

Article

Baryon-Antibaryon Annihilation in the Evolution of Antimatter Domains in Baryon-Asymmetric Universe

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Abstract: Non-trivial baryosynthesis scenarios can lead to the existence of antimatter domains in a baryon-asymmetrical Universe. The consequences of antibaryon-baryon annihilation at the border of antimatter domains is investigated. Low-density antimatter domains are further classified according to the boundary interactions. A similar classification scheme is also proposed for higher-densities antimatter domains. The antiproton-proton annihilation interactions are therefore schematized and evaluated. The antinuclei-nuclei-interaction patterns are investigated. The two-point correlation functions for antimatter domains are studied in the case of baryon-antibaryon boundary interactions, which influence the space and time evolution. The space-time evolution of antimatter domains after the photon thermalization epoch is analyzed.

Keywords: classical general relativity; fundamental problems and general formalism; exact solutions; antibaryons; antibaryon annihilation



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1. Introduction

The now standard cosmological scenario is based on inflation, baryosynthesis, and includes dark matter/energy in its sufficiently successful description of the data of precision cosmology on the structure and evolution of the Universe [1–9]. These basic elements of the modern cosmology involve Physics beyond the Standard model (BSM), which, on its turn, addresses cosmological probes for its study [5–10]. It makes necessary to use model-dependent cosmological messengers of new Physics to make a proper choice among possible BSM models [8]. A specific choice of parameters of BSM Physics model can, in particular, lead to nonhomogeneous baryosynthesis, so that the mechanism of generation of baryon asymmetry of the Universe simultaneously generates macroscopic regions with excess of antimatter in a baryon asymmetrical Universe [5–14].

Severe observational constraints still leave room for the existence of up to $10^5 M_{\odot}$ of macroscopic antimatter in our Galaxy, which can form a globular cluster of antimatter stars [15]. The existence of such a globular cluster should lead to antihelium components of cosmic rays, which can be a challenge for search at the AMS02 experiment [10,15,16]. The first results of this experiment [17,18] indicate the possible existence of such component, which cannot be explained as secondaries of cosmic-ray interactions or products of dark matter annihilation [19]. To confront the definite results of the analysis of the AMS02 data, expected for 2024, their interpretation in terms of primordial antimatter is of special interest, by linking them to mechanisms of nonhomogeneous baryosynthesis and parameters of the underlying BSM Physics models. Such interpretation inevitably involves development of

analytical methods of statistical analysis of cosmological evolution of antimatter domains, which is the subject of the present paper. The evolution of such domains depends on the antibaryon density within them and on the effects of baryon-antibaryon annihilation with the surrounding matter [12,13,15,19–27].

The interaction of antimatter from antimatter domains with matter in the surrounding medium is studied to determine the boundary conditions also in the case of the non-disappearance of the antimatter domains in the limiting processes.

In different cosmological settings, the appearance of domains with antibaryon excess can be predicted.

Within the framework of non-trivial baryosynthesis scenarios, the formation of antimatter domains containing antibaryons, such as antiprotons, antinuclei, and both possibilities are studied according to their dependence on their antimatter densities within the domains.

The boundary conditions for antimatter domains are determined through the interaction with the surrounding baryonic medium.

The consistency schemes for the non-annihilation of antimatter domains are due to both the appropriate characterization of the interaction regions at the boundary of the antimatter domains, in which the interaction with ordinary matter takes place, and the proper codification of the evolution of the antimatter properties.

The characterization of antimatter, in particular, has to be modeled such that the energy loss due to the boundary interactions (i.e., antibaryons/baryons annihilation) allows for the non-annihilation of the antimatter domain. Antimatter domains at the plasma epoch are treated as a perfect fluid in the low-energy limit.

A turbulence regime is postulated in the low-energy limit of the perfect-fluid solution. The appropriate turbulence scale l_T has to be imposed as $0.1d < l_T < d$, with d the domain size; the fluid viscosity implies an energy dissipation.

A chemical potential can be assigned for the boundary–interaction region, responsible for keeping baryons and antibaryons separated, as a function of the antimatter density. A baryon/antibaryon-symmetrical Universe with large domains is demonstrated not to be compatible with a γ -ray background; the difference in the amount of antimatter can therefore be ascribed to a possible contribution to the definition of the chemical potential.

The excess of antimatter is therefore theoretized as shifting to the antimatter-domain boundary regions; thus, the antimatter domain is investigated also according to the properties due to the turbulent regime and to the fluid viscosity.

A γ -ray-background is predicted as due to the interaction products of the π^0 mesons, as products of the antibaryon/baryon annihilation; the features of the spectrum can be traced.

The consequences of antimatter matter annihilation can be further experimentally looked for in the processes of heating of the baryonic medium, in those of momentum transfer in high-energy particles, and in those of heating due to energy transfer.

Within the analysis, new classifications for antibaryon domains, which can evolve in antimatter globular clusters, are in order.

Differences must be discussed within the relativistic framework chosen, the nucleosynthesis processes, the description of the surrounding matter medium, the confrontation with the experimental data within the observational framework. The spacetime-evolution of antimatter domains and the two-point correlation functions are described within the nucleon-antinucleon boundary interactions.

A classification of antimatter domains according to the densities, and to the type(s) of antimatter involved within the non-trivial baryosyntheses processes considered will be proposed.

Definitions of boundary interactions for antimatter domains will be studied.

Further solutions are found for the space-time evolution of antimatter domains, i.e., the study of the evolution of the interaction regions, and the two-point correlation functions.

Perspective investigations are envisaged as far as different limiting processes are concerned, as well as for the characterization of the corresponding celestial bodies. The necessity for further classifications for the correlation functions is outlined, and the comparison with the thermal history of the Universe within a relativistic background is discussed. For this, classifications of antimatter domains according to the densities, and to the type(s) of antimatter involved within the *non-trivial baryosynthesis processes* considered, as well as the definition of boundary interactions for antimatter domains, have been proposed for the sake of the investigation of the space-time evolution of antimatter domains (two-point correlation functions and number of antibaryons), as well as for those of further studies and comparisons.

The paper is organized as follows.

In Section 1, the properties of antimatter domains are outlined, and the antimatter domains are characterized for perfect-fluid solutions within the plasma description, in the thermal history of a FRW (Friedmann–Robertson–Walker) Universe. In Section 2, the cosmological scenario in which the model is set is introduced.

In Section 3, antimatter domains and antibaryon interactions are categorized, according to the thermal history of a FRW Universe.

In Section 4, nucleon-antinucleon interactions are studied for the sake of the characterization of the boundaries of antimatter domains.

