	398.04	437.25	475.66	513.18	549.74	585.27	619.68	652.9	684.85	715.46	744.64								
¹⁹³ Pb(b6)																			
$\mathcal{J}_0$	0.0928																		
Estimated Spin	11.5	13.5	15.5	17.5	19.5	21.5	23.5	25.5	27.5	29.5	31.5	33.5	35.5	37.5					
$E_{\gamma}Exp.(keV)$	235	276	316	356	394	433	471	507	544	580	615	650	684	718					
$E_{\gamma}Cal.(keV)$	234.93	276.64	317.86	358.50	398.49	437.75	476.20	513.77	550.38	585.96	620.42	653.69	685.69	716.35					

Table 4. Cont.

¹⁹⁸ Pb(<i>b</i> 3)																
$\mathcal{J}_0$	0.0873															
Estimated Spin	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
$E_\gamma Exp.(keV)$	216	260	303	345	386	429	469	548	586	624	660	696				
$E_\gamma Cal.(keV)$	215.56	259.79	303.35	346.13	388.02	428.90	468.65	507.17	544.33	580.02	614.13	646.54				
¹⁹⁶ Pb(<i>b</i> 1)																
$\mathcal{J}_0$	0.0872															
Estimated Spin	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
$E_\gamma Exp.(keV)$	171	216	260	303	346	388	429	469	509	547	584	621	655	689	720	752
$E_\gamma Cal.(keV)$	170.99	215.81	260.08	303.69	346.52	388.45	429.38	469.17	507.73	544.93	580.66	614.8	647.23	677.85	706.54	733.17
¹⁹⁶ Pb(<i>b</i> 1)																
$\mathcal{J}_0$	0.0872															
Estimated Spin	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
$E_\gamma Exp.(keV)$	171	216	260	303	346	388	429	469	509	547	584	621	655	689	720	752
$E_\gamma Cal.(keV)$	170.99	215.81	260.08	303.69	346.52	388.45	429.38	469.17	507.73	544.93	580.66	614.8	647.23	677.85	706.54	733.17
¹⁹² Pb(<i>b</i> 1)																
$\mathcal{J}_0$	0.0870															
Estimated Spin	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
$E_\gamma Exp.(keV)$	215	262	304	345	385	424	462	499	535	570	605	640				
$E_\gamma Cal.(keV)$	215.62	259.47	302.43	344.36	385.09	424.48	462.37	498.61	533.04	565.51	595.87	623.96				

In Figures 2 and 3, the dynamic moment of inertia $\mathcal{J}^{(2)}$ is plotted with respect to the rotational frequency ($\hbar\omega$). From Figure 2, it is observed that, in most of the cases, there is a good agreement in experimental and calculated dynamic moment of inertia. Furthermore, the (calculated) dynamic moment of inertia shows the same behaviour within the pair.

It is very clear from Figures 2 and 3 that the model reproduces the experimental trend of the moment of inertia $\mathcal{J}^{(2)}$ in the low frequency region; however, the trend deviates largely at the high frequency region with the theoretical trend of the moment of inertia $\mathcal{J}^{(2)}$. The modified VMI model is mainly composed of two term, the potential energy term and rotational energy term, and it does not contain any Coriolis effect in the calculations. Hence, the VMI model reproduces the generic rotational properties of identical SD bands very well. Since the band-head moment of inertia intimately depends upon the intrinsic structure of rotational bands, it was mentioned that for truly identical bands, it seems necessary that both bands must have the same moment of inertia. Thus, in Figures 2 and 3, the theoretical values of the moment of inertia in the $A = 190$ mass region display the same smooth rise in the dynamic moment of inertia with an increasing rotational frequency, while the experimental plot shows a staggering pattern, and not a smooth rise, in most of the cases. The two plots (experimental and calculated) will have good matching when we consider the Coriolis effect due to nearby levels in calculations. The focus of the present work is to predict the spins of SD bands for which this model is significantly valid, and we can define identical bands on the basis of level spins, transition energies and (dynamic) moment of inertia, while the coriolis effect is required to explain the staggering in experimental data.

