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Higher-Spin Currents, Operator Mixing and UV Asymptotics in Large-N QCD-like Theories

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Abstract: We extend to operator mixing—specifically, to higher-spin twist-2 operators—the asymptotic theorem on the ultraviolet asymptotics of the spectral representation of 2-point correlators of multiplicatively renormalizable operators in large-N confining QCD-like theories. The extension is based on a recent differential geometric approach to operator mixing that involves the Poincaré-Dulac theorem and allows us to reduce generically the operator mixing to the multiplicatively renormalizable case, provided that $\frac{\gamma_0}{\beta_0}$ is diagonalizable and a certain nonresonant condition for its eigenvalues holds according to the Poincaré-Dulac theorem, with γ_0 and β_0 the one-loop coefficients of the anomalous dimension matrix and beta function respectively. Relatedly, we solve a conundrum about the generic nonconservation of higher-spin currents versus the conservation—up to contact terms—of the corresponding free propagators in the spectral representation of 2-point correlators of higher-spin operators of pure integer spin to the leading large-N order.

Keywords: higher-spin currents; spectral representation; large-N limit; ultraviolet asymptotics; Poincaré-Dulac theorem

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1. Introduction and Physics Motivations

The solution of SU(N) QCD with N_f flavors [1,2] of massless quarks (massless QCD for short)—whose elementary fields are massless to every order in perturbation theory and whose perturbation theory is weakly coupled in the ultraviolet (UV) but strongly coupled in the infrared because of the asymptotic freedom (AF)—must unavoidably be nonperturbative.

Indeed, the AF and renormalization group (RG) require that every physical mass of the theory must be proportional to the RG-invariant scale $\Lambda_{QCD} = \Lambda_{RGI}$—the only free parameter in the massless QCD S matrix [3]—with

$$\Lambda_{RGI} \sim c_0 \, \Lambda \, e^{-\frac{1}{2\beta_0 g^2(\Lambda)}} \, g^{-\frac{\beta_1}{\beta_0^2}}(\Lambda)(1 + \sum_{n=1} c_n g^{2n}(\Lambda)) \,, \tag{1}$$

where $g(\Lambda) = g_{YM}(\Lambda)\sqrt{N}$ is the 't Hooft bare gauge coupling [1][1] and β_0, β_1 are the renormalization-scheme independent first-two coefficients of the beta function

$$\frac{\partial g(\Lambda)}{\partial \log \Lambda} = \beta(g(\Lambda)) = -\beta_0 g^3(\Lambda) - \beta_1 g^5(\Lambda) + \cdots \,, \tag{2}$$

while the remaining coefficients in the dots and c_0, c_n are scheme dependent.

Λ_{RGI} vanishes to every order of perturbation theory according to Equation (1).

Thus, the solution for the physical mass spectrum—and a fortiori for the S matrix—is equivalent to solving a nonperturbative weak-coupling problem—for Λ_{RGI} in terms of $g(\Lambda)$—of the finest asymptotic accuracy, since

$$g^2(\Lambda) \sim \frac{1}{\beta_0 \log(\frac{\Lambda}{\Lambda_{RGI}})^2} \left(1 - \frac{\beta_1}{\beta_0^2} \frac{\log\log(\frac{\Lambda}{\Lambda_{RGI}})^2}{\log(\frac{\Lambda}{\Lambda_{RGI}})^2}\right) \tag{3}$$

vanishes as the cutoff Λ diverges, in order for Λ_{RGI} to stay finite.

In fact, the preceding equations and our further considerations apply—mutatis mutandis—not only to massless QCD but to confining asymptotically free gauge theories with a single coupling massless in perturbation theory (massless QCD-like theories for short).

In relation to the above nonperturbative problem a considerable simplification occurs in the 't Hooft large-N limit with N_f fixed [1].

In such a limit the aforementioned nonperturbative solution of large-N QCD would appear as a theory of an infinite number of glueballs and mesons [1,4]—the real asymptotic states in the S matrix to the leading large-N order—weakly coupled at all energy scales as opposed to perturbation theory, with coupling of order $\frac{1}{N}$ for the glueballs and $\frac{1}{\sqrt{N}}$ for the mesons [1] and masses proportional to Λ_{QCD}.

Consequently, to the lowest nontrivial $\frac{1}{N}$ order, i.e., in the 't Hooft planar limit [1], the 2-point connected correlators—normalized to be of order 1 for large N—of single-trace gauge-invariant operators in the glueball sector and of quark bilinears in the meson sector must be an infinite sum of propagators of free fields [4]. For example, in Minkowskian space-time for Hermitian scalar operators $\mathcal{O}^{(0)}$ of canonical dimension D to the leading large-N order

$$\begin{aligned}\int \langle \mathcal{O}^{(0)}(x)\mathcal{O}^{(0)}(0)\rangle^{P}_{conn}\, e^{-ip\cdot x} d^4x &= i\sum_{n=1}^{\infty} \frac{m_n^{(0)2D-4} r_n^{(0)} \Lambda^{P2}_{QCD}}{|p|^2 - m_n^{(0)2} + i\epsilon} \\ &= i|p|^{2D-4} \sum_{n=1}^{\infty} \frac{r_n^{(0)} \Lambda^{P2}_{QCD}}{|p|^2 - m_n^{(0)2} + i\epsilon} + \cdots, \end{aligned} \tag{4}$$

where Λ^{P}_{QCD} is the planar limit of the RG-invariant scale, $r_n^{(0)}$ are dimensionless positive numbers and the dots denote contact terms [5], i.e., polynomials in the momentum squared $|p|^2 = g^{\alpha\beta} p_\alpha p_\beta$, with $g_{\alpha\beta}$ the mostly minus metric.

