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Measuring the Modified Gravitational Wave Propagation beyond General Relativity from CMB Observations

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Abstract: In modified gravity theories, gravitational wave propagations are presented in nonstandard ways. We consider a friction term different from GR and constrain the modified gravitational waves propagation from observations. The modified gravitational waves produce anisotropies and polarization, which generate measurable tensor power spectra. We explore the impact of the friction term on the power spectrum of B-modes and the impact on the constraints on the other parameters (e.g., r or A_t) when ν_0 is allowed to vary in the Monte Carlo analyses from Planck+BK18 datasets. If we assume the result of the scalar perturbations is unchanged, the inflation consistency relation alters with the friction term. In the ΛCDM+r+ν_0 model, the tensor-to-scalar ratio and the amplitude of the tensor spectrum are obviously influenced.

Keywords: primordial gravitational waves; modified friction term; CMB observations

Citation: Li, J. Measuring the Modified Gravitational Wave Propagation beyond General Relativity from CMB Observations. *Universe* **2022**, *8*, 367. https://doi.org/10.3390/universe8070367

Academic Editors: Pedro Pina Avelino and Zhiqi Huang

Received: 13 May 2022
Accepted: 29 June 2022
Published: 3 July 2022

1. introduction

In recent decades, general relativity (GR) has become an integral and indispensable part of modern physics. While GR describes the dynamics of space-time and governs the behaviors of our universe extremely successfully, the rapid development of observations inspires us to explore new physics beyond GR. Modified gravity theories provide plausible possibilities, which are worth pursuing and could give a better understanding of observations. In modified gravity theories, the evolution equations of gravitational waves are presented in nonstandard ways [1–13]. The friction term is a fundamental issue in the propagation of gravitational waves. When the gravitational waves propagate with a friction term different from GR, this scenario generates a variety of modified gravity theories [1–8]. Probing the friction term is an important way to explore modified gravity and underlying new physics. Hence, we need to investigate the possible deviations from GR and search for corresponding observable effects at cosmological scales.

The friction term in gravitational-wave propagation is time-dependent variation and represents a damping effect. Modified gravity models provide the motivation for the deviations of the friction term, such as the Horndeski theories [14–17], the nonlocal infrared modification of gravity [18,19] or the quantum chromodynamic (QCD) phase transition [20,21]. Recently, there have been some works that discuss and constrain the friction terms from the LIGO-Virgo Collaboration [5,22,23]. Furthermore, some papers assume a model-independent friction term and search for corresponding signatures. Instead of testing individual modified gravity models, they parametrize and investigate the departures from GR. Parametrization has been used widely in physics, such as the ΛCDM model, which can highlight the direction of modified gravity by the deviations of parameters. The tensor-mode parametrization for modified gravity has been proposed in [3], which discusses a general form of the modified tensor-mode propagation, including several physical effects. The inflation consistency relation is modified with the friction term, but they do not consider this when updating the constraints on friction.

In this paper, we consider the same friction term as [3] and constrain the modified gravitational-wave propagation from observations. When we consider the modified friction term, the behavior of gravitational waves changes. The modified gravitational waves produce anisotropies and polarization, which generate measurable tensor power spectra. Here, we explore the impact of the friction term on the power spectrum of B-modes and the impact of the constraints on the other parameters (e.g., r or A_t) when ν_0 is allowed to vary in the Monte Carlo analyses from Planck observations [24] and BICEP/Keck observations through the 2018 observing season (BK18) [25]. Owing to the modified inflation consistency relation, the tensor-to-scalar ratio and the amplitude of the tensor spectrum are obviously influenced.

The modified gravity may have an impact on both tensor and scalar perturbations. If all of the modified terms are considered together, it is hard to figure out the effects of the friction term. Here, we parametrize the friction term only and investigate corresponding observable effects.