In Section 5, the number of antimatter objects is calculated, according to different space-time antimatter statistical distribution for low-density antimatter domains. For different antimatter statistical spacetime distributions, a different number of objects are evaluated after the definition of probability; in the theoretical cases, the limiting processes of two neighboring objects are defined.

In Section 6, studies of interaction probabilities are exposed, for the definition of the boundary interactions for antimatter domains. In particular, the interaction probabilities and the annihilation probabilities are defined for antibaryons and baryons.

In Section 7, observational constraints are outlined. More in detail, antimatter domains are constrained as non-vanishing within the limiting processes, different from other astrophysical objects.

In Section 8, the equation for the number density of antiprotons is solved according to the perfect fluid paradigm. To do so, different aspects of the plasma characterization of the perfect fluid are analyzed, and the solution is found analytically and expanded, according to the thermal history of a FRW Universe.

In Section 9, correlation functions are studied; exact solutions are integrated analytically in some specific cases.

In Section 10, further developments and comparison are forecast.

In Section 11, a comparison with the forecast experimental evidence concludes the paper.

2. Cosmological Scenario

In several cosmological scenarios, the appearance of domains with an antibaryon excess can be predicted.

Within the framework of non-trivial baryosynthesis scenarios, the formation of antimatter domains containing antibaryons, such as antiprotons, antinuclei, and both possibilities, are studied, according to their dependence on their antimatter densities within the domains.

The boundary conditions for antimatter domains are determined through the interaction with the surrounding baryonic medium.

Within the analysis, new classifications for antibaryon domains, which can evolve in antimatter globular clusters, are in order.

Differences must be discussed within the relativistic framework chosen, the nucleosynthesis processes, the description of the surrounding matter medium, and the confrontation with the experimental data within the observational framework. The spacetime-evolution

of antimatter domains and the two-point correlation functions are described within the nucleon-antinucleon boundary interactions.

If the density is so low that nucleosynthesis is not possible, low density antimatter domains contain only antiprotons (and positrons).

High density antimatter domains contain antiprotons and antihelium.

Heavy elements can appear in stellar nucleosynthesis or in the high-density antimatter domains.

Strong non-homogeneity in antibaryons might imply (probably as a necessary condition) strong non-homogeneity for baryons, and produce some exotic results in nucleosynthesis.

3. Symmetry-Breaking Scenario

The cosmological scenario which is apt to imply the formation of antimatter domains is investigated here. The evaluation of the number of objects will be determined from a statistical point of view within the definition of probabilities deriving from the definition of the standard deviation for the appropriate fields.

Throughout the paper, the units $\hbar = c = 1$ are used.

3.1. Spontaneous CP Violation

The Lagrangian potential density

$$\begin{aligned} V(\phi_1, \phi_2, \chi) = & -\mu_1^2(\phi_1^+ \phi_1 + \phi_2^+ \phi_2) + \lambda_1[(\phi_1^+ \phi_1)^2 + (\phi_2^+ \phi_2)^2] + 2\lambda_3(\phi_1^+ \\ & \phi_1)(\phi_2^+ \phi_2)(\phi_1^+ \phi_2) + 2\lambda_4(\phi_1^+ \phi_2)(\phi_2^+ \phi_1) + \lambda_5[(\phi_1^+ \phi_2)^2 + h.c.] + \\ & \lambda_6(\phi_1^+ \phi_1 + \phi_2^+ \phi_2)(\phi_1^+ \phi_2 + \phi_2^+ \phi_1) - \mu_2^2 \chi^+ \chi + \delta(\chi^+ \\ & \chi)^2 + 2\alpha(\chi^+ \chi)(\phi_1^+ \phi_1 + \phi_2^+ \phi_2) + 2\beta[(\phi_1^+ \chi)(\chi^+ \phi_1) + (\phi_2^+ \chi)(\chi^+ \phi_2)] \end{aligned} \quad (1)$$

can be used for an effective low-energy electroweak $SU(2) \otimes U(1)$ theory, following [28], for the three scalar fields ϕ_1, ϕ_2, χ , with the potential parameters $\mu_1, \mu_2, \lambda_1, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \alpha, \beta$.

Within a GUT spontaneous CP violation, the formation of vacuum structures is separated from the rest of the matter Universe by domain walls, whose size is calculated to grow with the evolution of the Universe.

The behavior is calculated not to affect the evolution of the Universe if the volume energy $\rho(\tilde{V})$ density of the walls for $\rho(\tilde{V}) \sim \sigma_\phi^2 T^4 / \hbar$, \hbar value of the scalar coupling constant.

A CP-invariant Lagrangian is chosen in [11] of the form

$$L = (\partial\phi)^2 - \lambda^2(\phi^2 - \chi^2)^2 + \bar{\psi}(i\partial - m - ig\gamma_5\phi)\psi, \quad (2)$$

in which ϕ is a pseudoscalar, ψ the baryon field, λ and η real parameters, and where the vacuum is characterized by $\langle \phi \rangle = \sigma\eta$, with $\sigma = \pm 1$. For the Lagrangian density,

$$L = (\partial\chi)^2 - \frac{1}{2}m_\chi^2\chi^2 - 4\sigma\lambda 2\chi\eta^3 - \lambda^2\chi^4 + \bar{\psi}(i\hat{\partial} - M - i\frac{g^m}{M}\gamma_5\chi - \frac{g^2\sigma\eta}{M}\chi)\psi, \quad (3)$$

a CP violation is achieved after the substitution $\phi = \chi + \sigma\eta$.

The domain wall problem can be solved after the Kuzmin–Sapozhnikov–Tkachev mechanism [14].

Investigation within these research lines can be reconducted to the research for antinuclei in cosmic rays, and to research for annihilation products; annihilation at rest on a relativistic background is also to be investigated.

The annihilation of small-scale domains can be also conducted within the thin-boundary approximation; at different times, the diffusion of the baryon charge is determined after different processes.

To survive to the present time in baryon surroundings, an antibaryon domain should be sufficiently large. Creation of such domains inevitably involves a combination of nonhomogeneous baryosynthesis with inflation. In the model of spontaneous CP violation,

it can be arranged by an intermediate inflationary stage [13]. Another possibility is to provide initial conditions of nonhomogeneous baryosynthesis, as it takes place in the model of spontaneous baryosynthesis.