Khalaf et al. [29] mentioned three pairs of low spin identical bands in Normal Deformed nuclei using the NpNn scheme, which motivates us to apply the same scheme to SD bands. To verify our calculations, we have carried out the calculations of identical band correlation parameters using the NpNn scheme, and found one pair of SDIB [$^{195}\text{Ti}(b1)$, $^{197}\text{Bi}(b1)$] in which all the parameters are identical, as shown in Table 5. In other cases, we also applied the same procedure, but the approach is unsuccessful. We observe that there is a difference in the NpNn values for the identical band pairs in the rest of the cases. Figure 3 gives the plot of the dynamic moment of inertia with rotational frequency for the mentioned pair. Overall, the problem of identical bands in Super-Deformed nuclei is treated successfully.

Table 5. Correlation parameters for [$^{195}\text{Ti}(b1)$ and $^{197}\text{Bi}(b1)$] using the NpNn Scheme.

	^{195}Ti	^{197}Bi
J_0	0.0955	0.0958
(N_π, N_ν)	(0.5,6)	(0.5,6)
$N_p N_n$	12	12
F	3.25	3.25
F_0	−2.75	−2.75
P	0.9231	0.9231
SF	156	156
SP	0.71942	0.71942
$J_{SF}^{(2)}$	0.095	0.095
$R(4/2)$	1.436	1.354
$R(6/2)$	1.8617	1.7074