Indeed, as observed qualitatively in the early days of large-N QCD [4,6], the number of glueballs and mesons must be infinite in order for the exact spectral representation of 2-point correlators to match to the lowest nontrivial $\frac{1}{N}$ order the UV asymptotics of perturbation theory. The asymptotic theorem [5] on the UV asymptotics of the spectral representation of large-N 2-point correlators of multiplicatively renormalizable operators refines quantitatively the preceding statement by taking into account the RG improvement of the Euclidean perturbation theory.

Similarly, for spin-1 conserved currents with $D = 3$

$$\begin{aligned}\int \langle \mathcal{O}^{(1)}_{\alpha}(x)\mathcal{O}^{(1)}_{\beta}(0)\rangle^{P}_{conn}\, e^{-ip\cdot x} d^4x &= -i\sum_{n=1}^{\infty} \Pi_{\alpha\beta}(m_n^{(1)}) \frac{m_n^{(1)2} r_n^{(1)} \Lambda^{P2}_{QCD}}{|p|^2 - m_n^{(1)2} + i\epsilon} \\ &= -i|p|^2 \Pi_{\alpha\beta}(p) \sum_{n=1}^{\infty} \frac{r_n^{(1)} \Lambda^{P2}_{QCD}}{|p|^2 - m_n^{(1)2} + i\epsilon} + \cdots \end{aligned} \tag{5}$$

and for the conserved stress-energy tensor with $D = 4$

$$\begin{aligned}&\int \langle \mathcal{O}^{(2)}_{\alpha\beta}(x)\mathcal{O}^{(2)}_{\gamma\delta}(0)\rangle^{P}_{conn}\, e^{-ip\cdot x} d^4x \\ &= i\sum_{n=1}^{\infty} \left(\frac{1}{2}\Pi_{\alpha\gamma}(m_n^{(2)})\Pi_{\beta\delta}(m_n^{(2)}) + \frac{1}{2}\Pi_{\beta\gamma}(m_n^{(2)})\Pi_{\alpha\delta}(m_n^{(2)}) - \frac{1}{3}\Pi_{\alpha\beta}(m_n^{(2)})\Pi_{\gamma\delta}(m_n^{(2)})\right) \\ &\quad \frac{m_n^{(2)4} r_n^{(2)} \Lambda^{P2}_{QCD}}{|p|^2 - m_n^{(2)2} + i\epsilon} \\ &= i|p|^4 \left(\frac{1}{2}\Pi_{\alpha\gamma}(p)\Pi_{\beta\delta}(p) + \frac{1}{2}\Pi_{\beta\gamma}(p)\Pi_{\alpha\delta}(p) - \frac{1}{3}\Pi_{\alpha\beta}(p)\Pi_{\gamma\delta}(p)\right)\end{aligned}$$

$$\sum_{n=1}^{\infty} \frac{r_n^{(2)} \Lambda_{QCD}^{P2}}{|p|^2 - m_n^{(2)2} + i\epsilon} + \cdots , \tag{6}$$

where

$$\Pi_{\alpha\beta}(m) = g_{\alpha\beta} - \frac{p_\alpha p_\beta}{m^2} \tag{7}$$

and

$$\Pi_{\alpha\beta}(p) = g_{\alpha\beta} - \frac{p_\alpha p_\beta}{|p|^2} . \tag{8}$$

Each massive propagator is conserved only on the corresponding mass shell $|p|^2 = m^2$ because of the factors $\Pi_{\alpha\beta}(m)$. However, after subtracting the sum of contact terms denoted by the dots, the resulting massless projector $\Pi_{\alpha\beta}(p)$ implies off-shell conservation, as it must be for conserved currents.

Yet, for integer higher-spin currents with $s > 2$ a conundrum occurs: The above argument about the off-shell conservation up to contact terms applies to the spectral representation of large-N 2-point correlators of operators of pure spin s, so that it would follow the wrong conclusion that higher-spin operators are conserved as well to the leading large-N order.

The way-out is operator mixing of nonconserved currents. For example, higher-spin twist-2 operators mix with derivatives of twist-2 operators of lower spins [7], so that the above conservation is spoiled in the spectral representation of planar correlators of renormalized higher-spin operators.

Hence, the aim of the present paper is to extend to operator mixing the aforementioned asymptotic theorem [5] for multiplicatively renormalizable operators in large-N confining massless QCD-like theories.

Such an extension involves the differential geometry of operator mixing and the Poincaré-Dulac theorem [8], by constructing generically a renormalization scheme where the operator mixing may be reduced to the multiplicatively renormalizable case [8], provided that a certain nonresonant condition [8] for the eigenvalues of the ratio $\frac{\gamma_0}{\beta_0}$ of the one-loop coefficient γ_0 of the anomalous dimension matrix $\gamma(g) = \gamma_0 g^2 + \cdots$ and β_0 holds according to the Poincaré-Dulac theorem.

The aforementioned scheme may generically apply to the mixing of higher-spin twist-2 operators [9], specifically in SU(N) YM theory [9,10], so that we work out the corresponding extension of the asymptotic theorem.

2. Plan of the Paper

In Section 3 we recall recent results on the differential geometry of operator mixing and the corresponding UV asymptotics of 2-point correlators that involve the Poincaré-Dulac theorem.

In Section 4 we recall the asymptotic theorem for scalar multiplicatively renormalizable operators in large-N QCD-like theories.

In Section 5 we recall standard results on the free propagators of higher-spin particles both in Euclidean and Minkowskian space-time.

In Section 6—employing results in Sections 3–5—we work out the asymptotic theorem on the UV asymptotics of the spectral representation of 2-point correlators of higher-spin twist-2 operators that mix under renormalization in large-N QCD-like theories.

In Section 7 we summarize our conclusions and outline future developments.

