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Review

Thermal Radiation from Compact Objects in Curved Space-Time

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Abstract: We highlight here the fact that the distantly observed luminosity of a spherically symmetric compact star radiating thermal radiation isotropically is higher by a factor of $(1+z_b)^2$ compared to the corresponding flat space-time case, where z_b is the surface gravitational redshift of the compact star. In particular, we emphasize that if the thermal radiation is indeed emitted isotropically along the respective normal directions at each point, this factor of increment $(1+z_b)^2$ remains unchanged even if the compact object would lie within its *photon sphere*. Since a canonical neutron star has $z_b \approx 0.1$, the actual X-ray luminosity from the neutron star surface could be $\sim 20\%$ higher than what would be interpreted by ignoring the general relativistic effects described here. For a static compact object, supported by only isotropic pressure, compactness is limited by the Buchdahl limit $z_b < 2.0$. However, for compact objects supported by anisotropic pressure, z_b could be even higher ($z_b < 5.211$). In addition, in principle, there could be ultra-compact objects having $z_b \gg 1$. Accordingly, the general relativistic effects described here might be quite important for studies of thermal radiation from some ultra-compact objects.

Keywords: relativistic astrophysics; gravitational redshift; neutron stars, X-ray luminosity

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1. Introduction

Stellar remnants, such as white dwarfs (WDs), neutron stars (NSs) and black holes (BHs), mostly formed through the gravitational collapse of normal stars at the end of their life, are referred to as compact objects in astrophysics. It is well known that although general relativity may play a minor role for most of the observational astrophysics, it can become significant for studies of NSs and BH candidates (BHCs) or Ultra-Compact Objects (UCOs). A rough measure of the "compactness" of an astrophysical object may be found by the parameter $GM/R_b c^2$, where G is the constant of gravitation, c is the speed of light, M is the gravitational mass and R_b is the radius of the object. For the Sun, the compactness is very small $\approx 2 \times 10^{-6}$. Since a WD can be as massive as the Sun and yet have a radius of $\sim$1% of the Sun, its compactness can be two orders higher $\sim 2 \times 10^{-4}$. On the other hand, it is usually believed that for an NS, this compactness is $\sim$0.1.

The "compactness" can be empirically obtained by measuring the gravitational redshift of pertinent spectral lines emitted from the surface of a compact star. Thus, from a general relativistic point of view, it is z_b and not $GM/R_b c^2$ that is the intrinsic measure of the gravitational compactness of a star. The gravitational redshift (z_b) of radiation emitted from the surface of the body is given by the follwing equation [1–3]

$$1 + z_b = \left(1 - \frac{R_S}{R_b}\right)^{-1/2} \tag{1}$$

where $R_S = 2GM/c^2$ is the Schwarzschild radius of the compact object. Note that, in the limit $GM/R_b c^2 \ll 1$, the foregoing equation leads to

$$z_{\rm b} \approx \frac{GM}{R_{\rm b}c^2} = \frac{R_{\rm S}}{2R_{\rm b}} \tag{2}$$

and that explains why $GM/R_{\rm b}c^2$ is usually a measure of the gravitational compactness. As already mentioned, it is believed that an NS may have $z_{\rm b}\sim 0.1$, which is almost three orders higher than that of a typical WD. However, in principle, there can be compact objects with much higher gravitational redshift ($z_{\rm b}\sim 1$ or higher), and we shall call them Ultra Compact Objects (UCOs). The central aim of this short paper is to study the effect of curved space-time around hot UCOs on the actual value of the luminosity of the distantly observed thermal radiation *vis-a-vis* ($L_{\infty}^{\rm curved}$) and the luminosity erroneously estimated by ignoring such space-time curvature ($L_{\infty}^{\rm flat}$).

In general, for static compact objects that are supported by isotropic internal pressure, i.e., pressure is the same in both radial and transverse directions, $p_{\rm r} = p_{\perp}$, there is a well-known theoretical upper limit on the compactness, given by Buchdahl [4] as

$$\frac{2GM}{R_{\rm b}c^2} = \frac{8}{9} \tag{3}$$

and the corresponding gravitational redshift would be

$$z_{\rm Buch} = 2.0 \tag{4}$$

It is pertinent here to note that for a static compact object of mass M and radius $R_{\rm b}$, we assume that its exterior is described by the Schwarzschild metric possessing

