

Dark Matter in Supersymmetry

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Abstract: Supersymmetry is a well-motivated theory for physics beyond the Standard Model. In particular, supersymmetric models can naturally possess dark matter candidates that can give rise to the measured dark matter content of the universe. We review several models that have been analyzed with regard to dark matter by groups based in Spain in recent years. These models include, in particular, the Minimal Supersymmetric Standard Model (MSSM) and the ‘ μ from ν ’ Supersymmetric Standard Model ($\mu\nu$ SSM) in various versions.

Keywords: MSSM; $\mu\nu$ SSM; dark matter



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1. Introduction

Searches for Dark Matter (DM) is one of the main objectives in today’s particle and astroparticle physics. Searches at the Large Hadron Collider (LHC) (or other collider experiments) are complementary to the searches in “direct detection” (DD) experiments. Among the Beyond the Standard Model (BSM) theories that predict a viable DM particle, the Minimal Supersymmetric Standard Model (MSSM) [1–4] is one of the leading candidates. Supersymmetry (SUSY) predicts two scalar partners for all Standard Model (SM) fermions as well as fermionic partners to all SM bosons. Furthermore, contrary to the SM case, the MSSM requires two Higgs doublets.

This results in five physical Higgs bosons instead of the single Higgs boson in the SM: the light and heavy charge conjugation parity symmetry ($\mathcal{CP}$)-even Higgs bosons, h and H , the $\mathcal{CP}$ -odd Higgs boson, A and the charged Higgs bosons, $H^\pm$. The neutral SUSY partners of the (neutral) Higgs and electroweak (EW) gauge bosons give rise to the four neutralinos, $\tilde{\chi}_{1,2,3,4}^0$.

The corresponding charged SUSY partners are the charginos, $\tilde{\chi}_{1,2}^\pm$. The SUSY partners of the SM leptons and quarks are the scalar leptons and quarks (sleptons and squarks), respectively. The lightest SUSY particle (LSP) is naturally the lightest neutralino, $\tilde{\chi}_1^0$. It can make up the full DM content of the universe [5,6], or—depending on its nature—only a fraction of it. In the latter case, e.g., a SUSY axion [7] could be an additional DM component, bringing the total DM density into agreement with the experimental measurement.

Another particularly well motivated SUSY model is the ‘ μ from ν ’ Supersymmetric Standard Model ($\mu\nu$ SSM) [8,9], (see also [10] for a recent review of the $\mu\nu$ SSM and [11] for an vacuum structure analysis of the $\mu\nu$ SSM. Other information about the model can be found in <http://dark.ft.uam.es/muniverse> (accessed on 28 July 2022)). Beyond the well-known appealing features of commonly studied SUSY models, in the $\mu\nu$ SSM, the tiny neutrino masses and their mixings can be accommodated via an EW seesaw mechanism, where it is required that the matter content is enlarged with regard to the SM by right-handed neutrinos [8,12–15]. Their superpartners, the “right-handed” (This name is used also for the scalar particles in order to indicate that they are the superpartners of right-handed fermions) scalar neutrinos (sneutrinos) are gauge singlet scalar fields.

If the right-handed sneutrinos acquire vacuum expectation values (vevs), the so-called μ -term of the MSSM can effectively be generated, analogous to the Z_3 symmetric Next-to-MSSM (NMSSM) [16,17]. Consequently, the $\mu\nu$ SSM also offers a solution to the so-called μ -problem [18]. By construction, the $\mu\nu$ SSM does not permit a consistent assignment of conserved R -parity charges. It is thus a R -parity violating (RPV) model and therefore has no stable LSP.

Therefore, compared to the (N)MSSM, the collider constraints from the LHC are substantially weaker [19–26]. Regarding the DM content of the universe, it is possible to accommodate the measured relic abundance by means of the decaying—but long-lived—gravitino [27–31] (For an analysis of gravitino DM in the context of the bilinear RPV model, see Ref. [32]) or axino [33], interestingly producing gamma-rays potentially detectable in gamma-ray telescopes. Multicomponent DM scenarios with the axino/gravitino as the LSP and the gravitino/axino as the next-to-LSP (NLSP) were also discussed in the $\mu\nu$ SSM [31,33]. Concerning cosmology, baryon asymmetry might be realized in the $\mu\nu$ SSM through EW baryogenesis [34].

In this review, we give an overview of DM analyses in various SUSY models that have been obtained in the recent years by groups (to a relevant degree) based in Spain. (In the past, other DM candidates were proposed, such as the right-handed sneutrino in the NMSSM extended with right-handed neutrino superfields [35,36]. For a recent analysis, see Ref. [37]) The models comprise the pMSSM11 (the phenomenological MSSM with 11 free parameters); the electroweak (EW) MSSM, where the colored sector is assumed to be heavy and does not enter into the low-energy phenomenology; the $\mu\nu$ SSM with gravitino and/or axino as DM candidates (also the possibility of a sterile right-handed neutrino will be presented); and finally the recently proposed $U\mu\nu$ SSM [38,39].

The latter is a $U(1)'$ extension of the $\mu\nu$ SSM, where the presence of weakly interacting massive particles (WIMPs) as DM candidates dictated by the anomaly cancellation conditions was proven [40]. This is a remarkable result, given that the $U\mu\nu$ SSM is a RPV scenario.

2. Results in the pMSSM11

In this section, we review the results of the DM analysis in the pMSSM11 as obtained in Ref. [41]. The main idea is to investigate phenomenological models that have soft SUSY-breaking parameters that are not constrained by any universality condition (at the Grand Unified Theory (GUT) scale), though subject to milder constraints emanating, in particular, from upper limits on SUSY contributions to flavor-changing processes. These phenomenological MSSM (pMSSM) [42] models that have been studied in the literature contain up to 19 free parameters.

2.1. The Parameter Space

Here, we employ a model with 11 free parameters, given at the EW scale. These are three independent gaugino masses, $M_{1,2,3}$; a common scalar mass parameter for the first- and second-generation squarks, $m_{\tilde{q}}$; a distinct mass parameter for the third-generation squarks, $m_{\tilde{q}_3}$; a common mass parameter $m_{\tilde{\ell}}$ for the sleptons of the first- and second-generation; a distinct mass parameter for the scalar taus, $m_{\tilde{\tau}}$; a single trilinear mixing parameter A , which is taken to be universal at the electroweak scale; the Higgsino mass parameter μ ; the mass of the pseudoscalar Higgs, M_A and $\tan\beta = v_2/v_1$; and the ratio of the Higgs vevs.

The renormalization scale M_{SUSY} given by the geometric mean of the masses of the two scalar tops, $M_{\text{SUSY}} \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, which is also the scale at which the EW symmetry breaking conditions are imposed. The ranges of the sampled parameter space are summarized in Table 1.

Table 1. The ranges of the sampled pMSSM11 parameters.

Parameter	Range
M_1	$(-4, 4)$ TeV
M_2	$(0, 4)$ TeV
M_3	$(-4, 4)$ TeV
$m_{\tilde{q}}$	$(0, 4)$ TeV
$m_{\tilde{q}_3}$	$(0, 4)$ TeV
$m_{\tilde{\ell}}$	$(0, 2)$ TeV
$m_{\tilde{\tau}}$	$(0, 2)$ TeV
M_A	$(0, 4)$ TeV
A	$(-5, 5)$ TeV
μ	$(-5, 5)$ TeV
$\tan \beta$	$(1, 60)$

2.2. The Analysis Framework

A global likelihood analysis of the pMSSM11 was performed, including constraints from direct searches for SUSY particles at the LHC, measurements of the Higgs boson mass and signal strengths, LHC searches for SUSY Higgs bosons, precision electroweak observables, flavor constraints from B - and K -physics observables, the cosmological constraint on the overall cold DM (CDM) density, and upper limits on spin-independent and -dependent LSP-nuclear scattering. Furthermore, $(g - 2)_\mu$ is included as an additional constraint.