3.2. Spontaneous Baryosynthesis

A Lagrangian potential can be chosen of the form [27]

$$V(\chi) = -m_\chi^2 \chi^* \chi + \lambda_\chi (\chi^* \chi)^2 + V_0 \quad (4)$$

with $\chi = \frac{f}{\sqrt{2}} e^{\frac{i\alpha}{f}}$. After the usual $U(1)$ symmetry breaking, the choice $\theta = \alpha/f$ can be set.

Spontaneous baryosynthesis processes, allowing for the possibility of sufficiently large domains through proper combination of effects of inflation and baryosynthesis, can be achieved after the variance

$$\langle \delta\theta \rangle = \frac{H^3 t}{4\pi^2 f^2} \quad (5)$$

through the expression

$$\chi \equiv \frac{f}{\sqrt{2}} e^\theta, \quad (6)$$

as from [27]. The aforementioned variance is one for the Brownian motion of a Pseudo-Nambu–Goldstone field around the minimum of the potential after the $U(1)$ symmetry breaking, for which the number of antimatter objects can be evaluated [29–32].

The number of objects $\tilde{N}(t) - \tilde{N}(t_0)$ is evaluated through the probability of a baryon/antibaryon excess, $P(\chi)$, which is due to the initial conditions for the field χ , and therefore through the definition

$$\tilde{N}(t) - \tilde{N}(t_0) \equiv \int_{t_0}^t P(\chi) \ln \chi d\chi(t), \quad (7)$$

where the definition of $P(\chi)$ includes the variance Equation (5) [33,34] Δf_{eff} effective (time-dependent) phase function

$$f_{eff} = f \sqrt{1 + \frac{g\phi_\chi M_{Pl}}{12\pi\lambda} (N_c - N)}, \quad (8)$$

as in [27], with N e-foldings at inflation. The Hubble-radius function \tilde{H} will be used in the following, i.e., for the definition of effective quantities.

4. Antimatter Domains and Antibaryon Interactions

In the **radiation-dominated era**, within the cosmological evolution, the dominant contribution to the total energy is due to photons.

Within *low density antimatter domains*, the contribution of the density of antibaryons $\rho_{\bar{B}}$ is smaller than the contribution due to the radiation ρ_γ even at the matter-dominated stage.

In the **matter-dominated era and following**, within a *non-homogeneous* scenario, the following description holds:

$$\rho_{DM} > \rho_B, \text{ with } \rho \equiv \rho(x);$$

the creation of high density antibaryon domains can be accompanied by a similar increase in baryon density in the surrounding medium.

Therefore, outside high-density antimatter domains, the baryonic density may be also higher than DM density, i.e.,

$$\rho_B(x) > \rho_{DM}(x).$$

Within *low density antimatter domains*, the total density obeys the minorizations

$$\rho_{\bar{B}} + \rho_\gamma,$$

$$\rho_{\bar{B}} < \rho_{\gamma}$$

$$\rho_{dm} > \rho_B.$$

Within the matter-dominated era and following, within a *non-homogeneous* scenario, the following description holds:

$$\rho_{DM} > \rho_B, \quad (9)$$

with

$$\rho \equiv \rho(x); \quad (10)$$

the creation of high-density antibaryon domains can be accompanied by a similar increase in baryon density in the surrounding medium. Therefore, outside high density antimatter domain, baryonic density may be also higher than DM density

$$\rho_B(x) > \rho_{DM}(x); \quad (11)$$

for low density antimatter domains, the total density

$$\rho_{\bar{B}} + \rho_{\gamma}, \quad (12)$$

for which

$$\rho_{\bar{B}} < \rho_{\gamma} \quad (13)$$

$$\rho_{dm} > \rho_B. \quad (14)$$

5. Space-Time Antimatter Statistical Distribution for Low-Density Antimatter Domains

Antimatter domains are described as an antibaryon gas with positrons in the FRW Universe within the thermal history.

In low density antimatter domains, neither nucleosynthesis nor recombination takes place.

An antimatter domain can be analyzed according to the antimatter spacetime distribution.

5.1. Low-Density Antimatter Domains Described by a Binomial Space-Time Statistical Distribution

Let k be the binomial probability of the limiting process (approximation) of the existence of a domain containing only two antibaryons, and let p^k be the binomial probability of the limiting process of existence of a domain containing $k - 1$ neighboring antibaryons. The relativistic expression for the antimatter domain density, $\tilde{N}_{bin}(k) - \tilde{N}_{0 bin}(k)$, is given as

$$\begin{aligned} \tilde{N}_{bin}(k) - \tilde{N}_{0 bin}(k) \simeq \sum_k \frac{1}{(k!)(1-k)!} \frac{\chi_{t_a}}{\chi_{t_0}} \frac{2}{\Delta f_{eff}(t; t_a, t_i, t_0)} \left(\frac{-2}{4\pi^2} \right) \cdot \\ \cdot \left[\ln \frac{L_u e^{H_c(t_c - t_0)} - e^{H_0 t_0}}{l} \right]^k (t)^{k-3} - \tilde{N}_{bin}(k)(t_0). \end{aligned} \quad (15)$$

Let $2\hat{n}$ be the antibaryon (and/or antinuclei) density (at least considered for the neighboring antibaryon-antibaryon interaction); it is expressed by the effective expression of H for \hat{n} antibaryons, n being the number of antibaryons surrounding the domain. This way, $n - k$ relates the number of antibaryons neighboring with a baryon, and $(1 - p)^{n-k}$ is the binomial probability that a baryon is neighboring with an antibaryon, i.e., at the domain boundary.

In (15), the effective Hubble-radius function H_j has been specified for the indicated times values.

The space-time evolution of the defined probabilities will be demonstrated in the following to also be dependent on the size of the spherical shell, in which interaction with the surrounding medium takes place, enclosing the antimatter domain.

5.2. Low-Density Antimatter Domains Described by a Bernoulli Space-Time Statistical Distribution

Low-density antimatter domains described by a Bernoulli space-time statistical distribution are described through the relativistic expression for the antimatter domain density, $\tilde{N}_{bin}(k) - \tilde{N}_{0\ bin}(k)$,

$$\tilde{N}_{Ber}(k) - \tilde{N}_{0\ Ber}(k) \simeq \frac{1}{(k!)(1-k)!} \frac{\chi_{t_a}}{\chi_{t_0}} \frac{2}{\Delta f_{eff}(t; t_a, t_i, t_0)} \left(\frac{-2}{4\pi^2} \right) \cdot \left[\ln \frac{L_u e^{H_c(t_c - t_0)} - e^{H_0 t_0}}{l} \right]^k (t)^{k-3} - \tilde{N}_{Ber\ (k)}(t_0), \quad (16)$$

where k is the Bernoulli probability of finding two neighboring objects.