3. Differential Geometry of Operator Mixing and UV Asymptotics

In massless QCD-like theories 2-point correlators in Euclidean space-time at different points ($x \neq 0$)

$$G_{ik}(x) = \langle O_i(x) O_k(0) \rangle \tag{9}$$

of renormalized local gauge-invariant operators $O_i(x)$ that mix under renormalization

$$O_i(x) = \sum_k Z_{ik} O_{Bk}(x)\,, \tag{10}$$

with $O_{Bk}(x)$ the bare operators[2] and Z the bare mixing matrix, satisfy the Callan-Symanzik equation [14–18]

$$\left(x \cdot \frac{\partial}{\partial x} + \beta(g) \frac{\partial}{\partial g} + 2D \right) G + \gamma(g)\, G + G\, \gamma^T(g) = 0 \tag{11}$$

in matrix notation, with γ^T the transpose of γ, D the canonical dimension of the operators,

$$\gamma(g) = -\frac{\partial Z}{\partial \log \mu} Z^{-1} = \gamma_0 g^2 + \gamma_1 g^4 + \cdots \tag{12}$$

the matrix of the anomalous dimensions[3] and

$$\beta(g) = -\beta_0 g^3 - \beta_1 g^5 + \cdots \tag{13}$$

the beta function, where $g \equiv g(\mu)$ is the renormalized coupling at the scale μ, whose UV asymptotics for large μ is given by Equation (3) with $\mu = \Lambda$. The general solution of Equation (11) has the form

$$G(x) = Z(x,\mu) \mathcal{G}(x, g(\mu), \mu) Z^T(x,\mu)\,, \tag{14}$$

with $\mathcal{G}(x, g(\mu), \mu)$ RG invariant

$$\left(x \cdot \frac{\partial}{\partial x} + \beta(g) \frac{\partial}{\partial g} + 2D \right) \mathcal{G} = 0 \tag{15}$$

and

$$Z(x,\mu) = P \exp\left(- \int_{g(x)}^{g(\mu)} \frac{\gamma(g)}{\beta(g)} dg \right), \tag{16}$$

where $Z(x,\mu)$ is the renormalized mixing matrix in the coordinate representation, P denotes the path ordering of the exponential and, by an abuse of notation, $g(x) \equiv g(1/\sqrt{x^2})$, with the leading and next-to-leading universal, i.e., scheme independent, UV asymptotics as $x^2 \to 0$

$$g^2(x) \sim \frac{1}{\beta_0 \log(\frac{1}{x^2 \Lambda^2_{RGI}})} \left(1 - \frac{\beta_1}{\beta_0^2} \frac{\log\log(\frac{1}{x^2 \Lambda^2_{RGI}})}{\log(\frac{1}{x^2 \Lambda^2_{RGI}})} \right) \tag{17}$$

and $x^2 = \delta_{\alpha\beta} x^\alpha x^\beta$.

If $Z(x,\mu)$ is diagonalizable, Equation (14) reads in the diagonal basis

$$G_{ik}(x) \;\; = \;\; Z_i(x,\mu) \mathcal{G}_{ik}(x, g(\mu), \mu) Z_k(x,\mu) \tag{18}$$

that implies

$$G_{ii}(x) \;\; = \;\; Z_i(x,\mu) \mathcal{G}_{ii}(x, g(\mu), \mu) Z_i(x,\mu) \tag{19}$$

and consequently within the universal UV asymptotics as $\lambda \to 0$ for $x \neq 0$

$$G_{ii}(\lambda x) \quad \sim \quad Z_i(\lambda x, \mu) G_{ii}^{conf}(\lambda x) Z_i(\lambda x, \mu) \tag{20}$$

provided that $G_{ii}^{conf}(x)$—the conformal correlator in the diagonal basis to the lowest order of perturbation theory—does not vanish. Therefore, we may wonder under which conditions an operator basis exists where $Z(x, \mu)$ is diagonalizable.

The key observation in [8] for answering the above question is that renormalization can be interpreted in a differential geometric setting, where a change of operator basis, due to a change of renormalization scheme, is interpreted as a formal[4] real analytic invertible gauge transformation $S(g)$

$$O_i'(x) = \sum_k S_{ik}(g) O_k(x)\,. \tag{21}$$

Accordingly, the matrix

$$\begin{aligned} A(g) = -\frac{\gamma(g)}{\beta(g)} &= \frac{1}{g}\left(A_0 + \sum_{n=1}^{\infty} A_{2n} g^{2n}\right) \\ &= \frac{1}{g}\left(\frac{\gamma_0}{\beta_0} + \cdots\right) \end{aligned} \tag{22}$$

that occurs in the system of ordinary differential equations defining $Z(x, \mu)$ by Equations (12) and (13)

$$\left(\frac{\partial}{\partial g} + \frac{\gamma(g)}{\beta(g)}\right) Z = 0 \tag{23}$$

is interpreted as a (formal) real-analytic connection, with a simple pole at $g = 0$, that for the gauge transformation in Equation (21) transforms as

$$A'(g) = S(g) A(g) S^{-1}(g) + \frac{\partial S(g)}{\partial g} S^{-1}(g)\,. \tag{24}$$

Morevover,

$$\mathcal{D} = \frac{\partial}{\partial g} - A(g) \tag{25}$$

is interpreted as the corresponding covariant derivative that defines the linear system

$$\mathcal{D} X = \left(\frac{\partial}{\partial g} - A(g)\right) X = 0 \tag{26}$$

whose solution with a suitable initial condition is $Z(x, \mu)$.

As a consequence $Z(x, \mu)$ is interpreted as a Wilson line associated to the aforementioned connection

$$Z(x, \mu) = P \exp\left(\int_{g(x)}^{g(\mu)} A(g)\, dg\right) \tag{27}$$

that transforms as

$$Z'(x, \mu) = S(g(\mu)) Z(x, \mu) S^{-1}(g(x)) \tag{28}$$

for the gauge transformation $S(g)$.

Besides, by allowing the coupling to be complex valued, everything that we have mentioned applies in the (formal) holomorphic setting, instead of the real-analytic one.