The calculation of the observables contributing to the likelihood is performed with the MasterCode tool [41,43–52]. This tool combines consistently and interfaces various private and public codes employing the SUSY Les Houches Accord (SLHA) [53]. The analysis within MasterCode uses the following codes: SoftSusy 3.3.9 [54] for the spectrum, FeynWZ [55–57] for the electroweak precision observables, FeynHiggs 2.11.3 [58–67] for the Higgs sector and $(g - 2)_\mu$, SuFla [68,69] and SuperIso [70–72] for the flavor physics observables, Micromegas-3.2 [73] for the DM relic density, SSARD [74] for the spin-independent and -dependent elastic scattering cross-sections σ_p^{SI} and σ_p^{SD} .

The uncertainties in the cross-sections are derived from a straightforward propagation of errors in the input quantities that determine the cross-section. The dominant uncertainties are discussed below in more detail, SDECAY 1.3b [75] for calculating sparticle branching ratios and HiggsSignals 1.4.0 [76–78] and HiggsBounds 4.3.1 [79–83] for calculating constraints on the SUSY Higgs sector. The experimental values used for the analysis are given in [41,51] (and references there in).

2.3. DM Results in the pMSSM11

The first set of results for the $\tilde{\chi}_1^0$, which is assumed to yield the full amount of CDM, is presented in Figure 1. The upper plot shows the profile likelihood functions in one dimension for $m_{\tilde{\chi}_1^0}$ in the pMSSM11 with (without) the $(g - 2)_\mu$ constraint in blue (green), as well as dashed (solid) for (not) applying the constraints from LHC Run II. Including all constraints, one can see that a clear preference is found for a relatively low value of $m_{\tilde{\chi}_1^0}$. At the 2σ level, it is restricted to be $m_{\tilde{\chi}_1^0} \lesssim 500$ GeV.

Comparing the blue and the green lines, it becomes apparent that this result strongly relies on the $(g - 2)_\mu$ bound (see Section 3 for a detailed discussion). The lower plot of Figure 1 shows the triangular presentations of the composition of the $\tilde{\chi}_1^0$ in the fit with LHC 13-TeV and with the $(g - 2)_\mu$ constraint. The color coding indicates the $\Delta\chi^2$. The best-fit point is marked with a green star. One can observe that a small Wino fraction $N_{12}^2 < 0.1$ is strongly favored, while the relative proportions of the Bino fraction N_{11}^2 and the Higgsino fraction $N_{13}^2 + N_{14}^2$ are relatively unconstrained at the 95% confidence level (CL). The best-fit point is found as a pure bino LSP.

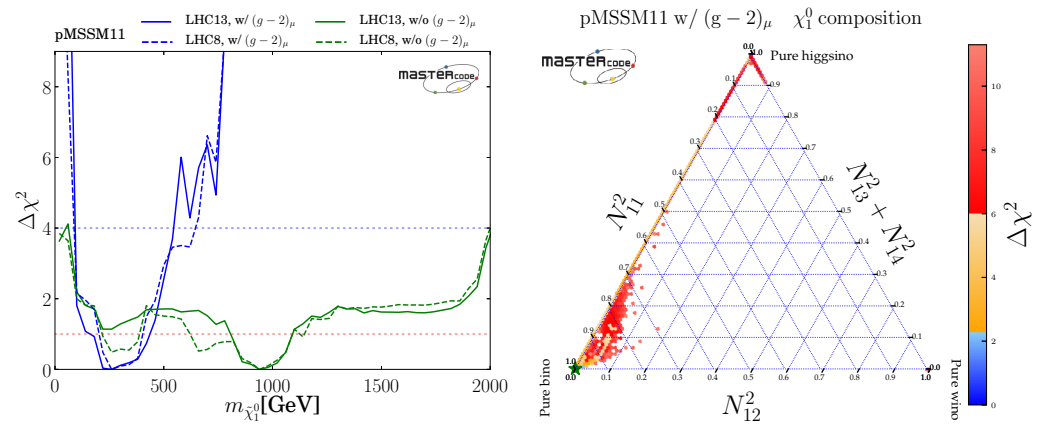


Figure 1. Upper plot: one-dimensional profile likelihood functions for the $\tilde{\chi}_1^0$ mass in the pMSSM11 with (blue) and without the $(g-2)_\mu$ constraint (green) and with (solid) and without (dashed) applying the constraints from LHC Run II. Lower plot: triangular presentations of the composition of the $\tilde{\chi}_1^0$ in the fit with LHC 13-TeV and with the $(g-2)_\mu$ constraint. The best-fit point is marked with a green star. Taken from [41].

In Figure 2, we show the preferred parameter regions in the $m_{\tilde{\chi}_1^0}-\sigma_p^{\text{SI}}$ plane (upper plot) and the $m_{\tilde{\chi}_1^0}-\sigma_p^{\text{SD}}$ plane (lower plot). σ_p^{SI} and σ_p^{SD} denote the spin-independent and spin-dependent DM-nucleon cross sections, respectively. In the upper plot, the upper limits established by the PandaX-II [84,85], XENON1T [86] and LUX [87] Collaborations are shown as blue, magenta and green contours, respectively. The combined limit (with green shading above) is indicated by a black line.

The future projected 90% CL exclusion sensitivities of the XENON1T/nT [88] and LUX-Zeplin (LZ) [89] experiments are shown as dashed blue and solid purple lines, respectively. The background neutrino ‘floor’ (with a yellow shading below) is shown as a dashed orange line. Although the DAMA/LIBRA collaboration observed an annual modulation in the detection rate [90], it is difficult to reconcile it with the negative results from the other experiments. It is true that most of them use different detection techniques than DAMA/LIBRA; however, ANAIS-112 using the same technique and target material, has not reported of hints of the presence of modulation after three-year exposure analysis [91].

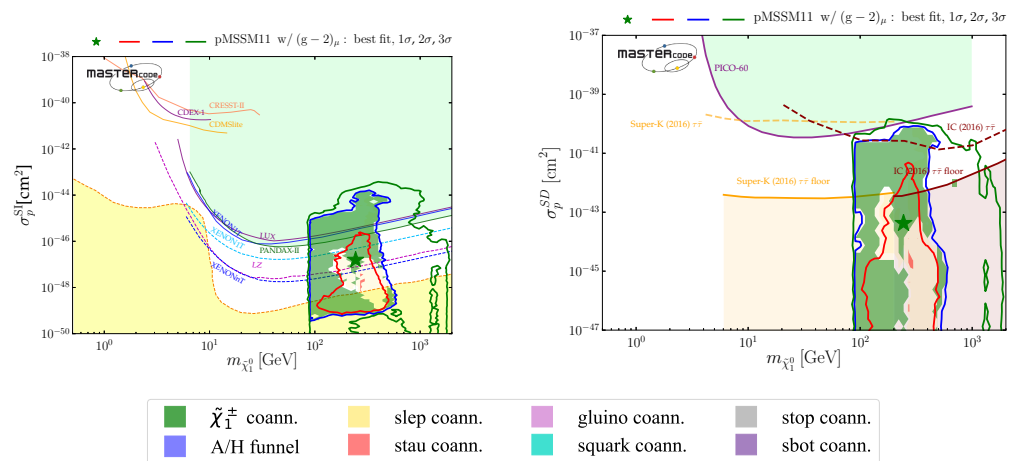


Figure 2. Upper plot: $m_{\tilde{\chi}_1^0}-\sigma_p^{\text{SD}}$ plane; for the various lines and shaded areas: see text. Lower plot: $m_{\tilde{\chi}_1^0}-\sigma_p^{\text{SI}}$ plane; for the various lines and shaded areas: see text. In both plots the red, blue and green solid lines indicate the 1, 2, 3 σ preferred parameter regions, respectively, with the best-fit point shown as a green star. The color coding within the 2 σ area correspond to the DM relic density mechanism. Taken from [41].