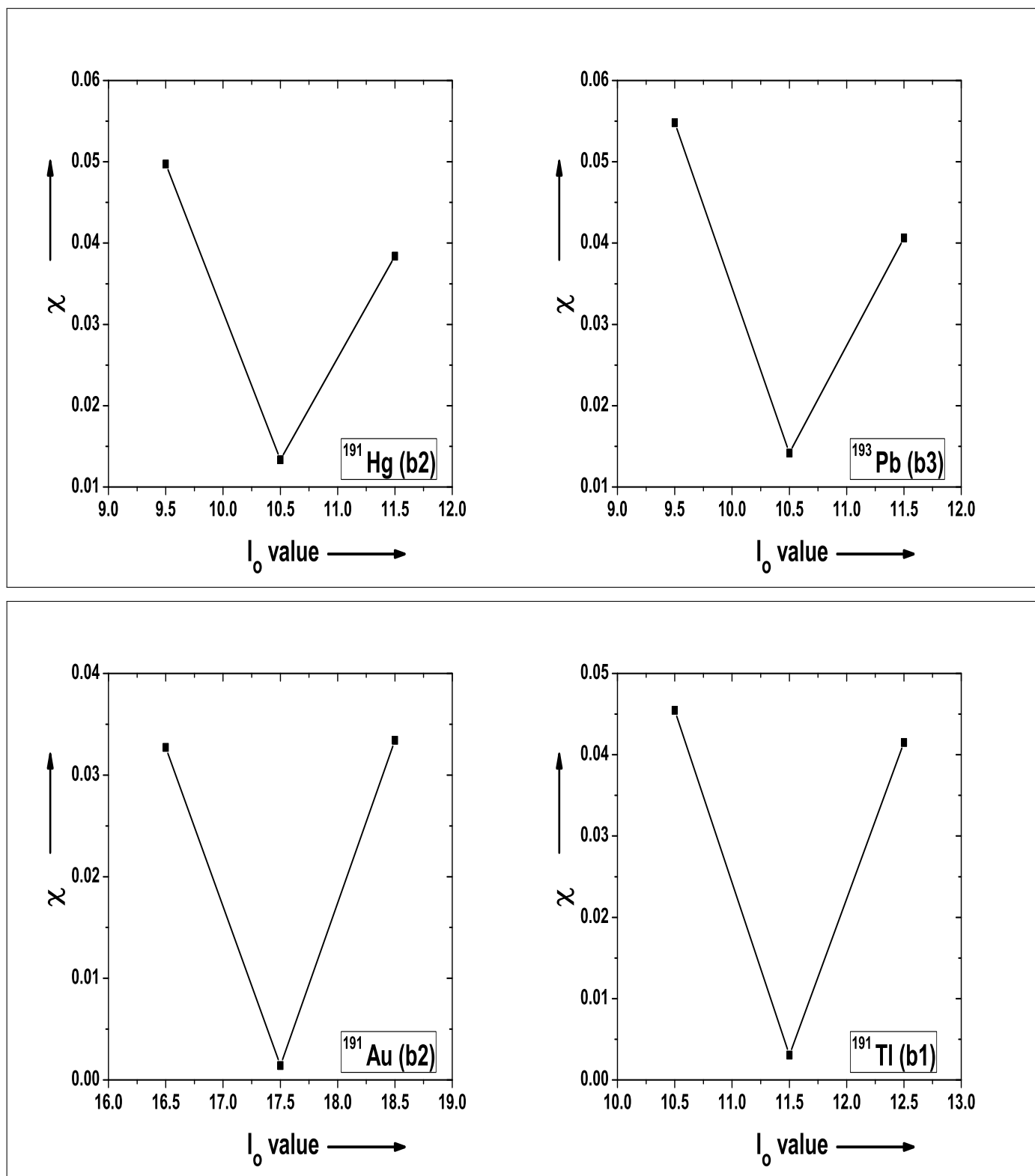


Figure 1. The RMS deviation plots for spin assignments in the $[^{191}\text{Hg}(b2), ^{193}\text{Pb}(b3)]$ pair and the $[^{191}\text{Au}(b2), ^{191}\text{Tl}(b1)]$ pair are shown. The band-head spins predicted using VMI equation is in agreement with the experimentally known band-head spins for both pairs. For the $[^{191}\text{Hg}(b2), ^{193}\text{Pb}(b3)]$ pair, the estimated band-head spin is consistent, while for the $[^{191}\text{Au}(b2), ^{191}\text{Tl}(b1)]$ pair, the estimated band-head spin is different. I_0 is the spin value prescribed to the lowest level observed.