According to the interpretation above, the easiest way to compute the UV asymptotics of $Z(x,\mu)$ consists in setting the meromorphic connection in Equation (22) in a canonical form by a suitable (formal) holomorphic gauge transformation [8].

As a consequence of the Poincaré-Dulac theorem [8], if the eigenvalues $\lambda_1, \lambda_2, \cdots$ of the matrix $\frac{\gamma_0}{\beta_0}$, in nonincreasing order $\lambda_1 \geq \lambda_2 \geq \cdots$, do not differ by a positive even integer, i.e.,

$$\lambda_i - \lambda_j - 2k \neq 0 \tag{29}$$

for $i \leq j$ and k a positive integer, then it exists a renormalization scheme where

$$-\frac{\gamma(g)}{\beta(g)} = \frac{\gamma_0}{\beta_0}\frac{1}{g} \tag{30}$$

is one-loop exact to all orders of perturbation theory. Hence, if γ_0 is diagonalizable, the UV asymptotics of $Z(x,\mu)$ as $x^2 \to 0$ reduces essentially to the multiplicatively renormalizable case

$$Z_i(x,\mu) \sim \exp\left(\int_{g(x)}^{g(\mu)} \frac{\gamma_{0i}}{\beta_0 g} dg\right) = \left(\frac{g(\mu)}{g(x)}\right)^{\frac{\gamma_{0i}}{\beta_0}} \tag{31}$$

in the aforementioned scheme, where $Z_i(x,\mu)$ and γ_{0i} denote the eigenvalues of the corresponding matrices.

This is the nonresonant diagonal case (I) of the classification in [8], where Equation (30) is satisfied and actually implied by the nonresonat condition for the eigenvalues of $\frac{\gamma_0}{\beta_0}$ in Equation (29) and $\frac{\gamma_0}{\beta_0}$ is diagonalizable. The remaining cases, where $Z(x,\mu)$ is not diagonalizable, are worked out in [19].

4. Asymptotic Theorem for Scalar Multiplicatively Renormalizable Operators

In a massless QCD-like theory the planar connected 2-point Euclidean correlators of multiplicatively renormalizable scalar local gauge-invariant single-trace operators or fermion bilinears, $\mathcal{O}^{E(0)}$, that are Hermitian in Minkowskian space-time satisfy within the universal UV asymptotics as $x^2 \to 0$ in the scheme of Equation (30) [5]

$$\begin{aligned}
\langle \mathcal{O}^{E(0)}(x)\mathcal{O}^{E(0)}(0)\rangle^{P}_{conn} &= \int \sum_{n=1}^{\infty} \frac{m_n^{(s)2D-4} R_n^{(0)}}{p^2 + m_n^{(0)2}} e^{ip\cdot x} \frac{d^4p}{(2\pi)^4} \\
&= \sum_{n=1}^{\infty} \int \left(\frac{(-p^2)^{D-2} R_n^{(0)}}{p^2 + m_n^{(0)2}} + \cdots \right) e^{ip\cdot x} \frac{d^4p}{(2\pi)^4} \\
&\sim \frac{2^{2D-4}\Gamma(D-1)\Gamma(D)}{\pi^2} \frac{1}{x^{2D}} \left(\frac{g(\mu)}{g(x)}\right)^{2\gamma'}
\end{aligned} \tag{32}$$

provided that, according to Equation (32), $\mathcal{O}^{E(0)}$ is normalized in such a way that to the lowest order in perturbation theory

$$\langle \mathcal{O}^{E(0)}(x)\mathcal{O}^{E(0)}(0)\rangle^{P}_{conn} = \frac{2^{2D-4}\Gamma(D-1)\Gamma(D)}{\pi^2} \frac{1}{x^{2D}}, \tag{33}$$

where $\gamma' = \frac{\gamma_0^{\mathcal{O}^{E(0)}}}{\beta_0}$, $R_n^{(s)} = r_n^{(s)} \Lambda_{QCD}^{P2} > 0$ by unitarity and the dots in Equation (32) are contact terms, i.e., polynomials in $p^2 = \delta^{\alpha\beta} p_\alpha p_\beta$ of maximum degree $D-2$ that occur by the identity [5]

$$m_n^{(0)2} = (m_n^{(0)2} + p^2) - p^2 . \tag{34}$$

The asymptotic theorem [5] states that for $D \geq 2$, under mild assumptions, from Equation (32) it follows asymptotically for large n

$$R_n^{(0)} \sim Z^{(0)2}(m_n^{(0)})\rho_0^{-1}(m_n^{(0)}) \sim \left(\frac{g(\mu)}{g(m_n^{(0)})}\right)^{2\gamma'} \rho_0^{-1}(m_n^{(0)}), \tag{35}$$

with $Z^{(0)}(m_n^{(0)})$ the multiplicative renormalization factor at the scale $m_n^{(0)}$ (Section 3) and $\rho_0(m_n^{(0)})$ the asymptotic spectral density [5]

$$\rho_0^{-1}(m_n^{(0)}) = \frac{dm_n^{(0)2}}{dn}. \tag{36}$$

5. A Detour on Higher-Spin Propagators

5.1. Propagators in Euclidean Space-Time

The propagator of a free massive bosonic field $\phi^E_{\mu_1\ldots\mu_s}(x)$ of integer spin s and canonical dimension $D = 1$ in Euclidean space-time reads

$$\langle \phi^E_{\mu_1\ldots\mu_s}(x)\phi^E_{\nu_1\ldots\nu_s}(0)\rangle = \int \frac{d^4p}{(2\pi)^4}\, e^{ip\cdot x}\frac{P^E_{\mu_1\ldots\mu_s,\nu_1\ldots\nu_s}(m)}{p^2+m^2}, \tag{37}$$