The solid lines in red, blue and green indicate the 1, 2, 3 σ preferred parameter regions of the global fit, respectively, with the best-fit point shown as a green star. The color coding within the 2σ area correspond to the DM relic density mechanism. We see that $m_{\tilde{\chi}_1^0} \gtrsim 100$ GeV, with upper limit $m_{\tilde{\chi}_1^0} \lesssim 550$ at the 95% CL, in agreement with Figure 1. One furthermore sees that, at the best-fit point, the nominal prediction for σ_p^{SI} is at the level of the sensitivities projected for the planned XENON1T/nT and LUX-Zeplin (LZ) experiments (solid purple line). The ranges of the nominal values of σ_p^{SI} at the 68% and 95% CL slightly extend below the neutrino floor.

Large values of σ_p^{SI} are found in the chargino coannihilation region (indicated by green shading), with other DM mechanisms, including squark coannihilation, yielding large values of σ_p^{SI} for $m_{\tilde{\chi}_1^0} \gtrsim 1$ TeV. This and the other DM mechanisms indicated, however, also allow much smaller values of σ_p^{SI} .

In the lower plot of Figure 2, the spin-dependent cross sections are analyzed. Here, the upper limit as reported by the PICO Collaboration [92] is shown as a purple contour (with a green shading). For this case, the neutrino floor for σ_p^{SD} is taken over from [93]. Furthermore, we show the indicative upper limits from IceCube [94] and SuperKamiokande [95] searches for energetic solar neutrinos obtained, which assume a predominant annihilation of the LSPs into $\tau^+\tau^-$. However, these are subject to larger theoretical uncertainties and will not be further discussed (see, however, Ref. [41]).

The preferred regions of the parameter space are indicated as in the upper figure. Similar to the case of σ_p^{SI} , one can observe that the permitted values of $m_{\tilde{\chi}_1^0}$ range from ~ 100 GeV to ~ 550 GeV. The uncertainties in the σ_p^{SD} calculation are substantially smaller than those found for σ_p^{SI} . One can see that the regions of the 68 and 95% CL in the nominal σ_p^{SD} calculations are found below the upper limit of PICO [92] (as indicated by the solid purple line). Concerning the best-fit point the nominal predictions is found ~ 3 orders of magnitude below the limit given currently by PICO.

Overall, the global analysis of the pMSSM11 shows that DD experiments, in particular via the spin-independent searches, can cover a relevant part of the parameter space. However, even the 1σ preferred regions reach below the neutrino floor and may thus escape the discussed DD experiments. In this case either novel techniques for the DM DD experiments are needed, or collider experiments will be necessary to cover the full parameter space (see also the discussion in Section 3).

3. Results in the EW-MSSM

In this section, we review recent DM analyses in the EW-MSSM [96]. In this analysis all relevant constraints on the EW sector of the MSSM are considered, which, in particular, includes the anomalous magnetic moment of the muon, $(g-2)_\mu$.

3.1. The EW-MSSM

The EW-MSSM is characterized by a light EW SUSY sector, where all other particles are assumed to be heavy. The masses and mixings of the neutralinos are defined by (on top of SM parameters) the $SU(2)_L$ and $U(1)_Y$ gaugino masses, M_2 and M_1 , the Higgsino parameter μ , as well as $\tan\beta := v_2/v_1$, the ratio of the two vevs of the two Higgs doublets. After the diagonalizing the mass matrix the four eigenvalues yield the four neutralino masses $m_{\tilde{\chi}_1^0} < m_{\tilde{\chi}_2^0} < m_{\tilde{\chi}_3^0} < m_{\tilde{\chi}_4^0}$. Similarly, the chargino masses and mixings are given (on top of SM parameters) by μ , M_2 and $\tan\beta$. The diagonalizing the mass matrix gives the two chargino-mass eigenvalues $m_{\tilde{\chi}_1^\pm} < m_{\tilde{\chi}_2^\pm}$.

For the sleptons a common soft SUSY-breaking parameters for all three generations was chosen. The mass matrices of the charged sleptons are given (on top of SM parameters) by the soft SUSY-breaking diagonal parameters $m_{\tilde{l}_L}^2$ and $m_{\tilde{l}_R}^2$, as well as A_l ($l = e, \mu, \tau$), the trilinear Slepton-Higgs coupling A_l , where, however, the later are set to zero. The mixing between the “right-handed” and “left-handed” scalar leptons is only relevant for scalar taus, since the off-diagonal entry of the mass matrix is dominated by $-m_\tau \tan\beta\mu$.

Therefore, in the first two generations, the masses can be approximated as $m_{\tilde{l}_1} \simeq m_{\tilde{l}_L}, m_{\tilde{l}_2} \simeq m_{\tilde{l}_R}$ (where small D -terms are assumed). In general, we follow the convention that $\tilde{l}_2$ ($\tilde{l}_1$) has the large “right-handed” (“left-handed”) component, i.e., no mass ordering is imposed. In addition $m_{\tilde{l}_1}$ and $m_{\tilde{l}_2}$, the symbols are equal for all three generations, we also use $m_{\tilde{e}_{1,2}}, m_{\tilde{\mu}_{1,2}}$ and $m_{\tilde{\tau}_{1,2}}$ for the masses of the scalar electrons, muons and taus. The charged slepton and the sneutrino masses are connected by the usual $SU(2)$ relation.

Overall, at the tree level, the EW sector is described in our analysis by six parameters: $M_2, M_1, \mu, m_{\tilde{l}_L}, m_{\tilde{l}_R}$ and $\tan \beta$. Here, we furthermore assume M_1, M_2, μ to be positive. It was shown in [97] that these positive parameter choices cover the relevant parameter space when the $(g - 2)_\mu$ limits are considered (see the discussion below).

As mentioned above, for strongly interacting particles, we make the assumption that the sector of colored particles in the MSSM is substantially heavier than the EW sector and, therefore, does not play a relevant role in our analysis. Concerning the Higgs-boson sector, we make the assumption that the higher-order corrections to M_h , the light $\mathcal{CP}$ -even Higgs boson mass, largely originating from the stop/top sector, give a value of $M_h \sim 125$ GeV, in agreement with the experimental data. This implies scalar top masses that are naturally in the TeV range [41,98], which are thus in agreement with the LHC bounds. Concerning the heavy Higgs-boson mass scale, M_A , it was shown in [97,99,100] that A -pole annihilation is largely excluded. For simplicity, we therefore assume M_A to be sufficiently large so as to not play any relevant role in our analysis.

3.2. The Relevant Constraints

The SM prediction of a_μ is given by [101]. The comparison with the combined experimental new world average, based on Refs. [102,103] yields a deviation of $\Delta a_\mu = (25.1 \pm 5.9) \times 10^{-10}$, corresponding to 4.2σ . This result is used as a hard 2σ cut on our results. The prediction of $(g - 2)_\mu$ in the MSSM is calculated using GM2Calc [104], implementing two-loop corrections from [105–107] (see also [108,109]). Vacuum stability constraints are considered with the public code Evade [110,111]. All relevant SUSY searches for EW particles are considered, mostly via CheckMATE [112–114] (see Ref. [97] for details on many analyses newly implemented by our group).

For the DM relic density constraints we use the latest result from Planck [115], either as a direct measurement, or as an upper bound. The relic density in the MSSM is evaluated with MicrOMEGAs [73]. For the DD DM constraints, we use the results for the spin-independent DM scattering cross-section σ_p^{SI} from XENON-1T [86] experiment. The theoretical predictions are evaluated using the public code MicrOMEGAs.