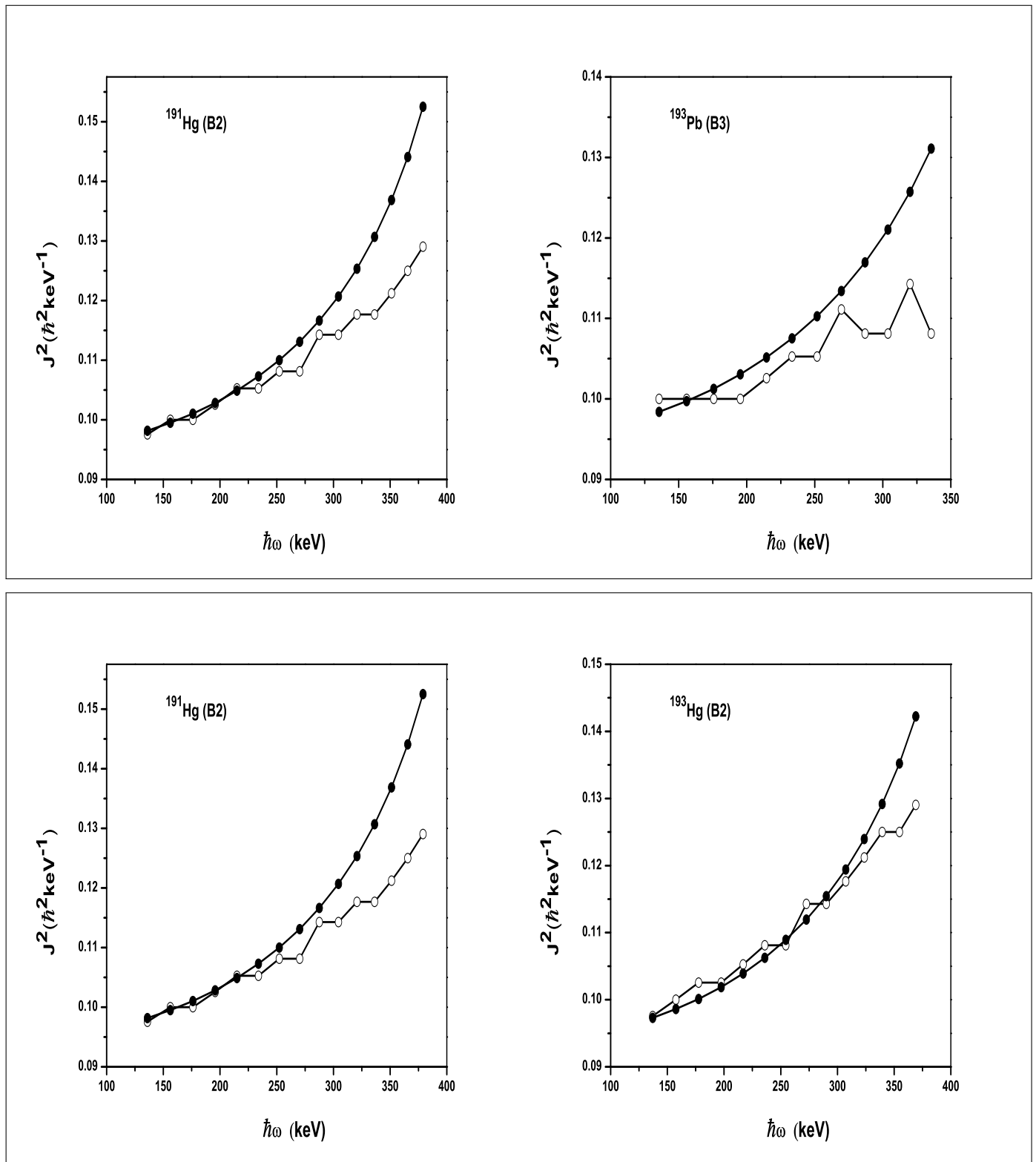


Figure 2. Cont.

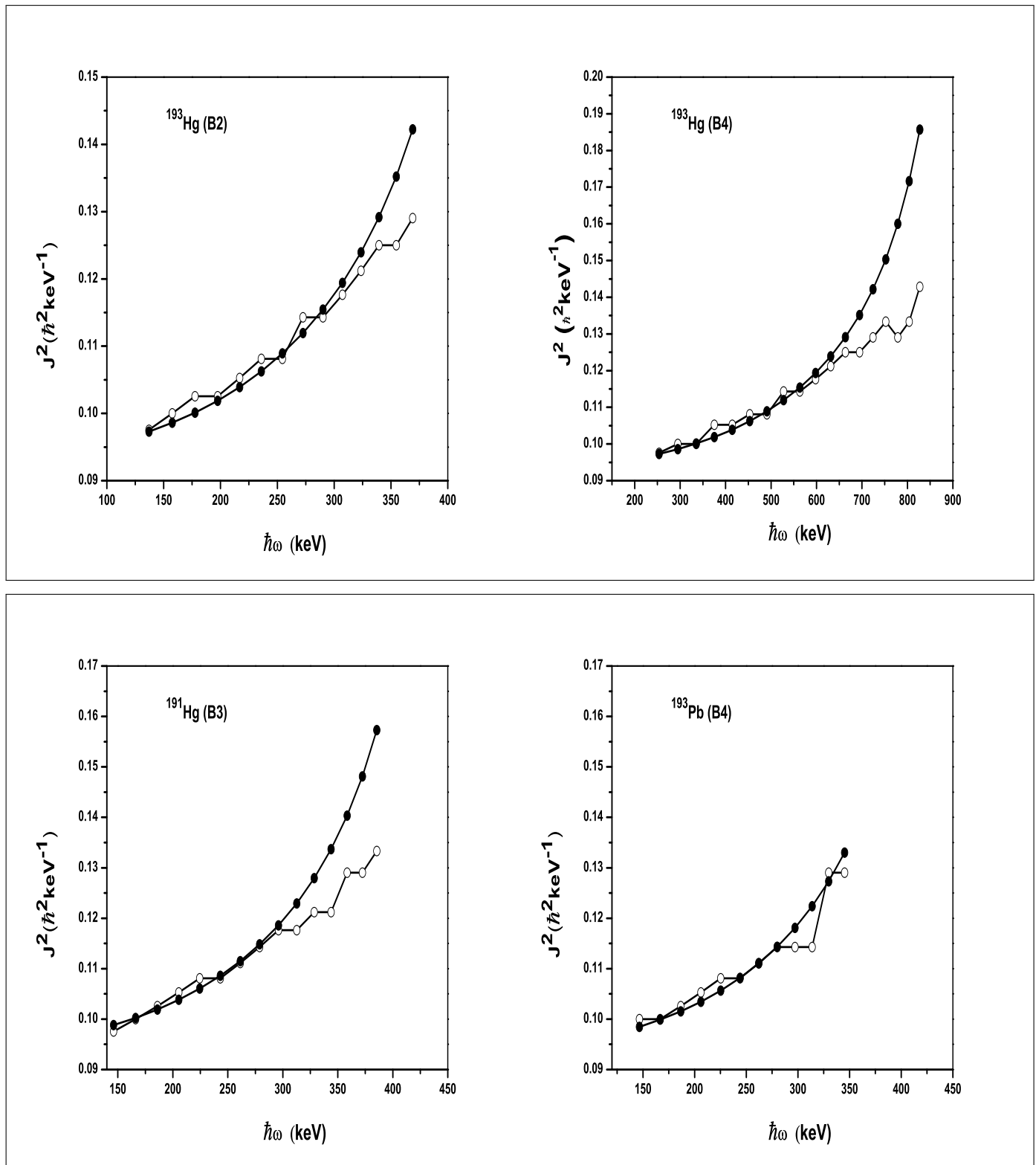


Figure 2. Experimental and theoretical dynamic moment of inertia plots with rotational frequency for the 4 pairs of SDIBs are presented. The plots show a good agreement between two isotopes in most of the cases. The circle ($\circ$) represents the experimental values and the solid circle ($\bullet$) represents the theoretical values.