2. The Modified Gravitational Waves Propagation

In the conformal Newtonian gauge, the metric about the Friedmann–Robert–Walker background is taken as

$$ds^2 = a^2\left\{-(1+2\Phi)d\eta^2 + \left[(1-2\Phi)\delta_{ij} + \frac{h_{ij}}{2}\right]dx^i dx^j\right\}, \tag{1}$$

where $a(\eta)$ is the scale factor, η is the conformal time, Φ is the scalar perturbation and h_{ij} is the gravitational wave perturbation. In GR, the gravitational waves satisfy the following wave equation

$$h_k'' + 2\frac{a'}{a}h_k' + k^2 h_k = 0, \tag{2}$$

where the prime denotes the derivative with respect to conformal time, and the source term is ignored. Tensor perturbations produce anisotropies and polarization, which could generate measurable tensor angular power spectra. The temperature and polarization perturbations satisfy the Boltzmann equations [26]

$$\tilde{\Delta}_T'^{(T)} + ik\mu\tilde{\Delta}_T^{(T)} = -h' - \tau'[\tilde{\Delta}_T^{(T)} - \Psi], \tag{3}$$

$$\tilde{\Delta}_P'^{(T)} + ik\mu\tilde{\Delta}_P^{(T)} = -\tau'[\tilde{\Delta}_P^{(T)} + \Psi], \tag{4}$$

where

$$\Psi = \frac{\tilde{\Delta}_{T0}^{(T)}}{10} + \frac{\tilde{\Delta}_{T2}^{(T)}}{7} + \frac{3\tilde{\Delta}_{T4}^{(T)}}{70} - \frac{3\tilde{\Delta}_{P0}^{(T)}}{5} + \frac{6\tilde{\Delta}_{P2}^{(T)}}{7} - \frac{3\tilde{\Delta}_{P4}^{(T)}}{70}, \tag{5}$$

the variables $\tilde{\Delta}_T^{(T)}$ and $\tilde{\Delta}_P^{(T)}$ describe the temperature and polarization perturbations generated by gravitational waves, the superscript (T) denotes contributions from tensor perturbations, $\mu = \hat{n}\cdot\hat{k}$ is the angle between photon direction and wave vector, τ' is the differential optical depth for Thomson scattering. The multipole moments of temperature and polarization are defined as $\Delta(k,\mu) = \sum_l (2l+1)(-i)^l \Delta_l(k) P_l(\mu)$, where $P_l(\mu)$ is the Legendre polynomial of order l. The polarization perturbations can be decomposed into E-mode and B-mode. The B-mode components mainly come from tensor perturbations on the small multipoles and contain information about gravitational waves. The polarization power spectra from tensor perturbations are given by [26]

$$C_{X\ell}^{(T)} = (4\pi)^2 \int k^2 dk P_h(k) \left|\Delta_{X\ell}^{(T)}(k, \eta = \eta_0)\right|^2, \tag{6}$$

where $P_h(k)$ is the primordial power spectrum of gravitational waves, X stands for E or B. The two-point correlations of polarization patterns at different points in the sky are presented by Equation (6). The tensor perturbations could generate a measurable BB tensor angular power spectrum.

The power spectrum from tensor perturbations is parameterized as

$$P_h(k) \quad = \quad A_t \left(\frac{k}{k_*} \right)^{n_t}, \tag{7}$$

where A_t is the tensor amplitude at the pivot scale $k_* = 0.05\,\mathrm{Mpc}^{-1}$, n_t is the tensor spectral index. In literature, the tensor-to-scalar ratio r is used to quantify the tensor amplitude compared to the scalar amplitude A_s at the pivot scale, namely

$$r \equiv \frac{A_t}{A_s}. \tag{8}$$

For the canonical single-field slow-roll inflation model, n_t is related to r by $n_t = -r/8$, which is called the inflation consistency relation in general relativity [27,28].

In this paper, we suggest the following form of the modified propagation equation for tensor perturbations

$$h_k'' + (2 + 3\nu_0)\frac{a'}{a}h_k' + k^2 h_k = 0, \tag{9}$$

where ν_0 is a constant parameter. If the background is exactly exponentially expanding with respect to the cosmic time as $a \propto e^{Ht}$, the tensor mode is given by [3]

$$\left| h_k^0 \right|^2 = \frac{G(2H)^{2+3\nu_0} \left[\Gamma(\frac{3}{2} + \frac{3}{2}\nu_0) \right]^2}{\pi^3 \cdot k^{3+3\nu_0}}, \tag{10}$$

where h_k^0 is the leading-order solution, H is the constant expansion rate during inflation, and G is the Newtonian constant. The case $\nu_0 = 0$ corresponds to the propagation in GR [29]. The power spectrum of gravitational waves is defined as

$$P_h(k) = \frac{k^3}{2\pi^2} \left| h_k^0 \right|^2. \tag{11}$$

Comparing with Equation (7), we can identify the tensor spectral index as

$$n_t = -3\nu_0. \tag{12}$$

For the slow-roll inflation, H is not a constant and is measured by the slow-roll parameter $\epsilon = -\dot{H}/H^2$. The tensor spectrum index becomes