3.3. Five Viable DM Scenarios

Five different scenarios were analyzed, classified by the mechanism that brings the LSP relic density into agreement with the measured values. The scenarios differ by the NLSP, or equivalently by the mass hierarchies between the mass scales determining the neutralino, chargino and slepton masses. These mass scales are the gaugino soft-SUSY breaking parameters M_1 and M_2 , the Higgs mixing parameter μ and the slepton soft SUSY-breaking parameters $m_{\tilde{l}_L}$ and $m_{\tilde{l}_R}$, see [96,97,99,100] for a detailed description. The five scenarios can be summarized as follows,

- (i) higgsino DM ($\mu < M_1, M_2, m_{\tilde{l}_L}, m_{\tilde{l}_R}$), DM relic density is only an upper bound (the relic density implies $m_{\tilde{\chi}_1^0} \sim 1$ TeV and $(g - 2)_\mu$ cannot be fulfilled), $m_{(\text{N})\text{LSP}} \lesssim 500$ GeV with $m_{\text{NLSP}} - m_{\text{LSP}} \sim 5$ GeV;
- (ii) wino DM ($M_2 < M_1, \mu, m_{\tilde{l}_L}, m_{\tilde{l}_R}$), DM relic density is only an upper bound, (the relic density implies $m_{\tilde{\chi}_1^0} \sim 3$ TeV and $(g - 2)_\mu$ cannot be fulfilled), $m_{(\text{N})\text{LSP}} \lesssim 600$ GeV with $m_{\text{NLSP}} - m_{\text{LSP}} \sim 0.3$ GeV;
- (iii) bino/wino DM with $\tilde{\chi}_1^\pm$ -coannihilation ($M_1 \lesssim M_2$), DM relic density can be fulfilled, $m_{(\text{N})\text{LSP}} \lesssim 650$ (700) GeV;

- (iv) bino DM with $\tilde{l}^\pm$ -coannihilation case-L ($M_1 \lesssim m_{\tilde{l}_L}$), DM relic density can be fulfilled, $m_{(N)\text{LSP}} \lesssim 650$ (700) GeV; and
- (v) bino DM with $\tilde{l}^\pm$ -coannihilation case-R ($M_1 \lesssim m_{\tilde{l}_R}$), DM relic density can be fulfilled, $m_{(N)\text{LSP}} \lesssim 650$ (700) GeV.

3.4. DM Results in the EW-MSSM

In this section, we review our results for the DM DD prospects in the five scenarios [96]. We take into account the projections for the exclusion reach of XENON-nT [116] and of the LZ experiment [117] (which effectively agree with each other). We also include the projections of the DarkSide [118] and Argo [119] experiments, which can go down to even lower cross sections, as well as the neutrino floor (NF) [120].

The results are summarized in Figure 3, where we show the $m_{\tilde{\chi}_1^0} - \sigma_p^{\text{SI}}$ planes for higgsino DM (upper left), wino DM (upper right), bino DM case-L (middle left) and case-R (middle right) and bino/wino DM with $\tilde{\chi}_1^\pm$ -coannihilation (lower plot). The color code indicates the DM relic density, where the red points are in full agreement with the Planck measurement. The black dashed, blue dashed, blue dot-dashed and black dot-dashed lines indicate the prospects for LZ/Xenon-nT, DarkSide, Argo and the NF, respectively.

One can observe that in for higgsino and wino DM all points will be covered by the next round of DD experiments, LZ and/or Xenon-nT are sufficient to cover the whole parameter space. The situation is different for bino DM case-L/R and bino/wino DM. In these three cases a large part of the allowed points cannot be probed by LZ/Xenon-nT. The Argon-based experiments can cover a substantially larger part of the allowed parameter space, where the parameter points giving the full DM relic density are mostly covered by LZ/Xenon-nT; however, DarkSide/Argo might be needed in the case of $\tilde{\chi}_1^\pm$ -coannihilation, as can be seen in the lower plot.

However, in all three scenarios, some parameter points are allowed even below the NF, which makes them inaccessible to current DD techniques (see [96] for a short discussion on future directional detection techniques). The allowed parameter spaces below the NF are relatively restricted in the LSP mass, which is bound to be $m_{\tilde{\chi}_1^0} \lesssim 400$ GeV.

At the High Luminosity LHC (HL-LHC), these points may still remain elusive due to the small mass splitting between the NLSP and the LSP (see the discussion in Ref. [96]). On the other hand, such small mass splittings do not pose a problem at e^+e^- colliders. The direct production of EW particles at e^+e^- colliders requires a sufficiently high center-of-mass energy, $\sqrt{s}$. Consequently, we focus here on a proposal for linear e^+e^- colliders, the ILC [121,122], which can reach energies up to 1 TeV, which we denote as ILC1000. We evaluate the cross-sections for the various LSP and NLSP pair production modes for $\sqrt{s} = 1$ TeV.

At the ILC1000 an integrated luminosity of 8ab^{-1} is foreseen [123,124]. The cross-section predictions are based on tree-level results, obtained as in Refs. [125,126]. Here, we do not attempt a rigorous experimental analysis, but follow analyses [127–129] that indicate that to a good approximation final states with the sum of the masses smaller than the center-of-mass energy can be detected.

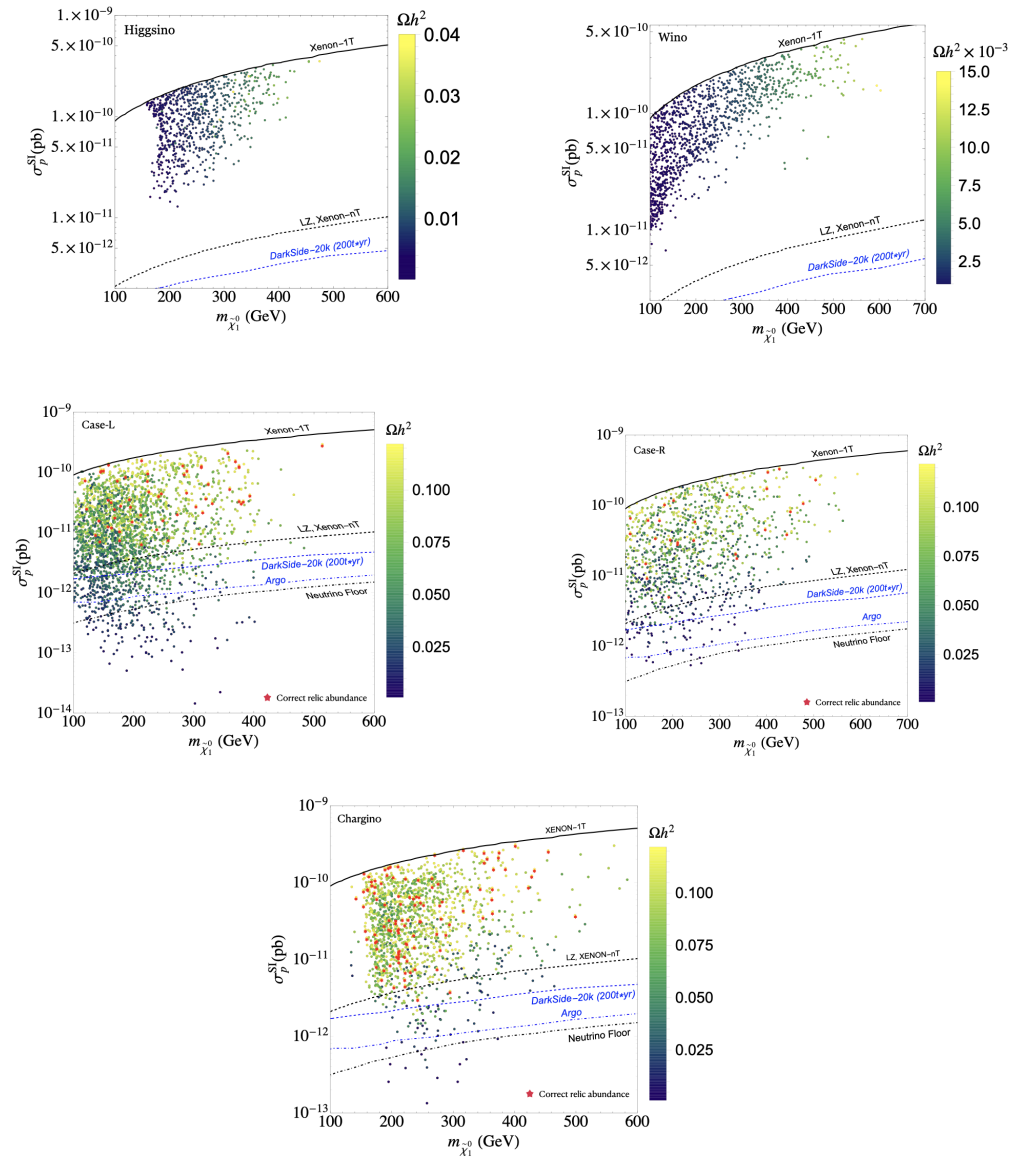


Figure 3. The results of our parameter scan in the five DM scenarios in the $m_{\tilde{\chi}_1^0} - \sigma_p^{SI}$ plane. The color code indicates the DM relic density. Red points are in full agreement with the Planck measurement.