5.3. Low-Density Antimatter Domains Described by a Poisson Space-Time Statistical Distribution

Let k be the Poisson probability of the existence of two neighboring domains with antibaryons.

Low-density antimatter domains described by a Bernoulli space-time statistical distribution are described through the relativistic expression for the antimatter domain density, $\tilde{N}_{Poiss}(k) - \tilde{N}_{0\ Poiss}(k)$,

$$\tilde{N}_{Poiss}(k) - \tilde{N}_{0\ Poiss}(k)(\Delta t) \simeq \sum_n \sum_k \frac{k^n e^k}{n!} \frac{\chi_{t_a}}{\chi_{t_0}} \frac{2}{\Delta f_{eff}(t; t_a, t_i, t_0)} \cdot \left(\frac{-2}{4\pi^2} \right) \left[\ln \frac{L_u e^{H_c(t_c - t_0)} - e^{H_0 t_0}}{l} \right]^k (t)^{k-3} - \tilde{N}_{Poiss\ (k)}(t_0), \quad (17)$$

5.4. Low-Density Antimatter Domains Described by a Gaussian Space-Time Statistical Distribution

For a Gaussian spacetime antimatter distribution, the following number of objects $\tilde{N} - \tilde{N}_0$ is calculated in the time interval:

$$\tilde{N}_{Gauss} - \tilde{N}_0 \simeq \left[\ln \frac{\theta_i}{\theta_c} + \ln(1 + \theta_c + \theta_c^2/2) - \ln(1 + \theta_i + \theta_i^2/2) \right] \cdot \left[\ln \frac{L_u e^{H_c(t_c - t_0)} - e^{H_0 t_0}}{l} \right] (e^{\theta_{60}} - e^{\theta_i}) \left[\frac{\theta_{60} - \theta_0}{\chi_{60} - \chi_0} [f(t_a) - f(t_0)] + f_{eff} \frac{1}{H_c^{3/2} - H_i^{(3/2)}} (t_c - t_i) \right] - N(0)_{Gauss}. \quad (18)$$

6. Nucleon-Antinucleon Interaction Studies: Boundary Interactions

Antibaryon-baryon boundary interactions are implied after the requirement $\tilde{N} > \Delta \tilde{N}$, i.e., s.t. the amount of annihilated antibaryons $\Delta \tilde{N}$ should not exceed the total amount \tilde{N} of antibaryons in domain.

Therefore, the domain can survive.

6.1. A Study: Proton-Antiproton Annihilation Probability, Limiting Process-Theoretical Formulation

Let $P(\bar{p})$ be the probability of existence of one antiproton of mass m_p , m_p being the proton mass, in the spherical shell, of section r_I , of (antimatter)-density ρ_I , delimiting the antimatter domain, in which the interaction takes place $P(\bar{p}) \equiv 3Nm_p/(r_I \rho_I)$ according to the Fisher's hypergeometrical non-central modified distribution; let $\tilde{P}_i \equiv \tilde{P}_{\bar{p} \rightarrow (d.c.i)}$ be probability of antiproton \bar{p} interaction with a proton p in a chosen i annihilation channel a.c., possibly also depending on the chemical potential; let Δt be the time interval considered. Under the most general hypotheses (most stringent constraint), $\Delta t \pm \delta t$, $\Delta t \simeq t_U \simeq 4 \cdot 10^{17} s$, with t_U the age of the Universe, δt to be set according to the particular phenomena considered; the probability of antiproton interaction $\tilde{P}_{\bar{p},i}(t, \Delta t)$, i.e., antiproton-proton annihilation (density) probability is obtained as

$$\tilde{P}_{\bar{p},i} \simeq \frac{1}{\Delta t} P_{\bar{p}} \tilde{P}_i, \quad (19)$$

for which δt can be neglected.

6.2. Nucleon-Antinucleon Interaction (Annihilation) Probabilities: Limiting Process-Theoretical Formulation

Given an antinucleus \bar{M} , the interaction probability $\bar{P}_{\bar{M},j}(t, \Delta t)$ through the annihilation channel(s) k is obtained as

$$\bar{P}_{\bar{M},k}(t, \Delta t) \simeq \frac{1}{\Delta t} P_A \bar{P}_{A,k}. \quad (20)$$

Equations (19) and (20) are normalized as $[t^{-1}]$.

6.3. Further Specifications of Boundary Interactions

The study of boundary interactions is further specified for non-trivial relativistic scenarios, such as *perturbed FRW solution with the thermal history of the Universe, i.e., also, according to the Standard Cosmological Principle* at large scales asymptotically isotropic and homogeneous.

More precise stipulations are in order for the case of non-trivial nucleosyntheses, possibilities of (interaction with) surrounding media, for which, in the case of antibaryon-baryon annihilation, the most stringent constraint is obtained as \bar{P} evaluated for present times in the description of reducing density in the limiting process of a low-density antimatter domain.

6.4. An Example

Low-density antimatter domains can be schematized as constituted of non-interacting antiprotons, for which boundary interactions take place, and the interaction with surrounding medium is considered. The possibility of low-density antimatter domains surrounded by low-density matter regions is to be considered.

7. Observational Evidence

The existence of low-density antimatter domains is *to be compared with the experimental data and numerical simulation of existence and non-disappearing of antimatter domains* based on the hypothesis that the examined antimatter domain has not undergone disappearance in the limiting process; in the case of higher densities antimatter domains, further schematizations allow one to describe processes.

8. Equation for the Number Density of Antiprotons

It is possible to evaluate the number of antibaryons after including both the interaction processes and the expansion of the Universe. The study is achieved at a temperature T , $4 \cdot 10^4 \text{ K} < T < 10^9 \text{ K}$; for low-density antimatter domains, the density of antimatter is considered within a domain three orders of magnitude less than the baryon density, and the interaction

$$\bar{n} + \bar{p} \rightarrow \bar{d} + \gamma \quad (21)$$

is studied: the cross section $\langle \sigma v \rangle$ does not depend on the temperature if below 1 MeV and implies the antideuterium production in the reaction only if the reaction rate exceeds the expansion rate of the Universe, the (integrated) Thomson cross section is studied through the diffusion coefficient $D(t)$.

Analytical solutions of the equation for the number density of antiprotons as a function of annihilation and expansion of the Universe can be looked for.

Further analyses can be brought on for antinuclei antimatter domains within the framework of non-trivial baryosyntheses products, for which further constraints and further studies are in order.