$$n_t = -3\nu_0 - 2\epsilon. \tag{13}$$

If we assume the result of the scalar perturbations is unchanged, the tensor-to-scalar ratio r is still related to the slow-roll parameter as $r = 16\epsilon$. The inflation consistency relation in the modified gravity becomes

$$n_t = -3\nu_0 - r/8. \tag{14}$$

In order to obtain the tensor angular power spectra for the modified gravitational waves propagation, we modify CAMB [30] by taking into account Equation (9). Our numerical results are presented in Figure 1. We show that the modified gravitational-wave propagation has impacts on the BB tensor angular power spectrum. The negative ν_0 enhances the BB tensor angular power spectrum, while the positive ν_0 reduces the BB tensor angular power spectrum.

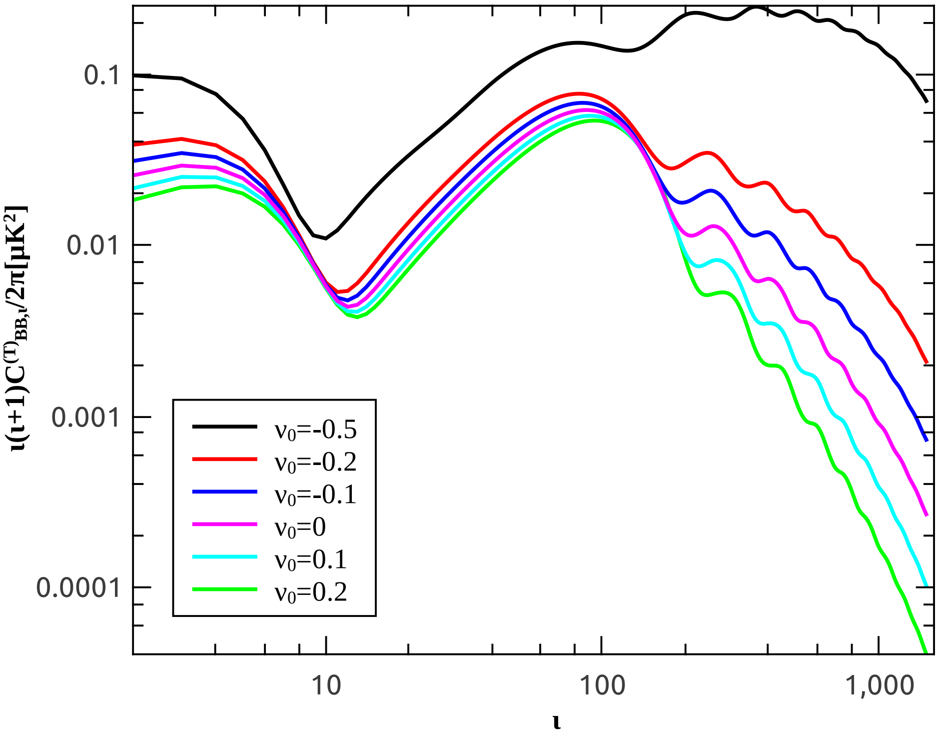

Figure 1. The plot of BB angular power spectrum from tensor perturbations for $\nu_0 = -0.5$, $\nu_0 = -0.2$, $\nu_0 = -0.1$, $\nu_0 = 0$, $\nu_0 = 0.1$ and $\nu_0 = 0.2$, respectively.

3. The Constraints on Friction Term from Planck+BK18 Datasets

In the standard ΛCDM model, the six parameters are the baryon density parameter $\Omega_b h^2$, the cold dark matter density $\Omega_c h^2$, the angular size of the horizon at the last scattering surface $\theta_{\rm MC}$, the optical depth τ, the scalar amplitude A_s and the scalar spectral index n_s. We extend this model by adding the tensor-to-scalar ratio r and the friction factor ν_0, and consider these eight parameters as fully free parameters, i.e., $r \in [0, 2]$, $\nu_0 \in [-0.5, 0.5]$. We use the publicly available codes Cosmomc [31] to constrain parameters, which adds the modified gravitational waves propagation in Equation (9) and the modified inflation consistency relation in Equation (14). The numerical results are presented in Figure 2.

