



Communication Towards the Explanation of Flatness of Galaxies Rotation Curves

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Abstract: We suggest a new explanation of the flatness of galaxies rotation curves without invoking dark matter. For this purpose, a new gravitational tensor field is introduced in addition to the metric tensor.

Keywords: modified theories of gravity; galaxies rotation curves; astrophysical and extragalactical scales; renormalizability; unitarity

1. Introduction

The velocities of stars and gas rotation around galaxies centers become independent of rotation radii at large enough radii; see, e.g., [1] and references therein. This asymptotical flatness of the galaxies rotation curves is in obvious contradiction with Newtonian dynamics, which demands that the velocities should decrease with radii as $1/\sqrt{r}$.

The contradiction is explained within the dark matter paradigm [2]. Within the paradigm, a galaxy is placed in a spherical halo of dark matter [3] with a mass density of dark matter decreasing as $1/r^2$. Hence, the Newtonian gravitational potential becomes logarithmically dependent on r providing a flat rotation curve. However, the hypothetical constituents of dark matter are still not discovered in direct experiments, despite numerous searches.

In this situation, it is worthwhile to develop explanations of the flatness of galaxies rotation curves that do not invoke dark matter. Probably the most well known attempt of this type is modified Newtonian dynamics, called MOND [4–6], where it is assumed that gravitational forces depend on the accelerations of objects participating in interactions. Modified Newtonian dynamics has essential phenomenological successes. In particular, it described the known Tully–Fisher relation [7], which establishes the correlation between a galaxy's luminosity and a corresponding flat rotation speed. However, modified Newtonian dynamics does not describe e.g., gravitational lensing by galaxies and galaxies clusters. Modified Newtonian dynamics is not a completely formalized theory, although there was an attempt to construct its complete Lagrangian version [8], which was not supported experimentally.

It should be mentioned that the Tully–Fisher relation is more precise in its barionic form, which states that a flat rotation speed in a galaxy correlates with its barionic mass, i.e., a sum of stars and gas masses, see [9] and references therein. The barionic Tully–Fisher relation states that $M_{bar} \propto V_{flat}^4$.

This tight correlation between the visible matter of a galaxy and a corresponding flat rotation speed is also a rather strong motivation to find the explanation of the flatness of galaxies rotation curves without dark matter. Also in [10], it is shown that there is a strong correlation between the observed radial star acceleration and the acceleration predicted by the observed distribution of baryons.

It is necessary to point out that, over the years, there have also been other ideas to replace the dark universe paradigm; for example, through the so-called de Sitter relativity [11–14], or through other approaches [15–17].

In the present paper, we suggest a new explanation of the flatness of galaxies rotation curves without invoking dark matter. For this purpose, a new tensor gravitational field is introduced in addition to the metric tensor.



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2. Main Part

We consider the General Relativity action, plus terms with a new gravitational tensor field, $f_{\mu\nu}$:

$$S = \int d^{4}x \sqrt{-g} (-M_{Pl}^{2}R + g_{\mu\nu}T^{\mu\nu} + M_{Pl}^{2}\Lambda$$
(1)
+ $f_{\mu\nu}(x) (D^{\lambda}D_{\lambda})^{3/2} f^{\mu\nu}(x) + G^{*}\sum_{i} \frac{1}{\sqrt{1 + m_{i}/m^{*}}} f_{\mu\nu}T_{i}^{\mu\nu}),$

Here, the *R*-term is the Einstein–Hilbert Lagrangian of General Relativity, the geometrical part of the action; $\sqrt{-g}$ is, as usual, the square root of the minus determinant of the metric tensor $g_{\mu\nu}(x)$. $M_{Pl}^2 = 1/(16\pi G)$ is the Planck mass squared.

 D_{λ} is the standard covariant derivative; for example:

$$D_{\lambda}f^{\mu\nu} = \frac{\partial}{\partial x^{\lambda}}f^{\mu\nu} + \Gamma^{\mu}_{\lambda\sigma}f^{\sigma\nu} + \Gamma^{\nu}_{\lambda\sigma}f^{\mu\sigma}, \qquad (2)$$

where the Christoffell symbols as usual are

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \big(\partial_{\nu} g_{\mu\beta} + \partial_{\mu} g_{\nu\beta} - \partial_{\beta} g_{\mu\nu} \big). \tag{3}$$

 $f_{\mu\nu}(x)$ is the introduced new tensor field additional to the metric tensor $g_{\mu\nu}(x)$. The field $f_{\mu\nu}(x)$ can be chosen to be the symmetric tensor, similar in this sense to the metric tensor $g_{\mu\nu}(x)$. It is necessary to ensure that the new gravitational field $f_{\mu\nu}(x)$ interacts with light, which has a traceless energy-momentum tensor $T_{light}^{\mu\nu}(x)$. The interaction of the field $f_{\mu\nu}$ with light is important, in order to produce extra gravitational lensing due to barionic matter as compared to General Relativity, since in the case of the absence of dark matter, General Relativity alone is not sufficient to describe the observed gravitational lensing. In principle, the new dynamical gravitational field $f_{\mu\nu}$ can also have an asymmetric part in addition to the symmetric one, but this is not essential at the present stage of considerations.