8.1. Time Evolution of Antimatter Domains

As in [25], the baryon/photon ratio s , $s \equiv n_b/n_\gamma$, can be studied after its diffusion equation

$$\frac{\partial s}{\partial t} = D(t) \frac{\partial^2 s}{\partial x^2}, \quad (22)$$

with the boundary conditions

$s(\mathbf{R}, t_0) = s_0, x < 0, s(\mathbf{R}, t_0) = 0, x > 0$, to compute the geodesics coordinate distance run across by atoms after the recombination age until the present time *within a suitable photon thermalization process* with s_0 the initial condition n_0/n_γ .

8.2. Number of Antibaryons in the Boundary Spherical Shell in Which the Antibaryon-Baryon Interaction Takes Place

The number of antibaryons in the boundary spherical shell in which the antibaryon-baryon interaction takes place obeys the equation

$$\frac{dn_{\bar{b}}}{dt} \simeq -\frac{R_d}{3} n_{\bar{b}} (< \sigma v > n_b + \beta), \quad (23)$$

with R_d the radius of the spherical antimatter domain, and $\beta > 0$ the growth rate of the photon density, where the latter is taken into account; it contributes to the solution, becomes almost negligible at the RD stage, and provides a very small contribution of antimatter in the total density.

After treating the number of baryons n_b as not changing with respect to the number of antibaryons $n_{\bar{b}}$, it is possible to evaluate the boundary of the antimatter domain as a spherical shell in which the baryon-antibaryon annihilation takes place as depending on whether the antibaryons in the low-density antimatter domains are not interacting.

Matter domains can be of interest only, if one takes into account that high density antimatter domains are associated with surrounding high density baryonic matter.

8.3. Perfect-Fluid Solution(s)

Further characterizations of the number of antibaryons in the boundary spherical shell in which the antibaryon-baryon interaction takes place are possible.

To obtain a further characterizations of the number of antibaryons in the boundary spherical shell in which the antibaryon-baryon interaction takes place, a **perfect-fluid solution** can be taken into account, i.e., a perfect-fluid solution

$$(\rho_E, \vec{p}). \quad (24)$$

The number of antibaryons in the boundary spherical shell in which the antibaryon-baryon interaction takes place becomes

$$\frac{dn_{\bar{b}}}{dt} \simeq -\frac{R_d}{3} n_{\bar{b}} (< \sigma v > n_b + \beta) - F(\rho_E, \vec{p}), \quad (25)$$

with F a function which encodes the properties of the perfect-fluid solution on a relativistic background at the time at which the number of baryons is evaluated.

8.4. Plasma Characterization

Under the hypotheses that annihilation products should induce isotropic pressure that stops the limiting process of the disappearance of the domain, at very small scales (smaller than the non-disappearance scale), the radiation pressure is not sufficient, i.e., further characterization of the antimatter domains has to be taken into account, as the following:

$$\frac{dn_{\bar{b}}}{dt} \simeq -\frac{R_d}{3} n_{\bar{b}} (< \sigma v > n_b + \beta) - \tilde{F}(\rho_E, \vec{p}; R_d, l_d; \vec{v}_T, v_f; \hat{i}), \quad (26)$$

with \tilde{F} an appropriate function to be specified in the solution of the differential ratio $dn_{\bar{b}}/dt$ at *plasma epoch* for the results to be holding at present times. In particular, the function \tilde{F} is

a function of the viscosity coefficient v_f , of the turbulent velocity \vec{v}_T , of the emulsion size l_d and of the radius of the antimatter domain R_d . The function \tilde{F} can also be a function of different i ages of the Universe \tilde{t} .

At the plasma epoch, the plasma behaves as a single fluid, such that fluid viscosity is determined by the radiation field.

In the annihilation region, the antibaryons migrate to the boundary of the antimatter domain; as density increases, the annihilation interactions become rapid, and the products of annihilation can express an isotropic pressure. The annihilation interactions cannot provide with the energy required to let the turbulent regime start; differently, the radiation spectrum is modified.

8.4.1. Turbulent Regime

As in [20], let v_T be the turbulent velocity; it is our purpose to calculate the corresponding energy dissipated ΔE_T .

Let ΔE_T be the *energy dissipated* per unit density per unit time at the effect of fluid viscosity within the interaction region of width ΔR ; therefore, the energy difference ΔE_T is calculated as

$$\Delta E_T \simeq \frac{v_T^3}{l_d} \frac{3(R_d^2 - \Delta R^2)}{R_D^3}, \quad (27)$$

v_T being the turbulent velocity.

8.4.2. Viscosity Coefficient

The *characteristic size of the emulsion region* is evaluated after the turbulence scale.

Subsequently, the turbulence scale is determined by the coefficient of viscosity v_f , as in the analysis of [21]; therefore, the energy difference dissipated due to the fluid viscosity, ΔE_v , is calculated as

$$\Delta E_v \simeq \frac{3(R_d^2 - \Delta R^2)}{R_D^3} n. \quad (28)$$

In the evaluation performed in Equation (28), the size ΔR is evaluated as $\Delta R \sim v_f(z)t(z)$ after the study of the redshift of the thermal photons. The coefficient of viscosity v_f relates to the choice of the time of the evolution of the Universe at which the exact analytical solution is calculated with the mass of the fluid moving, i.e., s.t.

$$t(z) = t_p(1+z)^{-3/2}, \quad (29)$$

with t_p being the present age of the Universe.

Furthermore, the inequalities for the size scales have to be introduced as

$$0.1d \leq \Delta R \leq d, \quad (30)$$

which is obtained after imposing a size d calculated as governed through the decay dynamics, i.e., the size d is calculated as larger than the mean-free path of the dynamics of the π -decay γ rays.

A nontrivial dependence on the redshift is outlined.

The differential equation for the number of antibaryons therefore acquires the two addends containing the energy differences ΔE_T and ΔE_v as

$$\frac{dn_{\bar{b}}}{dt} \simeq -\frac{R_d}{3} n_{\bar{b}} (<\sigma v> n_b - \beta) - f(\rho_E, \vec{p}) \equiv -\frac{R_d}{3} n_{\bar{b}} (<\sigma v> n_b + \beta) - (\Delta E_T + \Delta E_v) \frac{dn_{\bar{b}}}{dt}. \quad (31)$$

Equation (31) is therefore solved analytically exactly for the different antimatter spacetime distributions, and then expanded at present times.