$$g_{00} = 1 - \frac{2GM}{Rc^2}; \;\; R \geq R_{\rm b}, \tag{5}$$

and the gravitational redshift $z_{\rm b}$ is directly related to g_{00} and attendant bending of the exterior space-time due to the effect of the gravity of the compact object. The severe effect of general relativistic space-time bending around a compact object may be highlighted by the fact that even light and radiation may move in (unstable) *circular orbits* at $R = 1.5R_{\rm s}$ for compact stars having radii $R_{\rm b} < 1.5R_{\rm s}$ or $z_{\rm b} > 0.73$. The result follows from the studies on the propagation of photons or massless particles in the strong gravitational field around sufficiently compact objects [2]. The closed circular orbits of photons in various planes define what is known as a *photon sphere* with a corresponding gravitational redshift of $z_{\rm photon-sphere} = 0.73$. Note, since $z_{\rm photon-sphere} = 0.73$ is well below the Buchdahl limit of compactness of $z_{\rm Buch} = 2.0$, there can be static spherical compact objects that lie within their respective photon spheres. The most well-known compact object that lie within its photon sphere is the Schwarzschild black hole having $R = R_{\rm s}$ or $z_{\rm b} = \infty$. In the following section, we shall discuss the theoretical possibility of having ultra-compact spherical stars that may behave like *black hole mimickers* in view of their large $z_{\rm b} \gg 1$.

Stefan–Boltzmann Law

From the above discussion, it is clear that for an accurate study of emission of radiation from neutron stars and similar ultra-compact objects, the effect of curvature of space-time around such objects should be borne in mind. We shall highlight the fact that the faraway luminosity of such compact objects is higher by a factor of $(1 + z_{\rm b})^2$ than what one might estimate by applying the classical Stefan–Boltzmann law of the blackbody radiation, which tells us that a black body having a surface area $A_{\rm b}$ and temperature $T_{\rm b}$ has a thermal luminosity

$$L_{\rm b} = \sigma \, A_{\rm b} \, T_{\rm b}^4 \tag{6}$$

where $\sigma \approx 5.67 \times 10^{-8}$ kg s^{-3} K^{-4}.

In 1990, Lavenda & Dunning-Davies [5] argued that since black holes radiate with a thermal spectrum, they possess radiation pressure, and Boltzmann's derivation of Stefan's Law can be applied to black holes. Since the entropy is proportional to the surface area

of the black hole, they found that the corresponding pressure must be negative. If the second law is not to be violated, then the temperature must also be negative. Since a negative temperature of the black hole seems to be self-contradictory, these authors cast doubts on the validity of conventional black hole thermodynamics. Following this work linking this law with black hole thermodynamics, in 1995, de Lima & Santos [6] wanted to extend this law to the hypothetical cosmological fluids obeying the equation of states of the form $p = (\gamma - 1)\rho/c^2$, where ρ and p are the energy density and pressure of the fluid, respectively, and γ is a constant. Such discussions, however, have no relevance for the present study, which concerns only thermal radiation $p = (1/3)\rho/c^2$.

The Stefan–Boltzmann law concerns the thermal radiation of a black body, which is at rest with respect to the observer studying the radiation field. However, if the black body is moving with a velocity $\vec{v}$ with respect to the observer, one needs to extend the law by applying special relativity. This task was performed by Veitsman in 2013 [7]. It was found that the observer will find most of the radiation emitted by the fast moving black body to be emitted parallel to the velocity vector if $v \to c$. However, we are considering the case when there is no relative motion between the black body and the observer. Hence, the above study is also not applicable here. On the other hand, even for a static observer, in the presence of gravity, the space-time around the black body becomes curved, and one must invoke general relativity to reformulate the Stefan–Boltzmann law and also to interpret the result correctly. This is the aim of this study.

2. Ultra-Compact Objects

We have already mentioned that even a strictly static star supported by isotropic internal pressure may possess $z_b \approx 2.0$ and, accordingly, lie well within its photon sphere ($z_{\rm photon-sphere} = 0.73$). Now, we recall that if the compact star is supported by anisotropic internal pressure ($p_r \neq p_\perp$), its compactness may exceed the Buchdahl upper limit. Following the landmark paper by Buchdahl [4], the topic of study of self-gravitating static anisotropic spheres has been carried out by many authors by considering various restrictions on the degree of anisotropy, different equation of states and several latent physical conditions. One important point is that incorporation of pressure anisotropy not only raises the upper limit on z_b, but also the upper limit on the maximum mass of the compact stars. Nonetheless, here, we are primarily interested in the former aspect alone.