The main new point of the action in Equation (1) is the non-integer power 3/2 of the factor $D^{\lambda}D_{\lambda}$. This provides the $1/(k^2)^{3/2}$ behavior of the propagator of the new dynamical gravitational field $f_{\mu\nu}$ in the momentum space, where $k^2 = k_{\mu}k^{\mu}$ is the square of the four momentum k_{μ} . Due to such a behavior of the propagator, one can generate the logarithmic with the distance *r* gravitational log(r) potential, as will be shown below, which allows one to describe the flatness of galaxies rotation curves without invoking dark matter.

 G^* is the introduced new coupling constant of interaction of the new gravitatonal tensor field $f_{\mu\nu}$ with matter fields being described by energy-momentum tensors $T_i^{\mu\nu}$. Of course, it it assumed that the complete action also contains terms describing the propagations of matter fields, in addition to their interactions with gravitational fields.

 m^* is the introduced new mass parameter that is necessary to compensate for the dimensions of masses m_i in the corresponding square root xs.

The sum \sum_i in the action (1) goes over matter objects described by energy-momentum tensors $T_i^{\mu\nu}$ and having masses m_i . The couplings of them with the field $f_{\mu\nu}$ depend on m_i via $\frac{1}{\sqrt{1+m_i/m^*}}$; this property is quite essential and is used below to reproduce the famous barionic Tully–Fisher relation.

The numerical values of the new constants G^* and m^* should be fixed from the fitting experiments. It needs an analysis of a huge amount of empirical data for different galaxies, and that is why it is a subject for a separate publication.

The Λ -term in Equation (1) is not essential in perturbation theory, which we will consider. We will use the standard in the Quantum Field Theory system of units $\hbar = c = 1$.

To quantize the theory (1) self-consistently, one should add to the Lagrangian all possible quadratic terms in the Riemann tensor $R_{\mu\nu\rho\sigma}$; see [18,19], where the perturbatively reormalizable and unitary model of quantum gravity was formulated for the first time. However, these terms are not essential for the present considerations.

As has already been mentioned above, we will work within perturbation theory; hence, a linearized theory around the flat metric $\eta_{\mu\nu}$ is considered; that is, we make the following substitution

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},\tag{4}$$

Here, the generally accepted in Field Theory convention in four dimensions is chosen $\eta_{\mu\nu} = diag(+1, -1, -1, -1)$. Indexes are raised and lowered in the following via the flat metric tensor $\eta_{\mu\nu}$.

Within perturbation theory, one makes the standard shift of the metric field

$$h_{\mu\nu} \to M_{Pl} h_{\mu\nu}.$$
 (5)

Perturbative expansion proceeds as usual in the inverse powers of M_{Pl} , or in other words, in the powers of the Newton coupling constant $G = \frac{1}{16\pi M_{Pl}^2}$.

Let us now obtain the propagator of the new gravitational tensor field $f_{\mu\nu}$ in the momentum space. For this purpose, we take the quadratic in the field $f_{\mu\nu}$ part of the Lagrangian of the action in the Equation (1) and perform the Fourier transform to the momentum space:

$$Q = i(2\pi)^4 \int d^4k \, f^{\mu\nu}(-k) \left[(k^2)^{3/2} \eta_{\mu\rho} \eta_{\nu\sigma} \right] f^{\rho\sigma}(k), \tag{6}$$

To obtain the propagator $D_{\mu\nu\rho\sigma}$ of the field $f_{\mu\nu}$, one should, in the standard way, invert the matrix in the brackets of Equation (6):

$$[Q]_{\mu\nu\kappa\lambda}D^{\kappa\lambda\rho\sigma} = \frac{1}{2}(\delta^{\rho}_{\mu}\delta^{\sigma}_{\nu} + \delta^{\sigma}_{\mu}\delta^{\rho}_{\nu}).$$
⁽⁷⁾

Then, the propagator has the following form

$$D_{\mu\nu\rho\sigma} = \frac{1}{2i(2\pi)^4} \frac{\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}}{(k^2)^{3/2}}.$$
 (8)

The obtained propagator (8) generates the logarithmic in the gravitational potential of distance r.