In Figure 4, we show the LSP and NLSP pair production cross sections for an e^+e^- collider at $\sqrt{s} = 1000$ GeV as a function of the two (identical) final state masses. The upper plot shows $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 (+\gamma))$ production (Our tree level calculation does not include the photon radiation, which appears only starting from the one-loop level. However, such an ISR photon is crucial to detect this process due to the invisible final state. We take our tree-level cross section as a rough approximation of the cross section, including the ISR photon (see also [125]), and use the notation “ $(+\gamma)$ ”.) in green and $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ in blue. The open circles are the points below the anticipated XENON-nT/LZ limit, whereas the solid circles are the points below the neutrino floor. All points are within the reach of the ILC1000.

The cross sections range roughly from ~ 100 fb for low masses to ~ 100 fb for larger masses, with only a very few points have smaller cross sections. Overall, assuming an integrated luminosity of 8 ab^{-1} , this corresponds to $\sim 80,000$ – $800,000$ events. Consequently, in contrast to the HL-LHC, the e^+e^- colliders show a clear and conclusive complementarity

to the future DD experiments. The $\tilde{\chi}_1^\pm$ -coannihilation scenario will be fully covered by either DD experiments or by searches at the ILC1000.

The lower plots of Figure 4 show the LSP and NLSP production cross section in the $\tilde{l}^\pm$ -coannihilation scenario for case-L (left) and case-R (right). The green points show again $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0(+\gamma))$, whereas the violet points left and right show $\sigma(e^+e^- \rightarrow \tilde{\mu}_1\tilde{\mu}_1)$ (case-L) and $\sigma(e^+e^- \rightarrow \tilde{\mu}_2\tilde{\mu}_2)$ (case-R), respectively. Open and full circles denote, as above, the points below the anticipated XENON-nT/LZ limit and the neutrino floor.

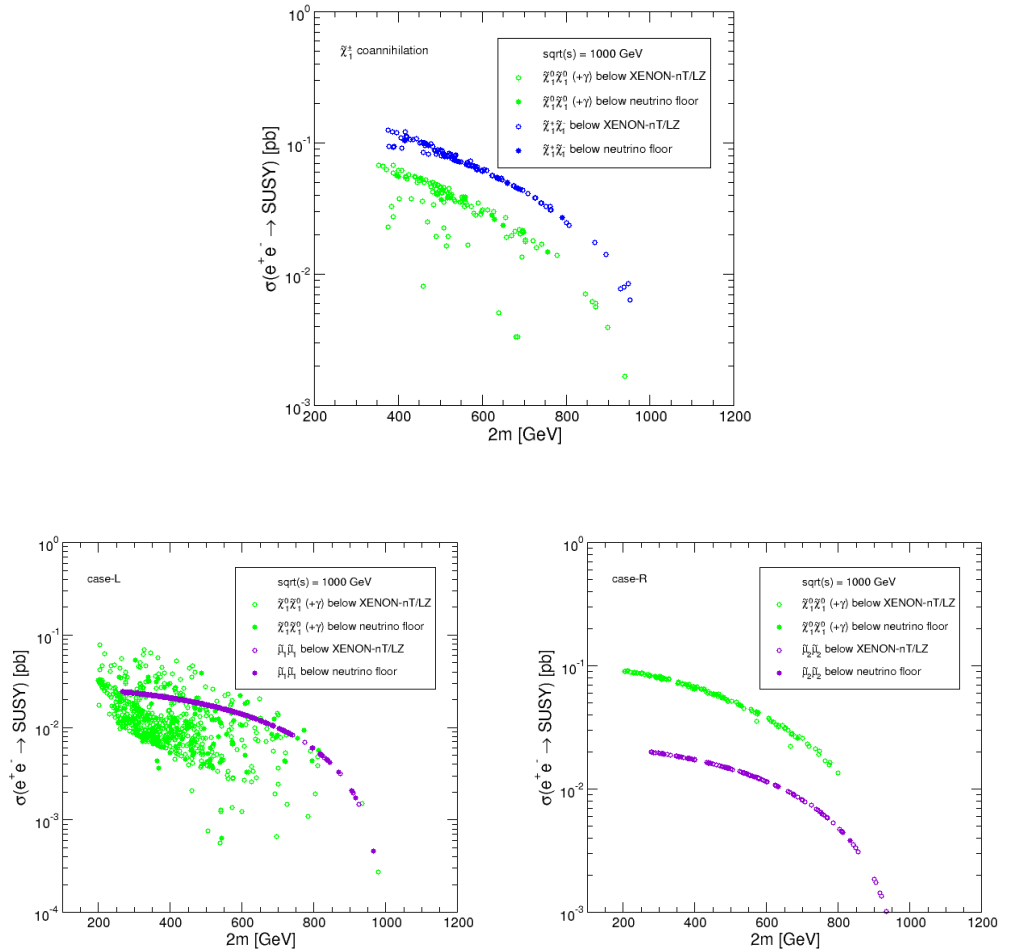


Figure 4. Cross section predictions at an e^+e^- collider with $\sqrt{s} = 1000$ GeV as a function of the sum of two final state masses. Upper plot: $\tilde{\chi}_1^\pm$ -coannihilation scenario; lower left plot: $\tilde{l}^\pm$ -coannihilation case-L; lower right plot: $\tilde{l}^\pm$ -coannihilation case-R. The color code indicates the final state, open circles are below the anticipated XENON-nT/LZ reach, and full circles are below the neutrino floor.

The visible spread in the $\tilde{\chi}_1^0\tilde{\chi}_1^0(+\gamma)$ production for case-L with regard to the case-R is a result of the more complex structure of the $e^\pm\tilde{e}_L\tilde{\chi}_1^0$ coupling as compared to the $e^\pm\tilde{e}_R\tilde{\chi}_1^0$ coupling, dominating the t -channel exchange diagram, respectively. In both cases, we see that, as for the $\tilde{\chi}_1^\pm$ -coannihilation case, all points result in particles that can be pair produced at the ILC1000 (except the very highest mass points in case-L). The cross sections range between 100 to 10 fb for $\tilde{\chi}_1^0\tilde{\chi}_1^0(+\gamma)$, and between 20 to 1 fb for smuon pair production. Even for the smallest production cross section, this corresponds to ~ 8000 events in the foreseen 1000 GeV ILC run. Furthermore, these two cases can conclusively be probed in the conjunction of DD experiments and an e^+e^- collider at $\sqrt{s} = 1000$ GeV, in contrast to the HL-LHC, where the prospects are less clear (see the discussion in [96]).