Two of the significant initial studies in this direction were by Bondi (1964) [8] and Bowers & Liang (1974) [9]. Additionally, two recent general studies on this topic are due to Herrera, Ospino & Prisco (2008) [10] and Herrera & Barreto (2013) [11]. However, since we are interested here in the numerical value of z_b, we may note an earlier detailed study by Ivanov (2002) [12], which shows that the upper limit on the gravitational redshift for an anisotropic static spherical star could be bounded as $z_b < 5.211$. Later, as Böhmer (2006) [13] studied the problem independently, he too found an upper limit of $z_b < 5.0$, which is more or less the same as what Ivanov (2002) had found earlier.

The above mentioned upper limits on gravitational redshift for anisotropic compact stars may not be the final word on this issue. A more recent study, on the possibilty that some of the black hole candidates might be ultra-compact objects, claims that for some choices of model parameters, an anisotropic star could not only be arbitrarily compact ($z_b \gg 1$), but could be as massive as many black holes too [14].

Quasi-Static Ultra-Compact Objects

In general relativity, as already mentioned, the exterior space-time of a *strictly static* spherically symmetric self-gravitating object is given by the *vacuum* Schwarzschild exterior metric. Thus, in a strict sense, the exterior space-time of a radiating star is non-static, even though the degree of non-staticity may be negligible. On the other hand, the Buchdahl upper limit ($z_b < 2.0$) is obtained by considering a strictly static non-radiating exterior space-time. Therefore, in principle, the surface gravitational redshift of a radiating star may exceed the Buchdahl limit, even if one would ignore any anisotropy due to inhomogeneity

or magnetic field. It was found that, accordingly, there could be radiating quasi-static ultra-compact objects having $z_b \gg 1$, which are supported against their intense self-gravity by the outward pressure of the radiation trapped by the same intense self-gravity [15–18].

3. Effects of Gravitational Redshift

The physical interpretation of gravitational redshift is that an amount of energy dE_b on the surface of a black body will be measured as [1–3]

$$dE_\infty = \frac{dE_b}{1+z_b} \tag{7}$$

by an infinitely faraway observer. Additionally, if an interval of time measured on the surface of the black body is dt, the same will become dilated to [1,2]

$$dt_\infty = (1+z_b)\, dt \tag{8}$$

to the same faraway observer.

Accordingly, the luminosity measured by the faraway observer is

$$\frac{dE_\infty}{dt_\infty} = (1+z_b)^{-2}\, \frac{dE_b}{dt}. \tag{9}$$

Therefore, the faraway luminosity will be smaller than the locally measured luminosity by a factor of $(1+z_b)^{-2}$, irrespective of the physical mechanism of the radiation, i.e, thermal or non-thermal [3]:

$$L_\infty = (1+z_b)^{-2}\, L_b \tag{10}$$

UCOs Lie within Their Photon Spheres

The foregoing equation implicitly assumes that all the radiation quanta leaving the compact object reach up to the faraway observer and do not return towards the compact object. Suppose the emission of the radiation is taking place from a given patch or few patches on the surface of the compact object and not from the entire surface; and in such a case, *the assumption of spherical symmetry will become violated*. Even for such anisotropic cases, as long as the compact object is larger than its photon sphere, $R_b \geq R_{photon-sphere} = 1.5R_S$, no radiation quanta bends backwards during their outward journey irrespective of their direction of emission, i.e., even if they would be emitted in a non-radial direction.

Does Equation (10) change for compact objects lying within their respective photon spheres? In order to probe, this we recall that all realistic gravitational collapse processes involve emission of radiation [15], and UCOs are formed by the emission of radiation. When the compact object lies within its photon sphere, $R_b < 1.5R_S$, the quanta emitted in the non-radial directions may bend backwards by the effect of strong gravity, and only the radiation emitted within a cone defined by a semi-angle θ_c [2]

$$\sin\theta_c = \frac{\sqrt{27}}{2}(1-R_S/R_b)^{1/2}(R_S/R_b) \tag{11}$$

can move away to infinity.

If $R_b \approx R_S$, the foregoing equation reduces to

$$\sin\theta_c \to \theta_c \approx \frac{\sqrt{27}}{2}(1+z_b)^{-1}. \tag{12}$$

In such a case, the solid angle of the radiation that can travel to infinity becomes

