universe

MDPI

Communication

Using Neutrino Oscillations to Measure H_0?

Luis A. Anchordoqui [1,2]

[1] Department of Physics and Astronomy, Lehman College, City University of New York, New York, NY 10468, USA; luis.anchordoqui@gmail.com
[2] Department of Physics, Graduate Center, City University of New York, New York, NY 10016, USA

Abstract: Recently, the idea of using neutrino oscillations to measure the Hubble constant was introduced. We show that such a task is unfeasible because for typical energies of cosmic neutrinos, oscillations average out over cosmological distances and so the oscillation probability depends only on the mixing angles.

Keywords: H_0 tension; neutrino oscillations

check for updates

Citation: Anchordoqui, L.A. Using Neutrino Oscillations to Measure H_0? *Universe* **2022**, *8*, 377. https://doi.org/10.3390/universe8070377

Academic Editor: Antonino Del Popolo

Received: 30 June 2022
Accepted: 6 July 2022
Published: 11 July 2022

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Copyright: © 2022 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

1. Introduction

In the course of the last decade, an intriguing inconsistency between measurements of the cosmic expansion rate based on early- and late-Universe probes has emerged. This inconsistency shows up as a discrepancy in the value of the Hubble constant H_0 [1], as inferred from measurements of the anisotropy of the cosmic microwave background, $H_0 = (67.27 \pm 0.60)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ at 68% CL [2], and as measured from a series of distance indicators in the local Universe. More precisely, the latest distance ladder measurement based on Type Ia supernovae (SNIa) calibrated by Cepheids gives $H_0 = (73.04 \pm 1.04)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ at 68% CL [3], whereas using the Tip of the Red Giant Branch to calibrate SNIa leads to $H_0 = (72.4 \pm 2.0)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [4] and $H_0 = (69.6 \pm 0.8\,(\mathrm{stat}) \pm 1.7\,(\mathrm{sys}))\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [5], both at 68% CL. Depending on which set of measurements one combines, the tension between the model-dependent and independent estimates of H_0 sits between 4.5σ to 6.3σ [6]. This so-called "H_0 tension" has become the new cornerstone of modern cosmology, and many new-physics setups are rising to the challenge [7].

In a recent publication, the possibility of using neutrino oscillations as a distance indicator to measure H_0 [8] was entertained. In this article, we show that such a task is unfeasible because for typical energies of neutrinos originating via cosmic-ray interactions in astrophysical sources, oscillations average out over cosmological distances and so the oscillation probability depends only on the angles of the leptonic mixing matrix.

The layout of the paper is as follows. In Section 2, we go through the formalism of neutrino oscillations and show that oscillation averaged probabilities for transition between flavors have no dependence on the travel distance to the astrophysical sources of cosmic neutrinos. In Section 3, we examine the conditions for the coherence loss of the neutrino wavepacket. Finally, in Section 4, we present our conclusions.

2. Oscillations of High-Energy Cosmic Neutrinos

Neutrino oscillations are the outcome of nonzero neutrino masses and the certainty that virtually all useful neutrino sources are coherent. In other words, the neutrinos produced via charged-current weak interactions associated with l_α charged leptons, $\alpha = e, \mu, \tau$, can be described as coherent superpositions of neutrino states ν_j with different masses m_j, $j = 1, 2, 3$, weighted by the elements $U_{\alpha j}$ of the Pontecorvo–Maki–Nagakawa–Sakata (PMNS) matrix [9–11], i.e.,

$$|\nu_\alpha\rangle = \sum_j U^*_{\alpha j}\,|\nu_j\rangle\,. \tag{1}$$

The superposition of mass eigenstates is valid in no small part because neutrino masses are tiny when compared with their laboratory/cosmic energies. For a neutrino of energy E_ν traveling a distance L, we can conveniently parameterize the oscillation phase Δ_{ij} as

$$\Delta_{ij} = \frac{\Delta m_{ij}\, L}{4E_\nu} \simeq 1.27 \left(\frac{\Delta m^2_{ij}}{\mathrm{eV}^2}\right) \left(\frac{L}{\mathrm{km}}\right) \left(\frac{E_\nu}{\mathrm{GeV}}\right)^{-1}, \tag{2}$$

where $\Delta m^2_{ij} \equiv m^2_i - m^2_j$ [12]. Now, for a relatively low redshift of $z = 0.05$, the distance traveled by the neutrinos is about 200 Mpc. Taking the highest energy measured at the IceCube facility $E_\nu \sim 10^7$ GeV [13] and the solar mass splitting, $\Delta m^2_{ij} \equiv \Delta m^2_\odot \simeq 7.42 \times 10^{-5}$ eV^2 [14], Equation (2) leads to $\Delta_{ij} \sim 6 \times 10^{10}$, which is the number of oscillation periods the neutrinos would experience. That is, the neutrinos would experience 60 billion oscillations to the Earth over a redshift $z = 0.05$. This implies that we would need to know $\Delta m^2_\odot$ to one part in 60 billion and we would also have to measure E_ν to one part in 60 billion. Both of these measurements are ridiculously unfeasible. After averaging over these uncertainties, we find the so-called oscillation averaged probability for transition of flavor α to β, which is given by

$$P(\nu_\alpha \to \nu_\beta) = \sum_i U^2_{\alpha i} U^2_{\beta i}\,, \tag{3}$$

and has no dependence on L [12]. Note that for $z = 0.05$, an oscillation phase $\Delta_{ij} \sim 1$ would require an unrealistic neutrino energy of roughly 10^{16} GeV.