with

$$P^{E\nu_1\ldots\nu_s}_{\mu_1\ldots\mu_s}(m) = \sum_{j=0}^{\left[\frac{s}{2}\right]}(-1)^j 2^{-j}\frac{s!}{j!(s-2j)!}\frac{(2s-2j-1)!!}{(2s-1)!!} \Pi^E{}_{(\mu_1\mu_2}\ldots\Pi^E{}_{\mu_{2j-1}\mu_{2j}}\Pi^{E(\nu_1\nu_2}\ldots\Pi^{E\nu_{2j-1}\nu_{2j}}\Pi^{E\nu_{2j+1}}_{\mu_{2j+1}}\ldots\Pi^{E\nu_s)}_{\mu_s}, \tag{38}$$

where the parentheses denote symmetrization of the indices in between, including a normalization factor that is the inverse of the number of terms occurring in the symmetrization, with

$$\Pi^E_{\mu\nu}(m) = \delta_{\mu\nu} + \frac{p_\mu p_\nu}{m^2} \tag{39}$$

and

$$(2k-1)!! = \frac{(2k)!}{2^k k!}. \tag{40}$$

5.2. Propagators in Minkowskian Space-Time

By the Wick rotation [20]

$$\begin{aligned} x^4 &\to ix^0 \\ p_4 &\to -ip_0 \end{aligned} \tag{41}$$

we get the propagator in Minkowskian space-time for the Hermitian field $\phi_{\mu_1\ldots\mu_s}(x)$ [21]

$$\langle \phi_{\mu_1\ldots\mu_s}(x)\phi_{\nu_1\ldots\nu_s}(0)\rangle = \int \frac{d^4p}{(2\pi)^4}\, e^{ip\cdot x}(-i)\,\frac{P^+_{\mu_1\ldots\mu_s,\nu_1\ldots\nu_s}(m)}{|p|^2_+ + m^2 - i\epsilon}, \tag{42}$$

where $|p|^2_+ = \eta^{\alpha\beta}p_\alpha p_\beta$, with $\eta_{\alpha\beta}$ the mostly plus metric and [21,22]

$$P^{+\nu_1\ldots\nu_s}_{\mu_1\ldots\mu_s}(m) = \sum_{j=0}^{\left[\frac{s}{2}\right]}(-1)^j 2^{-j}\frac{s!}{j!(s-2j)!}\frac{(2s-2j-1)!!}{(2s-1)!!} \Pi^+{}_{(\mu_1\mu_2}\ldots\Pi^+{}_{\mu_{2j-1}\mu_{2j}}\Pi^{+(\nu_1\nu_2}\ldots\Pi^{+\nu_{2j-1}\nu_{2j}}\Pi^{+\nu_{2j+1}}_{\mu_{2j+1}}\ldots\Pi^{+\nu_s)}_{\mu_s}. \tag{43}$$

Indeed, by the Wick rotation above

$$\langle \phi^E_{3\ldots3}(x)\phi^E_{3\ldots3}(0)\rangle \to \langle \phi_{3\ldots3}(x)\phi_{3\ldots3}(0)\rangle\,, \tag{44}$$

so that the space components of the Euclidean spin-1 projector $\Pi^E_{\mu\nu}(m)$ are analytically continued identically to the space-like components of the Minkowskian projector in terms of the mostly plus metric

$$\Pi^+_{\mu\nu}(m) = \eta_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}\,, \tag{45}$$

while by Equation (41) $\phi^E_{4\ldots4}(x) \to (-i)^s\phi_{0\ldots0}(x)$, so that

$$\langle \phi^E_{4\ldots4}(x)\phi^E_{4\ldots4}(0)\rangle \to (-1)^s\langle \phi_{0\ldots0}(x)\phi_{0\ldots0}(0)\rangle \tag{46}$$

consistently with the sign of the temporal components of the projector in Equation (45). In terms of the mostly minus metric $g_{\mu\nu}$ we define

$$\Pi^+_{\mu\nu}(m) = -(g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2}) = -\Pi_{\mu\nu}(m)\,. \tag{47}$$

Hence, by setting

$$\begin{aligned} P^{\nu_1\ldots\nu_s}_{\mu_1\ldots\mu_s}(m) = &\sum_{j=0}^{\left[\frac{s}{2}\right]}(-1)^j 2^{-j}\frac{s!}{j!(s-2j)!}\frac{(2s-2j-1)!!}{(2s-1)!!} \\ &\Pi_{(\mu_1\mu_2}\ldots\Pi_{\mu_{2j-1}\mu_{2j}}\Pi^{(\nu_1\nu_2}\ldots\Pi^{\nu_{2j-1}\nu_{2j}}\Pi^{\nu_{2j+1}}_{\mu_{2j+1}}\ldots\Pi^{\nu_s)}_{\mu_s)} \end{aligned} \tag{48}$$

we identically obtain

$$\langle \phi_{\mu_1\ldots\mu_s}(x)\phi_{\nu_1\ldots\nu_s}(0)\rangle = (-1)^s\int\frac{d^4p}{(2\pi)^4}\,e^{ip\cdot x}i\,\frac{P_{\mu_1\ldots\mu_s,\nu_1\ldots\nu_s}(m)}{|p|^2-m^2+i\epsilon}\,. \tag{49}$$

5.3. Propagators Restricted to the + Components

Restricting $P_{\mu_1\ldots\mu_s,\nu_1\ldots\nu_s}(m)$ to the + components, we get

$$P_{+\ldots+,+\ldots+}(m) = (-1)^s\frac{p_+^{2s}}{m^{2s}}\sum_{j=0}^{\left[\frac{s}{2}\right]}(-1)^j 2^{-j}\frac{s!}{j!(s-2j)!}\frac{(2s-2j-1)!!}{(2s-1)!!}\,, \tag{50}$$

where $p_+ = \frac{1}{\sqrt{2}}(p_0+p_3)$. By employing

$$(2n-1)!! = \frac{2^n\Gamma(n+\frac{1}{2})}{\sqrt{\pi}} \tag{51}$$

the above sum becomes

$$P_{+\ldots+,+\ldots+}(m) = (-1)^s\frac{p_+^{2s}}{m^{2s}}\frac{s!\Gamma(\frac{1}{2})}{\Gamma(s+\frac{1}{2})}\sum_{j=0}^{\left[\frac{s}{2}\right]}(-1)^j 2^{-2j}\frac{\Gamma(s-j+\frac{1}{2})}{\Gamma(\frac{1}{2})j!(s-2j)!}\,. \tag{52}$$