4. Results in the $\mu\nu$ SSM

In the $\mu\nu$ SSM [8], the presence in the superpotential of the RPV couplings $\lambda_i \hat{\nu}_i^c \hat{H}_d \hat{H}_u$ and $\kappa_{ijk} \hat{\nu}_i^c \hat{\nu}_j^c \hat{\nu}_k^c$, where $\hat{\nu}_i^c$ denote the right-handed neutrino superfields, solves dynamically both the μ - and ν -problems. After the successful EW symmetry breaking, the right-handed sneutrinos $\tilde{\nu}_{iR}$ develop vevs of the order of TeV, and therefore the λ couplings generate an effective mass term for Higgsinos, $\mu = \lambda_i \langle \tilde{\nu}_{iR} \rangle$, solving the μ -problem. In addition, the κ couplings, generate effective Majorana masses for right-handed neutrinos, $\mathcal{M}_{ij} = 2\kappa_{ijk} \langle \tilde{\nu}_{kR} \rangle$, solving the ν -problem through an EW-scale seesaw, i.e., it is possible to accommodate the correct neutrino masses and mixing angles with $Y_{ij}^\nu \lesssim 10^{-6}$ [8,12–15].

As of these small neutrino Yukawa couplings, the RPV is small in the $\mu\nu$ SSM. Note that, in the limit $Y_{ij}^\nu \rightarrow 0$ $\hat{\nu}_i^c$ can be identified as pure singlet superfields without lepton number, and, as a consequence, R -parity is conserved similarly to the NMSSM. Thus, Y_{ij}^ν are the parameters determining RPV in the $\mu\nu$ SSM. Since the LSP is not stable, it decays into SM particles with a low decay width related to the smallness of neutrino masses.

This connection between neutrino masses and RPV is relevant to have the gravitino or the axino as DM candidates, with large lifetimes due to the fact that their interactions are further suppressed by the small RPV parameters. In what follows, we briefly analyze the gravitino and/or axino LSP as DM in the $\mu\nu$ SSM, as well as discuss another DM candidate from the neutrino sector.

4.1. Gravitino

If R -parity is conserved the gravitino LSP is stable and therefore a DM candidate in the framework of supergravity [130–133] (see also Ref. [134] and references therein). Nevertheless, in the case of RPV the gravitino LSP can also be a (decaying) DM candidate [135,136] and can be detected through the observation of gamma-ray lines (and a smooth spectral signature).

Alternatively, superheavy gravitino DM from Starobinsky supergravity was recently proposed [137], which could also give rise to RPV decays. Decaying gravitino DM was studied in the $\mu\nu$ SSM for the case of the *Fermi* Large Area Telescope (*Fermi*-LAT) [138] in Refs. [27–31]. The prospects for detecting $\mu\nu$ SSM gravitino DM in future gamma-ray missions, such as enhanced ASTROGAM (e-ASTROGAM) [139] and All-sky Medium Gamma-ray Observatory (AMEGO) [140], were analyzed in Ref. [31].

The gravitino interacts with photon and photino; however, photino and left-handed neutrinos are mixed, and—as a consequence—the gravitino LSP decays into photon and neutrino as shown in Figure 5.

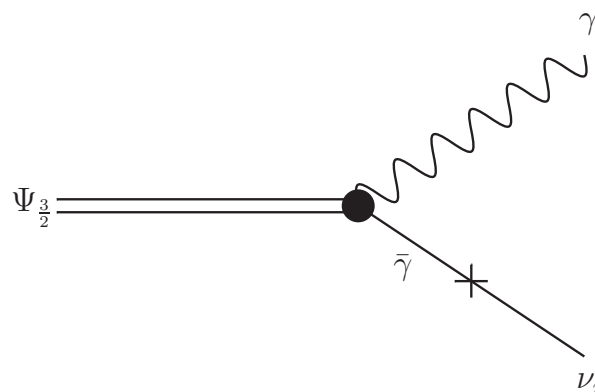


Figure 5. Tree-level diagram for the two-body decay of a gravitino into a photon and a neutrino, via photino-neutrino mixing.

The signals are gamma-ray lines with energies half of the gravitino mass $m_{3/2}$. The gravitino decay width is given by [135,136]:

$$\Gamma(\Psi_{3/2} \rightarrow \gamma \nu_i) \simeq \frac{m_{3/2}^3}{32\pi M_P^2} |U_{\tilde{\gamma}\nu}|^2, \quad (1)$$

where $\Gamma(\Psi_{3/2} \rightarrow \gamma \nu_i)$ denotes a sum of the partial decay widths into ν_i and $\bar{\nu}_i$, $M_P \approx 2.43 \times 10^{18}$ GeV is the reduced Planck mass, and the mixing parameter $U_{\tilde{\gamma}\nu}$ determines the photino content of the neutrino,

$$|U_{\tilde{\gamma}\nu}|^2 = \sum_{i=1}^3 |N_{i1} \cos \theta_W + N_{i2} \sin \theta_W|^2. \quad (2)$$

Here, N_{i1} (N_{i2}) is the bino (wino) component of the i -th neutrino, and θ_W is the weak mixing angle. One can easily estimate the value of $|U_{\tilde{\gamma}\nu}|$ in the $\mu\nu$ SSM [27], with the result

$$|U_{\tilde{\gamma}\nu}| \sim \frac{g' v_{iL}}{M_1}, \quad (3)$$

where v_{iL} are the vevs of the left-handed sneutrinos. For typical electroweak-scale values for the bino mass M_1 and $v_{iL} \lesssim 10^{-4}$ GeV in the $\mu\nu$ SSM since their values are determined by the small neutrino Yukawas, one obtains

$$10^{-8} \lesssim |U_{\tilde{\gamma}\nu}| \lesssim 10^{-6}. \quad (4)$$

This was confirmed in Refs. [27,30] by performing scans in the low-energy parameters of the $\mu\nu$ SSM in order to reproduce the observed neutrino masses and mixing angles. Relaxing some of the assumptions, such as an approximate GUT relation for gaugino masses and/or TeV scales for them, the lower bound can even be smaller: $10^{-10} \lesssim |U_{\tilde{\gamma}\nu}| \lesssim 10^{-6}$. As we can see from Equation (1), the gravitino decay is suppressed both, by the small RPV mixing parameter $|U_{\tilde{\gamma}\nu}|$ and by the scale of the gravitational interaction, making its lifetime much longer than the age of the Universe $\tau_{3/2} \gg t_{\text{today}} \sim 10^{17}$ s, with

$$\tau_{3/2} = \Gamma^{-1}(\Psi_{3/2} \rightarrow \gamma \nu_i) \simeq 3.8 \times 10^{33} \text{ s} \left(\frac{10^{-8}}{|U_{\tilde{\gamma}\nu}|} \right)^2 \left(\frac{0.1 \text{ GeV}}{m_{3/2}} \right)^3. \quad (5)$$

There are also three-body decays producing a smooth spectral signature. These processes were included in the analysis of Ref. [30]. The results imply that gravitino mass must be smaller than about 17 GeV and its lifetime larger than 4×10^{25} s.

4.2. Axino

The Axino LSP is another decaying DM candidate in the $\mu\nu$ SSM. Similar to the gravitino, it has an interaction term with photon and photino. The interaction is suppressed by the small RPV parameters as in the gravitino case and by the Peccei-Quinn scale instead of the gravitational one. As a consequence, the axino can have a lifetime greater than the age of the Universe and produces a signal with a gamma-ray line with energy half of the axino mass.

This framework was studied in Ref. [33] for Kim–Shifman–Vainshtein–Zakharov (KSVZ) [141,142] and Dine–Fischler–Srednicki–Zhitnitsky (DFSZ) [143,144] axion scenarios. The result implies that *Fermi*-LAT constraints impose that the axino mass must be smaller than about 3 GeV. Furthermore, a significant region of the parameter space lies in the ballpark of the proposed e-ASTROGAM. This would allow the exploration of masses in the range 2 MeV–3 GeV, as well as lifetimes of about 2×10^{26} – 8×10^{30} s.

4.3. Right-Handed Neutrino

Since the number of right-handed neutrinos is a free parameter in the $\mu\nu$ SSM, some of them might behave as sterile neutrinos and be viable candidates for warm DM [145]. This is similar to the (non-SUSY) neutrino minimal standard model (ν MSSM) [146]. In the case of the $\mu\nu$ SSM, we need some of the right-handed neutrinos to have small couplings, $Y^\nu \sim 10^{-13}$ and $\kappa \sim 10^{-8}$, in order to obtain keV masses and lifetimes long enough to be DM candidates [147].