To derive the logarithmic potential from the propagator (8), one should consider in the standard way the Fourier transform of the propagator in three space dimensions:

$$D(r) \propto \int d^3k \frac{e^{ik\vec{r}}}{k^3} = -\frac{4\pi}{r} \int_{\kappa}^{\infty} dk \frac{\sin(kr)}{(k)^2},\tag{9}$$

where the expression in the right hand side is obtained after integrations over angles are done. $\kappa > 0$ is a small regularizing parameter, which regularizes the infrared divergency in the integral. *k* is the length of the three vector \vec{k} .

Then, one obtains after performing an integration on the right hand side of the Equation (9):

$$D(r) \propto -Ci(r\kappa) - \frac{sin(r\kappa)}{r\kappa} = -log(r\kappa) + constant + O(\kappa^2),$$
(10)

where $Ci(r\kappa)$ is the standard cosine integral function, and on the right hand side, the expansion in the limit of small κ is completed.

Thus, one obtains the logarithmic in *r* potential, due to the field $f_{\mu\nu}$. The regularizing parameter κ is inessential when one takes the derivative in *r* in order to produce the gravitational force from the obtained logarithmic potential.

At large enough distances of r, that is, at galactic and extragalactic scales, this log r-potential starts to dominate over the Newtonian 1/r-potential that is generated by the metric field $h_{\mu\nu}$ of General Relativity for small potentials.

Thus, the gravitational field $f_{\mu\nu}$ generates a 1/r- force between a point object with a mass M_{bar} , having the energy-momentum tensor $T_{\mu\nu} = \delta^0_{\mu} \delta^0_{\nu} M_{bar} \delta^3(x)$ (describing a galaxy

with the baarionic mass M_{bar} of stars plus gas) and an analogous object with a mass M_{star} (describing a star with the mass M_{star}):

$$F = (G^*)^2 \frac{M_{bar}}{\sqrt{1 + M_{bar}/m^*}} \frac{M_{star}}{\sqrt{1 + M_{star}/m^*}} \frac{1}{r}.$$
 (11)

At a galaxy mass of M_{bar} that is large compared to the mass parameter m^* , the square root $\sqrt{1 + M_{bar}/m^*}$ becomes approximately just $\sqrt{M_{bar}/m^*}$, and the above relation (11) takes the form $E_{abs} (C^*)^2 \sqrt{M_{bar}/m^*} = \frac{M_{star}}{1}$ (12)

$$F \approx (G^*)^2 \sqrt{M_{bar} \cdot m^*} \frac{M_{star}}{\sqrt{1 + M_{star}/m^*}} \frac{1}{r}.$$
 (12)

From the other side, according to Newton's second law, one obtains

$$F = M_{star} \frac{V_{flat}^2}{r}.$$
(13)

Equating Expressions (12) and (13), we obtain the following relation

$$M_{bar} \approx \frac{V_{flat}^4}{(G^*)^4} \frac{1 + M_{star}/m^*}{m^*}.$$
 (14)

The expression (14) reproduces the barionic Tully–Fisher relation, which states that $M_{bar} \propto V_{flat}^4$.

It is interesting to note that the right-hand side of the expression (14) has dependence on the star mass M_{star} .

Thus, the flat rotation speeds of stars satisfy the barionic Tully–Fisher relation in our model.

We should also mention once more that the introduced tensor field $f_{\mu\nu}$ interacts with light, and that is why it adds additional—as compared to the metric field $h_{\mu\nu}$ of General Relativity—gravitational lensing due to barionic matter.

3. Discussion

The Newtonian theory of gravity could be assumed to be a perfect theory at galactic and extragalactic distances. However, the velocities of stars and gas rotation, as is known from experimental observations, are usually essentially larger than the velocities generated by visible barionic matter as they are estimated according to the Newtonian dynamics. It is presently commonly accepted to explain this paradox via the presence of the necessary amount of dark matter in galaxies. Also observed is gravitational lensing by galaxies and clusters of galaxies, which is larger then the lensing that can be produced within General Relativity due to visible matter only. This is again traditionally explained by the presence of the appropriate amount of dark matter. However, the constituents of dark matter are still not found, in spite of numerous experimental efforts. In this situation, it is worthwhile to develop models that are alternative to General Relativity, although it is clear that the approximations of these models for the solar system scales should coincide with General Relativity, which has been perfectly experimentally tested in the solar system.

4. Conclusions

We suggest a new explanation for the flatness of galaxies rotation curves without invoking dark matter. For this purpose, a new tensor gravitational field is introduced in addition to the metric tensor. The flat rotation speeds of stars in our model satisfy the known barionic Tully–Fisher relation.

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