From the definition of the Gegenbauer polynomials

$$C^{(\alpha)}_n(x) = \sum_{k=0}^{\left[\frac{n}{2}\right]}(-1)^k\frac{\Gamma(n-k+\alpha)}{\Gamma(\alpha)k!(n-2k)!}(2x)^{n-2k} \tag{53}$$

we obtain for $n = s$, $\alpha = \frac{1}{2}$ and $x = 1$

$$C_s^{(\frac{1}{2})}(1) = \sum_{k=0}^{\left[\frac{s}{2}\right]} (-1)^k \frac{\Gamma(s-k+\frac{1}{2})!}{\Gamma(\frac{1}{2})k!(s-2k)!} 2^{s-2k}, \tag{54}$$

so that

$$P_{+\ldots+,+\ldots+}(m) = (-1)^s \frac{p_+^{2s}}{m^{2s}} \frac{s!\Gamma(\frac{1}{2})}{\Gamma(s+\frac{1}{2})} 2^{-s} C_s^{(\frac{1}{2})}(1)\,. \tag{55}$$

The Rodrigues formula

$$C_s^{(\frac{1}{2})}(x) = \frac{(-1)^s}{2^s s!} \frac{d^s}{dx^s}(1-x^2)^s \tag{56}$$

yields

$$C_s^{(\frac{1}{2})}(1) = \frac{(-1)^s}{2^s s!} \frac{d^s}{dx^s}(1-x^2)^s \Big|_{x=1} = 1\,. \tag{57}$$

Thus, employing

$$\Gamma(s+\frac{1}{2}) = 2^{-s}\sqrt{\pi}\frac{(2s)!}{2^s s!} \tag{58}$$

we obtain

$$P_{+\ldots+,+\ldots+}(m) = (-1)^s \frac{p_+^{2s}}{m^{2s}} 2^s \frac{(s!)^2}{(2s)!} = (-1)^s \frac{p_+^{2s}}{m^{2s}} \frac{2^s}{\binom{2s}{s}}\,. \tag{59}$$

Hence, the canonically normalized propagator restricted to the + components is

$$\begin{aligned} \langle \phi_{+\ldots+}(x)\phi_{+\ldots+}(0)\rangle &= (-1)^s \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot x} i \frac{P_{+\ldots+,+\ldots+}(m)}{|p|^2 - m^2 + i\epsilon} \\ &= \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot x} i \frac{2^s \frac{(s!)^2}{(2s)!} \frac{p_+^{2s}}{m^{2s}}}{|p|^2 - m^2 + i\epsilon}\,. \end{aligned} \tag{60}$$

5.4. Analytic Continuation to the z Components

From the Wick rotation

$$p_+ = \frac{1}{\sqrt{2}}(p_0 + p_3) \to \frac{1}{\sqrt{2}}(ip_4 + p_3) = i\frac{1}{\sqrt{2}}(p_4 - ip_3) = ip_z \tag{61}$$

in Equation (60) we obtain the corresponding Euclidean propagator [20]

$$\begin{aligned} \langle \phi_{+\ldots+}(x)\phi_{+\ldots+}(0)\rangle &\to (-1)^s \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot x} \frac{2^s \frac{(s!)^2}{(2s)!} \frac{p_z^{2s}}{m^{2s}}}{p^2 + m^2} \\ &\equiv \langle \phi^{\mathbf{E}}_{z\ldots z}(x)\phi^{\mathbf{E}}_{z\ldots z}(0)\rangle \\ &= (-1)^s \langle \phi^{E}_{z\ldots z}(x)\phi^{E}_{z\ldots z}(0)\rangle\,, \end{aligned} \tag{62}$$

where the Euclidean field $\phi^{\mathbf{E}}_{z\ldots z}(x)$ is defined by the analytical continuation $\phi_{+\ldots+}(x) \to \phi^{\mathbf{E}}_{z\ldots z}(x) = i^s \phi^{E}_{z\ldots z}(x)$ that follows from Equations (39) and (61).

6. Extension of the Asymptotic Theorem to Higher-Spin Twist-2 Operators

The asymptotic theorem [5] (Section 4) may be straightforwardly extended from multiplicatively renormalizable operators to operators that mix under renormalization in the basis where they become multiplicatively renormalizable, provided that it exists.

If γ_0 is diagonalizable, for the existence of the aforementioned basis it suffices that the nonresonant condition for the eigenvalues of $\frac{\gamma_0}{\beta_0}$ is satisfied (Section 3).

Actually, the nonresonant condition is sufficiently generic, so that it may hold for large sets of operators that mix among themselves.

Twist-2 operators of integer higher spins in Minkowskian space-time mix [23] between themselves under renormalization. Indeed, the maximal-spin components $\mathcal{O}_{+^s}(x) \equiv \mathcal{O}_{+\ldots+}(x)$ along the $+$ direction of the Hermitian twist-2 operators in Minkowskian space-time $\mathcal{O}_s(x) \equiv \mathcal{O}_{\mu_1\ldots\mu_s}(x)$ of higher spin s in the conformal basis [23] mix with derivatives of twist-2 operators of lower spins [7]

$$(-i\partial)_+^k \mathcal{O}_{+^s} = \sum_i^s Z_{si}(-i\partial)_+^{k+s-i}\mathcal{O}_{B+^i}\,, \tag{63}$$

with the mixing matrix Z and the matrix of the anomalous dimensions $\gamma(g)$ lower triangular [7]. Moreover, γ_0 is diagonal [23] in the Minkowskian conformal basis.