4.4. Multicomponent DM

The framework of a DM scenario made of gravitinos and axinos was studied in Refs. [31,33]. If the axino is the LSP, a gravitino NLSP can live enough as to contribute to the relic abundance. Then, both particles can produce a signal detectable by future MeV-GeV gamma-ray telescopes, such as e-ASTROGAM. In addition, a well-tempered mixture of both particles can be found in a particular region producing a double-line signal as a smoking gun. Similar results are obtained with a gravitino LSP and an axino NLSP. Other candidates could also contribute to the total amount of relic abundance, such as the sterile neutrino, the axion, etc.

Summarizing, the $\mu\nu$ SSM is an interesting framework where the problem of DM can be solved through different candidates. The potential tensions between the standard Λ CDM model and cosmological observations (see, e.g., Ref. [148]) might be relaxed if several of these candidates contribute to DM.

5. Results in the $U\mu\nu$ SSM

We review, in this section, a specific WIMP DM realization in the framework of the $U\mu\nu$ SSM [40]. The $U\mu\nu$ SSM [38] is a $U(1)'$ extension of the $\mu\nu$ SSM, where baryon-number-violating operators as well as explicit mass terms are forbidden, and the potential domain wall problem is avoided. In order to ensure an anomaly free theory, states charged under the new gauge symmetry are introduced: exotic quarks $\mathbb{K}_i$ and additional singlets $\hat{\xi}_\alpha$ under the SM gauge group, such that we consider the following relevant superpotential [38]:

$$W = Y_{ij}^e \hat{H}_d \hat{L}_i \hat{e}_j^c + Y_{ij}^d \hat{H}_d \hat{Q}_i \hat{d}_j^c - Y_{ij}^u \hat{H}_u \hat{Q}_i \hat{u}_j^c - Y_{ij}^\nu \hat{H}_u \hat{L}_i \hat{\nu}_j^c + \lambda_i \hat{\nu}_i^c \hat{H}_u \hat{H}_d + k_{i\alpha\beta} \hat{\nu}_i^c \hat{\xi}_\alpha \hat{\xi}_\beta + Y_{ij}^{\mathbb{K}} \hat{\nu}_i^c \mathbb{K}_j \mathbb{K}_j^c \quad (6)$$

where the summation convention is implied on repeated indexes, with $i, j, k = 1, 2, 3$, the usual family indexes of the SM and $\alpha, \beta = 1, 2$ in the simplest construction. Our convention for the contraction of two $SU(2)$ doublets is, e.g., $\hat{H}_u \hat{H}_d \equiv \epsilon_{ab} \hat{H}_u^a \hat{H}_d^b$, $a, b = 1, 2$ and ϵ_{ab} the totally antisymmetric tensor with $\epsilon_{12} = 1$.

Masses for these new states are generated dynamically once the right-handed sneutrino acquires a vev, simultaneously generating the μ -term and masses for right-handed neutrinos.

5.1. The $U\mu\nu$ SSM and WIMP DM

A discrete Z_2 symmetry is present in the superpotential term of Equation (6) containing the superfields of type $\hat{\xi}_\alpha$, under which they have a charge -1 and the rest of the particle content a charge $+1$. This symmetry is not an extra requirement but arises from the charge assignment of the model and is a consequence of the gauge anomaly cancellation conditions. Such symmetry remains intact after the spontaneous symmetry breaking of the extra $U(1)'$ by the VEVs of right-handed sneutrinos.

As of this Z_2 symmetry, the superfields of type $\hat{\xi}_\alpha$ can only appear in pairs in the Lagrangian. As a consequence, it is straightforward to realize that vanishing vevs for their scalar components, $\langle \xi_\alpha \rangle = 0$, is a solution of the minimization equations. Thus, the Z_2 symmetry is exact, and therefore we can have either the bosonic or the fermionic components of $\hat{\xi}_\alpha$ as WIMP DM, without introducing R -parity.

Since the hierarchy $m_{\xi_\alpha^{\mathbb{R}}} > m_{\xi_\alpha^{\mathbb{I}}} > m_{\xi_\alpha^{\mathbb{F}}}$ can be naturally satisfied [38], we use, in our analysis, the lightest of the fermionic components of the superfields, say $\tilde{\xi}_1$, as the DM

particle. The heaviest state $\tilde{\xi}_2$ can decay for example to $\tilde{\xi}_2 \rightarrow \tilde{\xi}_1 \bar{q} q$ and therefore does not play any role in the phenomenology of interest here. In what follows, we denote our DM candidate by $\tilde{\xi} \equiv \tilde{\xi}_1$.

5.2. Constraints on the Parameter Space

This kind of DM interacts with the SM particle content via exchange of a new massive gauge boson Z' , right sneutrinos, SM-like Higgs via scalar mixing, as well as DM exchange (see Ref. [38]). In this setup, SI (SD) DM-nucleon scatterings are mediated by Higgs via scalar mixing (Z'), by interactions with light quarks within nucleons (see Figure 6). Therefore, DM direct detection experiments can probe regions of our parameter space.

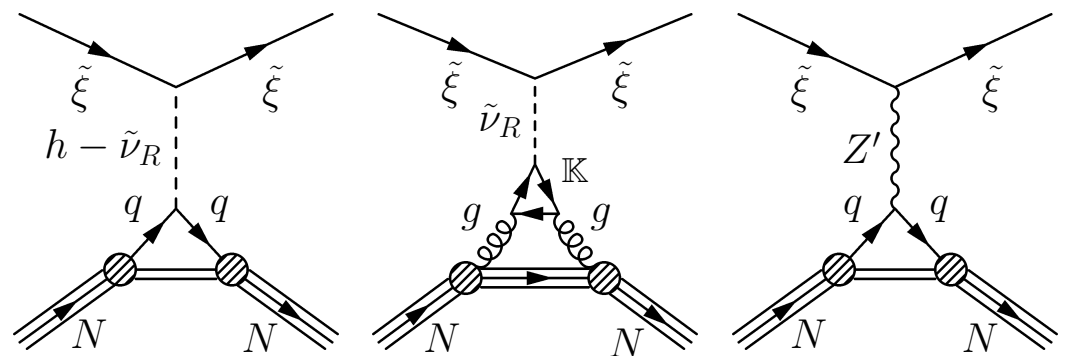


Figure 6. Diagrams contributing to DM scattering with nucleons mediated by (left panel) SM-like Higgs h and right sneutrinos $\tilde{\nu}_R$ via mixing, (middle panel) right-handed sneutrinos $\tilde{\nu}_R$ and induced by loop effects involving heavy exotic quarks $\mathbb{K}$, (right panel) Z' gauge boson.

We also point out that the exotic quarks offer an additional channel for SI scatterings by interacting directly with the gluons present in the nucleons and with DM by right sneutrino mediation. As the presence of these exotics is required by the anomaly cancellation conditions, their contribution is a rather general prediction of the $U\mu\nu$ SSM. Although it turns out to be significant only in specific corners of the parameter space of our scan range, it offers nevertheless the possibility of testing a part of the parameters in the future in the case of low values of the scalar mixing.

Additional constraints on this scenario are imposed by Z' LHC searches, which can exclude masses $m_{Z'} \simeq 1 - 5$ TeV depending on the value of the $U(1)'$ gauge coupling (see Ref. [38]), as well as R-hadron searches, which provide a lower bound on the masses of exotic quarks of the order of the TeV scale. Concerning LHC signals of the DM particle itself, the direct production is quite suppressed because it is a SM singlet. However, in regions of the parameter space where the singlets $\tilde{\xi}$ are lighter than $m_{Z'}/2$, they could be produced in Z' decays. The decay $Z' \rightarrow \tilde{\xi}_2 \tilde{\xi}_2$ with subsequent decay $\tilde{\xi}_2 \rightarrow \tilde{\xi}_1 \ell^+ \ell^-$ produces two pairs of collimated leptons, which can give striking signatures [149]. Other decay modes, such as $\tilde{\xi}_2 \rightarrow \tilde{\xi}_1 q \bar{q}$ are likely unobservable, as are the decays $Z' \rightarrow \tilde{\xi}_1 \tilde{\xi}_1$.