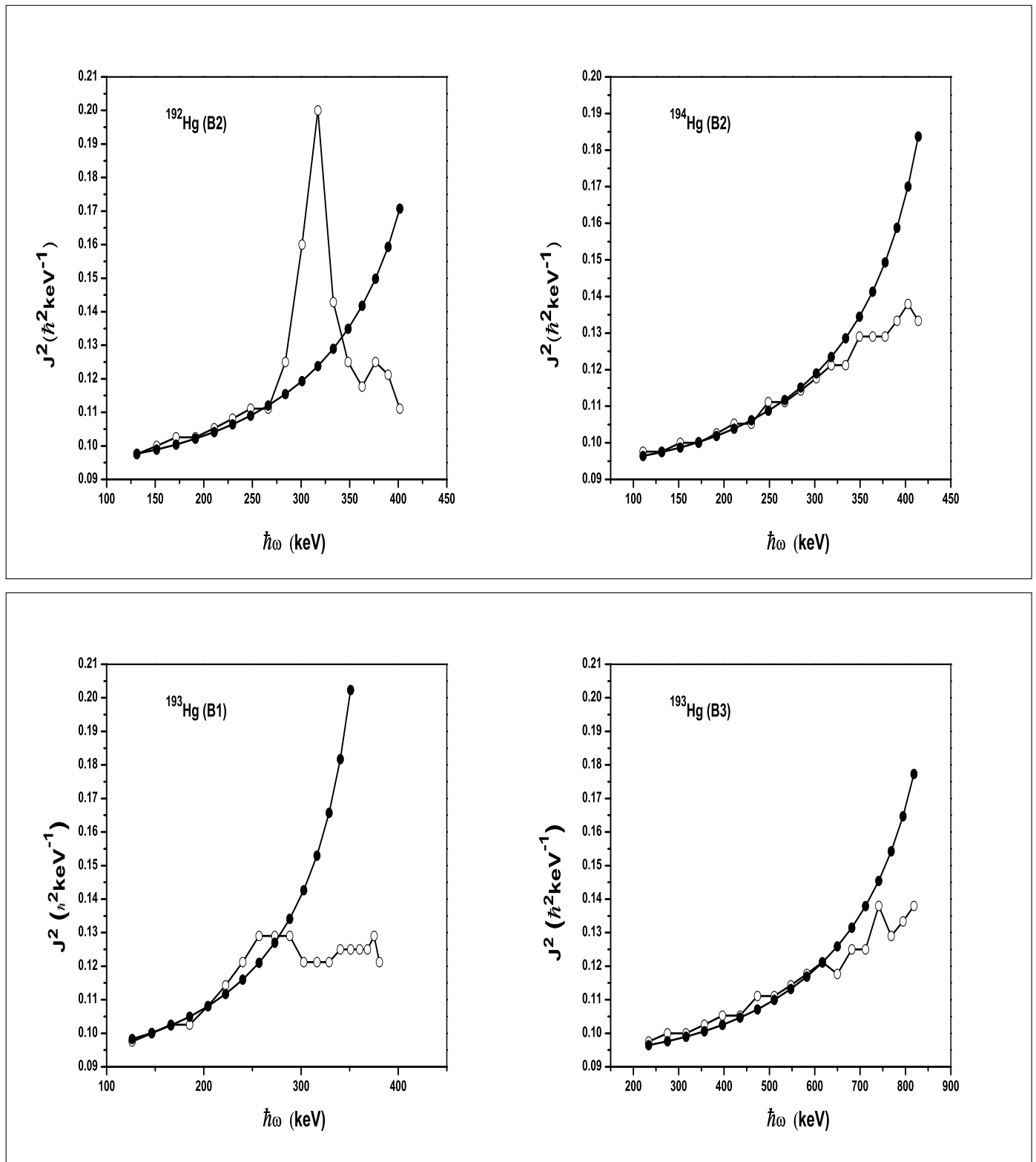
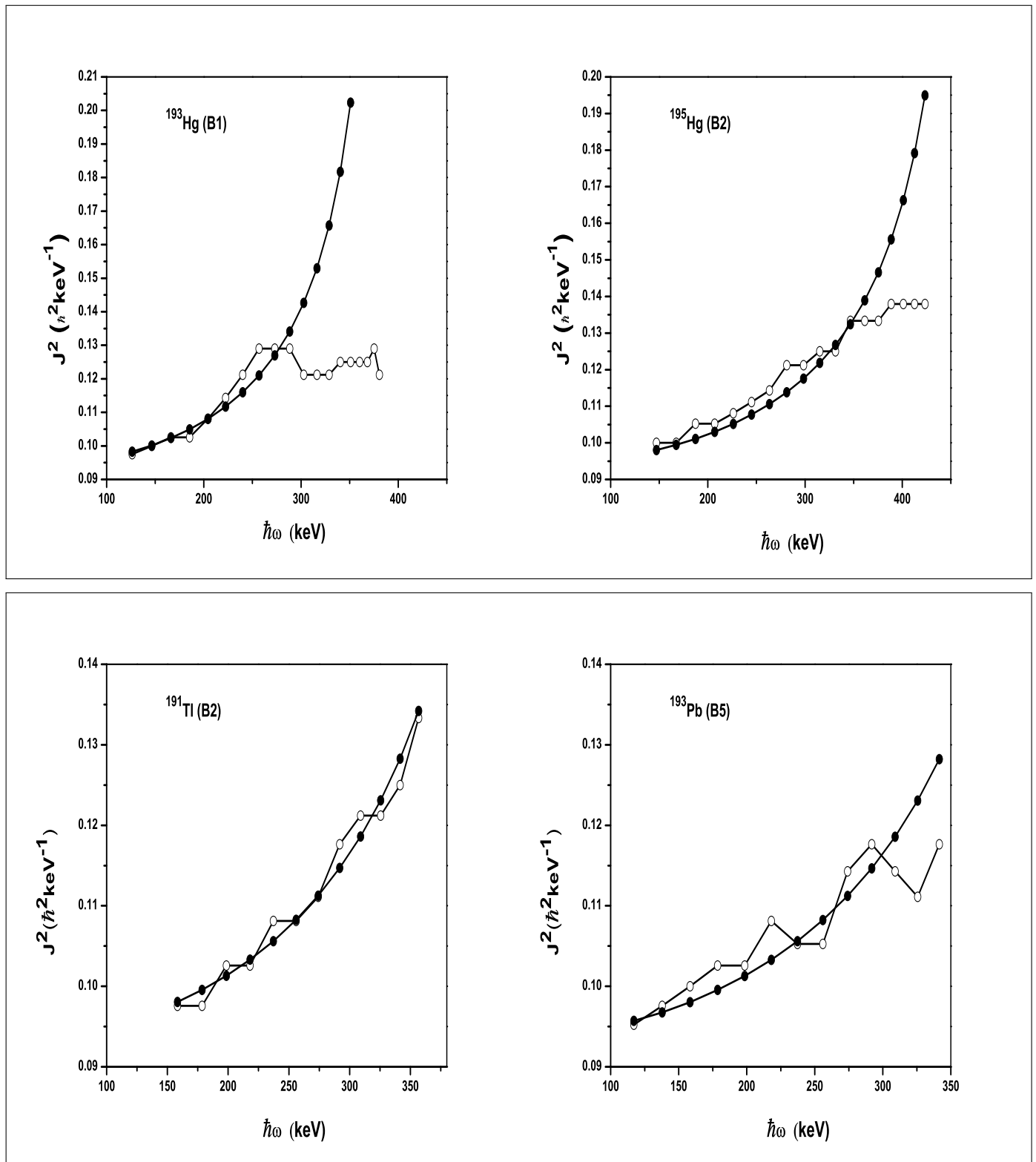