As a consequence, after the Wick rotation to Euclidean space-time, if the nonresonant diagonal basis of Equation (30) exists, it restricts [9,10] to the lowest order of perturbation theory to the Euclidean conformal basis because γ_0 is diagonal in this basis, so that $G_{ss}^{conf}(x)$ in Equation (20) is computed in the aforementioned basis. Hence, by assuming that the nonresonant condition is satisfied, the asymptotic theorem for the Euclidean twist-2 operators in the nonresonant diagonal basis reads within the universal UV asymptotics as $\lambda \to 0$

$$\begin{aligned}
\langle \mathcal{O}_s^E(\lambda x)\mathcal{O}_s^E(0)\rangle_{conn}^P &= \sum_{n=1}^{\infty}\int P_s^E\left(m_n^{(s)}\right)\frac{m_n^{(s)2s}R_n^{(s)}}{p^2+m_n^{(s)2}}\,e^{i\lambda p\cdot x}\frac{d^4p}{(2\pi)^4}+\cdots\\
&= \sum_{n=1}^{\infty}\int\left(P_s^E(p)\frac{(-p^2)^sR_n^{(s)}}{p^2+m_n^{(s)2}}+\cdots\right)e^{i\lambda p\cdot x}\frac{d^4p}{(2\pi)^4}+\cdots\\
&\sim \frac{1}{\pi^2}\frac{R_s^E(\lambda x)}{(\lambda x)^{2(2+s)}}\left(\frac{g(\mu)}{g(\lambda x)}\right)^{2\gamma'}
\end{aligned} \tag{64}$$

provided that, according to Equation (64), $\mathcal{O}_s^E(x)$ is normalized in such a way that to the lowest order of perturbation theory

$$\langle \mathcal{O}_s^E(x)\mathcal{O}_s^E(0)\rangle_{conn}^P = \frac{1}{\pi^2}\frac{R_s^E(x)}{x^{2(2+s)}}\,, \tag{65}$$

with $P_s^E\left(m\right) \equiv P^E_{\mu_1\ldots\mu_s,\nu_1\ldots\nu_s}(m)$ and $P_s^E\left(p\right)$ obtained from $P_s^E\left(m\right)$ by the substitution $m^2 \to -p^2$ as in the scalar case (Section 4).

The asymptotic equality in Equation (64) follows from Equations (19) and (20), where the conformal contribution reads

$$G_{ss}^{conf}(x) = \frac{1}{\pi^2}\frac{R_s^E(x)}{x^{2(2+s)}}\,, \tag{66}$$

with $R_s^E(x)$ a suitably normalized (Equation (71)) and symmetrized [24] dimensionless polynomial in

$$R_{\mu\nu}^E(x) = \delta_{\mu\nu} - 2\frac{x_\mu x_\nu}{x^2}\,. \tag{67}$$

Then, it follows from the asymptotic theorem [5] for multiplicatively renormalizable operators

$$R_n^{(s)} \sim Z^{(s)2}(m_n^{(s)})\rho_s^{-1}(m_n^{(s)}) \sim \left(\frac{g(\mu)}{g(m_n^{(s)})}\right)^{2\gamma'}\rho_s^{-1}(m_n^{(s)})\,, \tag{68}$$

where $Z^{(s)}(m_n^{(s)})$ is the multiplicative renormalization factor at the scale $m_n^{(s)}$ (Section 3) and $\rho_s(m_n^{(s)})$ is the asymptotic spectral density [5]

$$\rho_s^{-1}(m_n^{(s)}) = \frac{dm_n^{(s)2}}{dn}\,. \tag{69}$$

Indeed, since twist-2 operators are conserved [25] to the leading order of perturbation theory in a massless QCD-like theory, the universal UV asymptotics in Equation (64) arises only from particles of pure spin s, while the dots in the first line of Equation (64) include subleading contributions of nonconserved propagators involving derivatives of propagators of lower-spin particles. Hence, the asymptotic theorem for twist-2 operators can be reduced to the scalar case by employing Equation (32) with $D = 2$ within the universal UV asymptotics as $\lambda \to 0$

$$\begin{aligned}
&\sum_{n=1}^{\infty} \int P_s^E\big(m_n^{(s)}\big) \frac{m_n^{(s)2s} R_n^{(s)}}{p^2 + m_n^{(s)2}}\, e^{i\lambda p\cdot x} \frac{d^4p}{(2\pi)^4} \\
&= \sum_{n=1}^{\infty} \int \left(P_s^E(p) \frac{(-p^2)^s R_n^{(s)}}{p^2 + m_n^{(s)2}} + \cdots \right) e^{i\lambda p\cdot x} \frac{d^4p}{(2\pi)^4} \\
&\sim \frac{1}{\pi^2} \lambda^{-2s} \Delta^s P_s^E(-i\partial) \left(\frac{1}{(\lambda x)^4} \left(\frac{g(\mu)}{g(\lambda x)} \right)^{2\gamma'} \right) \\
&\sim \frac{1}{\pi^2} \lambda^{-2s} \left(\Delta^s P_s^E(-i\partial) \frac{1}{(\lambda x)^4} \right) \left(\frac{g(\mu)}{g(\lambda x)} \right)^{2\gamma'} \\
&= \frac{1}{\pi^2} \frac{R_s^E(\lambda x)}{(\lambda x)^{2(2+s)}} \left(\frac{g(\mu)}{g(\lambda x)} \right)^{2\gamma'}.
\end{aligned} \tag{70}$$