For a Bernoulli space-time statistical distribution of antimatter, the following expansion is found at the time t_1

$$\begin{aligned} \ln n_{\bar{b} \text{ Ber}} &\simeq \ln n_{\bar{b} \text{ Ber } 0} + \\ &- \left[\frac{R_d}{3} \tilde{n}_{\text{Ber}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_1^{k-1} + \right. \\ &- \frac{R_d}{3} \tilde{n}_{\text{Ber}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_0^{k-1} \left. \right] (\langle \sigma v \rangle > n_b + \beta) + \\ &- (\Delta E_T + \Delta E_v) n \Delta t. \end{aligned} \quad (32)$$

This way, the solution is demonstrated to depend on the effective quantities through \tilde{n}_{Ber} from Equation (16), i.e., the exact time dependence has been spelled out explicitly.

For a Poisson antimatter spacetime distribution, the following solution is found:

$$\begin{aligned} \ln n_{\bar{b} \text{ Poiss}} &\simeq \ln n_{\bar{b} \text{ Poiss } 0} + \\ &- \left[\frac{R_d}{3} \tilde{n}_{\text{Pois}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_1^{k-1} + \right. \\ &- \frac{R_d}{3} \tilde{n}_{\text{Pois}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_0^{k-1} \left. \right] (\langle \sigma v \rangle > n_b + \beta) + \\ &- (\Delta E_T + \Delta E_v) n \Delta t. \end{aligned} \quad (33)$$

Thus, the solution is calculated to depend on the effective quantities through \tilde{n}_{Pois} from Equation (17), i.e., the exact time dependence has been extracted explicitly.

For a binomial antimatter spacetime distribution, the following expansion holds:

$$\begin{aligned} \ln n_{\bar{b} \text{ bin}} &\simeq \ln n_{\bar{b} \text{ bin } 0} + \\ &- \left[\frac{R_d}{3} \tilde{n}_{\text{bin}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_1^{k-1} + \right. \\ &- \frac{R_d}{3} \tilde{n}_{\text{bin}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_0^{k-1} \left. \right] (\langle \sigma v \rangle > n_b + \beta) + \\ &- (\Delta E_T + \Delta E_v) n \Delta t. \end{aligned} \quad (34)$$

Accordingly, the solution is evaluated to depend on the effective quantities through \tilde{n}_{bin} from Equation (15), i.e., the exact time dependence has been quantified explicitly.

The solution found in the cases of different antimatter statistical space-time distributions differs from the functional dependence on the effective quantities; nevertheless, after the definition of the effective quantities, the time dependence is shown.

In the case of a Gaussian antimatter spacetime distribution, the following solution is calculated:

$$\begin{aligned} \ln n_{\bar{b} \text{ Gauss}} &\simeq \ln n_{\bar{b} \text{ Gauss } 0} + \\ &- \left[\frac{R_d}{3} \tilde{n}_{\text{Gauss}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) (\Delta t - 1) e^{\Delta t} (\langle \sigma v \rangle > n_b + \beta) + \right. \\ &- (\Delta E_T + \Delta E_v) n \Delta t \end{aligned} \quad (35)$$

Hence, the solution is shown to depend on the effective quantities through \tilde{n}_{Gauss} from Equation (18), i.e., the exact time dependence has been determined explicitly.

As a result, in the solutions, the overall time dependence of the effective quantities has been extracted. In this case, the effective-phase function $\Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b}))$ and the Hubble-radius function from the F equation, $\tilde{H}(\rho_E(b), \rho_E(\bar{b}))$, are modified by the interaction between radiation and antibaryons.

In the case of a one-parameter Gaussian antimatter spacetime distribution, the time dependence of the solution is different from the found previous cases, as the definition of probability involved in the corresponding differential equation exhibits a different time dependence itself.

8.5. Chemical-Potential Characterization

The thermal radiation implies separation of nucleons and antinucleons through the expression of a chemical potential.

The following specification for the differential equation of the number of antibaryons in the interaction region is obtained for a μ chemical potential

$$\frac{dn_{\bar{b}}}{dt} \simeq -\frac{R_d}{3} n_{\bar{b}} (< \sigma v > n_b + \beta) - (\Delta E_T + \Delta E_v) \frac{dn_{\bar{b}}}{dt} - \mu \nabla^2 n. \quad (36)$$

The thermal radiation implies separation of nucleons and antinucleons through a chemical-potential term.

The solution is found at the radiation-dominated era, in which the antimatter space-time statistical distributions are defined; the solution has to be expanded after the time of the surface of last scattering, according to the effective quantities.

In the case of a Bernoulli space-time statistical distribution of antimatter, the following solution is found at the time t_1 :

$$\begin{aligned} \ln n_{\bar{b} \text{ Ber}} &\leq \ln n_{\bar{b} \text{ Ber } 0} + \\ &- \left[\frac{R_d}{3} \tilde{n}_{\text{Ber}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_1^{k-1} + \right. \\ &- \left. \frac{R_d}{3} \tilde{n}_{\text{Ber}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_0^{k-1} \right] (< \sigma v > n_b + \beta) + \\ &- (\Delta E_T + \Delta E_v + \tilde{\mu}) n \Delta t. \end{aligned} \quad (37)$$

For a Poisson antimatter spacetime distribution, the following expansion holds:

$$\begin{aligned} \ln n_{\bar{b} \text{ Poiss}} &\leq \ln n_{\bar{b} \text{ Poiss } 0} + \\ &- \left[\frac{R_d}{3} \tilde{n}_{\text{Pois}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_1^{k-1} + \right. \\ &- \left. \frac{R_d}{3} \tilde{n}_{\text{Pois}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_0^{k-1} \right] (< \sigma v > n_b + \beta) + \\ &- (\Delta E_T + \Delta E_v + \tilde{\mu}) n \Delta t. \end{aligned} \quad (38)$$

In the case of a binomial antimatter spacetime distribution, the following expansion is calculated:

$$\begin{aligned} \ln n_{\bar{b} \text{ bin}} &\leq \ln n_{\bar{b} \text{ bin } 0} + \\ &- \left[\frac{R_d}{3} \tilde{n}_{\text{bin}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_1^{k-1} + \right. \\ &- \left. \frac{R_d}{3} \tilde{n}_{\text{bin}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_0^{k-1} \right] (< \sigma v > n_b + \beta) + \\ &- (\Delta E_T + \Delta E_v + \tilde{\mu}) n \Delta t \end{aligned} \quad (39)$$

In the case of a Gaussian antimatter spacetime distribution, the following expansion is evaluated:

$$\begin{aligned} \ln n_{\bar{b} \text{ Gauss}} &\simeq \ln n_{\bar{b} \text{ Gauss } 0} + \\ &- \left[\frac{R_d}{3} \tilde{n}_{\text{Gauss}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) (\Delta t - 1) e^{\Delta t} (< \sigma v > n_b + \beta) + \right. \\ &- (\Delta E_T + \tilde{\mu}) n \Delta t + \\ &- (\Delta E_T + \Delta E_v + \tilde{\mu}) n \Delta t. \end{aligned} \quad (40)$$

The results in the above hold after the immediate condition $\tilde{\mu} < \Delta E_T + \Delta E_v$, with $\Delta t \equiv t_1 - t_0$.