3. Coherence Loss of Cosmic Neutrino Oscillations

There is also a loss of coherence during neutrino propagation [15,16]. This is because different neutrino mass eigenstates of the same energy have different velocities, and so the wavepackets of the mass eigenstates composing a neutrino state will come apart as they propagate. For neutrinos traveling over cosmological distances, the flying path is so large that these components completely separate from each other. In the case of coherence loss, the oscillatory terms of the oscillation probabilities disappear.

Following [17], we define $\Delta E_{\nu,L}$ as the energy difference for which

$$\Delta_{ij}(E_\nu - \Delta E_{\nu,L}, L) - \Delta_{ij}(E_\nu, L) = \pi\,. \tag{4}$$

A straightforward calculation leads to

$$\Delta E_{\nu,L} \simeq \frac{4\pi E^2_\nu}{\Delta m^2_{ij} L} = E_\nu \frac{\ell_{ij}}{L}\,, \tag{5}$$

where $\ell_{ij} = 4\pi E_\nu / \Delta m^2_{ij}$ is the vacuum oscillation length. As neutrinos travel cosmological distances between their origins and us, they are essentially on their mass shells, satisfying $E^2_\nu = p^2 + m^2_j$. The relativistic dispersion relation implies that $E_\nu \sigma_{E_\nu} = p\sigma_p$, where σ_{E_ν} and σ_p are the uncertainties in the energy and the width of the wavepacket in the longitudinal direction. Because neutrinos are ultrarelativistic $E_\nu \simeq p$, and hence $\sigma_{E_\nu} \simeq \sigma_p$. It is easily seen that the interference between the effects of different mass eigenstates disappears if

$$\sigma_p > \Delta E_{\nu,L} = E_\nu \frac{\ell_{ij}}{L}\,; \tag{6}$$

an equivalent expression can be found in configuration space, where the size of the wavepacket at the production point is given by the inverse of the uncertainty σ_p, i.e., $\sigma_x \sim \sigma_p^{-1}$ [17].

If the parent pion does not undergo any interaction with matter or with the magnetic field before decay, the width of the wavepacket in the configuration space is estimated to be

$$\sigma_x \sim 2 \times 10^{-6} \left(\frac{E_\nu}{10^7 \text{ GeV}} \right)^{-1} \text{cm} \tag{7}$$

whereas for a parent muon,

$$\sigma_x \sim 2 \times 10^{-4} \left(\frac{E_\nu}{10^7 \text{ GeV}} \right)^{-1} \text{cm}\,, \tag{8}$$

i.e., for the neutrinos produced by muons, σ_x is larger by the ratio of the lifetime of the muon to that of the pion [17]. The dominant modification of σ_x at the sources comes from the interaction of the parent charged particle with the magnetic field B, which can be parameterized as

$$\sigma_x \sim 6 \times 10^{-19} \left(\frac{\Gamma}{100} \right)^{1/2} \left(\frac{B}{10^7 \text{ GeV}} \right)^{-1/2} \left(\frac{E_\nu}{10^7 \text{ GeV}} \right)^{-3/2} \text{cm}\,, \tag{9}$$

for pions and

$$\sigma_x \sim 5 \times 10^{-17} \left(\frac{\Gamma}{100} \right) \left(\frac{B}{10^7 \text{ G}} \right)^{-1} \left(\frac{E_\nu}{10^7 \text{ GeV}} \right)^{-2} \text{cm}\,, \tag{10}$$

for muons, where Γ is the Lorentz boost of the plasma [17].

A straightforward calculation shows that for typical B fields of cosmic neutrino sources, the mass states will decohere after propagation over cosmological distances, i.e., the mass eigenstates will be separated enough that there is no overlap of the wavefunctions and we detect on Earth the three mass states each with their own probability given by Equation (3). Indeed, as shown in [18], oscillation averaging and full decoherence yield the same probability. This implies that it does not even matter if we measure E_ν and Δm^2_{ij} to arbitrary precision.

4. Conclusions

We have shown that for typical energies of cosmic neutrinos, oscillations average out over cosmological distances and so the oscillation probability depends only on the mixing angles of the PMNS matrix. As a consequence, neutrino oscillations cannot be used to estimate cosmological distances and therefore cannot be adopted as a probe to measure H_0.