In the ΛCDM+r+ν_0 model, the constraints on the tensor-to-scalar ratio r and the friction factor ν_0 are

$$r \quad < \quad 0.243 \quad (95\%\ \text{C.L.}), \tag{15}$$

$$\nu_0 \quad < \quad 0.042 \quad (95\%\ \text{C.L.}), \tag{16}$$

from Planck+BK18 datasets (Planck: TTTEEE+lowE+lensing). These results show that the friction factor ν_0 refers to the negative region, and negative ν_0 enhances the upper limits on the tensor-to-scalar ratio. We cut out the small ν_0 parameter space because a smaller ν_0 leads to a larger tensor-to-scalar ratio.

Then, we fix the friction factor ν_0 and consider several cases to test the variation on tensor parameters. As we assume the result of the scalar perturbations is unchanged, we fix the standard ΛCDM parameters based on Planck observations: $\Omega_b h^2 = 0.02242$, $\Omega_c h^2 = 0.11933$, $100\theta_{\rm MC} = 1.04101$, $\tau = 0.0561$, $\ln(10^{10} A_s) = 3.047$ and $n_s = 0.9665$.

The numerical results are presented in Figure 3. In the ΛCDM+r+ν_0 model, the constraints on the tensor-to-scalar ratio r and the tensor amplitude are

$$r < 0.349 \quad (95\% \text{ C.L.}), \tag{17}$$
$$\ln\left(10^{10}A_t\right) = 1.17^{+0.81}_{-0.34} \quad (68\% \text{ C.L.}), \tag{18}$$

from BK18 for $\nu_0 = -0.5$. The constraints on the tensor-to-scalar ratio r and the tensor amplitude are

$$r < 0.100 \quad (95\% \text{ C.L.}), \tag{19}$$
$$\ln\left(10^{10}A_t\right) = -0.18^{+0.90}_{-0.37} \quad (68\% \text{ C.L.}), \tag{20}$$

from BK18 for $\nu_0 = -0.2$. The constraints on the tensor-to-scalar ratio r and the tensor amplitude are

$$r < 0.060 \quad (95\% \text{ C.L.}), \tag{21}$$
$$\ln\left(10^{10}A_t\right) = -0.68^{+0.89}_{-0.38} \quad (68\% \text{ C.L.}), \tag{22}$$

from BK18 for $\nu_0 = -0.1$. The constraints on the tensor-to-scalar ratio r and the tensor amplitude are

$$r < 0.037 \quad (95\% \text{ C.L.}), \tag{23}$$
$$\ln\left(10^{10}A_t\right) = -1.20^{+0.93}_{-0.38} \quad (68\% \text{ C.L.}), \tag{24}$$

from BK18 for $\nu_0 = 0$. The constraints on the tensor-to-scalar ratio r and the tensor amplitude are

$$r < 0.023 \quad (95\% \text{ C.L.}), \tag{25}$$
$$\ln\left(10^{10}A_t\right) = -1.71^{+0.93}_{-0.40} \quad (68\% \text{ C.L.}), \tag{26}$$

from BK18 for $\nu_0 = 0.1$. The constraints on the tensor-to-scalar ratio r and the tensor amplitude are

$$r < 0.013 \quad (95\% \text{ C.L.}), \tag{27}$$
$$\ln\left(10^{10}A_t\right) = -2.30^{+0.94}_{-0.43} \quad (68\% \text{ C.L.}), \tag{28}$$

from BK18 for $\nu_0 = 0.2$. We show that the modified gravitational waves propagation and the inflation consistency relation in the modified gravity have impacts on the tensor-to-scalar ratio r and the tensor amplitude. The negative ν_0 enhances the upper limits on the tensor-to-scalar ratio r, while the positive ν_0 reduces the upper limits. The related behaviors of the tensor-to-scalar ratio r and the friction factor ν_0 are in agreement with the variations in Figure 2.