Figure 3. Cont.



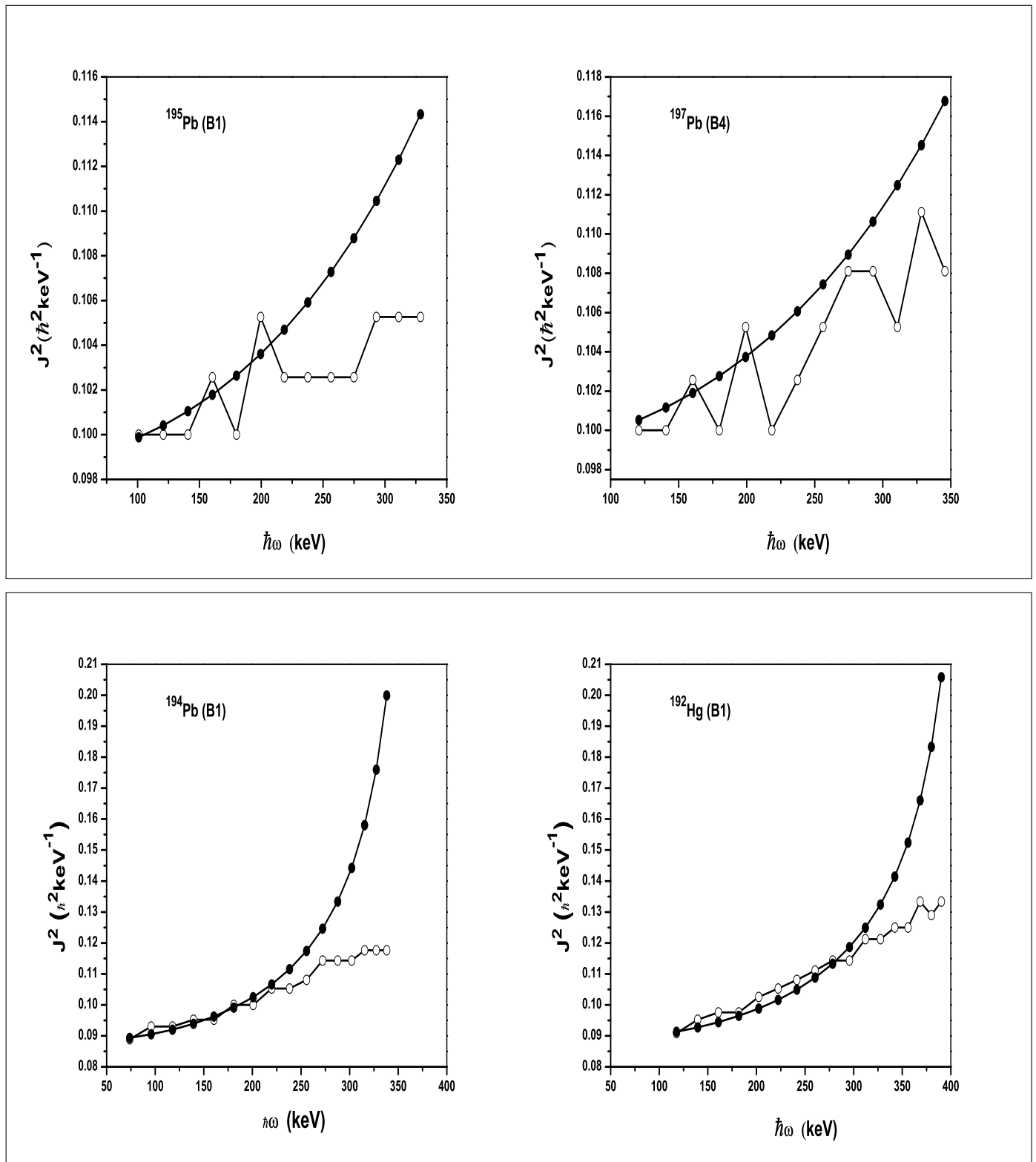


Figure 3. Cont.