Funding: Work supported by the U.S. National Science Foundation (NSF Grant PHY-2112527).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Hubble, E. A relation between distance and radial velocity among extra-galactic nebulae. *Proc. Nat. Acad. Sci. USA* **1929**, *15*, 168–173. [CrossRef] [PubMed]
2. Aghanim, N. et al. [Planck Collaboration]. Planck 2018 results VI: Cosmological parameters. *Astron. Astrophys.* **2020**, *641*, A6; Erratum in *Astron. Astrophys.* **2021**, *652*, C4. [CrossRef]
3. Riess, A.G.; Yuan, W.; Macri, L.M.; Scolnic, D.; Brout, D.; Casertano, S.; Jones, D.O.; Murakami, Y.; Breuval, L.; Brink, T.G.; et al. A comprehensive measurement of the local value of the Hubble constant with 1 km/s/Mpc uncertainty from the Hubble Space Telescope and the SH0ES Team. *arXiv* **2021**, arXiv:2112.04510.

4. Yuan, W.; Riess, A.G.; Macri, L.M.; Casertano, S.; Scolnic, D. Consistent calibration of the Tip of the Red Giant Branch in the Large Magellanic Cloud on the Hubble Space Telescope photometric system and a re-determination of the Hubble constant. *Astrophys. J.* **2019**, *886*, 61. [CrossRef]
5. Freedman, W.L.; Madore, B.F.; Hoyt, T.; Jang, I.S.; Beaton, R.; Lee, M.G.; Monson, A.; Neeley, J.; Rich, J. Calibration of the Tip of the Red Giant Branch (TRGB). *Astrophys. J.* **2020**, *891*, 57. [CrossRef]
6. Abdalla, E.; Abellán, G.F.; Aboubrahim, A.; Agnello, A.; Akarsu, Ö.; Akrami, Y.; Alestas, G.; Aloni, D.; Amendola, L.; Anchordoqui, L.A.; et al. Cosmology intertwined: A review of the particle physics, astrophysics, and cosmology associated with the cosmological tensions and anomalies. *J. High Energy Astrophys.* **2022**, *34*, 49–211. [CrossRef]
7. Valentino, E.D.; Mena, O.; Pan, S.; Visinelli, L.; Yang, W.; Melchiorri, A.; Mota, D.F.; Riess, A.G.; Silk, J. In the realm of the Hubble tension—A review of solutions. *Class. Quant. Grav.* **2021**, *38*, 153001. [CrossRef]
8. Khalifeh, A.R.; Jimenez, R. Using neutrino oscillations to measure H_0. *Phys. Dark Univ.* **2022**, *37*, 101063. [CrossRef]
9. Pontecorvo, B. Neutrino experiments and the problem of conservation of leptonic charge. *Zhur. Eksptl. Teoret. Fiz.* **1967**, *53*, 1717–1725.
10. Pontecorvo, B. Inverse beta processes and nonconservation of lepton charge. *Zhur. Eksptl. Teoret. Fiz.* **1957**, *34*, 247.
11. Maki, Z.; Nakagawa, M.; Sakata, S. Remarks on the unified model of elementary particles. *Prog. Theor. Phys.* **1962**, *28*, 870–880. [CrossRef]
12. Anchordoqui, L.A.; Barger, V.; Cholis, I.; Goldberg, H.; Hooper, D.; Kusenko, A.; Learned, J.G.; Marfatia, D.; Pakvasa, S.; Paul, T.C.; et al. Cosmic Neutrino Pevatrons: A Brand New Pathway to Astronomy, Astrophysics, and Particle Physics. *J. High Energy Astrophys.* **2014**, *1–2*, 1–30. [CrossRef]
13. Abbasi, R. et al. [IceCube Collaboration]. The IceCube high-energy starting event sample: Description and flux characterization with 7.5 years of data. *Phys. Rev. D* **2021**, *104*, 022002. [CrossRef]
14. Gonzalez-Garcia, M.C.; Maltoni, M.; Schwetz, T. NuFIT: Three-flavour global analyses of neutrino oscillation experiments. *Universe* **2021**, *7*, 459. [CrossRef]
15. Kiers, K.; Nussinov, S.; Weiss, N. Coherence effects in neutrino oscillations. *Phys. Rev. D* **1996**, *53*, 537–547. [CrossRef] [PubMed]
16. Stodolsky, L. The unnecessary wave packet. *Phys. Rev. D* **1998**, *58*, 036006. [CrossRef]
17. Farzan, Y.; Smirnov, A.Y. Coherence and oscillations of cosmic neutrinos. *Nucl. Phys. B* **2008**, *805*, 356–376. [CrossRef]
18. Ohlsson, T. Equivalence between neutrino oscillations and neutrino decoherence. *Phys. Lett. B* **2001**, *502*, 159–166. [CrossRef]