$$\Omega_c \approx \pi\theta_c^2 = \frac{27\pi}{4}(1+z_b)^{-2} \tag{13}$$

Accordingly, if the *radiation from the given patch* is emitted in a solid angle of 2π and not exclusively in a radially outward direction, the chance of escape of radiation from the given patch would decrease by a factor of $\Omega_c/2\pi \approx (27/8)(1+z_b)^{-2}$. In fact, the radiation emitted within the body of the compact object always interacts with trillions of atoms and electrons and moves only diffusively by performing random walk. As a result, once a compact object dips within its photon sphere, the radiation in the interior of the body may become gravitationally trapped [16–19]. However, the radiation quanta emitted from the surface of the compact object into the exterior *vacuum* moves unhindred without any random walk. In particular, *if a quanta is ejected radially outward along the local normal direction* $\theta_c = 0$, then *it can continue to move radially outward howsoever large* z_b *might be*. Thus, if the compact object is emitting radiation truly *isotropically in a radially outward direction at each point of emission*, all the outwardly moving quanta reach infinity even if $z_b \gg 1$. In such a case, Equation (10) connecting local luminosity and distantly measured luminosity will remain valid irrespective of the value of z_b. This important fact can be confirmed by noting the pertinent equation for the continued spherical gravitational contraction of stars [2,20–24]

$$L_\infty = (U_b + \Gamma_b)^2 L_b \tag{14}$$

where the parameter

$$U_b = \frac{\partial R_b}{\partial \tau} \tag{15}$$

is the rate of contraction of the outer radius of the star, with respect to local comoving proper time τ. On the other hand, the other parameter is

$$\Gamma_b = \left(1 + U_b^2 - \frac{2GM}{R_b c^2}\right)^{1/2} \tag{16}$$

(see, e.g., Equation (5.10) in [20], Equation (37) in [21], Equation (18) in [22], Equation (20) in [23] or Equation (17) in [24]). Now suppose that the continued gravitational collapse has resulted in the formation of a static UCO ($U_b = 0$) or quasi-static UCO ($U_b \to 0$). In such a case, one will have

$$\Gamma_b = \left(1 - \frac{2GM}{R_b c^2}\right)^{1/2} = (1+z_b)^{-1}, \tag{17}$$

and Equation (14), connecting local and distant luminosity, will reduce to

$$L_\infty = (1+z_b)^{-2}\, L_b. \tag{18}$$

Since Equation (14) is valid all the way up to $R_b \geq R_S = 2GM/c^2$ and not merely up to $R_b \geq 1.5R_S = 3GM/c^2$, for a spherically symmetrical compact object radiating isotropically along the local outward normal direction, Equation (18) is valid all the way up to $z_b \to \infty$. Therefore, as before, the faraway luminosity of a compact object lying well within its photon sphere will be smaller than the locally measured luminosity by a factor of $(1+z_b)^{-2}$, irrespective of the physical mechanism behind the radiation. The fact that a black hole is not visible to any faraway observer may also be physically understood by noting that $(1+z_b)^{-2} = 0$ for $z_b = \infty$.

4. Distant Luminosity of Thermal Radiation

It is now clear that if the surface temperature of a spherical black body is T_b, the faraway thermal luminosity will accordingly be

$$L_\infty = (1+z_b)^{-2}\, L_b = (1+z_b)^{-2}\, \sigma A_b T_b^4 \tag{19}$$

irrespective of the value of z_b. From the foregoing equation, one may be tempted to conclude that the temperature of the black body as perceived by a faraway observer would be

$$T_\infty = \left(\frac{L_\infty}{\sigma A_b}\right)^{1/4} = \frac{T_b}{\sqrt{1+z_b}} \text{ (incorrect)}. \tag{20}$$

Now we come to the crucial aspect of this paper and expain below that the foregoing conclusion would be incorrect.

In the curved space-time, *the temperature of the blackbody radiation varies spatially* in a manner that was discovered by Tolman & Ehrenfest in 1930 [25–27], 14 years after the formulation of general relativity and 50 years after the formulation of the Stefan–Boltzmann law. This important problem was extended for any stationary space-time by Buchdahl in 1949 [28]. It follows that in the curved space-time around the compact object, the temperature varies following the rule

$$T\sqrt{|g_{00}|} = \text{constant}. \tag{21}$$

From Equations (1) and (5), since $g_{00} = 1$ at $R = \infty$, one finds that the temperature of the black body radiation at $R = \infty$ is lower by a factor of $(1 + z_b)$:

$$T_\infty = \frac{T_b}{1+z_b}. \tag{22}$$

By combining Equations (20) and (22), it is found that in terms of the faraway temperature T_∞, the faraway luminosity is

$$L_\infty = (1+z_b)^2\, \sigma A_b T_\infty^4. \tag{23}$$

However, if the effect of gravitation around the compact object would be ignored, the exterior space-time would be flat with no gravitational redshift at all ($z_b = 0$), and one would have

$$T_\infty^{\text{flat}} = T_b \tag{24}$$

and accordingly

$$L_\infty^{\text{flat}} = L_b = \sigma A_b T_b^4. \tag{25}$$

These two equations lead us to our conclusion that for a spherically symmetric black body emitting radiation isotropically along local normal directions, one has

$$L_\infty^{\text{curved}} = (1+z_b)^2\, L_\infty^{\text{flat}}. \tag{26}$$

Does the Radius of the Blackbody Change by Gravity?