5.3. Analysis

We analyzed the possibility of reproducing the observed DM relic abundance via the freeze-out mechanism in this setup, performing a numerical study of the viable parameter space respecting all constraints. Results from Xenon1T experiment already exclude a subdominant portion of the allowed parameter space. We identified two main regions allowed by Xenon1T, at large DM masses $m_{\tilde{\xi}} \gtrsim 2-3$ TeV and at smaller DM masses $200 \text{ GeV} \lesssim m_{\tilde{\xi}} \lesssim 2-3$ TeV (see Figure 7).

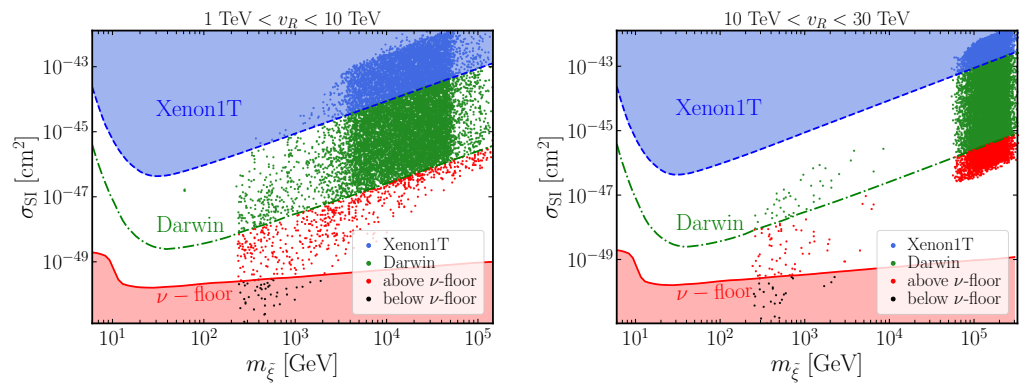


Figure 7. Spin-independent DM-nucleon cross section σ_{SI} versus DM mass $m_{\tilde{\xi}}$ in the parameter space of our model for the relevant two intervals of right-handed sneutrino VEVs (left panel) $1 \text{ TeV} < v_R < 10 \text{ TeV}$ and (right panel) $v_R > 10 \text{ TeV}$. Blue dots are excluded by the Xenon1T experiment [150]. Green dots will be probed by the upcoming Darwin experiment [151]. Red (black) dots correspond to points above (below) the neutrino floor [152].

For the case of large masses, the dynamical generation of the DM mass implies relatively large couplings with the right sneutrino. For this regime, new bosonic (right-handed sneutrinos $\tilde{\nu}_R$, heavy gauge boson Z') and fermionic (exotic quarks $\mathbb{K}$, neutralinos $\tilde{\chi}_i$ and charginos $\tilde{\chi}^{\pm}$) states are the most frequent particles present in the final states of DM annihilation and therefore essential to reproduce the correct relic abundance.

For the highest masses, $m_{\tilde{\xi}} \gtrsim 10^5 \text{ GeV}$, the viable part of the parameter space requires couplings that are typically on the edge of perturbative unitarity. This part of the parameter space offers optimistic detection prospects as the Darwin experiment should probe the majority of the viable parameters in the following year and the remaining part should be accessible with an increased exposure.

For the region with lower masses, achieving the correct relic abundance is less frequent as most of the annihilation channels mentioned previously are kinematically forbidden after imposing constraints on the new states. This regime typically relies on s -channel $\tilde{\nu}_R$ or Z' resonances with SM particles in the final states, such as quarks. Relatively low couplings are typically required for such masses and therefore the direct detection prospects are less optimistic. Nevertheless, a substantial part of the parameter space will be accessible by the Darwin experiment.

Interestingly, as many annihilation channels are usually required to achieve the correct relic abundance, non-velocity suppressed DM annihilation within large astrophysical structures could offer complementary detection prospects by indirect gamma-ray searches with the upcoming Cherenkov Telescope Array (CTA) [153].

6. Conclusions

We reviewed DM analyses in various SUSY models that have been obtained in the recent years by groups (to a relevant part) based in Spain. The first model was the pMSSM11, the phenomenological MSSM with 11 free parameters, based on Ref. [41]. We found that this model gives a perfect description of all relevant experimental data, including LHC searches, the anomalous magnetic moment of the muon, the DM relic abundance and the DD limits. A large part of the parameter space will be accessible to future spin-independent DD experiments. However, even at the 1σ level, points below the neutrino floor were found, making them difficult to access in future DD experiments.

The second model analyzed was the electroweak (EW)-MSSM [96], where the colored and the Higgs sector are assumed to be heavy, so that they do not play a role in the phenomenological analysis. Five different scenarios, depending on the mechanism that brings the DM relic abundance in agreement with the Planck limit, were identified. The wino and higgsino DM case can be covered by the next round of DD experiments. In the

other three scenarios, two cases of bino DM and mixed bino/wino DM, large parts of the allowed parameter space can be covered by future Argon-based DD experiments. However, all three scenarios contain parameter points below the neutrino floor. For these points, it was shown that possible future e^+e^- colliders with $\sqrt{s} \lesssim 1$ TeV can conclusively test these scenarios (while this may not be possible at the HL-LHC).

The third model analyzed was the $\mu\nu$ SSM [8], based on Ref. [27]. We showed first that the gravitino LSP is an interesting decaying DM candidate, with a lifetime larger than the age of the Universe. In order to avoid too large gamma-ray fluxes that are incompatible with *Fermi*-LAT observations, gravitino masses must be smaller than about 17 GeV and lifetimes larger than 4×10^{25} s [29,30].

We also applied this constraint to the case of axino DM, obtaining that its mass must be smaller than about 3 GeV. In addition, a significant region of the parameter space of axino DM lies in the ballpark of future gamma-ray missions, such as the proposed e-ASTROGAM, allowing the exploration of masses and lifetimes in the ranges 2 MeV–3 GeV and 2×10^{26} – 8×10^{30} s, respectively, [33]. The possibility of a multicomponent DM scenario made of gravitinos and axinos was also discussed [31,33].

We found that an axino or the gravitino can produce a signal detectable by future MeV-GeV gamma-ray telescopes, such as e-ASTROGAM. In addition, there is a parameter region where a well-tempered mixture of both particles is obtained, with a double-line signal arising as a smoking gun. Finally, we suggested that other DM candidates could be available in the $\mu\nu$ SSM in certain regions of the parameter space, such as a sterile neutrino [147].

The last model analyzed was the $U\mu\nu$ SSM [38], based on Ref. [40]. This model is a $U(1)'$ extension of the $\mu\nu$ SSM, where the gauge anomaly-cancellation conditions impose the presence of exotic quark superfields in the spectrum and allow the presence of several singlet superfields under the SM gauge group, in addition to the right-handed neutrino superfields. The gauge structure implies an additional discrete Z_2 symmetry in the superpotential, ensuring the stability of a singlet, which behaves as WIMP DM without invoking R -parity.

We analyzed this novel possibility in detail, using the fermionic component of the singlet as the DM candidate. In particular, we computed its amount of relic density via Z' , Higgs-right sneutrino and DM mediated annihilations and its potential signals in DM direct detection experiments. The constraints on the parameter space due to Z' direct searches at the LHC were imposed in the analysis as well as those from the hadronization inside the detector of the exotic quarks. Large regions of the parameter space were found to be in the reach of the upcoming Darwin experiment.

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