Specifically, for the analytically continued operators $\mathcal{O}_{+^s}(x) \to \mathcal{O}_{z^s}^{\mathbf{E}}(x)$ we get from Equations (62) and (70) within the universal UV asymptotics as $\lambda \to 0$

$$\begin{aligned}
\langle \mathcal{O}_{z^s}^{\mathbf{E}}(\lambda x) \mathcal{O}_{z^s}^{\mathbf{E}}(0) \rangle_{conn}^{P} &= (-1)^s \sum_{n=1}^{\infty} \int \left(2^s \frac{(s!)^2}{(2s)!} \frac{p_z^{2s} R_n^{(s)}}{p^2 + m_n^{(s)2}} + \cdots \right) e^{i\lambda p\cdot x} \frac{d^4p}{(2\pi)^4} + \cdots \\
&\sim (-1)^s \frac{2^s \frac{(s!)^2}{(2s)!}}{\pi^2} \frac{2^s (2s+1)! (\frac{-2(\lambda x_z)^2}{x^2})^s}{(\lambda x)^{2(2+s)}} \left(\frac{g(\mu)}{g(\lambda x)} \right)^{2\gamma'} \\
&= \frac{(2s+1) 2^s (s!)^2}{\pi^2} \frac{2^{2s} (\lambda x_z)^{2s}}{(\lambda x)^{2(2+2s)}} \left(\frac{g(\mu)}{g(\lambda x)} \right)^{2\gamma'},
\end{aligned} \tag{71}$$

where we have employed

$$\partial_z^{2s} \frac{1}{x^4} = 2^{2s} (2s+1)! \frac{x_z^{2s}}{x^{2(2+2s)}} \tag{72}$$

with $x_z = \frac{1}{\sqrt{2}}(x_4 - ix_3)$ [20].

Moreover, with the normalization in Equation (35), for $\gamma' \neq 0, -1$ it follows from [5] within the universal UV asymptotics as $p^2 \to +\infty$

$$\begin{aligned}
&\sum_{n=1}^{\infty} P_s^E(p) \frac{(-p^2)^s R_n^{(s)}}{p^2 + m_n^{(s)2}} \\
&\sim -(-p^2)^s P_s^E(p) \frac{g^{-2}(p)}{\beta_0(\gamma' + 1)} \left(\frac{g(\mu)}{g(p)} \right)^{2\gamma'} + \cdots
\end{aligned}$$

$$= -(-p^2)^s P_s^E(p) \frac{g^{-2}(\mu)}{\beta_0(\gamma'+1)} \left(\frac{g(\mu)}{g(p)}\right)^{2\gamma'+2} + \cdots \tag{73}$$

up to possibly divergent contact terms in the dots. For $\gamma' = -1$ the rhs of Equation (73) is computed by means of the limit

$$-\lim_{\gamma' \to -1} \frac{g^{-2}(\mu)}{\beta_0(\gamma'+1)} (\frac{g(\mu)}{g(p)})^{2\gamma'+2} = \frac{1}{\beta_0} g^{-2}(\mu) \log \frac{g^2(p)}{g^2(\mu)} + \cdots . \tag{74}$$

For conserved currents $\gamma' = 0$ and

$$\sum_{n=1}^{\infty} P_s^E(p) \frac{(-p^2)^s R_n^{(s)}}{p^2 + m_n^{(s)2}}$$
$$\sim -(-p^2)^s P_s^E(p) \log(\frac{p^2}{\Lambda_{QCD}^{P2}}) + \cdots . \tag{75}$$

In [9,10] it has been verified that in SU(N) YM theory the nonresonant condition in Equation (29) is satisfied for the gluonic twist-2 operators in the conformal basis up to $s = 10^4$. Hence, the nonresonant diagonal basis exists for twist-2 operators in SU(N) YM theory and the above asymptotic theorem applies.

7. Conclusions and Outlook

We have extended to operator mixing—specifically, to the mixing of twist-2 operators—the asymptotic theorem in the UV for the residues of the poles of the propagators in the spectral representation of 2-point correlators of multiplicatively renormalizable gauge-invariant operators in large-N massless QCD-like theories.

Apart from the intrinsic interest, one of the main applications of the above asymptotic theorem is for the program of the asymptotically-free bootstrap [26] that we summarize as follows.

Suppose that in some way we are given a candidate nonperturbative partial solution of a large-N QCD-like theory, not for the correlators of gauge-invariant operators, but for certain S-matrix amplitudes only, where gauge-invariant operators are employed as interpolating fields for the asymptotic states. We need to verify that the proposed S-matrix amplitudes arise from an asymptotically free theory.

In principle we may check the AF reconstructing the correlators from the S-matrix amplitudes by working out the LSZ reduction formulae the other way around, attaching to the amplitudes the canonically normalized propagators multiplied by the appropriate (square root of the) residues and summing on the corresponding tower of states.

Yet, the residues of the poles of the propagators are not actually included in a candidate solution for the S matrix amplitudes only.

However, if we are interested in checking the AF in the UV, which is indeed the only a-priori information that we can verify on the basis of the RG improvement of perturbation theory, it suffices to know the residues of the poles only asymptotically for large masses, i.e., in the UV, that is precisely the information provided by the aforementioned asymptotic theorem.

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Notes

[1] The passage to the 't Hooft coupling makes the subsequent large-N limit nontrivial, since the corresponding β_0 and β_1 admit a finite nonzero large-N limit.

[2] In fact, gauge-invariant operators also mix with BRST-exact operators and with operators that vanish by the equations of motion (EoM) [11–13]. But correlators of gauge-invariant operators with BRST-exact operators vanish, while correlators with EoM operators reduce to contact terms. Hence, for our purposes it suffices to take into account the mixing of gauge-invariant operators only.

[3] The sign of the coefficient matrices in Equation (12), $\gamma_0, \gamma_1, \cdots$, is the standard one, but opposite with respect to the convention employed in [3,5].

[4] A formal series is not assumed to be convergent and, indeed, we do not assume that the series in Equations (12) and (13) are convergent, since they arise from perturbation theory.

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