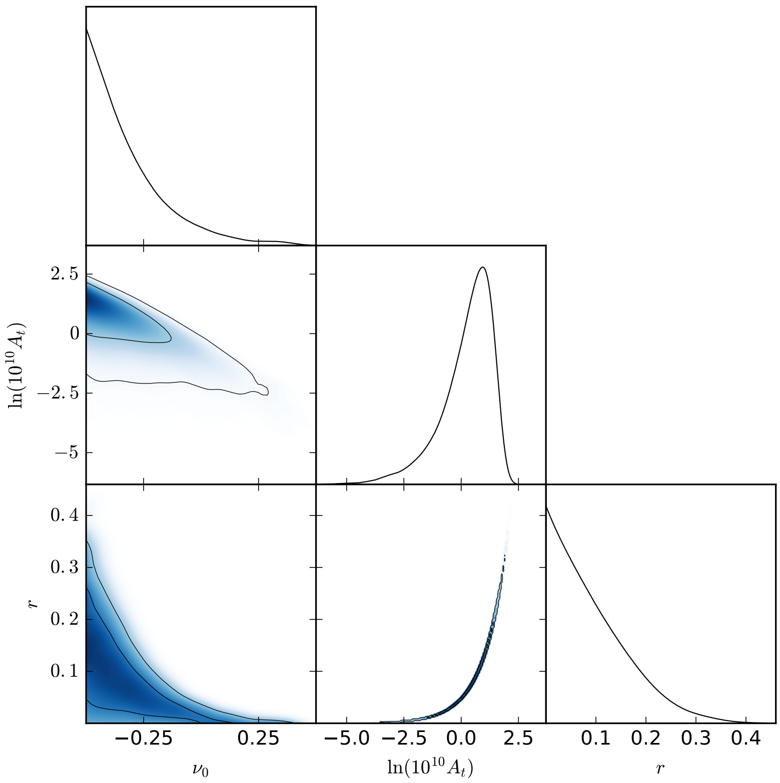

Figure 2. The contour plots and the likelihood distributions for parameters r, ν_0 and $\ln(10^{10}A_t)$ in the ΛCDM+r+ν_0 model at the 68% and 95% CL from Planck+BK18 datasets.

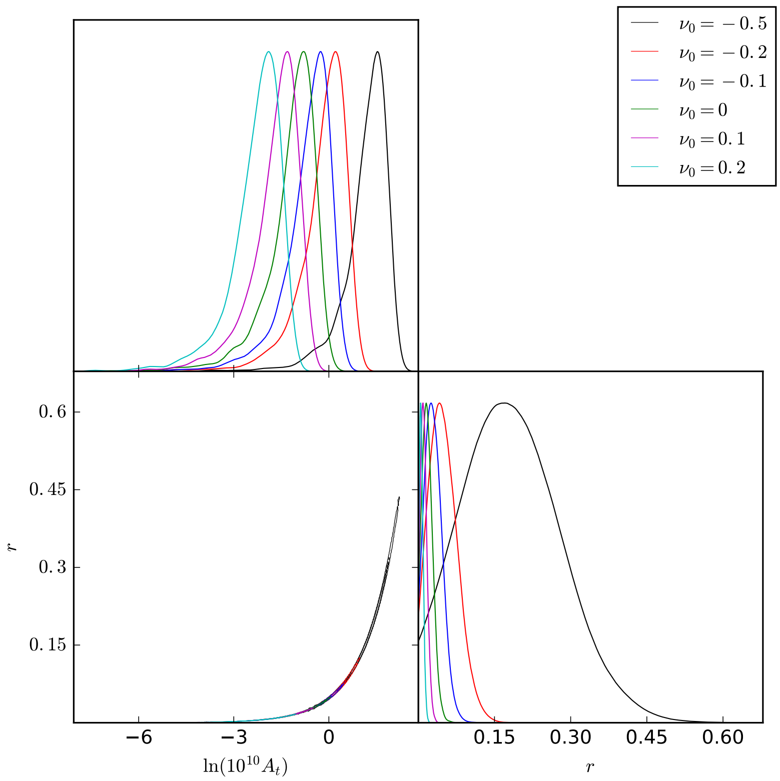

Figure 3. The contour plot and the likelihood distributions for parameters r and $\ln(10^{10}A_t)$ in the ΛCDM+r+ν_0 model at the 68% and 95% CL from BK18 with the modified gravitational waves propagation for $\nu_0 = -0.5$, $\nu_0 = -0.2$, $\nu_0 = -0.1$, $\nu_0 = 0$, $\nu_0 = 0.1$ and $\nu_0 = 0.2$, respectively.

4. Summary

In this paper, we consider a friction term different from GR and constrain the modified gravitational-wave propagation from observations. We consider the impact of the friction term on the power spectrum of B-modes and the impact of the constraints on the other parameters (e.g., r or A_t) when ν_0 is allowed to vary in the Monte Carlo analyses from the Planck+BK18 datasets. In the ΛCDM+r+ν_0 model, the tensor-to-scalar ratio and the amplitude of the tensor spectrum are obviously influenced.

Funding: This work is supported by the Natural Science Foundation of Shandong Province (grant No. ZR2021QA073) and the Research Start-up Fund of QUST (grant No. 1203043003587).

Conflicts of Interest: The authors declare no conflict of interest.

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