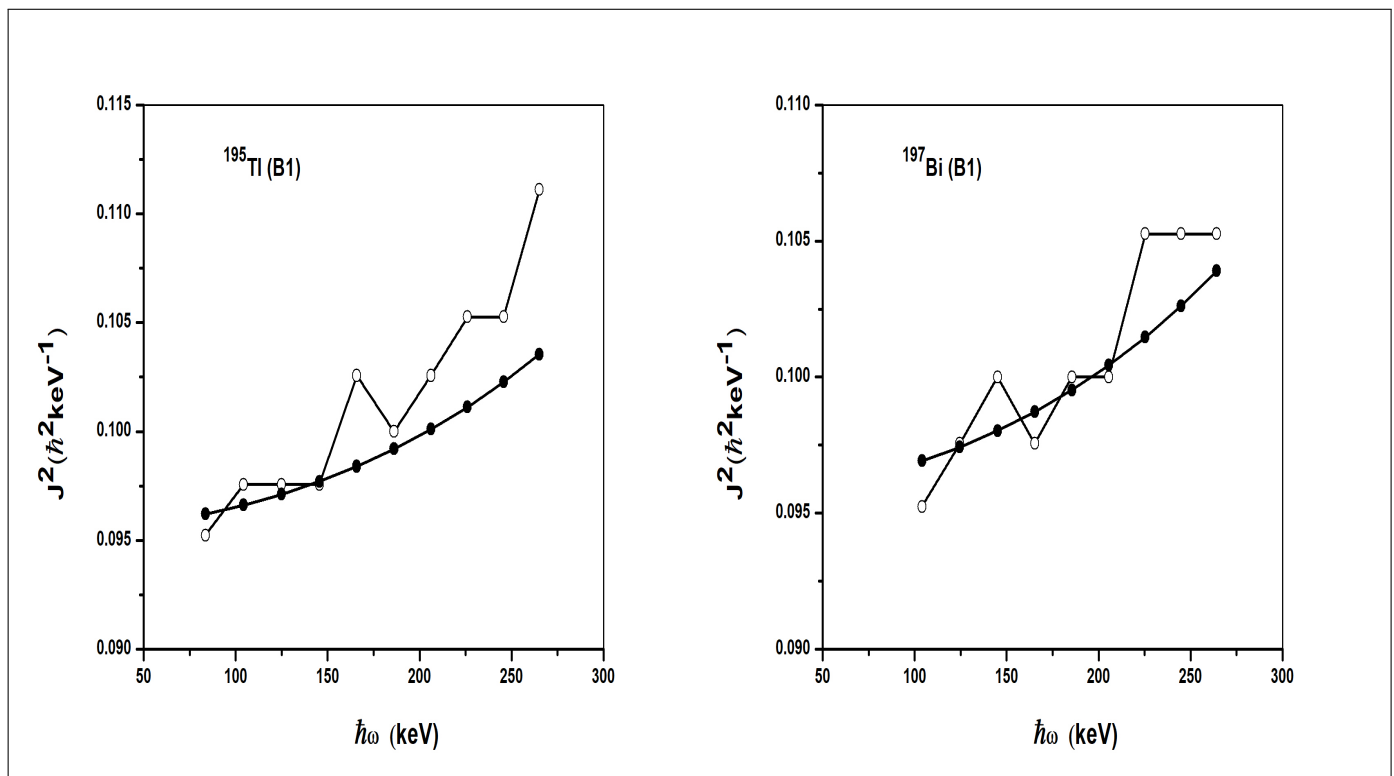


Figure 3. Experimental and theoretical dynamic moments of inertia are plotted with rotational frequency for 7 pairs of SDIBs. The plots show inconsistency between two isotopes in most of the cases. The cases mentioned have a band-head spin difference of one or two spins. The circle (○) represents the experimental values and the solid circle (●) represents the theoretical values.

5. Conclusions

To conclude, we have carried out an analysis of all the known SD bands in the $A = 190$ mass region for the identical bands. A total of 16 pairs of SD bands have been analysed. The band-head spin is estimated for all cases in which the experimental data is available. We find that we are able to predict the spin in all cases successfully. We are also able to give the SDIBs on the basis of the spins of the level of SD band, along with the moment of inertia and transition energies, which was not mentioned in earlier studies.

By using the modified VMI model, we find agreement between experimental transition energies and the theoretical ones. Further, we tested the existence of identical bands using the NpNn scheme. We extracted all the IB symmetry parameters like P-factor, saturation factor SF , structure factor SP , etc. successfully, which all depends on the valence proton and neutron numbers in one case. Pb is a doubly magic nuclei (both neutron and proton are a magic number), which suggests that an NpNn of zero could be one of the possible reasons that we are not able to extract the IB symmetry parameters in all other cases. Furthermore, for nuclei with $z \geq 78$ i.e., in case of Hg and Au, N breaks into $N = 82-114$ and $N = 114-126$. Thus, Hg and Au isotopes have lower valence neutron numbers, which could be another reason for the lack of success with the current approach.

SDIBs are very important to study the nuclear structure in heavy nuclei, in order to improve the fitting of dynamic moment of inertia. Our next step is to develop the VMI model with the Coriolis effect to explain the staggering observed in rotational bands of Super-Deformed nuclei. This analysis may help us in future work to improve the fit and the comparison between theoretical and experimental moments of inertia.

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