Suppose the faraway observer is unmindful of the general relativistic effect described here and chooses to define the thermal luminosity recorded by him/her as

$$L_\infty = 4\pi\sigma R_{bb}^2 T_\infty^4. \tag{27}$$

Since $A_b = 4\pi R_b^2$, then by comparing the above equation with Equation (23), he/she might conclude that the effect of gravity has increased the black body radius as

$$R_{bb} = (1+z_b)R_b. \tag{28}$$

However, let us explain why the above interpretation about the enhancement of radius is incorrect. Note that in a spherically symmetric space-time, surface area of a sphere $A = 4\pi R^2$ is defined covariantly [29], and thus, the enhancement of the luminosity over the corresponding flat space-time must not be interpreted in terms of an enhanced effective radius

$$R_\infty = R_{bb} = (1+z_b)R_b \text{ (incorrect)}. \tag{29}$$

It will be important here to recall that for a positively curved homogeneous space-time described by the Friedmann metric, while the proper volume of a sphere $V_{\rm GR} = 2\pi^2 R^3$ differs significantly from the Euclidean formula $V_{Euc} = (4\pi/3)R^3$, the surface area of a sphere of radius R continues to be unchanged $4\pi R^2$ [2,30]. One may also appreciate the fact that the surface area of a sphere $A = 4\pi R^2$ is an invariant by noting that for all spherically symmetric space-time metrics, be it Euclidean or non-Euclidean, the angular part of the metric always contain the term $R^2 d\Omega^2$, where $d\Omega^2$ is the metric of a unit sphere.

Thus, the radius of a spherical blackbody does not differ for two observers because *area of a sphere is an invariant even for a curved space-time*. In fact, it is because this invariance of the area of a spherical surface that the radial coordinate R appears as the "luminosity distance" even for a (static) curved space-time.

5. Discussions and Conclusions

For the curved space-time around a hot compact object having a surface gravitational redshift of $z_{\rm b}$, the actual luminosity infinitely faraway from the source is higher by a factor of $(1 + z_{\rm b})^2$ than what one may naively estimate by a tacit flat space-time computation by using the far away local temperature of the radiation field (Equations (23) and (26)). The effect described here is absent or negligble or not significant in most of the standard astrophysical discussions probably because:

- Most of the radiation high energy that astrophysicists consider are *non-thermal* while the present discussion is pertinent only for *thermal radiation emitted spherically from the surface* of a compact object.
- For WDs, compactness or $z_{\rm b} \sim 10^{-4}$ is extremely low, and therefore, this effect can be safely ignored.
- For hot NSs, this effect is indeed relevant for thermal emission due to a hot surface or Type I X-ray bursts, with thermal flashes resulting from runaway thermonuclear burning of accreted matter [31]. However, since that is usually believed for NSs, $z_{\rm b} \sim 0.1$, and this effect is very significant: $L_\infty^{\rm curved} = 1.21\ L_\infty^{\rm flat}$.
- For X-ray binaries containing BHs, the thermal X-ray emission mostly originates from extended hot accretion disks and not from the surface of the BHs. Thus, this effect is not pertinent for thermal X-ray emission from BH X-ray binaries.

A closer inspection, however, shows that for a canonical NS having $R = 10$ km and $M = 1.4$ solar-mass, actually, $z_{\rm b} \approx 0.15$, and this effect may be important because the distantly observed thermal luminosity will be larger than the flat space-time luminosity by a factor of 1.32 or 32% higher.

As already stressed, the enhancement factor of the distant luminosity remains unaltered even when the compact object lies within its photon sphere $z_{\rm b} > 0.73$ because for truly isotropic emission, at each point, radiation quanta is emitted along the local normal or radial direction: $\sin\theta_{\rm c} = 0$.

Accordingly, this effect could be highly significant for UCOs and Black Hole Mimickers, which are theoretically admissible compact objects with $z_{\rm b} \sim 1$ or even $z_{\rm b} \gg 1$ [14–16,18].

If we consider the theoretical upper limit of $z_{\rm b} < 5.211$ [12,13] of static UCOs, the actual faraway thermal luminosity could be nearly 36 times higher than the flat space-time case. Even if we consider the Buchdahl upper limit of $z_{\rm b} < 2.0$, the actual thermal luminosity could still be nine times higher than the corresponding flat space-time case.

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