It is possible to analyze the time dependence of the solution for the number of antibaryons after the time dependence of E_v from Equation (28). In the case of a Bernoulli antimatter spacetime distribution, the following expansion is calculated:

$$\begin{aligned}
\ln n_{\bar{b} \text{ Ber}} &\leq \ln n_{\bar{b} \text{ Ber } 0} + \\
&- \left[\frac{R_d}{3} \tilde{n}_{\text{Ber}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_1^{k-1} + \right. \\
&- \left. \frac{R_d}{3} \tilde{n}_{\text{Ber}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_0^{k-1} \right] (< \sigma v > n_b + \beta) + \\
&- (\Delta E_T + \tilde{\mu}) n \Delta t - \frac{3(R_d^2 - (v_f(z) t_1 (1+z)^{-3/2})^2)}{R_D^3} n t_1 + \\
&+ \frac{3(R_d^2 - (v_f(z) t_1 (1+z)^{-3/2})^2)}{R_D^3} n t_0.
\end{aligned} \tag{41}$$

For a Poisson antimatter spacetime distribution, the following solution holds:

$$\begin{aligned}
\ln n_{\bar{b} \text{ Poiss}} &\leq \ln n_{\bar{b} \text{ Poiss } 0} + \\
&- \left[\frac{R_d}{3} \tilde{n}_{\text{Poiss}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_1^{k-1} + \right. \\
&- \left. \frac{R_d}{3} \tilde{n}_{\text{Poiss}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_0^{k-1} \right] (< \sigma v > n_b + \beta) + \\
&- (\Delta E_T + \tilde{\mu}) n \Delta t - \frac{3(R_d^2 - (v_f(z) t_1 (1+z)^{-3/2})^2)}{R_D^3} n t_1 + \\
&+ \frac{3(R_d^2 - (v_f(z) t_1 (1+z)^{-3/2})^2)}{R_D^3} n t_0.
\end{aligned} \tag{42}$$

For a binomial antimatter spacetime distribution, the following solution is evaluated:

$$\begin{aligned}
\ln n_{\bar{b} \text{ bin}} &\leq \ln n_{\bar{b} \text{ bin } 0} + \\
&- \left[\frac{R_d}{3} \tilde{n}_{\text{bin}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_1^{k-1} + \right. \\
&- \left. \frac{R_d}{3} \tilde{n}_{\text{bin}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \frac{1}{k-2} t_0^{k-1} \right] (< \sigma v > n_b + \beta) + \\
&- (\Delta E_T + \tilde{\mu}) n \Delta t - \frac{3(R_d^2 - (v_f(z) t_1 (1+z)^{-3/2})^2)}{R_D^3} n t_1 + \\
&+ \frac{3(R_d^2 - (v_f(z) t_1 (1+z)^{-3/2})^2)}{R_D^3} n t_0.
\end{aligned} \tag{43}$$

In the case of a Gaussian antimatter spacetime distribution, the following expansion is calculated:

$$\begin{aligned}
\ln n_{\bar{b} \text{ Gauss}} &\simeq \ln n_{\bar{b} \text{ Gauss } 0} - \left[\frac{R_d}{3} \tilde{n}_{\text{Gauss}}(k; \Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b})), \tilde{H}(\rho_E(b), \rho_E(\bar{b})); \Delta t) \right] (t-1) e^{\Delta t} (< \sigma v > n_b + \beta) + \\
&- (\Delta E_T + \tilde{\mu}) n \Delta t - \frac{3(R_d^2 - (v_f(z) t_1 (1+z)^{-3/2})^2)}{R_D^3} n t_1 + \\
&+ \frac{3(R_d^2 - (v_f(z) t_1 (1+z)^{-3/2})^2)}{R_D^3} n t_0.
\end{aligned} \tag{44}$$

The energy difference caused through the properties of the viscosity coefficient at the turbulent-regime epoch produces a non-trivial time role of the time variable in the solution for the differential equation for the number of antibaryons, where the non-trivial dependence on the redshift is demonstrated.

As a result, similarly to the results obtained in the previous subsection, in the solutions, the overall time dependence of the effective quantities has been worked out. In this case, the effective-phase function $\Delta f_{\text{eff}}(\rho_E(b), \rho_E(\bar{b}))$ and the Hubble-radius function from the F equation, $\tilde{H}(\rho_E(b), \rho_E(\bar{b}))$, are formulated by the interaction between radiation and antibaryons.

9. Two-Point Correlation Functions

The two-point correlation functions \tilde{C}_2 for two antimatter domains α_1 and α_2 of size $> 10^3 M_\odot$ each, on a (homogeneous, isotropic) Minkowski-flat background, where antimatter densities $\rho \equiv \tilde{N}/V$ follow a Poisson space-time statistical distribution, reads

$d\tilde{C}_2(\alpha_1, \alpha_2) \equiv \rho^2(1 + \xi(|\vec{r}_{\alpha_1\alpha_2}|))dV_1dV_2$ where the estimator ξ is defined, $\xi(|\vec{r}_{\alpha_1\alpha_2}|) \equiv |\vec{r}_{\alpha_1\alpha_2}|$, with $\vec{r}_{\alpha_1\alpha_2}$ the distance of the two antimatter domains for the two antimatter domains of volume $V_{\alpha_l} \equiv \frac{4}{3}\pi r_l^3$ separated for a distance $|\vec{r}_{\alpha_1\alpha_2}|$.

The study of the correlation functions is framed within an FRW Universe with its thermal history; in the radiation-dominated epoch, antimatter is subdominant and therefore does not separate from the ordinary evolution of the considered Universe in its cosmological model.

9.1. Davis–Peebles Estimator

The **Davis–Peebles estimator** is used for the correlation function for an antimatter domain and another object.

It is important to consider the limiting example of the correlation function between an antimatter domain α_1 and an antibaryon α_3 through the Davis–Peebles estimator as a theoretical investigation in the limiting process of existence of one antibaryon.

The Davis–Peebles estimator is defined as

$$\xi_{l,l'} \equiv \frac{\tilde{N}_{bin}}{\tilde{N}} \frac{D_l(|\vec{r}|)}{D_{l'}(|\vec{r}|)} - 1, \quad (45)$$

\tilde{N} being the number of antibaryons in a low-density antimatter domain in which antibaryons are distributed according to a Poisson space-time statistical distribution, and \tilde{N}_{bin} being the number of antibaryons in a low-density antimatter domain in which antibaryons are distributed according to a binomial space-time statistical distribution; both quantities are evaluated as a function of the defined effective quantities.

In Equation (45), $D_l(|\vec{r}|)$ is the number of pairs of low-density antimatter domains within the geodesics (coordinate) interval distance $[r - \frac{dr}{2}, r + \frac{dr}{2}]$, and for which the theoretical framework of the limiting process of the existence of a single antibaryons is defined through $D_{l'}(|\vec{r}|)$, i.e., the number of pairs of objects between an antimatter domain and a (Poisson-distributed) antibaryon lying on the coordinate geodesics.

9.2. Analytical Solution of the Davis–Peebles Estimator

The Davis–Peebles estimator Equation (45) is *analytically* calculated as

$$\xi_{l,l'} \equiv \frac{\tilde{n}_{bin}(n, k; \Delta f_{eff}, \tilde{H}; \Delta t)}{\tilde{n}(n, k; \Delta f_{eff}, \tilde{H}; \Delta t)} \frac{D_l(|\vec{r}|)}{D_{l'}(|\vec{r}|)} - 1. \quad (46)$$

Within the use of statistical estimators, the typical time dependence $\tilde{H}_c^{2k} t^{4k-4}$ is thus simplified out. The Davis–Peebles estimator Equation (46) is therefore evaluated after the effective quantities \tilde{n}_{bin} and \tilde{n} form the binomial spacetime antimatter distribution, and from the Poisson spacetime antimatter distribution, respectively. Indeed, the *time dependence* is written explicitly in the ratio $\frac{\tilde{n}_{bin}(n, k; \Delta f_{eff}, \tilde{H}; \Delta t)}{\tilde{n}(n, k; \Delta f_{eff}, \tilde{H}; \Delta t)}$; both quantities are evaluated after the defined effective quantities, i.e., on the different statistical antimatter space-time distributions and on their dependence on \tilde{H} , i.e., the Hubble-radius function, and on Δf_{eff} , i.e., the effective (time-dependent) phase function.

9.3. Hamilton Estimator

The **Hamilton estimator** $\xi_{l,l'}$ accounts for the difference in (coordinate) distances among the Binomial antimatter spacetime distribution and the Poisson antimatter space-time distribution.

10. Outlook and Perspectives

After [29–31], the baryon asymmetry obtained after the symmetry violation of a pseudo-Nambu–Goldstone field reflected in the asymmetry of the number of antimatter objects, can be evaluated. In the present analysis, the evaluation has been performed

analytically. In particular, the variance of the field whose initial condition leads to the number of antimatter objects has been exploited in the definition of the related probabilities.

As from the analyses in [25], the number of antimatter objects within an antimatter domain has been considered. In particular, the diffusion equation for the boundary of the antimatter domain has been rewritten and specified for the evaluation of the number of antimatter objects as a function of further terms, due to the viscosity properties, the turbulent regime and the chemical potential at the boundaries as a perfect-fluid solution.

The correlation functions for antimatter objects have been calculated analytically. A comparison with the distribution of galaxies and with the corresponding scaling law(s) [35] can shed light on the distribution of antimatter objects with respect to the initial conditions of the field χ and related quantities.

Comparison with DM objects of different masses would allow one to register the differences in the survival of such objects of cosmological origin, according to the different (limiting) processes. Neutralino clumps of mass M_{cl} are estimated to survive the Galaxy evolution if their mass is within the range $10^{-8}M_{\odot} \leq M_{cl} \leq 10^{-6}M_{\odot}$ in [36,37]; further classifications of neutralino clumps also allow for further comparison with the case of antimatter domains [38].

Analyses of the limiting processes of disappearance of the antimatter domains are possible, after [39], and the characterization of the results follow [40].

The space-time evolution of antimatter domains separated in a small angular distance can be further studied through the Rubin–Limber correlation functions for small angles [41,42].

An analysis of the metric requiring a time evaluation after the time of the surface of last scattering to ensure a FRW Universe within its thermal history after the possible perturbations to the Ricci tensor being negligible is proposed in [43–48].

11. Conclusions

The evidence for antihelium component of cosmic rays cannot find explanation by natural astrophysical sources [19] and can imply the existence of macroscopic antimatter regions in a baryon asymmetrical Universe. The minimal mass of such a domain, surviving in a baryon asymmetrical Universe, is $10^3 M_{\odot}$, and it can form a globular cluster of antimatter stars in our Galaxy. Antimatter star evolution and activity can give rise to antinuclear components of cosmic rays and rough estimation of the corresponding antihelium flux, proportional to the ratio masses of antimatter to matter in our Galaxy, is about 10^{-8} of the helium flux of cosmic rays [10,15,16]. The realistic prediction of antihelium flux accessible to AMS02 experiment is now under way involving analysis of possible sources of energetic antinuclei from antimatter globular cluster and their propagation in galactic magnetic fields [49].

Such domains can be predicted in models with nonhomogeneous baryosynthesis under a specific choice of model parameters [12]. Detailed analysis of evolution of antimatter domains in a baryon asymmetrical Universe and prediction of their space distribution is important for revealing the observable signatures of such nonstandard cosmological scenario based on the specific choice of a Beyond the Standard Model [9]. Confirmation of such signatures would involve the development of statistical methods, which is started now in order to specify the observable features of a nonstandard cosmological scenario and strongly narrow the possible range of BSM models and their parameters, on which such a scenario is based. This development will inevitably take into account evolution of antimatter island distribution within the cosmological structures.

It should be noted that formation of antimatter domains can be accompanied by domain walls, which either disappear, as it takes place in [13,28], or collapse in black holes [10], leaving gravitational wave background or primordial black holes, which deserve special study in the context of multimessenger probes of the considered scenarios. Similarly to the analysis of [50] brought in the case of a model of baryon islands in the Universe, it

is possible, in the case of an antimatter domain, to evaluate the background annihilation radiation.

In [51], antimatter domains are analyzed as not undergoing annihilation only in the presence of a suitable mechanism, which should be apt to keep a separation between antimatter and ordinary matter; in the presented analyses, such a mechanism is constituted by the chemical potential.

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